COORDINATION FAILURES AND THE LENDER OF LAST RESORT: WAS BAGEHOT RIGHT AFTER ALL?

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Abstract
The classical doctrine of the Lender of Last Resort (LOLR), elaborated by Bagehot (1873), asserts that the central bank should lend to "illiquid but solvent" banks under certain conditions. Several authors have argued that this view is now obsolete: in modern interbank markets, a solvent bank cannot be illiquid. This paper provides a possible theoretical foundation for rescuing Bagehot's view. Our theory does not rely on the multiplicity of equilibria that arises in classical models of bank runs. We built a model of banks' liquidity crises that possesses a unique Bayesian equilibrium. In this equilibrium, there is a positive probability that a solvent bank cannot find liquidity assistance in the market. We derive policy implications about banking regulation (solvency and liquidity ratios) and interventions of the Lender of Last Resort. Furthermore, we find that public (bailout) and private (bail-in) involvement are complementary in implementing the incentive efficient solution and that Bagehot's Lender of Last Resort facility must work together with institutions providing prompt corrective action and orderly failure resolution. Finally, we derive similar implications for an International Lender of Last Resort (ILOLR). (JEL: G21, G28)

1. Introduction

There have been several recent controversies about the need for a Lender of Last Resort (LOLR) both within national banking systems (central bank) and at an international level (International Monetary Fund, IMF).1 The concept of a LOLR was elaborated in the nineteenth century by Thornton (1802) and Bagehot (1873).

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An essential point of the ‘classical’ doctrine associated with Bagehot asserts that
the LOLR role is to lend to “solvent but illiquid” banks under certain conditions.2

Banking crises have been recurrent in most financial systems. The LOLR
facility and deposit insurance were instituted precisely to provide stability to the
banking system and avoid unfavorable consequences for the real sector. Indeed,
financial distress may cause important damage to the economy, as the example of
the Great Depression makes clear.3 Traditional banking panics were eliminated
with the LOLR facility and deposit insurance by the end of the nineteenth cen-
tury in Europe, after the crisis of the 1930s in the United States and, by now,
also mostly in emerging economies, which have suffered numerous crises until
today.4 Modern liquidity crises associated with securitized money or capital mar-
kets have also required the intervention of the LOLR. Indeed, the Federal Reserve
intervened in the crises provoked by the failure of Penn Central in the U.S. com-
mercial paper market in 1970, by the stock market crash of October 1987, and
by Russia’s default in 1997, and subsequent collapse of LTCM (in the latter case a
“lifeboat” was arranged by the New York Fed). For example, in October 1987
the Federal Reserve supplied liquidity to banks through the discount window.5

The LOLR’s function of providing emergency liquidity assistance has been
criticized for provoking moral hazard on the banks’ side. Perhaps more impor-
tantly, Goodfriend and King (1988) (see also Bordo 1990; Kaufman 1991; and
Schwartz 1992) remark that Bagehot’s doctrine was elaborated at a time when
financial markets were underdeveloped. They argue that, whereas central bank
intervention on aggregate liquidity (monetary policy) is still warranted, individual
interventions (banking policy) are not anymore: with sophisticated interbank mar-
kets, banking policy has become redundant. Open-market operations can provide
sufficient liquidity, which is then allocated by the interbank market. The discount
window is not needed. In other words, Goodfriend and King argue that, when
financial markets function well, a solvent institution cannot be illiquid. Banks
can finance their assets with interbank funds, negotiable certificates of deposit
(CDs), and repurchase agreements (repos). Well-informed participants in this
interbank market will distinguish liquidity from solvency problems. This view
has consequences also for the debate about the need for an international LOLR.
Indeed, Chari and Kehoe (1998) claim, for example, that such an international
LOLR is not needed because joint action by the Federal Reserve, the European

2. The LOLR should lend freely against good collateral, valued at precrisis levels, and at a penalty
rate. These conditions are due to Bagehot (1873) and are also presented, for instance, in Humphrey
(1975) and Freixas et al. (1999).
other IMF member countries.
5. See Folkerts-Landau and Garber (1992). See also Freixas, Parigi, and Rochet (2004) for a
modeling of the interactions between the discount window and the interbank market.
Central Bank, and the Bank of Japan can take care of any international liquidity problem.\(^6\)

Those developments have led qualified observers to dismiss bank panics as a phenomenon of the past and to express confidence in the efficiency of financial markets—especially the interbank market—for resolving liquidity problems of financial intermediaries. This is based on the view that participants in the interbank market are the best-informed agents to ascertain the solvency of an institution with liquidity problems.\(^7\)

The main objective of this article is to provide a theoretical foundation for Bagehot’s doctrine in a model that fits the modern context of sophisticated and presumably efficient financial markets. We are thinking of a short time horizon that corresponds to liquidity crises. We shift emphasis from maturity transformation and liquidity insurance of small depositors to the ‘modern’ form of bank runs, where large well-informed investors refuse to renew their credit (CDs for example) on the interbank market. The decision not to renew credit may arise as a result on an event (e.g., failure of Penn Central, October 1987 crash, LTCM failure) that puts in doubt the repayment capacity of an intermediary or a number of intermediaries. The central bank may then decide to provide liquidity to those troubled institutions. The question arises of whether such intervention is warranted.

Since Diamond and Dybvig (1983) and Bryant (1980), banking theory has insisted on the fragility of banks due to possible coordination failures between depositors (bank runs). However it is hard to base any policy recommendation on their model, since it systematically possesses multiple equilibria. Furthermore, a run equilibrium needs to be justified with the presence of sunspots that coordinate the behavior of investors. Indeed, otherwise no one would deposit in a bank that would be subject to run. This view of banking instability has been disputed by Gorton (1985) and others who argue that crises are related to fundamentals and not to self-fulfilling panics. In this view, crises are triggered by bad news about the returns to be obtained by the bank. Gorton (1988) studies panics in the U.S. National Banking Era and concludes that crises were predictable by indicators of the business cycle. The phenomenon has been theorized in the literature on

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\(^6\) Jeanne and Wyplosz (2003) compare the required size of an international LOLR under the “open market monetary policy” and the “discount window banking policy” views.

\(^7\) For example, Tommaso Padoa-Schioppa, member of the European Central Bank’s executive committee in charge of banking supervision, has gone as far as saying that classical bank runs may occur only in textbooks, precisely because measures like deposit insurance and capital adequacy requirements have been put in place. Furthermore, despite recognizing that “rapid outflows of uninsured interbank liabilities” are more likely, Padoa-Schioppa (1999) states: “However, since interbank counterparties are much better informed than depositors, this event would typically require the market to have a strong suspicion that the bank is actually insolvent. If such a suspicion were to be unfounded and not generalized, the width and depth of today’s interbank market is such that other institutions would probably replace (possibly with the encouragement of the public authorities as described previously) those which withdraw their funds.”
information-based bank runs; see Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998). There is an ongoing empirical debate about whether crises are predictable and their relation to fundamentals.8

Our approach is inspired by Postlewaite and Vives (1987), who present an incomplete information model featuring a unique Bayesian equilibrium with a positive probability of bank runs. However, the model of Postlewaite and Vives (1987) differs from our model here in several respects. In particular, in Postlewaite and Vives there is no uncertainty about the fundamental value of the banks’ assets (no solvency problems) but incomplete information about the liquidity shocks suffered by depositors. The uniqueness of equilibrium in their case comes from a more complex specification of technology and liquidity shocks for depositors than in Diamond and Dybvig (1983). The present model is instead adapted from the “global game” analysis of Carlsson and Van Damme (1993) and Morris and Shin (1998).9 This approach builds a bridge between the “panic” and “fundamentals” views of crises by linking the probability of occurrence of a crisis to the fundamentals. A crucial property of the model is that, when the private information of investors is precise enough, the game among them has a unique equilibrium. Moreover, at this unique equilibrium there is an intermediate interval of values of the bank’s assets for which, in the absence of intervention by the central bank, the bank is solvent but still can fail if too large a proportion of investors withdraw their money. In other words, in this intermediate range for the fundamentals there exists a potential for coordination failure. Furthermore, the range in which such a coordination failure occurs diminishes with the ex ante strength of fundamentals.

Given that this equilibrium is unique and is based on the fundamentals of the bank, we are able to provide some policy recommendations on how to avoid such failures. More specifically, we discuss the interaction between ex ante regulation of solvency and liquidity ratios and ex post provision of emergency liquidity assistance. It is found that liquidity and solvency regulation can solve the coordination problem, but typically the cost is too high in terms of foregone returns. This means that prudential measures must be complemented with emergency discount-window loans.

To complete the policy discussion we introduce moral hazard and endogenize banks’ short-term debt structure as a way to discipline bank managers. This framework allows us to discuss early closure policies of banks as well as the interaction of the LOLR, prompt corrective action, and orderly resolution of failures. We can then study the adequacy of Bagehot’s doctrine in a richer environment and derive the complementarity between public (LOLR and other facilities) and private (market) involvement in crisis resolution.

8. See also Kaminsky and Reinhart (1999) and Radelet and Sachs (1998) for perspectives on international crises.
9. See also Goldstein and Pauzner (2003), Heinemann and Illing (2000), and Corsetti et al. (2004).
Finally, we provide a reinterpretation of the model in terms of the banking sector of a small open economy and derive lessons for a international LOLR facility.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses runs and solvency. Section 4 characterizes the equilibrium of the game between investors. Section 5 studies the properties of this equilibrium and the effect of prudential regulation on coordination failure. Section 6 makes a first pass at the LOLR policy implications of our model and its relation to Bagehot’s doctrine. Section 7 shows how to endogenize the liability structure and proposes a welfare-based LOLR facility with attention to crisis resolution. Section 8 provides the international reinterpretation of the model and discusses the role of an international LOLR and associated facilities. Concluding remarks end the paper in Section 9.

2. The Model

Consider a market with three dates: \( \tau = 0, 1, 2 \). At date \( \tau = 0 \) the bank possesses own funds \( E \) and collects uninsured wholesale deposits (CDs for example) for some amount \( D_0 \) that is normalized to unity. These funds are used in part to finance some investment \( I \) in risky assets (loans), the rest being held in cash reserves \( M \). Under normal circumstances, the returns \( R_I \) on these assets are collected at date \( \tau = 2 \), the CDs are repaid, and the stockholders of the bank receive the difference (when it is positive). However, early withdrawals may occur at an interim date \( \tau = 1 \), following the observation of private signals on the future realization of \( R \). If the proportion \( x \) of these withdrawals exceeds the cash reserves \( M \) of the bank then the bank is forced to sell some of its assets. To summarize our notation, the bank’s balance sheet at \( \tau = 0 \) is represented as:

\[
\begin{array}{ccc}
I & D_0 & = 1 \\
M & & E \\
\end{array}
\]

The terms in this representation are defined as follows.

1. \( D_0 (= 1) \) is the volume of uninsured wholesale deposits that are normally repaid at \( \tau = 2 \) but can also be withdrawn at \( \tau = 1 \). The nominal value of deposits upon withdrawal is \( D \geq 1 \) independently of the withdrawal date. Thus, early withdrawal entails no cost for the depositors themselves (when the bank is not liquidated prematurely).
2. $E$ represents the value of equity (or, more generally, long-term debt; it may also include insured deposits\(^{10}\))

3. $I$ denotes the volume of investment in risky assets, which have a random return $R$ at $\tau = 2$.

4. $M$ is the amount of cash reserves (money) held by the bank.

We assume that the withdrawal decision is delegated to fund managers who typically prefer to renew the deposits (i.e., not to withdraw early) but are penalized by the investors if the bank fails. This is consistent with the fact that the vast majority of wholesale deposits are held by collective investment funds and the empirical evidence on the remuneration of fund managers (see, for example, Chevalier and Ellison 1997, 1999). Indeed, the salaries of fund managers depend on the size of their funds and are not directly indexed on the returns on these funds. Instead, managers are promoted (i.e., get more funds to manage) if they build a good reputation, and demoted otherwise. Accordingly, we suppose that fund managers payoffs depend on whether they take the “right decision.” When the bank does not fail the differential payoff of withdrawing with respect to rolling over the CD, is negative and equal to $-C < 0$. When the bank does fail the differential payoff of withdrawing with respect to rolling over the CD, is positive and equal to $B > 0$. Therefore, fund managers adopt the following behavioral rule: withdraw if and only if they anticipate $PB - (1 - P)C > 0$ or $P > \gamma = C/(B + C)$, where $P$ is the probability that the bank fails.

This payoff structure is obtained, for example, if fund managers obtain a benefit $B > 0$ if they get the money back or if they withdraw and the bank fails. They get nothing otherwise. However, to withdraw involves a cost $C > 0$ for the managers (for example because their reputation suffers if they have to recognize that they have made a bad investment) as well as not withdrawing and the bank failing.

At $\tau = 1$, fund manager $i$ privately observes a signal $s_i = R + \varepsilon_i$, where the $\varepsilon_i$ are i.i.d. and also independent of $R$. As a result, a proportion $x$ of them decides to “withdraw” (i.e., not to renew their CDs). By assumption there is no other source of financing for the bank (except perhaps the central bank, see next), so if $x > M/D$ then the bank is forced to sell a volume $y$ of assets.\(^{11}\) If the needed volume of sales $y$ is greater than the total of available assets $I$, the bank fails at $\tau = 1$; if not, the bank continues until Date 2. Failure occurs at $\tau = 2$ whenever

$$R(I - y) < (1 - x)D. \tag{1}$$

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\(^{10}\) If they are fully insured, these deposits have no reason to be withdrawn early and can therefore be assimilated into stable resources.

\(^{11}\) These sales are typically accompanied with a repurchase agreement (or repo). They are thus equivalent to a collateralized loan.
Our modeling tries to capture, in the simplest possible way, the main institutional features of modern interbank markets. In our model, banks essentially finance themselves by two complementary sources: stable resources (equity and long-term debt) and uninsured short-term deposits (or CDs), which are uncollateralized and involve fixed repayments. However, in case of a liquidity shortage at Date 1, banks may also sell some of their assets (or equivalently borrow against collateral) on the repo market. This secondary market for bank assets is assumed to be informationally efficient in the sense that the secondary price aggregates the decentralized information of investors about the quality of the bank’s assets.\textsuperscript{12}

Therefore, we assume that the resale value of the bank’s assets depends on $R$. However, banks cannot obtain the full value of these assets but only a fraction $1/(1 + \lambda)$ of this value, with $\lambda > 0$. Accordingly, the volume of sales needed to face withdrawals $x$ is given by

$$y = (1 + \lambda) \frac{[xD - M]_+}{R},$$

where $[xD - M]_+ = \max(0, xD - M)$. The parameter $\lambda$ measures the cost of “fire sales” in the secondary market for bank assets. It is crucial for our analysis and can be explained via considerations of asymmetric information or liquidity problems.\textsuperscript{13}

Indeed, asymmetric information problems may translate into either limited commitment of future cash flows (as in Hart and Moore 1994 or Diamond and Rajan 2001), or moral hazard (as in Holmstrom and Tirole 1997), or adverse selection (as in Flannery 1996). We have chosen to stress the last explanation because it gives a simple justification for the superiority of the central bank over financial markets in the provision of liquidity to banks in trouble. This adverse selection premium comes from the fact that a bank may want to sell its assets for two reasons: because it needs liquidity or because it wants to get rid of its bad loans (with a value normalized to zero). Investors only accept to pay $R/(1 + \lambda)$ because they assess a probability $1/(1 + \lambda)$ to the former case. The superiority of the central bank comes from its supervisory knowledge of banks.\textsuperscript{14}

The presence of an adverse selection discount in credit markets is well established (see, e.g., Broecker 1990; Riordan 1993). Flannery (1996) presents a specific mechanism that explains why the secondary market for bank assets may be plagued by a

\textsuperscript{12} We can imagine, for instance, that the bank organizes an auction for the sale of its assets. If there is a large number of bidders and their signals are (conditionally) independent, then the equilibrium price $p$ of this auction will be a deterministic function of $R$.

\textsuperscript{13} For a similar assumption in a model of an international lender of last resort, see Goodhart and Huang (1999).

\textsuperscript{14} The empirical evidence points at the superiority of the central bank information because of its access to supervisory data (Peek, Rosengren, and Tootell 1999, for example). Similarly, Romer and Romer (2000) find evidence of a superiority of the Federal Reserve over commercial forecasters in forecasting inflation.
winner’s curse, inducing a fire-sale premium. He argues, furthermore, that this fire-sale premium is likely to be higher during crises, given that investors are then probably more uncertain about the precision of their signals. This makes the winner’s curse more severe because it is then more difficult to distinguish good from bad risks.

The parameter \( \lambda \) can also be interpreted as a liquidity premium—that is, the interest margin that the market requires for lending on short notice. (See Allen and Gale (1998) for a model in which costly asset sales arise due to the presence of liquidity-constrained speculators in the resale market.) In a generalized banking crisis we would have a liquidity shortage, implying a large \( \lambda \). Interpreting it as a market rate, \( \lambda \) can also spike temporarily in response to exogenous events, such as September 11.

In our model we will be thinking mostly of the financial distress of an individual bank (a bank is close to insolvency when \( R \) is small), although—for sufficiently correlated portfolio returns of the banks—the interpretation could be broadened (see also the interpretation in an international context in Section 8).

Operations on interbank markets do not involve any physical liquidation of bank assets. However, we will show that, when a bank is close to insolvency (\( R \) small) or when there is a liquidity shortage (\( \lambda \) large), the interbank markets do not suffice to prevent early closure of the bank. Early closure involves the physical liquidation of assets, and this is costly. We model this liquidation cost (not to be confused with the fire-sale premium \( \lambda \)) as being proportional to the future returns on the bank’s portfolio. If the bank is closed at \( \tau = 1 \) then the (per-unit) liquidation value of its assets is \( \nu R \), with \( \nu \ll 1/(1 + \lambda) \).

3. Runs and Solvency

We focus in this section on some features of banks’ liquidity crises that cannot be properly taken into account within the classical Bryant–Diamond–Dybvig (BDD) framework. In doing so we take the banks’ liability structure (and, in particular, the fact that an important fraction of these liabilities can be withdrawn on demand) as exogenous. A possible way to endogenize the bank’s liability structure is to introduce a disciplining role for liquid deposits. In Section 7 we explore such an extension.

We adopt explicitly the short time horizon (say, two days) that corresponds to liquidity crises. This means that we shift the emphasis from maturity transformation and liquidity insurance of small depositors to the “modern” form of bank runs—that is, large investors refusing to renew their CDs on the interbank market.

A second element that differentiates our model from BDD is that our bank is not a mutual bank but rather a corporation that acts in the best interest of its stockholders. This allows us to discuss the role of equity as well as the articulation
between solvency requirements and provision of emergency liquidity assistance. In Section 7 we endogenize the choice of assets by the bank through the monitoring effort of its managers (first-order stochastic dominance), but we take as given the amount of equity $E$. It would be interesting to extend our model by endogenizing the level of equity to capture the impact of leverage on the riskiness of assets chosen by banks (second-order stochastic dominance). In this model, however, both the amount of equity and the riskiness of assets are taken as given.

As a consequence of our assumptions, the relation between the proportion $x$ of early withdrawals and the failure of the bank is different from that in BDD. To see this, let us recapitulate the different cases.

1. $x D \leq M$: there is no sale of assets at $\tau = 1$. In this case, there is failure at $\tau = 2$ if and only if

$$RI + M < D \iff R < R_s = \frac{D - M}{I} = 1 - \frac{1 + E - D}{I}.$$ 

Here $R_s$ can be interpreted as the solvency threshold of the bank. Indeed, if there are no withdrawals at $\tau = 1$ ($x = 0$) then the bank fails at $\tau = 2$ if and only if $R < R_s$. The threshold $R_s$ is a decreasing function of the solvency ratio $E/I$.

2. $M < x D \leq M + RI/(1+\lambda)$: there is a partial sale of assets at $\tau = 1$. Failure occurs at $\tau = 2$ if and only if

$$RI - (1 + \lambda)(x D - M) < (1 - x)D$$

$$\iff R < R_s + \lambda \frac{x D - M}{I} = R_s \left[ 1 + \lambda \frac{x D - M}{D - M} \right].$$

This formula illustrates how, because of the premium $\lambda$, solvent banks can fail when the proportion $x$ of early withdrawals is too large.\(^{15}\) Note, however, an important difference with BDD: when the bank is “supersolvent” ($R > (1 + \lambda)R_s$) it can never fail, even if everybody withdraws ($x = 1$).

3. $x D > M + RI/(1+\lambda)$, the bank is closed at $\tau = 1$ (early closure).

The failure thresholds are summarized in Figure 1.

A few comments are in order. In our model, early closure is never ex post efficient because physically liquidating assets is costly. However, as discussed in Section 7, early closure may be ex ante efficient for disciplining bank managers and inducing them to exert effort. The perfect information benchmark of our model (where $R$ is common knowledge at $\tau = 1$) has different properties than in

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\(^{15}\) We may infer that to obtain resources $x D - M > 0$ we must liquidate a fraction $\mu = [(x D - M)/(RI)](1 + \lambda)$ of the portfolio; hence, $\tau = 2$ we have $R(1 - \mu)I = RI - (1 + \lambda)(x D - M)$ remaining.
BDD: here, the multiplicity of equilibria arises only in the median range $R_s \leq R \leq (1 + \lambda)R_s$. When $R < R_s$ everybody runs ($x = 1$), when $R > (1 + \lambda)R_s$ nobody runs ($x = 0$), and only in the intermediate region do both equilibria coexist.\textsuperscript{16} This pattern is crucial for being able to select a unique equilibrium via the introduction of private noisy signals (when noise is not too important, as in Morris and Shin (1998)).\textsuperscript{17}

The different regimes of the bank are represented in Figure 2 as a function of $R$ and $x$. The critical value of $R$ below which the bank is closed early (early closure threshold $R_{ec}$) is given by

$$R_{ec}(x) = (1 + \lambda)\left(\frac{xD - M}{I}\right)_{+},$$

and the critical value of $R$ below which the bank fails (failure threshold $R_f$) is given by

$$R_f(x) = R_s + \lambda\left(\frac{xD - M}{I}\right).$$

The parameters $R_s$, $M$, and $I$ are not independent. Since we want to study the impact of prudential regulation on the need for central bank intervention, we will focus on $R_s$ (a decreasing function of the solvency ratio $E/I$) and $m = M/D$ (the liquidity ratio). Replacing $I$ by its value $(D - M)/R_s$, we obtain:

$$R_{ec}(x) = R_s(1 + \lambda)\left(\frac{x - m}{1 - m}\right)_{+}; \text{ and}$$

$$R_f(x) = R_s\left(1 + \lambda\left(\frac{x - m}{1 - m}\right)_{+}\right).$$

16. When $R < R_s$, the differential payoff of withdrawing for fund managers is $B$. When $R > (1 + \lambda)R_s$, the differential payoff of withdrawing is $-C$.

17. Goldstein and Pauzner (2003) adapt the same methodology to the BDD model, in which the perfect information game always has two equilibria, even for very large $R$. Accordingly, they have to make an extra assumption, namely that short-term returns are high when the value of the bank’s assets is large, or that there exists a potential lender who can cover the liquidity needs of the bank, to get an upper dominance region. Goldstein and Pauzner (2003) provides a very useful extension of the methodology of global games to cases where strategic complementarity is not satisfied. See also Morris and Shin (2000).
It should be obvious that $R_{ec}(x) < R_f(x)$, since early closure implies failure whereas the converse is not true (see Figure 2).

4. Equilibrium of the Investors’ Game

To simplify the presentation we concentrate on “threshold” strategies, in which each fund manager decides to withdraw if and only if his signal is below some threshold, $t$. As we shall see, this is without loss of generality. For a given $R$, a fund manager withdraws with probability

$$\Pr[R + \varepsilon < t] = G(t - R),$$

where $G$ is the c.d.f. of the random variable $\varepsilon$. Given our assumptions, this probability also equals the proportion of withdrawals $x(R, t)$.

A fund manager withdraws if and only if the probability of failure of the bank (conditional on the signal $s$ received by the manager and the threshold $t$ used by

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18. It is assumed that the decision on whether to withdraw is taken before the secondary market is organized and thus before fund managers have the opportunity to learn about $R$ from the secondary price. (On this issue see Atkeson’s comments on Morris and Shin (2000).)
other managers) is large enough. That is, \( P(s, t) > \gamma \), where

\[
P(s, t) = \Pr[\text{failure} | s, t] = \Pr[R < R_F(x(R, t)) | s].
\]

Before we analyze the equilibrium of the investor’s game, let us look at the region of the plane \((t, R)\) where failure occurs. For this, transform Figure 2 by replacing \( x \) by \( x(R, t) = G(t - R) \); this yields Figure 3. Notice that \( R_F(t) \), the critical \( R \) that triggers failure, is equal to the solvency threshold \( R_s \) when \( t \) is low and fund managers are confident about the strength of fundamentals:

\[
R_F(t) = R_s \quad \text{if} \quad t \leq t_0 = R_s + G^{-1}(m).
\]

For \( t > t_0 \), however \( R_F(t) \) is an increasing function of \( t \) and is defined implicitly by

\[
R = R_s \left(1 + \lambda \left[ \frac{G(t - R) - m}{1 - m} \right] \right).
\]

Let us denote by \( G(\cdot | s) \) the c.d.f. of \( R \) conditional on signal \( s \):

\[
G(r | s) = \Pr[R < r | s].
\]

Then, given the definition of \( R_F(t) \),

\[
P(s, t) = \Pr[R < R_F(t) | s] = G(R_F(t) | s)
\]

(3)

\[\text{Figure 3. The different regimes in the (R, t) plane.}\]
It is natural to assume that $G(r|s)$ is decreasing in $s$: the higher $s$, the lower the probability that $R$ lies below any given threshold $r$. Then it is immediate that $P$ is decreasing in $s$ and nondecreasing in $t$: $\partial P / \partial s < 0$ and $\partial P / \partial t \geq 0$. This means that the depositors’ game is one of strategic complementarities. Indeed, given that other fund managers use the strategy with threshold $t$, the best response of a manager is to use a strategy with threshold $\bar{s}$: withdraw if and only if $P(s, t) > \gamma$ or (equivalently) if and only if $s < \bar{s}$, where $P(\bar{s}, t) = \gamma$. Let $\bar{s} = S(t)$. Now we have that $S' = - (\partial P / \partial t) / (\partial P / \partial s) \geq 0$: a higher threshold $t$ by others induces a manager to use a higher threshold also.

The strategic complementarity property holds for general strategies. For a fund manager, all that matters is the conditional probability of failure for a given signal, and this depends only on aggregate withdrawals. Recall that the differential payoff to a fund manager for withdrawing versus not withdrawing is given by $PB - (1 - P)C$. A strategy for a fund manager is a function $a(s) \in \{\text{not withdraw, withdraw}\}$. If more managers withdraw, then the probability of failure conditional on receiving signal $s$ increases. This just means that the payoff to a fund manager displays increasing differences with respect to the actions of others. The depositor’s game is a supermodular game that has a largest and a smallest equilibrium. In fact, the game is symmetric (i.e., exchangeable against permutations of the players) and hence the largest and smallest equilibria are symmetric. At the largest equilibrium, every fund manager withdraws on the largest number of occasions; at the smallest equilibrium, withdrawal is on the smallest number of occasions. The largest (smallest) equilibrium can be identified then with the highest (lowest) threshold strategy, $\bar{t} (t)$. These extremal equilibria bound the set of rationalizable outcomes. That is, strategies outside this set can be eliminated by iterated deletion of dominated strategies. We will make assumptions so that $\bar{t} = \bar{t}$ and equilibrium will be unique.

The threshold $t = t^\prime$ corresponds to a (symmetric) Bayesian–Nash equilibrium if and only if $P(t^\prime, t^\prime) = \gamma$. Indeed, suppose that funds managers use the threshold strategy $t^\prime$. Then for $s = t^\prime$ we have $P = \gamma$ and, since $P$ is decreasing

\begin{enumerate}
\item See Remark 15, in Vives (1999, p. 34). See also Chapter 2 in the same reference for an exposition of the theory of supermodular games.
\item The extremal equilibria can be found with the usual algorithm in a supermodular game (Vives 1990), starting at the extremal points of the strategy sets of players and iterating using the best responses. For example, to obtain $\bar{t}$, let all investors withdraw for any signal received (i.e., start from $t_0 = +\infty$ and $x = 1$) and apply iteratively the best response $S(t)$ of a player to obtain a decreasing sequence $t_i$ that converges to $\bar{t}$. Note that $S(+\infty) = t_1 < +\infty$, where $t_1$ is the unique solution to $P(t, +\infty) = G(R(1 + \lambda)t) = \gamma$ given that $G$ is (strictly) decreasing in $t$. The extremal equilibria are in strategies that are monotone in type, which with two actions means that the strategies are of the threshold type. The game among mutual fund managers is an example of a “monotone supermodular game” for which, according to Van Zandt and Vives (2003), extremal equilibria are monotone in type.
\item See Morris and Shin (2000) for an explicit demonstration of the outcome of iterative elimination of dominated strategies in a similar model.
\end{enumerate}
in $s$ for $s < t^*$, it follows that $P(s, t^*) > \gamma$ and the manager withdraws. Conversely, if $t^*$ is a (symmetric) equilibrium then, for $s = t^*$, there is no withdrawal and hence $P(t^*, t^*) \leq \gamma$. If $P(t^*, t^*) < \gamma$ then, by continuity, for $s$ close to but less than $t^*$ we would have $P(s, t^*) < \gamma$ a contradiction. It is clear then that the largest and smallest solutions to $P(t^*, t^*) = \gamma$ correspond respectively to the largest and smallest equilibrium.

An equilibrium can also be characterized by a couple of equations in two unknowns (a withdrawal threshold $t^*$ and a failure threshold $R^*$):

$$G(R^*|t^*) = \gamma; \quad (4)$$

$$R^* = R_s \left(1 + \lambda \left[\frac{G(t^* - R^*) - m}{1 - m}\right]_+\right). \quad (5)$$

Equation (4) states that, conditional on observing a signal $s = t^*$, the probability that $R < R^*$ is $\gamma$. Equation (5) states that, given a withdrawal threshold $t^*$, $R^*$ is the critical return (i.e., the one below which failure occurs). Equation (5) implies that $R^*$ belongs to $[R_s, (1 + \lambda)R_s]$. Notice that early closure occurs whenever $x(R, t^*)D > M + I R/(1 + \lambda)$, where $x(R, t^*) = G(t^* - R)$. This happens if and only if $R$ is smaller than some threshold $R_{EC}(t^*)$. We will have that $R_{EC}(t^*) < R^*$, because early closure implies failure whereas the converse is not true, as remarked before.

To simplify the analysis of this system, we shall make distributional assumptions on returns and signals. More specifically, we will assume that the distributions of $R$ and $\epsilon$ are normal, with respective means $\bar{R}$ and 0 and respective precisions (i.e., inverse variances) $\alpha$ and $\beta$. Denoting by $\Phi$ the c.d.f. of a standard normal distribution allows us to characterize the equilibrium by a pair $(t^*, R^*)$ such that

$$\Phi\left(\sqrt{\alpha + \beta}R^* - \frac{\alpha \bar{R} + \beta t^*}{\sqrt{\alpha + \beta}}\right) = \gamma \quad (6)$$

and

$$R^* = R_s \left(1 + \lambda \left[\frac{\Phi(\sqrt{\beta}(t^* - R^*)) - m}{1 - m}\right]_+\right). \quad (7)$$

We now can now state our first result.

**PROPOSITION 1.** When $\beta$ (the precision of the private signal of investors) is large enough relative to $\alpha$ (prior precision), there is a unique $t^*$ such that $P(t^*, t^*) = \gamma$. The investor’s game then has a unique (Bayesian) equilibrium. In this equilibrium, fund managers use a strategy with threshold $t^*$.

**Proof.** We show that $\varphi(s) \overset{\text{def}}{=} P(s, s)$ is decreasing for $\beta \geq \beta_0 \overset{\text{def}}{=} \frac{1}{2\pi} \left(\frac{\lambda \alpha D}{I}\right)^2$.
with $I = D - M/R_s$. Under our assumptions, $R$ conditional on signal realization $s$ follows a normal distribution

$$N\left(\frac{\alpha \bar{R} + \beta s}{\alpha + \beta}, \frac{1}{\alpha + \beta}\right).$$

Denoting by $\Phi$ the c.d.f. of a standard normal distribution, it follows that

$$\varphi(s) = P(s, s) = \Pr[R < R_F(s)|s] = \Phi\left[\sqrt{\alpha + \beta} R_F(s) - \frac{\alpha \bar{R} + \beta s}{\sqrt{\alpha + \beta}}\right]. \quad (8)$$

This function is clearly decreasing for $s < t_0$ since, in this region, we have $R_F(s) \equiv R_s$. Now, if $s > t_0$ then $R_F(s)$ is increasing and its inverse is given by

$$t_F(R) = R + \frac{1}{\sqrt{\beta}} \Phi^{-1}\left(\frac{I}{\lambda D} (R - R_s) + m\right).$$

The derivative of $t_F$ is

$$t'_F(R) = 1 + \frac{1}{\sqrt{\beta}} \frac{I}{\lambda D} \left[\Phi'\left(\Phi^{-1}\left(\frac{I}{\lambda D} (R - R_s) + m\right)\right)\right]^{-1}. \quad (9)$$

Since $\Phi'$ is bounded above by $1/\sqrt{2\pi}$, it follows that $t'_F$ is bounded below:

$$t'_F(R) \geq 1 + \sqrt{\frac{2\pi}{\sqrt{\beta}} \frac{I}{\lambda D}}.$$

Thus

$$R'_F(s) \leq \left[1 + \sqrt{\frac{2\pi}{\sqrt{\beta}} \frac{I}{\lambda D}}\right]^{-1}. \quad (10)$$

Given formula (8), $\varphi(s)$ will be decreasing provided that

$$\sqrt{\alpha + \beta} \left(1 + \sqrt{\frac{2\pi}{\sqrt{\beta}} \frac{I}{\lambda D}}\right)^{-1} \leq \frac{\beta}{\sqrt{\alpha + \beta}},$$

which (after simplification) yields:

$$\beta \geq \frac{1}{2\pi} \left(\frac{\lambda \alpha D}{I}\right)^2.$$

If this condition is satisfied then there is at most one equilibrium. Existence is easily shown. When $s$ is small, $R_F(s) = R_s$ and formula (8) implies that
lim_{s \to -\infty} \varphi(s) = 1. On the other hand, if \( s \to +\infty \) then \( R_F(s) \to (1 + \lambda) R_s \) and \( \varphi(s) \to 0. \)

The limit equilibrium when \( \beta \) tends to infinity can be characterized as follows:

From equation (6) we have that \( \lim_{s \to +\infty} \sqrt{\beta}(R^* - t^*) = \Phi^{-1}(\gamma). \) Given that \( \Phi[-z] = 1 - \Phi[z], \) we obtain from formula (7) that, in the limit,

\[
t^* = R^* = R_s \left( 1 + \frac{\lambda}{1 - m} \max\{1 - \gamma - m, 0\} \right).
\]

The critical cutoff \( R^* \) is decreasing with \( \gamma \) and ranges from \( R_s \) for \( \gamma \geq 1 - m \) to \( (1 + \lambda) R_s \) for \( \gamma = 0. \) It is also nonincreasing in \( m. \) As we establish in the next section, these features of the limit equilibrium are also valid for \( \beta \geq \beta_0. \)

It is worth noting also that, with a diffuse prior \( (\alpha = 0 \) \), the equilibrium is unique for any private precision of investors (indeed, we have that \( \beta_0 = 0. \) From (6) and (7) we obtain immediately that

\[
R^* = R_s \left( 1 + \frac{\lambda}{1 - m} \max\{1 - \gamma - m, 0\} \right)
\]

and \( t^* = R^* - \Phi^{-1}(\gamma)/\sqrt{\beta}. \) Both the cases \( \beta \to +\infty \) and \( \alpha = 0 \) have in common that each investor faces maximal uncertainty about the behavior of other investors at the switching point \( s_i = t^*. \) Indeed, it can be easily checked that in either case the distribution of the proportion \( x(R, t^*) = \Phi(\sqrt{\beta}(t^* - R)) \) of investors withdrawing is uniformly distributed over \([0, 1]\) conditional on \( s_i = t^*. \) This contrasts with the certainty case with multiple equilibria when \( R \in (R_s, (1 + \lambda) R_s) \), where (for example) in a run equilibrium an investor thinks that with probability one all other investors will withdraw. It is precisely the need to entertain a wider range of behavior of other investors in the incomplete information game that pins down a unique equilibrium, as in Carlsson and Van Damme (1993) or Postlewaite and Vives (1987).

The analysis could be easily extended to allow for fund managers to have access to a public signal \( v = R + \eta, \) where \( \eta \sim N(0, 1/\beta_p) \) is independent from \( R \) and from the error terms \( \varepsilon_i \) of the private signals. The only impact of the public signal is to replace the unconditional moments \( \overline{R} \) and \( 1/\alpha \) of \( R \) by its conditional moments, taking into account the public signal, \( v. \) A disclosure of a signal of high enough precision will imply the existence of multiple equilibria—much in the same manner as a sufficiently precise prior.

The public signal could be provided by the central bank. Indeed, the central bank typically has information about banks that the market does not have
and, conversely, market participants also have information that is unknown to the central bank.\(^2\) The model allows for the information structures of the central bank and investors to be nonnested. Our discussion then has a bearing on the slippery issue of the optimal degree of transparency of central bank announcements. Indeed, Alan Greenspan has become famous for his oblique way of saying things, fostering an industry of “Greenspanology” or interpretation of his statements. Our model may rationalize oblique statements by central bankers that seem to add noise to a basic message. Precisely because the central bank may be in a unique position to provide information that becomes common knowledge, it has the capacity to destabilize expectations in the market (which in our context means to move the interbank market to a regime of multiple equilibria). By fudging the disclosure of information, the central bank makes sure that somewhat different interpretations of the release will be made, preventing destabilization.\(^2\) Indeed, in the initial game (without a public signal) we may well be in the uniqueness region, but adding a precise enough public signal will mean we have three equilibria. At the interior equilibrium we have a result similar to that with no public information, but run and no-run equilibria also exist. We may therefore end up in an “always run” situation when disclosing (or increasing the precision of) the public signal while the economy is in the interior equilibrium without public disclosure. In other words, public disclosure of a precise enough signal may be destabilizing. This means that a central bank that wants to avoid entering in the “unstable” region may have to add noise to its signal if that signal is otherwise too precise.\(^2\)

5. Coordination Failure and Prudential Regulation

For \(\beta\) large enough, we have just seen that there exists a unique equilibrium whereby investors adopt a threshold \(t^*\) characterized by

\[
\Phi \left( \sqrt{\alpha + \beta R_F(t^*)} - \frac{\alpha \hat{R} + \beta t^*}{\sqrt{\alpha + \beta}} \right) = \gamma,
\]

or

\[
R_F(t^*) = \frac{1}{\sqrt{\alpha + \beta}} \left( \Phi^{-1}(\gamma) + \frac{\alpha \hat{R} + \beta t^*}{\sqrt{\alpha + \beta}} \right).
\]

For this equilibrium threshold, the failure of the bank will occur if and only if

\[
R < R_F(t^*) = R^*.
\]

\(^2\) See Peek, Rosengren, and Tootell (1999), DeYoung et al. (1998), and Berger, Davies, and Flannery (2000).

\(^2\) The potential damaging effects of public information is a theme also developed in Morris and Shin (2001).

This means that the bank fails if and only if fundamentals are weak, $R < R^*$. When $R^* > R_s$ we have an intermediate interval of fundamentals $R \in [R_s, R^*)$ where there is a coordination failure: the bank is solvent but illiquid. The occurrence of a coordination failure can be controlled by the level of the liquidity ratio $m$, as the following proposition shows.

**Proposition 2.** There is a critical liquidity ratio $\bar{m}$ of the bank such that, for $m \geq \bar{m}$, we have $R^* = R_s$; this means that only insolvent banks fail (there is no coordination failure). Conversely, for $m < \bar{m}$ we have $R^* > R_s$; this means that, for $R \in [R_s, R^*)$, the bank is solvent but illiquid (there is a coordination failure).

**Proof.** For $t^* \leq t_0 = R_s + 1/(\sqrt{\beta})\Phi^{-1}(m)$, the equilibrium occurs for $R^* = R_s$. By replacing in formula (6) we obtain

$$(\alpha + \beta)R_s \leq \sqrt{\alpha + \beta} \Phi^{-1}(\gamma) + \alpha \tilde{R} + \beta R_s + \sqrt{\beta} \Phi^{-1}(m),$$

which is equivalent to:

$$\Phi^{-1}(m) \geq \frac{\alpha}{\sqrt{\beta}}(R_s - \tilde{R}) - \sqrt{1 + \frac{\alpha}{\beta} \Phi^{-1}(\gamma)}.$$

Therefore, the coordination failure disappears when $m \geq \bar{m}$, where

$$\bar{m} = \Phi \left( \frac{\alpha}{\sqrt{\beta}}(R_s - \tilde{R}) - \sqrt{1 + \frac{\alpha}{\beta} \Phi^{-1}(\gamma)} \right).$$

Observe that, since $R_s$ is a decreasing function of $E/I$, the critical liquidity ratio $\bar{m}$ decreases when the solvency ratio $E/I$ increases.\textsuperscript{25}

The equilibrium threshold return $R^*$ is determined (when (10) is not satisfied) by the solution to

$$\phi(R) \equiv \alpha(R - \tilde{R}) - \sqrt{\beta} \Phi^{-1}\left( \frac{1 - m}{\lambda R_s} (R - R_s) + m \right) - \sqrt{\alpha + \beta} \Phi^{-1}(\gamma) = 0.$$  

When $\beta \geq \beta_0$ we have $\phi'(R) < 0$ and the comparative statics properties of the equilibrium threshold $R^*$ are straightforward. Indeed, it follows that $\partial \phi / \partial m < 0$, $\partial \phi / \partial R_s > 0$, $\partial \phi / \partial \lambda > 0$, $\partial \phi / \partial \gamma < 0$, and $\partial \phi / \partial \tilde{R} < 0$. The following proposition states the results.

\textsuperscript{25} More generally, it is easy to see that the regulator in our model can control the probabilities of illiquidity ($\Pr(R < R^*)$) and insolvency ($\Pr(R < R_s)$) of the bank by imposing appropriately high ratios of minimum liquidity and solvency.
PROP 3. The comparative statics of $R^*$ (and of the probability of failure) can be summarized as follows:

1. $R^*$ is a decreasing function of the liquidity ratio $m$ and the solvency $(E/I)$ of the bank, of the critical withdrawal probability $\gamma$, and of the expected return on the bank's assets $\bar{R}$.
2. $R^*$ is an increasing function of the fire-sale premium $\lambda$ and of the face value of debt $D$.

We have thus that stronger fundamentals, as indicated by a higher prior mean $\bar{R}$, also imply a lower likelihood of failure. In contrast, a higher fire-sale premium $\lambda$ increases the incidence of failure. Indeed, for a higher $\lambda$, a larger portion of the portfolio must be liquidated in order to meet the requirements of withdrawals. We also have that $R^*$ is decreasing with the critical withdrawal probability $\gamma$ and that $R^* \rightarrow (1 + \lambda)\bar{R}$ as $\gamma \rightarrow 0$.

A similar analysis applies to changes in the precision of the prior $\alpha$ and the private information of investors $\beta$. Assume that $\gamma = C/B < 1/2$. Indeed, we should expect that the cost $C$ of withdrawal is small in relation to the continuation benefit $B$ for the fund managers. If $\gamma < 1/2$ then it is easy to see that:

1. For large $\beta$ and bad prior fundamentals ($\bar{R}$ low), increasing $\alpha$ increases $R^*$ (more precise prior information about a bad outcome worsens the coordination problem).
2. Increasing $\beta$ decreases $R^*$.

6. Coordination Failure and LOLR Policy

The main contribution of our paper so far has been to show the theoretical possibility of a solvent bank being illiquid as a result of coordination failure on the interbank market. We shall now explore the Lender of Last Resort (LOLR) policy of the central bank and present a scenario where it is possible to give a theoretical justification for Bagehot’s doctrine.

We start by considering a simple central bank objective: eliminate the coordination failure with minimal involvement. The instruments at the disposal of the central bank are the liquidity ratio $m$ and intervention in the form of open-market or discount-window operations.26

We have shown in Section 5 that a high enough liquidity ratio $m$ eliminates the coordination failure altogether by inducing $R^* = \bar{R}$. This is so for $m \geq \bar{m}$. However, it is likely that imposing $m \geq \bar{m}$ might be too costly in terms of

26. Open-market operations typically involve performing a repo operation with primary security dealers. The Federal Reserve auctions a fixed amount of liquidity (reserves) and, in general, does not accept bids by dealers below the Federal Funds rate target.
foregone returns (recall that $I + M = 1 + E$, where $I$ is the investment in the risky asset). In Section 7 we analyze a more elaborate welfare-oriented objective and endogenize the choice of $m$. We look now at forms of central bank intervention that can eliminate the coordination failure when $m < m$.

Let us see how central bank liquidity support can eliminate the coordination failure. Suppose the central bank announces it will lend at rate $r \in (0, \lambda)$—and without limits—but only to solvent banks. The central bank is not allowed to subsidize banks and is assumed to observe $R$. The knowledge of $R$ may come from the supervisory knowledge of the central bank or perhaps by observing the amount of withdrawals of the bank. Then the optimal strategy of a (solvent) commercial bank will be to borrow exactly the liquidity it needs, that is, $D[x - m]_+$. Whenever $x - m > 0$, failure will occur at Date 2 if and only if

$$\frac{RI}{D} < (1 - x) + (1 + r)(x - m).$$

Given that $D/I = R_s/1 - m$, we obtain that failure at $t = 2$ will occur if and only if

$$R < R_s \left(1 + r \frac{[x - m]_+}{1 - m}\right).$$

This is exactly analogous to our previous formula giving the critical return of the bank, except here the interest rate $r$ replaces the liquidation premium $\lambda$. As a result, this type of intervention will be fully effective (yielding $R^* = R_s$) only when $r$ is arbitrarily close to zero. It is worth remarking that central bank help in the amount $D[x - m]_+$ whenever the bank is solvent ($R > R_s$), and at a very low rate, avoids early closure, and the central bank loses no money because the loan can be repaid at $\tau = 2$. Note also that, whenever the central bank lends at a very low rate, the collateral of the bank is evaluated under “normal circumstances”—that is, as if there were no coordination failure. Consider as an example the limit case of $\beta$ tending to infinity. The equilibrium with no central bank help is then

$$t^* = R^* = R_s \left(1 + \frac{\lambda}{1 - m} [\max\{1 - \gamma - m, 0\}]\right).$$

Suppose that $1 - \gamma > m$ so that $R^* > R_s$. Then withdrawals are $x = 0$ for $R > R^*$, $x = 1 - \gamma$ for $R = R^*$, and $x = 1$ for $R < R^*$. Whenever $R > R_s$, the central bank will help to avoid failure and will evaluate the collateral as if $x = 0$. This effectively changes the failure point to $R^* = R_s$.

Central bank intervention can take the form of open-market operations that reduce the fire-sale premium or of discount-window lending at a very low rate. The intervention with open-market operations makes sense if a high $\lambda$ is due to a temporary spike of the market rate (i.e., a liquidity crunch). In this situation, a liquidity injection by the central bank will reduce the fire-sale premium.
After September 11, for example, open-market operations by the Federal Reserve accepted dealers’ bids at levels well below the Federal Funds Rate target and pushed the effective lending rate to lows of zero in several days.27

Intervention via the discount window—perhaps more in the spirit of Bagehot—makes sense when $\lambda$ is interpreted as an adverse selection premium. The situation when a large number of banks is in trouble displays both liquidity and adverse selection components. In any case, the central bank intervention should be a very low rate, in contrast with Bagehot’s doctrine of lending at a penalty rate.28 This type of intervention may provide a rationale for the Fed’s apparently strange behavior of lending below the market rate (but with a “stigma” associated to it, so that banks borrow there only when they cannot find liquidity in the market).29 In Section 7 we provide a welfare objective for this discount-window policy.

In some circumstances the central bank may not be able to infer $R$ exactly because of noise (in the supervisory process or in the observation of withdrawals). Then the central bank will obtain only an imperfect signal of $R$. In this case, the central bank will not be able to distinguish perfectly between illiquid and insolvent banks (as in Goodhart and Huang 1999) and so, whatever the lending policy chosen, taxpayers’ money may be involved with some probability. This situation is realistic given the difficulty in distinguishing between solvency and liquidity problems.30

It may be argued also that our LOLR function could be performed by private banks through credit lines. Banks that provide a line of credit to another bank would then have an incentive to monitor the borrowing institution and reduce the fire-sale premium. The need for a LOLR remains, but it may be privately provided. Goodfriend and Lacker (1999) draw a parallel between central bank lending and private lines of credit, putting emphasis on the commitment problem.

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27. See Markets Group of the Federal Reserve Bank of New York (2002). Martin (2002) contrasts the classical prescription of lending at a penalty rate with the Fed’s response to September 11—namely, to lend at a very low interest rate. He argues that penalty rates were needed in Bagehot’s view because the gold standard implied limited reserves for the central bank.

28. Typically, the lending rate is kept at a penalty level to discourage arbitrage and perverse incentives. Those considerations lie outside the present model. For example, in a repo operation the penalty for not returning the cash on loan is to keep paying the lending rate. If this lending rate is very low then the incentive to return the loan is small. See Fischer (1999) for a discussion of why lending should be at a penalty rate.

29. The discount-window policy of the Federal Reserve is to lend at 50 basis points below the target Federal Funds Rate.

30. We may even think that the central bank cannot help ex post once withdrawals have materialized but that it receives a noisy signal $s_{CB}$ about $R$ at the same time as investors. The central bank then can act preventively and inject liquidity into the bank contingent on the received signal $L(s_{CB})$. In this case, also the risk exists that an insolvent bank ends up being helped. The game played by the fund managers changes, obviously, after liquidity injection by a large actor like the central bank.
of the central bank to limit lending.\footnote{If this commitment problem is acute then the private solution may be superior. However, Goodfriend and Lacker (1999) do not take a position on this issue. They state: “We are agnostic about the ultimate role of CB lending in a welfare-maximizing steady state.”} However, the central bank typically acts as LOLR in most economies, presumably because it has a natural superiority in terms of financial capacity and supervisory knowledge.\footnote{One of the few exceptions is the Liquidity Consortium in Germany, in which private banks and the central bank both participate.} For example, in the LTCM case it may be argued that the New York Fed had access to information that the private sector—even the members of the lifeboat operation—did not. This unique capacity to inspect a financial institution might have made possible the lifeboat operation orchestrated by the New York Fed. An open issue is whether this superior knowledge continues to hold in countries where the supervision of banks is basically in the hands of independent regulators like the Financial Services Authority of the United Kingdom.\footnote{See Vives (2001) for the workings of the Financial Services Authority and its relation with the Bank of England.} 

7. Endogenizing the Liability Structure and Crisis Resolution

In this section we endogenize the short-term debt contract assumed in our model, according to which depositors can withdraw at $\tau = 1$ or otherwise wait until $\tau = 2$. We have seen that the ability of investors to withdraw at $\tau = 1$ creates a coordination problem. We argue here that this potentially inefficient debt structure may be the only way that investors can discipline a bank manager subject to a moral hazard problem.

Suppose, as seems reasonable, that investment in risky assets requires the supervision of a bank manager and that the distribution of returns of the risky assets depends on the effort undertaken by the manager. For example, the manager can either exert or not exert effort, $\varepsilon \in \{0, 1\}$; then $R \sim N(\bar{R}_0, \alpha^{-1})$ when $\varepsilon = 0$, and $R \sim N(\bar{R}, \alpha^{-1})$ when $\varepsilon = 1$, where $\bar{R} > \bar{R}_0$. That is, exerting effort yields a return distribution that first-order stochastically dominates the one obtained by not exerting effort. The bank manager incurs a cost if he chooses $\varepsilon = 1$; if he chooses $\varepsilon = 0$, the cost is 0. The manager also receives a benefit from continuing the project until Date 2. Assume for simplicity that the manager does not care about monetary incentives. The manager’s effort cannot be observed, so his willingness to undertake effort will depend on the relationship between his effort and the probability that the bank continues at Date 1. Thus, withdrawals may enforce the early closure of the bank and so provide incentives to the bank manager.\footnote{This approach is based on Grossman and Hart (1982) and is followed in Gale and Vives (2002). See also Calomiris and Kahn (1991) and Carletti (1999).}
In the banking contract, short-term debt/demandable deposits can improve upon long-term debt/nondemandable deposits. With long-term debt, incentives cannot be provided to the manager because liquidation never occurs; therefore, the manager does not exert effort. Furthermore, neither can incentives be provided with renegotiable short-term debt, because early liquidation is ex post inefficient. Dispersed short-term debt (i.e., uninsured deposits) is what is needed.

Let us assume that it is worthwhile to induce the manager to exert effort. This will be true if \( \mathcal{R} - R_0 \) is large enough and the (physical) cost of asset liquidation is not too large. Recall that the (per-unit) liquidation value of its assets is \( \nu R \), with \( \nu \ll 1/(1 + \lambda) \), whenever the bank is closed at \( \tau = 1 \). We assume, as in previous sections, that the face value of the debt contract is the same in periods \( \tau = 1, 2 \) (equal to \( D \)), and we suppose also that investors—in order to trust their money to fund managers—must be guaranteed a minimum expected return, which we set equal to zero without loss of generality.

The banking contract will have short-term debt and will maximize the expected profits of the bank by choosing to invest in risky and safe assets and deposit returns subject to: the resource constraint \( 1 + E = I + Dm \) (where \( Dm = M \) is the amount of liquid reserves held by the bank); the incentive compatibility constraint of the bank manager; and the (early) closure rule associated with the (unique) equilibrium in the investors’ game. This early closure rule is defined by the property \( x(R, t^*)D > M + IR/(1 + \lambda) \), which is satisfied if and only if \( R < R_{EC}(t^*) \). As stated before, \( R_{EC}(t^*) < R^* \), because early closure implies failure whereas the converse is not true. Let \( R^o \) be the smallest \( R \) that fulfills the incentive compatibility constraint of the bank manager. We thus have \( R_{EC}(t^*) \geq R^o \). The banking program will maximize the expected value of the bank assets which consists of two terms: (i) the product of the size \( I = 1 + E - Dm \) of the bank’s investments by the net expected return on these investments, taking into account expected liquidation costs; and (ii) the value of liquid reserves \( Dm \). Hence the optimal banking contract will solve

\[
\max_m \{ (1 + E - Dm)(\mathcal{R} - (1 - \nu)E(R \mid R < R_{EC}(t^*(m))) \times \Pr(R < R_{EC}(t^*(m))) + Dm) \}
\]

subject to:

1. \( t^*(m) \) is the unique equilibrium of the fund managers’ game; and
2. \( R_{EC}(t^*(m)) \geq R^o \).

Given that \( t^*(m) \), and thus \( R_{EC}(t^*(m)) \), decrease with \( m \), the optimal banking contract is easy to characterize. If the net return on banks’ assets is always larger than the opportunity cost of liquidity (even when the banks have no liquidity at all)—that is, when

\[
\mathcal{R} - (1 - \nu)E(R \mid R < R_{EC}(t^*(0)))\Pr(R < R_{EC}(t^*(0))) > 1
\]
then it is clear that $m = 0$ at the optimal point. If, on the contrary,
\[ \overline{R} - (1 - \nu)E(R \mid R < R_{EC}(t^*(0)))\Pr(R < R_{EC}(t^*(0))) < 1, \]
then there is an interior optimum. An interesting question is how the banking contract compares with the incentive efficient solution, which we now describe.

Given that the pooled signals of investors reveal $R$, we can define the incentive-efficient solution as the choice of investment in liquid and risky assets and probability of continuation at $\tau = 1$ (as a function of $R$) that maximizes expected surplus subject to the resource constraint and the incentive compatibility constraint of the bank manager.\footnote{We disregard here the welfare of the bank manager and that of the funds’ managers.} Furthermore, given the monotonicity of the likelihood ratio $\phi(R|e = 0)/\phi(R|e = 1)$, the optimal region of continuation is of the cutoff form. More specifically, the optimal cutoff will be $R^o$, the smallest $R$ that fulfills the incentive compatibility constraint of the bank manager. The cutoff $R^o$ will be (weakly) increasing with the extent of the moral hazard problem that bank managers face.

The incentive-efficient solution solves
\[ \max_m \{(1 + E - Dm)(\overline{R} - (1 - \nu)E(R \mid R < R^o))\Pr(R < R^o) + Dm\}, \]
where $R^o$ is the minimal return cutoff that motivates the bank manager. If
\[ \overline{R} - (1 - \nu)E(R \mid R < R^o)\Pr(R < R^o) > 1 \]
then $m^o = 0$. Thus, at the incentive-efficient solution it is optimal not to hold any reserves. This should come as no surprise, since we assume there is no cost of liquidity provision by the central bank. A more complete analysis would include such a cost and lead to an optimal combination of LOLR policy with ex ante regulation of a minimum liquidity ratio.

Since $R_{EC}(t^*)$ must also fulfill the incentive compatibility constraint of the bank manager, it follows that, at the optimal banking contract with no LOLR, $R_{EC}(t^*) \geq R^o$. In fact, we will typically have a strict inequality, because there is no reason for the equilibrium threshold $t^*$ to satisfy $R_{EC}(t^*) = R^o$. This means that the market solution will entail too many early closures of banks, since the banking contract with no LOLR intervention uses an inefficient instrument (the liquidity ratio) to provide indirect incentives for bankers through the threat of early liquidation.

The role of a modified LOLR can be viewed, in this context, as correcting these market inefficiencies while maintaining the incentives of bank managers. By announcing its commitment to provide liquidity assistance (at a zero rate) in order to avoid inefficient liquidation at $\tau = 1$ (i.e., for $R > R^o$), the LOLR
can implement the incentive-efficient solution. When offered help, the bank will borrow the liquidity it needs, $D[x - m]_+$.\(^3\)

To implement the incentive-efficient solution, the modified LOLR must be more concerned with avoiding inefficient liquidation at $\tau = 1$ in the range $(R^o, R_{EC})$ than about avoiding failure of the bank. Now the solvency threshold $R_s$ has no special meaning. Indeed, $R^o$ will typically be different from $R_s$. The reason is that $R_s$ is determined by the promised payments to investors, cash reserves, and investment in the risky asset, whereas $R^o$ is just the minimum threshold that motivates the banker to behave. We will have that $R^o > R_s$ when the moral hazard problem for bank managers is severe and $R^o < R_s$ when it is moderate.

This modified LOLR facility leads to a view on the LOLR that differs from Bagehot’s doctrine and introduces interesting policy questions. Whenever $R^o > R_s$ there is a region (specifically, for $R$ in $(R_s, R^o)$) where there should be early intervention (or “prompt corrective action”, to use the terminology of banking regulators). Indeed, in this region the bank is solvent but intervention is needed to control moral hazard of the banker. On the other hand, in the range $(R^o, R_{EC})$ a LOLR policy is efficient if the central bank can commit. If it cannot and instead optimizes ex post (whether because building a reputation is not possible or because of weakness in the presence of lobbying), it will intervene too often. Some additional institutional arrangement is needed in the range $(R_s, R^o)$ in order to implement prompt corrective action (i.e., early closure of banks that are still solvent).

When $R^o < R_s$, there is a range $(R^o, R_{EC})$ where the bank should be helped even though it might be insolvent (and in this case money is lost). More precisely, for $R$ in the range $(R^o, \min\{R_s, R_{EC}\})$, the bank is insolvent and should be helped. If the central bank’s charter specifies that it cannot lend to insolvent banks, then another institution (deposit insurance fund, regulatory agency, treasury) financed by other means (insurance premiums or taxation) is needed to provide an “orderly resolution of failure” when $R$ is in the range $(R^o, \min\{R_s, R_{EC}\})$. This could be interpreted, as in corporate bankruptcy practice, as a way to preserve the going-concern value of the institution and to allow its owners and managers a fresh start after the crisis.

An important implication of our analysis is the complementarity between bail-ins (interbank market) and bailouts (LOLR) as well as other regulatory

\(^3\) We could also envision help by the central bank in an ongoing crisis to implement the incentive-efficient closure rule. The central bank would then lend at a very low interest rate to illiquid banks for the amount that they could not borrow in the interbank market in order to meet their payment obligations at $\tau = 1$. It is easy to see that in this case the equilibrium between fund managers is not modified. This is so because central bank intervention does not change the instances of failure of the bank (indeed, when a bank is helped at $\tau = 1$ because $s(R, t^\ast)D > M + IR/(1 + \lambda)$, it will fail at $\tau = 2$). In this case the coordination failure is not eliminated, but its effects (on early closure) are neutralized by the intervention of the central bank. The modified LOLR helps the bank in the range $(R^o, R_{EC}(t^\ast))$ in the amount $Dx(t^\ast, R) - (M + IR/(1 + \lambda)) > 0$. Thus LOLR help (bailout) complements the money raised in the interbank market $IR/(1 + \lambda)$(bail-in).
facilities (prompt corrective action, orderly resolution of failure) in crisis management. We can summarize by comparing different organizations as follows.

1. With neither a LOLR nor an interbank market, liquidation takes place whenever \( x > m D \), which inefficiently limits investment \( I \).

2. With an interbank market but no LOLR (as advocated by Goodfriend and King), the closure threshold is \( R_{EC} \) and there is excessive failure whenever \( R_{EC} > R^o \).

3. With both a LOLR facility and an interbank market:
   
   (i) If \( R^o > R_s \) (severe moral hazard problem for the banker) then the incentive-efficient solution can be implemented, complementing the LOLR with a policy of prompt corrective action in the range \((R_s, R^o)\);

   (ii) If \( R^o < R_s \) (moderate moral hazard problem for the banker), then a different institution (financed by taxation or by insurance premiums) is needed to complement the central bank and implement the incentive-efficient solution. The central bank helps whenever the bank is solvent, and the other institution provides an “orderly resolution of failure” in the range \((R^o, \min\{R_s, R_{EC}\})\).

8. An International Lender of Last Resort (ILOLR)

In this section we reinterpret the model in an international setting and provide a potential rationale for an international lender of last resort (ILOLR) à la Bagehot. Financial and banking crises, usually coupled with currency crises, have been common in emerging economies in Asia (Thailand, Indonesia, Korea), Latin America (Mexico, Brazil, Ecuador, Argentina) and in the periphery of Europe (Turkey). These crisis have proved costly in terms of output. The question is whether an ILOLR can help alleviate, or even avoid, such crises. An ILOLR could follow a policy of injecting liquidity in international financial markets—by actions that range from establishing the proposed global central bank that issues an international currency to merely coordinating the intervention of the three major central banks— or could act to help countries in trouble, much like a central bank acts to help individual banking institutions. The latter approach is developed in several proposals that adapt Bagehot’s doctrine to international lending; see, for example, the Meltzer Report (IFIAC 2000) and Fischer (1999). As pointed out by Jeanne and Wyplosz (2003), a major difference between the approaches concerns the required size of the ILOLR. The former (global CB) approach requires an issuer of international currency; in the latter, the intervention is bounded by the difference between the short-term foreign exchange liabilities of the banking sector and the foreign reserves of the country in question. We will look here at

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See Eichengreen (1999) for a survey of the different proposals.
the second approach. The main tension identified in the debate is between those who emphasize the effect of liquidity support on crisis prevention (Fischer 1999) and those who are worried about generating moral hazard in the country being helped (IFIAC 2000).

8.1. A Reinterpretation of the Model

Suppose now that the balance sheet of Section 2 corresponds to a small open economy for which $D_0$ is the foreign-denominated short-term debt, $M$ is the amount of foreign reserves, $I$ is the investment in risky local entrepreneurial projects, $E$, equity and long-term debt (or local resources available for investment), and $D$ is the face value of the foreign-denominated short-term debt.\(^{38}\) Our fund managers are now international fund managers operating in the international interbank market. The liquidity ratio $m = M/D$ is now the ratio of foreign reserves to foreign short-term debt—a crucial ratio, according to empirical work, in determining the probability of a crisis in the country.\(^{39}\) The parameter $\lambda$ now represents the fire-sale premium associated with early sales of domestic bank assets in the secondary market. Furthermore, for a given amount of withdrawals by fund managers $x > m$ at $\tau = 1$, there are critical thresholds for the return $R$ of investment below which the country is bankrupt ($R_f(x)$) or will default at $\tau = 1$ ($R_{cf}(x) < R_f(x)$). The effort $e$ necessary to improve returns could be understood to be exerted by bank managers, entrepreneurs, or even the government. According to Section 7, effort has a cost and the actors exerting effort are interested in continuing in their job. Default by the country at $\tau = 1$ deprives those actors from their continuation benefits (for example, because of restructuring of the banking and/or private sectors or because the government is removed from office), and consequently “default” at $\tau = 1$ for some region of realized returns is the only disciplining device.

8.2. Results

(i) There is a range of realizations of the return $R$, $(R_s, R^*)$ for which a coordination failure occurs. This happens when the amount of withdrawals by foreign fund managers is so large that the country is bankrupt even though it is (in principle) solvent.

(ii) For a high enough foreign reserve ratio $m$, the range $(R_s, R^*)$ collapses and there is no coordination failure of international investors.

\(^{38}\) The balance sheet corresponds to the consolidated private sector of the country. In some countries, local firms borrow from local banks and then the latter borrow in international currency.

\(^{39}\) Indeed, Radelet and Sachs (1998) as well as Rodrik and Velasco (1999) find that the ratio of short-term debt to reserves is a robust predictor of financial crisis (in the sense of a sharp reversal of capital flows). The latter also find that a greater short-term exposure aggravates the crisis once capital flows reverse.
(iii) The probability of bankruptcy of the banking sector is:

- decreasing in the foreign reserve ratio, the solvency ratio, the critical withdrawal probability $\gamma$, and the expected mean return of the country investment;
- increasing in the fire-sale premium and the face value of foreign short-term debt; and
- increasing in the precision of public information about $R$ when public news is bad and decreasing in the precision of private information (both provided $\gamma < 1/2$).

(iv) An ILOLR that follows Bagehot’s prescription can minimize the incidence of coordination failure among international fund managers—provided it is well informed about $R$. One possibility is that the ILOLR performs in-depth country research and has supervisory knowledge of the banking system of the country where the crisis occurs. 40

(v) The disclosure of a public signal about country return prospects may introduce multiple equilibria. A well-informed international agency may want to be cautious and not publicly disclose too precise information in order to avoid a rally of expectations in a run equilibrium.

(vi) In the presence of the moral hazard problem associated with eliciting high returns, foreign short-term debt serves the purpose of disciplining whoever must exert effort to improve returns. Note that domestic currency–denominated short-term debt will not have a disciplining effect because it can be inflated away. There will be an optimal cutoff point $R^\circ$ below which restructuring (of either the private sector or government) must occur in order to provide incentives to exert effort.

The following scenarios can be considered.

No Bail-In and No Bailout. With no ILOLR and no access to the international interbank market, country projects are liquidated whenever withdrawals by foreign fund managers are larger than foreign reserves. This inefficiently limits investment.

Bail-In but No Bailout. With no ILOLR but with access to the international interbank market, some costly project liquidation is avoided by having fire sales of assets, but still there will be excessive liquidation of entrepreneurial projects.

Bail-In and Bailout. With ILOLR and access to the international interbank market, we have two cases as follows:

- The moral hazard problem in the country is severe ($R^\circ > R_s$). In this case a policy of prompt corrective action in the range $(R_s, R^\circ)$ is needed to

40. Although this seems more far-fetched than in the case of a domestic LOLR, the IMF (for example) is trying to enhance its monitoring capabilities by way of “financial sector assessment” programs.
complement the ILOLR facility. A solvent country may need to ‘restructure’ when returns are close to the solvency threshold.

- The moral hazard problem in the country is moderate ($R^o < R_s$). Then, in addition to the ILOLR help for a solvent country, an orderly “resolution of failure” process is needed in the range ($R^O, \min\{R_s, R_{EC}\}$). An insolvent country should be helped when it is not too far away from the solvency threshold. This may be interpreted as a mechanism similar to the sovereign debt restructuring mechanism (SDRM) of the sort currently studied by the IMF with the objective of restructuring unsustainable debt.\footnote{See Bolton (2003) for a discussion of SDRM-type facilities from the perspective of corporate bankruptcy theory and practice.} In our case, this would be the foreign short-term debt. In the range ($R^o, \min\{R_s, R_{EC}\}$), an institution like an international bankruptcy court could help.

As before, an important insight from the analysis is the complementarity between the market (bail-ins) and an ILOLR facility (bailout)—together with other regulatory facilities—can provide for prompt corrective action and orderly failure resolution. Our conclusion is that an ILOLR facility à la Bagehot can help to implement the incentive-efficient solution, provided that it is complemented with provisions of prompt corrective action and orderly resolution of failure.

9. Concluding Remarks

In this paper we have provided a rationale—in the context of modern interbank markets—for Bagehot’s doctrine of helping illiquid but solvent banks. Indeed, investors in the interbank market may face a coordination failure and so intervention may be desirable. We have examined the impact of public intervention along the following three dimensions:

(i) solvency and liquidity requirements (at $\tau = 0$);
(ii) lender-of-last-resort policy (at the interim date $\tau = 1$); and
(iii) closure rules, which can consist of two types of policy: prompt corrective action or the orderly resolution of bank failures.

The coordination failure can be avoided by appropriate solvency and liquidity requirements. However, the cost of doing so will typically be too large in terms of foregone returns, and ex ante measures will only help partially. This means that prudential regulation needs to be complemented by a LOLR policy. The paper shows how discount-window loans can eliminate the coordination failure (or alleviate it, if for incentive reasons some degree of coordination failure is optimal). It also sheds light on when open-market operations will be appropriate.

A main insight of the analysis is that public and private involvement are both necessary in implementing the incentive-efficient solution. Furthermore,
implementation of this solution may also require complementing Bagehot’s LOLR facility with prompt corrective action (intervention on a solvent bank) or orderly failure resolution (help to an insolvent bank).

The model, when given an interpretation in an international context, provides a rationale for an international LOLR à la Bagehot, complemented with prompt corrective action and provisions for orderly resolution of failures, and it points to the complementarity between bail-ins and bailouts in crisis resolution.

References


