

Strategic Complementarity, Fragility, and Regulation

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Revised June 2014

ABSTRACT

Fragility is affected by how the balance sheet composition of financial intermediaries, the precision of information signals, and market stress parameters all influence the extent of strategic complementarity among investors' strategies. A solvency and a liquidity ratio are required to control the likelihood of insolvency and illiquidity. The solvency requirement must be strengthened in the face of increased competition, whereas the liquidity requirement must be strengthened under more conservative fund managers and higher penalties for fire sales. Greater disclosure may aggravate fragility and require an increase in the liquidity ratio, so regulators should establish prudential and disclosure policies in tandem.

Keywords: crises, illiquidity risk, insolvency risk, prudential policy, leverage ratio, liquidity ratio, disclosure, transparency, opaqueness, panic, run, derivatives market, shadow banking, sovereign debt.

JEL codes: G21, G28

* An earlier version of this paper was circulated under the title "Stress, Crises, and Policy". I am grateful to Elena Carletti, Gabriella Chiesa, Falco Fecht, Todd Keister, Arvind Krishnamurthy, Frédéric Malherbe, Jean-Charles Rochet, Philipp Schnabl, Anatoli Segura, Javier Suarez, Wolf Wagner, and to participants at the following conferences: 2010 ESMFM at Gerzensee, 2010 FIRS, 2011 Banca d'Italia-CEPR, 2011 EFA and EEA, 2011 CAREFIN, 2012 CESifo-Bundesbank, 2013 Barcelona GSE Summer Forum, 2013 DFG/MPI at Bonn, 2013 EBC at Tilburg, and 2014 IESE-CEPR-ERC Banking Conference, as well as seminars at the New York Fed, Northwestern Finance, the University of Oslo, IIES (Stockholm University), and EIEF for helpful comments, and to Rodrigo Escudero and Jorge Paz for able research assistance. The research leading to these results has received funding from the European Research Council under the European Advanced Grants scheme, Grant Agreement no. 230254 as well as from project ECO2011-29533 of the Spanish Ministry of Education and Science at the PPSRC of IESE.

Introduction

In a crisis situation—and the recent financial crisis is a good example—things seem to go wrong at the same time, and an adverse shock is magnified by the actions and reactions of investors.¹ In particular, liquidity evaporates when short-term investors rush to exit, after which a solvency problem may arise. The financial system's fragility has been attributed to the increased reliance, by investment banks and commercial banks both, on market funding. The demise of Northern Rock in 2007, of Bear Stearns and Lehman Brothers in 2008, and of IKB and Hypo Real State in Germany are all cases in point: each institution's short-term leverage was revealed as a crucial weakness of its balance sheet.² In this context it has proved difficult to disentangle liquidity risk from solvency risk. The debate has revolved around the opaqueness of financial products, the impact of public news (as provided by, e.g., the ABX index on residential mortgage-backed securities, public statements about the health of banks,³ and stigma associated with known borrowing from the discount window),⁴ and the influence of derivative markets on stability. A disclosure requirement such as the FAS rule 157, a mark to market accounting legislation implemented in 2007, has been credited with aggravating the consequences of the bust of the real state bubble since it forced banks to disclose large losses on their portfolios of mortgage-based securities.⁵ The crisis has put regulatory reform in the agenda. Policy makers and regulators are struggling with how to reform capital requirements, introduce liquidity requirements, and improve disclosure requirements.⁶

¹ See Brunnermeier (2009) and Krishnamurthy (2009).

² In June 2007, wholesale funds represented about 26% of liabilities in Northern Rock (Shin 2009); before the crisis, short-term financing represented a high percentage of Lehman Brothers' total liabilities (Adrian and Shin 2010). Washington Mutual suffered the withdrawal of \$16.5 billion worth of large deposits just in the two weeks before its collapse (according to the Office of Thrift Supervision). See also the evidence in Ivashina and Scharfstein (2010).

³ Such was the case for the run on IndyMac Bancorp in June 2008, which followed shortly after public release of letters by Senator Schumer of the Banking Committee.

⁴ See Armantier et al. (2011).

⁵ See the testimony of former FDIC chairman W. Isaac before the Subcommittee on Capital Markets, Insurance, and Government-Sponsored Enterprises, U.S. House of Representatives Committee on Financial Services of March 12, 2009.

⁶ See, for example, Financial Services Authority (2009) and Bank for International Settlements (BIS; 2009). The Dodd-Frank Act of 2010 introduced a leverage limitation for financial holding companies larger than a certain size. The BIS proposed two new liquidity ratios: a *liquidity coverage ratio* to cover short-term cash outflows with highly liquid assets; and a *net stable funding ratio* to cover required stable funding with available stable funds. Bear Stearns was regulated by the SEC and was subject to a liquidity requirement—one, however, that proved ineffective in the crisis.

Lack of attention to liquidity issues is believed to be part of why policy responses were inadequate during the Great Depression. Friedman and Schwartz (1963) argue that many bank failures arose out of panics—that is, because of liquidity rather than solvency problems. This explanation is consistent with the “self-fulfilling” view of crises described by Bryant (1980) and Diamond and Dybvig (1983), although that view of crisis has been disputed by Gorton (1985, 1988) and others.⁷ During the Great Recession triggered by the subprime mortgage crisis in 2007, liquidity issues have received a great deal of attention. For example, it has been claimed that the sovereign debt crisis in the euro area is driven by a self-fulfilling panic.⁸ Yet clearly the euro area also has important solvency problems. In general, the evidence points to issues of solvency and liquidity being intertwined in any crisis.⁹ A corollary to these observations is that regulations should address both solvency and liquidity concerns.

This paper presents a general framework for studying crises as well as an initial exploration of how regulations addressing solvency, liquidity, and transparency should be related. The key novel ingredient is showing how the degree of *strategic complementarity* of investors’ actions is the crucial parameter not only for characterizing equilibrium and fragility (where the latter is understood as equilibrium sensitivity to small changes in parameters, including the possibility of discrete jumps within a changing equilibrium set) but also for policy analysis. More specifically, the paper relates information structure, balance sheet, and market parameters to the degree of strategic complementarity of investors’ actions and fragility. Main findings are that (i) better public information increases liquidity problems when fundamentals are weak; (ii) liquidity requirements should be tightened in the presence of higher disclosure levels, if assets are opaque and investors conservative; and (iii) solvency requirements must be strengthened in the face of increased competition.

⁷ Gorton (1988) disputes the view that crises are panic driven with a study of crises in the US National Banking Era; he concludes that the crises were predictable (see also Schotter and Yorulmazer 2009). The “information” view of crises has been developed by, among others, Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998). Postlewaite and Vives (1987) present a model with incomplete information about the liquidity shocks suffered by depositors that features a unique Bayesian equilibrium in which there is a positive probability of bank runs. In that model, there is no uncertainty about the fundamental value of the banks’ assets and there are no solvency problems.

⁸ See De Grauwe and Ji (2013).

⁹ Calomiris and Mason (2003) also dispute the analysis of Friedman and Schwartz (1963); the latter authors conclude that some episodes of 1930s US banking crises can be explained by deteriorating fundamentals whereas others can be explained by the domination of a panic component (as in January and February of 1933). Starr and Yilmaz (2007) claim that both fundamentals and elements of panic figure into the dynamics of bank runs in Turkey.

In the model considered here, investors must decide whether to keep an investment or run (sell it or withdraw). The investment may be in a currency, a bank, or short-term debt. A financial intermediary will be our leading example. The model is based on the theory of games with strategic complementarities and incomplete information; an example are the “global games” of Carlsson and van Damme (1993) and Morris and Shin (1998).¹⁰ I provide a general framework that bridges the panics and fundamentals views of crises, characterizes illiquidity and insolvency risk, and nests among others, the models of Morris and Shin (1998, 2004), Rochet and Vives (2004), and Bebchuk and Goldstein (2011). This paper delivers predictions under both unique and multiple equilibria; describes how a regulator should set solvency and liquidity requirements in conjunction with requirements concerning degree of transparency, thereby reducing the likelihood of both insolvency and illiquidity; and applies the model to interpret the 2007 run on structured investment vehicles (SIVs). The model can also be applied to sovereign debt crises and currency attacks.

I shall characterize how the degree of strategic complementarity depends on the balance sheet composition of a financial intermediary, parameters of the information structure of investors (i.e, the precision of public and private information), and the level of stress indicators in the market. Strategic complementarity increases with a weaker balance sheet (higher leverage), with greater competitive pressure for funding, and with higher fire-sale penalties for early asset liquidation. All those parameter changes make the resistance level that must be overcome by the mass of running investors in order for a run to succeed less sensitive to the fundamentals. This diminished sensitivity of the resistance threshold induces an investor to react more strongly to changes in the strategy of other investors. Furthermore, strategic complementarity also increases when public information is more precise and when private information is less precise (when it is worse than public information). These information factors render an investor less uncertain about the behavior of others, which in turn leads that investor to react more strongly to any change in the decision rules of other investors.

A weaker balance sheet or an increase in stress indicators makes a crisis more likely, and there is a range of fundamentals that indicate coordination failure from the viewpoint of the

¹⁰ See Vives (2005) for a general overview. The importance of strategic complementarities in macro models is highlighted by Cooper and John (1988). Liu and Mello (2011) apply the global games methodology to study the behavior of hedge funds in the crisis.

institution under attack (i.e., when it is solvent but illiquid). A key observation is that the effect of bad news—say, a public signal about weak fundamentals—is magnified when strategic complementarity is high. This means that the public signals coming from, for instance, a derivatives market could be destabilizing, and the more so the more precise they are since they generate a larger range of illiquidity. These results are consistent with the 2007 run on SIVs (structured investment vehicles) and ABCP (asset-backed commercial paper) conduits featuring a high level of strategic complementarity among investors given that leverage was high and the crisis heightened fire-sale penalties. The results are also consistent with the potential destabilizing effect of FAS rule 157, making banks' balance sheets more transparent at a time of stress, as well as with the practice of clearinghouses in the US National Banking Era (1863-1913) of suspending the requirement for banks to publish their financial statements during a panic situation.¹¹

The policy message that follows from the analysis presented here is that a regulator must pay attention to the balance sheet composition of financial intermediaries—in particular, the regulator should manage the ratio of liquid assets to unsecured short-term debt as well as the short-term leverage ratio (ratio of unsecured short-term debt to equity or, more generally, stable funds). Those two ratios can be used to control the probability of insolvency or illiquidity, and they are partially substitutable. The reason why they are not perfectly substitutable is that a solvency ratio is more effective in controlling the probability of insolvency while a liquidity ratio is more effective in controlling the probability of illiquidity. The liquidity requirement is needed when fire-sale penalties are high and investors are conservative, the regulator is not willing to allow a large probability of illiquidity, and the precision of public information high enough relative to the precision of private information. This paper establishes that, in a more competitive environment (i.e., with higher returns offered on short-term debt), the solvency requirement should be strengthened. This means that financial liberalization should not proceed without increased solvency requirements, in contrast with the practice contributing to such banking crises as the US S&Ls in the 1980s. The expectation of turbulent environments, high fire-sale penalties, and conservative investors calls for the liquidity requirement to be strengthened but for the solvency requirement to be relaxed. Finally, when potent public signals are anticipated (e.g., introduction of a derivatives market such as the ABX index, or a new disclosure requirement such as the FAS rule 157), illiquidity may increase; then a strengthened liquidity requirement

¹¹ See Gorton (2010).

might be necessary, especially when assets are opaque. These results have implications also for the design of prudential policy with respect to sovereign debt.

The paper proceeds as follows. Section 1 sets up a framework of analysis with the basic model; this section also characterizes the equilibrium and discusses its links to both strategic complementarity and its comparative statics properties. The model is specialized to a bank run, a leading example that illustrates the results. Section 2 deals with regulation of a financial intermediary, and Section 3 extends the model for application to the 2007 run on SIVs. Section 4 covers connections in the literature as well as empirical issues. Section 5 considers other applications, including a sovereign debt run, and Section 6 offers some concluding remarks. The Appendix gives proofs of all the propositions.

1. A stylized crisis model: The link between strategic complementarity and fragility

This section presents a general framework for analysis, including the game played by investors. Also presented are characterizations of the equilibrium and its comparative static properties.

1.1. The investors' game

Consider the following binary action game among a continuum of investors with unit mass. The action set of player (investor) i is $\{0,1\}$, where $y_i = 1$ is interpreted as “acting” and $y_i = 0$ as “not acting”. Possible actions include attacking a currency, refusing to roll over debt, participating in a run on a bank or SIV, and declining to renew a certificate of deposit in the interbank market.

Let $\pi^1 = \pi(y_i = 1, y; \theta)$ and $\pi^0 = \pi(y_i = 0, y; \theta)$ denote, respectively, the payoffs to acting and not acting; here y is the fraction of investors acting and θ is the state of the world. The differential payoff to acting is $\pi^1 - \pi^0 = B > 0$ if $y \geq h(\theta)$ and is $\pi^1 - \pi^0 = -C < 0$ if $y < h(\theta)$, where $h(\theta)$ is the *resistance* function—that is, the critical fraction (of investors) above which it pays to act. These relationships are summarized in the following matrix.

| | | |
|-----------------|-----------------|--------------------|
| | $y > h(\theta)$ | $y \leq h(\theta)$ |
| $\pi^1 - \pi^0$ | $B > 0$ | $-C < 0$ |

It pays to act if enough investors act and so resistance is overcome (i.e., if $y > h(\theta)$). Let p be the probability that enough investors act. Then there is a critical success probability p of the collective action such that an agent is indifferent between acting and not acting: $pB + (1-p)(-C) = 0$. This probability is $\gamma \equiv C/(B+C)$, $\gamma \in (0,1)$, the ratio of the cost C of acting to $B+C$, the incremental benefit of acting with success versus acting with failure. An investor will act if his assessed probability of successful mass action exceeds γ . We shall assume that $h(\cdot)$ is strictly increasing, is smooth on $(\underline{\theta}, \tilde{\theta})$, crosses 0 at $\theta = \underline{\theta}$ (with $\lim_{\theta \uparrow \underline{\theta}} h(\theta) = 0$), and crosses 1 at $\theta = \tilde{\theta}$.¹²

The investors' game is one of strategic complementarities because the differential payoff to acting $\pi^1 - \pi^0$ is increasing in the mass of players acting y .¹³ Suppose that θ , the state of the world, is known; in other words, we have a game of complete information among investors. It follows from the stated payoffs that if $\theta < \underline{\theta}$ then acting is a dominant strategy but if $\theta \geq \tilde{\theta}$ then not acting is a dominant strategy. For $\theta \in [\underline{\theta}, \tilde{\theta})$ there are multiple equilibria, including one in which all investors act and one in which no investors act. The threshold $\underline{\theta}$ can therefore be understood as an institution's "solvency" threshold (when $\theta < \underline{\theta}$, the institution is overrun even if no investor attacks). Similarly, $\tilde{\theta}$ is the "supersolvency" threshold (when $\theta \geq \tilde{\theta}$, the institution resists even if all investors attack).

Since this is a game of strategic complementarities, there is both a largest and a smallest equilibrium—that is, we have extremal equilibria. The largest equilibrium is a "run" equilibrium in which only "supersolvent" institutions survive: $y_i = 1$ for all i if $\theta < \tilde{\theta}$, and $y_i = 0$ for all i if $\theta \geq \tilde{\theta}$. The smallest equilibrium is a "no-run" equilibrium where only "solvent" institutions fail: $y_i = 1$ for all i if $\theta < \underline{\theta}$, and $y_i = 0$ for all i if $\theta \geq \underline{\theta}$.¹⁴

From now on I consider an incomplete information version of the game where investors have a Gaussian prior on the state of the world $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$ and investor i observes a private

¹² Note that this allows the function $h(\cdot)$ to be discontinuous at $\theta = \underline{\theta}$ with $h(\underline{\theta}) > 0$ and $h(\theta) = 0$ for $\theta < \underline{\theta}$.

¹³ In a game of strategic complementarities, the marginal return of a player's action is increasing in the level of the rivals' actions. As a result, best replies are monotonically increasing; see Vives (2005).

¹⁴ Note that both equilibria are (weakly) decreasing in θ ; the reason is that $\pi^1 - \pi^0$ is decreasing in θ .

signal $s_i = \theta + \varepsilon_i$ (with Gaussian noise that is independent and identically distributed, $\varepsilon_i \sim N(0, \tau_\varepsilon^{-1})$).¹⁵ It is worth noting that the prior mean μ_θ of θ can be understood as a public signal of precision τ_θ ; under this interpretation, μ_θ can be negative.

The model encompasses several crisis situations studied in the literature: currency attacks (Morris and Shin 1998), loan foreclosures (Morris and Shin 2004), credit freezes (Bebchuk and Goldstein 2011), and bank runs (Rochet and Vives 2004). I next present the bank run model, which will underpin the policy exercise described in Section 2. Other applications of the model framework are examined in Section 5.

1.2. The bank run model

Consider a bank run model that fits into the general framework already introduced (based on Rochet and Vives 2004). Traditional bank runs resulted from massive withdrawals of deposits by individual depositors. Modern bank runs result from the nonrenewal of short-term credit in the interbank market; examples include the case of Northern Rock, the 2007 run on SIVs, and the 2008 run by short-term creditors on Bear Stearns and on Lehman Brothers.

Consider a market with three dates: $t = 0, 1, 2$. At date $t = 0$, the bank has its own funds E (including such stable resources as equity, long-term debt, and even insured deposits) as well as uninsured short-term debt (e.g., uninsured wholesale deposits and certificates of deposit, CDs) in amount $D_0 \equiv 1$. These funds are used as cash reserves M and also to finance risky investment I . So at $t = 0$, the balance sheet constraint is $E + D_0 = I + M$. The returns θI on these assets are collected at date $t = 2$; if the bank can meet its obligations, then the short-term debt is repaid at face value D and the bank's equity holders receive any residual amounts. Investors are also entitled to the face value D if they withdraw in the interim period $t = 1$. Let $m \equiv M/D$ be the liquidity ratio, $\ell = D/E$ the short-term leverage ratio, and $d = D/D_0$ the return on short-term debt.¹⁶

¹⁵ The incomplete information game is referred to in the literature (e.g., Carlsson and van Damme 1993) as a "global game".

¹⁶ The distinction of stable funds within liabilities is made also in a BIS (2009) document that addresses liquidity risk. In fact, the two liquidity ratios proposed by BIS are equivalent in our simple formulation: the BIS short-term funds ratio would correspond to m and the BIS net stable funds ratio to E/I . Note also that leverage $D/(D + E)$ equals $1/(1 + \ell^{-1})$ and is monotonic in ℓ .

Fund managers (recall that we view them as being on a continuum) make investment decisions about whether or not to extend short-term debt to the bank. At $t = 1$ each fund manager, after observing a private signal regarding the future realization of θ , decides whether to cancel ($y_i = 1$) or to renew ($y_i = 0$) her position. The claims on the bank at $t = 1$ are yD , since y is the mass of “acting” fund managers. If $yD > M$ then the bank can meet its payments only by selling some of its assets¹⁷ in a secondary market. The value of bank assets that are sold early reflects a fire-sale penalty $\lambda > 0$ (i.e., retrieving only $\theta/(1 + \lambda)$ for each unit invested).¹⁸

A fund manager is rewarded for making the right decision (viz., withdrawing if and only if the bank fails). The cost of canceling the investment is C , and the benefit from getting the money back or canceling when the bank fails is $\hat{B} = B + C$. The payoffs define (as in Section 1.1) the critical probability of failure $\gamma \equiv C/(B + C)$, above which a fund manager does not renew credit. What is crucial is that investors and fund managers both adopt, for whatever reason, a behavioral rule of this type: Cancel the investment if and only if the likelihood of bank failure exceeds some threshold γ . This rule is followed also by investors who expect a fixed return when withdrawing but expect nothing if they do not withdraw and the bank subsequently fails, and there is a (small) cost to withdrawing. Larger values of γ are associated with *less* conservative investors, from which it follows that risk management rules may influence γ .¹⁹

Suppose that all fund managers renew credit to the bank (i.e., $y = 0$). Then there are no fire sales at $t = 1$ and the bank fails at $t = 2$ if and only if $M + I\theta < D$ or $\theta < \varrho \equiv (D - M)/I$. Hence ϱ can be viewed as the bank’s solvency threshold because, below that level, the bank fails even if all fund managers renew credit.²⁰ Given the balance sheet constraint, we have

¹⁷ Or borrowing against collateral in the repo market.

¹⁸ The parameter λ could be related to the Libor–OIS spread (a measure of the difference between interbank rates and the rates paid on overnight index swaps, instruments that are not exposed to the default risk of intermediaries), which increased substantially both in August 2007 (start of the crisis) and September 2008 (collapse of Lehman Brothers). In the case of secured collateral, λ captures the haircut required. Haircuts on asset-backed securities rose dramatically after the collapse of Lehman Brothers (see e.g. Gorton and Metrick 2010).

¹⁹ According to Krishnamurthy’s (2010) review of debt markets during the crisis, investor conservatism is an important determinant of short-term lending behavior.

²⁰ The solvency threshold is related to what the FDIC calls (as part of its CAMELS assessment) the “Net Non-core Funding Dependence Ratio”, which is computed as noncore liabilities less short-term investments divided by long-term assets.

$\underline{\theta} = (1 - m) / (\ell^{-1} + d^{-1} - m)$.²¹ Suppose now that, at $t = 1$, the bank must liquidate some (but not all) assets, $M + I\theta(1 + \lambda)^{-1} \geq yD > M$. In this case, the bank will fail at $t = 2$ if and only if $I\theta - (1 + \lambda)(yD - M) < (1 - y)D$ (or, equivalently, if and only if $y > (I\theta + (1 + \lambda)M - D) / \lambda D$).²² Under the balance sheet constraint, we derive an equivalent inequality that identifies bank failure for $\theta \geq \underline{\theta}$:

$$y > h(\theta) = m + \frac{\ell^{-1} + d^{-1} - m}{\lambda}(\theta - \underline{\theta});$$

for $\theta < \underline{\theta}$ we have $h(\theta) < 0$ (see Figure 1). Note that, for $\theta = \tilde{\theta} \equiv (1 + \lambda)\underline{\theta}$, the bank does not fail even if fund managers do not renew credit (i.e., $y = 1$) because $h(\tilde{\theta}) = 1$. Therefore, $\tilde{\theta}$ is the “supersolvency” threshold. We have that $h(\underline{\theta}) = m > 0$ and that h is decreasing in λ and in d . If $1 - \ell^{-1} - d^{-1} < 0$ then, since $\text{sign}\{\partial \underline{\theta} / \partial m\} = \text{sign}\{1 - \ell^{-1} - d^{-1}\}$, it follows that h is increasing in m .²³

²¹ Note that $\ell^{-1} + d^{-1} - m = I/D > 0$ for a positive risky investment.

²² If $M + I\theta(1 + \lambda)^{-1} < yD$ then the bank fails at $t = 1$. It is worth noting that, whereas the US liquidity requirements of broker-dealers are based on *unsecured* funding, the demise of Bear Stearns followed from its failure to renew *secured* funding. See the US Securities and Exchange Commission’s report on its oversight of Bear Stearns “and related entities” (SEC 2008).

²³ We have $\partial h / \partial m = \lambda^{-1}(1 + \lambda - \theta) > 0$ because $\underline{\theta} < 1$ is implied by $1 - \ell^{-1} - d^{-1} < 0$ and then $\theta < (1 + \lambda)\underline{\theta} < 1 + \lambda$.

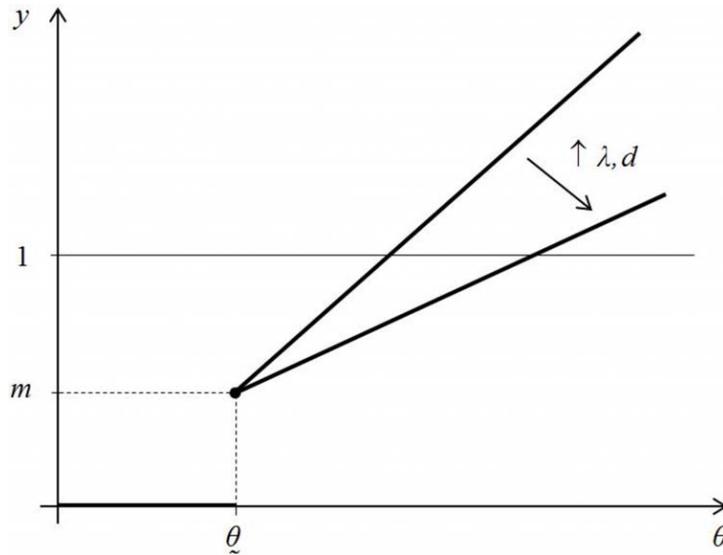


Figure 1. The resistance function in the bank runs model: $h(\theta) = m + (\ell^{-1} + d^{-1} - m)\lambda^{-1}(\theta - \underline{\theta})$ if $\theta \geq \underline{\theta}$, and $h(\theta) < 0$ if $\theta < \underline{\theta}$. Above the bold line $y > h(\theta)$ for $\theta \geq \underline{\theta}$ and the bank fails.

For the balance sheet of a financial intermediary, it is normally the case that $1 - \ell^{-1} - d^{-1} < 0$. Indeed, the ratio of (uninsured) short-term debt to stable funds (equity, long-term debt, and insured deposits), $\ell = D/E$, is below 1 for commercial banks. Although ℓ is above 1 for investment and wholesale banks, typically $d^{-1} \geq 0.9$ (given interest rates not exceeding 10%) and so we would need $\ell^{-1} < 0.1$ or $\ell > 10$ to have $1 - \ell^{-1} - d^{-1} > 0$.²⁴ For a typical SIV, $\ell < 1$.²⁵

An alternative interpretation of the bank model would be to consider the aggregate banking sector of a country and treat θ as a macroeconomic fundamental, where all investors are outside the banking sector. In this interpretation the ratios are the aggregate ratios, we abstract from any externalities among banks, and a crisis is a crisis of the banking sector.

²⁴ See Figure A in the Appendix for data on some US banks.

²⁵ According to the April 2008 *Global Financial Stability Report* of the International Monetary Fund (IMF), the typical funding profile of an SIV in October 2007 was 27% in asset-backed commercial paper and the rest in medium-term notes and capital notes. That profile would translate into a value of $\ell = 0.34$.

1.3. Equilibrium and strategic complementarity

In this section we return to the general model and study equilibria of the investors' game as well as the factors affecting strategic complementarity. We seek a symmetric equilibrium in threshold strategies under which an investor will run (or act) if and only if he receives a signal whose value is below a certain threshold.

In order to gain some intuition of the game's structure, let us think in terms of the best reply of a player $r(\cdot)$ to the (common) signal threshold \hat{s} used by the other players. An equilibrium then is a signal threshold s^* such that $s^* = r(s^*)$. This signal threshold will imply a critical world state $\theta^* \in [\underline{\theta}, \tilde{\theta})$, below which the "acting mass" is successful.

The best reply can be computed in two steps. Given a signal threshold \hat{s} at which other investors run, the investor computes the failure threshold $\hat{\theta}$ such that the institution fails if and only if $\theta < \hat{\theta}$: $\Pr(s < \hat{s} | \hat{\theta}) = h(\hat{\theta})$ if $\hat{\theta} > \underline{\theta}$ and $\hat{\theta} = \underline{\theta}$ otherwise. This yields the failure threshold curve $\hat{\theta} = \theta_f(\hat{s})$, which is increasing in \hat{s} . On this curve, whenever $\hat{\theta} > \underline{\theta}$, the fraction of acting players, $y = \Pr(s < \hat{s} | \hat{\theta})$, equals the critical fraction above which it pays to act, $h(\hat{\theta})$. Now, given a failure threshold $\hat{\theta}$, the investor can compute the signal threshold \hat{s} below which it is optimal to run: $\Pr(\theta < \hat{\theta} | \hat{s}) = \gamma$. That expression yields the signal threshold curve $\hat{s} = s_T(\hat{\theta})$, which is linear with slope $(\tau_\theta + \tau_\varepsilon)/\tau_\varepsilon$ and increasing in $\hat{\theta}$, and on which the expected payoff from acting or not acting is the same (see Figure 2 and Claim 1 in the Appendix).

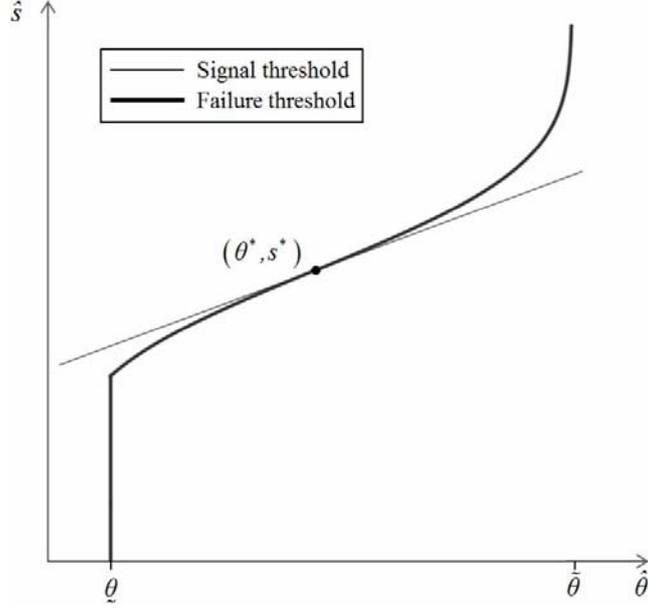


Figure 2: Equilibrium characterization in the bank run model. Parameters: $\gamma = 0.5$, $m = 0.1$, $\ell^{-1} = 0.2$, $d^{-1} = 0.9$, $\lambda = 1$, $\theta = 0.9$, $\tilde{\theta} = 1.8$, $\tau_\theta = 0.5$, $\tau_\varepsilon = 0.05$, $\mu_\theta = 1.3$.

The best-response function is the composition of the two thresholds: $r(\hat{s}) = s_T(\theta_F(\hat{s}))$,

$$r(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta_F(\hat{s}) - \frac{\tau_\theta}{\tau_\varepsilon} \mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon} \Phi^{-1}(\gamma),$$

and it is increasing. Indeed, the game is one of strategic complementarities; a higher threshold \hat{s} by others induces a player to use a higher threshold, too (see Figure 3). Equilibria are fixed points of this best response or, equivalently, are given by the intersection points of the two curves in space of failure and signal thresholds: $\hat{\theta} = \theta_F(\hat{s})$ and $\hat{s} = s_T(\hat{\theta})$.

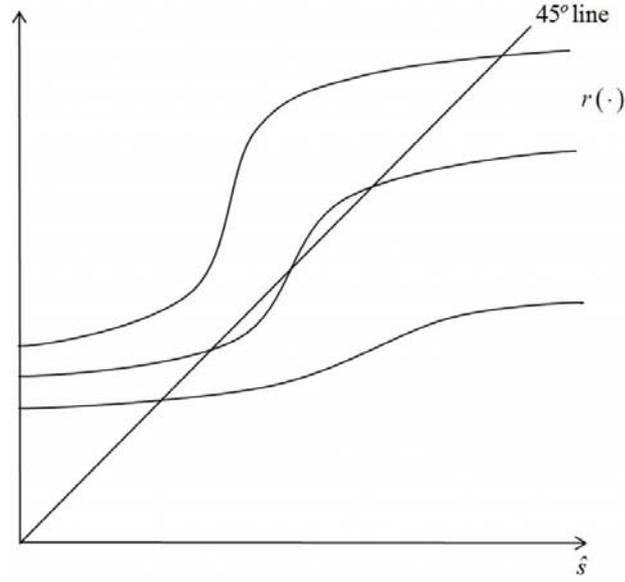


Figure 3: Possible best responses of a player to the threshold strategy \hat{s} used by rivals for an intermediate range of γ . The lower (resp., middle, upper) plot corresponds to high (resp., intermediate, low) values of h_1 .

The following proposition provides the equilibrium characterization and generalizes the results in the literature. Let h_1 denote the smallest slope of $h(\cdot)$ on $(\underline{\theta}, \tilde{\theta})$.

Proposition 1.

(i) *An equilibrium is characterized by two thresholds (s^*, θ^*) , where s^* is the signal threshold below which an investor acts and $\theta^* \in [\underline{\theta}, \tilde{\theta})$ is the state-of-the-world critical threshold, below which the acting mass is successful. The probability of a crisis conditional on $s = s^*$ is γ .*

(ii) *There is a critical $\bar{h}_0(\underline{\theta}) \in (0,1)$ such that:*

- *if $h(\underline{\theta}) \geq \bar{h}_0(\underline{\theta})$ then at the smallest equilibrium $\theta^* = \underline{\theta}$;*
- *if $h(\underline{\theta}) < \bar{h}_0(\underline{\theta})$ then $\theta^* > \underline{\theta}$ for any equilibrium.*

(iii) *The equilibrium is unique if $(\tau_\theta / \sqrt{\tau_\epsilon}) \leq h_1 \sqrt{2\pi}$.*

The proof of this proposition is given in the Appendix, and it exploits the game's monotonicity properties (i.e., the game is monotone supermodular). In this type of game there are extremal (largest and smallest) equilibria and they are in threshold strategies. Any other

equilibrium is bound by the largest and smallest equilibria. The uniqueness condition $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$ implies that $r'(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta'_F(\hat{s}) \leq 1$. In this case the largest and the smallest equilibrium coincide at the unique equilibrium. It is worth noting that r' is increasing in τ_θ because $\theta'_F(\hat{s})$ is independent of τ_θ ; hence strategic complementarity increases with better public information. This relation ensures that $r(\cdot)$ crosses the 45° line only once and that the equilibrium is unique. A sufficient condition for the existence of multiple equilibria (a necessary condition for regular equilibria when $r'(\hat{s}) \neq 1$) is that $r'(s) > 1$ for $r(s) = s$.

In sum: a *necessary* condition for multiple equilibria is that strategic complementarity be strong enough; a *sufficient* condition is that strategic complementarity be strong enough at relevant points (candidate equilibria). So if strategic complementarity is always moderate (case $r'(\cdot) \leq 1$) then there is a unique equilibrium, but when it is not moderate there may be multiple equilibria (see Figure 3). If h_1 (the smallest slope of $h(\cdot)$) is large enough, then there is a unique equilibrium. In the linear case, for an intermediate range of γ we find that reducing h_1 yields multiple equilibria (generically, three); if h_1 is reduced further then we return to a unique equilibrium, since in that case strategic complementarity is strong but in an irrelevant range (see Figure 3). Furthermore, if $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$ then there is always a range of γ in which there are three equilibria.²⁶

The extent of strategic complementarity among the players' actions depends on the slope of the best response. The maximal value of that slope is given by $\bar{r}' \equiv \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon + h_1\sqrt{2\pi\tau_\varepsilon}}$, which is increasing in h_1^{-1} . The degree of strategic complementarity will be higher whenever h is less sensitive to θ (larger h_1^{-1}), the prior is more precise (larger τ_θ), and/or the signals are imprecise (low τ_ε when $\tau_\varepsilon < \tau_\theta$). When h_1^{-1} is large, a change in fundamentals θ has little effect on the critical threshold $h(\theta)$. The implication is that a change in the strategy threshold \hat{s} used by

²⁶ Indeed, changes in γ move vertically the best reply, and with $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$, there is a range with multiple equilibria. This happens, for example, around $\gamma = 0.5$ when $h(\theta) = \theta/10$, $\tau_\theta = 1$, $\tau_\varepsilon = 5$, $\bar{\theta} = 5$, while the equilibrium is unique with $\gamma = 0.1$ and $\gamma = 0.9$.

other investors leads to a larger optimal reaction owing to the greater induced change in the conditional probability that the acting investors succeed.

Furthermore, \bar{r}' is indeed increasing in τ_θ ; with respect to τ_ε , \bar{r}' is first decreasing (in particular when $\tau_\varepsilon < \tau_\theta$ with $\bar{r}' \rightarrow \infty$ as $\tau_\varepsilon \rightarrow 0$) and then increasing (with $\bar{r}' \uparrow 1$ as $\tau_\varepsilon \rightarrow \infty$). When noise in the signals is large (small τ_ε), a player is relatively certain about the behavior of others and so strategic complementarity is increased. In the limit case of $\tau_\varepsilon \rightarrow \infty$ (or with a diffuse prior $\tau_\theta \rightarrow 0$), investors face maximal strategic uncertainty; in this case, the distribution of the proportion of acting players y is uniformly distributed over $[0,1]$ conditional on $s_i = s^*$. Yet at any of the multiple equilibria with complete information when $\theta \in (\underline{\theta}, \tilde{\theta})$, investors face no strategic uncertainty. For example, in the equilibrium where everyone acts, an investor has a point belief that all other investors will act.

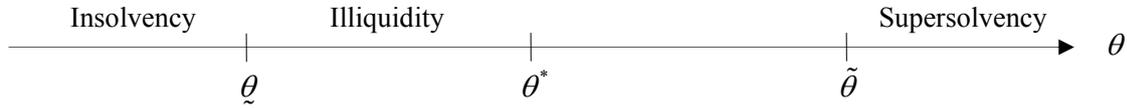
In short, strategic complementarity increases with less sensitivity of the resistance function to fundamentals, with more precise public information, and with less precise private information (when it is worse than public information).

In the bank run case, $h_1^{-1} = \lambda / (\ell^{-1} + d^{-1} - m)$; therefore, strategic complementarity among investors is increasing in short-term leverage ℓ , in the face value of short-term debt d , in the fire-sale penalty λ , and in the liquidity ratio m . All these factors make the resistance function less sensitive to θ . Thus we see that the strength of strategic complementarity, as measured by the maximal slope of the best response \bar{r}' , is affected by information parameters (τ_θ and τ_ε), by balance sheet structure (ℓ and m), and by market stress parameters (d and λ).

1.4. Coordination failure, illiquidity risk, and insolvency risk

At equilibrium with threshold θ^* , there is a crisis when $\theta < \theta^*$. In the range $[\theta^*, \tilde{\theta})$ there is coordination failure from the point of view of investors, because if all of them were to act (say, in the case of the currency peg) then they would succeed. In the range $[\underline{\theta}, \theta^*)$ there is coordination failure from the point of view of the institution attacked. Recall that with our bank run model the bank is solvent but illiquid in the range $[\underline{\theta}, \theta^*)$; that is, the bank would be spared

any problems if investors renewed their short-term debt, but in this range they do not and so the bank is illiquid. We can compare these concepts graphically as follows:



The risk of illiquidity is given by $\Pr(\underline{\theta} \leq \theta < \theta^*)$ and the risk of insolvency by $\Pr(\theta < \underline{\theta}) = \Phi(\sqrt{\tau_\theta}(\underline{\theta} - \mu_\theta))$, where Φ denotes the standard Normal cumulative distribution. Hence $\Pr(\theta < \underline{\theta})$ is the probability that the bank is insolvent when there is no coordination failure from the bank's perspective. The overall probability of a crisis is $\Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$. Note that $\Pr(\underline{\theta} \leq \theta < \theta^*) = \Pr(\theta < \theta^*) - \Pr(\theta < \underline{\theta})$. A crisis occurs for low values of the fundamentals. In contrast, in the complete information model there are multiple self-fulfilling equilibria in the range $(\underline{\theta}, \tilde{\theta})$.

1.5. Comparative statics

We shall develop the comparative statics properties of both unique and multiple equilibria. The comparative statics results that follow hold when the equilibrium is unique and also when there are multiple equilibria for the extremal (largest and smallest) equilibria. Furthermore, the results hold for any equilibrium—even the unstable, intermediate one, if out-of-equilibrium adjustment is adaptive. With best-reply dynamics at any stage after the parameter perturbation from equilibrium, a new state of the world θ is drawn independently and a player responds to the strategy threshold used by other players at the previous stage. Then a parameter change that monotonically alters the best reply will induce a monotone adjustment process with an unambiguous prediction. For instance, if we are at the higher equilibrium of the middle plot in Figure 3, then an increase in h_1 may induce a movement to the lower plot, in which case the best-reply dynamics would settle at the unique (and lower) equilibrium.²⁷ The results are stated formally in Proposition 2. Observe that, contrary to most of the literature, the comparative statics

²⁷ See Echenique (2002) and Vives (2005).

results presented here are not restricted to parameter configurations that determine a *unique* equilibrium.

In the applications it is useful to parameterize the resistance function $h(\theta; \alpha)$, where the parameter α represents an index of vulnerability or stress (with $\partial h / \partial \alpha < 0$). A larger α signifies more vulnerability or a more stressful environment for the institution under attack, since then there is a lower threshold for the attack to be successful. In the bank run model, for instance, $h(\theta; \alpha) = m > 0$ and we can identify the parameter α with λ , d , ℓ , or m^{-1} when $1 - \ell^{-1} - d^{-1} < 0$, since h is decreasing in all these variables.

Proposition 2 (Comparative statics). *Let $h(\theta; \alpha) < \bar{h}_0$. At extremal equilibria or under adaptive dynamics, the following statements hold.*

- (i) *The thresholds θ^* and s^* and the probability of crisis $\Pr(\theta < \theta^*)$ are all decreasing in γ (i.e., with less conservative investors) and in the expected value of the state of the world μ_θ ; they are increasing in the stress indicator α .*
- (ii) *The release of a public signal μ_θ has a multiplier effect on equilibrium thresholds (i.e., beyond its impact on an investor's best response), which is enhanced when τ_θ is higher.*
- (iii) *Let $\gamma < 1/2$. If μ_θ is low enough (weak fundamentals with $\mu_\theta \leq \theta$ is sufficient), then a more precise public signal increases θ^* , the probability $\Pr(\theta < \theta^*)$, and the range $[\theta, \theta^*)$ whereas a more precise private signal reduces them. If μ_θ is high enough (strong fundamentals with $\mu_\theta > \tilde{\theta} - (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)$ is sufficient), then the results are reversed.*

Remark 1. It is immediate from part (i) of the proposition that the range $[\theta, \theta^*)$ is decreasing in both γ and μ_θ .

Remark 2. The region of potential multiplicity $(\tau_\theta / \sqrt{\tau_\varepsilon}) > h_1 \sqrt{2\pi}$ is enlarged with an increase in payoff complementarity (decrease in h_1) and/or an increase in the precision of the public signal in relation to the private one, $\tau_\theta / \sqrt{\tau_\varepsilon}$.

Remark 3. The conditions for the comparative statics results in Proposition 2(iii) are sufficient but by no means necessary. When the equilibrium is unique it can be checked that part

(iii) can be strengthened as follows: There are thresholds $\hat{\mu}_\theta$ and $\bar{\mu}_\theta$ with $\hat{\mu}_\theta < \bar{\mu}_\theta$ such that (a) θ^* increases with τ_θ if and only if $\mu_\theta < \hat{\mu}_\theta$ and (b) θ^* increases with τ_ε if and only if $\mu_\theta > \bar{\mu}_\theta$ (see proof in Appendix).²⁸

Let us now apply the results of the proposition to illustrating the comparative static results in the bank run model. First of all, in this model \bar{h}_0 is given by a liquidity ratio $\bar{m} \in (0,1)$. This critical ratio \bar{m} depends on the parameters of the model and in particular it increases with ℓ . When $m \geq \bar{m}$ there is always an equilibrium in which only insolvent banks fail, $\theta^* = \underline{\theta}$. For $m < \bar{m}$, there is always a range of illiquidity, with $\theta^* > \underline{\theta}$ at any equilibrium. When $1 - \ell^{-1} - d^{-1} < 0$, the comparative statics of λ , d , ℓ , and m^{-1} follow from those of α in Proposition 2(i). The following statements are then immediate (see Claim 2 in the Appendix):

- The probability of insolvency $\Pr(\theta < \underline{\theta})$ is decreasing in the liquidity ratio m and in the expected return on the bank's assets μ_θ , increasing in the short-term leverage ratio ℓ and the face value of debt d , and independent of the fire-sale penalty λ and the critical withdrawal probability γ .
- At extremal equilibria or under adaptive dynamics, the probability of failure $\Pr(\theta < \theta^*)$, the critical θ^* , and the range of illiquidity $\theta^* - \underline{\theta}$ are all decreasing in m , γ , and μ_θ but increasing in λ , ℓ , and d . The probability of illiquidity $\Pr(\underline{\theta} \leq \theta < \theta^*)$ is decreasing in γ and increasing in λ and ℓ .

The equilibrium is unique if $\tau_\theta / \sqrt{2\pi\tau_\varepsilon} \leq h_1$ where $h_1 = (\ell^{-1} + d^{-1} - m) / \lambda = I / \lambda D$. In the limit case where $\tau_\varepsilon \rightarrow \infty$ the equilibrium is unique and allows for a closed-form solution. Then it is easy to see that

$$s^* = \theta^* = \underline{\theta} \left(1 + \frac{\lambda}{1-m} (\max\{1 - \gamma - m, 0\}) \right)$$

and $\bar{m} = 1 - \gamma$.²⁹ Indeed, when $m < 1 - \gamma$, both θ^* and $\theta^* - \underline{\theta}$ are decreasing in γ and in m (provided $1 - \ell^{-1} - d^{-1} < 0$) and increasing in λ , ℓ , and d .

²⁸ See Metz (2002) for a closely related result.

²⁹ In this case, θ^* and s^* are independent of μ_θ and τ_θ .

Table 1 summarizes the results. Note that the qualitative comparative statics of the probability of failure and of the range of illiquidity are identical.

Table 1. Comparative Statics of Solvency and Liquidity Risk for $m < \bar{m}$

| | $\Pr(\theta < \underline{\theta})$ (Insolvency) | $\Pr(\theta < \theta^*)$ (Failure) | $\theta^* - \underline{\theta}$ (Range of illiquidity) |
|--|---|------------------------------------|---|
| m (Liquidity ratio) | (-) | (-) | (-) |
| μ_θ (Strength of fundamentals) | (-) | (-) | (-) |
| ℓ (Leverage ratio) | (+) | (+) | (+) |
| d (Cost of funds) | (+) | (+) | (+) |
| λ (Fire-sale penalty) | (0) | (+) | (+) |
| γ^{-1} (Investor conservatism) | (0) | (+) | (+) |

The likelihood of failure rises with an increase in balance sheet stress (lower m whenever $1 - \ell^{-1} - d^{-1} < 0$ or higher ℓ) or market stress (higher d ; the return of deposits, which can be interpreted as an increase in competitive pressure; higher λ , the fire-sale penalty for early liquidation; or lower γ , more conservative investors). The probability of failure is also increasing in bad fundamentals (lower μ_θ). The prior mean μ_θ can be interpreted as a public signal. According to Proposition 2, the release of such a public signal has a multiplier effect on the equilibrium (and the more so the more precise it is). We turn to those effects now.

Part (iii) of Proposition 2 indicates that releasing more public information need not be beneficial, as when fundamentals are weak (μ_θ low enough). In this case the public signal is a coordinating device for investors to act (attack), since each one knows that others will weight the public signal heavily. This effect is reinforced when the public signal is more precise—and also

with *less* precise private information, since then the public signal’s value is enhanced (note that strategic complementarity is maximized for high τ_θ and low τ_ε).³⁰ The opposite occurs when fundamentals are sound. Figure 4 illustrates how an increase in public precision raises strategic complementarity and helps investors coordinate on a run when fundamentals are weak (although the bank expects to remain solvent provided $\mu_\theta > \underline{\theta}$). We see how increasing τ_θ from $\tau_\theta^L = 0.05$, corresponding to the flat best response in Figure 4’s top panel, moves the equilibrium threshold toward the “run” equilibrium (i.e., moves θ^* toward $\tilde{\theta}$) and generates more illiquidity. This is true even when increasing τ_θ causes multiple equilibria to appear, among them a “no-run” equilibrium (the smallest one; see Figure 4’s top panel, where the intersection of the 45° line with the flat part of the best response marks an equilibrium with $\theta^* = \underline{\theta}$ when $\tau_\theta^L = 1.75$). Indeed, when starting from $\tau_\theta^L = 0.05$, as τ_θ increases we would move—according to best-reply dynamics—to the largest (not the smallest) equilibrium; this yields an unambiguous comparative statics result.

³⁰ If μ_θ is low then fund managers would tend to withdraw but if private signals are very precise then they will not pay much attention to the public signal and will moderate their actions.

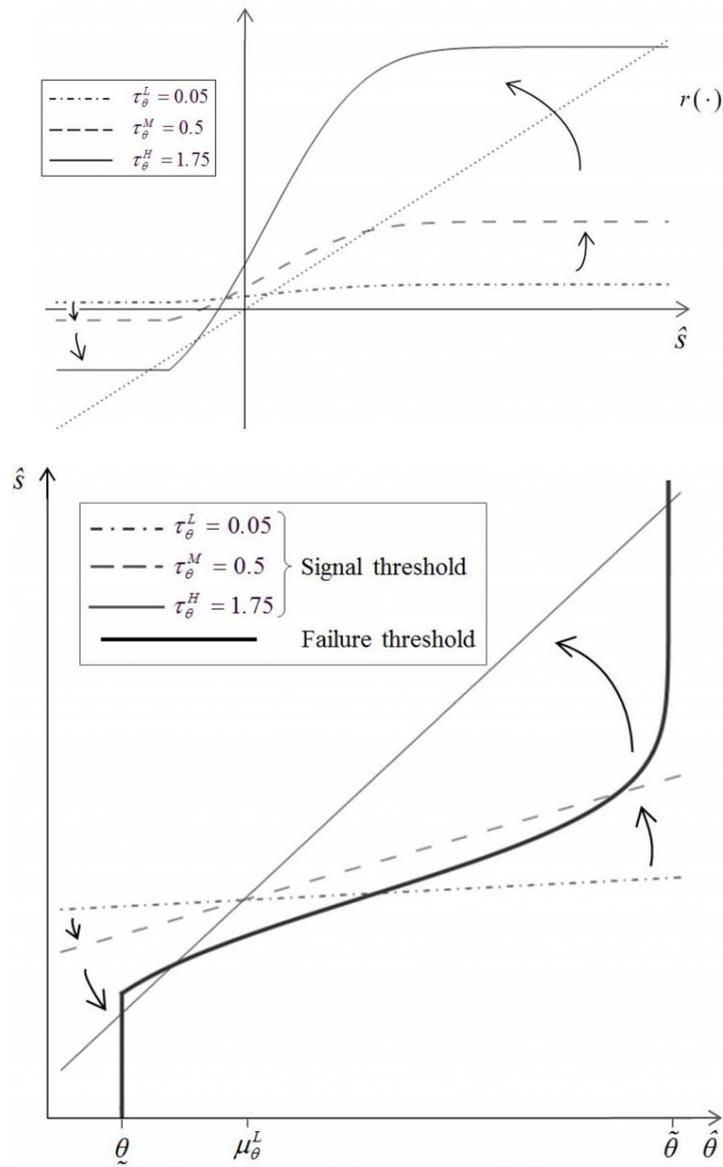


Figure 4. Increase in public precision ($\tau_\theta^L = 0.05$, $\tau_\theta^M = 0.5$, $\tau_\theta^H = 1.75$) with weak fundamentals (low public signal, $\mu_\theta^L = 1.1$). Other parameters: $\tau_\varepsilon = 0.05$, $\gamma = 0.5$, $m = 0.1$, $\ell^{-1} = 0.2$, $d^{-1} = 0.9$, $\lambda = 1$, $\underline{\theta} = 0.9$, $\tilde{\theta} = 1.8$.

Remark 4. The precision τ_θ is literally the inverse of the variance of θ , so a lower τ_θ corresponds to a higher riskiness of the fundamentals (the bank's risky long-term investment). The previous result can be reinterpreted as stating that, as long as we start in an equilibrium with

$\mu_\theta < \theta^*$, an increase in the riskiness of the assets τ_θ^{-1} (a mean-preserving spread) will reduce illiquidity risk. That is, starting in the “run” equilibrium $\tau_\theta^H = 1.75$, by reducing τ_θ we move to safer equilibria.³¹

Figure 5 illustrates how the effect of bad news (when the public signal goes from μ_θ^H to $\mu_\theta^L < \mu_\theta^H$) differs markedly when public precision is low versus high. In the latter case (right panel), strategic complementarity is high and the best response has a slope close to 1 since τ_θ is high, and the impact is dramatic; when τ_θ is low (left panel), the effect is relatively small because the best response curve is flatter.

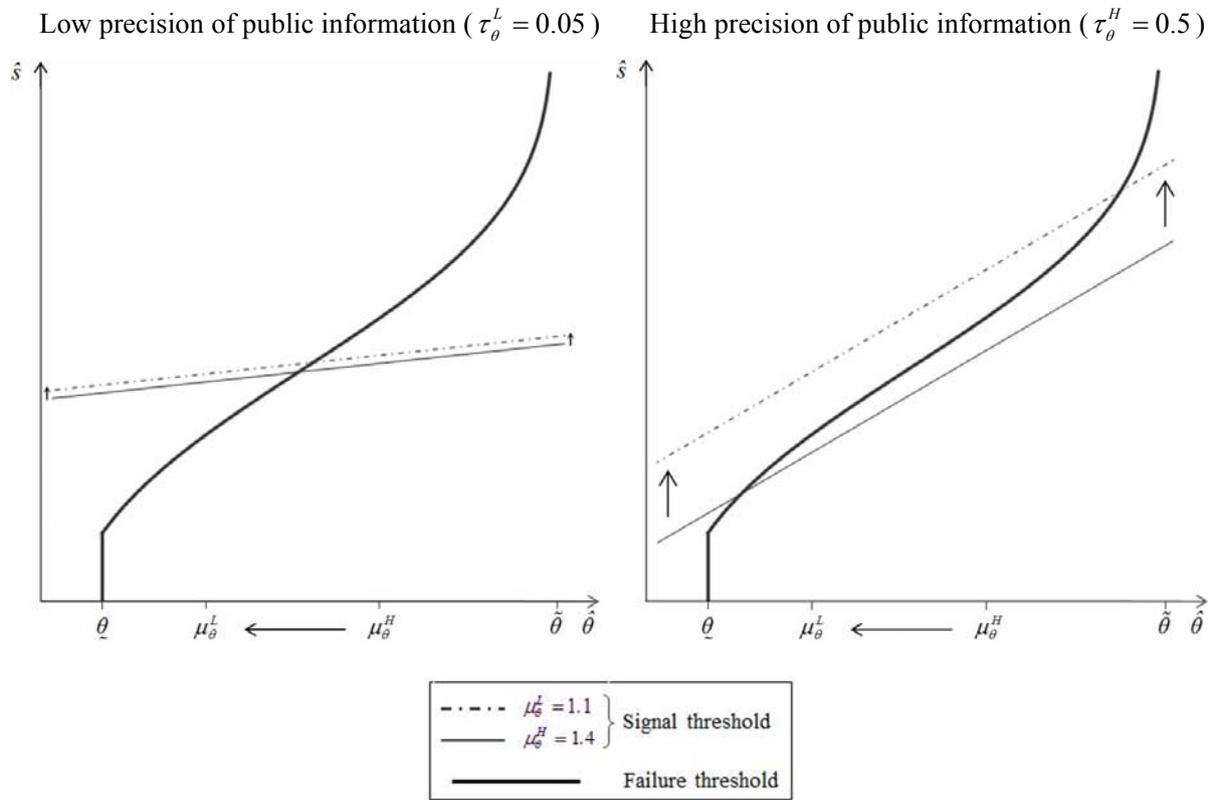


Figure 5. The effect of bad news (from $\mu_\theta^H = 1.4$ to $\mu_\theta^L = 1.1$). Other parameters: $\tau_\epsilon = 0.05$, $\gamma = 0.5$, $m = 0.1$, $\ell^{-1} = 0.2$, $d^{-1} = 0.9$, $\lambda = 1$, $\underline{\theta} = 0.9$, $\tilde{\theta} = 1.8$, $\tau_\theta^L = 0.05$, $\tau_\theta^H = 0.5$.

³¹ In a dynamic model related to the debt run model of He and Xiong (2012), Cheng and Milbradt (2012) find that—during a freeze—a bank that increases its portfolio risk may thereby increase the confidence of creditors.

Let us turn now to regulation in the bank runs model and to controlling the likelihood of crises.

2. Liquidity and solvency regulation of financial intermediaries

Two common objectives of regulators are to control the probabilities of insolvency and illiquidity (see e.g. Freixas and Rochet 2008). The potential for insolvency is typically controlled by reducing the incentives for investors to take risks on banks with limited liability. The potential for illiquidity is reduced by alleviating fragility and addressing the coordination and contagion problems associated with bank financing. In other words, the first objective is the prudential requirement that sets a lower bound on solvency and the second objective is to prevent a solvent institution from failing. The liquidity concern can be addressed by the central bank's "lender of last resort" facility and by liquidity regulation.³² In this paper it is assumed that the regulator's objectives include the control of both insolvency and illiquidity and that its instruments are leverage and liquidity requirements.

So suppose that the regulator wants to bound the maximum probability of insolvency at the level q and that of failure at the level p , thus bounding the probability of illiquidity. That is, q is the highest probability of insolvency $\Pr(\theta < \underline{\theta})$ allowed by the regulator and p is the highest allowed probability of failure $\Pr(\theta < \theta^*)$. This approach can be rationalized by presuming the regulator has a loss function, with appropriate weights on the likelihood of insolvency and illiquidity, that is minimized subject to an efficiency constraint (in terms of the expected value of bank assets). It is noteworthy that the strategic behavior of investors does not affect the determination of the solvency threshold $\underline{\theta}$; in fact, $\underline{\theta}$ depends only on balance sheet parameters and the cost of funds. However, the behavior of investors is crucial in determining the equilibrium failure threshold θ^* (even though θ^* turns out to be a function also of the model's balance sheet, market, and information parameters).

The analysis in the paper builds on the connection between balance sheet parameters, market stress parameters, and information parameters with the degree of strategic

³² Rochet and Vives (2004) study the central bank's lender of last resort (LOLR) facility. A central bank with perfect information on the focal bank's fundamentals could provide the appropriate liquidity—at the possible cost of fostering moral hazard. Yet a central bank with only imperfect information will certainly make errors, which reinforces the role of liquidity requirements. Repullo (2005) shows how the existence of a LOLR induces banks to hold lower levels of liquid assets.

complementarity of the actions of investors and the probabilities of insolvency and failure. The degree of strategic complementarity is quantified by the (maximal) slope of the best response function of an investor to the signal threshold used by other investors \bar{r}' (as explained in Section 1.3) and is the key determinant of fragility (understood as equilibrium sensitivity to small changes in parameters). Balance sheet parameters, such as m and ℓ affect \bar{r}' through the slope of the resistance function h (i.e. h_1). The probability of failure depends directly on the critical threshold θ^* which is affected by the function h . Increasing m (whenever $1 - \ell^{-1} - d^{-1} < 0$) or ℓ^{-1} relaxes the resistance function h and consequently decreases θ^* . The end result is that the likelihood of insolvency and of bank failure can be altered by the application of liquidity and leverage ratios and that we can ascertain how those move with the deep parameters of the model. For the sake of simplicity, let us restrict our attention to the parameter range in which there is a unique equilibrium.³³

A regulator that wants to eliminate illiquidity must set $p = q$. Yet that approach often proves too costly (in terms of forgone returns) owing to the reduced level of investment in the risky asset, so the regulator will set $p > q$ and thus allow some solvent but illiquid banks to fail.

The regulator wants to ensure $\Pr(\theta < \underline{\theta}) \leq q$ and $\Pr(\theta < \theta^*) \leq p$ where θ^* is the critical threshold at the unique equilibrium. The goal is accomplished by fulfilling two constraints:

$$\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta} \quad (\text{constraint S})$$

and

$$\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta} \quad (\text{constraint L}).$$

Note that if $p > q$ then $\theta_p^* > \underline{\theta}_q$. It can be shown that the boundaries of both constraints are linear in the space (m, ℓ^{-1}) and, as long as $0 < \underline{\theta}_q < 1$,³⁴ downward sloping with constraint L having a larger slope (in absolute value) than constraint S. We have

³³ Recall that a *sufficient* condition for the equilibrium to be unique is that $\lambda\tau_\theta/\sqrt{2\pi\tau_\epsilon} \leq \ell^{-1} + d^{-1} - m$; therefore, for $\lambda\tau_\theta/\sqrt{\tau_\epsilon}$ small a wide range of combinations of (m, ℓ^{-1}) will fulfill the inequality. In particular, if $\tau_\epsilon \rightarrow \infty$ the inequality will always hold.

³⁴ Suppose, for instance, that $q = 0.05$ and $\mu_\theta = 1.2$. Then $\Phi^{-1}(q) = -1.65$, and $(0.19, 68)$ is the range of τ_θ required to ensure that $0 < \underline{\theta}_q < 1$.

$$\ell^{-1} \geq (\underline{\theta}_q)^{-1} - d^{-1} - \left((\underline{\theta}_q)^{-1} - 1 \right) m$$

for constraint S (solvency) and

$$\ell^{-1} \geq \frac{1+k\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m$$

for constraint L (liquidity), where $k \in (0,1)$. The constant k is the maximal critical fraction of investors that may not renew credit in order for $\Pr(\theta < \theta^*) \leq p$ (i.e., $k = h(\theta_p^*)$). It is increasing in p , decreasing in γ , ambiguously related to $\tau_\theta/\tau_\varepsilon$, and independent of the model's other parameters (see Claim 3 in the Appendix). Also, when $p = q$ is required and $\theta^* = \underline{\theta}_q$ has been induced, k is precisely the liquidity ratio \bar{m} that would eliminate the illiquidity region. If $\lambda k > \theta_p^*/\underline{\theta}_q - 1$ then $(1+k\lambda)/\theta_p^* - (\underline{\theta}_q)^{-1} > 0$ and so L intersects S from above, as shown in Figure 6.

It follows from constraint S and $0 < \underline{\theta}_q < 1$ that $1 - \ell^{-1} - d^{-1} < 0$ for $m < 1$. From Section 1.5 (and Claim 2 in the Appendix), both $\underline{\theta}$ and θ^* are decreasing in m (since $1 - \ell^{-1} - d^{-1} < 0$) and in ℓ^{-1} . Therefore, the regulator can make sure that $\Pr(\theta < \underline{\theta}) \leq q$ and $\Pr(\theta < \theta^*) \leq p$ inducing a choice of (m, ℓ^{-1}) in the upper contour of the constraints S and L. Figure 6 depicts the upper contour set, for ratios of leverage and liquidity, with respect to which the probabilities of insolvency and overall crisis are bound by q and p , respectively.

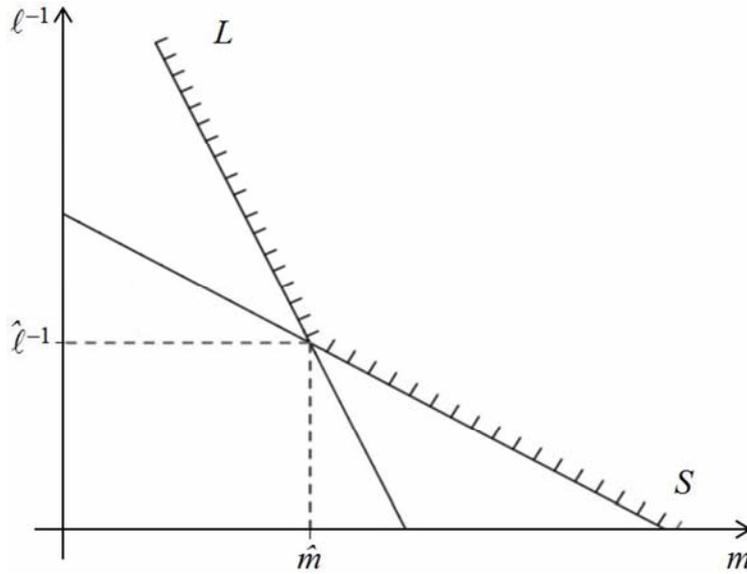


Figure 6: Solvency constraint S and liquidity constraint L used to reduce the likelihood of insolvency and crisis with (respectively) a short-term leverage ratio ($\ell = D/E$) and a liquidity ratio ($m = M/D$). The upper contour sets consist of the leverage and liquidity ratios for which the probabilities of insolvency and overall crisis are bound, respectively, by q and p .

The regulator can set an upper bound q on the maximum allowed likelihood of insolvency and an upper bound p on the maximum allowed likelihood of a crisis (and therefore of illiquidity) by the appropriate choice of the ratios of liquidity m and leverage ℓ . Thus a regulator must propose a region of (m, ℓ^{-1}) space in which the bank ratios must lie. This region is the upper contour of the constraints S and L , which is limited by a kinked downward-sloping schedule that reflects the (partial) substitutability between m and ℓ^{-1} (Figure 6). The kink arises because, even though both solvency and liquidity ratios can be used to reduce the likelihood of insolvency or crisis, the solvency (resp., liquidity) ratio is naturally more effective at curtailing insolvency (resp., illiquidity). That is, the slope of the S constraint is smaller, in absolute value, than the slope of the L constraint.

It is assumed that both constraints are binding. This is reasonable when the constraints are downward sloping. That is, unregulated banks would choose liquidity and solvency ratios (m, ℓ^{-1}) below the appropriate levels. For the case of constraint L this means, for example, that the level of p is not so large so that for any equilibrium and combination of ratios (m, ℓ^{-1}) ,

$\theta^* \leq \theta_p^*$, making L non-binding.³⁵ When faced with the upper-contour constraint set, the bank will choose the least-cost combination (m, ℓ^{-1}) , which will necessarily lie on the frontier of one of the constraints. Given the kink in the constraint set, however, often there will be no loss of efficiency if the regulator sets minimum levels for (m, ℓ^{-1}) . The reason is that the constrained optimization of the financial intermediary will lead to the kink in the constrained set for ample price ranges of liquidity and capital. The minimal $(\hat{m}, \hat{\ell}^{-1})$ ratios are given by the intersection of the boundaries of the solvency and liquidity constraints.

The regulator's problem can be seen in another, equivalent, way. Given that the solvency constraint S is binding at the set level $\underline{\theta}_q$ it is easy to see that the resistance function h is increasing in m (with ℓ^{-1} adjusting accordingly to fulfill S).³⁶ Along S, therefore, increasing m will reduce θ^* and the regulator can choose m to fulfill $\theta^* \leq \theta_p^*$.

When $\lambda k \leq (\theta_p^*/\underline{\theta}_q) - 1$, the L constraint lies below the S constraint (and the absolute value of its slope is greater); then typically $\hat{m} = 0$. The inequality will tend to be satisfied when the fire-sale premium λ is low or investors are not too conservative (γ large, since k is decreasing in γ). For example, if $\lambda = 0$ then there is no fire-sale penalty and so in equilibrium there is no illiquidity, $\theta^* = \underline{\theta}$. In this case it is clear that the liquidity requirement serves no purpose; since insolvency can be controlled with the solvency requirement, the constraints L and S collapse into one.

When $\lambda k > (\theta_p^*/\underline{\theta}_q) - 1$, the liquidity requirement is necessary and $\hat{m} = 1 - (1 - k)(1 - \lambda^{-1}((\theta_p^*/\underline{\theta}_q) - 1))^{-1} > 0$. This is so when fire sales penalties are significant (λ high), investors conservative (γ low), and the regulator not willing to allow a large probability of illiquidity (p not much larger than q , since $\theta_p^*/\underline{\theta}_q \rightarrow 1$ as $p \rightarrow q$). The liquidity requirement is also necessary when the precision of public information τ_θ is high enough relative

³⁵ For example, for $\tau_\epsilon \rightarrow \infty$ and (m, ℓ^{-1}) fulfilling the constraint S, the largest possible equilibrium is $\theta^* = \underline{\theta}_q (1 + \lambda(1 - \gamma))$ for $m = 0$. For $\underline{\theta}_q (1 + \lambda(1 - \gamma)) \leq \theta_p^*$ constraint L is non-binding.

³⁶ We have that along S, $h(\theta) = m + (1 - m)(\theta - \underline{\theta}_q)/\lambda \underline{\theta}_q$ and, therefore, $\partial h/\partial m = \lambda^{-1}(1 + \lambda - \theta/\underline{\theta}_q) > 0$ because $\theta < (1 + \lambda)\underline{\theta}_q$.

to the precision of private information τ_ε and investors are more conservative than the regulator ($1/2 > p > \gamma$, since in this case as $\tau_\theta/\tau_\varepsilon \rightarrow \infty$, $k \rightarrow 1$ and $\hat{m} \rightarrow 1$).³⁷

Proposition 3 summarizes the main conclusions of our analysis and gives the formal statement.

Proposition 3.

(i) *The solvency constraint S and the liquidity constraint L, which are both linear and acting on the ratios ℓ^{-1} and m , must be satisfied in order to control for the likelihood (respectively) of insolvency at level q and of a crisis at level p .*

(ii) *Let $0 < \underline{\theta}_q < 1$, then constraint S and constraint L are both downward sloping and constraint L always has a larger slope (in absolute value) than constraint S. Furthermore,*

$$\hat{m} = \max \left\{ 1 - (1 - k) \left(1 - \lambda^{-1} \left(\frac{\theta_p^*}{\underline{\theta}_q} - 1 \right) \right)^{-1}, 0 \right\}$$

and

$$\hat{\ell}^{-1} = (\underline{\theta}_q)^{-1} - d^{-1} - \left((\underline{\theta}_q)^{-1} - 1 \right) \hat{m} > 1 - d^{-1},$$

where the constant $k \in (0,1)$ is increasing in p, γ^{-1} , and, whenever $1/2 > p > \gamma$ and τ_ε is small, in $\tau_\theta/\tau_\varepsilon$. When $\tau_\varepsilon \rightarrow \infty$ we have $k = 1 - \gamma$.

(iii) *If $p = q$ then $\hat{m} = k$. If $p > q$ and $\lambda k > (\theta_p^*/\underline{\theta}_q) - 1$, then $k > \hat{m} > 0$ and the comparative statics of the regulatory ratios is given in Table 2.*

³⁷ However, if $p < \gamma$ then as $\tau_\theta \rightarrow \infty$, we have that $k \rightarrow 0$ and $(\theta_p^*/\underline{\theta}_q) \rightarrow 1$, and therefore $\hat{m} \rightarrow 0$.

Table 2. Comparative Statics of Regulatory Ratios When $p > q$ and $\lambda k > (\theta_p^* / \theta_q) - 1$

| | λ (Fire-sale penalty) | γ^{-1} (Investor conservatism) | d (Cost of funds) | μ_θ (Strength of fundamentals) | τ_θ (Precision of public information) |
|---------------------------------------|----------------------------------|--|------------------------|--|--|
| \hat{m} (Liquidity ratio) | (+) | (+) | 0 | (+) | (+)* |
| $\hat{\ell}^{-1}$ (Solvency ratio) | (-) | (-) | (+) | (-) | (-)* |

* Provided $p < 1/2$ and either $\tau_\varepsilon \rightarrow \infty$ or τ_ε is small and $p > \gamma$.

The minimal regulatory ratios \hat{m} and $\hat{\ell}^{-1}$ move in opposite directions in response to parameter changes: the liquidity requirement \hat{m} must be increased, and the solvency requirement $\hat{\ell}^{-1}$ decreased, with increased fire-sale penalty λ , with more conservative fund managers γ^{-1} , with better fundamentals μ_θ , and (provided either $\tau_\varepsilon \rightarrow \infty$ or τ_ε is small and $p > \gamma$) with more precise public information τ_θ . More funding pressure on intermediaries (a higher d) calls for an increased $\hat{\ell}^{-1}$ and a constant \hat{m} . An explanation of the comparative statics results using the movements of the regulatory constraints when parameters change follows.

A higher return d on short-term debt will increase the solvency requirement but leave the liquidity requirement unaffected. This is because both constraints are tightened in a vertical way when d increases (see Figure 7(a)). Therefore, liberalized policies that increase competition should be combined with increased solvency requirements. Those requirements were not (or were only partially) increased in several liberalization episodes that lead to subsequent crises, including the US Savings & Loans crisis in the 1980s, the 1990s crisis in the Nordic countries, and the banking liberalization in Spain that began in the late 1970s. Proposition 3 implies that, for a cross section of banks, intermediaries with higher funding costs (higher d) should have a higher solvency requirement and the same liquidity requirement.

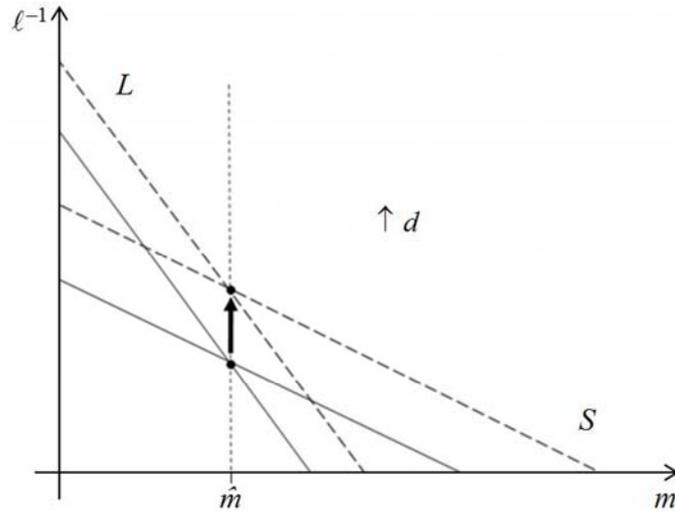


Figure 7(a). Effects of an increase in the cost of funds.

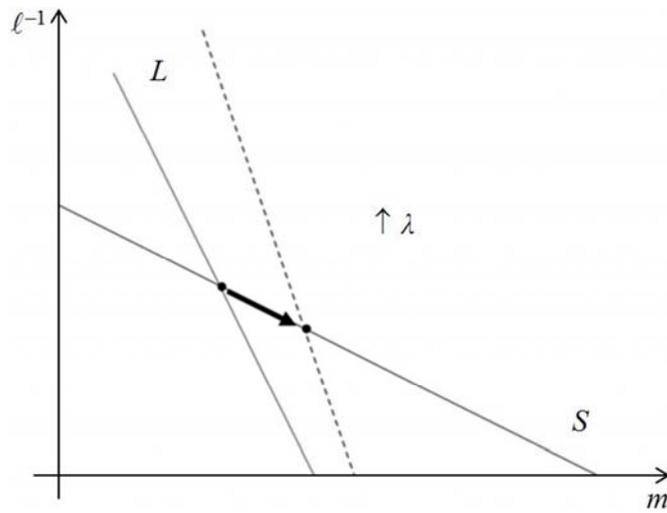


Figure 7(b). Effects of an increase in the fire-sale penalty.

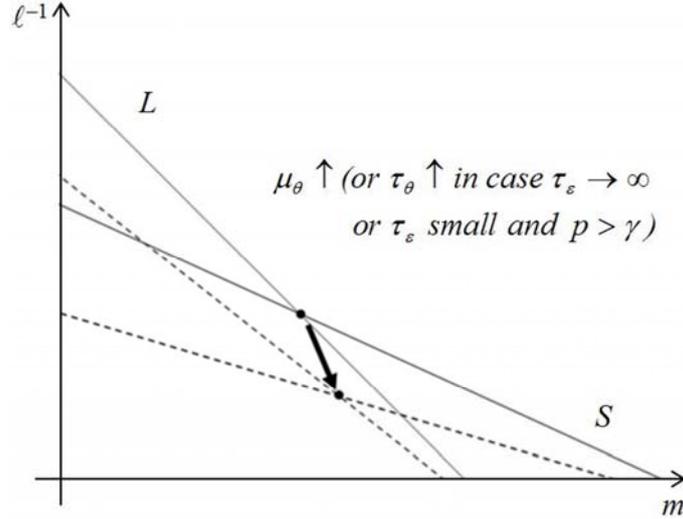


Figure 7(c). Effects of an increase in the strength of fundamentals (good news) or in the precision of public information (transparency) under certain conditions.

A larger fire-sale penalty λ and more conservative investment managers (lower γ) will raise the liquidity requirement and lower the solvency requirement. The reason is that increases in λ and γ^{-1} have no effect on the solvency constraint but do tighten the liquidity constraint; see Figure 7(b) for the effect of an increase in λ (an increase in γ^{-1} has a similar effect). So in an environment where λ is high and γ is low, the liquidity requirement should be strengthened but the solvency requirement can be relaxed.

An increase in return prospects μ_θ calls for a higher liquidity requirement and a lower solvency requirement. Although both constraints are more relaxed when μ_θ increases, if $p > q$ then θ_p^*/θ_q is decreasing in μ_θ and so constraint L is relaxed relatively less than is constraint S; hence \hat{m} increases with μ_θ (indeed, \hat{m} is decreasing in θ_p^*/θ_q) and $\hat{\ell}^{-1}$ decreases with μ_θ (since k is independent of μ_θ). The same is true for an increase in τ_θ whenever τ_ε is small enough and $p > \gamma$ (or when $\tau_\varepsilon \rightarrow \infty$, in which case $k = 1 - \gamma$); see Figure 7(c). In general, if τ_θ increases then a sufficient condition for L to be relaxed less than S (if at all) is that k be increasing in τ_θ , which it is when τ_ε is small enough and $p > \gamma$. This sufficient condition requires that the regulator be more conservative than investors ($p > \gamma$) and that investors be poorly informed (low τ_ε). Then $\partial \hat{m} / \partial \tau_\theta > 0$ and $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$. If $p \leq \gamma$ then k is decreasing in τ_θ . In this case it is

typical for \hat{m} to exhibit a hump-shaped pattern: increasing for low values of τ_θ and then slowly decreasing for large values of τ_θ . Then we may have $\partial\hat{m}/\partial\tau_\theta < 0$ for a range of τ_θ values, in which case increasing τ_θ leads to a reduction in both regulatory ratios.³⁸

These results suggest that the regulator should set disclosure and prudential policies in tandem, since prudential requirements depend on the precision τ_θ of the public signal. When $p < 1/2$, for example, if either $\tau_\varepsilon \rightarrow \infty$ or τ_ε is small enough and $p > \gamma$ (conservative investors), then \hat{m} increases with τ_θ and $\hat{\ell}^{-1}$ decreases with τ_θ . Requiring more disclosure must therefore be accompanied by a higher liquidity requirement and a lower solvency requirement. As we will see in Section 3, the existence of a derivatives market may entail the availability of a precise public signal; this should be taken into account by prudential regulation.

It is noteworthy that our policy prescription of relaxing solvency requirements when return prospects are higher (i.e., higher μ_θ) does *not* contradict the BIS macroprudential recommendation that capital requirements should be tightened in good times. The results of our static model should be understood to hold on average during the cycle. For example, if the regulator expects that a strong public signal will be available in the future (owing to disclosure or the presence of a derivatives market) then it should tighten the liquidity requirement when the conditions are fulfilled (i.e., when either $\tau_\varepsilon \rightarrow \infty$ or τ_ε is small enough and $p > \gamma$) while taking into account the average value of parameters in the cycle (or perhaps the worst-case scenario in terms of return prospects when assessing the probabilities of failure and illiquidity).

Remark 5. If parameters are such that multiple equilibria may appear then the analysis is more complex. We know that given that the solvency constraint S is binding at the set level θ_q , increasing m will reduce the largest equilibrium $\bar{\theta}^*$. However, if the required threshold θ_p^* is low enough then it may be that the liquidity ratio to induce $\bar{\theta}^* \leq \theta_p^*$, instead of \hat{m} as given in Proposition 3, is given by a higher level of m . Let $\bar{\bar{m}}$ be the liquidity ratio that makes the failure threshold just tangent to the signal threshold (see Figure 8). We have that $\bar{\bar{m}} > \bar{m}$ where for $m \geq \bar{\bar{m}}$ we have that $\theta^* = \theta_q$ at the smallest equilibrium. Then for $m > \bar{\bar{m}}$ there is a *unique*

³⁸ For example, $\partial\hat{m}/\partial\tau_\theta < 0$ for $\tau_\theta > 8.4$ when $p = 0.2$, $\gamma = 0.29$, $q = 0.1$, $\lambda = 1$, $\tau_\varepsilon = 5$, $d^{-1} = 0.9$, and $\mu_\theta = 1.25$.

equilibrium at $\theta^* = \underline{\theta}_q$. Increasing m increases strategic complementarity and, although it always decreases the largest equilibrium $\bar{\theta}^*$, it may induce multiple equilibria. Even though with $m \geq \bar{m}$ the smallest equilibrium is always $\theta^* = \underline{\theta}_q$, other equilibria may appear for intermediate values of m .

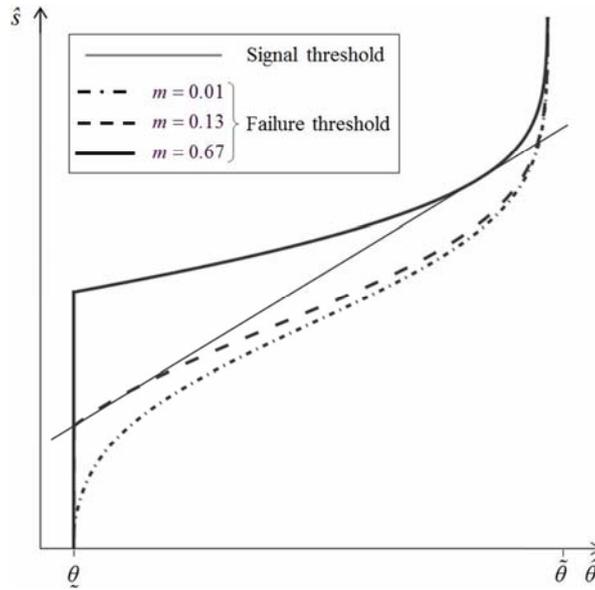


Figure 8. Effect of an increase in the liquidity ratio along the solvency constraint S ($m = 0.01$, $m = \bar{m} \approx 0.1307$, $m = \bar{\bar{m}} \approx 0.6749$ with corresponding values of ℓ^{-1} , respectively and approximately, of 0.55, 0.49, 0.25). Let $\tilde{\theta}_q = (1 + \lambda)\underline{\theta}_q$. For $m = 0.01$ there is a unique (and high) equilibrium; for $m = \bar{m}$ there are three equilibria with the smallest one with $\theta^* = \underline{\theta}_q$; and for $m > \bar{m}$ the equilibrium $\theta^* = \underline{\theta}_q$ is the unique one. Other parameters: $\tau_\varepsilon = 0.1$, $\tau_\theta = 2.5$, $\mu_\theta = 1.5$, $q = 0.1$, $\underline{\theta}_q = 0.69$, $\gamma = 0.15$, $d^{-1} = 0.9$, $\lambda = 0.25$.

Let us explore briefly what happens in the extreme case of the regulator allowing leverage so high that $1 - \ell^{-1} - d^{-1} > 0$. For banks with intense investment banking or wholesale activity we have $1 - \ell^{-1} > 0$ because $\ell = D/E$ is usually greater than 1; therefore, $1 - \ell^{-1} - d^{-1} > 0$ is possible. If leverage is high enough and if $1 - \ell^{-1} - d^{-1} > 0$, then $\underline{\theta} > 1$ and $\partial \underline{\theta} / \partial m > 0$ and so increasing liquid reserves makes insolvency *more* probable. The reason is that, if more liquid reserves M are retained, then fewer are available for investment in the risky asset, $I = 1 + E - M$. The effect is to lower the solvency threshold for low equity. Suppose, for example, that the bank has no equity ($E = 0$ and so $\ell^{-1} = 0$); in this case, the solvency threshold

would be $\underline{\theta} = (1-m)/(d^{-1}-m)$. Furthermore, if $1-\ell^{-1}-d^{-1} > 0$ then $\partial\theta^*/\partial m > 0$ if either γ or λ is small. Now increasing the liquidity ratio leads not only to a greater likelihood of insolvency and crisis but also to a higher range of illiquidity $\theta^* - \underline{\theta}$. For instance, let $\tau_\varepsilon \rightarrow \infty$ with $m < 1-\gamma$; then $\theta^* = \underline{\theta} \left(1 + \frac{\lambda(1-\gamma-m)}{1-m} \right)$ and $\partial\theta^*/\partial m > 0$ for $1-\ell^{-1}-d^{-1} > \gamma/(1+\lambda^{-1})$, and $\theta^* - \underline{\theta}$ increases with m if γ is small ($1-\ell^{-1}-d^{-1} > \gamma$). Increasing the liquidity ratio may actually *increase* the range of illiquidity. This occurs because keeping more cash reserves drains resources for investment and hence reduces the liquidation value of investments during the interim period, which may be needed to meet debt obligations.

Suppose the regulator allows that $\underline{\theta}_q > 1$ (and assume that $(1+k\lambda)(\theta_p^*)^{-1} - d^{-1} > (\underline{\theta}_q)^{-1} - d^{-1} > 0$). Then the solvency constraint is upward sloping, since $(\underline{\theta}_q)^{-1} - 1 < 0$, and the liquidity constraint will also be upward sloping if $\theta_p^* > 1 + \lambda$. If the liquidity constraint is downward sloping then the bank's choices when faced with the constraints are $(m, \ell^{-1}) = (0, (1+k\lambda)(\theta_p^*)^{-1} - d^{-1})$ and $(m, \ell^{-1}) = (\hat{m}, \hat{\ell}^{-1})$. If both constraints are upward sloping then the first equality is chosen. If $(1+k\lambda)(\theta_p^*)^{-1} < (\underline{\theta}_q)^{-1}$ then $(m, \ell^{-1}) = (0, (\underline{\theta}_q)^{-1} - d^{-1})$, since the intercept of the L constraint is below that of the S constraint. This case could arise if the fire-sale penalty λ is so low that the risk of illiquidity is extremely small and thus the only concern is solvency.

The findings can be summarized as follows. When the regulator's upper bound q for the probability of insolvency is close enough to $1/2$ (so that, e.g., $\underline{\theta}_q = \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta} > 1$ for $\mu_\theta > 1$ since $\Phi^{-1}(1/2) = 0$), it may be optimal—to control the likelihood of insolvency and illiquidity—to induce the intermediary to keep no liquid reserves and simply impose a leverage limit (while allowing enough leverage that $1-\ell^{-1}-d^{-1} > 0$). It is somewhat surprising that, when the regulator allows for a high chance of insolvency, it may propose no liquidity requirement but a solvency requirement that helps meet the liquidity constraint needed to control for the overall probability of crisis: $(m, \ell^{-1}) = (0, (1+k\lambda)(\theta_p^*)^{-1} - d^{-1})$.

3. An interpretation of the 2007 run on structured investment vehicles

A slowdown in housing prices combined with a tightening of monetary policy led to increasing doubts about subprime mortgages; these doubts were reflected in 2007's sharp decline in ABX, the asset-based securities index. This index had been launched in January 2006 to track the evolution of residential mortgage-backed securities (RMBS).³⁹ The decline in the ABX index during 2007 seems to have played a major role in unfolding the crisis and especially in the run on SIV and ABCP conduits. Indeed, at year-end 2006 the subindexes for triple-B securities began, after trading at par, to move downward; these subindexes then dropped dramatically in 2007 (see Figure 9).⁴⁰ A similar phenomenon was evident with CMBX, a synthetic ABX-like index based on a set of 25 commercial mortgage-backed securities (CMBS).

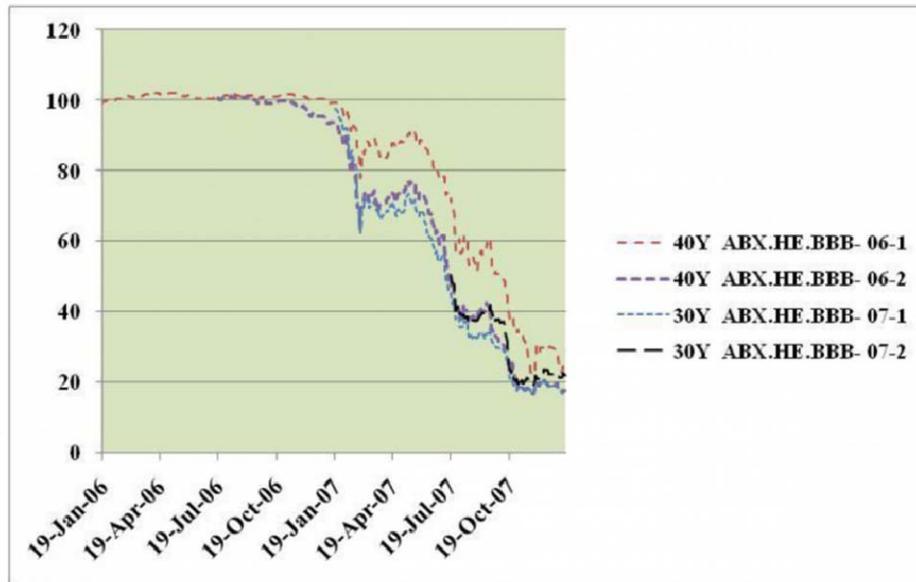


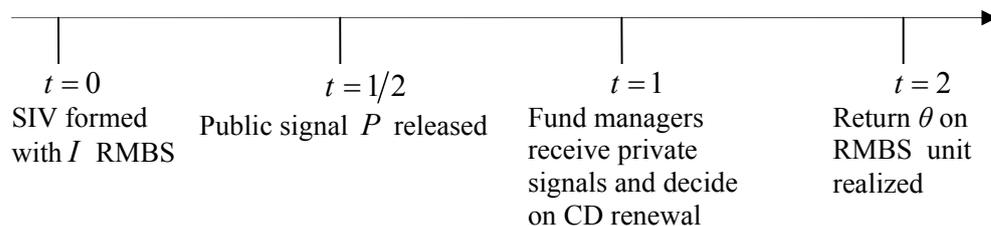
Figure 9: Prices of the 2006:1, 2006:2, 2007:1, and 2007:2 vintages of the ABX index for the BBB-tranche. *Source:* Gorton (2008).

³⁹ The index is a credit derivative based on an equally weighted index of 20 RMBS tranches; there are also subindexes of tranches with different ratings and for different vintages of mortgages. The ABX index filled two important functions: providing information about the aggregate market valuation of subprime risk; and serving as an instrument to cover positions in asset-based securities—for example, by shorting the index itself (Gorton 2008, 2010). In fact, trading in the ABX indexes by Paulson & Co. and Goldman Sachs delivered two of the largest payouts in the history of financial markets. See Stanton and Wallace (2011), who argue that the ABX index is an imperfect measure of subprime security values.

⁴⁰ The index starts trading at par. The only exception is the 2007:2 index, which opened significantly below par.

The ABX (and CMBX) indexes were highly visible and had a strong influence on markets. These indexes evolved in response to a sequence, from January to August 2007, of bad news on subprime mortgages—bankruptcies of and earnings warnings from originators, downgrading of ratings for RMBS bonds and collateralized debt obligations (CDOs), and large losses for hedge funds. The accumulated bad news reflected in the ABX indexes culminated in the panic of August 2007, when BNP Paribas froze a fund because liquidity had evaporated completely in some segments of the US securitized market. A spike in the overnight spread in ABCP and in the Libor–OIS spread followed, and the outstanding ABCP plummeted. The runs began on ABCP conduits and on SIVs that held some percentage of securities backed by subprime mortgages. These vehicles were funded with short-maturity paper, and the run manifested as investors not rolling over that paper.⁴¹ Such investment vehicles may not have had a high proportion of their assets directly contaminated by subprime mortgages, but their indirect exposure was substantial. As short-term financing dried up, bank sponsors intervened and absorbed many of these vehicles onto their balance sheets.⁴²

Consider the following time line in the basic banking model. At time $t = 0$, mortgage loans are awarded and securitized. At $t = 0$, an SIV is formed; it holds I loans and M reserves financed by equity E (or stable funds) and short-term debt (CDs) D_0 . At $t = 1/2$, a public signal P about θ is released. At $t = 1$ each fund manager, having received a private signal about θ , decides whether to cancel ($y_i = 1$) or renew ($y_i = 0$) her CD. At $t = 2$, the returns θI on the RMBS assets are collected; if the bank can meet its obligations then the CDs are repaid at their face value D and the SIV’s equity holders receive the residual (if any).



⁴¹ At the end of 2007, ABCP liabilities amounted to little more than a fourth of the typical SIV’s total yet to nearly all of the typical conduit’s total (see April’s report by the IMF (2008)).

⁴² See Acharya and Schnabl (2010) and Covitz et al. (2013) for evidence on runs in the ABCP market. It is important to distinguish between conduits that were motivated by regulatory arbitrage (and were fully insured by large commercial banks) and conduits that were motivated to transfer risk by off–balance sheet considerations. There is little scope for strategic complementarities among investors in the first case but substantial scope in the second. There was a corresponding larger decline in ABCP conduits of the second type, relative to those of the first type, starting in August 2007. See Acharya, Schnabl, and Suarez (2013).

The public signal P could be the value of the ABX index or the price quoted by a derivatives market with a package of RMBS as an underlying asset (such as the ABX index itself). Denote by τ the precision (accuracy) in P 's estimation of θ . Consider a scenario where neither the SIV nor fund managers in the short-term debt market participate in the derivatives market. Introduction of the ABX index implies a discrete increase in the precision τ of the public signal,⁴³ which increases strategic complementarity. A high level of noise in the signals will also tend to increase complementarity. Recall that, when τ_ε is already low, a still lower τ_ε increases strategic complementarity (since the maximal slope of $r(\cdot)$ tends to infinity as τ_ε approaches zero). In this case, $\tau/\sqrt{\tau_\varepsilon}$ will tend to be large and so multiple equilibria may appear (Proposition 1). Signals from SIV investors are likely to be imprecise (τ_ε low) given the opaqueness of structured subprime products and their distance from loan origination (as when the German Landesbank invested in US structured subprime products).⁴⁴

When bad news strikes (i.e., when $\mu_\theta \equiv E(\theta|P)$ declines), the probability of a crisis occurring increases and we may move, as in the right panel of Figure 5, from a relatively safe equilibrium to a “run” equilibrium (with θ^* close to $\tilde{\theta}$). Observe that the effect of bad news would be small if the precision τ of public signals were low (left panel of the figure). However, strategic complementarity was high not only because τ was high but also because short-term leverage ℓ was high and, in the crisis situation, the cost of funds d and the fire-sale penalty λ also increased (all these factors tend to increase h_1^{-1} and thus strategic complementarity). In fact, the fire-sale penalty increased dramatically: the market became practically illiquid, and SIVs had to be reabsorbed by parent banking institutions. Thus the impact of bad news was even more dramatic owing to it being combined with the increased precision of public signals (Proposition 2).

The model of Section 2 still applies, so both solvency requirements and liquidity requirements will be necessary for “shadow” banks such as SIVs. In particular: Proposition 3 and Table 2 together show the necessity of a liquidity requirement when the fire-sale premium is high

⁴³ The precision τ of *public* signals in the derivatives market is positively affected by the precision of the *private* signals of sophisticated traders in that market. For related models, see Angeletos and Werning (2006), Tarashev (2007), and Vives (2008, Sec. 4.4).

⁴⁴ See Pagano and Volpin (2012) for a model in which issuers of structured bonds choose opaque ratings in order to enhance the liquidity of their primary market—though at the cost of diminishing (perhaps drastically) the liquidity of the secondary market.

and investors are conservative and furthermore when τ_ε is low, $\gamma < p$, then a higher precision of the public signal should lead to an increased liquidity requirement. In other words, the introduction of a strong public signal—such as that provided by a derivatives market—should be met with tightened liquidity obligations.⁴⁵

4. Connections to the literature; empirical implications and evidence

Previous literature has obtained results related to those presented here, and a growing body of evidence is consistent with these results. We shall discuss both topics in turn.

4.1. Connections to the literature

Insolvency risk and illiquidity risk. Morris and Shin (2009) also study how insolvency risk and illiquidity risk vary with the balance sheet composition of a financial institution in a model where uncertainty about future fundamentals interacts with uncertainty about present strategies. In that model, there would be no illiquidity risk in the absence of future insolvency risk (i.e. ex post uncertainty) since the authors assume that the partial liquidation of assets has no long-run effect. Morris and Shin demonstrate that illiquidity risk is (i) decreasing in the ratio of cash plus interim realizable assets to short-term liabilities, (ii) increasing in the “outside option ratio” (opportunity cost of the funds of short-term debt holders), and (iii) increasing in the ex post variance of the asset portfolio (“fundamental risk ratio”). Both the particular results and the broad message of that paper are consistent with ours: regulation must focus on the balance sheet composition of any financial intermediary. Liu and Mello (2011) explain how the fragile capital structure of hedge funds, which is due to a coordination problem in redemptions, limits their arbitrage capabilities. The authors show that this fragility induces hedge funds to invest relatively more in cash assets and to use relatively more leverage ex ante. These predictions are borne out by our model, as when a regulator seeks to reduce the likelihood of insolvency or illiquidity since a decrease in stable funds (E) leads to more cash holdings (M).

Competitive pressure and fragility. The links between fragility and the pressure of competition have been analyzed in several papers. Chang and Velasco (2001), in a model of financial crisis in emerging markets that follows the Diamond and Dybvig (1983) tradition, find

⁴⁵ If parameters are such that multiple equilibria may appear then the liquidity ratio may need to be tightened to the point where the illiquidity region vanishes (see Remark 5).

that financial liberalization increases the expected welfare of depositors but may also increase fragility. Matutes and Vives (1996) present a model that combines the banking model of Diamond (1984) with a differentiated duopolistic structure à la Hotelling. These authors find that an increase in rivalry increases the probability of failure in an interior equilibrium of the depositor's game in which banks have positive market shares. Cordella and Yeyati (1998) report that disclosing a bank's risk exposure (by parties beyond the bank manager's control) may increase fragility by increasing the deposit rates demanded by investors. Goldstein and Pauzner (2005) also show, in a model of the global games type, how increasing the deposit rate increases the probability of a run of depositors. In the model presented here, increasing the deposit rate increases the likelihood of both insolvency and illiquidity.

Negative feedback loops. In the bank crisis model, the fire-sale penalty is related to adverse selection. Vives (2010) establishes that the asset fire-sale penalty is increasing in the noise component of the bidders' signals and in the amount auctioned and also that it is decreasing in the number of bidders. In a crisis scenario it is plausible to expect noisier signals, an increased amount auctioned, and fewer bidders. As a result, a bank that tries to sell more assets (because it is in distress) will face a larger discount, which in turn will induce more sales (to honor the previous commitments) and still further discounts. In the extreme, the market may collapse because adverse selection is so severe in relation to the number of bidders. A similar dynamic may prevail with the face value of debt, since a distressed bank that needs refinancing will be offered worse terms (than other banks would be) and this further aggravates the distressed bank's fragility.⁴⁶

Liquidity regulation. Liquidity requirements may have the unintended consequences of aggravating adverse selection and drying up markets for liquidity (Malherbe 2014). The reason is that, when intermediaries hoard liquidity, their selling behavior is interpreted as wanting to unload "lemon" assets. Perotti and Suárez (2011) explore how liquidity requirements and Pigouvian taxes can help internalize the systemic externality induced by short-term funding of intermediaries. In our model, the regulator implicitly minimizes a loss function that includes a cost for illiquidity stemming from systemic (macroprudential) causes. The interbank market may

⁴⁶ See Brunnermeier and Pedersen (2009) for a model of a liquidity spiral that combines market and funding liquidity; see Bernardo and Welch (2004) for a model in which the fear of having to liquidate assets after a crisis may contribute to the frenzy by incentivating sales in the middle of the run. See also Eisenbach (2010) for a feedback model of short-term debt and rollover risk with endogenous fire-sale penalties.

be a source of coinsurance for liquidity risk. Battacharya and Gale (1987) show how banks, when subject to shocks which are private information, have an incentive to underinvest in liquid assets since they can free-ride on the liquidity pool in the interbank market. Acharya, Shin and Yorulmazer (2010) show how liquidity support to failed banks, or unconditional support to survivors, induces banks to hold less liquidity. Calomiris, Heider and Hoerova (2013) show how the group of banks participating in the interbank market will impose cash requirements to prevent free riding.

Large players and market power. Market power may facilitate liquidity provision in the interbank market; liquidity is a public good, so sound banks may have an incentive to provide a troubled bank with liquidity in order to avoid contagion (Allen and Gale 2004; Sáez and Shi 2004). Market power can also impede the provision of liquidity, as when banks with surplus funds strategically underprovide lending in order to induce fire sales of the bank-specific assets of needy intermediaries (Acharya, Gale, and Yorulmazer 2011).⁴⁷

Strategic complementarity and the “many to fail” problem. The literature has also found strategic complementarities in the decisions of individual banks that, with other banks, force the central bank to bail them out collectively. Farhi and Tirole (2012) show how the private leverage choices of financial intermediaries display strategic complementarities through the response of monetary policy, which makes it optimal for banks to adopt a risky balance sheet. Acharya and Yorulmazer (2007) identify the regulator’s ex post incentives to bail out failed banks when many are failing. Hence banks, especially small ones, are induced to “herd” their investment policies and thereby increase the risk of collective failure.

4.2. Empirical implications and evidence

The model yields a rich array of empirical implications. To start with, this model is consistent with the notion that crises are driven both by fundamentals and by panic components. It also predicts a multiplier effect of public information that is increasing in the accuracy of that information. Table 1 provides testable predictions that link the probability of an intermediary’s failure or insolvency with the strength of fundamentals (μ_θ), balance sheet ratios (m and ℓ), and market stress parameters (λ , γ , and d). We find that the strength of strategic complementarities

⁴⁷ See Corsetti et al. (2004, 2006) for other effects of the presence of large players.

among investors, with its associated multiplier effect on comparative statics, increases with ℓ , λ , and d .

There is evidence that banking crises are driven by both solvency and liquidity issues. Gorton (1988), in his study of crises during the US National Banking Era (1865–1914), concluded that panics were triggered when the “fundamentals” (a leading indicator of recession) reached a certain level. Calomiris and Mason (2003) argue that some episodes of US banking crises in the 1930s can be explained by deteriorating fundamentals but that other episodes (e.g., the crises in January and February of 1933) are open to being interpreted as cases of the panic component dominating. Starr and Yilmaz (2007) study bank runs in Turkey and conclude that both fundamentals and panic elements are needed to account for the dynamics of those crises. In an experimental study, Schotter and Yorulmazer (2009) find that the severity of bank runs depends on fundamentals (i.e., the state of the economy).

Identifying and estimating strategic complementarities is difficult because the strategic aspect must be disentangled from investor responses to common shocks. Chen et al. (2010) identify strategic complementarities in mutual fund investment by relying on the fact that illiquid funds (in our model, those with a higher fire-sale penalty λ) result in greater strategic complementarities among investors than do liquid funds; the reason is that redemptions impose higher costs on illiquid funds. Hertzberg et al. (2011) use a natural experiment with a credit registry expansion in Argentina to identify complementarities.

Consistently with Proposition 2(i), there is experimental evidence that bank runs occur less frequently when banks face less stress (in our model, a lower α) in the sense of a larger amount of withdrawals being required to induce insolvency (Madies 2006; Garratt and Keister 2009). There is also evidence of public information’s multiplier effect, Proposition 2(ii), from the credit registry expansion in Argentina (Hertzberg et al. 2011) and from the so-called discount window stigma (Armantier et al. 2011; Acharya and Merrouche 2012). The latter refers to the reluctance of banks to borrow from the discount window because of fear that the bad news will become publicly known. Armantier et al. (2011) report that, during the crisis, many banks borrowed from the Term Auction Facility (where the borrowing bank is just one of many) at higher rates than those available at the discount window; they also show that this rate spread was increasing in the stress that affected the interbank market. The spread indicates how much a bank is willing to pay to *avoid* the public release of a bad signal, and it is consistent with the positive

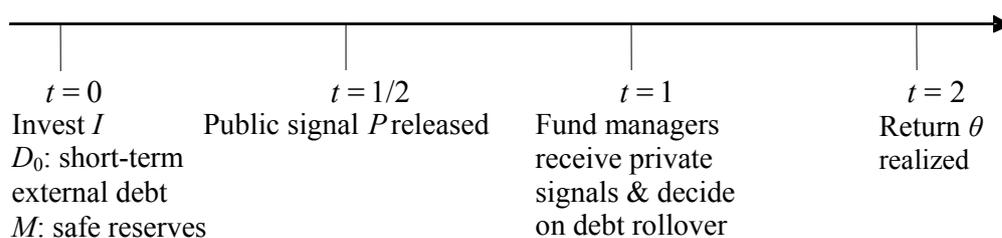
association (in the interbank market) between a higher publicity multiplier and a higher fire-sale penalty λ .

In accordance with these results, specializing Proposition 2 to the bank run model indicates that—among the 72 largest commercial banks in OECD countries—those that relied less on wholesale funding, and had higher capital cushions and liquidity ratios, fared better during the crisis in terms of enduring smaller declines in equity value and being less subject to government intervention; see Ratnovski and Huang (2009). Berger and Bouwman (2013) also find that capital improves the performance of banks during crisis periods. Finally, Fahlenbrach et al. (2012) document that banks with more short-term funding performed worse not only in the 2007 crisis but also in the 1998 crisis (when Russia defaulted on its debt).

5. Other applications

This section details three more illustrations of our approach: a sovereign debt crisis, a currency attack, and a credit market freeze.

Sovereign debt crisis. The bank run model of Section 1.2 can be reinterpreted as a sovereign debt run. So at $t = 0$, the country has some local resources E as well as external (public and private) short-term debt D_0 . The resources are invested in entrepreneurial projects I and foreign cash reserves M (understood to be in a strong currency). As before, investors in short-term debt can withdraw during the interim period at $t = 1$ and receive the face value D .



The public signal now can be interpreted as a credit rating. Under this interpretation, $m \equiv M/D$ is the reserves liquidity ratio, $\ell = D/E$ is the short-term external leverage ratio, and $d = D/D_0$ is the risk premium on short-term debt. The regions of insolvency and illiquidity identified in Section 1.4 still apply.

We can derive some prudential policy recommendations from these regulatory results. Namely, if the international regulator's objective is to reduce the risk of country insolvency and illiquidity, then constraints must be placed on short-term external leverage and the ratio of reserves. The solvency requirement should be higher for countries with a higher risk premium; however, if fire-sale penalties (say, on real estate assets) are higher then the liquidity requirement should be raised and the solvency requirement lowered. Of course, if both the risk premium and the fire-sale penalty are relatively high then probably both requirements should be raised. Prudential constraints must be adjusted in the presence of sovereign credit ratings with strengthened reserve requirements—provided that fire-sale penalties are high and that foreign investors are both conservative and not well informed.

Currency attacks. A streamlined version of the currency attacks model of Morris and Shin (1998) fits our general model. Here θ denotes central bank reserves, and $\theta \leq 0$ means that those reserves are depleted. Each speculator has one unit of resources to attack the currency ($y_i = 1$) at a cost C . Let $h(\theta; \alpha) = \alpha^{-1}\theta$, where $\alpha > 0$ is the mass of attackers and α^{-1} is the proportion of central bank reserves that are uncommitted; then the attack succeeds if $y > \alpha^{-1}\theta$. Alternatively, α could be interpreted as the wealth available to a fixed mass of speculators. See Figure 10(a). In the event of depreciation, the capital gain is fixed and equal to $\hat{B} = B + C$. We find that $\gamma \equiv C/(B + C)$ is likely to be small.

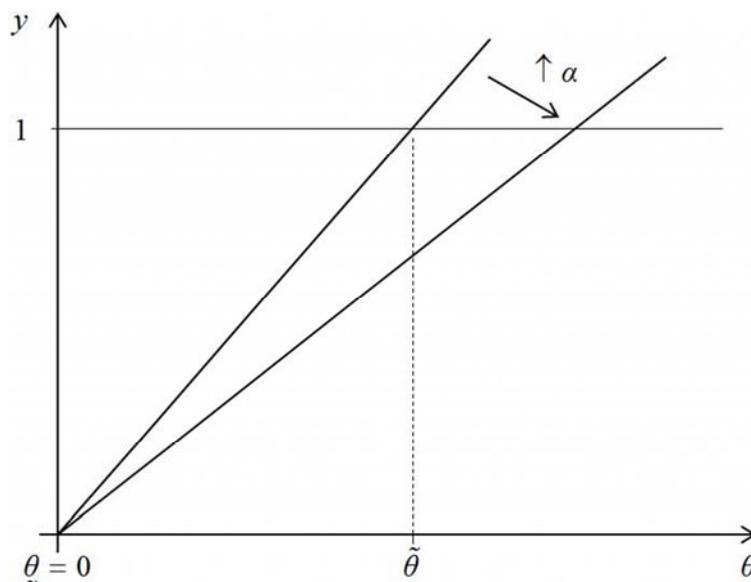


Figure 10(a). The resistance function $h(\theta; \alpha) = \alpha^{-1}\theta$ in the model of currency crises.

In the range $[\theta^*, \tilde{\theta})$, if currency speculators coordinated their attack then they would succeed; in fact, however, the currency holds. From Proposition 2(i) we have immediately that the likelihood of a currency crisis is decreasing in γ , the relative cost of the attack, and in μ_θ , the expected value of the central bank reserves. Also, the probability of illiquidity $\Pr(\underline{\theta} \leq \theta < \theta^*)$ is increasing in α whereas the probability of insolvency $\Pr(\theta < \underline{\theta})$ is unaffected by α .⁴⁸

Credit market freezes. Consider Bebchuk and Goldstein's (2011) model of self-fulfilling freezes in credit markets, which is a variation of the loan foreclosure model proposed by Morris and Shin (2004).⁴⁹ The bank's action is to foreclose on a loan made previously to a firm. In this case, θ denotes the fundamentals of the firms and y denotes the proportion of banks that do *not* renew credit to firms. Firms with good projects—those in which banks prefer to invest—exhibit returns above the risk-free rate only if $\theta \geq \underline{\theta}(\alpha) + \alpha^{-1}y$, where $\underline{\theta}(\alpha) = \tilde{\theta} - \alpha^{-1}$ with $\tilde{\theta} > 0$. Firms with bad projects return nothing, and banks can distinguish good projects from bad ones. Here the parameter α is interpreted as the inverse of the product of the mass of banks and a “complementarity” parameter that captures the performance of firms. We have that $h(\theta; \alpha) = \alpha(\theta - \underline{\theta}(\alpha)) = \alpha(\theta - \tilde{\theta}) + 1$ (see Figure 10(b)). Observe that in this case an increase in stress, $\partial h / \partial \alpha = -\tilde{\theta} < 0$, is associated with *reduced* strategic complementarity, $\partial h_1 / \partial \alpha > 0$. The parameter $1 - \gamma$ equals the ratio of the gross risk-free return to the gross return of good projects. The authors use their model to assess government responses in the present financial crisis. The comparative static results in Proposition 2 apply in this scenario as well. For example, an increase in the precision of a public signal about bad fundamentals (low μ_θ) will likewise increase the range of fundamentals within which a panic credit freeze may occur. An increase in

⁴⁸ If $h(\theta; \alpha) = \alpha^{-1}\theta$ then it is always the case that $\theta^* > \underline{\theta} = 0$ and $\Pr(\underline{\theta} \leq \theta < \theta^*)$ increases with α (since θ^* increases with α).

⁴⁹ The loan foreclosure model is formally equivalent to the currency attacks model. Here θ is the firm's ability to meet short-term claims (and $\theta \leq 0$ signifies no such ability). The action is to foreclose on a loan. In this case we have $h(\theta; \alpha) = \alpha^{-1}\theta$, where $\alpha > 0$ is the mass of creditors (and α^{-1} is the proportion of the firm's liquid resources that are not committed) and so the project fails if $y > \alpha^{-1}\theta$. The face value of the loan is L , and the value of collateral (at interim liquidation) is $K < L$. If we put $B = K$ and $C = L - K$, then $\gamma \equiv 1 - K/L$.

the mass of banks increases strategic complementarity and thus also increases the impact of public signals.⁵⁰

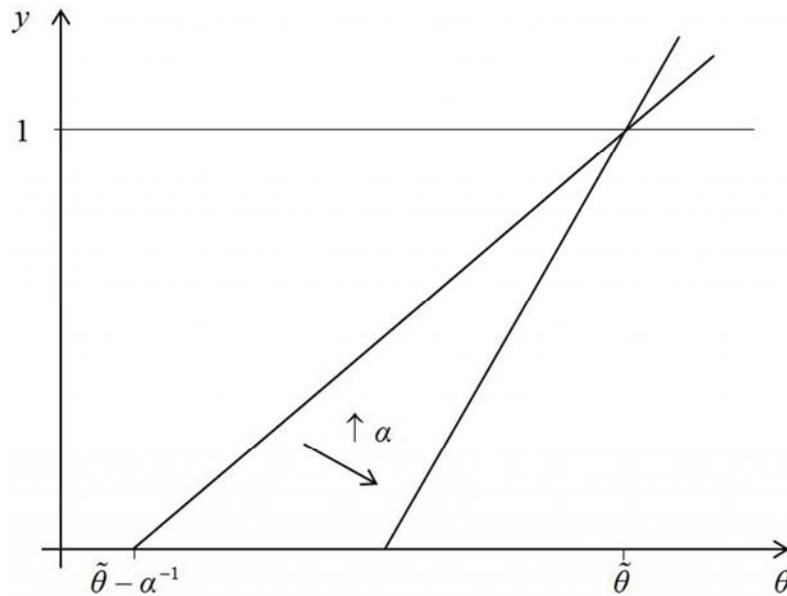


Figure 10(b). The resistance function $h(\theta; \alpha) = \alpha(\theta - \theta(\alpha))$ in the model of credit market freezes.

6. Concluding remarks

This paper presents a stylized model of a financial crisis that characterizes solvency and liquidity risk and highlights how extensive strategic complementarity among investors' actions can increase system fragility (understood as high sensitivity of equilibrium to small changes in parameters and the possibility of discrete jumps in the presence of multiple equilibria). Strategic complementarity increases with short-term leverage, greater competition for funds, higher fire-sale penalties, and public information that is more precise. If the asset side of a financial intermediary is relatively opaque, then a strong public signal (say, from a derivatives market) will increase both strategic complementarity and fragility. The analysis also characterizes how the probability of failure or crisis depends on balance sheet structure (leverage and liquidity), market stress parameters (extent of competition, fire-sale penalty for early liquidation of investments), and the informativeness of public and private signals.

⁵⁰ Gala and Volpin (2012) demonstrate how public information may reduce welfare since by correlating the investment decisions of agents and exacerbating the negative externality that one borrower imposes on others in the presence of credit rationing.

The main general policy conclusion on regulatory reform is that a piecemeal approach will not work. As it seeks to reduce the likelihood of insolvency or illiquidity, a regulator should pay attention to the balance sheet composition of financial intermediaries and to the prevailing level of disclosure. The regulator will need the option of imposing a leverage limitation and also (if fire-sale penalties are high and investors are conservative) the option of imposing a liquidity requirement. The solvency and liquidity requirements are partial substitutes, and both must be set while accounting for the level of transparency. So in an environment characterized by low market liquidity and conservative investors, the liquidity requirement should be tightened and the solvency requirement relaxed; note that prudential constraints may require modification under higher levels of disclosure (e.g., with stricter liquidity requirements and relaxed solvency requirements). Competition policy and prudential regulation are not independent: in a more competitive environment, leverage limits should be strengthened.⁵¹

The analysis presented here has several important limitations which have to be taken into account when considering policy prescriptions. First, both the intermediary's balance sheet and the regulator's objectives are exogenous. Each could be endogenized—for example, by introducing a moral hazard problem on the part of the intermediary which would rationalize the short-term debt structure and indicate an optimal closure policy for the regulator.⁵² Second, the analysis is basically static even though the aim is to capture dynamic phenomena.⁵³ Third, the investors are symmetric.⁵⁴ Finally, the analysis focuses on a single institution or a consolidated banking sector and takes market parameters (e.g., the fire-sale penalty) as given. This approach is therefore unable to account for either externalities among banks or contagion effects.⁵⁵ These issues are left for further research.

⁵¹ This theme is developed in Vives (2011) from a policy perspective.

⁵² For related approaches to the issue, see Calomiris and Kahn (1991), Diamond and Rajan (2000), Gale and Vives (2002), and Rochet and Vives (2004).

⁵³ See, for example, the dynamic analysis of panic debt runs in He and Xiong (2012) and in Cheng and Milbradt (2012).

⁵⁴ Sákovics and Steiner (2012) study a global game in which players are ex ante asymmetric.

⁵⁵ See Acharya and Yorulmazer (2007) and Farhi and Tirole (2012).

Appendix

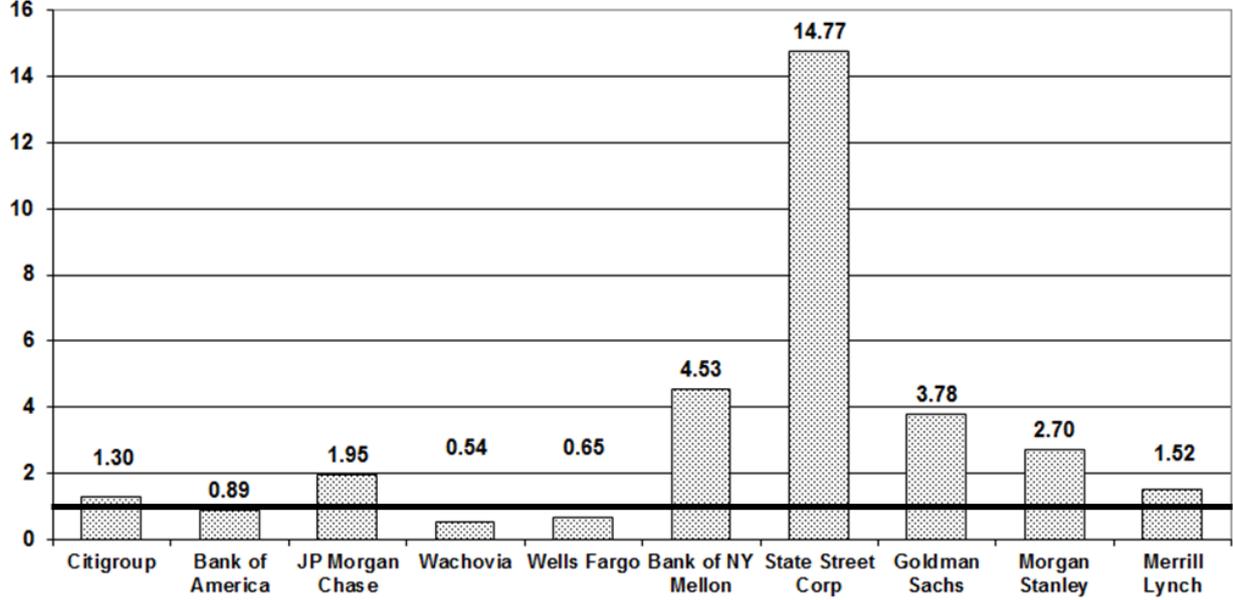


Figure A: Short-term leverage ratio of US banks, $\ell = D/E = (\text{Deposits (Uninsured)} + (\text{Short Term Debt} + \text{Other Liabilities})) / (\text{Equity} + \text{Long Term Debt} + \text{Deposits (Insured)})$. *Source:* Veronesi and Zingales (2010); data for Goldman Sachs and Morgan Stanley as of August 31, 2008.⁵⁶

Claim 1: Characterization of the best reply. *The best reply is given by*

$$r(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta_F(\hat{s}) - \frac{\tau_\theta}{\tau_\varepsilon} \mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon} \Phi^{-1}(\gamma). \text{ The maximal value of the slope is}$$

$$\bar{r}' \equiv \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon + h_1 \sqrt{2\pi\tau_\varepsilon}}, \text{ which is increasing in } h_1^{-1} \text{ and in } \tau_\theta \text{ and is first decreasing and then}$$

increasing in τ_ε ($\text{sign}\{\partial \bar{r}' / \partial \tau_\varepsilon\} = \text{sign}\{(1/2)h_1 \sqrt{2\pi\tau_\varepsilon} (1 - \tau_\theta \tau_\varepsilon^{-1}) - \tau_\theta\}$). If $(\tau_\theta / \sqrt{\tau_\varepsilon}) \leq h_1 \sqrt{2\pi}$

$$\text{then } r'(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta'_F(\hat{s}) \leq 1.$$

⁵⁶ Only State Street Corp. would end up exceeding $\ell = 10$ (with $\ell = 14.77$, which would yield $1 - \ell^{-1} - d^{-1} = 0$ if the interest rate were 7.26%). For Lehman Brothers, $\ell = 3.76$ at the end of 2007 (this figure derived from Adrian and Shin 2010).

Proof: From the equality $\Phi(\sqrt{\tau_\varepsilon}(\hat{s} - \theta)) = h(\theta)$ in the range $(\underline{\theta}, \tilde{\theta})$ we can solve for \hat{s} as a function of θ ; thus we obtain $s_F(\theta) = \theta + (1/\sqrt{\tau_\varepsilon})\Phi^{-1}(h(\theta))$ with derivative $s'_F = 1 + (1/\sqrt{\tau_\varepsilon})h'(\phi(\Phi^{-1}(h(\theta))))^{-1} > 0$, where ϕ is the density of the standard normal distribution. The threshold $\hat{\theta} = \theta_F(\hat{s})$ is obtained as the inverse of $s_F(\cdot)$ whenever $\hat{\theta} \in (\underline{\theta}, \tilde{\theta})$ and $\hat{\theta} = \underline{\theta}$ otherwise. Since ϕ is bounded above by $1/\sqrt{2\pi}$ and since h_1 is the smallest slope of $h(\cdot)$, it follows that s'_F is bounded below: $s'_F \geq 1 + \sqrt{2\pi/\tau_\varepsilon}h_1$. Hence $\theta'_F(\hat{s}) \leq (1 + \sqrt{2\pi/\tau_\varepsilon}h_1)^{-1}$ with strict inequality except possibly when $h(\theta) = 1/2$, because then $\Phi^{-1}(1/2) = 0$ and ϕ attains its maximum: $\phi(0) = 1/\sqrt{2\pi}$.

At the critical signal threshold \hat{s} , for a given failure threshold $\hat{\theta}$ the expected payoffs for acting and for not acting should be the same:

$$\begin{aligned} & E[\pi(1, y(\theta, s); \theta) - \pi(0, y(\theta, s); \theta) | s = \hat{s}] \\ &= \Pr(\theta < \hat{\theta} | \hat{s})B + \Pr(\theta \geq \hat{\theta} | \hat{s})(-C) = 0 \end{aligned}$$

or

$$\Pr(\theta < \hat{\theta} | \hat{s}) \equiv \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\hat{\theta} - \frac{\tau_\theta\mu_\theta + \tau_\varepsilon\hat{s}}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma,$$

where $\gamma \equiv C/(B+C) < 1$. It follows that the signal threshold curve is given by

$$s_T(\hat{\theta}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon}\hat{\theta} - \frac{\tau_\theta}{\tau_\varepsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon}\Phi^{-1}(\gamma),$$

and

$$r(\hat{s}) = s_T(\theta_F(\hat{s})) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon}\theta_F(\hat{s}) - \frac{\tau_\theta}{\tau_\varepsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon}\Phi^{-1}(\gamma).$$

Therefore, the maximal value of the slope is

$$\bar{r}' \equiv \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon + h_1\sqrt{2\pi\tau_\varepsilon}},$$

which is increasing in h_1^{-1} and in τ_θ and, with respect to τ_ε , is first decreasing and then increasing. Note that $\bar{r}' \leq 1$ if and only if $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$. We have that $\bar{r}' \rightarrow \infty$ as $\tau_\varepsilon \rightarrow 0$

and $\bar{r}' \uparrow 1$ as $\tau_\varepsilon \rightarrow \infty$. It is easily checked that $\text{sign}\{\partial\bar{r}'/\partial\tau_\varepsilon\} = \text{sign}\{(1/2)h_l\sqrt{2\pi\tau_\varepsilon}(1-\tau_\theta\tau_\varepsilon^{-1})-\tau_\theta\}$; in particular, $\partial\bar{r}'/\partial\tau_\varepsilon < 0$ for $\tau_\varepsilon < \tau_\theta$. ■

Proof of Proposition 1: (i) The game is “monotone supermodular” (i.e., a game of strategic complementarities with a monotone information structure) because $\pi(y_i, y; \theta)$ has increasing differences in $(y_i, (y, -\theta))$ —that is, the differential payoff to act ($\pi^1 - \pi^0$) is increasing in the aggregate action and in the negative of the state of the world $(y, -\theta)$ —and signals are affiliated. This means that extremal equilibria exist, are symmetric (because the game is symmetric), and are in monotone (decreasing) strategies in type (Van Zandt and Vives 2007). Since there are only two possible actions, the strategies at an extremal equilibrium must be of the threshold form: $y_i = 1$ if and only if $s_i < \hat{s}$, where \hat{s} is the threshold. It follows also that the extremal equilibrium thresholds, denoted \bar{s} and \underline{s} , bound the set of strategies that result from the iterated elimination of strictly dominated strategies. In sum, the extremal equilibria are in thresholds strategies and those bound any other possible equilibrium. If $\bar{s} = \underline{s}$ then the game is “dominance solvable” and the equilibrium is unique. Such an equilibrium is characterized by two thresholds: s^* denotes the signal threshold to act; and θ^* denotes the state-of-the-world critical threshold, below which the acting mass is successful and an acting player receives payoff $B - C > 0$. In equilibrium, the fraction of acting players $y(\theta^*, s^*) = \Pr(s < s^* | \theta^*) \equiv \Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*))$ must be no larger than the critical fraction above which it pays to act, $h(\theta^*)$. Note that $\theta^* \in [\underline{\theta}, \tilde{\theta})$ because $\lim_{\theta \uparrow \underline{\theta}} h(\theta) = 0$, $h(\cdot)$ is strictly increasing on $(\underline{\theta}, \tilde{\theta})$, and $h(\tilde{\theta}) = 1$. Furthermore, at the critical signal threshold, the expected payoffs for acting and for not acting should be the same:

$$\Pr(\theta < \theta^* | s^*) \equiv \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta\mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma,$$

where $\gamma \equiv C/(B + C) < 1$.

(ii) Consider an equilibrium (s^*, θ^*) . We show that if $\Phi(\sqrt{\tau_\varepsilon}(s^* - \underline{\theta})) \leq h(\underline{\theta})$ then $\theta^* = \underline{\theta}$. This will be so if $s_F(\underline{\theta}) \geq s_T(\underline{\theta})$ (see, for example, lower panel of Figure 4), where

$$s_F(\underline{\theta}) \equiv \underline{\theta} + \Phi^{-1}(h(\underline{\theta}))/\sqrt{\tau_\varepsilon} \quad \text{and} \quad s_T(\underline{\theta}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon}\underline{\theta} - \frac{\tau_\theta}{\tau_\varepsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon}\Phi^{-1}(\gamma). \quad \text{It is}$$

immediate then that $s_F(\underline{\theta}) \geq s_T(\underline{\theta})$ if $h(\underline{\theta}) \geq \bar{h}_0(\underline{\theta})$, where

$$\bar{h}_0(\underline{\theta}) \equiv \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\underline{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}}\Phi^{-1}(\gamma)\right) > 0.$$

For $h(\underline{\theta}) < \bar{h}_0(\underline{\theta})$ we have that $\theta^* > \underline{\theta}$ at any equilibrium; in that case, $\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) = h(\theta^*)$ and $\theta^* > \underline{\theta}$ (here $s_F(\underline{\theta}) < s_T(\underline{\theta})$). In summary, there is a critical $\bar{h}_0(\underline{\theta}) \in (0,1)$ such that for $h(\underline{\theta}) < \bar{h}_0(\underline{\theta})$ we have $\theta^* > \underline{\theta}$ and for $h(\underline{\theta}) \geq \bar{h}_0(\underline{\theta})$ there is always one equilibrium with $\theta^* = \underline{\theta}$.

(iii) From Claim 1 we have that if $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$ then $r'(\hat{s}) \leq 1$ and the equilibrium is unique. In this case, the game is dominance solvable because $\bar{s} = \underline{s}$. Furthermore, it should be clear that the critical thresholds θ^* and s^* move together.

Let $h(\underline{\theta}) < \bar{h}_0(\underline{\theta})$. Then we can combine

$$\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) = h(\theta^*) \quad (1)$$

and

$$\Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta\mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma \quad (2)$$

to obtain

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\varepsilon}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\varepsilon}\Phi^{-1}(\gamma) = 0 \quad (3)$$

by substituting the value of s^* from (1) into (2). Equation (3) may have multiple solutions in θ^* . As $\theta \rightarrow \tilde{\theta}$ we have that $\Phi^{-1}(h(\theta)) \rightarrow +\infty$ and $\varphi \rightarrow -\infty$; as $\theta \rightarrow \underline{\theta}$ we have that $h(\theta) \rightarrow h(\underline{\theta})$ and $\varphi \rightarrow \varphi(\underline{\theta}) > 0$ whenever $h(\underline{\theta}) < \bar{h}_0$. Hence there is at least one solution $\theta^* \in [\underline{\theta}, \tilde{\theta})$. That solution will be unique if $\varphi' < 0$; if $\varphi'(\theta) > 0$ for a potential solution $\varphi(\theta) = 0$ then there will be multiple solutions (three when $h(\cdot)$ is linear or two when there is a θ such that $\varphi(\theta) = 0$ and $\varphi'(\theta) = 0$). As γ tends to 0 (resp., 1) we have that $\Phi^{-1}(\gamma)$ tends to $-\infty$ (resp., $+\infty$), s^* tends to $+\infty$ (resp., $-\infty$), and θ^* tends to $\tilde{\theta}$ (resp., $\underline{\theta}$). These claims follow from equation (2) and because $\theta^* \in [\underline{\theta}, \tilde{\theta})$. There is indeed a unique solution if $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$. (We have that $\varphi'(\theta) = \tau_\theta - \sqrt{\tau_\varepsilon}h'(\theta)/\phi(\Phi^{-1}(h(\theta)))$ and φ' is bounded

above: $\varphi' \leq \tau_\theta - h_1\sqrt{2\pi\tau_\varepsilon}$ with strict inequality except possibly when $h(\theta) = 1/2$. Therefore, if $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$ then $\varphi' \leq 0$.) ■

Proof of Proposition 2: If $h(\varrho) < \bar{h}_0$, then θ^* is determined by

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\varepsilon}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\varepsilon}\Phi^{-1}(\gamma) = 0.$$

We obtain the results by looking at how changes in the parameters affect $\varphi(\cdot)$. When $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$, we have that $\varphi' < 0$ and there is a unique equilibrium. The usual comparative static analysis applies. When $\tau_\theta/\sqrt{\tau_\varepsilon} > h_1\sqrt{2\pi}$ there may be multiple equilibria, in which case our results will apply to the extremal ones. With adaptive dynamics the results apply in general.

(i) The result for θ^* follows because φ is decreasing in both γ and μ_θ and is increasing in α (since $\partial h/\partial \alpha < 0$). The threshold s^* moves with θ^* .⁵⁷ The result for $\Pr(\theta < \theta^*)$ is immediate for γ and α , and also for μ_θ given that increases in μ_θ shift the distribution of θ rightward.

(ii) Suppose the equilibrium is unique. The equilibrium signal threshold is determined by $r(s^*; \mu_\theta) - s^* = 0$. From this it follows that, for a marginal change in μ_θ ,

$$\left| \frac{ds^*}{d\mu_\theta} \right| = \frac{|\partial r/\partial \mu_\theta|}{1 - r'} > \left| \frac{\partial r}{\partial \mu_\theta} \right|$$

whenever $0 < r' < 1$. In consequence, an increase in μ_θ will have a larger effect on the equilibrium threshold s^* than does the direct impact on a player's best response $\partial r/\partial \mu_\theta = -\tau_\theta/\tau_\varepsilon$. The same is true for discrete changes—even with multiple equilibria, provided we restrict our attention to extremal equilibria or allow for adaptive dynamics. The multiplier effect is greatest when r' is close to 1 at equilibrium (i.e., when strategic complementarities are strong) and we approach the region of multiple equilibria (and this happens when τ_θ is large because r' is increasing in τ_θ).

⁵⁷ The result for μ_θ follows also from a general argument in monotone supermodular games. We know that extremal equilibria of monotone supermodular games are increasing in the posteriors of the players (Van Zandt and Vives 2007). A sufficient statistic for the posterior of a player under normality is the conditional expectation $E[\theta | s] = (\tau_\theta\mu_\theta + \tau_\varepsilon s)/(\tau_\theta + \tau_\varepsilon)$, which is increasing in μ_θ . Thus it follows that extremal equilibrium thresholds $(-\theta^*, -s^*)$ are increasing in μ_θ .

(iii) First consider changes in τ_θ . We have the equality $\partial\varphi/\partial\tau_\theta = \theta^* - \mu_\theta - (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)/2$ and therefore $\partial\varphi/\partial\tau_\theta > 0$ when $\underline{\theta} \geq \mu_\theta$, since $\theta^* > \underline{\theta} \geq \mu_\theta$ and $\Phi^{-1}(\gamma) < 0$ for $\gamma < 1/2$. Note that, for $\theta^* > \mu_\theta$, if τ_θ increases then $\Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$ also increases. Consider now changes in τ_ε . With our current assumptions and using the equality $\varphi(\theta^*) = 0$ in $\partial\varphi/\partial\tau_\varepsilon$, we obtain $\partial\varphi/\partial\tau_\varepsilon = -(\theta^* - \mu_\theta - (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma))\tau_\theta/2\tau_\varepsilon$ and $\partial\varphi/\partial\tau_\varepsilon < 0$. The results follow. If $\mu_\theta \geq \tilde{\theta} - (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)$ then $\theta^* < \mu_\theta$ and the results are reversed because $\theta^* < \tilde{\theta} \leq \mu_\theta$ (since $\Phi^{-1}(\gamma) < 0$, we have $-\Phi^{-1}(\gamma) > -\Phi^{-1}(\gamma)/2$). ■

Proof of Remark 3: Let $\theta^*(\mu_\theta)$ be the unique equilibrium. We have that $\hat{\mu}_\theta$ is the unique solution in μ_θ to $\partial\varphi/\partial\tau_\theta(\theta^*(\mu_\theta), \mu_\theta) = 0$ or to $\theta^*(\mu_\theta) = \mu_\theta + (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)/2$ and that $\hat{\mu}_\theta$ is the unique solution in μ_θ to $\partial\varphi/\partial\tau_\varepsilon(\theta^*(\mu_\theta), \mu_\theta) = 0$ or to $\theta^*(\mu_\theta) = \mu_\theta + (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)$. The solutions are unique since $\theta^*(\mu_\theta)$ is decreasing in μ_θ . This implies also that $\hat{\mu}_\theta < \tilde{\mu}_\theta$ since $\Phi^{-1}(\gamma) < 0$ because $\gamma < 1/2$. It follows that there are thresholds $\hat{\mu}_\theta$ and $\tilde{\mu}_\theta$ with $\hat{\mu}_\theta < \tilde{\mu}_\theta$ such that (a) θ^* increases with τ_θ if and only if $\mu_\theta < \hat{\mu}_\theta$ and (b) θ^* increases with τ_ε if and only if $\mu_\theta > \tilde{\mu}_\theta$. ■

Claim 2: Comparative static results in the bank run model. *There is a liquidity ratio $\bar{m} \in (0,1)$ such that when $m \geq \bar{m}$ there is always an equilibrium in which $\theta^* = \underline{\theta}$. For $m < \bar{m}$, $\theta^* > \underline{\theta}$ at any equilibrium and if $1 - \ell^{-1} - d^{-1} < 0$,*

- $\Pr(\theta < \underline{\theta})$ is decreasing in m and in μ_θ , increasing in ℓ and d , and independent of λ and γ .
- *At extremal equilibria or under adaptive dynamics,*
 - $\Pr(\theta < \theta^*)$, the critical θ^* , and $\theta^* - \underline{\theta}$ are all decreasing in m , γ , and μ_θ but increasing in λ , ℓ , and d ;
 - $\Pr(\underline{\theta} \leq \theta < \theta^*)$ is decreasing in γ and increasing in λ and ℓ .

Proof: From the proof of Proposition 1 it follows that the critical ratio \bar{m} satisfies

$m = \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\underline{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}\Phi^{-1}(\gamma)}\right)$, where $\underline{\theta} \equiv (1-m)/(\ell^{-1} + d^{-1} - m)$. Note that the right-hand side is decreasing in m if $\partial\underline{\theta}/\partial m < 0$, which obtains according to our prevailing assumption $1 - \ell^{-1} - d^{-1} < 0$. We have also that $\partial\underline{\theta}/\partial d > 0$, $\partial\underline{\theta}/\partial \ell > 0$, and $\underline{\theta}$ is independent of λ and γ . Note also that \bar{m} depends on the parameters of the model and in particular it increases with ℓ since $\partial\underline{\theta}/\partial \ell > 0$. Moreover, $\partial h/\partial m > 0$ (under the maintained assumption), $\partial h/\partial \ell^{-1} > 0$, and $\partial h/\partial d^{-1} > 0$. The other results follow from Proposition 2 and Remark 1 or by direct inspection of the equilibrium condition $\varphi(\theta) = 0$. ■

Proof of Proposition 3: (i) Given that $\Pr(\theta < \underline{\theta}) = \Phi(\sqrt{\tau_\theta}(\underline{\theta} - \mu_\theta))$, the solvency constraint S (namely, $\Pr(\theta < \underline{\theta}) \leq q$) holds if and only if $\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta}$, where $\underline{\theta} \equiv (1-m)/(\ell^{-1} + d^{-1} - m)$. Constraint S then follows immediately yielding $\ell^{-1} \geq (\underline{\theta}_q)^{-1} - d^{-1} - ((\underline{\theta}_q)^{-1} - 1)m$. Note that S is independent of γ and λ . Let θ^* be the critical threshold at the unique equilibrium; $\Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$ and, therefore, $\Pr(\theta < \theta^*) \leq p$ if and only if $\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta}$. Suppose that the constraint is binding. The constraint L follows from this inequality once (a) we note that $h(\theta^*) = h(\theta_p^*) = m + \frac{\ell^{-1} + d^{-1} - m}{\lambda} \theta_p^* - \frac{1-m}{\lambda}$ at the boundary $\theta^* = \theta_p^*$ and (b) we trade off ℓ^{-1} and m in a linear way to keep $h(\theta_p^*)$ constant at level k . Then constraint L is linear:

$$\ell^{-1} \geq \frac{1+k\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1\right)m; \text{ here, since } \varphi(\theta_p^*) = 0, \text{ we obtain}$$

$$k = h(\theta_p^*) = \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\theta_p^* - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}\Phi^{-1}(\gamma)}\right) = \Phi\left(\sqrt{\frac{\tau_\theta}{\tau_\varepsilon}}\Phi^{-1}(p) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}\Phi^{-1}(\gamma)}\right)$$

because $\theta_p^* - \mu_\theta = \Phi^{-1}(p)/\sqrt{\tau_\theta}$. Note that $k = 1 - \gamma$ when $\tau_\varepsilon \rightarrow \infty$. ■

Claim 3: We have that $k \in (0,1)$ is increasing in p , is decreasing in γ , and has an ambiguous dependence on $\tau_\theta/\tau_\varepsilon$:

1. If $1/2 > p > \gamma$ and if τ_ε is small, then k is increasing in $\tau_\theta/\tau_\varepsilon$, and $k \rightarrow 1$ as $\tau_\theta/\tau_\varepsilon \rightarrow \infty$.

2. If $1/2 > \gamma \geq p$ then k is decreasing in $\tau_\theta/\tau_\varepsilon$ and if, furthermore, $\gamma > p$ then $k \rightarrow 0$ as $\tau_\theta/\tau_\varepsilon \rightarrow \infty$.

Proof: The comparative statics with respect to p and γ follow immediately from the expression for k and the monotonicity of $\Phi(\cdot)$. Implication (1) follows because the expression

$$\text{sgn}\left\{\frac{\partial k}{\partial(\tau_\theta/\tau_\varepsilon)}\right\} = \text{sgn}\left\{\left(\frac{\tau_\theta}{\tau_\varepsilon}\right)^{-1/2}\Phi^{-1}(p) - \left(1 + \frac{\tau_\theta}{\tau_\varepsilon}\right)^{-1/2}\Phi^{-1}(\gamma)\right\}$$

is positive when $1/2 > p > \gamma$ (which implies that $-\Phi^{-1}(\gamma) > -\Phi^{-1}(p) > 0$) and τ_ε is small, since

$$\left(\frac{\tau_\theta}{\tau_\varepsilon}\right)^{-1/2} / \left(1 + \frac{\tau_\theta}{\tau_\varepsilon}\right)^{-1/2} = \sqrt{1 + \frac{\tau_\varepsilon}{\tau_\theta}}.$$

Furthermore then $k \rightarrow 1$ as $\tau_\theta/\tau_\varepsilon \rightarrow \infty$ from the expression for k since $\Phi^{-1}(\gamma) < \Phi^{-1}(p) < 0$. Implication (2) follows also from the expressions for $\text{sgn}\left\{\frac{\partial k}{\partial(\tau_\theta/\tau_\varepsilon)}\right\}$ and k . ■

(ii) Let $0 < \underline{\theta}_q < 1$. Then constraint S and constraint L are both downward sloping and constraint L always has a larger slope (in absolute value) than constraint S. This is because, in that case, $(\underline{\theta}_q)^{-1} - d^{-1} > (\underline{\theta}_q)^{-1} - 1 > 0$ and $(1 + \lambda)(\theta_p^*)^{-1} > (\underline{\theta}_q)^{-1} > 1$ since $(1 + \lambda)\underline{\theta}_q > \theta_p^*$ given that $\theta^* \in [\underline{\theta}_q, (1 + \lambda)\underline{\theta}_q]$. Furthermore, if $\lambda k > (\theta_p^*/\underline{\theta}_q) - 1$ then $(1 + k\lambda)/\theta_p^* - (\underline{\theta}_q)^{-1} > 0$, and so L intersects S from above (as in Figure 6).

Constraints S and L both become tighter when d increases. Constraint L becomes tighter if λ increases (since $k - m > 0$ for interior solutions for m) or if γ decreases (since k is decreasing in γ). Both constraints are more relaxed with higher μ_θ : S is looser because $m < 1$ and $\underline{\theta}_q$ is increasing in μ_θ ; and L is looser because $\partial k/\partial\mu_\theta = 0$, $m < 1$, $k - m > 0$, and θ_p^* is increasing in μ_θ . If $p > q$, then $\theta_p^*/\underline{\theta}_q$ is decreasing in μ_θ and so constraint L is relaxed less than is constraint S. A higher τ_θ increases both $\underline{\theta}_q$ and θ_p^* provided that $1/2 > p$, since then $\Phi^{-1}(q) < \Phi^{-1}(p) < 0$. The implication is that increases in τ_θ relax S but may have an ambiguous impact on L (when k is increasing in τ_θ ; if k decreases with τ_θ then L is also relaxed).

The minimal $(\hat{m}, \hat{\ell}^{-1})$ ratios are given by the intersection of the boundaries of the solvency constraint

$$\ell^{-1} = (\varrho_q)^{-1} - d^{-1} - \left((\varrho_q)^{-1} - 1 \right) m$$

and the liquidity constraint

$$\ell^{-1} = \frac{1+k\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m;$$

note that $\frac{1+\lambda}{\theta_p^*} - (\varrho_q)^{-1} > \frac{1+k\lambda}{\theta_p^*} - (\varrho_q)^{-1} > 0$ if $\lambda k > \left(\frac{\theta_p^*}{\varrho_q} - 1 \right)$. For positive solutions we obtain

$$\hat{m} = \frac{1+k\lambda - \theta_p^*/\varrho_q}{1+\lambda - \theta_p^*/\varrho_q} = 1 - \frac{1-k}{1-\lambda^{-1} \left((\theta_p^*/\varrho_q) - 1 \right)} \text{ and}$$

$$\hat{\ell}^{-1} = (\varrho_q)^{-1} - d^{-1} - \left((\varrho_q)^{-1} - 1 \right) \hat{m}.$$

It follows that

$$\hat{m} = \max \left\{ 1 - (1-k) \left(1 - \lambda^{-1} \left(\frac{\theta_p^*}{\varrho_q} - 1 \right) \right)^{-1}, 0 \right\}.$$

(iii) Observe that, if $p = q$ then $\hat{m} = k$, and for $p > q$, if $\lambda k > (\theta_p^*/\varrho_q) - 1$ then $k > \hat{m} > 0$. The comparative statics of the regulatory ratios given in Table 2 then follow: $\partial \hat{m} / \partial \lambda > 0$, $\partial \hat{m} / \partial \gamma < 0$, $\partial \hat{m} / \partial d = 0$, $\partial \hat{\ell}^{-1} / \partial \lambda < 0$, $\partial \hat{\ell}^{-1} / \partial \gamma > 0$, and $\partial \hat{\ell}^{-1} / \partial d > 0$. We have also that $\partial \hat{m} / \partial \mu_\theta > 0$ (since k is independent of μ_θ), $\partial(\theta_p^*/\varrho_q) / \partial \mu_\theta < 0$ (for $p > q$), and \hat{m} is decreasing in θ_p^*/ϱ_q . Correspondingly, and since $1 - \hat{m} > 0$, from $\partial(\varrho_q)^{-1} / \partial \mu_\theta < 0$ and $\partial \hat{m} / \partial \mu_\theta > 0$ we obtain that $\partial \hat{\ell}^{-1} / \partial \mu_\theta < 0$. The same results hold for increasing τ_θ ($\partial \hat{m} / \partial \tau_\theta > 0$ and $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$) as $\tau_\varepsilon \rightarrow \infty$ (in which case $k = 1 - \gamma$), since $\partial(\theta_p^*/\varrho_q) / \partial \tau_\theta < 0$ and $\partial(\varrho_q)^{-1} / \partial \tau_\theta < 0$ when $q < 1/2$. The same logic applies when $p > \gamma$ and τ_ε is small; then k increases with τ_θ and $\partial \hat{m} / \partial \tau_\theta > 0$. Hence it follows that $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$ because

$$\partial \hat{\ell}^{-1} / \partial \tau_\theta = - \left((\varrho_q)^{-1} - 1 \right) \partial \hat{m} / \partial \tau_\theta + (1 - \hat{m}) \partial (\varrho_q)^{-1} / \partial \tau_\theta < 0$$

since $\partial(\varrho_q)^{-1} / \partial \tau_\theta < 0$ for $1/2 > q$. ■

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