Price and quantity competition in a differentiated duopoly

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and

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This article analyzes the duality of prices and quantities in a differentiated duopoly. It is shown that if firms can only make two types of binding contracts with consumers, the price contract and the quantity contract, it is a dominant strategy for each firm to choose the quantity (price) contract, provided the goods are substitutes (complements).

1. Introduction

Two classical models in the theory of oligopoly are those of Cournot (1838) and Bertrand (1883). In both models the equilibrium concept is the noncooperative equilibrium of Nash (1950). In the former firms set quantities. In the latter prices are the strategy variables. In a duopoly situation where firms produce a homogeneous good and marginal costs are constant and equal for both firms, the Bertrand price equals marginal cost and the Cournot price is above it. With differentiated products, Bertrand prices are above marginal cost. In this case Cournot competition is still viewed as more “monopolistic” than Bertrand competition.1

We consider first a differentiated duopoly proposed by Dixit (1979). The demand structure is linear and allows the goods to be substitutes or complements. Firms have constant marginal costs and there are no fixed costs and no capacity limits. In this setup Cournot and Bertrand equilibria are unique. We show that Bertrand competition is more efficient than Cournot competition, in the sense that in equilibrium consumer surplus and total surplus are higher in the former regardless of whether the goods are substitutes or complements. Furthermore, profits are larger, equal, or smaller in Cournot than in Bertrand competition, according to whether the goods are substitutes, independent, or complements.

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1 This is not the case if one considers supergame equilibria. Price-setting supergame equilibria may support higher prices than quantity-setting equilibria for either homogeneous or differentiated products. See Brock and Scheinkman (1981) and Deneckere (1983).
Suppose now that each firm can make only two types of binding contracts with consumers: the price contract and the quantity contract. If a firm chooses the price contract, this means that it will have to supply the amount the consumers demand at a predetermined price, whatever action the competitor takes. If a firm chooses the quantity contract, it is committed to supply a predetermined quantity independently of the action of the competitor. Consider a two-stage game where firms first simultaneously commit themselves to a type of contract and afterwards compete contingent on the chosen types of contracts. Restricting attention to subgame perfect equilibria of this game, we show that if the goods are substitutes (complements), it is a dominant strategy for firm $i$ to choose the quantity (price) contract. When the goods are substitutes, if firms may commit themselves to offer consumers only a certain type of contract, they will always choose the quantity contract, and this will keep the prices high. When the goods are complements, firms want to keep the quantities produced high to reinforce each other’s market, and they will offer the price contract. This result generalizes, under certain assumptions, to a nonlinear demand structure. Furthermore, in the linear case the dominant strategy equilibrium turns out to be Pareto superior from the point of view of the firms, since Cournot (Bertrand) profits are the highest of all when the goods are substitutes (complements).

To get the results, we take advantage of the duality structure of Cournot and Bertrand competition in our differentiated commodity setting. This duality was first pointed out by Sonnenschein (1968) in a nondifferentiated framework. In our setup it turns out that Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. This means that they share similar strategic properties. For example, with linear demand, reaction functions slope downwards (upwards) in both cases. It is a matter of interchanging prices and quantities. A useful corollary is that one only needs to make computations or prove propositions for one type of competition (Cournot or Bertrand) or for one type of product (substitute or complement); the other cases follow by duality.

In Section 2 we present the linear model. Section 3 deals with the welfare properties of Bertrand and Cournot equilibria. The two-stage model is dealt with in Section 4. Section 5 extends the results to a nonlinear demand structure. Concluding remarks follow.

2. The linear model

We have an economy with a monopolistic sector with two firms, each one producing a differentiated good, and a competitive numéraire sector. There is a continuum of consumers of the same type with a utility function separable and linear in the numéraire good. Therefore, there are no income effects on the monopolistic sector, and we can perform partial equilibrium analysis. The representative consumer maximizes

$$U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i,$$

where $q_i$ is the amount of good $i$ and $p_i$ its price. $U$ is assumed to be quadratic and strictly concave $U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2)/2$, where $\alpha_i$ and $\beta_i$ are positive, $i = 1, 2$, $\beta_1 \beta_2 - \gamma^2 > 0$, and $\alpha_i \beta_j - \alpha_j \gamma > 0$ for $i \neq j$, $i = 1, 2$. This utility function gives rise to a linear demand structure. Inverse demands are given by

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2$$
$$p_2 = \alpha_2 - \gamma q_1 - \beta_2 q_2$$

in the region of quantity space where prices are positive. Letting $\delta = \beta_1 \beta_2 - \gamma^2$, $a_i = (\alpha_i \beta_j - \alpha_j \gamma)/\delta$, $b_i = \beta_i/\delta$ for $i \neq j$, $i = 1, 2$, and $c = \gamma/\delta$ (note that $a_i$ and $b_i$ are positive because of our assumptions), we can write direct demands as
provided that quantities are positive.\(^2\) The goods are substitutes, independent, or complements according to whether \(\gamma \equiv 0\). Demand for good \(i\) is always downward sloping in its own price and increases (decreases) with increases in the price of the competitor if the goods are substitutes (complements). When \(\alpha_1 = \alpha_2\) and \(\beta_1 = \beta_2 = \gamma\), the goods are perfect substitutes.\(^3\) When \(\alpha_1 = \alpha_2, \gamma^2/(\beta_1 \beta_2)\) expresses the degree of product differentiation, ranging from zero when the goods are independent to one when the goods are perfect substitutes. When \(\gamma\) is positive and \(\gamma^2/(\beta_1 \beta_2)\) approaches one, we are close to a homogeneous market.

Firms have constant marginal costs, \(m_1\) and \(m_2\). We consider from now on prices net of marginal cost. This is without loss of generality since if marginal costs are positive, we may replace \(\alpha_i\) and \(\beta_i\) by \(\alpha_i - m_i\) and \(a_i - b_i m_i + c m_j, i \neq j, i = 1, 2\), respectively.

Profits of firm \(i\), \(\Pi_i\), are given by \(\Pi_i = p_i q_i\), total surplus, \(TS\), with the quantity pair \((q_1, q_2)\) is just \(U(q_1, q_2)\), and consumer surplus, \(CS\), with prices \((p_1, p_2)\) and quantities \((q_1, q_2)\) is \(U(q_1, q_2) - (\Pi_1 + \Pi_2)\). Notice that \(\Pi_i\) is symmetric in \(p_i\) and \(q_i\).

3. Bertrand and Cournot equilibria

In Bertrand competition firms set prices, in Cournot competition, quantities. In both cases the equilibrium concept is the noncooperative Nash equilibrium. We have noticed above that profits are symmetric in prices and quantities. In the Cournot case firm 1 chooses \(q_1\) to maximize \((\alpha_1 - \beta_1 q_1 - \gamma q_2) q_1\), taking as given \(q_2\), and in the Bertrand case it chooses \(p_1\) to maximize \(p_1 (\alpha_1 - b_1 p_1 + c p_2)\), taking \(p_2\) as given. Both expressions are perfectly dual. We get one from the other by replacing \(q_1\) by \(p_1\), \(\alpha_1\) by \(a_1\), \(\beta_1\) by \(b_1\), and \(\gamma\) by \(-c\). We have thus that Cournot competition with substitute products \((\gamma > 0)\) is the dual of Bertrand competition with complements \((c < 0)\) (and similarly for the other possible combination). This means that they are going to share similar strategic properties and that we shall be able to derive the Bertrand reaction functions, equilibrium strategies, and profits from the Cournot ones. It is easily seen that the Cournot reaction of firm 1 to output \(q_2\) is \((\alpha_1 - \gamma q_2)/2\beta_1\), and correspondingly the Bertrand reaction to \(p_2\) is \((a_1 + cp_2)/2b_1\).\(^4\) Note that the reaction functions slope downwards in the Cournot case with substitutes and in the Bertrand case with complements. In any case, under our assumptions, reaction functions intersect only once, thereby yielding a unique equilibrium (Cournot and Bertrand). It is straightforward to compute the Cournot equilibrium. The Bertrand equilibrium follows by duality. Table 1 gives the equilibrium levels of prices and quantities under Cournot, \(C\), and Bertrand, \(B\), competition for firm 1. \(q_i^C\) and \(p_i^B\) denote, respectively, the Cournot quantity and the Bertrand price of firm \(i\). Let \(\Delta = 4\beta_1 \beta_2 - \gamma^2\) and \(D = 4b_1b_2 - c^2\).

To illustrate, take the symmetric case where \(\alpha_i = \alpha\) and \(\beta_i = \beta, i = 1, 2\) and consider two demand structures. When \(\gamma = 0\), the goods are independent, and we have the monopoly solution in any case, \(p_i^B = p_i^C = a/2b\) and \(q_i^B = q_i^C = \alpha/2\beta, i = 1, 2\). If instead \(\gamma = \beta > 0\), the goods are perfect substitutes and price equals marginal cost in Bertrand competition, \(p_i^B = 0\),\(^5\) and we have the usual Cournot solution \(q_i^C = \alpha/3\beta\) if there is quantity competition.

\(^2\) That is in the region \(\Lambda = \{p \in R^2: a_1 - b_1 p_1 + c p_2 > 0, a_2 - b_2 p_2 + c p_1 > 0\}\).

\(^3\) When \(\beta_1 = \beta_2 = -\gamma\) the demand system may not be well defined.

\(^4\) This is the Bertrand reaction function of firm 1, provided \(q_2\) is positive. When the reaction function reaches the boundary of the region \(\Lambda\) where \(q_2 = 0\), then \(p_1 = (b_2 p_2 - a_2)/c\) until \(p_1\) reaches \(\alpha_1/2\), the monopoly price for firm 1. For \(p_2\) larger than \((a_2 + c \alpha_1/2)/b_2\), firm 1 charges the monopoly price \(\alpha_1/2\).

\(^5\) The Bertrand quantity, \(q_i^B\), equals \(a\). When \(\gamma = \beta\), \(U\) is not strictly concave, \(\beta^2 - \gamma^2 = 0\), and \(b\) is infinite. \(bp^B\) tends to \(a\) as \(\gamma\) tends to \(\beta\).
TABLE 1  Equilibrium Levels of Price and Quantity

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bertrand</td>
<td>$\frac{2a_i b_i + a c}{D}$</td>
<td>$b_i p_i^B$</td>
</tr>
<tr>
<td>Cournot</td>
<td>$\beta_i q_i^C$</td>
<td>$\frac{2a_i b_i - \alpha_i \gamma}{\Delta}$</td>
</tr>
</tbody>
</table>

From Table 1, one gets $p_i^C - p_i^B = \alpha_i \gamma^2 / \Delta$ (and similarly $q_i^B - q_i^C = a_i c^2 / D$), $i = 1, 2$, which are nonnegative. Quantities are lower and prices higher in Cournot than in Bertrand competition, regardless of whether the goods are substitutes or complements. Cournot competition is more “monopolistic” than Bertrand competition. Firms have less capacity to raise prices above marginal cost in Bertrand competition because the perceived elasticity of demand of a firm when taking the price of the rival as given is larger than that which the same firm perceives when taking the quantity of the rival as given. In the first case the absolute value of the slope of the perceived demand function is $b_i$ and in the second $b_i - c_s / b_2$. The result is that in Bertrand competition firms quote lower prices than the Cournot ones. Furthermore, the difference in prices (or quantities) depends on the degree of product differentiation. When $\alpha_1 = \alpha_2$, this is given by $\gamma^2 / b_1 b_2$ and

$$p_i^C - p_i^B = \frac{\alpha_1}{4 \frac{\beta_1 \beta_2}{\gamma^2} - 1}$$

so that the more differentiated the products are, the smaller is the difference between the Cournot and Bertrand prices, and in the extreme situation of independent goods the difference is zero. The type of competition becomes less important, the less related the goods are.

Lower prices and higher quantities are always better in welfare terms. Consumer surplus is decreasing and convex as a function of prices, and total surplus equals $U(q_1, q_2)$, which is increasing and concave. Therefore, in terms of consumer surplus or total surplus, the Bertrand equilibrium dominates the Cournot one. For firms, if the goods are substitutes, low prices mean low profits, and Cournot profits are larger than Bertrand profits. The converse is true if the goods are complements since then to increase profits firms have to lower prices from the Cournot levels to gain market share. (See Appendix 1 for a proof of these assertions.) Proposition 1 summarizes the results thus far.

Proposition 1. Consumer surplus and total surplus are larger in Bertrand than in Cournot competition except when the goods are independent, in which case they are equal. Profits are larger, equal, or smaller in Cournot than in Bertrand competition, according to whether the goods are substitutes, independent, or complements.

4. The two-stage game

We suppose that firms can make two types of contracts with consumers: the price contract and the quantity contract. If firm $i$ chooses the price contract, this means that it

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6 Shubik (1980, pp. 68–78) showed that quantities are lower and prices are higher in Cournot than in Bertrand competition in a symmetric linear duopoly with substitute goods. Deneckere (1983) considered also the case of complements in a symmetric and linear duopoly model.

7 See Vives (1984) for an extension of Proposition 1 to an incomplete information setting where firms receive signals about the uncertain demand. Bayesian Cournot and Bertrand equilibria are compared.
will have to supply the amount the consumers demand at a predetermined price, whatever action the competitor, firm \( j, j \neq i \), takes. If firm \( i \) chooses the quantity contract, it is committed to supplying a predetermined quantity independently of the action of the competitor. That is, we restrict, in Grossman's terminology (1981), the supply curves firms can choose to two types: the vertical one, corresponding to quantity setting, and the horizontal one, corresponding to price setting. Furthermore, we assume that there are prohibitively high costs associated with changing the type of contract. Firms first choose what type of contract to offer consumers, and afterwards they compete contingent on the chosen types of contracts. Restricting attention to subgame perfect equilibria of this two-stage game, we shall see that if the goods are substitutes (complements), it is a dominant strategy for firm \( i \) to choose the quantity (price) contract. Denote the Cournot profits of firm \( i \) by \( \Pi_i^C \) and the Bertrand ones by \( \Pi_i^B \). At the second stage, if both firms choose the quantity contract, we have the Cournot outcome; if they choose the price contract, the Bertrand outcome prevails.

What happens if firm 1 chooses the price contract and firm 2 the quantity contract? In that case firm 2 chooses \( q_2 \) to maximize its profit, taking \( p_1 \) as given. That is, maximize \( p_2 q_2 \), where \( p_2 \) is a function of \( p_1 \) and \( q_2 \), derived from the demand equations, i.e., \( p_2 = (a_2 + c p_1 - q_2)/b_2 \). This yields the reaction function of firm 2, \( q_2 = (a_2 + c p_1)/2 \). It is just the quantity \( q_2 \) which corresponds to the Bertrand reaction to \( p_1 \). Notice that it is upward (downward) sloping for substitutes (complements). Firm 1 chooses \( p_1 \) to maximize its profit, taking \( q_2 \) as given. That is, maximize \( p_1 q_1 \), where \( q_1 \) is a function of \( p_1 \) and \( q_2 \).

Duality gives us firm's 1 reaction function: \( p_1 = (a_1 - \gamma q_2)/2 \). (It is the price \( p_1 \) that corresponds to the Cournot reaction to \( q_2 \)). We see it slopes down (up) for substitutes (complements). These reaction functions intersect once to yield a Nash equilibrium \((P, Q)\), where firm 1 chooses the price contract and firm 2 the quantity contract, with prices

\[
(p_1^P, p_2^Q) = \left( \frac{2a_1 b_2 + a_2 c}{E}, \frac{2a_2 b_1 + a_1 c - a_2 c^2/b_2}{E} \right)
\]

and quantities

\[
(q_1^P, q_2^Q) = \left( \frac{d}{b_2 p_1^P}, \frac{b_2 p_2^Q}{p_2^P} \right),
\]

where \( E = 4b_1 b_2 - 3c^2 \) and \( d = b_1 b_2 - c^2 \).

If firm 1 chooses the quantity contract and firm 2 the price contract, then firm 1 is on its Bertrand reaction function and firm 2 on its Cournot reaction function. By a dual argument of the above we get a Nash equilibrium with prices

\[
(p_1^Q, p_2^P) = \left( \frac{2a_1 b_2 + a_2 c - a_1 c^2/b_1}{E}, \frac{2a_2 b_1 + a_1 c}{E} \right)
\]

and quantities

\[
(q_1^Q, q_2^P) = \left( \frac{b_1 p_1^Q}{p_1^Q}, \frac{d}{b_1 p_2^P} \right).
\]

Let \( \Pi_i^P = p_i^P q_i^P \) and \( \Pi_i^Q = p_i^Q q_i^Q \). In Appendix 2 we show that \( \Pi_i^C > \Pi_i^Q > \Pi_i^P > \Pi_i^P \) if the goods are substitutes, and \( \Pi_i^P > \Pi_i^Q > \Pi_i^C > \Pi_i^P \) if they are complements. These inequalities have a clear interpretation. Suppose the goods are substitutes. If firm 1 sets prices and firm 2 quantities, then firm 1 is in the worst of the possible worlds since it faces a price cutter and takes as given the supply of the rival. Firm 1 would be better off by being a price cutter itself. The outcome would be then the Bertrand equilibrium which yields more profits to firm 1, \( \Pi_i^P > \Pi_i^P \). On the other hand, firm 1 would prefer to set

\^See footnote 4.
quantities and be the price cutter while facing a price-setting rival rather than to face another price cutter, $\Pi_q > \Pi_q^p$. The best of the possible worlds is when both firms set quantities and there is no price cutting. The Cournot outcome dominates in terms of profits the other outcomes. With complements we have the dual inequalities as expected, and the Bertrand equilibrium dominates in terms of profits the others. We have thus firm 1 facing the following payoff matrix at the first stage:

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$\Pi_1^p$</td>
<td>$\Pi_1^p$</td>
</tr>
<tr>
<td>Quantity</td>
<td>$\Pi_1^q$</td>
<td>$\Pi_1^q$</td>
</tr>
</tbody>
</table>

We see that it is dominant for firm 1 to choose the quantity contract if the goods are substitutes, since $\Pi_1^q > \Pi_1^p$ and $\Pi_1^q > \Pi_1^p$, and to choose the price contract if the goods are complements, since then $\Pi_1^p > \Pi_1^q$ and $\Pi_1^p > \Pi_1^q$. The same applies to firm 2. Proposition 2 states the result.

**Proposition 2.** In the two-stage game it is a dominant strategy for firm $i$ to choose the quantity (price) contract if the goods are substitutes (complements).

With substitute products, choosing the quantity contract is the best firm 1 can do, regardless of the competitor’s choice of contract. This is unfortunate from the welfare point of view since consumer surplus and total surplus are higher with price competition. It is, however, fortunate from the viewpoint of the firms since Cournot profits are larger than Bertrand profits. With complements, by choosing the price contracts firms enhance their profits and consumer surplus, and hence general welfare.

5. A nonlinear demand structure

To examine the case with nonlinear demand,\(^9\) suppose now that $U(q_1, q_2)$ is a differentially strictly concave utility function on $R^2$, which is (differentially) strictly monotone in a nonempty bounded region $Q$. Let $q = (q_1, q_2)$ and $p = (p_1, p_2)$. Our representative consumer by maximizing $U(q) - pq$ gives rise to an inverse demand system $p_i = f_i(q_i), i = 1, 2$, which is twice-continuously differentiable in the interior of $Q$. Inverse demands will be downward sloping,\(^10\) $\partial_i f_i < 0$, $i = 1, 2$, and the (symmetric) cross effect $\partial_f f_i, j \neq i$, will be negative or positive, depending on whether the goods are substitutes or complements. Under our assumptions $f$ can be inverted to yield a direct demand system $q_i = h_i(p), i = 1, 2$. The bounded region in price space where demands are positive will be denoted by $P$. The demand system $h$ will be twice-continuously differentiable in the interior of $P$. Direct demands are certainly going to be downward sloping, $\partial_i h_i < 0$, $i = 1, 2$, and $\partial_j h_i, j \neq i$, will be positive for substitute goods and negative for complements. We assume furthermore that the “own effect” is larger than the “cross effect,” that is, $|\partial_i f_i| > |\partial_f f_i|, j \neq i$ (or equivalently, $|\partial_i h_i| > |\partial_j h_i|, j \neq i$).

We suppose that there are no costs\(^11\) and that any firm can make positive profits

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\(^10\) $\partial_i f_i$ denotes the partial derivative of $f_i$ with respect to the $j$th variable.

\(^11\) If there are positive and constant marginal costs $m_1$ and $m_2$, then define new functions

$U(q) = U(q) - m_1 q_1 - m_2 q_2$, \hspace{1cm} $f_i(q) = f_i(q) - m_i$, \hspace{1cm} and \hspace{1cm} $h_i(p) = h_i(p)$, \hspace{1cm} $i = 1, 2$,

where $p_i = p_i - m_i$ to proceed with the analysis considering prices net of marginal cost.
even when the rival's price (quantity) is zero if the goods are substitutes (complements). This ensures interior solutions in equilibrium. Profits of firm $i$ in terms of prices are $\Pi_i(p) = p_i h_i(p)$ and in terms of quantities, $\Pi_i(q) = f_i(q) q_i$. Notice again that since $\partial h_i$ and $\partial f_j, j \neq i,$ have opposite signs, Cournot competition with substitutes is the dual of Bertrand competition with complements. We shall use this duality to infer results for the case of complements from the case of substitutes. For $i = 1, 2$, we make the following two assumptions:

Assumption 1. $\partial_i \Pi_i(p) + |\partial_q \Pi_i(p)| < 0$ for all $p$ in the interior of $P, j \neq i$.

Assumption 2. $\partial_i \Pi_i(q) + |\partial_q \Pi_i(q)| < 0$ for all $q$ in the interior of $Q, j \neq i$.

These assumptions ensure that the Bertrand and Cournot reaction functions are well behaved and have slope less than one in absolute value, and therefore there exist unique Bertrand and Cournot equilibria (Friedman, 1977). The assumptions put no restriction on the sign of the slope of the reaction functions. Intuitively though one expects reaction functions to slope up (down) in Bertrand (Cournot) competition if the goods are substitutes and conversely if the goods are complements.

We show in Appendix 2 that if Assumptions 1–3 hold and the goods are substitutes, then $\phi_1 > 0$, and the pairwise intersections of $R_1, R_2, \phi_1,$ and $\phi_2$ are unique. For example, the intersection of $\phi_1$ and $R_2$ yields the mixed equilibrium $(P, Q)$, where firm 1 chooses the price contract and firm 2 the quantity contract, and so on. Note that since the $\phi_i$'s are upward sloping and $\phi_i > R_i, i = 1, 2$, the Cournot price $p^C_2$ will be larger than $p^B_1$, $p^B_2$, or $p^B_1$. Therefore, in particular, Cournot prices are higher than Bertrand prices. What can we say about $\Pi_i^B$ and $\Pi_i^C$? Profits of firm 1 increase along the Bertrand reaction function $R_1$ as $p_2$ increases, because the goods are substitutes, and therefore $\Pi_i^B > \Pi_i^C$, since $p^B_2 > p^B_1$. Furthermore, it is easily seen by using the Cournot first-order condition that $d\Pi_i(\phi_i(p_2), p_2)/dp_2 = q_i \partial_2 h_i$, and therefore $\Pi_i$ increases along $\phi_1$ as $p_2$ increases. It follows that $\Pi_i^C > \Pi_i^B$, since $p^C_2 > p^C_1$. We conclude that if goods are substitutes, the outcome of the two-stage game is the Cournot equilibrium. If the goods are complements, we just replace prices by quantities and Cournot by Bertrand in the above argument. Proposition 3 states the results.

Proposition 3. If Assumptions 1–3 hold, all four subgames have unique equilibria, Cournot prices (quantities) are larger (smaller) than Bertrand prices (quantities), and in the two-stage game it is a dominant strategy for any firm to choose the quantity (price) contract if the goods are substitutes (complements).

Without Assumption 3 we cannot ensure the uniqueness of the mixed equilibria.

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12 Assumption 3 is stronger than necessary. To get uniqueness of the mixed equilibria it is enough to assume that the elasticity of the cross effect $\partial f_j$ with respect to $q_i$ minus the elasticity of the own effect $\partial f_i$ with respect to $q_i$ is less than one; and similarly replacing $\partial_1 f_i$ by $\partial h_i$, $\partial_2 f_i$ by $\partial h_i$, and $q_i$ by $p_i$. Assumption 3 implies this condition.
but the profits of firm 1 increase along \( R_1 \) and \( \phi_1 \) as \( p_2 \) increases and \( \phi_2 > R_2 \). Therefore, any outcome of the quantity contract yields more to firm 1 than the Bertrand outcome in the \((Q, P)\) case, and similarly the Cournot outcome dominates any outcome of the price contract in the \((P, Q)\) case.

Note that in the linear case the dominant strategy equilibrium was also Pareto superior to the others from the point of view of the firms since (with substitutes) Cournot profits were the largest of all. This is not necessarily so in the nonlinear case. If the demand structure is symmetric, it is shown in Vives (1985) that Cournot profits are larger than Bertrand profits. From the welfare point of view, with substitutes, an inefficient outcome obtains, since Cournot prices are the highest of all prices and therefore consumer surplus is lower at the Cournot equilibrium. With complements an efficient outcome obtains in terms of total surplus, since Bertrand quantities are the highest of all quantities. With substitutes the firms try to keep prices high and with complements they try to keep quantities high. In this way they reinforce each firm's market. If a firm produces nuts and its competitor produces bolts, the firm certainly wants the output of bolts to be high since otherwise it is going to have a low demand for nuts. On the other hand, if the competitor produces nuts also, the firm wants its competitor to charge a high price since this enhances the demand for the producer of nuts only. The welfare consequences in the two situations are very different indeed.

6. Concluding remarks

- We may summarize the results derived from the model as follows:

1. Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. Exchanging prices and quantities, we go from one to the other.

2. With a linear demand structure Bertrand competition is more efficient than Cournot competition (in consumer or total surplus terms), regardless of the nature of the goods (substitutes or complements) and independently of the degree of symmetry in the demand structure. With nonlinear demand and under certain assumptions Bertrand prices (quantities) are smaller (larger) than Cournot prices (quantities) if the goods are substitutes (complements).

3. If the firms can precommit to quantity or price contracts and the goods are substitutes (complements), it is a dominant strategy for a firm to choose the quantity (price) contract. Furthermore, in the linear case the dominant strategy equilibrium is also Pareto superior in terms of profits.

Appendix 1

- From the expressions of the equilibrium prices and quantities we have that

\[
\Pi_f^P = d(p_f^P)^2/b_2, \quad \Pi_f^q = b_1(p_f^q)^2, \quad \Pi_f^q = d(p_f^q)^2/b_2, \quad \text{and} \quad \Pi_f^q = b_1(p_f^q)^2.
\]

Let \( \tilde{c} = c^2/b_1b_2 \). We then obtain

\[
\frac{\Pi_f^P}{\Pi_f^q} = \frac{b_1b_2}{d} \left( \frac{4 - 3c^2}{4 - c^2(1 - c^2)} \right) = \frac{(4 - 3c^2)^2}{(4 - c^2)^2(1 - c^2)} = \frac{(4 - 3c^2)^2 - c^8}{(4 - c^2)^2 - c^8},
\]

which is larger than one. Furthermore,

\[
\frac{\Pi_f^q}{\Pi_f^q} = \left( \frac{D}{E} \left( 1 - \frac{ac^2}{b_1(2a_1b_2 + ac_2)} \right) \right) \quad \text{and} \quad \left( \frac{\Pi_f^q}{\Pi_f^q} \right)^{1/2} = \frac{1 - ac^2/(b_1(2a_1b_2 + ac_2))}{1 - 2c^2/D},
\]

which is larger than one if and only if \( c > 0 \). Finally,

\[
\frac{\Pi_f^q}{\Pi_f^q} = \left( \frac{2D}{b_2E(b_1 - ac^2/(2a_1b_2 + ac_2))} \right)^2.
\]
and

\[
\left( \frac{\Pi^i}{\Pi^j} \right)^{1/2} \leq \frac{1 - c^2/b_1b_2}{(1 - a_1c^2/(2a_1b_1 + a_2c))(1 - 2c^2/D)}
\]

which is larger than one if and only if \( c \) is negative. Therefore,

\[\Pi_i^1 > \Pi_i^2 > \Pi_i^0 \quad \text{if} \quad c > 0 \quad \text{and} \quad \Pi_i^0 > \Pi_i^2 > \Pi_i^1 \quad \text{if} \quad c < 0.\]

Appendix 2

Lemma 1 and its proof follow.

Lemma 1. If Assumptions 1-3 hold and the goods are substitutes, then \( \phi_i > 0, i = 1, 2, \) and the mixed equilibria are unique.

Proof. \( p_i = \phi_i(p_2) \) is defined implicitly by \( h_i(p) = r_i(q_2(p)) = 0 \) and therefore

\[\phi_i = (\partial h_1 - \partial h_2 r_i')/(\partial h_1 - \partial h_2 r_i').\]

The denominator is always negative since \( |r_i'| < 1 \) and \( |\partial h_1| > \partial h_1 > 0 \). The numerator is obviously positive if \( r_i' > 0 \), and if \( r_i' < 0, \partial h_1/|\partial h_2| = \partial h_2/\partial f_1 \) and \( |r_i'| = (\partial h_1 q_i/\partial f_1)/(2\partial h_1 + q_i/\partial f_1). \)

We conclude thus that \( \phi_i > 0. \) Suppose now that firm 1 chooses the price contract and firm 2 the quantity contract, and consider reaction curves in \( (p_1, q_2) \) space. We shall show that firm 1's reaction \( p_1 = f_1(r_1(q_2), q_2) \) is downward sloping and that firm 2's reaction \( q_2 = h_2(p_1, R_2(p_1)) \) is upward sloping, and therefore their intersection is unique. Differentiating the reaction functions, one gets,

\[\frac{dp_1}{dq_2} = -\frac{\partial h_1 + q_i/\partial f_1}{|\partial h_1|} \quad \text{and} \quad \frac{dq_2}{dp_1} = \frac{\partial h_2}{\partial f_2}.\]

If \( r_i' < 0, \) then \( dp_1/dq_2 \) is negative since we have seen that \( \partial h_1/|\partial h_2| > |r_i'| \) and if \( R_2 > 0, \) then \( dq_2/dp_1 \) is positive since \( \partial h_1/\partial h_2 > R_2 \) as \( \partial h_2 < 0 \) and \( \partial h_1 < 0 \), respectively, since the goods are substitutes and \( \partial h_2/\partial h_1 < 0. \)

Q.E.D.

References


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