Rand Journal of Economics
Vol. 15, No. 4, Winter 1984

Price and quantity competition in a differentiated duopoly

Nirvikar Singh*

and

Xavier Vives**

This article analyzes the duality of prices and quantities in a differentiated duopoly. It is shown that if firms can only make two types of binding contracts with consumers, the price contract and the quantity contract, it is a dominant strategy for each firm to choose the quantity (price) contract, provided the goods are substitutes (complements).

1. Introduction

■ Two classical models in the theory of oligopoly are those of Cournot (1838) and Bertrand (1883). In both models the equilibrium concept is the noncooperative equilibrium of Nash (1950). In the former firms set quantities. In the latter prices are the strategy variables. In a duopoly situation where firms produce a homogeneous good and marginal costs are constant and equal for both firms, the Bertrand price equals marginal cost and the Cournot price is above it. With differentiated products, Bertrand prices are above marginal cost. In this case Cournot competition is still viewed as more "monopolistic" than Bertrand competition.[1]

We consider first a differentiated duopoly proposed by Dixit (1979). The demand structure is linear and allows the goods to be substitutes or complements. Firms have constant marginal costs and there are no fixed costs and no capacity limits. In this setup Cournot and Bertrand equilibria are unique. We show that Bertrand competition is more efficient than Cournot competition, in the sense that in equilibrium consumer surplus and total surplus are higher in the former regardless of whether the goods are substitutes or complements. Furthermore, profits are larger, equal, or smaller in Cournot than in Bertrand competition, according to whether the goods are substitutes, independent, or complements.

* University of California, Santa Cruz.
** University of Pennsylvania.

We are indebted to Eddie Dekel, Andreu Mas-Colell, an anonymous referee, and the Editorial Board for helpful comments. Nirvikar Singh acknowledges financial support from the Faculty Research Committee at the University of California, Santa Cruz.

[1] This is not the case if one considers supergame equilibria. Price-setting supergame equilibria may support higher prices than quantity-setting equilibria for either homogeneous or differentiated products. See Brock and Scheinkman (1981) and Deneckere (1983).

Suppose now that each firm can make only two types of binding contracts with consumers: the price contract and the quantity contract. If a firm chooses the price contract, this means that it will have to supply the amount the consumers demand at a predetermined price, whatever action the competitor takes. If a firm chooses the quantity contract, it is committed to supply a predetermined quantity independently of the action of the competitor. Consider a two-stage game where firms first simultaneously commit themselves to a type of contract and afterwards compete contingent on the chosen types of contracts. Restricting attention to subgame perfect equilibria of this game, we show that if the goods are substitutes (complements), it is a dominant strategy for firm i to choose the quantity (price) contract. When the goods are substitutes, if firms may commit themselves to offer consumers only a certain type of contract, they will always choose the quantity contract, and this will keep the prices high. When the goods are complements, firms want to keep the quantities produced high to reinforce each other's market, and they will offer the price contract. This result generalizes, under certain assumptions, to a nonlinear demand structure. Furthermore, in the linear case the dominant strategy equilibrium turns out to be Pareto superior from the point of view of the firms, since Cournot (Bertrand) profits are the highest of all when the goods are substitutes (complements).

To get the results, we take advantage of the duality structure of Cournot and Bertrand competition in our differentiated commodity setting. This duality was first pointed out by Sonnenschein (1968) in a nondifferentiated framework. In our setup it turns out that Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. This means that they share similar strategic properties. For example, with linear demand, reaction functions slope downwards (upwards) in both cases. It is a matter of interchanging prices and quantities. A useful corollary is that one only needs to make computations or prove propositions for one type of competition (Cournot or Bertrand) or for one type of product (substitute or complement); the other cases follow by duality.

In Section 2 we present the linear model. Section 3 deals with the welfare properties of Bertrand and Cournot equilibria. The two-stage model is dealt with in Section 4. Section 5 extends the results to a nonlinear demand structure. Concluding remarks follow.

2. The linear model

■ We have an economy with a monopolistic sector with two firms, each one producing a differentiated good, and a competitive *numeraire* sector. There is a continuum of consumers of the same type with a utility function separable and linear in the *numeraire* good. Therefore, there are no income effects on the monopolistic sector, and we can perform partial equilibrium analysis. The representative consumer maximizes $U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i$, where q_i is the amount of good i and p_i its price. U is assumed to be quadratic and strictly concave $U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2)/2$, where α_i and β_i are positive, $i = 1, 2$, $\beta_1\beta_2 - \gamma^2 > 0$, and $\alpha_i\beta_j - \alpha_j\gamma > 0$ for $i \neq j$, $i = 1, 2$. This utility function gives rise to a linear demand structure. Inverse demands are given by

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2$$

$$p_2 = \alpha_2 - \gamma q_1 - \beta_2 q_2$$

in the region of quantity space where prices are positive. Letting $\delta = \beta_1\beta_2 - \gamma^2$, $a_i = (\alpha_i\beta_j - \alpha_j\gamma)/\delta$, $b_i = \beta_j/\delta$ for $i \neq j$, $i = 1, 2$, and $c = \gamma/\delta$ (note that a_i and b_i are positive because of our assumptions), we can write direct demands as

$$q_1 = a_1 - b_1p_1 + cp_2$$

$$q_2 = a_2 + cp_1 - b_2p_2,$$

provided that quantities are positive.[2] The goods are substitutes, independent, or complements according to whether $\gamma \gtreqless 0$. Demand for good i is always downward sloping in its own price and increases (decreases) with increases in the price of the competitor if the goods are substitutes (complements). When $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2 = \gamma$, the goods are perfect substitutes.[3] When $\alpha_1 = \alpha_2$, $\gamma^2/(\beta_1\beta_2)$ expresses the degree of product differentiation, ranging from zero when the goods are independent to one when the goods are perfect substitutes. When γ is positive and $\gamma^2/(\beta_1\beta_2)$ approaches one, we are close to a homogeneous market.

Firms have constant marginal costs, m_1 and m_2. We consider from now on prices net of marginal cost. This is without loss of generality since if marginal costs are positive, we may replace α_i and a_i by $\alpha_i - m_i$ and $a_i - b_im_i + cm_j$, $i \neq j$, $i = 1, 2$, respectively.

Profits of firm i, Π_i, are given by $\Pi_i = p_iq_i$, total surplus, TS, with the quantity pair (q_1, q_2) is just $U(q_1, q_2)$, and consumer surplus, CS, with prices (p_1, p_2) and quantities (q_1, q_2) is $U(q_1, q_2) - (\Pi_1 + \Pi_2)$. Notice that Π_i is symmetric in p_i and q_i.

3. Bertrand and Cournot equilibria

■ In Bertrand competition firms set prices, in Cournot competition, quantities. In both cases the equilibrium concept is the noncooperative Nash equilibrium. We have noticed above that profits are symmetric in prices and quantities. In the Cournot case firm 1 chooses q_1 to maximize $(\alpha_1 - \beta_1q_1 - \gamma q_2)q_1$, taking as given q_2, and in the Bertrand case it chooses p_1 to maximize $p_1(a_1 - b_1p_1 + cp_2)$, taking p_2 as given. Both expressions are perfectly dual. We get one from the other by replacing q_1 by p_1, α_1 by a_1, β_1 by b_1, and γ by $-c$. We have thus that Cournot competition with substitute products ($\gamma > 0$) is the dual of Bertrand competition with complements ($c < 0$) (and similarly for the other possible combination). This means that they are going to share similar strategic properties and that we shall be able to derive the Bertrand reaction functions, equilibrium strategies, and profits from the Cournot ones. It is easily seen that the Cournot reaction of firm 1 to output q_2 is $(\alpha_1 - \gamma q_2)/2\beta_1$, and correspondingly the Bertrand reaction to p_2 is $(a_1 + cp_2)/2b_1$.[4] Note that the reaction functions slope downwards in the Cournot case with substitutes and in the Bertrand case with complements. In any case, under our assumptions, reaction functions intersect only once, thereby yielding a unique equilibrium (Cournot and Bertrand). It is straightforward to compute the Cournot equilibrium. The Bertrand equilibrium follows by duality. Table 1 gives the equilibrium levels of prices and quantities under Cournot, C, and Bertrand, B, competition for firm 1. q_i^C and p_i^B denote, respectively, the Cournot quantity and the Bertrand price of firm i. Let $\Delta = 4\beta_1\beta_2 - \gamma^2$ and $D = 4b_1b_2 - c^2$.

To illustrate, take the symmetric case where $\alpha_i = \alpha$ and $\beta_i = \beta$, $i = 1, 2$ and consider two demand structures. When $\gamma = 0$, the goods are independent, and we have the monopoly solution in any case, $p_i^B = p_i^C = a/2b$ and $q_i^B = q_i^C = \alpha/2\beta$, $i = 1, 2$. If instead $\gamma = \beta > 0$, the goods are perfect substitutes and price equals marginal cost in Bertrand competition, $p_i^B = 0$,[5] and we have the usual Cournot solution $q_i^C = \alpha/3\beta$ if there is quantity competition.

---

[2] That is in the region $\Lambda = \{p \in R_+^2: a_1 - b_1p_1 + cp_2 > 0, a_2 - b_2p_2 + cp_1 > 0\}$.

[3] When $\beta_1 = \beta_2 = -\gamma$ the demand system may not be well defined.

[4] This is the Bertrand reaction function of firm 1, provided q_2 is positive. When the reaction function reaches the boundary of the region Λ where $q_2 = 0$, then $p_1 = (b_2p_2 - a_2)/c$ until p_1 reaches $\alpha_1/2$, the monopoly price for firm 1. For p_2 larger than $(a_2 + c\alpha_1/2)/b_2$, firm 1 charges the monopoly price $\alpha_1/2$.

[5] The Bertrand quantity, q_i^B, equals a. When $\gamma = \beta$, U is not strictly concave, $\beta^2 - \gamma^2 = 0$, and b is infinite. bp^B tends to a as γ tends to β.

TABLE 1 **Equilibrium Levels of Price and Quantity**

	Price	Quantity
Bertrand	$\frac{2a_1b_2 + a_2c}{D}$	$b_1p_1^B$
Cournot	$\beta_1q_1^C$	$\frac{2\alpha_1\beta_2 - \alpha_2\gamma}{\Delta}$.

From Table 1, one gets $p_i^C - p_i^B = \alpha_i\gamma^2/\Delta$ (and similarly $q_i^B - q_i^C = a_ic^2/D$), $i = 1, 2$, which are nonnegative. Quantities are lower and prices higher in Cournot than in Bertrand competition, regardless of whether the goods are substitutes or complements. Cournot competition is more "monopolistic" than Bertrand competition.[6] Firms have less capacity to raise prices above marginal cost in Bertrand competition because the perceived elasticity of demand of a firm when taking the price of the rival as given is larger than that which the same firm perceives when taking the quantity of the rival as given. In the first case the absolute value of the slope of the perceived demand function is b_1 and in the second $b_1 - c^2/b_2$. The result is that in Bertrand competition firms quote lower prices than the Cournot ones. Furthermore, the difference in prices (or quantities) depends on the degree of product differentiation. When $\alpha_1 = \alpha_2$, this is given by $\gamma^2/\beta_1\beta_2$ and

$$p_i^C - p_i^B = \frac{\alpha_1}{4\dfrac{\beta_1\beta_2}{\gamma^2} - 1}$$

so that the more differentiated the products are, the smaller is the difference between the Cournot and Bertrand prices, and in the extreme situation of independent goods the difference is zero. The type of competition becomes less important, the less related the goods are.

Lower prices and higher quantities are always better in welfare terms. Consumer surplus is decreasing and convex as a function of prices, and total surplus equals $U(q_1, q_2)$, which is increasing and concave. Therefore, in terms of consumer surplus or total surplus, the Bertrand equilibrium dominates the Cournot one. For firms, if the goods are substitutes, low prices mean low profits, and Cournot profits are larger than Bertrand profits. The converse is true if the goods are complements since then to increase profits firms have to lower prices from the Cournot levels to gain market share. (See Appendix 1 for a proof of these assertions.) Proposition 1 summarizes the results thus far.[7]

Proposition 1. Consumer surplus and total surplus are larger in Bertrand than in Cournot competition except when the goods are independent, in which case they are equal. Profits are larger, equal, or smaller in Cournot than in Bertrand competition, according to whether the goods are substitutes, independent, or complements.

4. The two-stage game

■ We suppose that firms can make two types of contracts with consumers: the price contract and the quantity contract. If firm i chooses the price contract, this means that it

[6] Shubik (1980, pp. 68–78) showed that quantities are lower and prices are higher in Cournot than in Bertrand competition in a symmetric linear duopoly with substitute goods. Deneckere (1983) considered also the case of complements in a symmetric and linear duopoly model.

[7] See Vives (1984) for an extension of Proposition 1 to an incomplete information setting where firms receive signals about the uncertain demand. Bayesian Cournot and Bertrand equilibria are compared.

will have to supply the amount the consumers demand at a predetermined price, whatever action the competitor, firm j, $j \neq i$, takes. If firm i chooses the quantity contract, it is committed to supplying a predetermined quantity independently of the action of the competitor. That is, we restrict, in Grossman's terminology (1981), the supply curves firms can choose to two types: the vertical one, corresponding to quantity setting, and the horizontal one, corresponding to price setting. Furthermore, we assume that there are prohibitively high costs associated with changing the type of contract. Firms first choose what type of contract to offer consumers, and afterwards they compete contingent on the chosen types of contracts. Restricting attention to subgame perfect equilibria of this two-stage game, we shall see that if the goods are substitutes (complements), it is a dominant strategy for firm i to choose the quantity (price) contract. Denote the Cournot profits of firm i by Π_i^C and the Bertrand ones by Π_i^B. At the second stage, if both firms choose the quantity contract, we have the Cournot outcome; if they choose the price contract, the Bertrand outcome prevails.

What happens if firm 1 chooses the price contract and firm 2 the quantity contract? In that case firm 2 chooses q_2 to maximize its profit, taking p_1 as given. That is, maximize p_2q_2, where p_2 is a function of p_1 and q_2, derived from the demand equations, i.e., $p_2 = (a_2 + cp_1 - q_2)/b_2$. This yields the reaction function of firm 2, $q_2 = (a_2 + cp_1)/2$. It is just the quantity q_2 which corresponds to the Bertrand reaction to p_1.[8] Notice that it is upward (downward) sloping for substitutes (complements). Firm 1 chooses p_1 to maximize its profit, taking q_2 as given. That is, maximize p_1q_1, where q_1 is a function of p_1 and q_2. Duality gives us firm's 1 reaction function: $p_1 = (\alpha_1 - \gamma q_2)/2$. (It is the price p_1 that corresponds to the Cournot reaction to q_2.) We see it slopes down (up) for substitutes (complements). These reaction functions intersect once to yield a Nash equilibrium (P, Q), where firm 1 chooses the price contract and firm 2 the quantity contract, with prices

$$(p_1^P, p_2^Q) = \left(\frac{2a_1b_2 + a_2c}{E}, \frac{2a_2b_1 + a_1c - a_2c^2/b_2}{E}\right)$$

and quantities

$$(q_1^P, q_2^Q) = \left(\frac{d}{b_2} p_1^P, b_2 p_2^Q\right),$$

where $E = 4b_1b_2 - 3c^2$ and $d = b_1b_2 - c^2$.

If firm 1 chooses the quantity contract and firm 2 the price contract, then firm 1 is on its Bertrand reaction function and firm 2 on its Cournot reaction function. By a dual argument of the above we get a Nash equilibrium with prices

$$(p_1^Q, p_2^P) = \left(\frac{2a_1b_2 + a_2c - a_1c^2/b_1}{E}, \frac{2a_2b_1 + a_1c}{E}\right)$$

and quantities

$$(q_1^Q, q_2^P) = \left(b_1 p_1^Q, \frac{d}{b_1} p_2^P\right).$$

Let $\Pi_1^P = p_1^P q_1^P$ and $\Pi_1^Q = p_1^Q q_1^Q$. In Appendix 2 we show that $\Pi_1^C > \Pi_1^Q > \Pi_1^B > \Pi_1^P$ if the goods are substitutes, and $\Pi_1^B > \Pi_1^P > \Pi_1^C > \Pi_1^Q$ if they are complements. These inequalities have a clear interpretation. Suppose the goods are substitutes. If firm 1 sets prices and firm 2 quantities, then firm 1 is in the worst of the possible worlds since it faces a price cutter and takes as given the supply of the rival. Firm 1 would be better off by being a price cutter itself. The outcome would be then the Bertrand equilibrium which yields more profits to firm 1, $\Pi_1^B > \Pi_1^P$. On the other hand, firm 1 would prefer to set

[8] See footnote 4.

quantities and be the price cutter while facing a price-setting rival rather than to face another price cutter, $\Pi_1^Q > \Pi_1^B$. The best of the possible worlds is when both firms set quantities and there is no price cutting. The Cournot outcome dominates in terms of profits the other outcomes. With complements we have the dual inequalities as expected, and the Bertrand equilibrium dominates in terms of profits the others. We have thus firm 1 facing the following payoff matrix at the first stage:

		Firm 2	
		Price	Quantity
Firm 1	Price	Π_1^B	Π_1^P
	Quantity	Π_1^Q	Π_1^C .

We see that it is dominant for firm 1 to choose the quantity contract if the goods are substitutes, since $\Pi_1^Q > \Pi_1^B$ and $\Pi_1^C > \Pi_1^P$, and to choose the price contract if the goods are complements, since then $\Pi_1^B > \Pi_1^Q$ and $\Pi_1^P > \Pi_1^C$. The same applies to firm 2. Proposition 2 states the result.

Proposition 2. In the two-stage game it is a dominant strategy for firm i to choose the quantity (price) contract if the goods are substitutes (complements).

With substitute products, choosing the quantity contract is the best firm 1 can do, regardless of the competitor's choice of contract. This is unfortunate from the welfare point of view since consumer surplus and total surplus are higher with price competition. It is, however, fortunate from the viewpoint of the firms since Cournot profits are larger than Bertrand profits. With complements, by choosing the price contracts firms enhance their profits and consumer surplus, and hence general welfare.

5. A nonlinear demand structure

■ To examine the case with nonlinear demand,[9] suppose now that $U(q_1, q_2)$ is a differentially strictly concave utility function on R_+^2, which is (differentially) strictly monotone in a nonempty bounded region Q. Let $q = (q_1, q_2)$ and $p = (p_1, p_2)$. Our representative consumer by maximizing $U(q) - pq$ gives rise to an inverse demand system $p_i = f_i(q)$, $i = 1, 2$, which is twice-continuously differentiable in the interior of Q. Inverse demands will be downward sloping,[10] $\partial_i f_i < 0$, $i = 1, 2$, and the (symmetric) cross effect $\partial_j f_i$, $j \neq i$, will be negative or positive, depending on whether the goods are substitutes or complements. Under our assumptions f can be inverted to yield a direct demand system $q_i = h_i(p)$, $i = 1, 2$. The bounded region in price space where demands are positive will be denoted by P. The demand system h will be twice-continuously differentiable in the interior of P. Direct demands are certainly going to be downward sloping, $\partial_i h_i < 0$, $i = 1, 2$, and $\partial_j h_i$, $j \neq i$, will be positive for substitute goods and negative for complements. We assume furthermore that the "own effect" is larger than the "cross effect," that is, $|\partial_i f_i| > |\partial_j f_i|$, $j \neq i$ (or equivalently, $|\partial_i h_i| > |\partial_j h_i|$, $j \neq i$).

We suppose that there are no costs[11] and that any firm can make positive profits

[9] The model in this section follows Vives (1985). Hathaway and Rickard (1979) consider also a nonlinear duopoly and Bylka and Komar (1975) "mixed" oligopolies. See Cheng (1984) for a graphical discussion of some of the issues.

[10] $\partial_j f_i$ denotes the partial derivative of f_i with respect to the jth variable.

[11] If there are positive and constant marginal costs m_1 and m_2, then define new functions

$$\tilde{U}(q) = U(q) - m_1 q_1 - m_2 q_2, \qquad \tilde{f}_i(q) = f_i(q) - m_i, \qquad \text{and} \qquad \tilde{h}_i(\hat{p}) = h_i(p), \qquad i = 1, 2,$$

where $\hat{p}_i = p_i - m_i$ to proceed with the analysis considering prices net of marginal cost.

even when the rival's price (quantity) is zero if the goods are substitutes (complements). This ensures interior solutions in equilibrium. Profits of firm i in terms of prices are $\Pi_i(p) = p_i h_i(p)$ and in terms of quantities, $\hat{\Pi}_i(q) = f_i(q)q_i$. Notice again that since $\partial_j h_i$ and $\partial_j f_i$, $j \neq i$, have opposite signs, Cournot competition with substitutes is the dual of Bertrand competition with complements. We shall use this duality to infer results for the case of complements from the case of substitutes. For $i = 1, 2$, we make the following two assumptions:

Assumption 1. $\partial_{ii}\Pi_i(p) + |\partial_{ij}\Pi_i(p)| < 0$ for all p in the interior of P, $j \neq i$.

Assumption 2. $\partial_{ii}\Pi_i(q) + |\partial_{ij}\hat{\Pi}_i(q)| < 0$ for all q in the interior of Q, $j \neq i$.

These assumptions ensure that the Bertrand and Cournot reaction functions are well behaved and have slope less than one in absolute value, and therefore there exist unique Bertrand and Cournot equilibria (Friedman, 1977). The assumptions put no restriction on the sign of the slope of the reaction functions. Intuitively though one expects reaction functions to slope up (down) in Bertrand (Cournot) competition if the goods are substitutes and conversely if the goods are complements.

We know that if firm 1 chooses the price contract, then firm 2 will be on its Bertrand reaction function, $R_2(\cdot)$, and that if firm 2 chooses the quantity contract, then firm 1 will be on its Cournot reaction function, $r_1(\cdot)$. The function $r_1(\cdot)$ defines implicitly in price space a curve $p_1 = \phi_1(p_2)$, and $R_2(\cdot)$ similarly defines a curve in quantity space, $q_2 = \psi_2(q_1)$. It is easily checked that $\partial_1\Pi_1(\phi_1(p_2), p_2) < 0$, and therefore $\phi_1(p_2) > R_1(p_2)$. That is to say, the translated Cournot reaction in price space is always larger than the Bertrand reaction. To ensure the uniqueness of the "mixed" equilibria we need a further assumption:

Assumption 3. $\partial_{ii}f_i \leqslant 0$, $\partial_{ii}h_i \leqslant 0$ and $\partial_{ij}f_i \geqslant 0$, $\partial_{ij}h_i \leqslant 0$ if the goods are substitutes, or $\partial_{ij}f_i \leqslant 0$, $\partial_{ij}h_i \geqslant 0$ if they are complements, $j \neq 1$, $i = 1, 2$.

We show in Appendix 2 that if Assumptions 1–3 hold and the goods are substitutes, then $\phi_1' > 0$, and the pairwise intersections of R_1, R_2, ϕ_1, and ϕ_2 are unique.[12] For example, the intersection of ϕ_1 and R_2 yields the mixed equilibrium (P, Q), where firm 1 chooses the price contract and firm 2 the quantity contract, and so on. Note that since the ϕ_i's are upward sloping and $\phi_i > R_i$, $i = 1, 2$, the Cournot price p_i^C will be higher than p_i^P, p_i^Q or p_i^B, and p_i^P larger than p_i^B. Therefore, in particular, Cournot prices are higher than Bertrand prices. What can we say about Π_1^Q and Π_1^P? Profits of firm 1 increase along the Bertrand reaction function R_1 as p_2 increases, because the goods are substitutes, and therefore $\Pi_1^Q > \Pi_1^B$, since $p_2^P > p_2^B$. Furthermore, it is easily seen by using the Cournot first-order condition that $d\Pi_1(\phi_1(p_2), p_2)/dp_2 = q_1\partial_2 h_1$, and therefore Π_1 increases along ϕ_1 as p_2 increases. It follows that $\Pi_1^C > \Pi_1^P$, since $p_2^C > p_2^Q$. We conclude that if goods are substitutes, the outcome of the two-stage game is the Cournot equilibrium. If the goods are complements, we just replace prices by quantities and Cournot by Bertrand in the above argument. Proposition 3 states the results.

Proposition 3. If Assumptions 1–3 hold, all four subgames have unique equilibria, Cournot prices (quantities) are larger (smaller) than Bertrand prices (quantities), and in the two-stage game it is a dominant strategy for any firm to choose the quantity (price) contract if the goods are substitutes (complements).

Without Assumption 3 we cannot ensure the uniqueness of the mixed equilibria,

---

[12] Assumption 3 is stronger than necessary. To get uniqueness of the mixed equilibria it is enough to assume that the elasticity of the cross effect $\partial_j f_i$ with respect to q_i minus the elasticity of the own effect $\partial_i f_i$ with respect to q_i is less than one; and similarly replacing $\partial_j f_i$ by $\partial_j h_i$, $\partial_j f_i$ by $\partial_i h_i$, and q_i by p_i. Assumption 3 implies this condition.

but the profits of firm 1 increase along R_1 and ϕ_1 as p_2 increases and $\phi_2 > R_2$. Therefore, any outcome of the quantity contract yields more to firm 1 than the Bertrand outcome in the (Q, P) case, and similarly the Cournot outcome dominates any outcome of the price contract in the (P, Q) case.

Note that in the linear case the dominant strategy equilibrium was also Pareto superior to the others from the point of view of the firms since (with substitutes) Cournot profits were the largest of all. This is not necessarily so in the nonlinear case. If the demand structure is symmetric, it is shown in Vives (1985) that Cournot profits are larger than Bertrand profits. From the welfare point of view, with substitutes, an inefficient outcome obtains, since Cournot prices are the highest of all prices and therefore consumer surplus is lower at the Cournot equilibrium. With complements an efficient outcome obtains in terms of total surplus, since Bertrand quantities are the highest of all quantities. With substitutes the firms try to keep *prices* high and with complements they try to keep *quantities* high. In this way they reinforce each firm's market. If a firm produces nuts and its competitor produces bolts, the firm certainly wants the output of bolts to be high since otherwise it is going to have a low demand for nuts. On the other hand, if the competitor produces nuts also, the firm wants its competitor to charge a high price since this enhances the demand for the producer of nuts only. The welfare consequences in the two situations are very different indeed.

6. Concluding remarks

■ We may summarize the results derived from the model as follows:

(1) Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. Exchanging prices and quantities, we go from one to the other.

(2) With a linear demand structure Bertrand competition is more efficient than Cournot competition (in consumer or total surplus terms), regardless of the nature of the goods (substitutes or complements) and independently of the degree of symmetry in the demand structure. With nonlinear demand and under certain assumptions Bertrand prices (quantities) are smaller (larger) than Cournot prices (quantities) if the goods are substitutes (complements).

(3) If the firms can precommit to quantity or price contracts and the goods are substitutes (complements), it is a dominant strategy for a firm to choose the quantity (price) contract. Furthermore, in the linear case the dominant strategy equilibrium is also Pareto superior in terms of profits.

Appendix 1

■ From the expressions of the equilibrium prices and quantities we have that

$$\Pi_1^C = d(p_1^C)^2/b_2, \qquad \Pi_1^B = b_1(p_1^B)^2, \qquad \Pi_1^P = d(p_1^P)^2/b_2, \qquad \text{and} \qquad \Pi_1^Q = b_1(p_1^Q)^2.$$

Let $\bar{c} = c^2/b_1b_2$. We then obtain

$$\frac{\Pi_1^C}{\Pi_1^Q} = \frac{\Pi_1^B}{\Pi_1^P} = \frac{b_1b_2}{d}\left(\frac{E}{D}\right)^2 = \frac{(4-3\bar{c}^2)^2}{(4-\bar{c}^2)^2(1-\bar{c}^2)} = \frac{(4-3\bar{c}^2)^2}{(4-3\bar{c}^2)^2-\bar{c}^6},$$

which is larger than one. Furthermore,

$$\frac{\Pi_1^Q}{\Pi_1^B} = \left(\frac{D}{E}\left(1-\frac{a_1c^2}{b_1(2a_1b_2+a_2c)}\right)\right) \qquad \text{and} \qquad \left(\frac{\Pi_1^Q}{\Pi_1^B}\right)^{1/2} = \frac{1-a_1c^2/(b_1(2a_1b_2+a_2c))}{1-2c^2/D},$$

which is larger than one if and only if $c > 0$. Finally,

$$\frac{\Pi_1^P}{\Pi_1^C} = \left(\frac{dD}{b_2E(b_1-a_1c^2/(2a_1b_2+a_2c))}\right)^2$$

and

$$\left(\frac{\Pi_1^P}{\Pi_1^C}\right)^{1/2} = \frac{1 - c^2/b_1b_2}{(1 - a_1c^2/(b_1(2a_1b_2 + a_2c)))(1 - 2c^2/D)},$$

which is larger than one if and only if c is negative. Therefore,

$$\Pi_1^C > \Pi_1^Q > \Pi_1^B > \Pi_1^P \quad \text{if} \quad c > 0 \quad \text{and} \quad \Pi_1^B > \Pi_1^P > \Pi_1^C > \Pi_1^Q \quad \text{if} \quad c < 0.$$

Appendix 2

■ Lemma 1 and its proof follow.

Lemma 1. If Assumptions 1–3 hold and the goods are substitutes, then $\phi_i' > 0$, $i = 1, 2$, and the mixed equilibria are unique.

Proof. $p_1 = \phi_1(p_2)$ is defined implicitly by $h_1(p) - r_1(h_2(p)) = 0$ and therefore

$$\phi_1' = -(\partial_2 h_1 - \partial_2 h_2 r_1')/(\partial_1 h_1 - \partial_1 h_2 r_1').$$

The denominator is always negative since $|r_1'| < 1$ and $|\partial_1 h_1| > \partial_1 h_2 > 0$. The numerator is obviously positive if $r_1' > 0$, and if $r_1' < 0$, $\partial_2 h_1/|\partial_2 h_2| > |r_1'|$ follows from Assumption 3 since $\partial_2 h_1/|\partial_2 h_2| = \partial_2 f_1/\partial_1 f_1$ and $|r_1'| = (\partial_2 f_1 + q_1\partial_{12} f_1)/(2\partial_1 f_1 + q_1\partial_{11} f_1)$. We conclude thus that $\phi_1' > 0$. Suppose now that firm 1 chooses the price contract and firm 2 the quantity contract, and consider reaction curves in (p_1, q_2) space. We shall show that firm 1's reaction $p_1 = f_1(r_1(q_2), q_2)$ is downward sloping and that firm 2's reaction $q_2 = h_2(p_1, R_2(p_1))$ is upward sloping, and therefore their intersection is unique. Differentiating the reaction functions, one gets, respectively, $dp_1/dq_2 = \partial_1 f_1 r_1' + \partial_2 f_1$ and $dq_2/dp_1 = \partial_2 h_2 R_2' + \partial_1 h_2$. If $r_1' < 0$, dp_1/dq_2 is negative since we have seen that $\partial_2 f_1/\partial_1 f_1 > |r_1'|$, and if $R_2' > 0$, then dq_2/dp_1 is positive since $\partial_1 h_2/|\partial_2 h_2| > R_2'$ as $\partial_{22} h_2 \leqslant 0$ and $\partial_{21} h_2 \leqslant 0$ and $R_2' = -(\partial_1 h_2 + p_2\partial_{21} h_2)/(2\partial_2 h_2 + p_2\partial_{22} h_2)$. Obviously, dp_1/dq_2 is negative and dq_2/dp_1 positive when $r_1' \geqslant 0$ and $R_2' \leqslant 0$, respectively, since the goods are substitutes and $\partial_2 f_1 < 0$, $\partial_2 h_1 > 0$. *Q.E.D.*

References

BERTRAND, J. "Révue de la *Théorie Mathématique de la Richesse Sociale et des Recherches sur les Principles Mathématiques de la Théorie des Richesses.*" *Journal des Savants* (1883), pp. 499–508.

BROCK, W. AND SCHEINKMAN, J.A. "Price Setting Supergames with Capacity Constraints." Research Paper No. 8130, S.S.R.I.U. of Wisconsin, Madison, 1981.

BYLKA, S. AND KOMAR, J. "Cournot-Bertrand Mixed Oligopolies" in M.W. Los *et al.*, eds., Warsaw Fall Seminars in Mathematical Economics, New York: Springer-Verlag, 1975, pp. 22–23.

CHENG, L. "Bertrand Equilibrium Is More Competitive Than Cournot Equilibrium: the Case of Differentiated Products." Mimeo, University of Florida, 1984.

COURNOT, A. *Researches into the Mathematical Principles of the Theory of Wealth.* English edition of Cournot (1838), translated by N.T. Bacon, New York: Kelley, 1960.

DENECKERE, R. "Duopoly Supergames with Product Differentiation." *Economic Letters,* Vol. 11 (1983), pp. 37–42.

DIXIT, A.K. "A Model of Duopoly Suggesting a Theory of Entry Barriers." *Bell Journal of Economics,* Vol. 10 (1979), pp. 20–32.

FRIEDMAN, J.W. *Oligopoly and the Theory of Games.* Amsterdam: North Holland, 1977.

GROSSMAN, S. "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs." *Econometrica,* Vol. 49 (1981), pp. 1149–1172.

HATHAWAY, N.J. AND RICKARD, J.A. "Equilibria of Price-Setting and Quantity-Setting Duopolies." *Economic Letters,* Vol. 3 (1979), pp. 133–137.

NASH, J. "Equilibrium Points in *n*-Person Games." *Proceedings of the National Academy of Sciences* (1950), pp. 48–49.

SHUBIK, M. with LEVITAN, R. *Market Structure and Behavior.* Cambridge: Harvard University Press, 1980.

SONNENSCHEIN, H. "The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One." *Journal of Political Economy,* Vol. 76 (1968), pp. 316–318.

VIVES, X. "Duopoly Information Equilibrium: Cournot and Bertrand." *Journal of Economic Theory,* Vol. 34 (1984), pp. 71–94.

———. "On the Efficiency of Cournot and Bertrand Equilibria with Product Differentiation." *Journal of Economic Theory* (forthcoming, 1985).