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# Private information, strategic behavior, and efficiency in Cournot markets

**Xavier Vives***

*When analyzing a Cournot market with strictly convex costs dependent on a private information parameter, do we err more by ignoring market power or by ignoring the impact of incomplete information? Is the welfare loss at the market outcome driven by private information or by market power? The answer to both questions is that in large enough markets, abstracting from market power provides a much better approximation than abstracting from private information. Let* n *index the size of the market and the (free entry) number of firms. Then the effect of market power (private information) is of the order of* $1/n$ $(1/\sqrt{n})$ *for prices and* $1/n^2$ $(1/n)$ *for per-capita deadweight loss.*

## 1. Introduction

■ Consider a Cournot market in which firms have both private information and market power. When is the modeller more likely to go wrong, by ignoring the impact of incomplete information or by ignoring strategic behavior? What are the relative weights of private information and market power in accounting for the welfare losses at the market outcome?

These closely related issues are of relevance for applied work, and from a positive viewpoint, because of the tendency to simplify when modelling situations with private information and potential strategic interaction. For example, many models in Industrial Organization abstract from private information and concentrate on strategic behavior while assuming players have full information about the environment. Conversely, many models of financial markets take into account the private information of traders but assume competitive (price-taking) behavior. Given that both private-information and market-power issues are present in most markets in actuality, one may wonder whether these simplifications are warranted. Furthermore, from a normative point of view the source of the welfare loss at the market allocation matters in terms of policy design. Indeed, if the weight of the welfare loss is on market power, then classical competition policy is called for, whereas if the weight is on information, then information-revelation and sharing issues are at center stage. Finally, from an empirical point of view the answer to the questions posed may help design ways to assess the total welfare loss (Harberger's triangle) at the market outcome.

Concentrating attention on the Cournot market mechanism seems justified because of its wide use in applied work. Indeed, the Cournot model is arguably the most commonly used

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theoretical construct to introduce imperfect competition in a market or an economy. This is certainly so in Industrial Organization analysis; it also has widespread use in, for example, international trade, macroeconomics, and public economics.[1] The Cournot outcome emerges in a range of circumstances as an upper bound to the exercise of static market power. For example, it is the least competitive of possible supply function equilibria (see Klemperer and Meyer (1989) and Vives (1999)). The relevance of private cost information is highlighted by the activities of trade associations and the increasingly popular practice of "benchmarking." Both tend to disseminate information about the costs of firms. For example, Armantier and Richard (2000) argue the importance of incomplete operating cost information in the airline industry. Those authors estimate a structural Bayesian Cournot equilibrium modelling the duopoly competition of American Airlines with United Airlines at Chicago O'Hare, and they examine the welfare implications of the exchange of cost information.

I take as the base model a Cournot market with independent values, corresponding to idiosyncratic cost parameters of the firms. As the market grows large, the deadweight loss tends to zero, and the effects of both market power and private information vanish. The aim is to disentangle the relative contribution of market power and private information when accounting for the rate at which welfare losses—arising in private-information economies with potentially strategic traders—vanish as the economy grows large. The examination of the rate at which the market outcome approaches the competitive outcome is relevant because convergence per se is not of practical use if the rate is very slow.[2] The rate analysis will be complemented with simulations to ascertain for what market size the rate results start to obtain and to check what happens in concentrated markets.

The base model is a free entry linear-quadratic Cournot market in which the intercept of the (increasing) marginal production cost of an active firm is subject to an idiosyncratic shock. There is a positive entry cost. Shocks are i.i.d. and are private information to the firms. The market outcome is taken to be the Bayesian Cournot-Nash equilibrium of the active firms. The size of the market is parameterized by the number of consumers $m$. It is possible to show that the order of magnitude of the free-entry number of firms $n$ is the same as that of consumers. That is, the ratio of consumers to firms is bounded away from zero and infinity for any market size. I am interested in large markets and, for simplicity of exposition, shall index the market size and the number of firms by $n$. As $n$ increases, the ($n$-replica Cournot) market grows large and firms become small in relation to the market.[3]

Four regimes are considered according to whether firms are strategic or nonstrategic (price-takers) and according to whether there is private or full information about costs. This setup allows me to disentangle market-power effects from private-information effects from both a positive and a normative point of view. The strategic private-information regime corresponds to the Bayesian Cournot-Nash equilibrium, the market outcome.

The central result in my linear-quadratic model is the following. I find that the price distortion induced by market power, keeping constant the information regime (full or private), vanishes at the rate of $1/n$ and the corresponding inefficiency at the rate of $1/n^2$. However, the price distortion induced by private information, keeping constant market power (Cournot Nash or price taking), vanishes more slowly, at the rate of $1/\sqrt{n}$, with the associated efficiency loss fading at the rate of $1/n$. The consequence is that the wedge between the market outcome (strategic firms with private information) and first-best efficiency (price taking with full information) is of the order of $1/\sqrt{n}$ for prices and $1/n$ for (per-capita) deadweight loss and is driven by private information and not by market power. In other words, increasing $n$ is more effective in reducing the welfare loss due to market power than the loss due to private information.

[1] It should be pointed out that even if firms compete in prices, the Cournot model can be seen as a reduced form for more complex competition modes (for example, in capacities and prices as argued by Kreps and Scheinkman (1983); see Vives (1999) for a discussion of related models).

[2] A notable exception is the work by Yosha (1997), which incorporates uncertainty but not private information and examines the tradeoff that the tension between diversification and competition in a Cournot-Walras economy with financial intermediaries imposes on the rate of convergence to the limit competitive outcome.

[3] This, indeed, ensures that there is a well-defined limit market as $n$ tends to infinity.

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The effect of market power thus fades away quickly without uncertainty, where the Cournot price tends to marginal cost at the rate of $1/n$ and the deadweight loss decreases at the rate of $1/n^2$. Productive inefficiency is of the same order.[4] The result also holds under private information because the same forces are at work. In both cases (with full information and with private information), market power affects the deadweight loss by decreasing expected output and by making firms too insensitive to their costs. The result is that both allocative and productive inefficiency are impaired (and by the same order of magnitude). When there is no market power difference, expected prices are the same independent of the information regime. Differences in information are of a different order because they are driven by variance terms. This is so because in a private information regime, a firm has to estimate the sample mean of cost parameters (the average cost of active firms). It follows then that information is aggregated at the rate associated to the law of large numbers, as the sample mean of i.i.d. cost parameters converges to its population mean, and this rate is $1/\sqrt{n}$.

Simulations with the model show that there is a critical $\bar{n}$, below 10 for a wide range of parameter values and significant uncertainty, such that the effect of private information dominates the effect of market power if and only if $n > \bar{n}$. This critical $\bar{n}$ is decreasing in the prior variability of the cost parameters.

The results have positive and normative implications for Cournot markets with increasing marginal costs. On the positive side, for large enough markets to approximate the market outcome, forgetting market power turns out to be much better than forgetting the effect of private information. On the normative side, the results suggest that in large enough Cournot markets, private information is a more important source of deadweight loss than market power. The analysis also provides a potential explanation for the low estimates typically obtained in the approximation of Harberger's triangles.

The plan of the rest of the article is as follows. Section 2 presents the free-entry Cournot market with private information. Section 3 characterizes equilibria in the four regimes considered. Section 4 shows that the rate of convergence of prices to marginal costs is driven by the effect of private information and not market power (Proposition 2). Section 5 states a welfare property of the Bayesian price-taking mechanism, decomposes the deadweight loss at the market outcome into private information and market power terms, and presents the welfare counterpart of Proposition 2 (Proposition 4). Section 6 develops implications for competition policy, and Section 7 deals with some extensions. Proofs are collected in the Appendix.

## 2. An independent-values Cournot market with free entry

■ Consider a market for a homogeneous product with $m$ consumers, each with quasi-linear preferences and maximizing the net benefit function $U(x) - px$ with $U(x) = \alpha x - \beta x^2/2$, where $\alpha > 0$ and $\beta > 0$ and $x$ is the consumption level. This gives rise to the inverse demand given by $P_m(X) = \alpha - \beta_m X$, where $X$ is total output and $\beta_m = \beta/m$. The parameter $m$ measures the size of the market.

There are potentially many firms that may enter the market. If firm $i$ enters, it produces according to a quadratic cost function $C(x_i; \theta_i) = \theta_i x_i + (\lambda/2)x_i^2$, where $\theta_i$ is a random parameter and $\lambda > 0$. The $\theta_i$'s parameters are i.i.d. with finite mean $\bar{\theta}$ and variance $\sigma_\theta^2$ (and this is common knowledge as well as all other parameters in the model).

Entry is modelled as a two-stage game. At the first stage, firms decide whether to enter the market or not. If a firm decides to enter, it pays a fixed cost $F > 0$. At the second stage, each active firm $i$, upon observing a cost realization $\theta_i$, sets an output level.[5]

Given that $n$ firms have entered, a Bayesian Cournot-Nash equilibrium (BNE) obtains.

[4] See the results in Dasgupta and Ushio (1981) and Ushio (1983), allowing for decreasing average costs; in Guesnerie and Hart (1985), allowing for U-shaped costs, all in homogeneous-product models; and Vives (1999) for differentiated-products models.

[5] Mankiw and Whinston (1986) study whether free-entry equilibria with no uncertainty entail too many firms from a welfare point of view. I will not pursue this line of enquiry in the presence of uncertainty and private information.

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Suppose that parameters are such that for any cost realization, a firm wants to produce. (See the Appendix for parameter conditions.) Given our assumptions, for any $n$ there is a unique BNE. This equilibrium is symmetric and linear, with firm $i$ producing according to $x_n(\theta_i) = b_n(\alpha - \bar{\theta}) - a_n(\theta_i - \bar{\theta})$, with $a_n = (\lambda + 2\beta_m)^{-1}$, $b_n = (\lambda + \beta_m(1+n))^{-1}$. This yields expected profits $\mathrm{E}\pi_n = (\beta_m + \lambda/2)\, \mathrm{E}(x_n(\theta_i))^2 = (b_n)^2(\alpha - \bar{\theta})^2 + (a_n)^2\sigma_\theta^2$. A free-entry equilibrium is then a subgame-perfect equilibrium of the two-stage game. Given a market of size $m$, the free-entry number of firms $n^*(m)$ is approximated by the solution to $\mathrm{E}\pi_n = F$ (provided $F$ is not so large that no entry is profitable). It can be checked that $n^*(m)$ is of the same order as $m$. This means that the ratio of consumers to firms is bounded away from zero and infinity for any market size.[6] For example, we could have $n^*(m)$ approximately equal to $m/k$, with $k$ positive and typically large.

For simplicity of exposition in the rest of the article, I index market size and the number of firms by $n$ and consider $n$ replica Cournot markets. We can think of consumer groups of size $k$ and have the same number of firms and consumer groups. As $n$ increases, the market grows large and firms become small in relation to the market. It should be understood, however, that the basic exogenous parameter is market size, with the number of firms adjusting with free entry.

## 3. Regimes and benchmarks

■ Consider thus an $n$ replica market. I denote the average of a variable by a tilde. For example, the average or per-capita output is $\tilde{x}_n = X/n$. The profit of firm $i$ is therefore given by $\pi_i = (\alpha - \theta_i - \beta\tilde{x}_n)x_i - (\lambda/2)x_i^2$, total surplus by $TS = nU(X/n) - \sum_i C(x_i;\theta_i)$, and per-capita surplus by $TS/n = U(X/n) - (\sum_i C(x_i;\theta_i))/n$. Without loss of generality and for most of the rest of the article, I let $\beta = 1$.

I shall consider four possible regimes according to whether firms are strategic or not and according to whether cost information is private or public (full information). The two strategic regimes are the Bayesian Cournot-Nash equilibrium of the private-information game and the full-information Cournot equilibrium. The Bayesian Cournot-Nash equilibrium is taken to be the market outcome; the other regimes are benchmarks for the analysis. The two nonstrategic regimes are the Bayesian price-taking equilibrium (with private information) and the standard competitive equilibrium (with full information). Those regimes are obviously not realistic, since despite there being a finite number of firms, they do not realize that they have market power. For example, at the Bayesian price-taking equilibrium, firms do not take into account any influence of their actions on the price ("price taking"), which they have to estimate with their private information ("Bayesian").[7]

Variables in the nonstrategic regimes are denoted by the superscript $c$ ("competitive"); variables in the full-information regimes are denoted by the superscript $f$ ("full information"). Table 1 presents the different regimes and the corresponding notation for strategies. At private-information equilibria, the strategy of firm $i$ depends only on its type $\theta_i$. At full-information equilibria, it will depend on the realization of all types, which given the structure of the model can be summarized by $(\theta_i, \tilde{\theta}_n)$, where $\tilde{\theta}_n = (1/n)\sum_{j=1}^n \theta_j$.

It is easy to check that there exists a unique (and symmetric) equilibrium in each regime. The equilibrium is linear in the information firms have. At the strategic BNE (Section 2), the firm realizes the effect of its output on the market price, equating expected marginal revenue with marginal cost $MC(x_i;\theta_i) = \theta_i + \lambda x_i$. At the Bayesian price-taking equilibrium, the firm does not perceive any influence of its output on the (random) price and equates the expected price with marginal cost: $\mathrm{E}(p \mid \theta_i) = MC(x_i;\theta_i)$. At the full information Cournot equilibrium, a firm equates marginal revenue to marginal cost, and at the full information price-taking equilibrium, the firm equates price to marginal cost, $p = MC(x_i;\theta_i)$. Given linearity, prices will depend on

[6] More precisely, we say that a sequence of real numbers $A_n$ converges to zero at the rate $1/n^r$ for some $r > 0$ (or that the sequence $A_n$ is of the order $1/n^r$) when there exist $N$ and positive constants $k_1$ and $k_2$ such that for $n \geq N$, $k_1 \leq |A_n|n^r \leq k_2$.

[7] Furthermore, at the Bayesian price-taking equilibrium, a firm, despite knowing that costs are i.i.d., conjectures—correctly—that the expected price conditional on the firm's cost parameter is not independent of that cost parameter.

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TABLE 1 **Regimes and Strategies**

| Information | Strategic | Price Taking |
|---|---|---|
| Private | Bayesian Cournot-Nash $x_n(\theta_i)$ | Bayesian price taking $x_n^c(\theta_i)$ |
| Full | Cournot Nash $x_n^f(\theta_i, \tilde{\theta}_n)$ | Competitive $x_n^{fc}(\theta_i, \tilde{\theta}_n)$ |

$\tilde{\theta}_n$ in the full-information regimes. From the marginal conditions in each case, it is immediate to characterize the equilibria. Let us assume also that the underlying parameters are such that for whatever realizations of the cost parameters, firms want to produce positive quantities in any regime (see the Appendix for parameter conditions). This means that the relative inefficiency of firms is never so large as to imply the shutdown of inefficient producers. Proposition 1 states the results.

*Proposition 1.* There is a unique and symmetric equilibrium in each regime. Equilibrium strategies are given by

$$\begin{aligned} x_n(\theta_i) &= b_n(\alpha - \bar{\theta}) - a_n(\theta_i - \bar{\theta}); & a_n &= (\lambda + 2/n)^{-1},\ b_n = (\lambda + (1+n)/n)^{-1} \\ x_n^c(\theta_i) &= b_n^c(\alpha - \bar{\theta}) - a_n^c(\theta_i - \bar{\theta}); & a_n^c &= (\lambda + 1/n)^{-1},\ b_n^c = (\lambda + 1)^{-1} \\ x_n^f(\theta_i, \tilde{\theta}_n) &= b_n^f(\alpha - \tilde{\theta}_n) - a_n^f(\theta_i - \tilde{\theta}_n); & a_n^f &= a_n^c,\ b_n^f = b_n \\ x_n^{fc}(\theta_i, \tilde{\theta}_n) &= b_n^{fc}(\alpha - \tilde{\theta}_n) - a_n^{fc}(\theta_i - \tilde{\theta}_n); & a_n^{fc} &= \lambda^{-1},\ b_m^{fc} = b_n^c. \end{aligned}$$

The proof is standard and is omitted. Average output in each case is given as follows: $\tilde{x}_n = b_n(\alpha - \bar{\theta}) - a_n(\tilde{\theta}_i - \bar{\theta})$; $\tilde{x}_n^c = b_n^c(\alpha - \bar{\theta}) - a_n^c(\tilde{\theta}_i - \bar{\theta})$; $\tilde{x}_n^f = b_n(\alpha - \tilde{\theta})$; and $\tilde{x}_n^{fc} = b_n^c(\alpha - \tilde{\theta})$.[8]

It is immediate that when there is no market power difference, expected outputs (and prices) are the same and that market power lowers (raises) expected output (price). That is, $\mathrm{E}\,\tilde{x}_n (= \mathrm{E}\,x_{ni}) = \mathrm{E}\,\tilde{x}_n^f (= \mathrm{E}\,x_{ni}^f) = b_n(\alpha - \bar{\theta})$, and $\mathrm{E}\,\tilde{x}_n^c (= \mathrm{E}\,x_{ni}^c) = \mathrm{E}\,\tilde{x}_n^{fc} (= \mathrm{E}\,x_{ni}^{fc}) = b_n^c(\alpha - \bar{\theta})$ and $b_n < b_n^c$. The Bayesian Cournot equilibrium is the least sensitive to own costs and the full-information price-taking equilibrium the most sensitive, with an equal sensitivity for both intermediate regimes ($a_n < a_n^f = a_n^c < a_n^{fc}$). Keeping market power constant, a firm reacts more to its cost realization with full information.

## 4. Market power and information aggregation in large markets

■ The following proposition characterizes the convergence of the BNE price to the full-information competitive equilibrium price as the market grows large and decomposes its rate into market-power and private-information effects. We will say that the sequence of random variables $Y_n$ converges to zero at the rate $1/\sqrt{n^r}$ (or $Y_n$ is of the order $1/\sqrt{n^r}$) if $\mathrm{E}(Y_n)^2$ converges to zero at the rate $1/n^r$ (or $\mathrm{E}(Y_n)^2$ is of the order $1/n^r$). For example, given that $\mathrm{E}(Y_n)^2 = (\mathrm{E}\,Y_n)^2 + \mathrm{var}\,Y_n$, a sequence $Y_n$, such that $\mathrm{E}\,Y_n = 0$ and $\mathrm{var}\,Y_n$ is of the order of $1/n$, converges to zero at the rate $1/\sqrt{n}$. This is the typical convergence rate for the sample mean to converge to the population mean associated with the law of large numbers.[9] In particular, $\tilde{\theta}_n - \bar{\theta}$ is of order $1/\sqrt{n}$ because $\mathrm{var}\,\tilde{\theta}_n = \sigma_\theta^2/n$.

*Proposition 2.* As the market grows large, the market price (at the BNE) $p_n$ converges to the full-information competitive price $p_n^{fc}$ at the rate of $1/\sqrt{n}$. This "slow" convergence is driven by

[8] In the full-information competitive case, individual supply can be expressed in terms of the classical supply function $x_n^{fc}(\theta_i, \tilde{\theta}_n) = \lambda^{-1}(p_n^{fc} - \theta_i)$, $p_n^{fc} = (\alpha\lambda + \tilde{\theta}_n)/(1+\lambda)$, and therefore $\tilde{x}_n^{fc} = \lambda^{-1}(p_n^{fc} - \tilde{\theta}_i)$.

[9] With i.i.d. drawings $Z_1, \ldots, Z_n$ from a distribution with mean $\mu$ and finite variance $\sigma^2$, we have that $Y_n = ((\sum_j Z_j)/n) - \mu$ converges (in mean square) to zero at the rate of $1/\sqrt{n}$ because $\mathrm{E}\,Y_n = 0$ and $\mathrm{var}\,Y_n = \sigma^2/n$.

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the rate at which information is aggregated $(1/\sqrt{n})$ and not by the rate at which market power is dissipated $(1/n)$.

Consider the decomposition $p_n - p_n^{fc} = p_n - p_n^c + p_n^c - p_n^{fc}$. The market-power difference (keeping information private) $p_n - p_n^c = \mathrm{E}(p_n - p_n^c) + (a_n - a_n^c)(\tilde{\theta}_n - \bar{\theta})$ inherits the order of the expected price difference $\mathrm{E}(p_n - p_n^c)$, which is $1/n$ because $(a_n - a_n^c)$ is of lower order than $1/n$. The information difference (with price-taking behavior) $p_n^c - p_n^{fc} = (a_n^c - b_n^c)(\tilde{\theta}_n - \bar{\theta})$ is of the order of $1/\sqrt{n}$ because $(a_n^c - b_n^c)$ is of the order of a constant. The interaction between both effects is negative ($\mathrm{E}\{(p_n - p_n^c)(p_n^c - p_n^{fc})\} < 0$) and of the same order as the market-power term.[10] This means that the difference $p_n - p_n^{fc}$ will inherit the order $1/\sqrt{n}$ of $p_n^c - p_n^{fc}$, which is higher than the order $1/n$ of $p_n - p_n^c$. A parallel analysis establishes the same result for the alternative decomposition $p_n - p_n^{fc} = p_n - p_n^f + p_n^f - p_n^{fc}$. Now $p_n - p_n^f$ is of the order $1/\sqrt{n}$ (information) and $p_n^f - p_n^{fc}$ is of the order $1/n$ (market power).

The intuition for the result is the following. The order of $p_n - p_n^c$ or $p_n^f - p_n^{fc}$ is driven by the difference in expected values, whereas the order of $p_n^c - p_n^{fc}$ or $p_n - p_n^{fc}$ is driven by the variance of the price difference (because expected prices are the same when there is no difference in market power). The difference in expected prices between a strategic and a price-taking regime (be it with private information, $p_n - p_n^c$, or with full information, $p_n^f - p_n^{fc}$), is of the order of $1/n$ as in markets with no uncertainty and is explained by the rate at which market power disappears. Indeed, with full information it is immediate from the first-order conditions for a Cournot equilibrium that

$$\frac{p_n^f - \left(\sum_i MC\left(x_{in}^f; \theta_i\right)\right) / n}{p_n^f} = \frac{1/n}{\eta}.$$

It follows that the order of magnitude of the margin over average marginal cost is $1/n$ provided the elasticity of demand $\eta$ is bounded away from zero and infinity.

The variance of the price difference keeping market power constant, $p_n^c - p_n^{fc}$ (price taking) or $p_n - p_n^f$ (Cournot), is driven by the discrepancy between the sample mean and the population mean of the cost parameters, $\tilde{\theta}_n - \bar{\theta}$, which is of the order of $1/\sqrt{n}$. A firm in a private-information regime has to estimate the market price (price-taking case) or residual demand (Cournot case), which depends on the average realization of the cost parameter, and his strategy will depend only on his cost realization (and the known population mean). In contrast, with full information the strategy of a firm depends on both his cost realization and on the average realization of the cost parameter.

## 5. Welfare

■ **Welfare characterization of market equilibria.** Let us start by providing a general welfare characterization of a Cournot market with private information allowing for a general information structure (there will be more on this in Section 7). Consider an $n$-firm Cournot market with smooth inverse downward-sloping demand $P(X)$ and smooth convex costs, $C(x_i; \theta_i)$ for firm $i$. Suppose that firm $i$ receives a private signal $s_i$ about $\theta_i$. The following result provides an analogue to the First Welfare Theorem for Bayesian price-taking equilibria. We say that firms use decentralized strategies if each firm can choose its output as a function only of its signal.

*Proposition 3.* In a Cournot private-information environment, Bayesian price-taking equilibria maximize expected total surplus (ETS) subject to the use of decentralized production strategies.

[10] The market-power effect (information effect) is decreasing (increasing) in $\tilde{\theta}_n - \bar{\theta}$ because $a_n - a_n^c < 0$ $(a_n^c - b_n^c > 0)$. The market-power term $p_n - p_n^c$ is decreasing in $\tilde{\theta}_n - \bar{\theta}$ because higher realizations of costs $\tilde{\theta}_n$ imply higher prices and a more elastic demand (demand is linear).

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We will be interested in the differences in terms of (per-capita) ETS in the different regimes. The following result for a linear-quadratic specification of the model (with $P$ linear and $C$ quadratic) will prove useful.

*Lemma 1.* In the linear-quadratic specification of the model, the difference in (per-capita) ETS between a price-taking regime, $R = fc$ or $R = c$, and another regime with strategies based on weakly less information (that is, any other with respect to $fc$ and BNE with respect to $c$) is given by $(ETS^R - ETS)/n = (\beta\,\mathrm{E}(\tilde{x}_n - \tilde{x}_n^R)^2 + \lambda(\sum_i \mathrm{E}(x_{in} - x_{in}^R)^2/n))/2$.

The result follows considering a Taylor series expansion of $TS$ (stopping at the second term due to the quadratic nature of the payoff) around price-taking equilibria ($R = fc$ or $R = c$). The key to simplifying the computations is to notice that at price-taking equilibria, total surplus is maximized. Note that if the strategies and the information structure are symmetric, then $\mathrm{E}(x_{in} - x_{in}^R)^2$ is independent of $i$ and therefore $\sum_i \mathrm{E}(x_{in} - x_{in}^R)^2/n = \mathrm{E}(x_{in} - x_{in}^R)^2$.

We can decompose the total inefficiency with respect to price-taking regime $R$ in allocative and productive inefficiency. The latter is associated with the production of an average output in a non-cost-minimizing way, the former with the loss in surplus when producing, in a cost-minimizing way, an average output different from the benchmark. Consider, for simplicity of notation, a symmetric information structure and strategies. When average outputs $\tilde{x}_n$ and $\tilde{x}_n^R$ are produced in a cost-minimizing way, then for all $i$, $x_{in} - x_{in}^R = \tilde{x}_n - \tilde{x}_n^R$.[11] This implies that pure allocative inefficiency is given by $(\beta + \lambda)\mathrm{E}(\tilde{x}_n - \tilde{x}_n^R)^2/2$. The residual is due to productive inefficiency and can be expressed as $\lambda\,\mathrm{E}(u_i - v_i)^2/2$, where $u_i = x_{in} - \tilde{x}_n$ and $v_i = x_{in}^R - \tilde{x}_n^R$. We can then write $(ETS^R - ETS)/n = ((\beta + \lambda)\mathrm{E}(\tilde{x}_n - \tilde{x}_n^R)^2 + \lambda\,\mathrm{E}(u_i - v_i)^2)/2$. Note that $\mathrm{E}(u_i - v_i)^2 = \mathrm{var}(u_i - v_i)$ because $\mathrm{E}\,u_i = \mathrm{E}\,v_i = 0$.

□ **Welfare losses and rates of convergence.** We are now ready to state the welfare equivalent of Proposition 2 for the independent-values model (Section 2) in which firms receive a perfect signal of their cost parameter and the parameters are independent draws of a common distribution. To compute welfare differences, I use Lemma 1 and consider departures from price-taking behavior.

*Proposition 4.* In the independent-values model, the (per-capita) expected deadweight loss at the market outcome $(ETS_n^{fc} - ETS_n)/n$ is of the order of $1/n$. This order of magnitude is driven by information aggregation and not by market power: $(ETS_n^{fc} - ETS_n^c)/n$ and $(ETS_n^f - ETS_n)/n$ are of the order of $1/n$, while $(ETS_n^c - ETS_n)/n$ and $(ETS_n^{fc} - ETS_n^f)/n$ are of the order of $1/n^2$.

We have the following:

$$\text{Deadweight loss } (DWL_n) = \text{Private information loss} + \text{Market power loss}$$
$$(ETS_n^{fc} - ETS_n)/n = (ETS_n^{fc} - ETS_n^c)/n + (ETS_n^c - ETS_n)/n$$

or

$$(ETS_n^{fc} - ETS_n)/n = (ETS_n^f - ETS_n)/n + (ETS_n^{fc} - ETS_n^f)/n,$$

and in terms of order of magnitude, $1/n \approx 1/n + 1/n^2$. (See Table 2.)

**TABLE 2 Deadweight Loss**

| | Deadweight Loss Due to: | |
|---|---|---|
| Benchmark | Market Power | Private Information |
| Positive: Bayes-Cournot ($ETS_n$) | $\Delta W_n^c = (ETS_n^c - ETS_n)/n$ | $\Delta W_n^f = (ETS_n^f - ETS_n)/n$ |
| Normative: Competitive ($ETS_n^{fc}$) | $\Delta \underline{W}_n^{fc} = (ETS_n^{fc} - ETS_n^f)/n$ | $\Delta \underline{W}_n^f = (ETS_n^{fc} - ETS_n^c)/n$ |
| | order $1/n^2$ | order $1/n$ |

[11] An immediate implication of cost minimization when producing average output $x$ is that $x_i = x + (\tilde{\theta} - \theta_i)/\lambda$.

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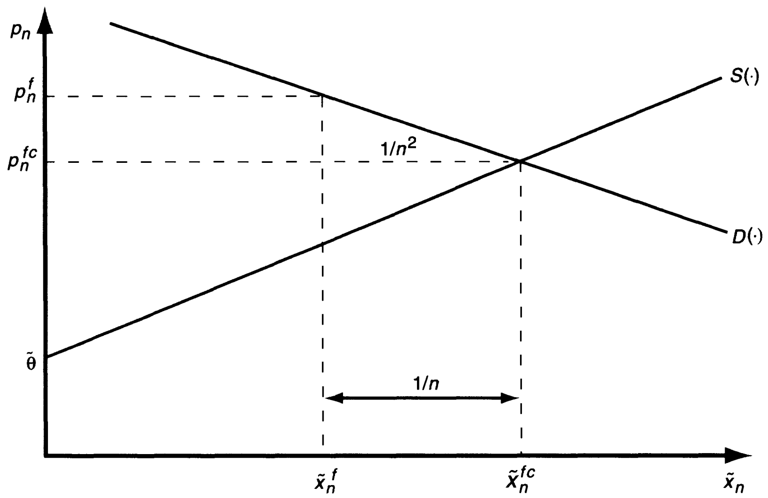
FIGURE 1

FULL-INFORMATION REGIMES




Market power dissipates according to the certainty rate. This yields an expected deadweight loss of the order of $1/n^2$, be it in the full-information or the private-information cases. In the full-information case, $((ETS_n^{fc} - ETS_n^f)/n)$, this is a well-known result. The full-information competitive equilibrium is determined by the intersection of (average) demand $D(p) = \alpha - p$ with aggregate (average) supply $S(p) = \lambda^{-1}(p - \tilde{\theta}_n)$, yielding an average output of $(\alpha - \tilde{\theta}_n)/(\lambda + 1)$. The full-information Cournot outcome yields an average output of $(\alpha - \tilde{\theta}_n)/(\lambda + (1 + n)/n)$. The difference, as we know, is of the order of $1/n$, and consequently the deadweight loss due to allocative inefficiency $(\beta + \lambda)\mathrm{E}(\tilde{x}_n^{fc} - \tilde{x}_n^{fc})^2/2$ (the area of the Harberger triangle) is of the order of $1/n^2$. (See Figure 1.) To this we should add the productive inefficiency $\lambda\ \mathrm{var}\{(x_{ni}^{fc} - \tilde{x}_n^{fc}) - (x_{ni}^{fc} - \tilde{x}_n^{fc})\}/2$ associated with not producing the Cournot output vector in a cost-minimizing way (inducing again a deadweight loss of the same order, $1/n^2$).

Similarly, in the private-information case the order of magnitude of both allocative and productive inefficiency with respect to the Bayesian price-taking equilibrium (or decentralized team solution) is $1/n^2$. Indeed, the deadweight loss due to allocative inefficiency is given by $(\beta+\lambda)\mathrm{E}(\tilde{x}_n - \tilde{x}_n^c)^2/2$ and the loss due to productive inefficiency by $\lambda\ \mathrm{var}\{(x_{ni} - \tilde{x}_n) - (x_{ni}^c - \tilde{x}_n^c)\}/2$.[12] Both are of the order $1/n^2$, and this implies that $(ETS_n^c - ETS_n)/n$ is of the order $1/n^2$. In the cases of both full and private information, market power affects the deadweight loss not only by decreasing expected output (because $b_n = b_n^f < b_n^c = b_n^{fc}$) but also by making firms too insensitive to their costs (respectively, $a_n < a_n^c$ and $a_n^f < a_n^{fc}$). Both allocative and productive inefficiency are impaired by the same order of magnitude.

When differences in information are at stake, keeping market power constant, the deadweight loss is driven by the variance terms because expected output does not change when comparing regimes. For example, with price-taking behavior, $(ETS_n^{fc} - ETS_n^c)/n = ((\beta + \lambda)\mathrm{var}(\tilde{x}_n^c - \tilde{x}_n^{fc}) + \lambda\ \mathrm{var}\{(x_{ni}^c - \tilde{x}_n^c) - (x_{ni}^{fc} - \tilde{x}_n^{fc})\})/2$. We know that $\tilde{x}_n^c - \tilde{x}_n^{fc} = (b_n^c - a_n^c)(\tilde{\theta}_n - \bar{\theta})$, $(b_n^c - a_n^c)$ is of the order of a constant and $\mathrm{var}(\tilde{\theta} - \bar{\theta})$ is of the order $1/n$. In a fashion parallel to Figure 1, we have that the discrepancy between $\tilde{x}_n^c$ and $\tilde{x}_n^{fc}$ is of the order of $1/\sqrt{n}$ and, correspondingly, the allocative deadweight loss of the order of $1/n$. Furthermore, $(x_{ni}^c - \tilde{x}_n^c) - (x_{ni}^{fc} - \tilde{x}_n^{fc}) = a_n^{fc}(\theta_i - \tilde{\theta}_n) - a_n^c(\theta_i - \bar{\theta})$, which again yields a productive efficiency loss of the order of $1/n$ because of the leading term $\tilde{\theta}_n$. This implies in particular that the leading terms in the expected deadweight loss at the Bayesian

[12] It is easy to see that $\mathrm{var}\{(x_{ni} - \tilde{x}_n) - (x_{ni}^c - \tilde{x}_n^c)\} = (a_n^c - a_n)^2\mathrm{var}(\tilde{\theta}_n - \theta_i)$, the first term of the order $1/n^2$ and the second of the order of a constant.

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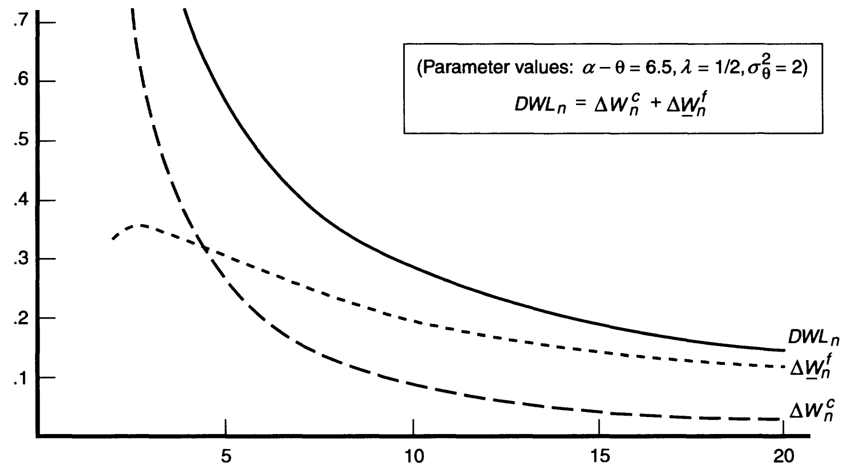
FIGURE 2

$DWL_n = \Delta W_n^c + \Delta \underline{W}_n^f$ (parameter values: $\alpha - \bar{\theta} = 6.5$, $\lambda = 1/2$, $\sigma_\theta^2 = 2$)




Cournot-Nash equilibrium (with respect to the first best) are the variance terms associated with differences in information.

□ **How large is large?** Proposition 4 implies that for $n$ large enough, the welfare loss due to private information is larger than that from market power. However, with a single firm there is no loss due to private information. A relevant question then arises as to how large a market is needed for the welfare loss from private information to dominate that from market power. Simulations with the model help to answer the question. I perform simulations in the following parameter range: $\beta = 1$, $\alpha - \bar{\theta}$ in $\{2, 6.5\}$, and $\lambda$ and $\sigma_\theta^2$ in $\{1/10, 1/2, 1, 2, 10\}$.[13]

The result is that whatever the decomposition of the welfare loss, $DWL_n = \Delta \underline{W}_n^f + \Delta W_n^c$ or $DWL_n = \Delta W_n^f + \Delta \underline{W}_n^c$, there is a critical $\bar{n}$ beyond which the loss from private information dominates and $\bar{n}$ decreases with more prior uncertainty ($\sigma_\theta^2$ larger).[14] For large uncertainty, $\bar{n}$ is below 10. The market-power terms $\Delta W_n^c$ or $\Delta \underline{W}_n^c$ are typically decreasing in $n$, whereas the information terms $\Delta \underline{W}_n^f$ or $\Delta W_n^f$ first increase and then decrease in $n$. This yields a total deadweight loss $DWL_n$, which is typically always decreasing in $n$. Figure 2 displays the first decomposition when $\alpha - \bar{\theta} = 6.5$, $\lambda = 1/2$, $\sigma_\theta^2 = 2$. This yields $\bar{n} = 4.4$. Figure 3 displays the second decomposition with $\lambda = 2$ and $\sigma_\theta^2 = 10$, and this yields $\bar{n} = 6.4$.

□ **Estimating Harberger triangles.** A typical empirical assessment of the deadweight loss due to market power uses an approximation at the industry level with data on profit returns and sales positing a certain value for the elasticity of demand.[15] Harberger and others have obtained low estimates for the welfare loss.[16] This should not be surprising from the perspective of the present article, because even in moderately sized markets, according to the simulations, the effect

[13] Only $\alpha - \bar{\theta}$ matters for the comparisons we perform. We get $\alpha - \bar{\theta} = 2$ with, for example, $\alpha = 3$, $\bar{\theta} = 1$ and $\alpha - \bar{\theta} = 6.5$ with, for example, $\alpha = 10$, $\bar{\theta} = 3.5$. Some of the parameter combinations in the simulations violate the sufficient conditions for positive outputs.

[14] In both decompositions the welfare loss due to market power and to private information increases with $\sigma_\theta^2$ and decreases with $\lambda$. The effect of $\sigma_\theta^2$ via private information is larger, and this explains why $\bar{n}$ decreases with $\sigma_\theta^2$. With respect to $\lambda$ in the first decomposition, the effect on the information term is larger and $\bar{n}$ increases with $\lambda$. In the second decomposition, $\bar{n}$ may be increasing, U-shaped, or decreasing with $\lambda$ (respectively, for low, intermediate, and high values of $\sigma_\theta^2$).

[15] More precisely, under constant returns the deadweight loss is approximated by $R\eta d^2/2$, where $R$ is revenue, $\eta$ the demand elasticity, and $d$ the relative price distortion from the competitive level.

[16] See the discussion in Scherer and Ross (1990).

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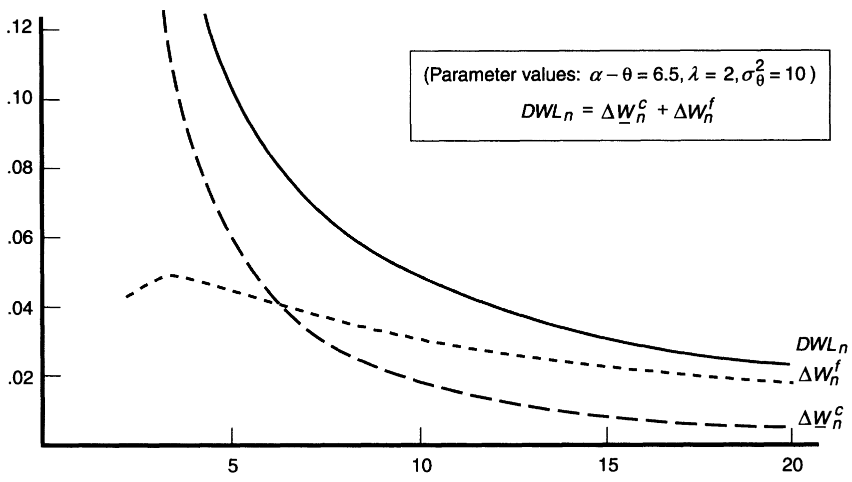
FIGURE 3

$DWL_n = \Delta \underline{W}_n^c + \Delta W_n^f$ (parameter values: $\alpha - \bar{\theta} = 6.5$, $\lambda = 2$, $\sigma_\theta^2 = 10$)




of market power fades quickly. However, accounting for incomplete information would increase the estimates. Indeed, the Harberger approach in the context of my model would approximate the true deadweight loss due to allocative inefficiency, $((\beta + \lambda)((\mathrm{E}(\tilde{x}_n - \tilde{x}_n^{fc}))^2 + \mathrm{var}(\tilde{x}_n - \tilde{x}_n^{fc})))/2$, by the first term in the sum, which is of order $1/n^2$, forgetting the second, which is of order $1/n$. This is perfectly all right under full information (because then $\mathrm{var}(\tilde{x}_n^{fc} - \tilde{x}_n^{fc})$ is of order $1/n^3$) but not with private information. Furthermore, to the allocative inefficiency measure the loss due to productive inefficiency $(\lambda \,\mathrm{var}\{(x_{ni} - \tilde{x}_n) - (x_{ni}^{fc} - \tilde{x}_n^{fc})\})/2$ should be added. The outcome of the new computation of the deadweight loss would be higher by an order of magnitude. As an example, with a market-concentration equivalent of ten firms ($n = 10$), while the true relative deadweight loss is of the order of 10%, the estimated one ignoring private information would be of the order of only 1%.

## 6. Competition policy implications

■ The results obtained have a bearing on the antitrust policy toward information exchange and communication among firms. Antitrust authorities look with suspicion on information exchanges of individual firms' prices and quantities because they can help the monitoring of deviations from collusive agreements.[17] In contrast, policy in both the United States and Europe is more permissive about exchange of cost and demand information (via trade associations, for example).[18]

Sharing cost information helps collusion by allowing firms to divide the market.[19] If firms are sufficiently impatient, restrictions on communication may decrease collusive profits (Athey and Bagwell, 2001).[20] The collusion concern is acute in the presence of a few players in the market, because the critical discount factor above which collusion is possible typically increases with the

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[17] Indeed, in the United States, for example, it is (close to) per se illegal to exchange information on current prices (*United States* v. *Container Corporation of America*). Similarly, the European Commission looks hard on information exchanges of individual firm data. (See Kühn and Vives (1995).)

[18] See Doyle and Snyder (1999) for an analysis of information exchange of production plans in the U.S. automobile industry.

[19] Asymmetric cost information is an obstacle that even a legal cartel, in which side payments are possible, has to confront because production has to be allocated efficiently among cartel members (see Cramton and Palfrey (1990) and Kihlstrom and Vives (1992)).

[20] However, the cost of not communicating may be productive inefficiency. With price competition it might be that this effect outweighs the price-reduction potential of the lack of communication. The reason is that colluding firms may have an ample tolerance of productive inefficiency before lowering prices. Athey and Bagwell conjecture nonetheless that this conclusion may not hold with competition à la Cournot.

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number of firms in the market. At the same time, sharing cost information in a static Cournot model improves welfare (as we have seen before; see also Vives (1984) and Shapiro (1986)). As a consequence, there is a tension between the collusion concern and the static efficiency benefits. Disallowing the exchange of cost information may come at a cost. The question then is how to devise a rule of thumb for policy absent a complete (and costly) estimate of the potential benefits of information sharing.

We have seen how, for a given degree of cost uncertainty, there is a critical number of firms, $\bar{n}(\sigma_\theta^2)$, with $\bar{n}$ decreasing in $\sigma_\theta^2$, such that for $n$ larger (smaller) than $\bar{n}$ the welfare loss derived from the lack of information aggregation is larger (smaller) than the welfare loss derived from market power. This critical $n$ is not very large, between 5 and 10 firms, for a wide range of parameter values. Therefore, the relative benefit of letting firms share cost information is large unless the number of firms is small. This suggests a safe-haven policy for information exchange on costs: It should be allowed in not very concentrated markets. In concentrated markets, the potential for collusion should be assessed, and if it is deemed high, the information exchange should be disallowed. A more drastic (and easier-to-implement) policy would call for a ban on cost information sharing in concentrated markets because the potential static efficiency gain is not large and the collusive potential significant.

The question then arises whether firms have incentives to reveal or share their cost information if this is allowed, and, if not, what measures can be taken to facilitate information exchange. In the Cournot market with independent costs, it is a dominant strategy to share *ex ante* information. That is, if firms can commit to share information before receiving their signals, they will do it (Fried, 1984; Shapiro, 1986; and Raith, 1996). A trade association may provide the mechanism to *ex ante* share information. But information about costs may be exchangeable in practice only at the interim stage—that is, after each firm learns its cost level but does not know the costs of the rival firms. In this case, if information is not verifiable and there are no other signalling possibilities, no information revelation is possible. The reason is that all types of firms would like to be perceived as being of a low-cost type. With verifiable information, full revelation obtains (Okuno-Fujiwara, Postlewaite, and Suzumura, 1990). This happens because the least-cost firm will reveal itself credibly and then all other types unravel. A way to promote information sharing therefore would be to facilitate the verification of information via "benchmarking" or the formation of trade associations that can audit and check the information reported by their members.[21]

Information could also be revealed with costly signalling, be it in the form of wasteful advertising (Ziv, 1993), for example, or with dynamic competition in which production levels are observable (Mailath, 1989). Then dynamic interaction may reveal part or all of it over time (depending on whether separating or semi-pooling equilibria obtain; see Vives (1999). This suggests that the welfare loss from incomplete information might dissipate while the one derived from market power remains. This type of dynamic revelation applies when costs change slowly or, in the extreme, for once-and-for-all shocks. In other situations, however, the types of firms change each period following a stochastic process, and the revelation of today's cost parameter provides only an estimate of tomorrow's cost. In the steady state, then, the welfare loss due to private information will remain significant.

Finally, the implications for merger policy remain to be studied. The analysis should be extended allowing for asymmetric industry configurations. The potential importance of the effect of private information in relation to market power may indicate changed private and social incentives for merger. This might expand the limited range of cases in which mergers without synergies are profitable in the Cournot world of Salant, Switzer, and Reynolds (1983) and Perry and Porter (1985).[22]

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[21] See Vives (1999) for a survey of the results about information revelation and sharing, and Azcuenaga (1996) for some of the antitrust concerns raised by benchmarking.

[22] It is worth noting that the cost structure used in the analysis is akin to one considered in Perry and Porter (1985) in which a total amount of capital $k$ is to be distributed among the firms in the industry. A firm with a share $s$ of the capital will have a cost function $C(x, \theta; s) = sk + \theta x + (\lambda/2s)x^2$.

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## 7. Extensions

■ We have considered an independent-values linear Cournot model. How far do the results extend to other specifications? We will consider relaxing, in turn, linearity, the information structure, the product structure, and the market microstructure. We will deal finally with the policy implications.

□ **Linearity.** I have restricted the analysis to cases in which, in equilibrium, all firms wish to produce. This is not the case, for example, with constant marginal costs. Then, as the number of firms grows, only the more efficient firms can survive. This means that we have not allowed for the selection effect of competition weeding out inefficient firms (Demsetz, 1973). It is an open question to assess the relative importance of market power versus information aggregation in this case. One can conjecture that, if anything, convergence to the first best will be slowed down because now productive inefficiency may loom larger at the market outcome.

Furthermore, I have assumed a linear structure to derive the results. However, the results should be extendible to a nonlinear frame. The reason is as follows. Our linear-quadratic payoff could be seen as an approximation to a general payoff (up to the second order). With this perspective, a version of Lemma 1 to compute welfare differences across regimes should hold (stopping the Taylor approximation at the second order), and similar results could, hopefully, be derived. Nonetheless, the approximation need not be good for a small number of firms unless the variance is low.

□ **Information structure.** In a Cournot market, information aggregation does not obtain asymptotically outside the independent-values case except under very special parameter configurations. For example, it does obtain with common-value uncertainty and constant marginal costs under some regularity conditions of the signal structure (Palfrey, 1985), but it does not obtain with increasing marginal costs (Vives, 1988). In general, with common values or with private values, as the market grows large, the BNE does not converge to the (full-information) competitive equilibrium. However, the BNE does converge to price-taking behavior, and this represents the best possible decentralized mechanism. According to Proposition 3, price-taking behavior maximizes ETS subject to the use of decentralized strategies (it is "second-best efficient"). Considering a (symmetric) information structure that allows as extremes private values (correlated types or cost parameters with perfect signals) and a common value (with noisy signals), and with a joint distribution of random variables assumed to yield affine conditional expectations, one can show that unique (and affine in information) Bayesian equilibria exist. Furthermore, in the Cournot market the BNE converges to the best decentralized outcome (or second-best outcome) fast, of the order of $1/n$ for prices and $1/n^2$ for (per-capita) ETS.[23]

□ **Product market.** The basic model presented can also be interpreted as a Cournot model with product differentiation, (idiosyncratic) demand uncertainty, and constant marginal costs, assumed to be zero for simplicity. Indeed, the profits of firm $i$ can be rewritten as

$$\pi_i = \left(\alpha - \theta_i - \left(\frac{\beta}{n} + \frac{\lambda}{2}\right) x_i - \frac{\beta}{n} \sum_{j \neq i} x_j\right) x_i.$$

The inverse demand function faced by firm $i$ is

$$p_i = \alpha - \theta_i - \left(\frac{\beta}{n} + \frac{\lambda}{2}\right) x_i - \frac{\beta}{n} \sum_{j \neq i} x_j,$$

with $(\beta/n) + (\lambda/2)$ and $\beta/n$, respectively, the own and the cross-demand effects, and $\theta_i$ a random

[23] See a web Appendix to this article for a precise statement and proof (available at http://faculty.insead.edu/vives).

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shock to demand. With this interpretation as the economy is replicated the limit market is monopolistically competitive, firm $i$ facing inverse demand $p_i = \alpha - \theta_i - (\lambda/2)x_i - \beta\tilde{x}$, where $\tilde{x}$ is the average output of the varieties present in the market. Now firms retain market power even in a large market, even though a single firm does not influence market aggregates.[24]

□ **Market microstructure.** In a $k$-double auction with $n$ suppliers and $n$ bidders (with traders having independent valuations), the expected inefficiency disappears fast, at the rate of $1/n^2$ (Rustichini, Satterthwaite, and Williams, 1994). Market microstructure matters. I can explain the result considering price competition in my model with i.i.d. constant marginal costs drawn from a compact interval. In this case sellers bid for the right to supply the market, that is, to face the downward-sloping demand curve. With constant marginal costs, the replication of demand only introduces a scale factor, which I ignore here. The firm that quotes the lowest price gets all the market (if more than one firm quotes the lowest price, they share the market). With complete information it is well known that the equilibrium price equals the cost of the second-most efficient firm ($\theta_{\text{next}}$), and there is no productive inefficiency because the lowest-cost firm supplies the market. With incomplete information there is a unique and symmetric Bayesian Bertrand equilibrium $p_n(\theta)$, which is increasing and differentiable (see Hansen, 1988, and Spulber, 1995). The equilibrium strategy is given by the solution to the following differential equation:

$$p_n(\theta) - \theta = p_n'(\theta)D(p_n(\theta))/((n-1)h(\theta)D(p_n(\theta)) - p_n'(\theta)D'(p_n(\theta))),$$

where $h(\theta)$ is the hazard rate of the distribution (the probability that the cost of a rival equals $\theta$ given that it is no less than $\theta$), with an appropriate boundary condition. Under standard boundedness conditions, $p_n(\theta) - \theta$ will be of the order of $1/n$ and therefore so will the margin of the winning firm $p_n(\theta_{\min}) - \theta_{\min}$. Given that $p_n(\theta)$ is increasing, there is no productive inefficiency; the lowest-cost firm supplies the market. Furthermore, the deadweight loss due to allocative inefficiency is of the order of $1/n^2$. The winner-take-all nature of competition implies that the equilibrium strategy depends on $(n-1)h(\theta)$, that is, on the probability that, conditional on having the lowest cost, a rival also has the lowest cost. The higher this probability, the lower the margin, *ceteris paribus*. The firm conditions on its cost realization and on the event of winning the contest. One can draw an analogy with supply function competition (Klemperer and Meyer, 1989), where a firm submits a supply function effectively conditioning on the information contained in the market price. The consequence is that in the auction/Bertrand game a firm is effectively conditioning on more information than in the Cournot game. In the latter, a firm has to estimate the average cost of (active) rivals and relies only on its cost realization. The auction/Bertrand mechanism better aggregates information than the Cournot one, and this explains the different rates at which prices converge to marginal cost in the presence of incomplete information.

□ **Policy implications.** In what directions is the competition policy recommendation robust? It is robust if competition is à la Cournot. This is so because information sharing on costs (or demand) is good for welfare with Cournot competition independent of whether uncertainty is of the private or common-value variety (see Vives, 1999). However, things get more complicated with Bertrand competition. Then information sharing tends to be bad for welfare, with the exception of the common-value cost uncertainty case. For example, in the price competition model of of the last subsection, the deadweight loss is larger under full than under incomplete information (Hansen, 1988). That is, contrary to the Cournot case, we have that $ETS_n^f < ETS_n$. The reason is that with incomplete information a firm tends to be more aggressive and sets a price below the

[24] Note also that switching the role of prices and quantities and letting $\beta < 0$, we obtain a Bertrand model with product differentiation and idiosyncratic demand uncertainty. Then the demand for product $i$ is given by $x_i = \alpha - \theta_i - (\lambda/2)p_i - \beta\tilde{p}$, where $\tilde{p}$ is the average price and products are gross substitutes (and competition of the strategic complements variety) because $\beta < 0$.

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expected price with complete information. The result is that $p(\theta_{\min}) < \mathrm{E}(\theta_{\text{next}} \mid \theta_{\min})$.[25] When we couple the above with the result in Athey and Bagwell (2001) that restricting communication about i.i.d. costs may be harmful to welfare when firms compete in prices and try to collude (because productive efficiency is impaired), we are left with no simple policy bottom line.

Leaving aside the winner-take-all case of auctions, where even with a few players the deadweight loss is small, what is a robust result is that, with significant uncertainty and private information, the welfare consequences of information exchange dominate those of market power in moderately sized and large markets.[26]

## Appendix

■ Proofs of Propositions 2–4 and Lemma 1, and a discussion of the parameters guaranteeing interior equilibria follow.

□ **Parameters guaranteeing interior equilibria.** A small $\lambda$ constrains the support of $\theta_i$ in order for outputs to be positive. Let the support of $\theta_i$ be $[\theta^m, \theta^M]$ and $\Delta = [\theta^m - \theta^M]/2$. Then, to ensure that in the limit (in which all regimes yield the same outcome because $\tilde{\theta}_n - \bar{\theta}$ converges almost surely to zero as $n$ tends to infinity) all types of firms produce positive outputs, it is sufficient that $x(\theta^M) > 0$. This holds if and only if $\lambda\alpha + \bar{\theta} > (1+\lambda)\theta^M$. If the distribution of $\theta_i$ is symmetric around the mean $\bar{\theta}$, then $\bar{\theta} = [\theta^m + \theta^M)/2$, $\theta^M = \bar{\theta} + \Delta$, and $\theta^m = \bar{\theta} - \Delta$. The inequality $\lambda\alpha + \bar{\theta} > (1+\lambda)\theta^M$ can be rewritten as $\lambda(1+\lambda)^{-1}(\alpha - \bar{\theta}) > \Delta$. This puts an upper bound of $(\lambda(1+\lambda)^{-1}(\alpha - \bar{\theta}))^2$ on $\sigma_\theta^2$ because $\sigma_\theta^2 \leq \Delta^2$ for a symmetric distribution with support $[\bar{\theta} - \Delta, \bar{\theta} + \Delta]$. Another upper bound of the form $\sigma_\theta \leq \bar{\theta}$ comes from the fact that $\theta^m = \bar{\theta} - \Delta \geq 0$ and therefore $\bar{\theta} \geq \Delta$. A sufficient condition for positive outputs in all the regimes close to the limit (when costs are still random) is that $\lambda\alpha + \theta^m > (1+\lambda)\theta^M$ or $\lambda(\alpha + \theta^M) > \theta^M - \theta^m$.

*Proof of Proposition 2.* We consider first (i) $p_n - p_n^c$, then (ii) $p_n^c - p_n^{fc}$ to derive the result for (iii) $p_n - p_n^{fc}$.

(i) Differences in prices correspond to differences (changing signs) in average outputs. Consider $p_n - p_n^c = \tilde{x}_n^c - \tilde{x}_n = (b_n^c - b_n)(\alpha - \bar{\theta}) + (a_n - a_n^c)(\tilde{\theta}_n - \bar{\theta})$ and recall that $\mathrm{E}(Y_n)^2 = (\mathrm{E}\,Y_n)^2 + \mathrm{var}\,Y_n$. We have that $\mathrm{E}(p_n - p_n^c) = (b_n^c - b_n)(\alpha - \bar{\theta})$ because $\mathrm{E}\,\tilde{\theta}_n = \mathrm{E}\,\theta_i = \bar{\theta}$, and that $\mathrm{var}(p_n - p_n^c) = (a_n - a_n^c)^2\sigma_\theta^2/n$. It is easily seen that both $(b_n^c - b_n)$ and $(a_n - a_n^c)$ are of order $1/n$ (indeed, $n(b_n^c - bn)$ tends to $1/(1+\lambda)^2$ and $n(a_n - a_n^c)$ tends to $-1/\lambda^2$ as $n$ tends to infinity). Therefore, $\mathrm{E}(p_n - p_n^c)^2$ inherits the order of $(\mathrm{E}(p_n - p_n^c))^2$, which is $1/n^2$ because $\mathrm{E}(p_n - p_n^c)$ is of order $1/n$ (and $\mathrm{var}(p_n - p_n^c)$ is of smaller order, $1/n^3$). We conclude that $p_n - p_n^c$ is of the order of $1/n$.

(ii) Consider now $p_n^c - p_n^{fc} = \tilde{x}_n^{fc} - \tilde{x}_n^c = (a_n^c - b_n^c)(\tilde{\theta}_n - \bar{\theta})$. We have that $\mathrm{E}(p_n^c - p_n^{fc}) = 0$ given that $\mathrm{E}\,\tilde{\theta}_n = \bar{\theta}$, and $\mathrm{var}(p_n^c - p_n^{fc}) = (a_n^c - b_n^c)^2 \mathrm{var}\,\tilde{\theta}_n$. Because $(a_n^c - b_n^c)$ converges to $(\lambda(1+\lambda))^{-1}$ and $\mathrm{var}\,\tilde{\theta}_n = \sigma_\theta^2/n$, $\mathrm{var}(p_n^c - p_n^{fc})$ and $\mathrm{E}(p_n^c - p_n^{fc})^2$ are of the order of $1/n$. We conclude that $p_n^c - p_n^{fc}$ is of the order of $1/\sqrt{n}$.

(iii) Given that $p_n - p_n^{fc} = p_n - p_n^c + p_n^c - p_n^{fc}$, $\mathrm{E}(p_n - p_n^{fc})^2 = \mathrm{E}(p_n - p_n^c)^2 + \mathrm{E}(p_n^c - p_n^{fc})^2 + 2\mathrm{E}(p_n - p_n^c)(p_n^c - p_n^{fc})$. The first summand is of order $1/n^2$, the second of order $1/n$, and the third of order $1/n^2$ (because $\mathrm{E}(p_n - p_n^c)(p_n^c - p_n^{fc}) = (a_n - a_n^c)(a_n^c - b_n^c)\mathrm{E}(\tilde{\theta}_n - \bar{\theta})^2$ is of order $1/n^2$ as $(a_n - a_n^c)$ is of order $1/n$, $(a_n^c - b_n^c)$ is of the order of a constant, and $\mathrm{E}(\tilde{\theta}_n - \bar{\theta})^2 = \mathrm{var}\,\tilde{\theta}_n$). It follows that $\mathrm{E}(p_n - p_n^{fc})^2$ is of order $1/n$ and $p_n - p_n^{fc}$ is of order $1/\sqrt{n}$. A parallel analysis establishes the same result for the alternative decomposition $p_n - p_n^{fc} = p_n - p_n^f + p_n^f - p_n^{fc}$. Now $p_n - p_n^f$ is of the order $1/\sqrt{n}$ and $p_n^f - p_n^{fc}$ is of the order $1/n$. *Q.E.D.*

*Proof of Proposition 3.* The maximization of ETS subject to decentralized production strategies is the problem of a team having as its objective

$$\mathrm{E}\{TS\} = \mathrm{E}\left\{\int_0^x P(Z)dZ - \sum_{j=1}^n C(x_j(s_j); \theta_j)\right\}$$

and choosing strategies $x_j(s_j)$, $j = 1, \ldots, n$, where $X = \sum_{j=1}^n x_j(s_j)$. Under our assumptions the optimal decision rules are determined (for interior solutions) by the set of first-order conditions (FOC) $\mathrm{E}((\partial TS/\partial x_i) \mid s_i) = 0$ or, equivalently, $\mathrm{E}(P(X) \mid s_i) = \mathrm{E}(MC(x_i; \theta_i) \mid s_i)$. Indeed, a set of decision rules are optimal if and only if they are person-by-person optimal given that the team function is concave and differentiable (Radner, 1962). The conditions are fulfilled in our case. Now, price-taking firm $i$ will maximize $\mathrm{E}(\pi_i \mid s_i) = \mathrm{E}(P(X) \mid s_i)x_i - \mathrm{E}(C(xi; \theta_i) \mid s_i)$, yielding a FOC, $\mathrm{E}(P(X) \mid s_i) = \mathrm{E}(MC(xi; \theta_i) \mid s_i)$. Those conditions are sufficient given our assumptions and therefore the solutions to both problems coincide. *Q.E.D.*

[25] With inelastic demand, the revenue equivalence theorem would apply (Bertrand competition with complete information as an open descending auction) and $p(\theta_{\min}) < \mathrm{E}(\theta_{\text{next}} \mid \theta_{\min})$. This balances exactly the fact that in the first-price auction, raising the bid a little bit increases profit when winning but reduces the probability of winning. If demand is elastic, then the profit increase when winning is strictly smaller and the optimum bid must therefore be smaller than before.

[26] The safe-haven policy advocated for a Cournot world raises the question of how to determine whether competition is of this type or not. This is a difficult issue beyond the scope of the present article.

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*Proof of Lemma 1.* In the linear-quadratic model, total surplus is given by (dropping the $n$ subscripts)

$$TS = \sum_j (\alpha - \theta_j)x_j - \frac{1}{2}\left(\frac{\beta}{n}\left(\sum_j x_j\right)^2 + \lambda \sum_j x_j^2\right).$$

A Taylor series expansion of $TS$ around a price-taking equilibrium $x^R = (x_1^R, \ldots, x_n^R)$, stopping at the second term due to the fact that $TS$ is quadratic, yields

$$TS(x) - TS(x^R) = \nabla TS(x^R)(x - x^R) + \frac{1}{2}(x - x^R)' D^2 TS(x^R)(x - x^R),$$

where $\nabla TS(x^R)$ and $D^2 TS(x^R)$ are, respectively, the gradient and the Hessian matrix of $TS$ evaluated at $x^R$. We have that

$$D^2 TS(x^R) = -B = -\begin{pmatrix} \frac{\beta}{n} + \lambda & \cdots & \frac{\beta}{n} \\ \frac{\beta}{n} & \cdots & \frac{\beta}{n} + \lambda \end{pmatrix}$$

and

$$(x - x^R)' B(x - x^R) = \frac{\beta}{n}\left(\sum_j \left(x_j - x_j^R\right)\right)^2 + \lambda \sum_j \left(x_j - x_j^R\right)^2.$$

If $R = fc$, the first best, it is immediate that $\nabla TS(x^R) = 0$. If $R = c$, the team optimum solution with decentralized strategies (Proposition 3), then it still holds that $\mathrm{E}\{\nabla TS(x^R)(x - x^R)\} = 0$. The argument for the latter equality is as follows. At the team optimum we have that $\mathrm{E}((\partial TS/\partial x_i) \mid s_i) = 0$ for all $i$. Therefore, since at the alternative regime strategies are based on a weakly coarser information partition (that is, $x_i$ at most will depend on $s_i$),

$$\mathrm{E}\left(\frac{\partial TS}{\partial x_i}(x^c) s_i\right) = 0.$$

We have then that

$$\mathrm{E}\left\{\sum_i \frac{\partial TS}{\partial x_i}(x^c)\right\} = 0.$$

In both cases, therefore, $\mathrm{E}(TS(x) - TS(x^R))/n = -(\beta\, \mathrm{E}(\tilde{x} - \tilde{x}^R)^2 + \lambda(\sum_i \mathrm{E}(x_i - x_i^R)^2/n)/2$. Note also that for symmetric strategies and information structure, $\mathrm{E}(x_i - x_i^R)^2$ is independent of $i$. *Q.E.D.*

*Proof of Proposition 4.* We consider first (i) $(ETS_n^c - ETS_n)/n$, then (ii) $(ETS_n^{fc} - ETS_n)/n$ and $(ETS_n^{fc} - ETS_n^f)/n$. All the other results (iii) then follow.

(i) According to Lemma 1 and given that equilibria are symmetric, we have that $(ETS_n^c - ETS_n)/n = (\beta\, \mathrm{E}(\tilde{x}_n - \tilde{x}_n^c)^2 + \lambda(\mathrm{E}(x_{in} - x_{in}^c)^2)/2$. We know that $\mathrm{E}(\tilde{x}_n - \tilde{x}_n^c)^2 = (\mathrm{E}(\tilde{x}_n - \tilde{x}_n^c))^2 + \mathrm{var}(\tilde{x}_n - \tilde{x}_n^c)$, and from the proof of Proposition 2, $\mathrm{E}(\tilde{x}_n - \tilde{x}_n^c)^2$ inherits the order of $(\mathrm{E}(\tilde{x}_n - \tilde{x}_n^c))^2$, which is $1/n^2$ given that $\mathrm{var}(\tilde{x}_n - \tilde{x}_n^c)$ is of smaller order, $1/n^3$.

Similarly, $\mathrm{E}(x_{in} - x_{in}^c)^2 = (\mathrm{E}(x_{in} - x_{in}^c))^2 + \mathrm{var}(x_{in} - x_{in}^c)$. Now, $\mathrm{E}(x_{in} - x_{in}^c)$ is of the same order as $\mathrm{E}(\tilde{x}_n - \tilde{x}_n^c)$, $1/n$, and $\mathrm{var}(x_{in} - x_{in}^c) = \sigma_\theta^2$, is of order $1/n^2$ because $(a_n^c - an)$ is of order $1/n$. Therefore, $\mathrm{E}(x_{in} - x_{in}^c)^2$ is of order $1/n^2$. We conclude that $(ETS_n^c - ETS_n)/n$ is of the order of $1/n^2$.

(ii) Consider now $(ETS_n^{fc} - ETS_n)/n$. As before, we have $(ETS_n^{fc} - ETS_n)/n = (\beta\, \mathrm{E}(\tilde{x}_n - \tilde{x}_n^{fc})^2 + \lambda(\mathrm{E}(x_{in} - x_{in}^{fc})^2)/2$. We know that the order of $\mathrm{E}(\tilde{x}_n - \tilde{x}_n^{fc})^2$ is the same as the order of $\mathrm{E}(p_n - p_n^{fc})^2$, which according to Proposition 2 is $1/n$ (and this is driven by the variance term $\mathrm{var}(\tilde{x}_n - \tilde{x}_n^{fc})$). Now, $\mathrm{E}(x_{in} - x_{in}^{fc})^2 = (\mathrm{E}(x_{in} - x_{in}^{fc}))^2 + \mathrm{var}(x_{in} - x_{in}^{fc})$. We have that $x_{in} - x_{in}^{fc} = b_n(\alpha - \bar{\theta}) - a_n(\theta_i - \bar{\theta}) - b_n^{fc}(\alpha - \tilde{\theta}_n) + a_n^{fc}(\theta_i - \tilde{\theta}_n)$. We have that $\mathrm{E}(x_{in} - x_{in}^{fc}) = \mathrm{E}(\tilde{x}_n - \tilde{x}_n^{fc})(= (b_n - b_n^{fc})(\alpha - \bar{\theta}))$ is of the order $1/n$. It follows also that $\mathrm{var}(x_{in} - x_{in}^{fc}) = (\mathrm{var}\, \tilde{\theta}_n)(\lambda(1 + \lambda))^{-2} + (a_n - a_n^{fc})^2\sigma_\theta^2 + 2(\lambda(1 + \lambda))^{-1}(a_n - a_n^{fc})\mathrm{cov}(\tilde{\theta}_n, \theta_i)$. Given that $\mathrm{cov}(\tilde{\theta}_n, \theta_i) = \mathrm{var}\, \tilde{\theta}_n = \sigma_\theta^2/n$, and that $(a_n - a_n^{fc})$ is of order $1/n$, the leading term is the first and $\mathrm{var}(x_{in} - x_{in}^{fc})$ is of the order of $1/n$. The conclusion is that $\mathrm{var}(x_{in} - x_{in}^{fc})$ is in turn the leading term in $\mathrm{E}(x_{in} - x_{in}^{fc})^2$, which is of order $1/n$. Similarly, we could show that $(ETS_n^{fc} - ETS_n^f)/n$ is of the order of $1/n^2$ (and that $(ETS_n^{fc} - ETS_n^c)/n$ is of the order of $1/n$).

(iii) From the stated results, all the others follow because $ETS_n^{fc} - ETS_n = (ETS_n^{fc} - ETS_n^c) + (ETS_n^c - ETS_n) = (ETS_n^{fc} - ETS_n^f) + (ETS_n^f - ETS_n)$. Indeed, of the three increments involved in any equality, we always know the order of at least two of them, and therefore the order of the third follows. *Q.E.D.*

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