



## Regulating Insider Trading When Investment Matters<sup>\*</sup>

LUIS ANGEL MEDRANO<sup>1</sup> and XAVIER VIVES<sup>2</sup>

<sup>1</sup>*Universitat Pompeu Fabra*; <sup>2</sup>*INSEAD and ICREA-UPF*

**Abstract.** We provide a general framework for analyzing the effects of insider trading on real investment and welfare as well as the consequences of different regulatory policies in a model where all traders are rational expected-utility maximizers and aware of their position in the market. We find that: with costly information acquisition, an “abstain-or-disclose” rule tends to be optimal; with free information acquisition, laissez-faire is better. This suggests enforcing an abstain-or-disclose rule with a high standard of proof for inside information. Our approach also uncovers the pitfalls of welfare analysis in the noise-trader model.

**Key words:** “abstain-or-disclose” rule, hedging, insider trading, noise traders, real investment, selective disclosure, speculation, welfare.

**JEL classification:** D82, G12, G14.

### 1. Introduction

The aim of this paper is to provide a general environment for conducting a welfare analysis of insider trading that supersedes the shortcomings of the received literature and identifies the trade-offs involved in forcing insiders to disclose their information before trading.

More and more countries are regulating insider trading, and there is evidence that enforcement of insider trading laws reduces the cost of equity in a country (Bhattacharya and Daouk, 2002). At the same time there is also

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evidence that insiders do trade in advance of information release and earn excess returns (see, Seyhun, 1986 or 1992; Damodaran and Liu, 1993; Aboody and Lev, 2000; for evidence in high-tech companies).<sup>1</sup> In fact, insider trading figures prominently in the recent wave of corporate scandals, as in the cases of Enron and ImClone.<sup>2</sup> Insider trading is perceived as being “unfair”, and many commentators think that insider profits should be curbed (see Fried, 1998). In contrast, others believe that private and social incentives are aligned and so firms should determine themselves what restrictions to impose (Carlton and Fischel, 1983).<sup>3</sup> Leading regulations of insider trading include an “abstain-or-disclose” rule in the U.S. and a prohibition against trading on inside (precise) information in the EU. Recently, tougher disclosure requirements have been imposed in the U.S. and in some European countries in order to avoid early selective disclosure of material information (to large investors, for example).<sup>4</sup> Furthermore, the recent scandals may prompt a rush to regulate insider trading more tightly.

In light of the concern about insider trading, it is somewhat surprising that the welfare consequences of regulating (or not) insider trading are less well understood. Indeed, despite an accumulation of work on the effects of insider trading, further progress on its welfare consequences has been prevented by the use of noise-trader models and other methodological problems related to the modelling of market power and information, as explained in Section 2. In order to help fill this gap, we examine the trade-offs associated with insider trading in a production economy in which all traders are rational expected-utility maximizers (doing away with noise traders) and aware of their position in the economy.

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<sup>1</sup> The authors find that insider gains in R&D-intensive firms are larger than in other firms. The rationale for this result is traced to the uniqueness of R&D capital to the firm, nontradability in organized markets, and poor disclosure (because of the accounting convention of expensing R&D).

<sup>2</sup> In the Im Clone case, Samuel Waksal (former CEO of the biotechnology firm) was charged with insider trading. According to the accusation, he attempted to sell shares of the company two days before it became public that the cancer drug Erbitux developed by the company would not pass the FDA test. Furthermore, he allegedly tipped family members (who sold about \$10 million of the stock over the following two days) and his friend Martha Stewart, who also sold shares the day before the announcement (*Financial Times*, June 13, 2002).

<sup>3</sup> In fact, the authors argue that there is no evidence that organizers of firms tried to restrict insider trading in the absence (or unenforcement) of regulations and therefore insider trading restrictions are not necessary to enhance firm value (and, indeed, could be distortionary).

<sup>4</sup> The “fair disclosure” rule of the U.S. Securities and Exchange Commission (SEC) states that “when an issuer, or person on its behalf, discloses material nonpublic information to certain enumerated persons (in general, securities market professionals and holders of the issuer’s securities who may well trade on the basis of the information), it must make public disclosure of that information”. See the SEC’s home page at <http://www.sec.gov/rules/final/33-7881.htm>.

Our analysis sheds some light on the appropriate regulation of insider trading:

- When is laissez-faire a sensible policy?
- When is it a good idea to enforce an “abstain-or-disclose” rule as in the U.S.?
- Does information need to be “precise”, in order to be considered inside information, as in the EU Directive on insider dealing?
- What are the effects of early selective disclosure of material information to large investors?

As a by-product of our modelling we put on solid ground the welfare analysis of the noise-trader model, which is a limiting case of our analysis. In particular, we can see when the welfare implications derived from the noise-trader model hold and when they are misleading.

The setup of our model is as follows. An entrepreneur (or a coalition of insiders, the initial owners of the firm) has a project requiring investment and – because he is risk averse – wants to hedge it partially by selling shares of the firm in the stock market. The market features competitive, risk-averse speculators/market makers and hedgers who have a random endowment of an asset correlated with the project of the firm. The entrepreneur/insider obtains information about the value of the project in the course of production at  $t = 1$ , after investment is made and the number of shares to be issued has been determined at  $t = 0$ . The stock market opens at  $t = 2$ . Neither the stock price nor private information have a chance to affect investment.

Different scenarios fit the model. A first interpretation would be trading in a secondary market (at  $t = 2$ ) where the firm is under the control of a coalition of insiders (the initial owners have sold a small fraction of the firm). For example, a coalition of insiders in a high-tech company learn valuable information about the effectiveness of a new drug (being developed by the firm) that shortly will be released to the market. Or perhaps the manager provides information or an early warning to the major shareholders – the initial owners – in exchange of a promise of nonintervention (Maug, 2002).<sup>5</sup>

We could also envision a second scenario of a venture capitalist starting a new project and deciding to go public (at  $t = 0$ ); the firm could be floated

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<sup>5</sup> The project cannot be floated at the investment stage because of agency problems (the manager must keep shares in order to lessen moral hazard). Alternatively, information disclosure associated with the flotation would tip competitors who could move and try to copy the product (this may be particularly relevant in high-tech industries; see Campbell (1979) and Yosha (1995)). At the same time, in high-tech industries outside investors would be reluctant to invest in a new project because they face very high risk with no information. Venture capital may then come to the rescue.

with an IPO auction (at  $t = 2$ ). The model would then be about trading in the primary market.<sup>6</sup> However, in our model we assume that all traders can short-sell the shares of the firm. This is not typical in a IPO auction (although feasible, for example, with forward contracts in the OTC market). Regardless, in equilibrium and on average, speculators and hedgers in our model buy shares that are sold by the entrepreneur.

Finally, the model fits a futures market. A producer wants to hedge in the futures market at  $t = 2$  part of his production and obtains private information at  $t = 1$  about the future value of the product once the seeds have been planted at  $t = 0$  (in Bray, 1985). This could be an agricultural producer or, say, a diamond producer with market power in the futures market.

We consider a model of the CARA-normal variety and characterize linear equilibria with and without insider trading. The insider is risk averse, has market power, and receives a noisy signal about the liquidation value of the firm. The rest of the agents are also risk averse and are competitive. The question then arises concerning the correct benchmark for the case with no insider trading (IT). We argue below that two regimes are of relevance: public disclosure (PD) and no information (NI). We characterize equilibria in the three regimes: IT, PD, and NI.

The regimes NI and PD arise in a context where a “abstain-or-disclose” rule is applied to corporate insiders (as in the U.S. with SEC rule 10b-5 of the 1934 Act). The insider in possession of “material” nonpublic information must abstain from trading (until the information becomes public) or disclose the information to the market and then trade.<sup>7</sup> What constitutes “material” information is left vague (Seyhun, 1992). In contrast, the 2003 EU Directive on insider dealing requires (Article 1) the information to be “precise” (similarly, U.K. law requires inside information to be precise or specific). Article 2 prohibits insider trading and defines broadly who is an insider.<sup>8</sup> In the context of our model (first interpretation), the entrepreneur/coalition of insiders – when trading on the basis of their acquired private information – would be subject to

<sup>6</sup> At  $t = 0$  is when the firm would file a preliminary prospectus, which must be approved by the SEC if in the US. IPO auctions are widely used. For a comparison with fixed-price offerings see Chemmanur and Liu (2002).

<sup>7</sup> This happens when the other party to the transaction is entitled to know the information owing to a fiduciary duty (*Chiarella v. U.S.* and *Dirk v. SEC*) or other similar relationship (according to the misappropriation theory adopted in *U.S. v. Newman*). Rule 10b-5 applies to insiders but not to outside shareholders who may possess information on the company (because they do not have a fiduciary duty to other shareholders) unless if the shareholder owns more than 10% of the equity of the firm, in which case the shareholder is considered an “insider”. The 1934 Securities and Exchange Act defines corporate insiders as corporate officers, directors, and owners of 10% or more of any equity class of securities.

<sup>8</sup> The European procedure can be explained in part by a larger reliance on criminal prosecution (Maug, 2002). See the appendix for the relevant articles of the EU Directive.

the U.S. “abstain-or-disclose” rule or the EU prohibition on insider dealing. In the case of an IPO, if there is new material information after the filing at  $t = 0$ , the firm must file again the prospectus. This is equivalent to disclosure.<sup>9</sup> Under the EU Directive, issuers have the obligation to disclose inside information.

The effects of an “abstain-or-disclose” rule will depend on whether information is acquired for free or at a cost. If the entrepreneur/insider learns the signal for free in the course of his activity, then—when faced with the choice of (a) disclosing the information and trading or (b) not disclosing and not being able to trade – he will choose to disclose because only by trading can he hedge the investment risk. The relevant welfare comparison is between a public disclosure regime and insider trading. If learning the signal has a cost, then the entrepreneur has no motive to learn it, since the information must be disclosed before use. The relevant welfare comparison is between a regime in which the entrepreneur has no private information (NI) and insider trading (IT).

In the IT regime it is found, not surprisingly, that adverse selection may prevent the existence of a linear equilibrium. However, a linear equilibrium always exists when the combined risk-weighted informational advantage of the insider and the hedgers is not very high (in particular, when insurance is the principal aim of hedger trading). Despite the complexity of the analysis in which market power is combined with the presence of hedgers, we are able to obtain analytical results for the case when the risk-adjusted informational advantage of the hedgers is small, and we extend these results with simulations to a broad region of the parameter space. Our analysis yields the following conclusions.

The level of investment is increasing in the hedging effectiveness of the asset market from the point of view of the entrepreneur, which is (in general) decreasing in the precision of the information of the insider. An insider with better information will be able to hedge less of his investment because of adverse selection, but will be able to speculate more profitably.

If the signal is public knowledge (and not perfect), then an equilibrium always exists. As the precision of the signal increases, we see an increase in market depth, price volatility, and the average stock price along with a decrease in investment and in the expected utility of the insider, speculators, and (for reasonable parameter values) hedgers. Public information revelation leads to less uncertainty about the payoff of the project and to a deeper market, as speculators do not ask for a large discount to trade on the risky asset. However, the hedging capacity of the market decreases from the entrepreneur’s point of view since the public signal is then more precise. This is because early revelation of information destroys insurance opportunities

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<sup>9</sup> In an IPO auction, the issuing firm is typically required to disclose (in the prospectus) all relevant material information that may affect the value of the shares.

(Hirshleifer effect), which dominates the potentially beneficial effect of an increased market depth.<sup>10</sup> The dark side of information revelation looms larger. Despite the increase in market depth hedgers are hurt by the increase in volatility and decrease of the risk premium.

If the insider has information yet no obligation to disclose it (“laissez-faire”), then some of his information is leaked into prices and becomes public. This revelation of information has effects similar to public disclosure, but they are weaker because there is only partial revelation. However, there is an additional effect: because of adverse selection, speculators will now demand a larger premium to accommodate orders. The net effect on welfare and investment depends on the benchmark of comparison.

When compared with the NI regime (costly information acquisition), insider trading increases the price precision and, for reasonable parameter values, price volatility; it also reduces market depth and investment (inducing an increase in the expected price and so reducing the risk premium), with the result that the expected utility of all traders decreases. Hedgers are hurt because market depth is reduced and price volatility is increased. Hence, insider trading is Pareto-inferior, and adverse selection is the culprit. The possibility remains that for an entrepreneur who is close to risk neutral (or in high -“noise” scenarios, where the aggregate endowment shock of hedgers is very volatile), the insider gains with insider trading. This is so because of the speculative gains he can make with his private information.

When compared with the PD regime (costless information acquisition), insider trading reduces market depth but, unlike before, also reduces price precision and price volatility. The result is that – except for a very large risk-adjusted informational advantage of the insider – insider trading reduces real investment (and risk premia) as well as the expected utility of the insider and the speculators. Two negative forces impinge upon the hedging effectiveness of the stock market and investment: adverse selection and public information revelation (Hirshleifer effect). The IT regime suffers from adverse selection but minimizes public information disclosure. The PD regime eliminates adverse selection but maximizes public information disclosure. For reasonable parameter values, adverse selection counts for more and investment decreases under the IT regime in relation to the PD regime. The effect on hedgers is ambiguous and depends on the precision of information of the insider.<sup>11</sup> For a low precision of the insider’s information, the reduction in

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<sup>10</sup> See Hirshleifer (1971). If uncertainty about risk factors not correlated with the endowment of the hedger is resolved early, then the stock is more correlated with the hedger’s endowment and hedging opportunities may be improved (Dow and Rahi, 2001).

<sup>11</sup> Bernhardt and Hughson (2002) find that in the NYSE the price impact of information is positive but small because “the quality of the information signals is quite poor, particularly in the middle of the trading day”. This would suggest a low precision of the signals for informed traders.

price volatility is small and hedgers are hurt by insider trading; the opposite happens when the precision of the insider's information is high. For a large risk-adjusted informational advantage of the insider, investment increases with insider trading (because with public disclosure of the signal the Hirshleifer effect is severe) and a Pareto-superior outcome is obtained.

An interesting by-product of the analysis is that we can ascertain exactly when hedgers have (in the aggregate) demands of the noise-trader form – that is, when noise-trader demands provide a good approximation to the demands of rational hedgers. This happens precisely when the risk-weighted informational advantage of a hedger is very low (in particular, when hedgers are very risk averse). At the same time, this allows us to check whether the implicit welfare analysis in the noise-trader model is accurate. In this model the welfare of noise traders is typically measured by their aggregate expected profits (i.e., by how much money they lose in expected terms) and their losses are inversely proportional to the market depth. That is: the greater the depth of the market, the less money noise traders lose. The surprising result is that there is a large range of cases for which the welfare analysis of the noise trader model is misleading *qualitatively*, even when hedgers' risk aversion is high and the demands of traders approach those of the noise-trading model. For example, for a high precision of the insider's information, hedgers improve with insider trading (compared with a PD regime) despite the fact that market depth decreases, contradicting the implicit welfare criterion of noise-trader models. Indeed, hedgers also care about the risk premium and price volatility.

The paper proceeds as follows. Section 2 reviews the literature and our contribution. In Section 3 we present the model. Section 4 derives the equilibrium with insider trading and Section 5 the equilibrium with public disclosure. Section 6 analyzes the effects of insider trading, taking as benchmarks the NI and PD regimes. Section 7 relates our hedgers to the usual noise traders, and Section 8 covers the implications for regulating insider trading. Most of the proofs are collected in the appendix.

## 2. Literature Review

Much progress has been made in the analysis of insider trading: understanding its effects in terms of creating adverse selection, accelerating the resolution of uncertainty, and modifying insurance and hedging opportunities (see e.g. Manne, 1966, 1980; Demsetz, 1986; King and Roell, 1988; Manove, 1989; Fishman and Hagerty, 1989; Ausubel, 1990; Leland, 1992; Dennert, 1992, 1993; Bernhardt, Hollifield and Hughson, 1995; Repullo, 1999; Dow and Rahi, 2001; Bhattacharya and Nicodano, 2001). However, further progress in the analysis of the effects of insider trading is hampered by one or more of the following:

- Assumption of exogenous noise traders.
- Assumption of competitive behavior by agents (insider, entrepreneur) with market power.
- Ill-defined incentives to float the firm/project (e.g., risk-neutral entrepreneur sells firm when the expected price is lower than its fundamental value).
- “Inside” information emanating from outside the firm or with no productive value.

In this paper we set out to model the impact of insider trading on the investment and welfare of market participants when all agents are rational and aware of their position in the market. Our modeling of insider trading emphasizes the effects on *ex ante* investment in line with the analysis of Ausubel (1990) and Bhattacharya and Nicodano (2001). This is in contrast with the work of Leland (1992), Dow and Rahi (2001), and Medrano and Vives (2002), who analyze the effect of insider information on interim investment. Other papers have also done away with noise traders: Ausubel (1990), Bernhardt et al. (1995), Dow and Rahi (2001), Qi (1996), and Bhattacharya and Nicodano (2001); the latter two papers model noise traders as agents that suffer an interim preference shock as in Diamond and Dybvig (1983).<sup>12</sup> However, the cited works all stay within the competitive paradigm, with the exception of the overlapping generations model of Bernhardt et al. (1995).<sup>13</sup>

Ausubel (1990) considers an exchange economy, with rational traders and a unique rational expectations equilibrium, in which private information is received after investment by both the (competitive) insider and outsiders. Forbidding of insider trading increases the expected return of outsiders. So the latter invest more and this may benefit insiders. The result is that a ban on insider trading may be Pareto-improving. However, in this model inside information has no productive value (and is unrelated to investment). Bhattacharya and Nicodano (2001) model liquidity traders as early diers in a Diamond–Dybvig frame: there is a risk-neutral insider (not one of the entrepreneurs) who has an endowment of the risky asset, and there are potentially multiple equilibria. The authors find, using simulations, that inside information may be beneficial by making the price more informative and improving risk sharing (making short-term traders better off). Bernhardt et al. (1995) examine the trade-offs associated with the adverse selection and price information effects of insider trading in an

<sup>12</sup> In order to rationalize noise trading in different contexts, Diamond and Verrecchia (1981), Bhattacharya and Spiegel (1991), and Spiegel and Subrahmanyam (1992), also consider traders suffering endowment shocks.

<sup>13</sup> Huddart et al. (2001) study the effects of ex post revelation of insider trades in the context of the Kyle (1985) model.



overlapping generations model. They show that, when investment is sufficiently “information elastic”, insider trading may be welfare enhancing. In principle, the net effect on outsiders is ambiguous. Outsiders prefer informative prices (due to past insider trading) but prefer not to trade with insiders in the future. With persistent production shocks, the first effect dominates.

Perhaps the paper that is closest is Leland (1992) (and the extensions in Repullo (1999)). However we depart from them in that our entrepreneur is the insider, is risk averse, and has market power. In Leland (1992) the insider is external to the firm and risk averse but perfectly informed, and he takes into account the effect of his trade on the price. The entrepreneur is risk neutral and price-taking and, in fact, is forced to float the firm because (in the model) the expected fundamental value is larger than the expected price. In contrast, our model provides a rationale – risk aversion – for the entrepreneur to float the project, and we assume that, being the only supplier of the asset, he is aware of his market power. Furthermore, in Leland’s model (as in ours), the firm does not learn anything from the stock price. Leland (1992) shows that the average investment level may be higher with insider trading because risk averse outsiders increase the demand for the risky asset associated to investment. Expected stock prices will tend to increase, decreasing the risk premium (decreasing the conditional volatility of returns and increasing the *ex ante* volatility of prices). Leland performs also a welfare analysis that is subject to the usual problems in noise-trader models. In any case, he concludes that liquidity traders and outside investors are hurt and that insiders and owners of firms issuing shares benefit (because of a higher issuing price). The net effect is ambiguous (positive if investment is very sensitive to the current price, risk aversion of investors is low, and liquidity trading has low volatility). Repullo (1999) shows that some of Leland’s results are not robust to the introduction of noise in the information of the insider and analyzes some variations of the model with investment prior to trading. For example, the insider has no effect when modeled as having a positive mass instead of zero mass as in Leland. In any case, Leland’s model depends on noise traders and thus it cannot provide a satisfactory welfare analysis.

In the models discussed so far, the presence of insiders tends to make prices more informative. However, Fishman and Hagerty (1992) show that the presence of insiders may discourage information collection by outside investors/analysts, possibly leading to less informative prices.<sup>14</sup> There is also a related literature on public disclosure by firms that exploits potential

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<sup>14</sup> In their model, stock prices guide the entry decisions of potential entrants. A related result is obtained by Khanna et al. (1994). There is also a literature on how information in stock prices help managerial incentives (see Holmstrom and Tirole, 1993).

trade-offs between adverse selection and Hirshleifer effects. For instance, Diamond (1985) focuses on the effect of public information on (endogenous) costly information collection, comparing a regime that features competitive privately informed investors with a regime in which, in addition to private information, the firm discloses a public signal. In the Diamond model, public information increases welfare because fewer investors collect costly information. In our model, when public disclosure increases welfare it does so by (a) increasing real investment, and (b) expanding opportunities to share risks by reducing the adverse selection problem caused by the presence of insiders.

The issues raised in our paper have a parallel in the literature on security design (see Demange and Laroque, 1995, 2002; Rahi, 1996). Demange and Laroque (2002) consider a setting similar to ours, albeit in a competitive stock market with risk-neutral market makers, and study how the entrepreneur might design the securities to be offered in the market, depending on different informational assumptions about the signals received by outsiders. An insight from their analysis is that the entrepreneur may want to favor projects for which the asymmetry of information is less pronounced.

### 3. The Model

Consider an economy where a single risky asset (with random *ex post* liquidation value  $v$ ) and a riskless asset (with unitary return) are traded among a continuum of risk-averse competitive uninformed speculators, a continuum of risk-averse competitive hedgers, and a large informed trader (the insider). The risky asset is traded at a price  $p$  and thus generates a return  $v - p$ .

*Insider.* The insider is an entrepreneur who undertakes a risky business. Let  $q$  denote the level of investment (and also the number of shares issued). The stochastic return per unit of investment (and the liquidation value per share) is given by  $v$ . The technology is represented by a deterministic quadratic cost function of the form  $C(q) = c_1q + c_2q^2/2$ , where  $c_1 \geq 0$  and  $c_2 \geq 0$ . The insider is risk averse and has CARA utility:  $U(W_i) = -\exp\{-\rho_i W_i\}$ , where  $\rho_i > 0$  is the coefficient of constant absolute risk aversion and  $W_i$  is the insider's final wealth. By virtue of his position, the entrepreneur has some privileged (inside) information  $s$  on the likely realization of the production return  $v$ . We assume that  $s$  is observed after choosing  $q$ , but before trading in the security market, and that it is a noisy version of  $v$ :  $s = v + \epsilon$ , where  $v$  and  $\epsilon$  are independent and  $E[\epsilon] = 0$ .<sup>15</sup> The final wealth of the insider, upon

<sup>15</sup> We will use  $\bar{x}$ ,  $Ex$ , or  $E[x]$  to denote the expected value of a random variable  $x$ ; we use  $\sigma_x^2$  and  $\sigma_{xy}$  to denote (respectively) the variance of  $x$  and the covariance between  $x$  and  $y$ .

choosing a level of investment  $q$  and buying  $x_i$  shares, is given by  $W_i = vq - C(q) + (v - p)x_i$ . The position of the entrepreneur in the market is therefore  $q + x_i$ .<sup>16</sup>

The insider has two motives to trade in the security market. First, he is interested in trading in order to hedge part of the risk coming from real investment  $q$  (here  $vq - C(q)$  is the random value of the entrepreneur's endowment before trading in the security market). Second, he may trade for speculative reasons in order to exploit his private information about  $v$ . The insider acts strategically (i.e., takes into account the effect his demand has on prices) and submits a demand schedule  $X_i(s, p)$  that is contingent on his private information  $s$ . If  $x_i$  is positive then the entrepreneur is a net buyer of shares; he is a net supplier if  $x_i$  is negative ( $-x_i$  will be the entrepreneur's net supply of the risky asset). In equilibrium we will see that  $E[x_i] < 0$  and the entrepreneur will sell shares on average. In the IPO scenario,  $E[-x_i] > 0$  could be interpreted as the firm's (net) supply of shares.

*Speculators.* There is a continuum of competitive uninformed speculators (or market makers) indexed in the interval  $[0, 1]$  (endowed with the Lebesgue measure). The final wealth of speculator  $k$  buying  $x_{sk}$  shares at price  $p$  is given by  $W_{sk} = (v - p)x_{sk}$ , where the buyer's initial nonrandom wealth is normalized to zero.<sup>17</sup> Speculators trade to profit by taking some of the risks that the entrepreneur and hedgers try to hedge (but their trades are not motivated by any informational advantage or any need for hedging). Speculators do not bear any risk before trading in the security market; they are risk averse and have CARA utilities:  $U(W_{sk}) = -\exp\{-\rho_s W_{sk}\}$ , where  $\rho_s > 0$ . Speculator  $k$  submits a demand schedule  $X_{sk}(p)$ . Since they have rational expectations, speculators use their observation of the price to update their beliefs about  $v$ .

*Hedgers.* There is a continuum of competitive hedgers indexed in the interval  $(1, 2]$  (endowed with the Lebesgue measure). Hedger  $j$  has an initial endowment  $u_j$  of an asset with future (random) value  $z$  correlated with  $v$ . For example, suppose the firm is in the telecommunications sector and the hedger owns stock in a nontraded firm in the same sector with liquidation value  $z$ ; or, alternatively, that  $u_j$  is linked to the human capital of workers of the firm. The final wealth of hedger  $j$  buying  $x_{hj}$  shares at price  $p$  is given by  $W_{hj} = u_j z + (v - p)x_{hj}$ . Hedgers are risk averse and have CARA utilities:

<sup>16</sup> Production and share issuing is modeled as in Leland (1992). See also Bray (1985) for a related model of futures markets, where  $p$  would correspond to the price in the futures market and  $v$  to the future random spot price.

<sup>17</sup> It is well known that, with constant absolute risk aversion, a trader's demand for a risky asset does not depend on his initial nonrandom wealth; hence we can assume (without loss of generality) that speculators have zero initial wealth.

$U(W_{hj}) = -\exp\{-\rho_h W_{hj}\}$ , where  $\rho_h > 0$ . Hedger  $j$  privately observes  $u_j$  and places a demand schedule  $X_{hj}(p, u_j)$  that is contingent on his private information  $u_j$ . We assume that  $u_j$  may be written as  $u_j = u + \eta_j$ , where  $u$  and  $\eta_j$  are independent (and  $\eta_j$  is independent of  $\eta_l$  for all  $j \neq l$ ). We will use the convention that errors cancel out in the aggregate,  $\int_1^2 \eta_j dj = 0$  a.s. Hence,  $\int_1^2 u_j dj = \int_1^2 (u + \eta_j) dj = u + \int_1^2 \eta_j dj = u$  a.s., so that  $u$  is the aggregate risky endowment of the hedgers. A hedger uses observed prices to update his beliefs about  $v$ . A hedger's primary motive for trading is to reduce risks. However, the endowment shock to hedger  $j$  is his private information, and each hedger places a demand schedule. Therefore, hedgers' demand has also a speculative component.

*Timing.* The timing of events in the model is as follows. At  $t = 0$ , the entrepreneur chooses the level of real investment  $q$  and also the number of shares issued (at this time he has no private information). The level of investment  $q$  is public information. At  $t = 1$ , the entrepreneur receives a private signal  $s$  about  $v$  and hedger  $j$  receives an endowment shock  $u_j$ . At  $t = 2$ , the entrepreneur, speculators, and hedgers all submit their demand schedules, the market-clearing price is set, and trade occurs. Finally, at  $t = 3$ , the terminal values  $z$  and  $v$  are realized and agents consume. It may be useful to view the insider as a coalition of the initial owners of the firm who face an investment opportunity. They (or their manager) decide the level of investment, knowing that the next round of trade will incorporate the expectations about the value of the project and that the coalition of insiders will by then have privileged information. Risk aversion provides the incentive to float the project.

*Distributional assumptions.* All random variables are assumed to be normally distributed:  $v \sim N(\bar{v}, \sigma_v^2)$ ,  $z \sim N(\bar{z}, \sigma_z^2)$ ,  $u \sim N(0, \sigma_u^2)$ ,  $\epsilon \sim N(0, \sigma_\epsilon^2)$ , and  $\eta_j \sim N(0, \sigma_\eta^2)$  for all  $j$ . Without loss of generality, we assume that  $z$  may be written as  $z = \sigma_z[(r_{vz}/\sigma_v)v + \sqrt{1-r_{vz}^2}\xi]$ , where  $r_{vz}$  is the correlation coefficient between  $z$  and  $v$  and where  $\xi \sim N(0, 1)$ . Moreover, we assume that  $\xi$  is independent of any other variable in the model, that  $\text{cov}(v, u) = \text{cov}(s, u) = \text{cov}(v, u_j) = \text{cov}(s, u_j) = \text{cov}(v, \epsilon) = \text{cov}(v, \eta_j) = \text{cov}(u, \eta_j) = \text{cov}(s, \eta_j) = \text{cov}(\epsilon, u) = \text{cov}(\epsilon, \eta_j) = 0$  for all  $j$ , and that  $\text{cov}(\eta_j, \eta_l) = 0$  for all  $j \neq l$ . Let  $R_{sv}$  denote the square of the correlation coefficient between  $s$  and  $v$ ,  $R_{sv} = \sigma_v^2 / (\sigma_v^2 + \sigma_\epsilon^2)$ , and let  $R_u$  denote the square of the correlation coefficient between  $u$  and  $u_j$ ,  $R_u = \sigma_u^2 / (\sigma_u^2 + \sigma_\eta^2)$ .

Throughout this paper, the subscripts  $i$ ,  $s$ , and  $h$  will refer to the insider, the speculators, and the hedgers, respectively.

*Linear equilibria and pricing.* We will restrict attention to perfect Bayesian linear (affine) equilibria. At these equilibria, agents' strategies at the market stage will be linear in the signals they observe.

- The insider's strategy will be a linear function of his private information  $s$ , the price  $p$  (since he submits a limit order), and  $q$ . The strategy may be written (without loss of generality)<sup>18</sup> as

$$X_i(s, p) = \alpha_i(s - \bar{v}) + \beta_i(\bar{v} - p) - \gamma_i q,$$

where  $\alpha_i$ ,  $\beta_i$ , and  $\gamma_i$  are endogenous nonrandom parameters.

- Speculator  $k$ 's strategy may be written (without loss of generality) as

$$X_{sk}(p) = \beta_s(\bar{v} - p) - \gamma_s q,$$

where  $\beta_s$  and  $\gamma_s$  are endogenous nonrandom parameters.

- Finally, hedger  $j$ 's strategy  $x_{hj}$  will depend on his endowment shock  $u_j$ , the price, and  $q$ . We can assume that it may be written as

$$X_{hj}(p, u_j) = \beta_h(\bar{v} - p) - \delta u_j - \gamma_h q,$$

where  $\beta_h$ ,  $\gamma_h$ , and  $\delta$  are endogenous nonrandom parameters.

The equilibrium price must satisfy the market-clearing condition

$$X_s(p) + X_h(p, u) + X_i(p, s) + q = q, \quad (1)$$

where  $X_s(p) = \int_0^1 X_{sk}(p) dk$  is the speculators' aggregate demand and  $X_h(p, u) = \int_1^2 X_{hj}(p, u_j) dj$  is the hedgers' aggregate demand. Given the linear strategies posited above,  $X_s(p) = \beta_s(\bar{v} - p) - \gamma_s q$ ,  $X_h(p, u) = \beta_h(\bar{v} - p) - \delta u - \gamma_h q$ , and the equilibrium price is given by

$$p = \bar{v} + \frac{1}{\beta_i + \beta_s + \beta_h} [\alpha_i(s - \bar{v}) - \delta u] - \frac{\gamma_i + \gamma_s + \gamma_h}{\beta_i + \beta_s + \beta_h} q,$$

or equivalently,

$$p = \bar{v} - \Gamma q + \frac{\alpha_i(s - \bar{v}) - \delta u}{\Lambda},$$

where  $\Gamma = (\gamma_i + \gamma_s + \gamma_h)/(\beta_i + \beta_s + \beta_h)$  and  $\Lambda = \beta_i + \beta_s + \beta_h$ . That is, the equilibrium price will be a linear function of the random inside information  $s$ , the hedgers' random aggregate endowment  $u$  (errors  $\eta_j$  cancel in the aggregate), and the level of real investment  $q$ .

*Reasonable parameter values.* We extend the analytical results with simulations. We take two variations of a base case for our simulations, assuming throughout that  $\rho_h \geq \rho_i > \rho_s$  and that volatilities are not too far from market

<sup>18</sup> We should write  $X_i(s, p) = \alpha_i s - \beta_i p - \gamma_i q + \varphi_i$ , but in equilibrium we will have  $\varphi_i = (-\alpha_i + \beta_i)\bar{v}$ .

values (similar to those in Leland (1992), for example). The base case has  $\rho_h = 3$ ,  $\rho_i = 2$ , and  $\rho_s = 1$ ; volatilities are given by  $\sigma_v = 0.2$ ,  $\sigma_u = 0.1$ ,  $\sigma_z = 0.2$  and covariances by  $r_u = \sqrt{R_u} = 0.1$ ,  $r_{vz} = (\sigma_{vz})/(\sigma_v\sigma_z) = 0.91$ , and  $\bar{v} = 1$ ; and  $R_{sv}$  ranges from 0 to 1. In variation 1 (BC1) we have  $c_1 = 0.9$  and  $c_2 = 0.02$  while in variation 2 (BC2) we set  $c_1 = c_2 = 0$ . We also consider variations in  $\rho_h$ ,  $\rho_i$ ,  $r_u$ ,  $\sigma_u$ , and  $\sigma_v$ . (For example, we consider a volatility of the fundamental value of  $\sigma_v = 0.6$ , which is of the Nasdaq type, in contrast with the base case of  $\sigma_v = 0.2$ , which is of the NYSE type; we also consider  $\rho_h = 6$ ,  $\rho_i \in \{0.1, 0.2, 0.5, 1.5\}$  high-noise scenarios with  $\sigma_u \in \{0.5, 0.6, 0.7\}$  and  $r_u = 0.4$  and  $0.5$ .) It is worth remarking that, in the parameter grid considered, the magnitude of the computed equilibria is on the order of  $2 \times 3 \times 2 \times 5 \times 4 \times 3 \times 101 = 72\,720$ .

*Market quality parameters.* We will analyze the following dimensions of market quality:

- (1) *Market depth* is measured by the inverse of the price impact of a one-unit increase in the hedging demand  $\delta u$ :

$$\Lambda = \left( \left| \frac{dp}{d(\delta u)} \right| \right)^{-1}.$$

The higher is  $\Lambda$ , the lower is the price impact of an increase in  $\delta u$  and the higher is the market depth. In equilibrium,  $\Lambda$  is equal to the sum of the price sensitivities of the demands of the insider, speculators, and hedgers:

$$\Lambda = \beta_s + \beta_h + \beta_i.$$

- (2) *Price volatility* is defined as the variance of the equilibrium price,  $\text{var}[p]$ . In our model, volatility is caused by information about the liquidation value  $v$  and by uncertainty about the hedgers' aggregate random endowment  $u$ . In equilibrium, price volatility will be increasing in the volatility of inside information, the volatility of hedgers' aggregate random endowment, and the trading intensities of the entrepreneur ( $\alpha_i$ ) and the hedgers ( $\delta$ ). It will be decreasing in the depth of the market  $\Lambda$ .
- (3) *Price informativeness*  $\tau$  is defined as the precision of the price in the estimation of the future liquidation value  $v$ :

$$\tau = \frac{1}{\text{var}[v | p]}.$$

The more informative the price is, the lower is the volatility of the liquidation value conditional on the price.

#### 4. Equilibrium with Insider Trading

In this section we first characterize equilibria in the securities market for a given investment level  $q$  when the entrepreneur/insider is allowed to trade on the basis of his private information. We then go on to characterize the optimal investment policy.

##### 4.1. EQUILIBRIUM IN THE SECURITIES MARKET

Speculator  $k$ 's objective function given his information  $\{p\}$  can be written as

$$E[-\exp\{-\rho_s W_{sk}\}|p] = -\exp\left\{-\rho_s\left(E[W_{sk}|p] - \frac{\rho_s}{2}\text{var}[W_{sk}|p]\right)\right\}.$$

This expression follows because we restrict ourselves to linear equilibria and so preserve the normality of  $W_{sk}$  conditional on  $p$ .<sup>19</sup> Since  $E[W_{sk}|p] = x_{sk}E[v-p|p]$  and  $\text{var}[W_{sk}|p] = x_{sk}^2\text{var}[v-p|p]$ , maximizing with respect to  $x_{sk}$  yields a demand function for the risky asset of

$$X_{sk}(p) = \frac{E[v-p|p]}{\rho_s\text{var}[v-p|p]}, \quad (2)$$

which is linear in  $p$  since  $\text{var}[v-p|p]$  is constant and since  $E[v-p|p]$  is linear in  $p$  (owing to the normality assumption). All the speculators will place the same demand schedule (since all of them have the same information), so the speculators' aggregate demand  $X_s(p)$  will be given by the same expression. This demand will depend on  $q$  because knowledge of  $q$  is needed to infer information about  $s$  from the price; it may be written as  $X_s(p) = \beta_s(\bar{v} - p) - \gamma_s q$ .

Similarly, hedger  $j$  will choose  $x_{hj}$  to maximize

$$E[U(W_{hj})|p, u_j] = -\exp\{-\rho_h(E[W_{hj}|p, u_j] - \rho_h\text{var}[W_{hj}|p, u_j]/2)\},$$

where  $E[W_{hj}|p, u_j] = u_j E[z|p, u_j] + (E[v|p, u_j] - p)x_{hj}$  and  $\text{var}[W_{hj}|p, u_j] = u_j^2\text{var}[z|p, u_j] + x_{hj}^2\text{var}[v-p|p, u_j] + 2u_j x_{hj}\text{cov}[z, v-p|p, u_j]$ .

<sup>19</sup> Using the fact, that for a normally distributed random variable  $e$ ,  $E[\exp\{e\}] = \exp\{E[e] + \text{var}[e]/2\}$ .

From the first-order condition, hedger  $j$ 's optimal demand for shares is given by

$$X_{hj}(p, u_j) = \frac{E[v - p|p, u_j] - \rho_h u_j \text{cov}[z, v - p|p, u_j]}{\rho_h \text{var}[v - p|p, u_j]}. \quad (3)$$

Hedger  $j$ 's demand may be decomposed into two terms as follows.

- *Speculative demand:*  $E[v - p|p, u_j]/(\rho_h \text{var}[v - p|p, u_j])$ , which will depend on  $q$  (because this helps reading the information about  $s$  in the price) and on  $u_j$  provided that  $R_u > 0$  (because then  $u_j$  contains information on  $u$ , which in turn helps to recover information about  $s$  in the price).
- *Hedge supply:*  $-(\text{cov}[z, v - p|p, u_j]/\text{var}[v - p|p, u_j])u_j = -(\sigma_{uz}/\sigma_v^2)u_j$ . The amount of the hedger's initial endowment ( $u_j$ ) that is hedged in the market is proportional to the correlation between the value of the hedger's asset  $z$  and the return of the risky security  $v - p$  conditional on the hedger's information  $\{p, u_j\}$ .

The demand of hedger  $j$  can be written then as  $X_{hj}(p, u_j) = \beta_h(\bar{v} - p) - \gamma_h q - \delta u_j$ . The hedger's aggregate demand will be given by

$$\begin{aligned} X_h(p, u) &= \int_1^2 x_{hj}(p, u_j) dj = \beta_h(\bar{v} - p) - \gamma_h q - \delta \int_1^2 u_j dj \\ &= \beta_h(\bar{v} - p) - \gamma_h q - \delta u. \end{aligned}$$

Now, from the market-clearing condition  $x_s + x_h + x_i = 0$ , the relation between the insider's trade  $x_i$  and the price is given by

$$p = \bar{v} + \lambda[x_i - \delta u - (\gamma_s + \gamma_h)q], \quad \text{where} \quad \lambda = \frac{1}{\beta_s + \beta_h}.$$

The entrepreneur's maximization problem at the market stage is

$$\max_{x_i} E[-\exp\{-\rho_i W_i\} | s, p]$$

$$\text{s.t.} \quad W_i = vq - Cq + (v - p)x_i$$

$$p = \bar{v} + \lambda[x_i - \delta u - (\gamma_s + \gamma_h)q].$$



Given normality, this is equivalent to maximizing

$$E[W_i|s, p] - \frac{\rho_i}{2} \text{var}[W_i|s, p] = \\ qE[v|s] - C(q) + x_i\{E[v|s] - p\} - \frac{\rho_i}{2}(x_i + q)^2 \text{var}[v|s].$$

Although the price has no information to aggregate, it is still useful from the insider's point of view because it allows him to infer the exact amount of noise created by hedge trading (and thus eliminate the price risk it creates). If the second order-condition holds,  $2\lambda + \rho_i \text{var}[v|s] > 0$ , then the insider has a well-defined demand function

$$X_i(s, p) = \frac{E[v|s] - p - \rho_i q \text{var}[v|s]}{\rho_i \text{var}[v|s] + \lambda}, \quad (4)$$

where  $E[v|s] = \bar{v} + R_{sv}(s - \bar{v})$  and  $\text{var}[v|s] = (1 - R_{sv})\sigma_v^2$ . We may write  $x_i$  as

$$X_i(s, p) = \alpha_i(s - \bar{v}) + \beta_i(\bar{v} - p) - \gamma_i q,$$

where

$$\alpha_i = \frac{R_{sv}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \quad \beta_i = \frac{1}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \quad \gamma_i = \frac{\rho_i(1 - R_{sv})\sigma_v^2}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}.$$

The entrepreneur's asset position can also be decomposed into two terms.

- *Speculative demand:*  $(E[v|s] - p)/(\rho_i \text{var}[v|s] + \lambda)$ , according to which the insider buys (sells) if his estimate of the asset liquidation value is greater (lower) than the price. Moreover, the weight placed on inside information  $\alpha_i$  is increasing in the precision of the information and is decreasing in the insider's risk aversion and the slope of residual supply.
- *Hedge supply:*  $HS_i = (\rho_i \text{var}[v|s]) / (\rho_i \text{var}[v|s] + \lambda) q = \gamma_i q$ . This depends on real investment, which determines the initial risk borne by the entrepreneur before trading on the asset market, but it is independent of the realization of his information. Note that  $\gamma_i > 0$  unless,  $R_{sv} = 1$  (perfect information) or  $\rho_i = 0$  (risk neutrality), in which case the entrepreneur does not want to hedge any part of the investment. Furthermore, it is never optimal to hedge all the risk due to real investment,  $\gamma_i < 1$ , provided the entrepreneur has market power,  $\lambda > 0$ . He reduces his hedge supply in order to get a higher (expected) price (a price-taking entrepreneur would have a hedge supply

$HS_i = q$ ). The entrepreneur undertakes a risky business and then sells part of the risky asset to obtain insurance. We think of  $\gamma_i = HS_i/q$  as the hedge ratio.

Following a standard procedure (Kyle, 1989), we can characterize a linear and imperfectly competitive rational expectations equilibrium (a linear Bayesian equilibrium in demand functions). A sketch of the proof can be found in the appendix.

**PROPOSITION 1.** If there is an imperfectly competitive rational expectations equilibrium in the security market, then it is given by

$$\begin{aligned} X_s(p) &= \beta_s(\bar{v} - p) - \gamma_s q, \\ X_h(u) &= \beta_h(\bar{v} - p) - \gamma_h q - \delta u, \\ X_i(s, p) &= \alpha_i(s - \bar{v}) + \beta_i(\bar{v} - p) - \gamma_i q, \\ p &= \bar{v} - \Gamma q + \frac{\alpha_i(s - \bar{v}) - \delta u}{\Lambda}, \end{aligned}$$

where

$$\begin{aligned} \Lambda &= \beta_s + \beta_h + \beta_i, \\ \Gamma &= (\gamma_s + \gamma_h + \gamma_i)/\Lambda, \\ \alpha_i &= \frac{R_{sv}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \\ \beta_i &= \frac{1}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \\ \gamma_i &= \frac{\rho_i(1 - R_{sv})\sigma_v^2}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \\ \beta_s &= \frac{1}{\rho_s\sigma_v^2} \frac{\delta^2\sigma_u^2 - (\alpha_i/\lambda)\sigma_v^2}{\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2}, \\ \gamma_s &= \frac{-\alpha_i}{\rho_s[\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2]} \gamma_i, \end{aligned}$$

$$\beta_h = \frac{1}{\rho_h \sigma_v^2} \frac{(1 - R_u) \delta^2 \sigma_u^2 - (\alpha_i / \lambda) \sigma_v^2}{(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

$$\gamma_h = \frac{-\alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2] (1 + \alpha_i E)} \gamma_i,$$

$$\delta = \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) \left\{ 1 + \frac{R_u \alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2]} \right\}^{-1},$$

and

$$\lambda = \frac{1}{\beta_s + \beta_h},$$

with

$$E = \frac{1}{\rho_s} \left[ \delta^2 \sigma_u^2 + \frac{R_{sv} (1 - R_{sv})}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda]^2} \sigma_v^2 \right]^{-1}$$

$$+ \frac{1}{\rho_h} \left[ (1 - R_u) \delta^2 \sigma_u^2 + \frac{R_{sv} (1 - R_{sv})}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda]^2} \sigma_v^2 \right]^{-1}.$$

In principle, to find the equilibrium we must solve a system of eleven equations with eleven unknowns ( $\Lambda, \Gamma, \alpha_i, \beta_i, \gamma_i, \beta_s, \gamma_s, \beta_h, \gamma_h, \delta, \lambda$ ). However, if we have the equilibrium values of  $\delta$  and  $\lambda$ , then (from the first nine equations) the equilibrium values of the first nine parameters ( $\Lambda, \Gamma, \alpha_i, \beta_i, \gamma_i, \beta_s, \gamma_s, \beta_h, \gamma_h$ ) are easy to compute. By substituting the equations

$$\alpha_i = \frac{R_{sv}}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda},$$

$$\beta_s = \frac{1}{\rho_s \sigma_v^2} \frac{\delta^2 \sigma_u^2 - (\alpha_i / \lambda) \sigma_v^2}{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

$$\beta_h = \frac{1}{\rho_h \sigma_v^2} \frac{(1 - R_u) \delta^2 \sigma_u^2 - (\alpha_i / \lambda) \sigma_v^2}{(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

into the last two equations in the proposition, we obtain the following two equations in  $(\delta, \lambda)$ :

$$\frac{R_u R_{sv}}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda} \delta = \rho_h \left[ \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) - \delta \right] \left\{ (1 - R_u) \delta^2 \sigma_u^2 + \frac{(1 - R_{sv}) R_{sv} \sigma_v^2}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda]^2} \right\}, \quad (5)$$

$$\lambda = \frac{\sigma_v^2}{\delta^2 \sigma_u^2} \frac{1 + \frac{\alpha_i}{\rho_s [\delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} + \frac{\alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]}}{1 + \frac{1}{\rho_s [\delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} + \frac{1 - R_u}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]}}. \quad (6)$$

There is a linear equilibrium if and only if there is a solution to this two-equation system in  $(\delta, \lambda)$  with  $\lambda \geq 0$ .

The expected price is equal to the prior expected liquidation value minus a *risk premium*,  $\bar{p} = \bar{v} - \Gamma q$ . The risk premium is positive and is directly proportional to the level of investment, where  $\Gamma = (\gamma_s + \gamma_h + \gamma_i)/\Lambda$ . The equilibrium parameter  $\Lambda = \beta_s + \beta_h + \beta_i$  is an index of *market depth*.

We define the *price precision* as  $\tau \equiv (\text{var}[v | p])^{-1}$ ; it is an index of the informativeness of the price about the liquidation value  $v$ . The price contains information about  $v$  if and only if traders with information on fundamentals trade on the basis of that information. Thus, it is natural to expect that the higher the trader's sensibility to fundamentals information, the more informative the price. This is true in equilibrium.

The entrepreneur, on average, is a net supplier of the risky asset. That is,  $E[x_i] = q(\beta_i \Gamma - \gamma_i) < 0$ . Because the risk premium is positive, the *ex ante* expected value of his speculative demand is positive ( $\beta_i(\bar{v} - \bar{p}) = \beta_i \Gamma q > 0$ ) and the entrepreneur sells a nonrandom quantity of the risky asset to hedge his real investment  $-\gamma_i q$ . In equilibrium, the entrepreneur's hedge supply is greater than his expected speculative demand. However, when the entrepreneur receives a perfect signal  $R_{sv} = 1$ , we have  $\gamma_s = \gamma_h = \gamma_i = \Gamma = 0$  and so the entrepreneur does not hedge any part of the investment. This does not mean that the stock market is not active. The entrepreneur still wants to trade for speculative reasons and to absorb hedge demand from hedgers (and his expected demand is zero because there is no risk premium).

*Existence* of a linear equilibrium cannot be guaranteed. It is easy to find values of the exogenous parameters such that no linear equilibrium exists. However, equilibrium always exists when the insider has no private information. The nonexistence result should not be surprising. For example, Bhattacharya and Spiegel (1991) show how the market may break down because of asymmetric information when there are informational and hedging motives for trade.<sup>20</sup> The reason is adverse selec-

<sup>20</sup> In their model, if a linear equilibrium fails to exist then nonlinear equilibria in some feasible class do not exist either, except for a degenerate no-trade equilibrium. In our model we do not examine nonlinear equilibria, and no-trade equilibria arise only when the entrepreneur does not hedge the project at all.

tion. When the insider has no information there is no adverse selection problem; when he has private information then hedgers have also an informational advantage with respect to uninformed speculators. For example, if the squared correlation coefficient between  $u$  and  $u_j$ ,  $R_u$ , is close to unity then hedger  $j$  can recover from the price essentially the information of the insider because he observes  $u_j$  and this is very close to  $u$ . A linear equilibrium always exists when the adverse selection problem is moderate. This happens when the combined informational advantage of the insider and the hedgers is not very high – more precisely, when the risk-adjusted informational advantage of the insider ( $R_{sv}/\rho_i$ ) and/or the hedgers ( $R_u/\rho_h$ ) is sufficiently small.<sup>21</sup> This means in particular that a linear equilibrium exists when the main trading motive for hedgers and the entrepreneur is insurance, which happens when they are very risk averse ( $\rho_h$  and/or  $\rho_i$  large) and/or their informational advantage is small ( $R_u$  and/or  $R_{sv}$  small). It is worth noting that, if the entrepreneur is risk neutral ( $\rho_i = 0$ ), then a linear equilibrium will not exist unless  $R_u/\rho_h$  and/or  $R_{sv}$  are very small.

The following proposition states the result.

**PROPOSITION 2.** Given fixed values of the exogenous parameters  $\sigma_{vz}$ ,  $\sigma_u^2$ ,  $\sigma_v^2$ ,  $\sigma_z^2$ , and  $\rho_s$ , a unique linear equilibrium exists if  $(R_{sv}/\rho_i)$  and/or  $(R_u/\rho_h)$  are small. As  $R_u/\rho_h$  or  $R_{sv}/\rho_i$  tend to zero, the equilibrium parameter  $\delta$  tends to  $\sigma_{vz}/\sigma_u^2$ .

*Proof.* See appendix. □

Simulations in the BC1 range of “reasonable parameter values” establish that the largest value of  $R_{sv}$ ,  $\bar{R}_{sv}(R_u/\rho_h)$ , for which there is equilibrium is decreasing in  $R_u/\rho_h$ . For example, we find that  $\bar{R}_{sv}(0.25/3) = 0.1156$ ,  $\bar{R}_{sv}(0.16/3) = 0.2601$ ,  $\bar{R}_{sv}(0.25/10) = 0.5625$ ,  $\bar{R}_{sv}(0.01/3) = 1$ .

#### 4.2. INVESTMENT

For a given  $q$ , the insider’s ex ante expected utility may be written (after cumbersome manipulations; see Section A.1 in the appendix for a summary) as

$$J_i(q) = E[-\exp\{-\rho_i W_i\}] = -|SG_i||IG_i|\exp\{-\rho_i[q\bar{v} - C(q) - (\rho_i/2)q^2\sigma_v^2]\},$$

<sup>21</sup> It is worth noting that equilibrium may exist even if  $(R_{sv} = 1)$ . This will be so if, for some appropriate values of other parameters,  $R_u/\rho_h$  is small enough.

where

$$|SG_i| = \left\{ 1 + \frac{\rho_i(R_{sv}\sigma_v^2 + \delta^2\lambda^2\sigma_u^2)}{\rho_i(1 - R_{sv})\sigma_v^2 + 2\lambda} \right\}^{-1/2},$$

$$|IG_i| = \exp\{-(\rho_i^2/2)\sigma_v^2 dq^2\},$$

$$d = \frac{\rho_i\sigma_v^2}{\rho_i\sigma_v^2 + 2\lambda + \rho_i\lambda^2\delta^2\sigma_u^2} \left\{ 1 + \frac{(1 - R_{sv})\lambda}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda} \frac{\alpha_i E}{1 + \alpha_i E} \right\}^2$$

and

$$E = \frac{1}{\rho_s} \left[ \delta^2\sigma_u^2 + \frac{R_{sv}(1 - R_{sv})\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right]^{-1} \\ + \frac{1}{\rho_h} \left[ (1 - R_u)\delta^2\sigma_u^2 + \frac{R_{sv}(1 - R_{sv})\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right]^{-1}.$$

The insider's *ex ante* expected utility is the product of three terms: the utility derived from the speculative demand  $|SG_i|$ ,<sup>22</sup> the utility coming from real investment, and the utility derived from the insurance achieved *via* the hedge supply  $|IG_i|$ .

The optimal investment level solves  $\max_q J_i(q)$ . This is equivalent to solving

$$\max_q [q\bar{v} - C(q) - 0.5\rho_i\sigma_v^2 q^2(1 - d)].$$

The optimal level of real investment is obtained by equating (expected) marginal value  $\bar{v}$  to marginal cost  $C'(q) + \rho_i\sigma_v^2(1 - d)q$ , which is the sum of the marginal production costs and the (opportunity) cost related to the riskiness of real investment (see Figure 1). The optimal level of real investment is increasing in  $d$ , which is a measure of hedging effectiveness of the asset market from the entrepreneur's point of view:

$$q^{IT} = \frac{\bar{v} - c_1}{c_2 + \rho_i\sigma_v^2(1 - d)}. \quad (7)$$

<sup>22</sup> The speculative term has two components: the term  $R_{sv}\sigma_v^2$  is associated with gains from private information and the term  $\delta^2\lambda^2\sigma_u^2$  with gains from market making. The private information gains disappear, obviously, when there is no private information ( $R_{sv} = 0$ ).

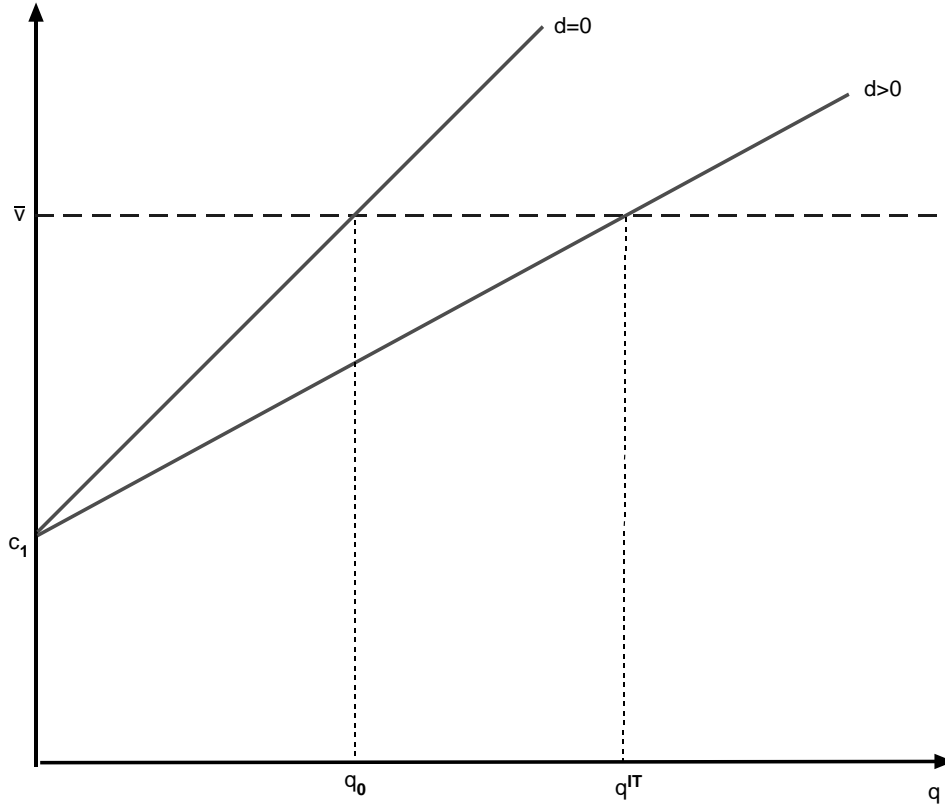


Figure 1. Investment ( $q$ ) and hedging effectiveness of the market ( $d$ ).

As expected, the direct impact of an increase in risk aversion  $\rho_i$  or risk  $\sigma_v^2$  is to decrease  $q^{IT}$ . There are other indirect effects operating through  $d$ , but the simulations we have performed with the model (see below) indicate that the direct effects prevail. An increase in the cost parameters  $c_1$  and  $c_2$  unambiguously decrease investment. If the market is totally ineffective in hedging or if there is no market, then  $d=0$  and investment  $q^{IT}$  hits its lowest level  $q_0 \equiv (\bar{v} - c_1)/(c_2 + \rho_i \sigma_v^2)$ . It is easy to check that, for  $R_{sv}$  close to 0 or close to 1, we have  $1 > d > 0$  (and in all our simulations the inequalities hold also). In particular, when  $R_{sv} = 1$ , it follows that  $d > 0$ . This may seem surprising because the insider has no hedging demand ( $\gamma_i = 0$ ) when  $R_{sv} = 1$ . However, by speculating the entrepreneur can partially hedge his investment provided that, conditional on the signal, the investment return and the speculative demand are correlated:

$$\text{cov} \left\{ E[v|s], \frac{E[v|s] - p}{\rho_i \text{var}[v|s] + \lambda} \right\} = \frac{R_{sv} \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda} > 0,$$

whenever  $R_{sv} > 0$ . If the insider is risk neutral then  $q^{IT} = (\bar{v} - c_1)/c_2$  and, obviously, the hedging effectiveness of the market  $d$  plays no role.

#### 4.3. EXPECTED UTILITIES

The insider's *ex ante* expected utility may be written as

$$EU_i \equiv J_i(q^{IT}) = -|SG_i| \exp\{-0.5\rho_i(\bar{v} - c_1)q^{IT}\},$$

the product of the speculative component with production and insurance gains.

The speculators' *ex ante* expected utility can be seen (see Section A.1 in the appendix) to be given by

$$\begin{aligned} EU_s &\equiv E[-\exp\{-\rho_s W_s\}] \\ &= -|SG_s| \exp\left\{-0.5 \frac{(\Gamma q)^2}{\text{var}[E(v|p) - p] + \text{var}[v|p]}\right\}, \end{aligned}$$

where

$$\begin{aligned} |SG_s| &= \left\{1 + \frac{\text{var}[E(v|p) - p]}{\text{var}[v|p]}\right\}^{-1/2}, \\ \text{var}[E(v|p) - p] &= \frac{[\delta^2 \sigma_u^2 + \alpha_i^2 R_{sv}^{-1} \sigma_v^2 - \alpha_i \Lambda \sigma_v^2]^2}{\Lambda^2 [\delta^2 \sigma_u^2 + \alpha_i^2 R_{sv}^{-1} \sigma_v^2]} \end{aligned}$$

and

$$\text{var}[v|p] = \sigma_v^2 \frac{\delta^2 \sigma_u^2 + \alpha_i^2 (1 - R_{sv}) R_{sv}^{-1} \sigma_v^2}{\delta^2 \sigma_u^2 + \alpha_i^2 R_{sv}^{-1} \sigma_v^2}.$$

Note that, for given  $\Lambda$  and  $\lambda$ ,  $EU_s$  increases with the risk premium  $\Gamma q$ , which is nothing else but the expected margin  $E(v - p) = \bar{v} - \bar{p} = \Gamma q$ .

The expressions for the expected utility of a hedger  $EU_h$  are complicated (see Section A.1) but simulations show that, for given  $\Lambda$  and  $\lambda$ ,  $EU_h$  also increases with the risk premium  $\Gamma q$ . To gain intuition as to why  $E[U(W_h)]$  decreases with  $\bar{p} = \bar{v} - \Gamma q$ , observe that when the hedger hedges his endowment the return is precisely  $p$ , and a higher expected level of  $p$  increases the risk borne by the agent. (If  $v = z$  so that  $\sigma_{vz} = \sigma_v^2$  and



if the endowment is completely hedged,  $x_{hj} = -u_j$ , then  $W_{hj} = u_j z + (v - p)x_{hj} = u_j p$ .) Furthermore, it is possible to show that if hedgers are sufficiently risk averse then  $EU_h$  increases with  $\Lambda$  and decreases in  $\text{var}[p]$ , *ceteris paribus*.<sup>23</sup>

### 5. Equilibrium With Public Disclosure

We wish to compare the equilibrium with insider trading, which is described in Proposition 1, with the equilibrium in the same market without insider trading. What is meant exactly by a “market without insider trading”? A wide variety of restrictions on insider trading could be considered. We will explore in Section 6 the consequences of prohibiting insider trading *via* an “abstain-or-disclose” rule. Two relevant benchmarks will be a PD regime in which the entrepreneur publicly reveals his inside information  $s$  and a NI regime in which the entrepreneur has no private information,  $R_{sv} = 0$ . In this section we explore the two benchmarks in which there is symmetric information about  $v$ .

If the insider publicly reveals his private information  $s$  before trading in the asset market, then all the agents share the same information about  $v$ . The next proposition describes the equilibrium in this case.

**PROPOSITION 3 (PD regime).** If  $R_{sv} < 1$  and the entrepreneur publicly discloses his inside information before trading on it, then there is a unique, linear, and imperfectly competitive rational expectations equilibrium in the asset market that is characterized by

$$\begin{aligned} X_s(s, p) &= \beta_s(\bar{v} - p) + \alpha_s(s - \bar{v}), \\ X_h(u) &= \beta_h(\bar{v} - p) + \alpha_h(s - \bar{v}) - \delta u, \\ X_i(s, p) &= \alpha_i(s - \bar{v}) + \beta_i(\bar{v} - p) - \gamma_i q, \\ p &= E[v|s] - \Gamma q - \frac{\delta u}{\Lambda}, \end{aligned}$$

where

$$\alpha_i = \frac{R_{sv}}{\left[\rho_i + \frac{\rho_h \rho_s}{\rho_h + \rho_s}\right](1 - R_{sv})\sigma_v^2}, \quad \beta_i = \frac{1}{\left[\rho_i + \frac{\rho_h \rho_s}{\rho_h + \rho_s}\right](1 - R_{sv})\sigma_v^2},$$

<sup>23</sup> It can also be seen that, in this case,  $EU_h$  increases with the risk premium  $\Gamma q$  for given  $\Lambda$  and  $\lambda$ .

$$\begin{aligned} \gamma_i &= \frac{\rho_i}{\rho_i + \frac{\rho_h \rho_s}{\rho_h + \rho_s}}, & \alpha_s &= \frac{R_{sv}}{\rho_s(1 - R_{sv})\sigma_v^2}, \\ \beta_s &= \frac{1}{\rho_s(1 - R_{sv})\sigma_v^2}, & \alpha_h &= \frac{R_{sv}}{\rho_h(1 - R_{sv})\sigma_v^2}, \\ \beta_h &= \frac{1}{\rho_h(1 - R_{sv})\sigma_v^2}, & \delta &= \sigma_{vz}/\sigma_v^2, & \lambda &= \frac{\rho_h \rho_s}{\rho_h + \rho_s}(1 - R_{sv})\sigma_v^2, \\ \Lambda &= \frac{1}{(1 - R_{sv})\sigma_v^2} \left( \frac{1}{[\rho_i + \frac{\rho_h \rho_s}{\rho_h + \rho_s}]} + \frac{1}{\rho_s} + \frac{1}{\rho_h} \right) \text{ and } \Gamma = \gamma_i/\Lambda. \end{aligned}$$

The level of real investment chosen by the entrepreneur is given by  $q = (\bar{v} - c_1)/(c_2 + \rho_i \sigma_v^2(1 - d))$ , where  $d = \rho_i(1 - R_{sv})/[\rho_i + 2\rho_s \rho_h/(\rho_s + \rho_h) + \rho_i(\rho_s \rho_h/(\rho_s + \rho_h))^2(1 - R_{sv})\sigma_v^2 \delta^2 \sigma_u^2]$ .

*Proof.* See appendix. □

**REMARK.** If everybody knows the exact value of  $v$  ( $R_{sv} = 1$ ), the security market breaks down, since as soon as the value of the signal is disclosed by the insider there is no longer any risk to be born. Otherwise equilibrium always exists because neither the entrepreneur nor the hedgers have an informational advantage. If  $R_{sv} = 1$  then the unique possible equilibrium price is  $p = v$  (at any price different from  $v$ , all the traders would like to buy or sell an unbounded amount of shares), but at that price a risk-averse trader has no incentive to trade. This is in contrast to the insider trading case, where the market need not break down even if the insider is perfectly informed ( $R_{sv} = 1$ ). This is so because the insider acts strategically and prevents the price from being fully revealing.

When  $s$  is public information, the hedgers do not have an informational advantage because  $u$  is revealed by the price. The outsiders (hedgers and speculators) do not face adverse selection, since the entrepreneur has no informational advantage either. In this case there is a linear equilibrium for all parameter configurations (unless  $R_{sv} = 1$ ). Outsiders are more willing to provide liquidity, and the price impact of the entrepreneur's demand will tend to be lower. Moreover, the asset is now less risky to outsiders because they have more information. Both effects tend to increase the outsiders' trading intensity and, as a result, the depth of the market increases.

The only equilibrium parameter affected by  $\sigma_{vz}$  is  $\delta$  (and consequently  $q^{PD}$ ). If  $\delta = 1$  then hedgers supply  $u$  in the aggregate. Furthermore, the entrepreneur's hedge ratio  $\gamma_i$  is independent of  $R_{sv}$  (and of any variance of the random variables in the model); it depends only on the degree of risk aversion of the agents in the economy.

The strategies of the outsiders do not depend on  $q$ .<sup>24</sup> This is because  $s$  is known and need not be inferred from the price. At the same time, since there is no informational asymmetry between the insider and the outsiders, the entrepreneur's speculative demand and the expected production return (conditional on the entrepreneur's information) are uncorrelated. That is,

$$\text{cov}\left\{\frac{E[v|s] - p}{\rho_i \text{var}[v|s] + \lambda}, E[v|s]\right\} = 0$$

because  $E[v|s] - p = \Gamma q + \delta u/\Lambda$ . This is in contrast to the case of insider trading.

The equilibrium of the model when the insider does not have any private information before trading in the asset market can be derived from Proposition 3 by letting  $R_{sv} = 0$ .

The next proposition and claim analyze the main effects of changes in the precision of information about  $v$  with public disclosure. It also allows us to compare the equilibria described in Proposition 3 (with public revelation of inside information) with the case of no inside information.

**PROPOSITION 4** (Comparative statics in PD regime). Assume that the entrepreneur publicly discloses his inside information before trading takes place. If  $R_{sv}$  increases, then:

1. Trading intensities ( $\alpha_i$ ,  $\beta_i$ ,  $\alpha_s$ ,  $\beta_s$ ,  $\alpha_h$ , and  $\beta_h$ ) increase; the market becomes deeper ( $\Lambda$  increases); the stock price becomes more informative and the risk premium ( $\gamma_i q/\Lambda$ ) is lower (the average stock price is higher).
2. The level of real investment decreases.
3. The expected utilities of both the insider and the speculators decrease.

*Proof.* See Appendix. □

**CLAIM 5.**<sup>25</sup> If  $R_{sv}$  increases then

- prices are more volatile and
- the expected utility of hedgers decreases.

Proposition 4 collects the main effects of public information on real investment and on the efficiency of the financial market. If the precision of public

<sup>24</sup> The speculator's demand is given by  $X_s(p, s) = (E[v|s] - p)/(\rho_s \text{var}[v|s])$ , and hedger  $j$ 's demand is given by  $X_{hj}(p, s, u_j) = [(E[v|s] - p)/(\rho_h \text{var}[v|s])] - (\sigma_{vz}/\sigma_v^2)u_j$ .

<sup>25</sup> This Claim is based on simulations BC1, BC2 and variations, in each case for the parameter ranges described in Section 3. Table 1 reports the result of the BC1 simulations. The result for BC2 are not reported but are available from the authors upon request.

information increases, then all the agents trade more aggressively because the *ex post* volatility of the risky asset is lower. As a direct consequence, the stock price will be more informative, since traders react more to their information. Moreover, the market depth is increased because it is proportional to the traders' aggregate price sensitivity. The impact on price volatility  $\text{var}[p] = R_{sv}\sigma_v^2 + (\delta/\Lambda)^2\sigma_u^2$  is ambiguous: more informative prices will tend to be more volatile, but at the same time the market is becoming deeper. The first effect dominates unless  $\sigma_v\sigma_u$  is very large. For reasonable parameter values,  $d\text{var}[p]/dR_{sv} > 0$  because  $(\delta/\Lambda)^2$  decreases less than linearly with  $R_{sv}$ .

The level of real investment  $q$  is decreasing in  $R_{sv}$ . As  $R_{sv}$  tends to unity the hedging effectiveness of the market tends to zero ( $d \rightarrow 0$ ). The reason is the Hirshleifer effect: public information reduces the risk-sharing opportunities provided by the financial market because it destroys insurance opportunities for the entrepreneur and hedgers (the latter by revealing information correlated with their endowment shock).

The risk premium  $(\gamma_i/\Lambda)q$  is decreasing in the precision of public information, since increases in  $R_{sv}$  cause market depth  $\Lambda$  to increase and  $q$  to decrease (and the hedge ratio  $\gamma_i$  of the entrepreneur is independent of  $R_{sv}$ ). If the precision of  $s$  increases, the entrepreneur's hedge supply  $\gamma_i q$  decreases, along with his expected demand:  $dE[x_i]/dR_{sv} < 0$  (indeed,  $E[x_i] = \gamma_i q(\beta_i/\Lambda - 1)$ , and  $\beta_i/\Lambda < 1$  is independent of  $R_{sv}$ ). This happens because the optimal level of real investment is decreasing in  $R_{sv}$ .

The effect of increases in  $R_{sv}$  on the expected utility of the entrepreneur is negative. This is so because both speculative and production-related gains diminish with better public information (the latter because  $q$  decreases). The same is true for the expected utility of speculators. The expected utility of hedgers decreases also in the simulations performed. The decrease in the risk premium, compounded with the increased volatility, outweigh the positive effect of an increased market depth. In summary, we obtain the result that, in a regime with public disclosure, increasing the precision of public information is typically Pareto-inferior.

## 6. Effects of Insider Trading

If the market is subject to an "abstain-or-disclose" rule, the entrepreneur should either publicly reveal his inside information  $s$  or abstain from trading on the basis of that information.<sup>26</sup> To examine the desirability of such an "abstain-or-disclose" rule we need to know what the effects are of imposing

<sup>26</sup> If the entrepreneur has private information then perhaps he can commit his portfolio to an independent trust with instructions to do as well as possible but, obviously, with no inside information. Then, instead of abstaining from trade, he (the trust's manager) will trade as if  $R_{sv} = 0$ .

it. Suppose first that the entrepreneur/insider learns  $s$  for free in the course of his activity. Then – when faced with the choice of (a) disclosing  $s$  and trading or (b) not disclosing and not being able to trade on the information – he will choose to disclose because only by trading can he hedge the investment risk.<sup>27</sup> This means that the relevant welfare comparison is between a public disclosure regime and insider trading (IT).<sup>28</sup>

Suppose now that learning  $s$  costs some effort. Then the entrepreneur will never spend any effort to learn  $s$  if the information must be disclosed before use. Indeed, if he obtains the information then he will like to disclose it and trade, but we know from Proposition 4 that the entrepreneur is better off when NI about  $v$  is available. Therefore, he will choose not to collect any information.<sup>29</sup> This means that the relevant welfare comparison is between a regime in which the entrepreneur has no private information (NI where  $R_{sv} = 0$ ) and insider trading (IT).

The relevant benchmarks of comparison are then the equilibrium with insider trading (IT), the equilibrium with public disclosure of  $s$  (PD), and the equilibrium with no private information on  $v$  (NI).

#### 6.1. INSIDER TRADING VERSUS NO INFORMATION ON FUNDAMENTALS

In this subsection we compare the equilibrium when insider trading is permitted (see Proposition 1) with the equilibrium in a similar market in which the entrepreneur does not have any inside information (see Proposition 3 with  $R_{sv} = 0$ ).

All we have to do is to analyze the comparative statics of the endogenous parameters that characterize the equilibrium when insider trading is allowed with respect to  $R_{sv}$ . We do so in the case for which we can ensure that there is a unique (linear) equilibrium ( $R_u/\rho_h$  small). The robustness of the results is checked with simulations.

In what follows we illustrate the comparative statics with respect to  $R_{sv}$  in the IT equilibrium. The formal statement (Proposition 9) and proof are in the appendix. If the risk-adjusted informational advantage of hedgers ( $R_u/\rho_h$ ) is small, then:

<sup>27</sup> It is possible to check that the expected utility of the insider and speculators strictly decreases if there is no trade (NT) either in relation to the public disclosure regime or the NI regime. This is also the case for hedgers provided that their expected utility does not diverge to  $-\infty$ . For the entrepreneur we have that  $EU_i^{NT} < EU_i^{PD} < EU_i^{NI}$ .

<sup>28</sup> However, if the entrepreneur can commit his portfolio to a trust then he will prefer to “abstain” rather than “disclose” because the expected utility of the insider decreases with  $R_{sv}$  when information is public. In this case, the relevant comparison would be between the no-information regime and insider trading.

<sup>29</sup> The result also holds if the entrepreneur can hire an agent to invest on his behalf. Then the entrepreneur will never pay for the information because he cannot use it.

Table 1. Results for BC1

Exogenous parameters										
$c_1$	0.9	$\rho_i$	2	$\rho_h$	3	$\text{var}(v)$	0.04	$R_u$	$0.1^2$	
$c_2$	0.02	$\rho_s$	1	$E(v)$	1	$\text{var}(u)$	0.01	$R_{vz}$	$0.91^2$	
Equilibrium with NI										
$R_{sv}$	$\lambda^{NI}$	$\alpha_i^{NI}$	$\gamma_i^{NI}$	$\delta^{NI}$	$\Lambda^{NI}$	$q^{NI}$	$\tau^{NI}$	$EU_i^{NI}$	$EU_s^{NI}$	$EU_h^{NI}$
0	0.030	0.00	0.7273	0.91	42.42	1.84	25.00	-0.83173	-0.987559	-42.55
Equilibrium with IT										
$R_{sv}$	$\lambda^{IT}$	$\alpha_i^{IT}$	$\gamma_i^{IT}$	$\delta^{IT}$	$\Lambda^{IT}$	$q^{IT}$	$\tau^{IT}$	$EU_i^{IT}$	$EU_s^{IT}$	$EU_h^{IT}$
0.05	0.495	0.09	0.1332	0.89	3.77	1.22	25.56	-0.88195	-0.999691	-57.39
0.10	0.711	0.13	0.0920	0.88	2.68	1.15	26.19	-0.88662	-0.999843	-60.73
0.15	0.880	0.16	0.0717	0.88	2.19	1.12	26.88	-0.88852	-0.999893	-63.46
0.20	1.026	0.18	0.0587	0.87	1.89	1.10	27.60	-0.88951	-0.999917	-65.99
0.25	1.157	0.21	0.0493	0.86	1.69	1.08	28.37	-0.89008	-0.999932	-68.43
0.30	1.278	0.22	0.0420	0.86	1.53	1.07	29.19	-0.89043	-0.999941	-70.83
0.35	1.392	0.24	0.0360	0.85	1.41	1.06	30.07	-0.89063	-0.999948	-73.23
0.40	1.500	0.26	0.0310	0.84	1.31	1.05	31.00	-0.89075	-0.999953	-75.64
0.45	1.604	0.27	0.0267	0.84	1.23	1.05	31.99	-0.89081	-0.999957	-78.08
0.50	1.705	0.29	0.0229	0.83	1.16	1.04	33.05	-0.89083	-0.999960	-80.55
0.55	1.804	0.30	0.0196	0.82	1.10	1.04	34.18	-0.89082	-0.999963	-83.06
0.60	1.902	0.31	0.0166	0.82	1.04	1.03	35.40	-0.89080	-0.999965	-85.63
0.65	1.998	0.32	0.0138	0.81	0.99	1.03	36.71	-0.89077	-0.999967	-88.24

Table I. Continued.

$R_{sv}$	$\lambda^{IT}$	$\alpha_i^{IT}$	$\gamma_i^{IT}$	$\delta^{IT}$	$\Lambda^{IT}$	$q^{IT}$	$\tau^{IT}$	$EU_i^{IT}$	$EU_s^{IT}$	$EU_h^{IT}$
0.70	2.094	0.33	0.0113	0.80	0.95	1.03	38.13	-0.89073	-0.999969	-90.92
0.75	2.191	0.34	0.0090	0.79	0.91	1.02	39.66	-0.89070	-0.999970	-93.65
0.80	2.289	0.35	0.0069	0.79	0.87	1.02	41.32	-0.89066	-0.999972	-96.44
0.85	2.390	0.35	0.0050	0.78	0.83	1.02	43.13	-0.89064	-0.999973	-99.29
0.90	2.492	0.36	0.0032	0.77	0.80	1.02	45.10	-0.89062	-0.999974	-102.21
0.95	2.599	0.36	0.0015	0.75	0.77	1.01	47.27	-0.89063	-0.999976	-105.19
1.00	2.711	0.37	0.0000	0.74	0.74	1.0116000	49.66	-0.89065	-0.999977	-108.23
Equilibrium with PD										
$R_{sv}$	$\lambda^{PD}$	$\alpha_i^{PD}$	$\gamma_i^{PD}$	$\delta^{PD}$	$\Lambda^{PD}$	$q^{PD}$	$\tau^{PD}$	$EU_i^{PD}$	$EU_s^{PD}$	$EU_h^{PD}$
0.05	0.029	0.48	0.727	0.91	45	1.77	26.32	-0.83794	-0.989104	-46.63
0.10	0.027	1.01	0.727	0.91	47	1.70	27.78	-0.84372	-0.990453	-51.14
0.15	0.026	1.60	0.727	0.91	50	1.64	29.41	-0.84909	-0.991636	-56.16
0.20	0.024	2.27	0.727	0.91	53	1.58	31.25	-0.85411	-0.992678	-61.76
0.25	0.023	3.03	0.727	0.91	57	1.52	33.33	-0.85881	-0.993599	-68.03
0.30	0.021	3.90	0.727	0.91	61	1.47	35.71	-0.86322	-0.994415	-75.09
0.35	0.020	4.90	0.727	0.91	65	1.42	38.46	-0.86735	-0.995142	-83.06
0.40	0.018	6.06	0.727	0.91	71	1.38	41.67	-0.87125	-0.995790	-92.10
0.45	0.017	7.44	0.727	0.91	77	1.34	45.45	-0.87493	-0.996370	-102.40
0.50	0.015	9.09	0.727	0.91	85	1.30	50.00	-0.87840	-0.996890	-114.17
0.55	0.014	11.11	0.727	0.91	94	1.26	55.56	-0.88168	-0.997358	-127.70
0.60	0.012	13.64	0.727	0.91	106	1.22	62.50	-0.88480	-0.997779	-143.32
0.65	0.011	16.88	0.727	0.91	121	1.19	71.43	-0.88775	-0.998160	-161.43
0.70	0.009	21.21	0.727	0.91	141	1.16	83.33	-0.89056	-0.998504	-182.55

Table I. Continued.

$R_{sv}$	$\lambda^{PD}$	$\alpha_i^{PD}$	$\gamma_i^{PD}$	$\delta^{PD}$	$\Lambda^{PD}$	$q^{PD}$	$\tau^{PD}$	$EU_i^{PD}$	$EU_s^{PD}$	$EU_h^{PD}$
0.75	0.008	27.27	0.727	0.91	170	1.13	100.00	-0.89323	-0.998816	-207.31
0.80	0.006	36.36	0.727	0.91	212	1.10	125.00	-0.89577	-0.999099	-236.51
0.85	0.005	51.52	0.727	0.91	283	1.07	166.67	-0.89819	-0.999356	-271.14
0.90	0.003	81.82	0.727	0.91	424	1.05	250.00	-0.90051	-0.999591	-312.48
0.95	0.002	172.73	0.727	0.91	848	1.02	500.00	-0.90272	-0.999805	-362.16
1.00	0.000	-	-	0.91	-	1.00	-	-	-	-

The Table presents the comparative statics for the endogenous market parameters with respect to the accuracy of private information  $R_{sv}$ , ranging from 0 to 1, obtained with the simulations for the base case 1 (BC1). The values of the exogenous parameters are displayed on top, then the equilibrium values for market parameters when there is no information (NI), then when there is private information and insider trading (IT), and finally when there is public disclosure (PD). The first column on the left for equilibrium values displays the range of values for  $R_{sv}$ .



- IT increases price precision, increases the insider's sensitivity to trading signals, and decreases the price responsiveness of speculators and hedgers; when  $\rho_i$  is not too large, it decreases market depth and increases price volatility.
- When  $R_{sv}$  is close to 1, insider trading decreases real investment and the risk premium, increases the insider's speculative gains (reducing his production and insurance gains), and decreases speculators' *ex ante* expected utility.
- When  $\rho_i$  is low enough, insider trading increases the expected utility of the insider.

Through simulations, we establish some further results on these comparative statics (for the range of parameters illustrated in Section 3):

- The results in the first two bullet points above hold for reasonable parameter values (cases BC1 and BC2 with variations  $R_u = 0.16$  and  $R_u = 0.25$ ).
- Insider trading decreases the expected utility of the insider and hedgers (cases BC1 and BC2). However, in high-noise environments (with  $\sigma_u \in \{0.5, 0.6, 0.7\}$  and  $r_u = 0.4$  or  $0.5$ ) the expected utility of the insider increases with insider trading.

The simulations performed (see Table I, where results for BC1 are displayed), provide estimates of the magnitude of the effects as well as a robustness check for parameters outside the range of the base cases.

*Quantitative effects.* (1)  $\Lambda^{NI}$  is much larger than  $\Lambda^{IT}$  even for  $R_{sv}$  small (from  $R_{sv} = 0$  to  $R_{sv} = 1/20$ ,  $\Lambda$  drops by 91%). (2) A similar effect (although not so drastic) holds for  $q$  (from  $R_{sv} = 0$  to  $R_{sv} = 1/20$ ,  $q$  drops by 34%). (3) Price precision doubles as  $R_{sv}$  increases from 0 to 1, and price volatility increases dramatically at the beginning (by 214% from  $R_{sv} = 0$  to  $R_{sv} = 1/20$ ). (4)  $E[U_i^{IT}]$  is U-shaped with  $R_{sv}$  and changes moderately as  $R_{sv}$  ranges from 0 to 1: it falls by at most 7%. (5)  $E[U_h^{IT}]$  falls by 35% from  $R_{sv} = 0$  to  $R_{sv} = 1/20$  and by a further 88% until  $R_{sv} = 1$ .

For BC2, the drop in traders' expected utilities when there is insider trading is phenomenal (from  $R_{sv} = 0$  to  $R_{sv} = 1/20$ ,  $E[U_i]$  drops by a factor of  $10^5$  and is decreasing in  $R_{sv}$ , and  $E[U_h]$  drops by a factor of 300; to  $R_{sv} = 1$  there is a further drop by a factor of 31 in the first case and by 300% in the second). In BC2, investment has no real cost and so  $q$  is much larger; hence the entrepreneur needs to hedge much more.

*Robustness.*

- The results hold also for  $R_u/\rho_h$  not close to zero (in particular, for  $R_u = 0.16$  and  $R_u = 0.25$ ).
- $E[U_i^{IT}] > E[U_i^{NI}]$  when  $\rho_i = 0.2$  or lower, with the rest of parameters as in BC1 (for  $\rho_i = 0.1$   $E[U_i^{IT}]$  increases with  $R_{sv}$  for all  $R_{sv}$ ).

- In high-noise environments but for otherwise “reasonable” parameter values (e.g., BC1 with  $\sigma_u = 0.7$ ), we have also that  $E[U_i^{IT}]$  increases with  $R_{sv}$  for all  $R_{sv}$ . (For  $\sigma_u = 0.5$ ,  $E[U_i^{IT}]$  is U-shaped with  $R_{sv}$ , and  $E[U_i^{IT}] > E[U_i^{NI}]$  for  $R_{sv}$  high.)

The intuition for these results is as follows. When insider trading is permitted, the insider trades more aggressively, and speculators and hedgers trade less aggressively because they face adverse selection. As a direct consequence, the marginal impact of the entrepreneur’s asset position on the current price will be higher when IT is permitted,  $\lambda^{IT} > \lambda^{NI}$ . For reasonable parameter values, market depth is reduced by IT. Market depth is equal to the sum of the price sensitivities of demands,  $\Lambda = \beta_s + \beta_h + \beta_i$ . In the simulations, market depth is reduced drastically.<sup>30</sup>

Price precision is increased by insider trading. The informativeness of current prices increases with the amount of trading motivated by fundamentals information ( $\alpha_i$ ) and decreases with the amount of noise created by hedging trades ( $\delta$ ).<sup>31</sup> If insider trading is permitted, then  $\alpha_i$  increases because the entrepreneur is better informed and market depth decreases; consequently,  $\delta$  is reduced by hedgers. Both effects tend to increase price precision. For reasonable parameter values, current prices will be more volatile with insider trading. Price volatility is a result of the noise created by hedge trading and by information about  $v$ . Moreover, when insider trading is permitted, price volatility due to the former factor increases if the market becomes thinner (and this happens for reasonable parameter values or if  $\rho_i$  is sufficiently close to zero).<sup>32</sup>

Given the effects of the main market parameters, we can now understand the effects of insider trading on investment, stock prices and the welfare of market participants.

- Permitting insider trading tends to reduce the level of real investment, since it reduces the risk-sharing opportunities provided by the asset market. This is because it creates an adverse selection problem that makes speculators and hedgers less willing to share the risk inherent

<sup>30</sup> This must always be true for an insider with low aversion to risk. Indeed, when  $R_{sv}$  increases there are two effects on  $\Lambda$ : a positive direct effect given  $\lambda$  (because the insider responds more to the price) that is weighted by  $\rho_i$  and a negative indirect effect that increases  $\lambda$ . (As another consequence, if insider trading is allowed then the amount of the hedgers’ initial endowment that is covered decreases; that is,  $\delta$  is decreasing in  $R_{sv}$ .)

<sup>31</sup> The equilibrium price is informationally equivalent to  $\alpha_i(s - \bar{v}) - \delta u$ , and price precision can be seen to be given by  $\tau = (\alpha_i^2 \sigma_v^2 + R_{sv} \delta^2 \sigma_u^2) / \{\sigma_v^2 [\alpha_i^2 \sigma_v^2 (1 - R_{sv}) + R_{sv} \delta^2 \sigma_u^2]\}$ , which is increasing in  $\alpha_i$  and decreasing in  $\delta$ .

<sup>32</sup> Price volatility is given by  $\text{var}[p] = [\sigma_u^2 / \Lambda^2] + R_{sv} \sigma_v^2 / \{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda\}^2 \Lambda^2$ . We have that  $1 / \{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda\} \Lambda = \lambda / (2\lambda + \rho_i (1 - R_{sv}) \sigma_v^2)$ , which is increasing in  $R_{sv}$  since  $d\lambda / dR_{sv} > 0$ . Therefore,  $d\text{var}[p] / dR_{sv} > 0$  if the conditions that make  $\Lambda$  decreasing in  $R_{sv}$  hold.

in real production. Furthermore, a more informative price decreases insurance opportunities and contributes to decreased hedging possibilities (Hirshleifer effect). When the insider is risk neutral, investment is unaffected by insider trading because (as stated before) in that case investment then does not depend on the hedging effectiveness of the market.

- The average stock price tends to increase with insider trading. Two effects explain this result. The insider's hedge supply is lower since the level of real investment is lower, and the insider's speculative demand is higher since it is increasing in the precision of inside information. A countervailing effect is that market depth may decrease, tending to increase the risk premium. The latter effect tends to be dominated for reasonable parameter values and when  $R_{sv}$  is close to unity.
- If insider trading is permitted, the insider's speculative gains will be higher ( $|SG_i^{IT}| < |SG_i^{NI}|$ ) whereas, his insurance and production gains will be lower ( $\exp\{-\rho_i(\bar{v} - c_1)q^{IT}/2\} > \exp\{-\rho_i(\bar{v} - c_1)q^{NI}/2\}$ ). Speculative gains increase with the precision of inside information, but inside information creates an adverse selection problem. As a result, the entrepreneur's gains coming from real investment and the level of real investment go down. The entrepreneur's expected utility may increase or decrease depending on whether or not the speculative effect dominates. If the entrepreneur is sufficiently risk averse, his expected utility will decrease when insider trading is allowed. The opposite result will hold if he is very close to risk neutral or in high-noise scenarios.<sup>33</sup> In our range of reasonable parameter values, the production and insurance losses from insider trading dominate the speculative gains.
- If inside information is sufficiently precise, the speculator's expected utility will be reduced by insider trading ( $EU_s^{IT} < EU_s^{NI}$ ), because in this situation the average stock price goes up (and, as a direct result, the expected return  $[\bar{v} - \bar{p}]$  goes down) and speculators trade less aggressively. The results also hold for reasonable parameter values.
- Three effects tend to reduce hedger's expected utility under insider trading. First, obtaining insurance is more costly when insider trading is allowed because the market is thinner. Second, the increase in price volatility also tends to hurt hedgers, especially if they are very risk averse. Finally, when insider trading reduces risk premia, this also hurts hedger's expected utility. All three effects occur for reasonable parameter values.

<sup>33</sup> In fact, if the entrepreneur is risk neutral, then his expected profits are always larger with insider trading and are increasing in  $R_{sv}$  (and the expected profits in the NI regime are in turn larger than in the PD regime).

The picture that emerges for reasonable parameter values is as follows: In other words, insider trading reduces market depth and investment while increasing the expected price (reducing the risk premium) and the price precision, with the result that the expected utility of all traders decreases. Insider trading is pareto-inferior. The possibility remains that, if  $\rho_i$  is very low (or in high-noise scenarios), the insider improves with insider trading.

## 6.2. INSIDER TRADING VERSUS PUBLIC DISCLOSURE

Now we compare the equilibrium when insider trading is permitted with the equilibrium when the entrepreneur publicly announces his private information before trading takes place (PD). That is, we compare the equilibria described in Propositions 1 and 3. As before, we present analytical results for the case in which we can ensure that there is a unique equilibrium ( $R_u/\rho_h$  small) and then check the robustness of the results with simulations. The formal statement (Proposition 10) and proof of the analytical results is in the appendix.

We establish analytically that, if the risk-adjusted informational advantage ( $R_u/\rho_h$ ) is small, then insider trading, compared with public disclosure:

- decreases the insider's sensitivity to trading signals, the price responsiveness of speculators and hedgers, the market depth, and price precision;
- when  $R_{sv}$  is close to 1, increases real investment and decreases price volatility;
- when  $R_{sv}$  is close to 1 (and/or  $\rho_i$  is close to 0), increases the expected utility of the insider;
- when  $R_{sv}$  is close to 1, increases the expected utility of speculators.

Simulations effected for reasonable parameter values (BC1 and BC2) show also that insider trading:

- reduces price volatility and the risk premium;
- except for  $R_{sv}$  very close to 1, decreases real investment and the expected utility of the insider and of speculators;<sup>34</sup>
- increases (decreases) the expected utility of hedgers if  $R_{sv}$  is high (low).

If inside information becomes public, the informational asymmetry and the adverse selection problem disappear. This makes outsiders (speculators and hedgers) more responsive to price. As a direct consequence, the residual supply faced by the insider is less sensitive to his demand ( $\lambda^{IT} > \lambda^{PD}$ ), and

<sup>34</sup> In high-noise scenarios,  $R_{sv}$  need not be so close to unity for  $q^{IT} > q^{PD}$  (for example, this holds for BC1 with  $\sigma_u = 0.7$  when  $R_{sv} \geq 14/20$ ).

hence the insider also trades more aggressively. The result is that market depth is greater with public disclosure ( $\Lambda^{\text{IT}} < \Lambda^{\text{PD}}$ ).

The asset price is more informative if the inside information is made public, since more traders are reacting more to information about  $v$ . Price precision increases a lot because with PD the competitive agents observe  $s$ . There are two sources of volatility: information and hedging  $u$ . With PD, the price is more informative but also the market is deeper. In the simulations we find that the informational effect always dominates and price volatility decreases in the IT regime.

The intuition for the impact on investment and welfare is as follows.

- The effect on real investment of disclosing the inside information depends on the trade-off between the following effects. If inside information is made public there are more risk-sharing opportunities because the adverse selection problem faced by outsiders is eliminated (and they are more willing to share the risk due to real investment). However, more information with PD destroys insurance opportunities (Hirshleifer effect). Here the Hirshleifer effect works in favor of IT because with IT less information is revealed. The adverse selection effect dominates unless the information of the insider is very precise and so real investment is reduced by insider trading,  $q^{\text{IT}} < q^{\text{PD}}$ . This tends to reduce the risk premium in the IT regime. Note that, when  $R_{sv} = 1$ , the market collapses with PD and  $q^{\text{PD}}$  equals the investment level with no trade,  $q^{\text{PD}} = q_0 \equiv (\bar{v} - c_1)/(c_2 + \rho_i \sigma_v^2) < q^{\text{IT}}$ . We then have that  $q^{\text{IT}} > q^{\text{PD}}$ , and the same result holds for  $R_{sv}$  very close to unity. When the information of the insider is precise, making it public basically destroys the hedging possibilities in the financial market; keeping it private allows speculation by the insider and retains some risk sharing. Finally, as before, if the insider is risk neutral then investment is unaffected by insider trading.
- The speculative gains of the entrepreneur are higher with IT, and his overall utility is higher  $EU_i^{\text{IT}} > EU_i^{\text{PD}}$  if the insider's informational advantage  $R_{sv}$  is sufficiently large. This holds a fortiori when IT increases  $q$  ( $R_{sv}$  very close to 1). However, typically the decrease in  $q$  in the IT regime is large and  $EU_i^{\text{IT}} < EU_i^{\text{PD}}$ .
- If  $R_{sv}$  is very close to unity (and/or  $\rho_i$  is small), the insurance gains for speculators in the IT and PD cases are close, though the speculative gains are higher in the IT case. The result is that, for those parameter configurations  $EU_s^{\text{IT}} > EU_s^{\text{PD}}$ . However, for reasonable parameter configurations, insider trading reduces  $q$  with the result that  $EU_s^{\text{IT}} < EU_s^{\text{PD}}$ .
- The net effect of insider trading on hedgers is ambiguous. Insider trading creates adverse selection and also reduces market depth and

(typically) risk premia, worsening the position of hedgers; at the same time, it (typically) reduces price volatility also, tending to improve their position. If  $R_{sv}$  is high, then  $q^{IT}$  and  $q^{PD}$  are close together (and  $q^{IT} > q^{PD}$  for  $R_{sv}$  very close to 1), the price volatility effect dominates, and hedgers are better off with insider trading. When  $R_{sv}$  is low,  $q^{IT}$  tends to be much smaller than  $q^{PD}$  and the price volatility effect is dominated.<sup>35</sup>

To summarize, the general picture is the following. Unless the insider has a very large risk-adjusted informational advantage ( $R_{sv}/\rho_i$ ), insider trading reduces market depth, price volatility, and risk premia as well as real production and the expected utility of the insider and the speculators. The effect on hedgers is ambiguous and depends on the precision of the insider's information. Thus for a low  $R_{sv}$  insider trading is Pareto-inferior, for a very high  $R_{sv}$  it is Pareto-superior and for intermediate levels it is only hedgers who benefit from insider trading.

## 7. Hedgers, Noise Traders, and Market Depth

Many market microstructure models assume the existence of noise traders, agents that trade randomly for unspecified reasons, which are typically taken to be liquidity shocks. Their utility is evaluated in terms of expected "trading costs", that is, the average losses that they bear. In the usual CARA-Normal models, the expected losses of noise traders (trading  $u$  in the aggregate) are  $(1/\Lambda)\sigma_u^2$ , where  $\Lambda$  is market depth.

Are there circumstances in which rational expected-utility maximizing agents give rise to demands for assets of the "noise trader" form? Are expected losses an appropriate measure of their welfare? Is depth a good proxy for welfare?

Our model has a response to these questions.<sup>36</sup> The aggregate demand of hedgers is  $X_h(p) = \beta_h(\bar{v} - p) - \gamma_h q - \delta u$ . A hedger speculates on his endowment information, takes the counterpart of the shares sold by the entrepreneur, and hedges his endowment shock. If we fix the hedger's risk aversion to a finite value ( $\rho_h < \infty$ ) and let their risk-adjusted informational advantage  $R_u/\rho_h$  tend to zero, then according to Proposition 3 their equilibrium demand tends to  $X_h(p) = \beta_h(\bar{v} - p) - \gamma_h q - u$  if  $\sigma_{vz} = \sigma_v^2$ , with  $\beta_h \geq 0$  and  $\gamma_h < 0$ . Hedgers have no informational advantage when there is no correlation between each

<sup>35</sup> It is also interesting to examine what happens to the expected utility of hedgers conditional on their endowment. For  $R_{sv}$  low, more than 50% of hedgers prefer the equilibrium in the PD regime (those with a shock less than some small  $u > 0$ ). For  $R_{sv}$  high, more than 50% of hedgers prefer the equilibrium in the IT regime. Only those hedgers with a shock close to zero prefer the PD equilibrium. This is true even though sometimes the (*ex ante*) expected utility of hedgers diverges to  $-\infty$  (e.g., in BC1 or BC2 with  $\rho_h = 6$ ).

<sup>36</sup> See Sarkar (1994) for results in related models.

individual endowment shock  $u_j$  and the average  $u$  ( $R_u \rightarrow 0$ ). In the aggregate, they supply  $u$  and take a speculative position like the speculators. Furthermore, as  $R_{sv}/\rho_i \rightarrow 0$  we have  $\gamma_h \rightarrow 0$  because (a) as  $R_{sv}/\rho_i \rightarrow 0$  the insider's responsiveness to his information  $\alpha_i$  tends to zero and (b) a hedger needs not condition on  $q$  when trying to read the information about  $s$  in the price. Finally, if  $\rho_h \rightarrow \infty$ , then  $\beta_h = \gamma_h = 0$  and hedgers simply eliminate of all the risk associated with their endowment and supply  $u$  in the aggregate.<sup>37</sup>

In summary, the order flow will contain an exogenous supply  $u$  (independent of any deep parameter of the model) whenever  $z$  is perfectly correlated with  $v$  and when the risk-adjusted informational advantage of a hedger is vanishingly small ( $R_u/\rho_h \rightarrow 0$ ). This happens if hedgers are infinitely risk averse ( $\rho_h \rightarrow \infty$ ) or if there is no correlation between each individual endowment shock  $u_j$  and the average  $u$  ( $R_u \rightarrow 0$ ). When hedgers are infinitely risk averse,  $\rho_h \rightarrow \infty$ , then we have exactly that  $X_h = -u$  as in the noise-trader models.

The proper modeling of liquidity traders has important implications for welfare analysis. Indeed, in a noise-trader model the welfare of so-called liquidity traders is measured by their expected losses, which are proportional to the inverse of market depth  $\Lambda$ . However, we have seen that in many circumstances the expected utility of hedgers and market depth do not move together:

- With public disclosure we have seen how increasing public information is typically Pareto-inferior and hence the “welfare” criterion of minimizing noise-trader losses is typically misleading. Indeed, as  $R_{sv}$  increases, market depth  $\Lambda$  increases at the same time that the expected utility of all traders goes down. This is because, for hedgers, the increase in  $\Lambda$  is more than compensated by an increase in the variance of prices and a decrease in the risk premium. This cannot happen in a noise-trader model.
- When comparing insider trading with the public disclosure regime we have that, for  $R_{sv}$  high, the expected utility of hedgers increases with insider trading despite the fact that market depth decreases—contradicting the implicit welfare criterion of noise-trader models. Even when  $\rho_h$  is high and the demands of traders approach those of the noise-trader model, the welfare analysis derived from that model and based on looking only at market depth is incorrect. Indeed, what matters is expected utility; then, risk premia and price volatility are important.

<sup>37</sup> Hedger  $j$ 's initial wealth may be written as  $W_{hj} = u_j z = (\sigma_{vz}/\sigma_v^2)u_j v + \sigma_z \sqrt{1 - r_{vz}^2} u_j \xi$ . Hedger  $j$  has an initial endowment  $u_j$  of an asset with future (random) value  $z$ ; equivalently, we may suppose that he has a portfolio consisting of  $(\sigma_{vz}/\sigma_v^2)u_j$  shares of stock with liquidation value  $v$  together with  $\sigma_z \sqrt{1 - r_{vz}^2} u_j$  shares of some stock with future value  $\xi$  (where  $\xi$  is independent of  $v$ ). In order to minimize risks, it is clear that hedger  $j$ 's optimal strategy consists of selling his shares  $(\sigma_{vz}/\sigma_v^2)u_j$  (since  $\xi$  is independent of  $v$  and there is no tradable security correlated with  $\xi$ ).

## 8. Conclusions

Our model can shed light on several public policy issues in relation to insider trading: (1) an abstain-or-disclose rule; (2) a laissez-faire policy; (3) the rationale for the EU directive considering only “precise” information; (4) new regulations concerning the early selective release of material information.

(1) The consequences of an abstain-or-disclose rule are as follows.

- If the entrepreneur/insider learns his private signal  $s$  for free in the course of his activity, then he will choose to disclose. The relevant welfare comparison is between a PD regime and insider trading (IT). The welfare consequences of IT depend then on the information precision of the insider. If the information is imprecise, then the adverse selection effect dominates, real investment  $q$  decreases, and IT will tend to be Pareto-inferior; if it is very precise then the Hirshleifer effect dominates,  $q$  increases, and IT will tend to be Pareto-superior. In an intermediate range, only hedgers benefit from insider trading (because it reduces price volatility).
- If the entrepreneur/insider can learn  $s$  only at some cost, then the entrepreneur will never spend any effort to learn  $s$ . This means that the relevant welfare comparison is between a regime in which the entrepreneur has no private information (NI) and insider trading (IT). Then  $q$  decreases with IT, which tends to be bad for everyone—except for the insider when he is very close to risk neutrality (see Table II).<sup>38</sup>

In the welfare analysis we may also take into account positive external effects of investment (i.e., on other agents in the economy). If we take this perspective and give little weight to the utility of the insider, then we should conclude that an abstain-or-disclose rule is optimal when information is costly to acquire. Indeed, then the relevant benchmark for comparison is the NI regime, and insider trading always decreases investment and decreases the utility of all participants (except for the insider when he has a large risk-weighted informational advantage). This conclusion is reinforced by the fact

<sup>38</sup> When the insider is risk neutral, investment is unaffected by insider trading but the welfare effects are as stated in Table II. Insider trading increases the entrepreneur’s expected utility (in the PD and NI regimes, the insider has no informational advantage and so his speculative gains are lower than in the IT regime). If we compare the IT regime with the PD regime, then insider trading increases the expected utilities of speculators and hedgers because public disclosure reduces insurance opportunities (hedgers trade less and, as a result, both speculators and the insider are worse off with public information). If we compare the IT regime with the NI regime, then insider trading decreases the expected utilities of speculators and hedgers. Obtaining insurance is more costly when insider trading is allowed; as a result, hedgers trade less. Speculators also trade less (because insider trading creates an adverse selection problem and hedging demand is lower).



Table II. Effects of insider trading

Information of insider	Free PD benchmark	Costly NI benchmark
Very precise (and/or close to risk neutral)	$q \uparrow$ Pareto-superior	$q \downarrow$ $EU_i \uparrow$ $EU_s, EU_h \downarrow$
Precise	$q \downarrow$ $EU_i \downarrow, EU_s \downarrow$ $EU_h \uparrow$	$q \downarrow$ Pareto-inferior
Imprecise	$q \downarrow$ Pareto-inferior	

The table displays the impact of insider trading on the level of investment  $q$  and the expected utilities of the insider  $EU_i$ , speculators  $EU_s$ , and hedgers  $EU_h$ , depending on the information precision of the insider (in the left column) and the benchmark for comparison (in the top row). With free information acquisition the benchmark is public disclosure, and with costly information acquisition the benchmark is no information.

that insider trading induces an increase in welfare only under extreme parameter values. However, if information is obtained by the entrepreneur at no cost, then an abstain-or-disclose rule may not be such a good idea, because if the insider information is of high quality then insider trading helps hedgers. Even worse, when the risk-adjusted informational advantage of the insider is very high, an abstain-or-disclose rule will hurt everyone. This situation is more likely to arise (that is, it arises for a larger range of the precision of insider information) in high noise scenarios (i.e., with high volatility of the endowments of hedgers).

(2) The consequences of laissez-faire may be described as follows.

In the absence of regulation, one would expect insider trading to arise only in those circumstances where it is favorable to the insider. Otherwise, corporate charters should take care of the problem. When insider trading hurts the entrepreneur/coalition of insiders, corporate charters should impose and enforce an abstain-or-disclose rule because it is in the interest of the initial owners of the firm.<sup>39</sup> This means that we will have an optimal outcome whenever private and social incentives about the desirability of insider trading are aligned. This happens when insider trading is either Pareto-superior or Pareto-inferior. However, when information is obtained by the entrepreneur at no cost and is of intermediate quality, then a laissez-faire policy will hurt hedgers (in relation to an abstain-or-disclose rule). A countervailing effect will be that investment will increase with public disclosure, in

<sup>39</sup> Laissez-faire outcomes can be approximated also with a default rule prohibiting insider trading only if the corporate charter has not stated a policy in this regard (see Bainbridge, 1999).

which case the net welfare result is likely to be ambiguous.<sup>40</sup> Therefore with costless information acquisition, a laissez-faire policy seems appropriate (the ambiguity in the welfare assessment for moderately precise information seems to call for prudence and no intervention).

The difference between a laissez-faire policy and a mandated abstain-or-disclose rule is in the type of errors they induce. The advantage of laissez-faire is that it avoids prohibiting insider trading when it turns out to be Pareto-superior (and this tends to happen more in highly volatile industries). The cost is that situations will arise where it pays for the firm to allow insider trading even though outsiders are hurt thereby.<sup>41</sup> Both laissez-faire and a mandated abstain-or-disclose rule will have the same effects when insider trading hurts the insider but benefits hedgers.

The preceding analysis suggests the following rule of thumb: With costly information acquisition a mandated abstain-or-disclose rule is appropriate; with free information acquisition, laissez-faire is better. Taking into account that costly information acquisition should be more easily verifiable and detectable, the rule of thumb can be put into practice in the following way:

Enforce an abstain-or-disclose rule with a high standard of proof for inside information. In essence, this would mean that insiders need only worry about information that is costly to acquire.

(3) Is there a basis for prohibiting insider trading only with precise information (as in the EU Directive)?

It does not seem so. Indeed, the only case where insider trading is potentially Pareto-superior is with precise information; whenever it is imprecise, insider trading is Pareto-inferior.

(4) Does selective early disclosure of material information diminish welfare?

The typical situation involving early selective disclosure of information (to large fund investors, for example) does not involve a cost of information acquisition for the insider. The rule of thumb would thus advocate a laissez-faire policy.

## A. Appendix

### A.1. EU Directive on Insider Dealing (2003, Excerpts)

Article 1 states that “‘Inside information’ shall mean information of a precise nature which has not been made public, relating, directly or indirectly, to one or more issuers of financial instruments or to one or more financial instru-

<sup>40</sup> If the precision of the signal of the insider is very high (and/or his risk aversion very low) and if information is obtained by the entrepreneur at a cost, then with a laissez-faire policy there will be insider trading (because the insider will choose to become informed, provided the cost of information is not too large) but hedgers and speculators will be hurt.

<sup>41</sup> Khanna et al. (1994), in a model with two informed agents (an insider and an outsider), find that private and social incentives with respect to insider trading may differ.

ments and which, if it were made public, would be likely to have a significant effect on the prices of those financial instruments or on the price of related derivative financial instruments.”

Article 2.1. “Member states shall prohibit any person referred to in the second subparagraph who possesses inside information from using that information by acquiring or disposing of, or by trying to acquire or dispose of, for his own account or for the account of a third party, either directly or indirectly, financial instruments to which that information relates. The first subparagraph shall apply to any person who possesses that information: (a) by virtue of his membership of the administrative, management or supervisory bodies of the issuer; or (b) by virtue of his holding in the capital of the issuer; or (c) by virtue of his having access to the information through the exercise of his employment, profession or duties; or (d) by virtue of his criminal activities.” The European procedure can be explained in part by a larger reliance on criminal prosecution (Maug, 2002).

In Article 6, paragraph 1: “Member states shall ensure that issuers of financial instruments inform the public as soon as possible of inside information which directly concerns the said issuers” (under paragraph 2, some delay in information release may be allowed in some circumstances). Furthermore, in paragraph 4 of Article 6 it is stated that: “Persons discharging managerial responsibilities within an issuer of financial instruments and, where applicable, persons closely associated with them, shall, at least, notify to the competent authority the existence of transactions conducted on their own account relating to shares of the said issuer, or to derivatives or other financial instruments linked to them. Member states shall ensure that public access to information concerning such transactions, on at least an individual basis, is readily available as soon as possible.”

## A.2. Proofs of Propositions in Section 4

*Proof of Proposition 1 (sketch).* Since we restrict attention to linear equilibria and since the level of investment  $q$  is public information, agents’ strategies may be written as  $X_i(s, p) = \alpha_i(s - \bar{v}) + \beta_i(\bar{v} - p) - \gamma_i q + \varphi_i$ ,  $X_{hj}(p, u_j) = \beta_h(\bar{v} - p) - \gamma_h q - \delta u_j + \varphi_h$ , and  $X_{sk}(p) = \beta_s(\bar{v} - p) - \gamma_s q + \varphi_s$ . From the market-clearing condition,  $X_s(p) + X_h(p, u) + X_i(p, s) + q = q$  (where  $X_s(p) = \int_0^1 X_{sk}(p) dk$  and  $X_h(p, u) = \int_1^2 X_{hj}(p, u_j) dj$ ), the equilibrium price is given by

$$p = \bar{v} - \Gamma q + \frac{\alpha_i(s - \bar{v}) - \delta u}{\Lambda} + \Upsilon,$$

where  $\Gamma = (\gamma_i + \gamma_s + \gamma_h)/(\beta_i + \beta_s + \beta_h)$ ,  $\Lambda = \beta_i + \beta_s + \beta_h$ , and  $\Upsilon = (\varphi_i + \varphi_s + \varphi_h)/(\beta_i + \beta_s + \beta_h)$ . We now compute the optimal strategies given

this price function. Since price  $p$  is informationally equivalent to  $\{\alpha_i(s - \bar{v}) - \delta u\} = \Lambda[p - \bar{v} + \Gamma q - \Upsilon]$ , we have that

$$E[v|p] = \bar{v} + \frac{\Lambda(\alpha_i \sigma_v^2)}{\delta^2 \sigma_u^2 + (\alpha_i^2 / R_{sv}) \sigma_v^2} [p - \bar{v} + \Gamma q - \Upsilon],$$

$$\text{var}[v|p] = \sigma_v^2 \frac{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2}{\delta^2 \sigma_u^2 + R_{sv}^{-1} \alpha_i^2 \sigma_v^2},$$

$$E[v|p, u_j] = \bar{v} + \left(\frac{1}{k}\right) \left\{ \alpha_i \left(\frac{\sigma_u^2}{R_u}\right) \sigma_v^2 [\alpha_i(s - \bar{v}) - \delta u] + \delta \alpha_i \sigma_u^2 \sigma_v^2 u_j \right\},$$

$$\text{var}[v|p, u_j] = \sigma_v^2 \frac{(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2}{(1 - R_u) \delta^2 \sigma_u^2 + R_{sv}^{-1} \alpha_i^2 \sigma_v^2},$$

$$\text{where } k = \frac{\sigma_u^2}{R_u} \left[ \alpha_i^2 \frac{\sigma_v^2}{R_{sv}} + \delta^2 \sigma_u^2 (1 - R_u) \right].$$

Substituting these expressions in speculator  $k$ 's optimal demand,  $X_{sk}(p) = E[v - p|p] / (\rho_s \text{var}[v - p|p])$ , hedger  $j$ 's optimal demand,  $X_{hj}(p, u_j) = (E[v - p|p, u_j] - \rho_h u_j \text{cov}[z, v - p|p, u_j]) / (\rho_h \text{var}[v - p|p, u_j])$ , and the insider's optimal strategy,  $X_i(s, p) = (E[v|s] - p - \rho_i q \text{var}[v|s]) / (\rho_i \text{var}[v|s] + \lambda)$  (where  $E[v|s] = \bar{v} + R_{sv}(s - \bar{v})$  and  $\text{var}[v|s] = (1 - R_{sv}) \sigma_v^2$ ), and then comparing with the linear strategy initially posited, we obtain (after making some simplifications)

$$\alpha_i = \frac{R_{sv}}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda},$$

$$\beta_i = \frac{1}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda},$$

$$\gamma_i = \frac{\rho_i (1 - R_{sv}) \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda}, \quad \varphi_i = 0,$$

$$\beta_s = \frac{1}{\rho_s \sigma_v^2} \frac{\delta^2 \sigma_u^2 - (\alpha_i / \lambda) \sigma_v^2}{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

$$\gamma_s = \frac{-1}{\rho_s} \frac{(\gamma_s + \gamma_h + \gamma_i) \alpha_i}{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

$$\begin{aligned}\varphi_s &= -\frac{\Lambda\alpha_i[\delta^2\sigma_u^2 + R_{sv}^{-1}\alpha_i^2\sigma_v^2]}{\rho_s[\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2][\delta^2\sigma_u^2 + (\alpha_i^2/R_{sv})\sigma_v^2]}\Upsilon, \\ \beta_h &= \frac{1}{\rho_h\sigma_v^2}\frac{(1 - R_u)\delta^2\sigma_u^2 - (\alpha_i/\lambda)\sigma_v^2}{(1 - R_u)\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2}, \\ \gamma_h &= \frac{-1}{\rho_h}\frac{(\gamma_s + \gamma_h + \gamma_i)\alpha_i}{(1 - R_u)\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2}, \\ \varphi_h &= -\frac{\Lambda\alpha_i[\delta^2\sigma_u^2 + R_{sv}^{-1}\alpha_i^2\sigma_v^2]}{\rho_h[\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2][\delta^2\sigma_u^2 + (\alpha_i^2/R_{sv})\sigma_v^2]}\Upsilon, \\ \delta &= \left(\frac{\sigma_{vz}}{\sigma_v^2}\right)\left\{1 + \left(\frac{\alpha_i}{\rho_h}\right)\frac{R_u}{(1 - R_u)\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2}\right\}^{-1}.\end{aligned}$$

The relation between the insider's asset position and the price is given by  $p = \bar{v} + \frac{\varphi_s + \varphi_h}{\beta_s + \beta_h} + \lambda[x_i - \delta u - (\gamma_s + \gamma_h)q]$ , so that  $\lambda = dp/dx_i = 1/(\beta_s + \beta_h)$ . Moreover, from the equations  $\varphi_i = 0$ ,

$$\begin{aligned}\varphi_s &= -\Lambda\alpha_i[\delta^2\sigma_u^2 + R_{sv}^{-1}\alpha_i^2\sigma_v^2]/(\rho_s[\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2][\delta^2\sigma_u^2 + (\alpha_i^2/R_{sv})\sigma_v^2])\Upsilon, \\ \varphi_h &= -\frac{\Lambda\alpha_i[\delta^2\sigma_u^2 + R_{sv}^{-1}\alpha_i^2\sigma_v^2]}{\rho_h[\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2][\delta^2\sigma_u^2 + (\alpha_i^2/R_{sv})\sigma_v^2]}\Upsilon \text{ and} \\ \Upsilon &= \frac{\varphi_i + \varphi_s + \varphi_h}{\beta_i + \beta_s + \beta_h},\end{aligned}$$

we directly have that, in any linear equilibrium,

$$\varphi_i = \varphi_s = \varphi_h = \Upsilon = 0.$$

Finally, the equations

$$\begin{aligned}\gamma_s &= \frac{-1}{\rho_s}\frac{(\gamma_s + \gamma_h + \gamma_i)\alpha_i}{\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2}, \\ \gamma_h &= \frac{-1}{\rho_h}\frac{(\gamma_s + \gamma_h + \gamma_i)\alpha_i}{(1 - R_u)\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2},\end{aligned}$$

may also be written (after simple manipulations) as

$$\begin{aligned}\gamma_s &= \frac{-\frac{\alpha_i}{(1 + \alpha_i E)}\gamma_i}{\rho_s[\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2]}, \\ \gamma_h &= \frac{-\frac{\alpha_i}{(1 + \alpha_i E)}\gamma_i}{\rho_h[(1 - R_u)\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2]},\end{aligned}$$

with

$$E = \frac{1}{\rho_s} \left[ \delta^2 \sigma_u^2 + \frac{R_{sv}(1 - R_{sv})}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \sigma_v^2 \right]^{-1} \\ + \frac{1}{\rho_h} \left[ (1 - R_u)\delta^2 \sigma_u^2 + \frac{R_{sv}(1 - R_{sv})}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \sigma_v^2 \right]^{-1}$$

$$\text{for } \alpha_i = \frac{R_{sv}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}. \quad \square$$

LEMMA 6. In equilibrium, for  $R_{sv} > 0$ , the endogenous parameters satisfy the following inequalities:

$$\lambda > \sigma_v^2 \left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right),$$

$$0 < \delta < \frac{\sigma_{vz}}{\sigma_v^2},$$

$$0 \leq \gamma_s + \gamma_h + \gamma_i \leq \gamma_i < 1.$$

Moreover,  $0 \leq |\gamma_s + \gamma_h| \leq \gamma_i < 1$ ,  $\gamma_s \leq 0$ ,  $\gamma_h \leq 0$ ,  $\Lambda > 0$ ,  $\Gamma \geq 0$ ,  $\alpha_i > 0$ ,  $\beta_i > 0$ ,  $\beta_s > 0$ , and  $\beta_h > 0$ . The equalities hold if and only if  $R_{sv} = 1$ . If  $R_{sv} = 0$ , then  $\lambda = \sigma_v^2(\rho_s \rho_h)/(\rho_s + \rho_h)$ ,  $\delta = \sigma_{vz}/\sigma_v^2$ ,  $\alpha_i = \gamma_s = \gamma_h = 0$ ,  $0 < \gamma_i < 1$ ,  $\Gamma \Lambda < 1$ ,  $\Lambda > 0$ ,  $\Gamma > 0$ ,  $\beta_i > 0$ ,  $\beta_s > 0$ , and  $\beta_h > 0$ .

*Proof.* First, let us show that  $\sigma_{vz}/\sigma_v^2 \geq \delta \geq 0$ . The second-order condition of the insider's optimization problem implies that (in any equilibrium)  $\lambda > 0$ . Now, for any given  $\lambda > 0$ , equation (5) is a cubic equation in  $\delta$ , so it has at least a real solution. It is easy to check that this equation,

$$\frac{R_u R_{sv}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda} \delta = \rho_h \left[ \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) - \delta \right] \\ \times \left\{ (1 - R_u)\delta^2 \sigma_u^2 + \frac{(1 - R_{sv})R_{sv}\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right\},$$

has no negative solution in  $\delta$  for any  $\lambda > 0$ . Suppose there is an equilibrium and that  $\hat{\lambda}$  is the equilibrium value of  $\lambda$ . For any  $\delta < 0$  the left-hand side,  $R_u R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \hat{\lambda})\delta$ , is negative while the right-hand side,

$$\rho_h \left[ \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) - \delta \right] \left\{ (1 - R_u) \delta^2 \sigma_u^2 + \frac{(1 - R_{sv}) R_{sv} \sigma_v^2}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \hat{\lambda}]^2} \right\},$$

is positive, so this equation has no negative solution and hence the equilibrium value of  $\delta$  cannot be negative. On the other hand, for any  $\delta \geq \sigma_{vz}/\sigma_v^2$  the left-hand side is positive while the right-hand side is negative, so that the equilibrium value of  $\delta$  cannot be greater than  $\sigma_{vz}/\sigma_v^2$ . Thus, for any candidate equilibrium (for which we know  $\lambda > 0$ ), the solutions of Equation (5) must be in the interval  $(0, \sigma_{vz}/\sigma_v^2]$ .

In fact, it is easy to show that  $\delta = \sigma_{vz}/\sigma_v^2$  for  $R_{sv} = 0$  and that  $\delta < \sigma_{vz}/\sigma_v^2$  for any  $R_{sv} \neq 0$ . On the other hand, note that Equation (5) may be written as

$$\delta = \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) \left\{ 1 + \frac{R_u \alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2]} \right\}^{-1},$$

so  $\delta$  cannot be equal to zero because  $R_u \alpha_i / (\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2]) \geq 0$ . Consequently, we have that  $\delta = \sigma_{vz}/\sigma_v^2$  for  $R_{sv} = 0$  and that

$$0 < \delta < \frac{\sigma_{vz}}{\sigma_v^2}$$

for every  $R_{sv} \neq 0$ .

Now let us show that, in equilibrium,  $\lambda \geq \sigma_v^2 (\rho_s \rho_h) / (\rho_s + \rho_h)$ . Taking into account that  $\alpha_i$  depends only on  $\lambda$  and some exogenous parameters, Equation (6) may be seen as an equation in  $(\delta, \lambda)$ ; for any given  $\delta \in [0, \sigma_{vz}/\sigma_v^2]$ , it may be written as a (fifth-degree) polynomial equation in  $\lambda$ . Moreover, it is easy to check that any positive solution is such that  $\lambda \geq \sigma_v^2 (\rho_s \rho_h) / (\rho_s + \rho_h)$ . Equation (6) may be written as

$$\frac{1}{\lambda} \{1 + K\} = \frac{1}{\rho_s \sigma_v^2} \frac{\delta^2 \sigma_u^2}{\delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2} + \frac{1}{\rho_h \sigma_v^2} \frac{(1 - R_u) \delta^2 \sigma_u^2}{(1 - R_u) \delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2},$$

where

$$K = \frac{\alpha_i}{\rho_s [\delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} + \frac{\alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \geq 0.$$

Since both

$$\frac{\delta^2 \sigma_u^2}{\delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2}, \quad \frac{(1 - R_u) \delta^2 \sigma_u^2}{(1 - R_u) \delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2}$$

are less than or equal to 1, it is obvious that (in any equilibrium)

$$\frac{1}{\lambda} \{1 + K\} \leq \frac{1}{\rho_s \sigma_v^2} + \frac{1}{\rho_h \sigma_v^2}.$$

And, since

$$K = \frac{\alpha_i}{\rho_s [\delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} + \frac{\alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \geq 0,$$

we have that

$$\frac{1}{\lambda} \leq \frac{1}{\lambda} \{1 + K\} \leq \frac{1}{\rho_s \sigma_v^2} + \frac{1}{\rho_h \sigma_v^2}.$$

If  $R_{sv} = 0$ , then  $\alpha_i = 0$ ,  $K = 0$ , and

$$\lambda = \left( \frac{1}{\rho_s \sigma_v^2} + \frac{1}{\rho_h \sigma_v^2} \right)^{-1}.$$

Otherwise, for any  $R_{sv} \neq 0$  we have  $\alpha_i > 0$ ,  $K > 0$ , and

$$\lambda > \left( \frac{1}{\rho_s \sigma_v^2} + \frac{1}{\rho_h \sigma_v^2} \right)^{-1} = \sigma_v^2 \left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right).$$

The other inequalities are obvious from the characterization of the equilibrium. For instance, in any equilibrium  $\lambda > 0$  it is clear that

$$\gamma_i = \frac{\rho_i (1 - R_{sv}) \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda}$$

is greater than zero and strictly less than unity,  $0 \leq \gamma_i < 1$  (where  $\gamma_i = 0$  if and only if  $R_{sv} = 1$ ). Moreover, since  $\alpha_i = R_{sv} / (\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda) \geq 0$  and  $\gamma_i \geq 0$ , it follows that both are (equal to or) less than zero:



$$\gamma_s = \frac{-\alpha_i}{\rho_s [\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2] (1 + \alpha_i E)} \gamma_i \leq 0,$$

$$\gamma_h = \frac{-\alpha_i}{\rho_h [(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2] (1 + \alpha_i E)} \gamma_i \leq 0.$$

On the other hand,

$$\gamma_i + \gamma_s + \gamma_h = \frac{\gamma_i}{(1 + \alpha_i E)} \geq 0,$$

where

$$E = \frac{1}{\rho_s} [\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2]^{-1} + \frac{1}{\rho_h} [(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2]^{-1} \geq 0.$$

It is then clear that

$$0 \leq \gamma_s + \gamma_h + \gamma_i \leq \gamma_i < 1.$$

In fact, if  $R_{sv} = 0$  then  $\gamma_s = \gamma_h = 0$  and  $0 < \gamma_i < 1$ . If  $R_{sv} = 1$ , then  $\gamma_s = \gamma_h = \gamma_i = 0$ . Furthermore, for any  $R_{sv} \in (0, 1)$ ,

$$0 < \gamma_s + \gamma_h + \gamma_i < \gamma_i < 1.$$

On the other hand,

$$0 \leq \Gamma \Lambda = (\gamma_s + \gamma_h + \gamma_i) < 1$$

and, since  $0 \leq \gamma_i < 1$  and  $0 \leq \gamma_s + \gamma_h + \gamma_i \leq \gamma_i < 1$ ,

$$0 \leq |\gamma_s + \gamma_h| \leq \gamma_i < 1.$$

The inequalities  $\Lambda > 0$ ,  $\Gamma \geq 0$ ,  $\alpha_i \geq 0$ ,  $\beta_i > 0$ ,  $\beta_s > 0$ , and  $\beta_h > 0$  follow directly from the characterization of equilibria in Proposition 1 and the foregoing results.  $\square$

*Proof of Proposition 2.* Let us check first that, if  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  tend to zero, then the equilibrium parameter  $\delta$  tends to  $\sigma_{vz}/\sigma_v^2$ . Equation (5) may be written as

$$\frac{R_u}{\rho_h} \frac{R_{sv}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda} \delta = \left[ \frac{\sigma_{vz}}{\sigma_v^2} - \delta \right] \left\{ (1 - R_u)\delta^2 \sigma_u^2 + \frac{(1 - R_{sv})R_{sv}\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right\}$$

or as

$$\frac{R_u}{\rho_h} \alpha_i \delta = \left[ \frac{\sigma_{vz}}{\sigma_v^2} - \delta \right] \left\{ (1 - R_u)\delta^2 \sigma_u^2 + \frac{(1 - R_{sv})R_{sv}\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right\},$$

since  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$ . If  $R_u/\rho_h$  is low, then it is obvious that  $\delta$  will be close to  $\sigma_{vz}/\sigma_v^2$  (since  $(1 - R_u)\delta^2 \sigma_u^2 + (1 - R_{sv})R_{sv}\sigma_v^2/[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2 > 0$ ). On the other hand, if  $R_{sv}/\rho_i$  is low, then  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$  will be close to zero (this is clear if  $R_{sv}$  is close to zero and also if  $\rho_i$  is sufficiently large, since  $\lambda \geq \sigma_v^2 \left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right) > 0$ ) and, as a direct consequence,  $\delta$  will also be close to  $\sigma_{vz}/\sigma_v^2$ . Therefore, if  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  tends to zero, the equilibrium parameter  $\delta$  tends to  $\sigma_{vz}/\sigma_v^2$ .

There is equilibrium if and only if there is a solution to Equations (5) and (6) satisfying the second-order condition  $\lambda > 0$ . Moreover, from Lemma 6 we know that (in any equilibrium)  $\lambda \geq \sigma_v^2(\rho_s \rho_h)/(\rho_s + \rho_h)$ , and  $\sigma_{vz}/\sigma_v^2 \geq \delta \geq 0$ .

Equation (5) may be written as  $g_1(\delta, \lambda) = g_2(\delta, \lambda)$ , where  $g_1(\delta, \lambda) = R_u R_{sv}/(\rho_h[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda])\delta$  and

$$g_2(\delta, \lambda) = \left[ \frac{\sigma_{vz}}{\sigma_v^2} - \delta \right] \left\{ (1 - R_u)\delta^2 \sigma_u^2 + \frac{(1 - R_{sv})R_{sv}\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right\}.$$

For any  $\lambda > 0$ , it is obvious that there exists a solution  $\delta$  of the equation  $g_1(\delta, \lambda) = g_2(\delta, \lambda)$ , since  $g_1(\delta, \lambda)$  is strictly increasing in  $\delta$ ,  $g_1(0, \lambda) = 0$ ,  $g_2(\delta, \lambda)$  is continuous,  $g_2(0, \lambda) > 0$ ,  $g_2\left(\frac{\sigma_{vz}}{\sigma_v^2}, \lambda\right) = 0$ . Is it possible to have more than one solution  $\delta$  in the interval  $[0, \sigma_{vz}/\sigma_v^2]$ ? If  $\partial g_2(\delta, \lambda)/\partial \delta < 0$  for all  $\delta \in [0, \sigma_{vz}/\sigma_v^2]$ , we know that there is only one solution  $\delta$  to  $g_1(\delta, \lambda) = g_2(\delta, \lambda)$  for each  $\lambda > 0$ . The foregoing partial derivative is given by

$$\frac{\partial g_2(\delta, \lambda)}{\partial \delta} = -\frac{(1 - R_{sv})R_{sv}\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} - \left[ 3\delta - 2\frac{\sigma_{vz}}{\sigma_v^2} \right] (1 - R_u)\sigma_u^2 \delta.$$

From this expression, if  $\delta > (2/3)(\sigma_{vz}/\sigma_v^2)$  then  $3\delta - 2(\sigma_{vz}/\sigma_v^2) > 0$  and, as a direct consequence,  $\partial g_2(\delta, \lambda)/\partial \delta < 0$ . But if  $\delta < 2/3(\sigma_{vz}/\sigma_v^2)$ , then  $\partial g_2(\delta, \lambda)/\partial \delta$  may be greater than zero (in fact, it is easy to have parameter values such that  $\partial g_2(\delta, \lambda)/\partial \delta > 0$ ). Therefore, in general we cannot guarantee uniqueness of

the solution of the equation  $g_1(\delta, \lambda) = g_2(\delta, \lambda)$  for any  $\lambda > 0$  and, as a result, we cannot establish existence of a well-defined implicit function  $\delta(\lambda)$  for every  $\lambda > 0$ .

However, if  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  is close to zero, then the candidate  $\delta$  is close to  $\sigma_{vz}/\sigma_v^2$  and  $\partial g_2(\delta, \lambda)/\partial \delta$  will be less than zero in any solution. Therefore, if  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  is close to zero, we can guarantee uniqueness of the solution and, consequently, we can establish existence of a well-defined implicit function  $\delta(\lambda)$  for every  $\lambda > 0$ . Substituting  $\delta(\lambda)$  in Equation (6) yields a single equation in  $\lambda$ :

$$\begin{aligned} & \lambda \delta^2(\lambda) \sigma_u^2 \left\{ 1 + \frac{1}{\rho_s[\delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right\} \\ & + \lambda \delta^2(\lambda) \sigma_u^2 \frac{(1 - R_u)}{\rho_h[(1 - R_u) \delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \\ & = \sigma_v^2 \left\{ 1 + \frac{\alpha_i}{\rho_s[\delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right. \\ & \quad \left. + \frac{(1 - R_u) \alpha_i}{\rho_h[(1 - R_u) \delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right\}, \end{aligned}$$

where  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$ . This equation may be written as  $h_1(\lambda) = h_2(\lambda)$ , where

$$\begin{aligned} h_1(\lambda) &= \lambda \delta^2(\lambda) \sigma_u^2 \left\{ 1 + \frac{1}{\rho_s[\delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right. \\ & \quad \left. + \frac{1 - R_u}{\rho_h[(1 - R_u) \delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right\}, \\ h_2(\lambda) &= \sigma_v^2 \left\{ 1 + \frac{\alpha_i}{\rho_s[\delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right. \\ & \quad \left. + \frac{(1 - R_u) \alpha_i}{\rho_h[(1 - R_u) \delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right\}, \end{aligned}$$

here  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$  and  $\delta(\lambda)$  is close to  $\sigma_{vz}/\sigma_v^2$ .

Each positive solution of this equation will characterize an equilibrium. Let us prove that this equation has positive solutions provided  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  is close to zero.

If  $\lambda = 0$ , then  $h_1(\lambda) = 0$  and  $h_2(\lambda) = \sigma_v^2 > 0$ . If  $\lambda \rightarrow +\infty$ , then  $\alpha_i \rightarrow 0$ ,  $h_2(\lambda) \rightarrow \sigma_v^2$ , and  $h_1(\lambda) \rightarrow +\infty$ . Moreover, the functions  $h_1(\lambda)$  and  $h_2(\lambda)$  are both continuous for  $\lambda > 0$ . As a result, we can guarantee that the equation  $h_1(\lambda) = h_2(\lambda)$  has a positive solution. Equilibria will be characterized by (a) the positive solutions  $\hat{\lambda}$  of the equation  $h_1(\lambda) = h_2(\lambda)$  and (b) the values of  $\hat{\delta}$  given by the implicit function  $\delta(\lambda)$ ,  $\hat{\delta} = \delta(\hat{\lambda})$ .

We have proved existence of equilibrium for  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  sufficiently close to zero. Now we shall prove uniqueness. There will be a unique equilibrium if  $dh_1(\lambda)/d\lambda > dh_2(\lambda)/d\lambda$  in any positive solution  $\lambda$  of the equation  $h_1(\lambda) = h_2(\lambda)$  where

$$h_1(\lambda) = \lambda \delta^2(\lambda) \sigma_u^2 \left\{ 1 + \frac{1}{\rho_s[\delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right\} + \lambda \delta^2(\lambda) \sigma_u^2 \frac{(1 - R_u)}{\rho_h[(1 - R_u) \delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]},$$

$$h_2(\lambda) = \sigma_v^2 \left\{ 1 + \frac{\alpha_i}{\rho_s[\delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} + \frac{(1 - R_u) \alpha_i}{\rho_h[(1 - R_u) \delta^2(\lambda) \sigma_u^2 + \alpha_i^2 (R_{sv}^{-1} - 1) \sigma_v^2]} \right\},$$

where  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$  and  $\delta(\lambda)$  is implicitly defined by the unique positive solution of

$$\frac{R_u R_{sv}}{\rho_h[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]} \delta = \left[ \frac{\sigma_{vz}}{\sigma_v^2} - \delta \right] \left\{ (1 - R_u) \delta^2 \sigma_u^2 + \frac{(1 - R_{sv}) R_{sv} \sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \right\}.$$

(Remember that function  $\delta(\lambda)$  is defined only for positive values of  $\lambda$  and is well-defined provided that  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  is sufficiently close to zero).

We have to check whether  $dh_1(\lambda)/d\lambda$  is greater or less than  $dh_2(\lambda)/d\lambda$  (for any possible positive solution of the equation  $h_1(\lambda) = h_2(\lambda)$ ).

It is obvious that  $d\alpha_i/d\lambda < 0$  (since  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$ ), and it can be easily proved that  $d\delta(\lambda)/d\lambda > 0$  and also (provided  $(R_u/\rho_h)(R_{sv}/\rho_i)$  is sufficiently close to zero) that  $d\delta(\lambda)/d\lambda$  is close to zero. The equation that (implicitly) defines  $\delta(\lambda)$  can be written as

$$G(\delta, \lambda) = \frac{R_u}{\rho_h} \alpha_i \delta - \left[ \frac{\sigma_{vz}}{\sigma_v^2} - \delta \right] \left\{ (1 - R_u) \delta^2 \sigma_u^2 + \frac{(1 - R_{sv}) \alpha_i^2 \sigma_v^2}{R_{sv}} \right\} = 0$$

with  $\alpha_i = R_{sv}/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$ . It is then clear that  $\partial G/\partial \delta > 0$  (for, provided  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  is close to zero,  $\sigma_{vz}/\sigma_v^2 - \delta$  is also close to zero) and

$$\frac{\partial G}{\partial \lambda} = \frac{-R_u R_{sv}}{\rho_h[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \delta + \left[ \frac{\sigma_{vz}}{\sigma_v^2} - \delta \right] \frac{2(1 - R_{sv})R_{sv}\sigma_v^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2}.$$

Since (for  $(R_u/\rho_h)(R_{sv}/\rho_i)$  sufficiently close to zero)  $\sigma_{vz}/\sigma_v^2 - \delta$  is close to zero, it follows that the first term of  $\partial G/\partial \lambda$  dominates the second term and therefore  $\partial G/\partial \lambda < 0$ . By applying the implicit function theorem, if  $(R_u/\rho_h)(R_{sv}/\rho_i)$  is close to zero then we have  $d\delta(\lambda)/d\lambda > 0$ . Yet we know that, as  $(R_u/\rho_h)(R_{sv}/\rho_i)$  tends to zero,  $\delta$  tends to  $\sigma_{vz}/\sigma_v^2$  (and, by Lemma 6,  $\delta$  is always equal to or less than  $\sigma_{vz}/\sigma_v^2$ ) and also  $d\delta(\lambda)/d\lambda > 0$ . If  $\lambda$  increases then  $\delta$  increases, but the increase of  $\delta$  is very small, because it is “always” close to  $\sigma_{vz}/\sigma_v^2$  and cannot exceed that value for  $\delta \leq \sigma_{vz}/\sigma_v^2$ . Therefore, if  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  is close to zero,  $d\delta(\lambda)/d\lambda$  is positive but very small. Put another way, as  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i \rightarrow 0$ ,  $d\delta(\lambda)/d\lambda$  tends to zero from the right  $d\delta(\lambda)/d\lambda \rightarrow 0^+$ .

Let us compare  $dh_1(\lambda)/d\lambda$  with  $dh_2(\lambda)/d\lambda$ . Since  $dx_i/d\lambda < 0$  and (as  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  tends to zero)  $d\delta(\lambda)/d\lambda \rightarrow 0^+$ , the terms  $1/(\rho_s[\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2])$  and  $(1 - R_u)/(\rho_h[(1 - R_u)\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2])$  are both increasing in  $\lambda$ . Moreover, it can be easily checked that

$$\begin{aligned} \frac{dh_2(\lambda)}{d\lambda} &= \sigma_v^2 \frac{d}{d\lambda} \left\{ \frac{\alpha_i}{\rho_s[\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right\} \\ &\quad + \sigma_v^2 \frac{d}{d\lambda} \left\{ \frac{(1 - R_u)\alpha_i}{\rho_h[(1 - R_u)\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right\} \\ &\leq \frac{d}{d\lambda} \left\{ \frac{1}{\rho_s[\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right. \\ &\quad \left. + \frac{1 - R_u}{\rho_h[(1 - R_u)\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right\} \\ &\leq \frac{d}{d\lambda} \left( \lambda \delta^2(\lambda) \sigma_u^2 \left\{ 1 + \frac{1}{\rho_s[\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right\} \right) \\ &\quad + \frac{d}{d\lambda} \left( \lambda \delta^2(\lambda) \sigma_u^2 \left\{ \frac{1 - R_u}{\rho_h[(1 - R_u)\delta^2(\lambda)\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right\} \right) \\ &= \frac{dh_1(\lambda)}{d\lambda}. \end{aligned}$$

The first inequality is due to the fact that  $dx_i/d\lambda < 0$ , and the second is obvious since  $[\lambda\delta^2(\lambda)]$  is increasing in  $\lambda$  (for  $d\delta(\lambda)/d\lambda \geq 0$ ).

To summarize: we have proved that, for  $R_u/\rho_h$  and/or  $R_{sv}/\rho_i$  sufficiently close to zero,  $dh_2(\lambda)/d\lambda \leq dh_1(\lambda)/d\lambda$  and, as a direct consequence, there is a unique linear equilibrium.  $\square$

LEMMA 7. Let  $x \sim N(0, \Sigma)$  and  $W = c + b'x + x'Ax$ , where  $c \in \mathbb{R}$ ,  $b \in \mathbb{R}^n$ , and  $A$  is an  $n \times n$  matrix. Then, if  $\Sigma^{-1} + 2aA$  is positive definite,

$$\begin{aligned} -E[\exp(-aW)] &= -|\Sigma|^{-1/2} |\Sigma^{-1} + 2aA|^{-1/2} \\ &\quad \times \exp\{-a[c - (1/2)ab'(\Sigma^{-1} + 2aA)^{-1}b]\}. \end{aligned}$$

*Proof.* See Danthine and Moresi (1993).  $\square$

COROLLARY 8. If  $x \sim N(\bar{x}, \sigma_x^2)$  and  $y \sim N(\bar{y}, \sigma_y^2)$ , then

$$E[\exp\{x - y^2\}] = \frac{1}{\sqrt{1 + 2\sigma_y^2}} \exp\left\{\bar{x} + \frac{\sigma_x^2}{2} - \frac{(\bar{y} + \text{cov}[x, y])^2}{1 + 2\sigma_y^2}\right\}.$$

*Proof.* See Demange and Laroque (1995).  $\square$

### A.3. Expected Utilities

#### A.3.1. EXPECTED UTILITY OF INSIDER IN IT REGIME

Conditional on  $(s, p)$ , the entrepreneur's expected utility is

$$E[-\exp\{-\rho_i W_i\}|s, p] = -\exp\left\{-\rho_i \left(E[W_i|s, p] - \frac{\rho_i}{2} \text{var}[W_i|s, p]\right)\right\},$$

where  $E[W_i|s, p] = qE[v|s] - C(q) + X_i(s, p)\{E[v|s] - p\}$  and  $\text{var}[W_i|s, p] = q^2 \text{var}[v|s] + \text{var}[v|s]\{X_i(s, p)\}^2 + 2qX_i(s, p)\text{var}[v|s]$ . Substituting into the above expression  $X_i(s, p) = (E[v|s] - p - \rho_i q \text{var}[v|s]) / (\rho_i \text{var}[v|s] + \lambda)$  and simplifying yields

$$\begin{aligned} E[U(W_i)|s, p] &= -\exp\left\{\rho_i C(q) + \frac{1}{2}\rho_i^2 q^2 \text{var}[v|s]\right\} \\ &\quad \times \exp\left\{-\rho_i q E[v|s] - \frac{\rho_i}{2} [\rho_i \text{var}[v|s] + 2\lambda][X_i(s, p)]^2\right\}, \end{aligned}$$

where the first exponential is nonrandom. To obtain the entrepreneur's *ex ante* (unconditional) expected utility, it suffices to apply Corollary 8 by taking  $x = -\rho_i q E[v|s]$  and  $y = \sqrt{\frac{\rho_i}{2} [\rho_i \text{var}[v|s] + 2\lambda]} X_i(s, p)$ . We know that  $E\{E[v|s]\} = \bar{v}$  and  $\text{var}[E(v|s)] = R_{sv} \sigma_v^2$ . We have also that

$$E[X_i(s, p)] = \frac{E[v - p]}{\rho_i \text{var}[v|s] + \lambda} - \gamma_i q = \left( \frac{\Gamma}{\rho_i \text{var}[v|s] + \lambda} - \gamma_i \right) q,$$

$$\text{var}[X_i(s, p)] = \left[ R_{sv} - \frac{\alpha_i}{\Lambda} \right]^2 \frac{\sigma_v^2}{R_{sv}} + \frac{\delta^2 \sigma_u^2}{\Lambda^2},$$

$$\text{cov}\{E[v|s], X_i(s, p)\} = \frac{R_{sv} \left[ R_{sv} - \frac{\alpha_i}{\Lambda} \right]}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda R_{sv}} \frac{\sigma_v^2}{R_{sv}}.$$

Since

$$\alpha_i = \frac{R_{sv}}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda}, \quad \gamma_i = \frac{\rho_i (1 - R_{sv}) \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda},$$

$$\Gamma = \frac{\gamma_i}{(1 + \alpha_i E) \Lambda} \quad \left( \text{for } \Gamma = \frac{\gamma_s + \gamma_h + \gamma_i}{\Lambda} \text{ and } \gamma_i + \gamma_s + \gamma_h = \frac{\gamma_i}{(1 + \alpha_i E)} \right),$$

$$\Lambda = \frac{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda] \lambda} \quad \left( \text{for } \Lambda = \beta_s + \beta_h + \beta_i, \beta_s + \beta_h = 1/\lambda, \right.$$

$$\left. \beta_i = \frac{1}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda} \right),$$

we may write

$$E[X_i(s, p)] = -\gamma_i q \left( 1 - \frac{1}{1 + \alpha_i E} \frac{\lambda}{[\rho_i \text{var}[v|s] + 2\lambda]} \right),$$

$$\text{var}[X_i(s, p)] = \frac{R_{sv} \sigma_v^2 + \lambda^2 \delta^2 \sigma_u^2}{[\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda]^2},$$

$$\text{cov}\{E[v|s], X_i(s, p)\} = \frac{R_{sv} \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda}.$$

Thus, by applying Corollary 8, we see that the entrepreneur's *ex ante* (unconditional) expected utility may be written as

$$E[U(W_i)] = -|SG_i| \exp \left\{ -\rho_i \left[ q\bar{v} - C(q) - \frac{1}{2} \rho_i q^2 \sigma_v^2 (1-d) \right] \right\}$$

(since  $\gamma_i = \rho_i(1 - R_{sv})\sigma_v^2 / [\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]$ ), where

$$|SG_i| = \left\{ 1 + \rho_i \frac{R_{sv}\sigma_v^2 + \lambda^2 \delta^2 \sigma_u^2}{\rho_i(1 - R_{sv})\sigma_v^2 + 2\lambda} \right\}^{-1/2}, \quad |IG_i| = \exp \left\{ -\frac{1}{2} \rho_i^2 d \sigma_v^2 q^2 \right\}$$

and

$$d = \frac{\rho_i \sigma_v^2}{\rho_i \sigma_v^2 + 2\lambda + \rho_i \lambda^2 \delta^2 \sigma_u^2} \left\{ \frac{[\rho_i \text{var}[v|s] + 2\lambda]}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]} \right. \\ \left. \times \left( 1 - \frac{1}{(1 + \alpha_i E)} \frac{\lambda}{[\rho_i \text{var}[v|s] + 2\lambda]} \right) (1 - R_{sv}) + R_{sv} \right\}^2$$

with

$$E = \frac{1}{\rho_s} \left[ \delta^2 \sigma_u^2 + \frac{R_{sv}(1 - R_{sv})}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \sigma_v^2 \right]^{-1} \\ + \frac{1}{\rho_h} \left[ (1 - R_u) \delta^2 \sigma_u^2 + \frac{R_{sv}(1 - R_{sv})}{[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]^2} \sigma_v^2 \right]^{-1}.$$

After some simple manipulations,  $d$  may also be written as

$$d = \frac{\rho_i \sigma_v^2}{\rho_i \sigma_v^2 + 2\lambda + \rho_i \lambda^2 \delta^2 \sigma_u^2} \left\{ 1 + \frac{\lambda}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda} \frac{\alpha_i E}{1 + \alpha_i E} (1 - R_{sv}) \right\}^2.$$

□

### A.3.2. EXPECTED UTILITY OF SPECULATORS IN IT REGIME

Speculator  $k$ 's expected utility conditional on his information ( $p$ ) is equal to

$$E[-\exp\{-\rho_s W_{sk}\} | p] = -\exp \left\{ -\rho_s \left( E[W_{sk} | p] - \frac{1}{2} \rho_s \text{var}[W_{sk} | p] \right) \right\},$$



where  $E[W_{sk}|p] = x_{sk}(p)\{E[v|p] - p\} = (E[v|p] - p)^2/(\rho_s \text{var}[v|p])$  and  $\text{var}[W_{sk}|p] = \text{var}[v|p]\{x_{sk}^2(p)\} = (E[v|p] - p)^2/(\rho_s^2 \text{var}[v|p])$ . Substituting into the above expression and simplifying yields

$$E[-\exp\{-\rho_s W_{sk}\}|p] = -\exp\left\{-\frac{(E[v|p] - p)^2}{2\text{var}[v|p]}\right\}.$$

To obtain speculator  $k$ 's *ex ante* (unconditional) expected utility, it suffices to apply Corollary 8 by taking  $x = 0$  and  $y = (E[v|p] - p)/\sqrt{2\text{var}[v|p]}$ , so that

$$E[U(W_{sk})] = -\left\{1 + \frac{\text{var}[E[v|p] - p]}{\text{var}[v|p]}\right\}^{-1/2} \\ \times \exp\left\{-\frac{1}{2} \frac{(E[E[v|p] - p])^2}{\text{var}[v|p] + \text{var}[E[v|p] - p]}\right\}.$$

Since  $E[p] = \bar{v} - \Gamma q$  and  $E(v|p) = \bar{v} + \alpha_i \sigma_v^2 / (\alpha_i^2 R_{sv}^{-1} \sigma_v^2 + \delta^2 \sigma_u^2) \{\alpha_i (s - \bar{v}) - \delta u\}$ , we have that  $E[E(v|p)] = \bar{v} + [\alpha_i \sigma_v^2 / (\alpha_i^2 R_{sv}^{-1} \sigma_v^2 + \delta^2 \sigma_u^2)] \delta E[u] = \bar{v}$  and  $E[E(v|p) - p] = \Gamma q$ , so speculator  $k$ 's *ex ante* (unconditional) expected utility is given by

$$EU_s \equiv E[-\exp\{-\rho_s W_s\}] = -|SG_s| \exp\left\{-\frac{1}{2} \frac{(\Gamma q)^2}{\text{var}[E(v|p) - p] + \text{var}[v|p]}\right\},$$

where

$$|SG_s| = \left\{1 + \frac{\text{var}[E(v|p) - p]}{\text{var}[v|p]}\right\}^{-1/2}, \\ \text{var}[E(v|p) - p] = \frac{[\delta^2 \sigma_u^2 + \alpha_i^2 R_{sv}^{-1} \sigma_v^2 - \alpha_i \Lambda \sigma_v^2]^2}{\Lambda^2 [\delta^2 \sigma_u^2 + \alpha_i^2 R_{sv}^{-1} \sigma_v^2]}, \\ \text{var}[v|p] = \sigma_v^2 \frac{\delta^2 \sigma_u^2 + \alpha_i^2 (1 - R_{sv}) R_{sv}^{-1} \sigma_v^2}{\delta^2 \sigma_u^2 + \alpha_i^2 R_{sv}^{-1} \sigma_v^2}. \quad \square$$

### A.3.3. HEDGER'S EXPECTED UTILITY WITHOUT TRADING

If hedger  $j$  is not allowed to trade in the security market, his final wealth would be given by  $W_{hj} = u_j z$ . Conditional on his private information ( $u_j$ ), hedger  $j$ 's expected utility is equal to

$$E[U(W_{hj})|u_j] = -\exp\{-\rho_h(E[W_{hj}|u_j] - \rho_h \text{var}[W_{hj}|u_j]/2)\}$$

where  $E[W_{hj}|u_j] = u_j E[z]$  and  $\text{var}[W_{hj}|u_j] = u_j^2 \sigma_z^2$ .

Substituting into the above expression and simplifying then yields

$$E[U(W_{hj})|u_j] = -\exp\left\{-\rho_h\left(u_j \bar{z} - \frac{\rho_h}{2} u_j^2 \sigma_z^2\right)\right\}.$$

Now applying Corollary 8, hedger  $j$ 's expected utility is given by

$$E[U(W_{hj})] = -\left[1 - \rho_h^2 \sigma_z^2 \frac{\sigma_u^2}{R_u}\right]^{-1/2} \exp\left\{\frac{\rho_h^2 \bar{z}^2 \sigma_u^2}{2(R_u - \rho_h^2 \sigma_z^2 \sigma_u^2)}\right\}$$

provided that  $\rho_h^2 \sigma_z^2 \sigma_u^2 = \rho_h^2 \sigma_z^2 (\sigma_\eta^2 + \sigma_u^2) = \rho_h^2 \sigma_z^2 (\sigma_u^2 / R_u) < 1$ . Otherwise, the expected utility diverges to  $-\infty$ .  $\square$

#### A.3.4. HEDGER'S EXPECTED UTILITY CONDITIONAL ON INFORMATION SET

$$E[U(W_{hj})|p, u_j] = -\exp\left\{-\rho_h \left(\frac{\sigma_{vz}}{\sigma_v^2}\right) p u_j - \frac{[E[v|p, u_j] - p]^2}{2 \text{var}[v|p, u_j]} + \frac{\rho_h^2 \sigma_z^2 (1 - R_{vz}) u_j^2}{2}\right\},$$

where

$$p = \bar{v} - \Gamma q + \frac{\alpha_i (s - \bar{v}) - \delta u}{\Lambda}. \quad \square$$

#### A.3.5. HEDGER'S EXPECTED UTILITY CONDITIONAL ON HIS ENDOWMENT

$$\begin{aligned} E[U(W_{hj})|u_j] &= -\left\{1 + 2K_1 \frac{\delta^2}{\Lambda^2} (1 - R_u) \sigma_u^2\right\}^{-1/2} \\ &\times \exp\left\{-\rho_h \left(\frac{\sigma_{vz}}{\sigma_v^2}\right) \bar{v} u_j + \frac{\rho_h^2 \sigma_z^2 (1 - R_{vz}) u_j^2}{2}\right\} \\ &\times \exp\left\{-\frac{\alpha_i^2 \Lambda^2 \sigma_v^2 \sigma_u^2}{2R_u [\alpha_i^2 (1 - R_{sv}) R_{sv}^{-1} \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u)]} \left[\Gamma q + \frac{\delta}{\Lambda} R_u u_j\right]^2\right\} \\ &\times \exp\left\{\frac{1}{2\Lambda^2} \delta^2 \sigma_u^2 (1 - R_u) K_2^2 + K_2 \left[\Gamma q + \frac{\delta}{\Lambda} R_u u_j\right]\right\} \\ &\times \exp\left\{-\frac{K_1}{1 + 2K_1 \frac{\delta^2}{\Lambda^2} (1 - R_u) \sigma_u^2} \left[\Gamma q + \frac{\delta}{\Lambda} R_u u_j + \frac{\delta^2}{\Lambda^2} (1 - R_u) \sigma_u^2 K_2\right]^2\right\}, \end{aligned}$$

where

$$K_1 = \frac{\left[ \alpha_i \left( \frac{\alpha_i}{R_{sv}} - \Lambda \right) \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u) \right]^2}{2\sigma_v^2 \left[ \alpha_i^2 (1 - R_{sv}) R_{sv}^{-1} \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u) \right] \left[ \alpha_i \frac{\alpha_i}{R_{sv}} \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u) \right]}$$

and where

$$K_2 = \rho_h \frac{\sigma_{vz}}{\sigma_v^2} u_j - \frac{\Lambda \alpha_i K_1 \sigma_v^2}{\alpha_i \left( \frac{\alpha_i}{R_{sv}} - \Lambda \right) \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u)} \left[ \Gamma q + \frac{\delta}{\Lambda} R_u u_j \right]$$

or

$$K_2 = - \frac{\Lambda \alpha_i K_1 \sigma_v^2}{\alpha_i \left( \frac{\alpha_i}{R_{sv}} - \Lambda \right) \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u)} \Gamma q + \left[ \rho_h \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) - \frac{\delta \alpha_i R_u K_1 \sigma_v^2}{\alpha_i \left( \frac{\alpha_i}{R_{sv}} - \Lambda \right) \sigma_v^2 + \delta^2 \sigma_u^2 (1 - R_u)} \right] u_j. \quad \square$$

### A.3.6. HEDGER'S *EX ANTE* EXPECTED UTILITY

We can apply Lemma 7 to compute hedger  $j$ 's (*ex ante*) expected utility by taking  $a = -1$ ,  $c = -(\Gamma q)^2 / \text{var}[v|p, u_j]$ ,  $x' = (u_j, p - E[p])$ ,

$$\Sigma = \begin{pmatrix} \frac{\sigma_u^2}{R_u} & \frac{-\delta \sigma_u^2}{\Lambda} \\ \frac{-\delta \sigma_u^2}{\Lambda} & \frac{\alpha_i^2 (\sigma_v^2 / R_{sv}) + \delta^2 \sigma_u^2}{\Lambda^2} \end{pmatrix},$$

$$A = \begin{pmatrix} \frac{\rho_h^2 \sigma_v^2 (1 - R_{vz})}{2} - \frac{(\alpha_i \delta \sigma_v^2 \sigma_u^2)^2}{k^2 \text{var}[v|p, u_j]} & \frac{1}{2} \left( -\rho_h \frac{\sigma_{vz}}{\sigma_v^2} - \frac{2\alpha_i \delta \sigma_v^2 \sigma_u^2}{k \text{var}[v|p, u_j]} \right) \\ \frac{1}{2} \left( -\rho_h \frac{\sigma_{vz}}{\sigma_v^2} - \frac{2\alpha_i \delta \sigma_v^2 \sigma_u^2}{k \text{var}[v|p, u_j]} \right) & \frac{-m^2}{\text{var}[v|p, u_j]} \end{pmatrix}$$

and

$$b = \begin{pmatrix} -\rho_h \frac{\sigma_{vz}}{\sigma_v^2} (\bar{v} - \Gamma q) - \frac{2\Gamma q \alpha_i \delta \sigma_v^2 \sigma_u^2}{k \text{var}[v|p, u_j]} \\ - \frac{2Aqm}{\text{var}[v|p, u_j] W_{hj} |u_j} \end{pmatrix}$$

where

$$k = R_u^{-1} \sigma_u^2 [(1 - R_u) \delta^2 \sigma_u^2 + R_{sv}^{-1} \alpha_i^2 \sigma_v^2] \text{ and } m = \alpha_i \sigma_v^2 \sigma_u^2 \Lambda / (R_u k) - 1.$$

If the matrix  $\Sigma^{-1} + 2aA$  is positive definite, then hedger  $j$ 's (*ex ante*) expected utility will be given by

$$\begin{aligned} EU_h &\equiv -E[\exp(-\rho_h W_{hj})] \\ &= -|\Sigma|^{-1/2} |\Sigma^{-1} + 2aA|^{-1/2} \exp\{-a[c - (1/2)ab'(\Sigma^{-1} + 2aA)^{-1}b]\}. \end{aligned}$$

Otherwise, hedger  $j$ 's expected utility diverges to  $-\infty$ .  $\square$

#### A.3.7. HEDGER'S *EX ANTE* EXPECTED UTILITY FOR LARGE RISK AVERSION

When  $\rho_h$  is large ( $\rho_h \rightarrow +\infty$ ), the following approximations hold (since in this case  $x_{hj}(p, u_j) \rightarrow -\left(\frac{\sigma_{vz}}{\sigma_v^2}\right)u_j$ ):

$$\begin{aligned} E[U(W_{hj})|u_j] &\cong -\exp\left\{\frac{\rho_h^2}{2} u_j^2 \sigma_z^2 (1 - r_{vz}^2)\right\} \\ &\quad \times \exp\left\{-\rho_h \frac{\sigma_{vz}}{\sigma_v^2} u_j \left(E[p|u_j] - \frac{\rho_h}{2} \frac{\sigma_{vz}}{\sigma_v^2} u_j \text{var}[p|u_j]\right)\right\}, \end{aligned}$$

where

$$E[p|u_j] = \bar{v} - \Gamma q - \frac{\delta}{\Lambda} E[u|u_j] = \bar{v} - \Gamma q - \frac{\delta}{\Lambda} R_u u_j$$

and

$$\text{var}[p|u_j] = \Lambda^{-2} \left\{ \alpha_i^2 \frac{\sigma_v^2}{R_{sv}} + \delta^2 (1 - R_u) \sigma_u^2 \right\}.$$

Note that  $E[U(W_{hj})|u_j]$  is increasing (decreasing) in  $E[p|u_j] - (\rho_h/2)(\sigma_{vz}/\sigma_v^2)u_j \text{var}[p|u_j]$  if  $u_j$  is positive (negative). Using Lemma 7 and the fact that  $E[U(W_{hj})] = E[E[U(W_{hj})|u_j]]$  (when  $[R_u(\sigma_u^2)^{-1} + 2\rho_h A] > 0$  with

$$A = -\left[\frac{\delta}{\Lambda} R_u \frac{\sigma_{vz}}{\sigma_v^2} + \frac{\rho_h}{2} \left(\frac{\sigma_{vz}^2}{\sigma_v^4} \text{var}[p|u_j] - \sigma_z^2 (1 - r_{vz}^2)\right)\right],$$

we find that

$$E[U(W_{hj})] \cong - (R_u/\sigma_u^2)^{-1/2} [R_u(\sigma_u^2)^{-1} + 2\rho_h A]^{-1/2} \exp \left\{ \frac{\rho_h^2 (\bar{v} - \Gamma q)^2}{2 R_u(\sigma_u^2)^{-1} + 2\rho_h A} \right\}.$$

If  $[R_u(\sigma_u^2)^{-1} + 2\rho_h A] \leq 0$  then  $E[U(W_{hj})]$  diverges to  $-\infty$ .

Note that  $EU_h$  increases with  $\Gamma q$  and with  $\Lambda$  but decreases with  $\text{var}[p]$ , keeping in each case the other equilibrium parameters fixed.  $\square$

#### A.4. Proofs of Propositions in Section 5

*Proof of Proposition 3.* If the insider publicly reveals his private information before trading on the asset market, speculator  $k$ 's information set becomes  $\{s, p\}$ . Maximizing  $E[U(W_{sk})|s, p]$  with respect to  $x_{sk}$  yields the demand function for the risky asset,

$$X_{sk}(p) = \frac{E[v|s] - p}{\rho_s \text{var}[v|s]},$$

where  $E[v|s] = \bar{v} + R_{sv}(s - \bar{v})$  and  $\text{var}[v|s] = (1 - R_{sv})\sigma_v^2$ . It is obvious that  $x_{sk}$  may be written as

$$X_s(s, p) = \beta_s(\bar{v} - p) + \alpha_s(s - \bar{v}),$$

where  $\alpha_s = R_{sv}/[\rho_s(1 - R_{sv})\sigma_v^2]$  and  $\beta_s = 1/[\rho_s(1 - R_{sv})\sigma_v^2]$ .

Similarly, hedger  $j$  will choose  $x_{hj}$  to maximize  $E[U(W_{hj})|p, s, u_j]$ . From the first-order condition, hedger  $j$ 's optimal demand for shares is given by

$$X_{hj}(p, s, u_j) = \frac{E[v|s] - p}{\rho_h \text{var}[v|s]} - \left( \frac{\sigma_{vz}}{\sigma_v^2} \right) u_j,$$

since  $E[v - p|p, s, u_j] = E[v|s]$ ,  $\text{var}[v - p|p, s, u_j] = \text{var}[v|s]$ , and  $\text{cov}[z, v - p|p, s, u_j] = \frac{\sigma_{vz}}{\sigma_v^2} \text{var}[v|s]$ . After integrating on the interval  $(1, 2]$ , the hedgers' aggregate demand will be given by

$$X_h(u) = \beta_h(\bar{v} - p) + \alpha_h(s - \bar{v}) - \delta u,$$

where  $\alpha_h = R_{sv}/[\rho_h(1 - R_{sv})\sigma_v^2]$ ,  $\beta_h = 1/[\rho_h(1 - R_{sv})\sigma_v^2]$ , and  $\delta = (\sigma_{vz}/\sigma_v^2)$ .

From the market-clearing condition, the relation between the insider's asset position and its price becomes  $p = \bar{v} + (\alpha_s + \alpha_h)/(\beta_s + \beta_h)(s - \bar{v}) + (\beta_s + \beta_h)^{-1}(x_i - \delta u)$ , so that  $\lambda = (\beta_s + \beta_h)^{-1} = [\rho_s \rho_h / (\rho_s + \rho_h)] \text{var}[v|s]$ . Thus, the insider's optimal demand schedule is given by

$$X_i(s, p) = \alpha_i(s - \bar{v}) + \beta_i(\bar{v} - p) - \gamma_i q,$$

where

$$\alpha_i = \frac{R_{sv}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \quad \beta_i = \frac{1}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda}, \quad \gamma_i = \frac{\rho_i(1 - R_{sv})\sigma_v^2}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda},$$

$\lambda = \frac{\rho_s \rho_h}{\rho_s + \rho_h} (1 - R_{sv}) \sigma_v^2$ . Moreover, it obviously satisfies the second-order condition  $2\lambda + \rho_i \text{var}[v|s] > 0$ , since  $\lambda$  is greater than zero.

From the market clearing condition and the optimal strategies of the insider, hedgers, and speculators, the equilibrium price is obtained as

$$p = E[v|s] - \frac{\gamma_i q + \delta u}{\Lambda},$$

where  $\Lambda = \beta_s + \beta_h + \beta_i$ .

Conditional on  $(s, p)$ , the entrepreneur's expected utility is

$$E[-\exp\{-\rho_i W_i\}|s, p] = -\exp\left\{-\rho_i \left(E[W_i|s, p] - \frac{\rho_i}{2} \text{var}[W_i|s, p]\right)\right\},$$

where  $E[W_i|s, p] = qE[v|s] - C(q) + x_i(s, p)\{E[v|s] - p\}$  and  $\text{var}[W_i|s, p] = q^2 \text{var}[v|s] + \text{var}[v|s]\{x_i(s, p)\}^2 + 2qX_i(s, p)\text{var}[v|s]$ .

Substituting into the above expression and simplifying yields

$$E[U(W_i)|s, p] = -\exp\left\{\rho_i C(q) + \frac{\rho_i^2 q^2}{2} \text{var}[v|s]\right\} \\ \times \exp\left\{-\rho_i q E[v|s] - \frac{\rho_i}{2} [\rho_i \text{var}[v|s] + 2\lambda] x_i^2\right\},$$

where the first exponential is nonrandom. To obtain the entrepreneur's *ex ante* (unconditional) expected utility, it suffices to apply Corollary 8 by

taking  $x = -\rho_i q E[v|s]$  and  $y = \sqrt{\frac{\rho_i}{2} [\rho_i \text{var}[v|s] + 2\lambda]} X_i(s, p)$ .

We know that  $E\{E[v|s]\} = \bar{v}$  and  $\text{var}[E(v|s)] = R_{sv} \sigma_v^2$ . We have also that  $E[X_i(s, p)] = -\gamma_i q / (\Lambda \lambda)$ ,  $\text{var}[X_i(s, p)] = (\beta_i \delta / \Lambda)^2 \sigma_u^2$ , and  $\text{cov}\{E[v|s], X_i(s, p)\} = 0$ .

Thus, for a given  $q$ , the entrepreneur's *ex ante* (unconditional) expected utility is given by

$$E[U(W_i)|s,p] = -|SG_i| \exp \left\{ \rho_i C(q) + \frac{\rho_i^2}{2} q^2 (1 - R_{sv}) \sigma_v^2 - \rho_i q \bar{v} + \frac{\rho_i^2}{2} q^2 R_{sv} \sigma_v^2 \right\} \\ \times \exp \left\{ -\frac{1}{2} \rho_i \left( \frac{\gamma_i}{\lambda \Lambda} \right)^2 q^2 \frac{\rho_i \text{var}[v|s] + 2\lambda}{1 + \rho_i [\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda] \left( \frac{\beta_i}{\Lambda} \right)^2 \delta^2 \sigma_u^2} \right\},$$

where  $|SG_i| = \{1 + \rho_i [\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda] \left( \frac{\beta_i}{\Lambda} \right)^2 \delta^2 \sigma_u^2\}^{-1/2}$ . Since  $\beta_i = 1 / (\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda)$  and  $\Lambda = \beta_s + \beta_h + \beta_i$  (where  $\beta_h + \beta_i = 1/\lambda$ ), we have  $\Lambda = (\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda) / (\lambda [\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda])$ . Therefore, the entrepreneur's *ex ante* (unconditional) expected utility may be written as

$$E[-\exp\{-\rho_i W_i\}] = -|SG_i| \exp\{-\rho_i [q \bar{v} - C(q) - 0.5 \rho_i \sigma_v^2 q^2 (1 - d)]\},$$

where

$$|SG_i| = \left\{ 1 + \frac{\rho_i \lambda^2 \delta^2 \sigma_u^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda} \right\}^{-1/2}$$

and

$$d = \frac{1}{\rho_i \sigma_v^2} \left( \frac{\gamma_i}{\lambda} \right)^2 \frac{\rho_i \text{var}[v|s] + 2\lambda}{\Lambda^2 + \rho_i [\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda] \beta_i^2 \delta^2 \sigma_u^2}.$$

Given that

$$\gamma_i = \frac{\rho_i (1 - R_{sv}) \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda}, \quad \beta_i = \frac{1}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda} \\ \text{var}[v|s] = (1 - R_{sv}) \sigma_v^2, \quad \Lambda = \frac{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda}{\lambda [\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda]}$$

$d$  may be written as

$$d = \frac{\rho_i [(1 - R_{sv}) \sigma_v^2]^2 / \sigma_v^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda + \rho_i \lambda^2 \delta^2 \sigma_u^2}.$$

By substituting  $\lambda = [\rho_s \rho_h / (\rho_s + \rho_h)](1 - R_{sv})\sigma_v^2$ , we may also write  $d$  as

$$d = \frac{\rho_i(1 - R_{sv})}{\rho_i + 2 \frac{\rho_s \rho_h}{\rho_s + \rho_h} + \rho_i \left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right)^2 (1 - R_{sv})\sigma_v^2 \delta^2 \sigma_u^2},$$

so that, finally, for a given  $q$ , the entrepreneur's *ex ante* (unconditional) expected utility is given by

$$E[-\exp\{-\rho_i W_i\}] = -|SG_i| \exp\{-\rho_i[q\bar{v} - C(q) - 0.5\rho_i\sigma_v^2 q^2(1 - d)]\},$$

where

$$|SG_i| = \left\{ 1 + \frac{\rho_i \lambda^2 \delta^2 \sigma_u^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + 2\lambda]} \right\}^{-1/2},$$

$$d = \frac{\rho_i(1 - R_{sv})}{\rho_i + 2 \frac{\rho_s \rho_h}{\rho_s + \rho_h} + \rho_i \left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right)^2 (1 - R_{sv})\sigma_v^2 \delta^2 \sigma_u^2}.$$

The entrepreneur chooses  $q$  to maximize the above *ex ante* expected utility, so it is obvious that the level of real investment will be given by  $q = (\bar{v} - c_1)/(c_2 + \rho_i\sigma_v^2(1 - d))$  with the above value of  $d$ .

Similarly, speculator  $k$ 's expected utility conditional in his information  $(s, p)$  is equal to

$$E[-\exp\{-\rho_s W_{sk}\}|s, p] = -\exp\{-\rho_s(E[W_{sk}|s, p] - \rho_s \text{var}[W_{sk}|s, p]/2)\},$$

where  $E[W_{sk}|s, p] = X_{sk}(s, p)\{E[v|s] - p\}$  and  $\text{var}[W_{sk}|s, p] = \text{var}[v|s] \times \{x_{sk}^2(s, p)\}$ .

Substituting into the above expression and simplifying yields

$$E[-\exp\{-\rho_s W_{sk}\}|s, p] = -\exp\left\{-\frac{(E[v|s] - p)^2}{2\text{var}[v|s]}\right\}.$$

To obtain speculator  $k$ 's *ex ante* (unconditional) expected utility, it suffices to apply Corollary 8 by taking  $x = 0$  and  $y = (E[v|s] - p)/\sqrt{2\text{var}[v|s]}$ , where  $E[v|s] - p = (\gamma_i q + \delta u)/\Lambda$ . By taking into account that  $E[E(v|s) - p] = \gamma_i q/\Lambda$  and  $\text{var}[E(v|s) - p] = (\delta/\Lambda)^2 \sigma_u^2$  (and making some simplifications) we have

$$E[-\exp\{-\rho_s W_{sk}\}] = -|SG_s| \exp\left\{-\frac{(\gamma_i q)^2}{2[\Lambda^2(1 - R_{sv})\sigma_v^2 + \delta^2 \sigma_u^2]}\right\},$$



where

$$|SG_s| = \left[ 1 + \frac{\delta^2 \sigma_u^2}{\Lambda^2 (1 - R_{sv}) \sigma_v^2} \right]^{-1/2}. \quad \square$$

*Proof of Proposition 4.* From the characterization of the equilibrium with public disclosure of  $s$  (see Proposition 3), we immediately obtain:

1.  $d\alpha_i/dR_{sv} > 0$ ,  $d\beta_i/dR_{sv} > 0$ ,  $d\gamma_i/dR_{sv} = 0$ ,  $d\alpha_s/dR_{sv} > 0$ ,  $d\beta_s/dR_{sv} > 0$ ,  $d\alpha_h/dR_{sv} > 0$ , and  $d\beta_h/dR_{sv} > 0$ ; also,  $d\Lambda/dR_{sv} > 0$ . Price precision is given by  $\tau = 1/\text{var}[v|p]$ . The price is informationally equivalent to  $R_{sv}(s - \bar{s}) - \delta u/\Lambda$ , so

$$\text{var}[v|p] = \sigma_v^2 \left[ 1 - \frac{R_{sv}^2 \sigma_v^2}{R_{sv} \sigma_v^2 + (\delta/\Lambda)^2 \sigma_u^2} \right].$$

Since  $\delta$  is independent of  $R_{sv}$  and  $d\Lambda/dR_{sv} > 0$ , we have that  $\frac{d\text{var}[v|p]}{dR_{sv}} < 0$  and  $d\tau/dR_{sv} > 0$ . Finally,  $(\gamma_i q)/\Lambda$  decreases in  $R_{sv}$  because  $\gamma_i$  is independent of  $R_{sv}$ ,  $d\Lambda/dR_{sv} > 0$ , and  $dq/dR_{sv} < 0$  (from Result 2).

2.  $dd/dR_{sv} < 0$  and consequently  $dq/dR_{sv} < 0$ .
3. The insider's *ex ante* expected utility is given by

$$E[-\exp\{-\rho_i W_i\}] = -|SG_i| \exp\{-0.5\rho_i(\bar{v} - c_1)q\},$$

where

$$\begin{aligned} |SG_i| &= \left\{ 1 + \frac{\rho_i \lambda^2 \delta^2 \sigma_u^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda} \right\}^{-1/2} \\ &= \left\{ 1 + \rho_i \frac{\left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right)^2}{\rho_i + 2 \frac{\rho_s \rho_h}{\rho_s + \rho_h}} (1 - R_{sv}) \sigma_v^2 \delta^2 \sigma_u^2 \right\}^{-1/2} \end{aligned}$$

given that

$$\lambda = \frac{\rho_s \rho_h}{\rho_s + \rho_h} (1 - R_{sv}) \sigma_v^2.$$

From the above expression,

$$\begin{aligned} \frac{dE[U_i(W_i)]}{dR_{sv}} &= -\exp\{-0.5\rho_i(\bar{v} - c_1)q\} \left( \frac{d|SG_i|}{dR_{sv}} \right) \\ &\quad - |SG_i| \left( \frac{d\exp\{-0.5\rho_i(\bar{v} - c_1)q\}}{dR_{sv}} \right). \end{aligned}$$

It follows immediately that  $d|SG_i|/dR_{sv} > 0$ . On the other hand,  $\text{sign}(d\exp\{\cdot\}/dR_{sv}) = -\text{sign}(dq/dR_{sv}) > 0$  since  $dq/dR_{sv} < 0$ . Thus,  $dE[U_i(W_i)]/dR_{sv} < 0$ .

4. The speculator's *ex ante* expected utility is given by

$$E[-\exp\{-\rho_s W_{sk}\}] = -|SG_s| \exp\left\{-\frac{(\gamma_i q)^2}{2[\Lambda^2(1 - R_{sv})\sigma_v^2 + \delta^2\sigma_u^2]}\right\},$$

where

$$|SG_s| = \left[1 + \frac{\delta^2\sigma_u^2}{\Lambda^2(1 - R_{sv})\sigma_v^2}\right]^{-1/2},$$

$$\Lambda^2\sigma_v^2 = \frac{1}{(1 - R_{sv})^2\sigma_v^2} \left(\frac{1}{\rho_i + \frac{\rho_h\rho_s}{\rho_h + \rho_s}} + \frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^2.$$

If  $R_{sv}$  increases then  $\Lambda^2(1 - R_{sv})\sigma_v^2$  increases,  $\delta$  does not change, and  $q$  decreases. As a result, both  $|SG_s|$  and the exponential term increase, so that  $dE[U_s(W_s)]/dR_{sv} < 0$ .  $\square$

### A.5. Propositions and proofs for Section 6

**PROPOSITION 9.** Let  $R_u/\rho_h$  be close to zero. If the entrepreneur has and trades on inside information, then the following statements hold.

1. The insider's trading intensity increases,  $\alpha_i^{\text{IT}} > \alpha_i^{\text{NI}}$  ( $d\alpha_i^{\text{IT}}/dR_{sv} > 0$ ).
2. Speculators and hedgers trade less aggressively,  $\beta_s^{\text{IT}} < \beta_s^{\text{NI}}$  ( $d\beta_s^{\text{IT}}/dR_{sv} < 0$ ) and provided that  $R_u$  is small,  $\beta_h^{\text{IT}} \leq \beta_h^{\text{NI}}$  ( $d\beta_h^{\text{IT}}/dR_{sv} \leq 0$ ). Furthermore,  $\lambda^{\text{IT}} > \lambda^{\text{NI}}$  ( $d\lambda^{\text{IT}}/dR_{sv} > 0$ ).
3. If  $\rho_i$  is not too large then market depth decreases,  $\Lambda^{\text{IT}} < \Lambda^{\text{NI}}$  ( $d\Lambda^{\text{IT}}/dR_{sv} < 0$ ).
4. Price precision increases,  $\tau^{\text{IT}} > \tau^{\text{NI}}$  ( $d\tau^{\text{IT}}/dR_{sv} > 0$ ).
5. If  $\rho_i$  is not too high then  $\text{var}[p^{\text{IT}}] > \text{var}[p^{\text{NI}}]$  ( $d\text{var}[p^{\text{IT}}]/dR_{sv} > 0$ ).
6. If  $R_{sv}$  is close to unity then real investment decreases,  $q^{\text{IT}} < q^{\text{NI}}$ .
7. If  $R_{sv}$  is close to unity then the average stock price will increase,  $\bar{p}^{\text{IT}} > \bar{p}^{\text{NI}}$ .
8. The insider's speculative gains will be higher ( $|SG_i^{\text{IT}}| < |SG_i^{\text{NI}}|$ ) while his production and insurance gains will be lower ( $\exp\{-0.5\rho_i(\bar{v} - c_1)q^{\text{IT}}\} > \exp\{-0.5\rho_i(\bar{v} - c_1)q^{\text{NI}}\}$ ) if  $R_{sv}$  is close to unity. We have that  $E[U_i^{\text{IT}}] > E[U_i^{\text{NI}}]$  for  $\rho_i$  low.

9. If  $R_{sv}$  is close to unity then the speculators' *ex ante* expected utility will be reduced,  $E[U_s^{IT}] < E[U_s^{NI}]$ .

*Proof*

The key to the proof is to show that, for  $R_u/\rho_h$  close to zero,  $d\lambda/dR_{sv} > 0$ . We will first show that if  $R_u/\rho_h \rightarrow 0$  then  $d\delta(R_{sv})/dR_{sv}$  remains strictly positive and tends to zero. We know that there is equilibrium if and only if there is a solution to the two-equation system (5) and (6), which may be written as

$$F(\lambda, \delta; R_{sv}) \equiv \lambda - \lambda \frac{R_u}{\rho_h} \frac{[(1 - R_u)\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]^{-1}}{\frac{1}{\rho_s[\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} + \frac{1}{\rho_h[(1 - R_u)\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]}} - \frac{\sigma_v^2}{\delta^2\sigma_u^2} \left\{ \frac{1}{\rho_s[\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} + \frac{1}{\rho_h[(1 - R_u)\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} \right\}^{-1} - \frac{\sigma_v^2}{\delta^2\sigma_u^2} \alpha_i = 0,$$

$$G(\lambda, \delta; R_{sv}) \equiv \frac{R_u}{\rho_h} \frac{\alpha_i}{(1 - R_u)\delta^2\sigma_u^2 + (1 - R_{sv})R_{sv}^{-1}\alpha_i^2\sigma_v^2} \delta + \left[ \delta - \frac{\sigma_{vz}}{\sigma_v^2} \right] = 0,$$

where  $\alpha_i = R_{sv}/[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]$ . If  $R_u/\rho_h$  is small then there is a unique solution and, by applying the implicit function theorem we can compute the derivatives  $d\lambda(R_{sv})/dR_{sv}$  and  $d\delta(R_{sv})/dR_{sv}$ :

$$\frac{d\lambda(R_{sv})}{dR_{sv}} = - \frac{\begin{vmatrix} \frac{\partial F}{\partial R_{sv}} & \frac{\partial F}{\partial \delta} \\ \frac{\partial G}{\partial R_{sv}} & \frac{\partial G}{\partial \delta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial \lambda} & \frac{\partial F}{\partial \delta} \\ \frac{\partial G}{\partial \lambda} & \frac{\partial G}{\partial \delta} \end{vmatrix}}, \quad \frac{d\delta(R_{sv})}{dR_{sv}} = - \frac{\begin{vmatrix} \frac{\partial F}{\partial \lambda} & \frac{\partial F}{\partial R_{sv}} \\ \frac{\partial G}{\partial \lambda} & \frac{\partial G}{\partial R_{sv}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial \lambda} & \frac{\partial F}{\partial \delta} \\ \frac{\partial G}{\partial \lambda} & \frac{\partial G}{\partial \delta} \end{vmatrix}}.$$

It is easy to check that if  $R_u/\rho_h \rightarrow 0$  then  $\partial G/\partial \lambda \rightarrow 0$ ,  $\partial G/\partial \delta \rightarrow 1$ ,  $\partial G/\partial R_{sv} \rightarrow 0$ , and  $\partial F/\partial \lambda > 0$  so

$$\frac{d\delta(R_{sv})}{dR_{sv}} = - \frac{\frac{\partial F}{\partial \lambda} \frac{\partial G}{\partial R_{sv}} - \frac{\partial G}{\partial \lambda} \frac{\partial F}{\partial R_{sv}}}{\frac{\partial F}{\partial \lambda} \frac{\partial G}{\partial \delta} - \frac{\partial F}{\partial \delta} \frac{\partial G}{\partial \lambda}} \rightarrow \frac{0}{\frac{\partial F}{\partial \lambda}} = 0$$

and

$$\frac{d\lambda(R_{sv})}{dR_{sv}} = - \frac{\frac{\partial F}{\partial R_{sv}} \frac{\partial G}{\partial \delta} - \frac{\partial F}{\partial \delta} \frac{\partial G}{\partial R_{sv}}}{\frac{\partial F}{\partial \lambda} \frac{\partial G}{\partial \delta} - \frac{\partial F}{\partial \delta} \frac{\partial G}{\partial \lambda}} \rightarrow - \frac{\frac{\partial F}{\partial R_{sv}}}{\frac{\partial F}{\partial \lambda}}$$

and therefore

$$\text{sign}\left(\frac{d\lambda(R_{sv})}{dR_{sv}}\right) = -\text{sign}\left(\frac{\partial F}{\partial R_{sv}}\right).$$

Moreover, after some tedious (but easy) manipulations we have that  $\partial F/\partial R_{sv} < 0$  (just note that

$$\text{sign}\left(\frac{\partial F}{\partial R_{sv}}\right) = -\text{sign}\left(\frac{\partial}{\partial R_{sv}}\left[\alpha_i + \left\{\frac{1}{\rho_s[\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]} + \frac{1}{\rho_h[(1 - R_u)\delta^2\sigma_u^2 + \alpha_i^2(R_{sv}^{-1} - 1)\sigma_v^2]}\right\}^{-1}\right]\right),$$

where the last partial derivative is positive because, by Lemma 6, the equilibrium value of  $\lambda$  satisfies the inequality  $(\lambda \geq \sigma_v^2(\rho_s\rho_h)/(\rho_s + \rho_h))$ .

Therefore if  $R_u/\rho_h$  is close to zero, then  $d\lambda(R_{sv})/dR_{sv}$  is positive and bounded away from zero and  $\frac{d\delta(R_{sv})}{dR_{sv}}$  is close to zero (because of continuity of the derivatives). Let then  $R_u/\rho_h$  be close to zero and let us proceed to the proof of the claims.

$$1. \beta_s = \frac{1}{\rho_s\sigma_v^2} \frac{\delta^2\sigma_u^2 - (\alpha_i/\lambda)\sigma_v^2}{\delta^2\sigma_u^2 + (R_{sv}^{-1} - 1)\alpha_i^2\sigma_v^2} \quad \text{and so}$$

$$\frac{d\beta_s}{dR_{sv}} = \frac{\partial\beta_s}{\partial\lambda} \frac{d\lambda}{dR_{sv}} + \frac{\partial\beta_s}{\partial\delta} \frac{d\delta}{dR_{sv}} + \frac{\partial\beta_s}{\partial\alpha_i} \frac{d\alpha_i}{dR_{sv}} + \frac{\partial\beta_s}{\partial R_{sv}},$$

where it is clear that  $\partial\beta_s/\partial\lambda \geq 0$ ,  $\partial\beta_s/\partial R_{sv} \geq 0$ , and  $\partial\beta_s/\partial\alpha_i < 0$ . Therefore, if  $R_u/\rho_h \rightarrow 0$ , then

$$\frac{d\beta_s}{dR_{sv}} \rightarrow \frac{\partial\beta_s}{\partial\lambda} \frac{d\lambda}{dR_{sv}} + \frac{\partial\beta_s}{\partial\alpha_i} \frac{d\alpha_i}{dR_{sv}} + \frac{\partial\beta_s}{\partial R_{sv}},$$

where the first and third terms are positive (because

$$\frac{\partial\beta_s}{\partial\lambda} \geq 0, \frac{d\lambda(R_{sv})}{dR_{sv}} > 0, \frac{\partial\beta_s}{\partial R_{sv}} \geq 0 \text{ and } \frac{d\lambda(R_{sv})}{dR_{sv}} > 0)$$

and  $d\beta_s/dR_{sv} < 0$ . As a direct consequence,  $(\partial\beta_s/\partial\alpha_i)(d\alpha_i/dR_{sv})$ , must be negative and, since  $\partial\beta_s/\partial\alpha_i < 0$  and  $d\delta(R_{sv})/dR_{sv} \rightarrow 0$ , we have that  $d\alpha_i^\Gamma/dR_{sv} > 0$ .

$$2. \beta_s = \frac{1}{\rho_s \sigma_v^2} \frac{\delta^2 \sigma_u^2 - (\alpha_i/\lambda) \sigma_v^2}{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

$$\beta_h = \frac{1}{\rho_h \sigma_v^2} \frac{(1 - R_u) \delta^2 \sigma_u^2 - (\alpha_i/\lambda) \sigma_v^2}{(1 - R_u) \delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2},$$

$$\lambda = \frac{1}{\beta_s + \beta_h}.$$

Here  $R_u/\rho_h$  is close to zero if  $R_u$  is sufficiently small or  $\rho_h \rightarrow \infty$  (or both). If  $R_u$  is close to zero, then

$$\beta_s \rho_s \cong \beta_h \rho_h \cong \frac{1}{\sigma_v^2} \frac{\delta^2 \sigma_u^2 - (\alpha_i/\lambda) \sigma_v^2}{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2}$$

and, as a result,  $\text{sign}(d\beta_s/dR_{sv}) = \text{sign}(d\beta_h/dR_{sv})$ . Since  $\lambda = 1/(\beta_s + \beta_h)$  and  $d\lambda(R_{sv})/dR_{sv} > 0$ , it follows that  $d\beta_s/dR_{sv}$  and  $d\beta_h/dR_{sv}$  must be strictly negative. If  $\rho_h \rightarrow \infty$ , then  $d\beta_h/dR_{sv} \rightarrow 0$  and  $\lambda \rightarrow 1/\beta_s$ . Since  $d\lambda(R_{sv})/dR_{sv} > 0$ , it is clear that  $d\beta_s/dR_{sv} < 0$  for  $\rho_h$  large enough.

3. Market depth may be written as  $\Lambda = 1/\lambda + \beta_i$  with  $\beta_i = 1/[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]$ , or as  $\Lambda = 1/\lambda + 1/[\rho_i(1 - R_{sv})\sigma_v^2 + \lambda]$ . If  $\rho_i$  is sufficiently close to zero, then,  $d\Lambda^{\text{IT}}/dR_{sv} < 0$  since  $d\lambda(R_{sv})/dR_{sv} > 0$ .

4. Price precision is given by

$$\tau^{\text{IT}} = \tau_v \left[ 1 + \frac{\alpha_i^2 \sigma_v^2}{\delta^2 \sigma_u^2 + (R_{sv}^{-1} - 1) \alpha_i^2 \sigma_v^2} \right].$$

If  $R_{sv} = 0$  then  $\tau^{\text{IT}} = \tau_v$ , yet on the other hand it is obvious that  $\tau^{\text{IT}} > \tau_v$  for any  $R_{sv} > 0$ . Therefore, it is clear that  $\tau^{\text{IT}}$  is strictly increasing in  $R_{sv}$  for values sufficiently close to zero. For  $R_{sv} > 0$ ,

$$\frac{d\tau^{\text{IT}}}{dR_{sv}} = \frac{\partial \tau^{\text{IT}}}{\partial \delta} \frac{d\delta}{dR_{sv}} + \frac{\partial \tau^{\text{IT}}}{\partial \alpha_i} \frac{d\alpha_i}{dR_{sv}} + \frac{\partial \tau^{\text{IT}}}{\partial R_{sv}},$$

where  $\partial \tau^{\text{IT}}/\partial R_{sv} > 0$  and  $\partial \tau^{\text{IT}}/\partial \alpha_i > 0$  (these partial derivatives are equal to zero for  $R_{sv} = 0$  but are strictly greater than zero for any value of  $R_{sv} > 0$ ). Moreover, we know that  $d\delta/dR_{sv} \rightarrow 0$  (if  $R_u/\rho_h$  is close to zero) and that  $d\alpha_i/dR_{sv} > 0$ . Consequently, if  $R_u/\rho_h$  is close to zero then  $d\tau^{\text{IT}}/dR_{sv} > 0$ .

5. We have that  $\text{var}[p^{\text{IT}}] = (\alpha_i^2 \sigma_v^2 / R_{sv} + \delta^2 \sigma_u^2) / \Lambda^2$ . Since  $\alpha_i^2 / R_{sv}$  is increasing in  $R_{sv}$  and since  $d\delta / dR_{sv} \rightarrow 0$  if  $R_u / \rho_h$  is close to zero, it is obvious that if  $\Lambda$  is decreasing in  $R_{sv}$  then  $d\text{var}[p^{\text{IT}}] / dR_{sv} > 0$ . Thus,  $d\text{var}[p^{\text{IT}}] / dR_{sv} > 0$  if  $\rho_i$  is sufficiently close to zero.

6. We have that  $q^{\text{IT}} = (\bar{v} - c_1) / (c_2 + \rho_i \sigma_v^2 (1 - d^{\text{IT}}))$ , where

$$d^{\text{IT}} = \frac{\rho_i \sigma_v^2}{\rho_i \sigma_v^2 + 2\lambda^{\text{IT}} + \rho_i (\lambda^{\text{IT}})^2 (\delta^{\text{IT}})^2 \sigma_u^2} \left\{ 1 + \frac{(1 - R_{sv}) \lambda^{\text{IT}} \alpha_i^{\text{IT}} E^{\text{IT}}}{\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda^{\text{IT}} (1 + \alpha_i^{\text{IT}} E^{\text{IT}})} \right\}^2$$

and

$$E^{\text{IT}} = \frac{1}{\rho_s} \left[ (\delta^{\text{IT}})^2 \sigma_u^2 + \frac{R_{sv} (1 - R_{sv}) \sigma_v^2}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda^{\text{IT}}]^2} \right]^{-1} \\ + \frac{1}{\rho_h} \left[ (1 - R_u) (\delta^{\text{IT}})^2 \sigma_u^2 + \frac{R_{sv} (1 - R_{sv}) \sigma_v^2}{[\rho_i (1 - R_{sv}) \sigma_v^2 + \lambda^{\text{IT}}]^2} \right]^{-1}.$$

On the other hand,  $q^{\text{NI}} = (\bar{v} - c_1) / (c_2 + \rho_i \sigma_v^2 (1 - d^{\text{NI}}))$ , where

$$d^{\text{NI}} = \frac{\rho_i \sigma_v^2}{\rho_i \sigma_v^2 + 2\lambda^{\text{NI}} + \rho_i (\lambda^{\text{NI}})^2 (\delta^{\text{NI}})^2 \sigma_u^2}.$$

If  $R_{sv} = 1$ , then  $d^{\text{IT}} = \rho_i \sigma_v^2 / (\rho_i \sigma_v^2 + 2\lambda^{\text{IT}} + \rho_i (\lambda^{\text{IT}})^2 \delta^2 \sigma_u^2)$ . Since  $\lambda^{\text{IT}} > \lambda^{\text{NI}}$  and  $\delta^{\text{IT}} \rightarrow \delta^{\text{NI}}$  as  $R_u / \rho_h \rightarrow 0$  (while  $\lambda^{\text{IT}}$  remains bounded away from  $\lambda^{\text{NI}}$ ), it follows that for  $R_u / \rho_h$  small we have

$$d^{\text{IT}} = \frac{\rho_i \sigma_v^2}{\rho_i \sigma_v^2 + 2\lambda^{\text{IT}} + \rho_i (\lambda^{\text{IT}})^2 (\delta^{\text{IT}})^2 \sigma_u^2} \\ < \frac{\rho_i \sigma_v^2}{\rho_i \sigma_v^2 + 2\lambda^{\text{NI}} + \rho_i (\lambda^{\text{NI}})^2 (\delta^{\text{NI}})^2 \sigma_u^2} = d^{\text{NI}},$$

which directly implies that  $q^{\text{IT}} < q^{\text{NI}}$  because  $q$  is strictly increasing in  $d$ . By continuity this is also true for  $R_{sv}$  close to unity.

7. In the equilibrium with insider trading the expected price is given by  $\bar{p}^{\text{IT}} = \bar{v} - \Gamma^{\text{IT}} q^{\text{IT}}$ , where  $\Gamma^{\text{IT}}$  may be written as  $\Gamma^{\text{IT}} = (\gamma_s^{\text{IT}} + \gamma_h^{\text{IT}} + \gamma_i^{\text{IT}}) / \Lambda^{\text{IT}} = \gamma_i^{\text{IT}} / (1 + \alpha_i^{\text{IT}} E^{\text{IT}}) (1 / \Lambda^{\text{IT}})$ . If  $R_{sv} = 1$  then  $\gamma_i^{\text{IT}} = 0$  and, as a direct consequence,  $\Gamma^{\text{IT}} = 0$  and

$$\bar{p}^{IT} = \bar{v} > \bar{v} - \Gamma^{NI} q^{NI} = \bar{p}^{NI},$$

since  $\Gamma^{NI} q^{NI} = \sigma_v^2 [2/\rho_i + 1/\rho_h + 1/\rho_s]^{-1} q^{NI} > 0$ . By continuity, this is also true for  $R_{sv}$  close to unity.

8. The insider's *ex ante* expected utility may be written as

$$E[-\exp\{-\rho_i W_i^{IT}\}] = -|SG_i^{IT}| \exp\{-0.5\rho_i(\bar{v} - c_1)q^{IT}\},$$

where

$$|SG_i^{IT}| = \left\{ 1 + \frac{\rho_i(R_{sv}\sigma_v^2 + \lambda^2\delta^2\sigma_u^2)}{\rho_i(1 - R_{sv})\sigma_v^2 + 2\lambda} \right\}^{-1/2}.$$

From this last expression,

$$\frac{d|SG_i^{IT}|}{dR_{sv}} = \frac{\partial|SG_i^{IT}|}{\partial R_{sv}} + \frac{\partial|SG_i^{IT}|}{\partial\lambda} \frac{d\lambda}{dR_{sv}} + \frac{\partial|SG_i^{IT}|}{\partial\delta} \frac{d\delta}{dR_{sv}}.$$

It can be shown that

$$\begin{aligned} \frac{\partial|SG_i^{IT}|}{\partial R_{sv}} < 0, \quad \frac{\partial|SG_i^{IT}|}{\partial\lambda} < 0, \quad \frac{d\delta}{dR_{sv}} \rightarrow 0 \text{ as } R_u/\rho_h \rightarrow 0 \text{ and} \\ \frac{d\lambda}{dR_{sv}} > 0, \text{ so that } \frac{d|SG_i^{IT}|}{dR_{sv}} < 0. \end{aligned}$$

As a direct consequence,  $|SG_i^{IT}| < |SG_i^{NI}|$ . On the other hand, for all  $\rho_i > 0$ , if  $R_{sv}$  is close to unity then the exponential term is higher when insider trading is permitted, since  $q^{IT} < q^{NI}$ . If  $\rho_i = 0$  then  $q^{IT} = q^{NI} = (\bar{v} - c_1)/c_2$ ; the insider maximizes expected profits and only the speculative gains matter. The insider benefits with IT:  $EU_i^{IT} > EU_i^{NI}$ . The same result holds for  $\rho_i$  close to zero, speculative gains then loom larger than production and insurance gains.

9. We will analyze separately the cases of  $1/\rho_h$  close to zero and  $R_u$  sufficiently small. In the IT equilibria, if  $R_{sv} = 1$  and  $1/\rho_h$  is close to zero, then  $\Gamma = 0$ ,  $\alpha_i = 1/\lambda$ ,  $\Lambda = 2/\lambda$ , and  $\lambda$  is implicitly defined by  $\delta^2\sigma_u^2 - \sigma_v^2/\lambda^2 = \delta^2\sigma_u^2\rho_s\sigma_v^2/\lambda$ . As a consequence, the speculators' *ex ante* expected utility is given by

$$E[U_{sk}^{IT}] = -|SG_s^{IT}| = \left\{ 1 + \frac{\delta^2\sigma_u^2\rho_s^2\sigma_v^2}{4} \right\}^{-1/2}.$$

In the NI equilibrium, if  $1/\rho_h$  is close to zero, then  $\Lambda^{\text{NI}} = (2\rho_s\sigma_v^2 + \rho_i\sigma_v^2)/[\rho_s\sigma_v^2(\rho_s\sigma_v^2 + \rho_i\sigma_v^2)]$ , and the speculators' *ex ante* expected utility may be written as

$$E[U_{sk}^{\text{NI}}] = E[-\exp\{-\rho_s W_{sk}^{\text{NI}}\}] = -|SG_s^{\text{NI}}| \exp\left\{-0.5 \frac{(\Gamma q)^2}{\delta^2 \sigma_u^2 / \Lambda^2 + \sigma_v^2}\right\},$$

where

$$|SG_s^{\text{NI}}| \rightarrow \left\{1 + \delta^2 \sigma_u^2 \rho_s^2 \sigma_v^2 \left[\frac{\rho_s \sigma_v^2 + \rho_i \sigma_v^2}{2\rho_s \sigma_v^2 + \rho_i \sigma_v^2}\right]^2\right\}^{-1/2}.$$

Since

$$\exp\left\{-0.5 \frac{(\Gamma q)^2}{\delta^2 \sigma_u^2 / \Lambda^2 + \sigma_v^2}\right\} < 1 \quad \text{and} \quad \frac{1}{2} \leq \frac{\rho_s \sigma_v^2 + \rho_i \sigma_v^2}{2\rho_s \sigma_v^2 + \rho_i \sigma_v^2} \leq 1$$

(which implies that  $[(\rho_s \sigma_v^2 + \rho_i \sigma_v^2)/(2\rho_s \sigma_v^2 + \rho_i \sigma_v^2)]^2 \geq \frac{1}{4}$  and hence that  $|SG_s^{\text{IT}}| > |SG_s^{\text{NI}}|$  and  $-|SG_s^{\text{IT}}| < -|SG_s^{\text{NI}}|$ ), it is obvious that

$$E[U_{sk}^{\text{NI}}] = -|SG_s^{\text{NI}}| \exp\left\{-0.5 \frac{(\Gamma q)^2}{\delta^2 \sigma_u^2 / \Lambda^2 + \sigma_v^2}\right\} > -|SG_s^{\text{NI}}| \geq -|SG_s^{\text{IT}}| = E[U_{sk}^{\text{IT}}].$$

By continuity, this is also true for  $R_{sv}$  close to unity.

A similar proof can be given for  $R_{sv} = 1$  and  $R_u$  close to zero. The speculators' *ex ante* expected utility is given by  $E[U_{sk}^{\text{IT}}] = \{1 + (1/\rho_s + 1/\rho_h)^{-2} \times \delta^2 \sigma_u^2 \sigma_v^2 / 4\}^{-1/2}$  in the IT equilibria and by

$$E[U_{sk}^{\text{NI}}] = -|SG_s^{\text{NI}}| \exp\left\{-0.5 \frac{(\Gamma q)^2}{\delta^2 \sigma_u^2 / \Lambda^2 + \sigma_v^2}\right\} \quad \text{with}$$

$$|SG_s^{\text{NI}}| \rightarrow \left\{1 + \frac{\delta^2 \sigma_u^2 \sigma_v^2}{\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^2} \left[\frac{\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^{-1} \sigma_v^2 + \rho_i \sigma_v^2}{2\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^{-1} \sigma_v^2 + \rho_i \sigma_v^2}\right]^2\right\}^{-1/2}$$

in the NI equilibrium. Since

$$\exp\left\{-0.5 \frac{(\Gamma q)^2}{\delta^2 \sigma_u^2 / \Lambda^2 + \sigma_v^2}\right\} < 1 \quad \text{and} \quad \frac{1}{2} \leq \frac{\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^{-1} \sigma_v^2 + \rho_i \sigma_v^2}{2\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^{-1} \sigma_v^2 + \rho_i \sigma_v^2} \leq 1$$



which implies that

$$\left[ \frac{\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^{-1} \sigma_v^2 + \rho_i \sigma_v^2}{2\left(\frac{1}{\rho_s} + \frac{1}{\rho_h}\right)^{-1} \sigma_v^2 + \rho_i \sigma_v^2} \right]^2 \geq \frac{1}{4}$$

and therefore that  $|SG_s^{IT}| > |SG_s^{NI}|$  and  $-|SG_s^{IT}| < -|SG_s^{NI}|$ ), it is obvious that  $E[U_{sk}^{NI}] > -|SG_s^{NI}| \geq -|SG_s^{IT}| = E[U_{sk}^{IT}]$ . By continuity, this is also true for  $R_{sv}$  close to unity.  $\square$

**PROPOSITION 10.** Let  $R_u/\rho_h$  be close to zero. Then with insider trading the following statements hold.

1. The insider trades less aggressively:  $\alpha_i^{IT} < \alpha_i^{PD}$ .
2. The outsiders are less responsive to price movements,  $\beta_s^{IT} < \beta_s^{PD}$  and  $\beta_h^{IT} < \beta_h^{PD}$  for  $R_{sv} > 0$  (and  $\beta_s^{PD} - \beta_s^{IT}$  and  $\beta_h^{PD} - \beta_h^{IT}$  are increasing in  $R_{sv}$ ).
3. The market becomes thinner:  $\Lambda^{IT} < \Lambda^{PD}$ .
4. Price precision is reduced:  $\tau^{IT} < \tau^{PD}$ . If  $R_{sv}$  is close to unity then  $\text{var}[p^{PD}] > 2\text{var}[p^{IT}]$ .
5. If  $R_{sv}$  is close to unity then  $q^{IT} > q^{PD}$ .
6. The speculative gains of the insider are larger: if  $R_{sv}$  is close to unity (or if  $\rho_i$  is close to zero), then  $E[U_i^{IT}] > E[U_i^{PD}]$ .
7. If  $R_{sv}$  is close to unity,  $E[U_{sk}^{IT}] > E[U_{sk}^{PD}]$ .

*Proof*

1. We know that  $d\lambda^{IT}/dR_{sv} > 0$  (see Proposition 9),  $d\lambda^{PD}/dR_{sv} < 0$  (since  $\lambda^{PD} = [\rho_h \rho_s / (\rho_h + \rho_s)](1 - R_{sv})\sigma_v^2$ , and (when  $R_{sv} = 0$ )  $\lambda^{IT} = \lambda^{PD} = [\rho_h \rho_s / (\rho_h + \rho_s)]\sigma_v^2$ ). Thus, we conclude that  $\lambda^{IT}(R_{sv})$  is strictly greater than  $\lambda^{PD}(R_{sv})$  for all  $R_{sv} > 0$ . Moreover, the difference  $\lambda^{IT}(R_{sv}) - \lambda^{PD}(R_{sv})$  is increasing in  $R_{sv}$ . On the other hand,  $\alpha_i^{IT} = R_{sv} / (\rho_i(1 - R_{sv})\sigma_v^2 + \lambda^{IT})$  and  $\alpha_i^{PD} = R_{sv} / (\rho_i(1 - R_{sv})\sigma_v^2 + \lambda^{PD})$ . Since  $\lambda^{IT} > \lambda^{PD}$  for all  $R_{sv} > 0$ , it follows that  $\alpha_i^{IT} < \alpha_i^{PD}$  for all  $R_{sv} > 0$ .

2. We know that  $d\beta_s^{IT}/dR_{sv} < 0$  and  $d\beta_h^{IT}/dR_{sv} \leq 0$  (see Proposition 9),  $d\beta_s^{PD}/dR_{sv} > 0$ , and  $d\beta_h^{PD}/dR_{sv} > 0$  (since  $\beta_s^{PD} = 1/[\rho_s(1 - R_{sv})\sigma_v^2]$  and  $\beta_h^{PD} = 1/[\rho_h(1 - R_{sv})\sigma_v^2]$ ). Thus,  $\beta_s^{PD} > \beta_s^{IT}$  and  $\beta_h^{PD} > \beta_h^{IT}$  for all  $R_{sv} > 0$ . Moreover, the differences  $\beta_s^{PD} - \beta_s^{IT}$  and  $\beta_h^{PD} - \beta_h^{IT}$  are increasing in  $R_{sv}$ .

3. Market depth is given by  $\Lambda = 1/\lambda + 1/(\rho_i(1 - R_{sv})\sigma_v^2 + \lambda)$ . Since  $\lambda^{IT} > \lambda^{PD}$  for all  $R_{sv} > 0$ , it is obvious that  $\Lambda^{IT} < \Lambda^{PD}$ .

4. In the IT equilibrium, the price is informationally equivalent to  $\alpha_i^{IT}(s - \bar{v}) - \delta^{IT}u$ . On the other hand, in the PD equilibrium the price is informationally equivalent to  $(\alpha_i^{PD} + \alpha_s^{PD} + \alpha_h^{PD})(s - \bar{v}) - \delta^{PD}u$ , since  $s$  is now public information. If  $R_u/\rho_h$  tends to zero,  $\delta^{IT} \rightarrow \delta^{PD} = (\sigma_{vz}/\sigma_v^2)$ .

Moreover,  $0 < \alpha_i^{\text{IT}} < \alpha_i^{\text{PD}}$  and  $\alpha_s^{\text{PD}} + \alpha_h^{\text{PD}} > 0$ , so it is clear that  $\tau^{\text{IT}} = 1/\text{var}[v|p^{\text{IT}}] < \tau^{\text{PD}} = 1/\text{var}[v|p^{\text{PD}}]$  for all  $R_{sv} > 0$ .

The volatility of prices is given by

$$\text{var}[p^{\text{IT}}] = \frac{(\lambda^{\text{IT}})^2}{[\rho_i(1 - R_{sv})\sigma_v^2 + 2\lambda^{\text{IT}}]^2} R_{sv}\sigma_v^2 + \frac{(\delta^{\text{IT}})^2\sigma_u^2}{(\Lambda^{\text{IT}})^2}$$

in the IT equilibrium and by

$$\text{var}[p^{\text{PD}}] = R_{sv}\sigma_v^2 + \frac{(\delta^{\text{PD}})^2\sigma_u^2}{(\Lambda^{\text{PD}})^2}$$

in the PD equilibrium.

If  $R_{sv} \rightarrow 1$  and  $R_u/\rho_h \rightarrow 0$ , then  $\delta^{\text{IT}} \rightarrow \delta^{\text{PD}} = (\sigma_{vz}/\sigma_v^2)$ ,  $\Lambda^{\text{PD}} \rightarrow \infty$ , and  $\Lambda^{\text{IT}} = 2/\lambda^{\text{IT}}$ , where the value of  $\lambda^{\text{IT}}$  is implicitly defined by  $(\delta^{\text{IT}})^2\sigma_u^2 - \sigma_v^2/(\lambda^{\text{IT}})^2 = (\delta^{\text{IT}})^2\sigma_u^2\rho_s\sigma_v^2/\lambda^{\text{IT}}$  (or equivalently, by  $(\delta^{\text{IT}})^2\sigma_u^2 = \sigma_v^2/\lambda^{\text{IT}} \times [\lambda^{\text{IT}} - \rho_s\sigma_v^2]$ ). As a result,  $\text{var}[p^{\text{PD}}] \rightarrow \sigma_v^2$  and

$$\text{var}[p^{\text{IT}}] \rightarrow \frac{1}{4}\sigma_v^2 + \frac{(\delta^{\text{IT}})^2\sigma_u^2}{(\Lambda^{\text{IT}})^2} = \frac{\sigma_v^2}{4} \left[ 1 + \frac{\lambda^{\text{IT}}}{\lambda^{\text{IT}} - \rho_s\sigma_v^2} \right] < \frac{\sigma_v^2}{2},$$

so that

$$\text{var}[p^{\text{PD}}] \rightarrow \sigma_v^2 > \frac{\sigma_v^2}{2} > \text{var}[p^{\text{IT}}].$$

5. In the equilibrium with IT,  $q^{\text{IT}} = (\bar{v} - c_1)/(c_2 + \rho_i\sigma_v^2(1 - d^{\text{IT}}))$ , where

$$d^{\text{IT}} = \frac{\rho_i\sigma_v^2}{\rho_i\sigma_v^2 + 2\lambda^{\text{IT}} + \rho_i(\lambda^{\text{IT}})^2\delta^2\sigma_u^2} \left\{ 1 + \frac{(1 - R_{sv})\lambda^{\text{IT}}}{\rho_i(1 - R_{sv})\sigma_v^2 + \lambda^{\text{IT}}} \frac{\alpha_i^{\text{IT}}E^{\text{IT}}}{1 + \alpha_i^{\text{IT}}E^{\text{IT}}} \right\}^2.$$

On the other hand, we have  $q^{\text{PD}} = (\bar{v} - c_1)/(c_2 + \rho_i\sigma_v^2(1 - d^{\text{PD}}))$ , where  $d^{\text{PD}}$  may be written (after some simple but tedious manipulations) as

$$d^{\text{PD}} = \frac{\rho_i(1 - R_{sv})}{\rho_i + 2\frac{\rho_s\rho_h}{\rho_s + \rho_h} + \rho_i\left(\frac{\rho_s\rho_h}{\rho_s + \rho_h}\right)^2(1 - R_{sv})\sigma_v^2\delta^2\sigma_u^2}.$$

If  $R_{sv} \rightarrow 1$ , then  $d^{\text{PD}} \rightarrow 0$  and  $d^{\text{IT}} \rightarrow \rho_i \sigma_v^2 / [\rho_i \sigma_v^2 + 2\lambda^{\text{IT}} + \rho_i (\lambda^{\text{IT}})^2 \delta^2 \sigma_u^2] > 0$ . Consequently,  $q^{\text{IT}} > q^{\text{PD}}$ .

6. In the PD equilibrium, the insider's *ex ante* expected utility is given by

$$E[-\exp\{-\rho_i W_i^{\text{PD}}\}] = -|SG_i^{\text{PD}}| \exp\{-0.5\rho_i(\bar{v} - c_1)q^{\text{PD}}\}$$

where  $|SG_i^{\text{PD}}|$  may be written as

$$|SG_i^{\text{PD}}| = \left\{ 1 + \frac{\rho_i (\lambda^{\text{PD}})^2 (\delta^{\text{PD}})^2 \sigma_u^2}{\rho_i (1 - R_{sv}) \sigma_v^2 + 2\lambda^{\text{PD}}} \right\}^{-1/2}$$

or as

$$|SG_i^{\text{PD}}| = \left\{ 1 + \rho_i \left( \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right)^2 \left[ \rho_i + 2 \frac{\rho_s \rho_h}{\rho_s + \rho_h} \right]^{-1} (1 - R_{sv}) (\delta^{\text{PD}})^2 \sigma_u^2 \sigma_v^2 \right\}^{-1/2},$$

since  $\lambda^{\text{PD}} = [\rho_s \rho_h / (\rho_s + \rho_h)] (1 - R_{sv}) \sigma_v^2$ . It is obvious that  $d|SG_i^{\text{PD}}|/dR_{sv} > 0$  and, from Proposition 9.8,  $d|SG_i^{\text{IT}}|/dR_{sv} < 0$ ; hence it is clear that for any  $R_{sv} > 0$  we have  $|SG_i^{\text{PD}}| > |SG_i^{\text{IT}}|$ . On the other hand, if  $R_{sv}$  is close to unity, then  $q^{\text{IT}} > q^{\text{PD}}$  and, as a direct consequence,  $\exp\{-0.5\rho_i(\bar{v} - c_1)q^{\text{PD}}\} > \exp\{-0.5\rho_i(\bar{v} - c_1)q^{\text{IT}}\}$ . Therefore, provided that  $R_{sv}$  is close to unity,  $E[U_i^{\text{PD}}] < E[U_i^{\text{IT}}]$ .

Moreover, if the insider is risk neutral ( $\rho_i = 0$ ), then he will maximize his expected wealth, which may be written as

$$E[W_i^{\text{IT}}] = \bar{v}q^{\text{IT}} - C(q^{\text{IT}}) + \frac{1}{4} \left[ \frac{R_{sv}}{\lambda^{\text{IT}}} \sigma_v^2 + \lambda^{\text{IT}} (\delta^{\text{IT}})^2 \sigma_u^2 \right],$$

$$E[W_i^{\text{PD}}] = \bar{v}q^{\text{PD}} - C(q^{\text{PD}}) + \frac{1}{4} \lambda^{\text{PD}} (\delta^{\text{PD}})^2 \sigma_u^2,$$

where  $\lambda^{\text{PD}} = [\rho_s \rho_h / (\rho_s + \rho_h)] (1 - R_{sv}) \sigma_v^2$ ,  $q^{\text{IT}} = q^{\text{PD}} = (\bar{v} - c_1)/c_2$ ,  $\delta^{\text{PD}} = \sigma_{vz}/\sigma_v^2$  and the values of  $\lambda^{\text{IT}}$  and  $\delta^{\text{IT}}$  are given by Proposition 1 with  $\rho_i = 0$ . Since  $q^{\text{IT}} = q^{\text{PD}}$ ,  $\delta^{\text{IT}} \cong \delta^{\text{PD}}$  (provided  $R_u/\rho_h$  is close to zero), and  $\lambda^{\text{IT}} > \lambda^{\text{PD}}$ , it is obvious that if  $\rho_i = 0$  then

$$\begin{aligned} E[W_i^{\text{IT}}] &= \bar{v}q^{\text{IT}} - C(q^{\text{IT}}) + \frac{1}{4} \lambda^{\text{IT}} (\delta^{\text{IT}})^2 \sigma_u^2 + \frac{1}{4} \frac{R_{sv}}{\lambda^{\text{IT}}} \sigma_v^2 \\ &> \bar{v}q^{\text{PD}} - C(q^{\text{PD}}) + \frac{1}{4} \lambda^{\text{PD}} (\delta^{\text{PD}})^2 \sigma_u^2 = E[W_i^{\text{PD}}]. \end{aligned}$$

That is, if  $\rho_i = 0$  then the insider's welfare is higher in the IT regime,  $E[W_i^{\text{IT}}] > E[W_i^{\text{PD}}]$ . By continuity, this result also holds for  $\rho_i$  close to zero (if  $\rho_i$  is close to zero then it is also true that  $E[U_i^{\text{PD}}] < E[U_i^{\text{IT}}]$ , since  $|SG_i^{\text{PD}}| > |SG_i^{\text{IT}}|$  and  $q^{\text{IT}} \cong q^{\text{PD}}$ ).

7. In the PD equilibrium, speculator  $k$ 's *ex ante* expected utility is given by  $E[U_{sk}^{\text{PD}}] = -|SG_{sk}^{\text{PD}}| |IG_{sk}^{\text{PD}}|$ , where

$$|SG_{sk}^{\text{PD}}| = \left\{ 1 + \left[ \frac{1}{\rho_s} + \frac{1}{\rho_h} + \frac{1}{\rho_i + \frac{\rho_s \rho_h}{\rho_s + \rho_h}} \right]^{-2} (1 - R_{sv}) (\delta^{\text{PD}})^2 \sigma_v^2 \sigma_u^2 \right\}^{-1/2},$$

$$|IG_{sk}^{\text{PD}}| = \exp \left\{ -\frac{1}{2} \frac{(\gamma_i^{\text{PD}} q^{\text{PD}})^2 (1 - R_{sv}) \sigma_v^2}{\left[ \frac{1}{\rho_s} + \frac{1}{\rho_h} + \frac{1}{\rho_i + \frac{\rho_s \rho_h}{\rho_s + \rho_h}} \right]^2 + (1 - R_{sv}) \sigma_v^2 (\delta^{\text{PD}})^2 \sigma_u^2} \right\}.$$

On the other hand, in the IT equilibria the *ex ante* expected utility of speculator  $k$  is given by

$$E[-\exp\{-\rho_s W_{sk}^{\text{IT}}\}] = -|SG_{sk}^{\text{IT}}| \exp \left\{ -0.5 \frac{(\Gamma^{\text{IT}} q^{\text{IT}})^2}{\text{var}[E(v|p^{\text{IT}}) - p^{\text{IT}}] + \text{var}[v|p^{\text{IT}}]} \right\},$$

where

$$|SG_{sk}^{\text{IT}}| = \left\{ 1 + \frac{\text{var}[E(v|p^{\text{IT}}) - p^{\text{IT}}]}{\text{var}[v|p^{\text{IT}}]} \right\}^{-1/2},$$

$$\text{var}[E(v|p^{\text{IT}}) - p^{\text{IT}}] = \frac{[(\delta^{\text{IT}})^2 \sigma_u^2 + (\alpha_i^{\text{IT}})^2 R_{sv}^{-1} \sigma_v^2 - \alpha_i^{\text{IT}} \Lambda^{\text{IT}} \sigma_v^2]^2}{(\Lambda^{\text{IT}})^2 [(\delta^{\text{IT}})^2 \sigma_u^2 + (\alpha_i^{\text{IT}})^2 R_{sv}^{-1} \sigma_v^2]},$$

and

$$\text{var}[v|p^{\text{IT}}] = \sigma_v^2 \frac{(\delta^{\text{IT}})^2 \sigma_u^2 + (\alpha_i^{\text{IT}})^2 (1 - R_{sv}) R_{sv}^{-1} \sigma_v^2}{(\delta^{\text{IT}})^2 \sigma_u^2 + (\alpha_i^{\text{IT}})^2 R_{sv}^{-1} \sigma_v^2}.$$

If  $R_{sv} = 1$  and  $1/\rho_h$  is close to zero, then  $E[U_{sk}^{\text{PD}}] = -1$ ,  $\Gamma^{\text{IT}} = 0$ ,  $\alpha_i^{\text{IT}} = 1/\lambda^{\text{IT}}$ , and  $\Lambda^{\text{IT}} = 2/\lambda^{\text{IT}}$  where the value of  $\lambda^{\text{IT}}$  is implicitly defined by  $(\delta^{\text{IT}})^2 \sigma_u^2 - \sigma_v^2 / (\lambda^{\text{IT}})^2 = (\delta^{\text{IT}})^2 \sigma_u^2 \rho_s \sigma_v^2 / \lambda^{\text{IT}}$ . As a direct consequence,

$$E[U_{sk}^{IT}] = - \left\{ 1 + \frac{(\delta^{IT})^2 \sigma_u^2 \rho_s^2 \sigma_v^2}{4} \right\}^{-1/2} > -1 = E[U_{sk}^{PD}].$$

Similarly, if  $R_{sv} = 1$  and  $R_u$  is close to zero,

$$E[U_{sk}^{IT}] = - \left\{ 1 + \left( \frac{1}{\rho_s} + \frac{1}{\rho_h} \right)^{-2} \frac{(\delta^{IT})^2 \sigma_u^2 \sigma_v^2}{4} \right\}^{-1/2} > -1 = E[U_{sk}^{PD}].$$

Thus, if  $R_{sv} = 1$  and  $R_u$  is close to zero,  $E[U_{sk}^{IT}] > E[U_{sk}^{PD}]$ . By continuity, if  $R_u/\rho_h$  is close to zero then this result,  $E[U_{sk}^{IT}] > E[U_{sk}^{PD}]$ , holds also for  $R_{sv}$  sufficiently close to unity.  $\square$

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