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Entry Deterrence and the Free Rider Problem

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The public good aspect of entry prevention is examined in an industry characterized by an established oligopoly facing a potential entrant. Although incumbent firms act noncooperatively, underinvestment in entry-deterrence does not occur and in fact incumbents may find themselves in a Pareto dominated arrangement (in terms of profits) by preventing entry.

1. INTRODUCTION

Beginning with the pioneering work of Bain (1956) and Sylos-Labini (1964), the entry-prevention literature has concentrated on the case of an incumbent firm (or colluding incumbents) facing potential entry. This includes, for example, Dixit (1979), (1980), Milgrom and Roberts (1982) and Spence (1977), and the papers surveyed in Gilbert (forthcoming). Yet examples where a single firm has maintained persistent control of a market that is not a natural monopoly are rare. More common are situations where one or a few firms have remained dominant in an industry over significant periods and where industry concentration levels have remained higher than could be justified by technological conditions. Hence a more realistic setting for examining incentives for entry prevention is that of an established oligopoly facing a potential entrant where all firms are strategic agents playing a noncooperative game.[1]

We consider a market for a homogeneous product with an established m firm oligopoly facing a potential entrant which must pay a fixed cost to enter the industry. Marginal costs are constant and equal for all firms. In the first stage of the entry game the incumbents decide independently how much to produce. In the second stage the potential entrant decides whether to enter or not, and if it enters how much to produce. For any level of the entry cost there is an associated entry preventing output, and incumbents can protect themselves from new competition by producing at least this amount. Restricting attention to subgame perfect equilibria, we find that if entry is not blockaded (i.e. if the limit output is larger than the m-firm Cournot output), three regions for the entry preventing output describe the possible outcomes. If the limit output is small, entry is prevented by incumbents and typically there is a continuum of entry preventing equilibria. If the limit output is large, entry is allowed by incumbents and total output is less than the entry preventing output. For limit outputs in an intermediate region both types of equilibria exist.

Like national defense, entry prevention in this model is a public good. If any firm produces enough to deter entry, all firms are protected from competition. Thus each firm could "free-ride" on the entry preventing activities of its competitors with the implication that there would be too little investment in entry deterrence. Another observation is that noncooperative oligopolists overproduce with respect to a coordinated cartel and this might be conducive to entry deterrence. An analysis of the incentives for entry prevention in an established oligopoly raises issues that are more subtle than these arguments may suggest. The first argument ignores the benefits that accrue to a firm that engages in entry deterrence while the second ignores the fact that to prevent entry, when it is not blockaded, the incumbents have to produce more than the oligopoly output (with no threat of entry).

In the paper we examine in detail the incentives for entry prevention in an established oligopoly. The results of this analysis are unambiguous given our assumptions. In our model an established oligopoly never under-invests and may over-invest in entry deterrence. In any oligopoly equilibrium where entry is allowed, the profits of incumbents would be lower if entry were prevented. In this sense incumbents do not under-invest in entry deterrence. There are situations where incumbents' profits are higher allowing entry, but the unique oligopoly equilibrium calls for entry prevention. Hence over-investment in entry deterrence is a distinct possibility. To make matters worse, when both types of equilibria coexist the profits of each incumbent firm are higher when entry is allowed. Thus incumbents may become trapped in a Pareto dominated arrangement (in terms of profits) by preventing entry. Even though entry prevention is costly, if entry is to be prevented and there is more than one incumbent each incumbent firm wants to be the entry preventer. Given that the limit output has to be produced to prevent entry and that marginal costs are constant, profits of any incumbent increase with investment up to the limit output. Furthermore rivalry among incumbents diminishes the value of allowing entry to occur, so that entry prevention becomes relatively more attractive to each firm as the number of incumbents increases.

The model we develop provides an equilibrium framework for the evaluation of the welfare effects of changes in the number of incumbents (which could be due to coalition formation) and in the entry conditions. If the entry cost is positive, entry is blockaded if the number of incumbents is large enough. Total output, and hence consumer surplus, is not always monotone nondecreasing in the number of established firms unless we restrict attention to undominated equilibria (in terms of profits). The number of established firms may increase and total output may fall because the type of equilibrium may switch from preventing entry to allowing entry.

These results are robust to the introduction of (symmetric) product differentiation. Furthermore, the assumption of a single potential entrant may be generalized to a sequence of potential entrants with little change in the conclusions, provided that demand is linear.[2]

The plan of the paper is as follows. Section 2 characterizes the equilibria of our noncooperative entry game. Section 3 addresses the public good problem in entry prevention. The comparative statics of entry deterrence are examined in Section 4 and concluding remarks follow.

2. ENTRY DETERRENCE AS A NONCOOPERATIVE GAME

Our objective is to analyse how firms in an established oligopoly, acting noncooperatively, respond to incentives for entry prevention. Entry can be deterred by guaranteeing an entrant a residual demand that yields revenues short of total costs. This means there is a critical limit output, Y, for the oligopoly. Any established firm can take the initiative and prevent entry by making up the difference between Y and the outputs of all other established firms.

We model the market as a two stage game with complete information. At the initial stage the incumbent firms make simultaneous and independent production decisions. In the second stage a potential entrant chooses whether or not to enter and if so what to produce, taking the incumbents' outputs as given. Incumbents are assumed to have available a mechanism which makes the first stage output level credible for the second stage when entry may occur, although the mechanism is not specifically addressed in this model.

We want to exclude from our analysis any industry equilibria that are the consequence of threats which, if called, would not be enforced. A potential entrant cannot threaten to enter at an output level which is not a rational choice after entry has occured. We restrict attention to subgame perfect equilibria where the potential entrant cannot make empty threats and the incumbents exercise foresight with respect to the actions of the potential entrant.

Incumbent firms belong to a finite set M, ($\# M = m$), and have a cost function for a single homogeneous good $C_i(x) = S + vx$ for $x \geqq 0$, $i \in M$ where $v \geqq 0$ is a constant marginal cost and S is a sunk cost. There is a single potential entrant with a cost function $C_e(x) = F + vx$ if $x > 0$ and zero otherwise, with $F \geqq 0$. F is an entry cost that is avoided if the firm does not enter.

Let $p = P(X)$ be the inverse demand function, with X total output. We assume that $P(\cdot)$ cuts both axes and is twice-continuously differentiable, downward sloping, $P' < 0$, and concave, $P'' \leqq 0$, whenever positive. $P(\cdot)$ is positive on the interval $[0, \bar{X}]$, for some $\bar{X} > 0$. From now on we c nsider prices net of the constant marginal cost v so that, without loss of generality, we may set $v = 0$.

The goal of this section is to characterize the equilibria of our game. Ignoring entry for the moment, let $r(Z)$ be the optimal output of firm i when the other incumbents produce a total output of Z. Thus $r(\cdot)$ is thus the Cournot best reply of firm i. It is easily seen that $r(\cdot)$ is positive and continuously differentiable with $-1 < r' < 0$ on $(0, \bar{X})$. If the potential entrant did not have to pay the fixed cost F its best response function also would be $r(\cdot)$. Revenue for the potential entrant along $r(\cdot)$ is decreasing in the output of the established firms X^0. When X^0 equals a critical level Y, revenue equals the fixed cost F and profits are zero. Assuming that the potential entrant enters if and only if it can make positive profits, its best response function is given by

$$q(X^0) = \begin{cases} r(X^0) & \text{if } X^0 < Y, \\ 0 & \text{otherwise.} \end{cases}$$

Let X^0_{-i} be the total output of incumbents omitting firm i. Incumbent i's profits ignoring sunk costs and taking into account the optimal response of the potential entrant are

$$\Pi_i(x_i, X^0_{-i}) = P(X^0_{-i} + x_i + q(X^0_{-i} + x_i))x_i.$$

Incumbents choose output levels given outputs of other established firms and the strategy function of the entrant. The entrant chooses its output optimally given any incumbent total output, X^0. In a subgame perfect equilibrium (S.P.E.) incumbent i produces x_i with x_i in arg max $\pi_i(x_i, X^0_{-i})$, and the potential entrant uses $q(\cdot)$. Let $\phi(Z)$ be the set of best replies for firm i when the other incumbents produce Z, that is,

$$\phi(Z) = \{x \in [0, \bar{X}]: \pi_i(x, Z) \geqq \pi_i(y, Z) \textit{ for all } y \in [0, \bar{X}]\}.$$

Then $(x_i)_{i \in M}$ is a S.P.E. if $x_i \in \phi(X^0_{-i})$ for all i. We derive the properties of ϕ to characterize the set of equilibria or our game.

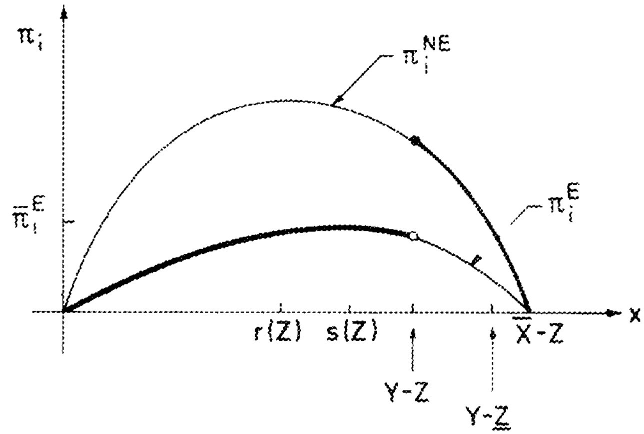

FIGURE 1

The thick line represents the profits of firm i as a function of its own output x_i given that the other incumbents produce a total output of Z.

Consider firm i and let other incumbents produce Z. Firm i may prevent entry by producing $x_i \geqq Y-Z$, in which case it gets $\pi_i^{NE} = P(x_i+Z)x_i$, or may allow entry with $x_i < Y-Z$ and earn $\pi_i^E = P(x_i+Z+r(x_i+Z))x_i$. At $x_i = Y-Z$, $\pi_i^{NE} > \pi_i^E$. Thus firm i's profit jumps up at $x_i = Y-Z$. (See Figure 1). We assume that $\pi_i^E(x_i, Z)$ is single peaked in x_i for all Z (A sufficient condition for this to be the case is that $r(\cdot)$ be convex). Let $s(Z) = \arg\max \pi_i^E(x, Z)$, the best reply of the firm when entry is to occur. Given our assumptions, $s(\cdot)$ is continuously differentiable and $-1 < s' < 0$ on $(0, \bar{X})$.

Let $\bar{\pi}_i^E(Z) = \pi_i^E(s(Z), Z)$, $\bar{\pi}_i^E$ is the maximum level of profits for firm i if entry occurs given that the other incumbents produce Z. Whenever $Z + r(Z) \geqq Y$, firm i blockades entry by producing according to its Cournot best response function (that is, by ignoring the threat of entry). Since $1 + r' > 0$, $Z + r(Z)$ is increasing in Z. Hence there is a unique solution $\bar{Z}(Y)$ to $Z + r(Z) = Y$ (let $\bar{Z}(Y) = 0$ if the solution is not positive). If $Z \geqq \bar{Z}(Y)$ firm i produces $r(Z)$ and entry does not occur.

What if $Z < \bar{Z}(Y)$? In this case firm i can prevent entry by producing $Y-Z$ and earn $\bar{\pi}_i^{NE}(Z) = \pi_i^{NE}(Y-Z, Z)$, or allow entry by producing $s(Z)$ and earn $\bar{\pi}_i^E(Z)$. At $Z = \bar{Z}(Y)$, $\bar{\pi}_i^{NE} > \bar{\pi}_i^E$ since $\pi_i^{NE} > \pi_i^E$ at $x_i = Y-Z$ and $r(\bar{Z}) = Y - \bar{Z}$. Now, $\bar{\pi}_i^{NE}(Z) = P(Y)(Y-Z)$, which is linear and downward sloping in Z, and in the appendix it is shown that $\bar{\pi}_i^E(Z)$ is strictly convex and downward sloping in Z (see Figure 2). Since $\bar{\pi}_i^{NE}(\bar{Z}) > \bar{\pi}_i^E(\bar{Z})$ there is a unique intersection where $\bar{\pi}_i^{NE}(Z) = \bar{\pi}_i^E(Z)$. Let $\underline{Z}(Y)$ ($\underline{Z}$ for short) solve $\bar{\pi}_i^{NE}(Z) = \bar{\pi}_i^E(Z)$ if there is a positive solution and let $\underline{Z}$ be zero otherwise. Referring to Figure 2, $\bar{\pi}_i^{NE} > \bar{\pi}_i^E$ for $\underline{Z} < Z \leqq \bar{Z}$ and $\bar{\pi}_i^{NE} < \bar{\pi}_i^E$ for $Z < \underline{Z}$. We have thus the best reply correspondence of firm i given by (see Figure 3)

$$\phi(Z) = \begin{cases} r(Z) & \text{if } Z \geqq \bar{Z}(Y) \quad \text{(Blockade entry)}, \\ Y - Z & \text{if } \bar{Z}(Y) > Z \geqq \underline{Z}(Y) \quad \text{(Prevent entry)}, \\ s(Z) & \text{if } \underline{Z}(Y) \geqq Z \geqq 0 \quad \text{(Allow entry)}. \end{cases}$$

Note that at $Z = \underline{Z}(Y)$ the firm is indifferent between allowing and preventing entry and that $s(Z) < Y - \underline{Z}$. The best response of firm i jumps up at $Z = \underline{Z}(Y)$ where the firm switches from allowing to preventing entry (see Figure 3). For the case of linear demand,

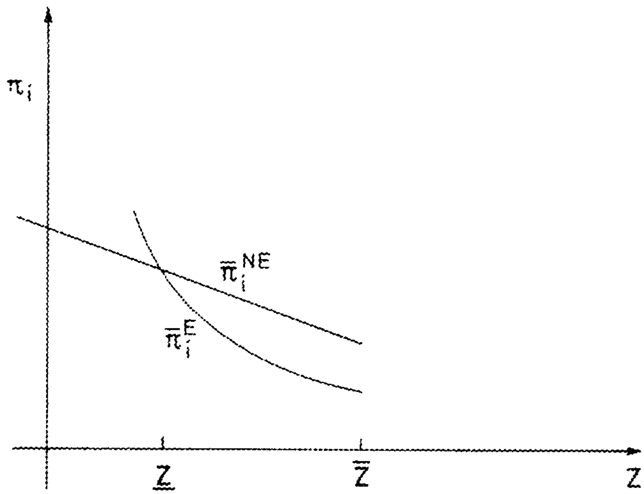

FIGURE 2

$\bar{\pi}_i^E(Z)$ is the maximum level of profits for firm i if entry occurs given that the other incumbents produce Z. $\bar{\pi}_i^{NE}(Z)$ is the level of profits firm i gets by producing $Y - Z$ and preventing entry.

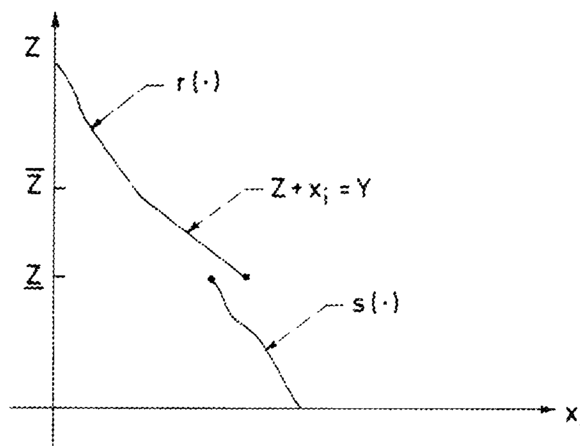

FIGURE 3

Best response correspondence of firm i.

$P(X) = a - X$, $a > 0$, the best response functions are $s(Z) = r(Z) = (a - Z)/2$. That is, the best response of the firm is the same whether entry is allowed or blockaded in this case. Simple computations yield $\bar{Z}(Y) = \max\{0, 2Y - a\}$ and $\underline{Z}(Y) = \max\{0, (2Y - (1 + \sqrt{\frac{1}{2}})a)/(1 - \sqrt{\frac{1}{2}})\}$. The properties of $\underline{Z}(\cdot)$ are given in Lemma 1 (see Figure 4, proof in appendix).

Lemma 1. *Let Y_1 be the limit output at which a monopolist is indifferent between allowing or preventing entry. Then $Y_1 > 0$ and $\underline{Z}(\cdot)$ is zero on $[0, Y_1]$ and for $Y > Y_1$ it increases with Y, with slope larger than one, up to $\bar{X}$.*

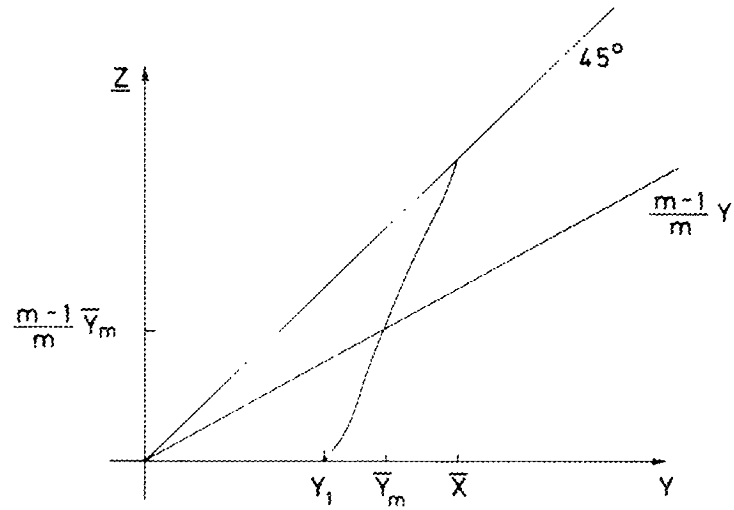

FIGURE 4

The total output of the other incumbents which makes firm i indifferent between allowing and preventing entry as a function of the entry preventing output Y, $\underline{Z}(Y)$

With Lemma 1 and the best response correspondence of firm i, ϕ, we are ready to characterize the equilibria of the game according to the level of the entry preventing output Y. Three possible types of equilibria emerge depending upon whether the ϕ's intersect in the blockade, prevent or allow entry regions. Recall that the slopes of the Cournot best reply function, r, and of the best reply function with entry, s, are between -1 and 0 and therefore their respective intersection are unique (see Szidarovsky and Yakowitz (1977)). Let x_i^C denote the Cournot output and x_i^E the equilibrium output with entry for firm i. By symmetry, $x_i^C = x^C$ and $x_i^E = x^E$ for all i. Let $X^C = mx^C$. If $X^C \geqq Y$ then entry is blockaded at the Cournot equilibrium.

Let $\underline{Y}_m$ be the smallest Y such that the ϕ's intersect at x^E. The limit output $\underline{Y}_m$ solves $\underline{Z}(Y) = (m-1)x^E$. That is, when all other incumbents produce x^E firm i is indifferent between allowing and preventing entry. Let $\bar{Y}_m$ be the largest Y such that the ϕ's intersect on the hyperplane $\sum_{i=1}^m x_i = Y$; that is $\bar{Y}_m$ is the largest Y for which to prevent entry is an equilibrium. $\bar{Y}_m$ solves $\underline{Z}(Y) = (m-1)Y/m$ (see Figure 4). The largest entry preventing output for which to prevent entry is an equilibrium requires equal shares for the incumbents, that is, $x_i = Y/m$ for $i \in M$. It follows that $\bar{Y}_m > \underline{Y}_m$ since $Y/m > x^E$ and $\underline{Z}(\cdot)$ is increasing in Y. Furthermore $\underline{Y}_m > X^C$ since we are considering Y's larger than X^C.

Proposition 1.

(i) *If $Y \leqq X^C$, then each incumbent produces its Cournot output x^C and the potential entrant stays. Incumbents blockade entry by producing at Cournot levels.*

(ii) *If $X^C < Y \leqq \bar{Y}_m$, then any $(x_i)_{i \in M}$ in the set $\{x \in R_+^m : \sum_{i \in M} x_i = Y,\ r(X_{-i}) \leqq x_i \leqq Y - \underline{Z},\ i \in M\}$ and the potential entrant staying out is an equilibrium. Incumbents prevent entry by producing a total output of Y.*

(iii) *If $Y \geqq \underline{Y}_m$, then each incumbent producing x^E and the potential entrant producing $r(mx^E)$ is an equilibrium. Incumbents allow entry.*

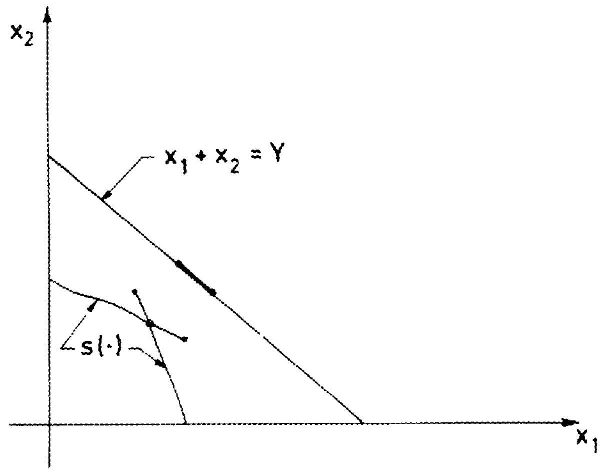

FIGURE 5

The intersection of the $s(\cdot)$ functions is the entry equilibrium. The thick segment represents entry preventing equilibria.

In summary, when the total Cournot output of the m established firms exceeds the entry preventing output Y, entry is blockaded. When Y is larger than the total Cournot output, but smaller than $\underline{Y}_m$, incumbents prevent entry. When Y is larger than $\bar{Y}_m$ entry is allowed. In the intermediate region where $\underline{Y}_m \leqq Y \leqq \bar{Y}_m$ we have both types of equilibria. When entry is prevented there is typically a continuum of entry preventing equilibria with incumbents producing at any point $x \in R_+^m$ above the Cournot reaction functions $r(\cdot)$ and below $Y - \underline{Z}$ on the hyperplane defined by $\sum_{i \in M} x_i = Y$. Figure 5 illustrates both types of equilibria for an established duopoly.

3. IS ENTRY PREVENTION A PUBLIC GOOD?

Imperfect coordination in oligopoly suggests the possibility that entry prevention may be a public good and hence competing firms may underinvest in entry-deterring capital investment (see, e.g. Waldman (1982)). We will show that this intuition is false, at least for the model in this paper. Indeed the opposite problem arises in our oligopoly setting.

Entry deterrence has the characteristics of a public good since if a group of incumbents prevent entry by producing an aggregate output at least as large as Y then entry will not occur whatever action incumbents outside the group might take. In this situation all incumbents enjoy the same amount of entry prevention and one incumbent's "consumption" of entry prevention does not decrease the amount of entry prevention enjoyed by other incumbents. The public good analogy for entry prevention suggests that incumbent firms in a noncooperative oligopoly would tend to underinvest in entry deterrence. Underinvestment in entry prevention would be associated with one or more of the following.

(a) Incumbents' total profits are higher preventing than allowing entry, but the (unique) industry equilibrium allows entry.
(b) Either entry prevention or entry may be an industry equilibrium, but incumbents' profits are higher when entry is prevented.
(c) An established monopoly (or colluding incumbents) prevents entry in more situations than an established, noncooperating, oligopoly.

We show in Proposition 2 below that in none of these respects is there underinvestment in entry prevention and that "too much" entry prevention definitely can occur. The basic idea is simple. Suppose we have two incumbents. Note that since marginal costs are constant the best entry preventing equilibrium for firm 1 is the one in which the firm produces the most (The price is $P(Y)$ and profit is proportional to x_i). When $Y = \underline{Y}_2$ and $x_2 = x^E$, firm 1 is indifferent between preventing or allowing entry. If firm 2 produces more then firm 1 prevents entry and is worse off. If $Y > \underline{Y}_2$, then firm 1 when preventing entry produces less and the price is lower. Therefore profits are lower than in the entry equilibrium. We have thus the opposite situation than (b): when both entry prevention and entry may be equilibria, profits when entry is allowed are larger than when entry is prevented for any firm.

Proposition 2. *Let* $m \geqq 2$, *then:*

(i) *When* $\underline{Y}_m \leqq Y \leqq \bar{Y}_m$, *entry prevention and allowing entry are both equilibria but the profits of each incumbent firm are higher when entry is allowed.*

(ii) *In all cases where an established monopoly prevents entry an oligopoly does so too and there are situations where an oligopoly prevents entry when a monopoly would allow entry to occur.*

Proof. (i) Let $\bar{Y}_m \geqq Y \geqq \underline{Y}_m$. Profits of incumbent i with entry are $P(X^E)x^E$ and with no entry, when it produces x_i, $P(Y)x_i$. By definition of $\underline{Y}_m$ when $Y = \underline{Y}_m$ both magnitudes are equal if all the other incumbents produce x^E and $x_i = \underline{Y}_m - (m-1)x^E$. This is the best entry preventing equilibrium from the point of view of firm i since it is the one where it produces the most. If firm i produces less then it is worse off. When $Y > \underline{Y}_m$, at an entry preventing equilibrium firm i produces $x_i \leqq Y - Z(Y)$ which is less than $\underline{Y}_m - Z(\underline{Y}_m) = \underline{Y}_m - (m-1)x^E$ since the slope of Z is larger than one. Therefore when $Y > \underline{Y}_m$ firm i, at an entry preventing equilibrium, produces less than $\underline{Y}_m - (m-1)x^E$ and the price is lower than when $Y = \underline{Y}_m$. We conclude that firm i obtains less profits at any entry preventing equilibrium than at the entry equilibrium, profits being equal only when $Y = \underline{Y}_m$ and $x_i = \underline{Y}_m - (m-1)x^E$.

(ii) Suppose entry is not blockaded either by a monopolist or by an m-firm oligopoly. That is, $Y > mx^C$ since mx^C is larger than the monopoly output for $m \geqq 2$. A monopolist would prevent entry if $Y \leqq \underline{Y}_1$ (Recall that $\underline{Y}_1 = \bar{Y}_1$). An entry prevention equilibrium with m-firms exists whenever $Y \leqq \bar{Y}_m$ and it is the unique equilibrium whenever $Y < \underline{Y}_m$. But $\underline{Y}_m > \underline{Y}_1$ since $\underline{Y}_m$ is increasing in m. Thus whenever a monopolist would prevent entry the established oligopoly would do so also and when $Y \in (\underline{Y}_1, \underline{Y}_m)$ the oligopoly would prevent entry when a monopoly would not. ||

From (ii) it follows that preventing entry cannot yield higher total profits for incumbents in any equilibrium where entry is allowed and in fact for limit outputs close to and smaller than $\underline{Y}_m$ incumbents' total profits would be higher allowing entry but the unique equilibrium calls for entry prevention. Note also that total output with entry, X^E is smaller than $\underline{Y}_m$ since when $Y = \underline{Y}_m$, $P(X^E)x^E = P(Y)(Y-(m-1)x^E)$, x^E is less than $Y - (m-1)x^E$ and demand is downward sloping. Therefore when $Y \geqq \underline{Y}_m$ total output is less and price is higher at the entry equilibrium. The intuition for this is clear. By allowing entry when $Y \geqq \underline{Y}_m$ incumbents exploit the tendency of the entrant to hold back output. Incumbents produce less than they would if entry were prevented and total production is less than the limit output.

The results in this section run against the presumption that there is a public good problem in entry prevention. With a single incumbent entry deterrence is costly and therefore one could think that with more established firms each incumbent would like the competitors to carry the burden of preventing entry. Nevertheless, given the marginal costs are constant, the profits of each incumbent when entry is prevented increase with investment up to the limit output and each incumbent wants to be "the entry preventer". Furthermore, rivalry amongst incumbent firms reduces the profitability of allowing entry relative to preventing entry and leads to greater investment in entry deterrence in the sense that relative to a monopoly or perfectly coordinated cartel, the oligopoly will prevent entry for larger values of Y.

We have seen that when $\underline{Y}_m \leqq Y \leqq \tilde{Y}_m$ there are two types of equilibria, one where entry is prevented and another where entry is allowed and that the entry equilibrium dominates in terms of profits those equilibria where entry is prevented. This means that incumbents may get trapped in a Pareto dominated arrangement preventing entry. On the other hand if we restrict attention to undominated equilibria then the equilibrium is unique in terms of total output for any given Y. If Y is less than the total Cournot output of the established firms entry is blockaded. If Y exceeds the Cournot output but is less than $\underline{Y}_m$, then entry is prevented. Otherwise entry is allowed.

4. COMPARATIVE STATICS

The model developed in the preceding sections provides an equilibrium framework for the evaluation of the effects of changes in the number of incumbents, m, and of changes in entry conditions on market prices and welfare. The fixed cost of entry, assumed sunk once incurred, is a measure of the height of entry barriers. Changes in m may come about through explicit or implicit coalition formation among incumbents. For example, two incumbents merge and act as a single unit and therefore the number of "effective" incumbents decreases by one. We do not discuss here the profitability of such coalition formation (See the article by Salant et al. (1983) for a discussion of the Cournot case.)

Suppose now that Y is fixed. The effect of changes in m in the equilibria of our model will depend on the induced changes in the critical values $\underline{Y}_m$ and $\tilde{Y}_m$. This suggests three critical numbers m_1, m_2, m_3 (not necessarily integers) defined implicitly as follows. $X^C_{m_1} = Y$, $\underline{Y}_{m_2} = Y$ and $\tilde{Y}_{m_3} = Y$. These three equations have unique solutions $m_i(Y)$, $i = 1$, 2, 3, since X^C_m, $\underline{Y}_m$ and $\tilde{Y}_m$ are all increasing in m. If the solution of equation i is less than one, let $m_i = 1$. Furthermore, $m_1(Y) > m_2(Y) > m_3(Y)$ provided that they are larger than one since we know that $X^C_m < \underline{Y}_m < Y_m$ for $m > 1$. $m_1(Y)$ is the lower bound for the number of incumbents m whose Cournot outputs blockade entry. $m_2(Y)$ is the upper bound on m for which allowing entry is an equilibrium. $m_3(Y)$ is the lower bound on m for which preventing entry is an equilibrium. Proposition 3 is just a restatement of Proposition 1.

Proposition 3. *For fixed Y,*
(i) *if $m > m_1(Y)$, then entry is blockaded;*
(ii) *if $m_3(Y) \leqq m \leqq m_1(Y)$, then preventing entry is an equilibrium;*
(iii) *if $m \leqq m_2(Y)$, then allowing entry is an equilibrium.*

For a sufficiently large number of incumbent firms, entry is blockaded by the Cournot output. As the number of incumbents decreases, perhaps reflecting the formation of coalitions, there is a region $[m_2(Y), m_1(Y)]$ where incumbents will prevent entry by setting the limit output. For m in $[m_3(Y), m_2(Y)]$ either preventing or allowing entry can be an equilibrium. For a smaller number of incumbents to allow entry is the only equilibrium.

In the linear case with $\lambda = Y/a$, $m_1 = \lambda/(1-\lambda)$, $m_2 = (\lambda - \sqrt{1/2})/(1-\lambda)$ and $m_3 = m_1(1-\sqrt{1/2})/(1+\sqrt{1/2})$ (provided all are larger than one). For example, when $\lambda = 0{\cdot}97$ then $m_1 \approx 32{\cdot}3$, $m_2 \approx 8{\cdot}8$ and $m_3 \approx 5{\cdot}6$, so that for $m \geqq 33$ entry is blockaded, for $33 \geqq m \geqq 9$ entry is prevented, for $m \leqq 5$ entry is allowed and if m equals 6, 7, or 8 both types of equilibria coexist.

Note that decreasing the number of incumbents may actually decrease the market price if both the initial and the final m are in $[m_3(Y), m_2(Y)]$ and the equilibrium switches from allowing entry to preventing entry. Recall that total output with entry, X_m^E, is less than the entry preventing output Y (see Figure 6) and therefore the market price with entry is higher than with no entry. If we restrict attention to undominated equilibria (in terms of profits) so that in the interval $[m_3, m_2]$ to allow entry is the unique equilibrium, then the market price is monotone nonincreasing in the number of established firms.

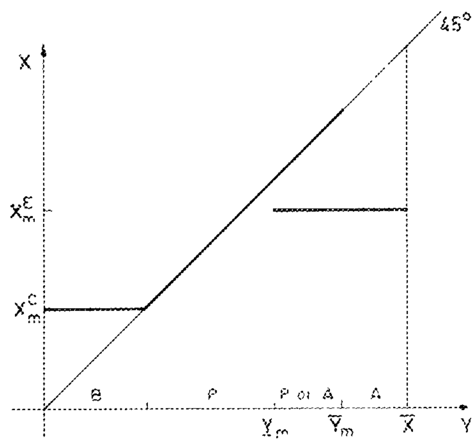

Figure 6

FIGURE 6

Total equilibrium output as a function of the entry preventing output Y. B: Blockaded, P: Prevent, A: Allow, entry

Finally we consider the effect of a change in the limit output, Y, holding the number of incumbents fixed. A change in the limit output could result, for example, from a technological development that lowers the cost of entry. An increase in the limit output from Y to Y' would have no effect on price or performance if entry is either blockaded or allowed at both values. If entry is prevented at Y and Y', the increase lowers the equilibrium price. But an increase in the limit output may cause a switch from an equilibrium where entry is prevented to an equilibrium where entry is allowed. (See Figure 6). In that case total output is lowered and costs are higher with entry, since there is a positive entry cost, and therefore total surplus is lower. When incumbents switch to allowing entry at Y', the lower entry cost has the perverse effect of reducing economic performance. In this case lowering the cost of entry, and thereby making potential competition a more credible threat, leads to an unambigous deterioration in market performance.

5. CONCLUDING REMARKS

We have explored a model where m incumbents face noncooperatively a potential entrant that must pay a cost F to enter the industry. Given m and F Proposition 1 specifies whether entry will be blockaded, prevented or allowed, except in a region where there are two types of equilibria, one where entry is prevented and another where entry is allowed.

Despite noncooperative behaviour among incumbent firms, we found no evidence of underinvestment in entry prevention. Indeed, the opposite result occurs in some situations: incumbents prevent entry even though their profits would be higher if entry were allowed (see Proposition 2). A main reason why this result obtains is that entry-preventing investment earns revenues and therefore confers direct benefits on any firm that invests to exclude rivals. With constant marginal costs, if entry is prevented, profits of an incumbent increase with investment up to the limit output. Thus each incumbent firm is better off if it carries the "burden" of entry prevention. Furthermore, competition among the incumbents facing potential entry lowers the return to each incumbent from allowing entry and makes entry prevention look more attractive. These incentives to be the entry-preventer can lead to excessive investment in entry prevention. However, if entry were allowed, incumbents' equilibrium outputs would fall and entrants would hold back output resulting in a market price higher than the limit price. Thus incumbents' profits with entry can be higher than if entry is prevented while each incumbent firm chooses to prevent entry.

Note that entry deterrence is still a public good in this model, yet its supply may be excessive. Our analysis may suggest the existence of a more general class of noncooperative games involving the supply of public goods where the free-rider problem need not arise. Indeed, public goods may be over-supplied, in the sense that the total net valuation may be negative. This is the case with entry deterrence, where the total benefit from entry prevention can be negative relative to allowing entry.

Entry prevention may have desirable welfare consequences, as consumers benefit from lower prices and, with constant marginal costs, the avoidance of entry expenditures is a net social gain Of course there are situations (low limit outputs in region P of Figure 6) where firms limit price when total output would be higher (and price would be lower) with entry. This occurs at relatively small limit outputs, corresponding to high costs associated with entry (and for which preventing entry is the unique equilibrium).

In the extreme these results suggest that policies which raise the cost of entry actually can have desirable welfare consequences (at least within the narrow focus of our model). If entry is easy, so that $Y \geqq \ddot{Y}_m$ in Figure 6, incumbents would allow entry and total output would be X_m^E. A tax on entry (or an entry fee) could lower the limit output to a level Y somewhat below $\underline{Y}_m$. Incumbents would then prevent entry. and total output Y would be greater (and price lower) than X_m^E.

The analysis could be extended to a symmetric product differentiation demand structure. If the incumbent firms face a pool of potential entrants a similar analysis may be carried out assuming that entry is sequential and that demand is linear. Nonlinear demand with sequential entry may cause difficulties.[2]

We have supposed from the start that outputs, once set, were maintained by the firms and that market price was the one which cleared the market.[3] We could refine the analysis and distinguish between capacity and output (as in Dixit (1980)). Capacity would have a constant unit cost; output would have a constant marginal cost up to capacity and infinite otherwise. The two stage game would be as before, substituting output by capacity except that once all capacities were set the profits accruing to each firm would be the ones of the Cournot equilibrium resulting from the cost functions (capacities) chosen by the firms. One may conjecture that (as in Dixit (1980) where he analyzes the case where $m=1$) there will not be excess capacity in equilibrium (S.P.E.) since capacity is costly and excess capacity does not deter entry; potential entrants ignore capacities that are not going to be used fully (i.e. capacities that are not credible). Although there will be no excess capacity in equilibrium, our reformulated game does not reduce to the original game since now firms have to worry not only about the *profitability* of deterring entry but also about the *feasibility* of doing so. There are going to be situations where it would be profitable to keep an entrant out but it is not possible to do so because the output needed to prevent entry cannot be induced in Cournot equilibrium by any capacity choice of the incumbents. The characterization of equilibria turns out to be complex, due to the feasibility (or credibility) constraint, but one thing is certain: incumbents will have a harder time preventing entry than suggested by our model.

APPENDIX

1. Let $P(\cdot)$ be C^2, $P'<0$ and $P''\leqq 0$ on $(0, \bar{X})$. We show that r is C^1, $-1<r'<0$ and that the maximum profits of firm i decrease with the output of the other firms.

Firm i's profit when other incumbents produce Z is $\pi_i(x_i, Z)=P(x_i+Z)x_i$. $r(Z)$ solves $P(x_i+Z)+x_iP'(x_i+Z)=0$ and therefore $r'=-(P'+x_iP'')/(2P'+x_iP'')$. Now $d\pi_i(r(Z), Z)/dZ$ equals $\partial\pi_i/\partial Z$ (using the first-order condition F.O.C.). $\partial\pi_i/\partial Z=x_iP'$, which is negative.

2. Assume furthermore that $\pi_i^E(x_i, Z)$ is single peaked in x_i for all Z then s is C^1, $-1<s'<0$ and $\bar{\pi}_i^E(Z)$ $(=\pi_i^E(s(Z), Z))$ is decreasing and strictly convex in Z.

$s(Z)$ solves $P(x_i+Z)+x_i(1+r'(x_i+Z))P'(x_i+Z)=0$. Therefore

$$s'=-\frac{(1+r')(P'+(1+r')x_iP'')+x_iP''r''}{(1+r')(P+(1+r')x_iP'')+x_iP''r''+(1+r')P'}$$

and $-1<s'<0$ since by single peakedness the denominator is negative. Now, $\bar{\pi}_i^E(Z)=P(Z+s(Z)+r(Z+s(Z)))s(Z)$ and $d\bar{\pi}_i^E/dZ=x_i(1+r')P$, which equals $-P$ using the F.O.C. Therefore $d^2\bar{\pi}_i^E/dZ^2=-(1+s')(1+r')P'$, which is positive since $1+s'$ and $1+r'$ are positive and P' is negative.

3. *Proof of Lemma* 1. $\underline{Z}(Y)$ is zero at least for Y's less than the monopoly output of firm i, x^M since in this case $\bar{Z}(Y)=0$ and $\bar{Z}\geqq\underline{Z}$ Y, is larger than x^M. $\underline{Z}(\bar{X})=\bar{X}$ since $P(\bar{X})=0$ and according to the definition of the function $\underline{Z}(\cdot)$. Now $\underline{Z}$ solves $P(Y)(Y-Z)-\bar{\pi}_i^E(Z)=0$. Therefore $d\underline{Z}/dY=-(P'(Y)(Y-Z)+P(Y))/(P(X^E)-P(Y))$ since $d\bar{\pi}_i^E/dZ=-P(X^E)$ where $X^E=Z+s(Z)+r(Z+s(Z))$. The numerator is negative since it is the marginal profit at the entry preventing output and the denominator is positive since $P(X^E)>P(Y)$. (At $\underline{Z}$ we know that $P(Y)(Y-\underline{Z})=P(X^E)s(Z)$ and $Y-\underline{Z}<s(\underline{Z})$, therefore $P(Y)<P(X^E)$). Furthermore $-P'(Y)(Y-\underline{Z})>P(X^E)$ since we know that $P(X^E)=-s(\underline{Z})P'(X^E)(1+r)$ and $-P'(Y)>-P'(X^E)$ (demand is concave and $Y>X^E$), $Y-\underline{Z}>s(\underline{Z})$ and $1>1+r'$. Therefore $d\underline{Z}/dY>1$. ||

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NOTES

1. Several papers have examined incentives for entry deterrence in oligopolistic situations. Examples of spatial competition and product differentiation are Prescott and Visscher (1977), Schmalensee (1978) and Judd (1983). Nti and Shubik (1981) analyze simultaneous move games of entry with mixed strategies, while Bernheim (1984) considers sequential entry.

2. See Vives (1985) for an analysis of sequential entry.

3. Alternatively one could think of the decision variables of the firms as capacities, with ensuing nonstrategic price competition which makes the market clearing price (where supply meets demand) prevail. Kreps and Scheinkman (1983) show in a two-stage duopoly game (with a particular rationing rule of unsatisfied demand) that competition in capacities followed by strategic price competition yields Cournot outcomes, that is, the market outcome is as if firms competed in quantities. Vives (1983) considers a model where firms choose capacities (corresponding to the efficient scale of operation) first and then a market clearing stage follows.

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