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AGGREGATION OF INFORMATION IN LARGE COURNOT MARKETS

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Consider a homogeneous product market where firms have private information about an uncertain demand parameter and compete in quantities. We examine the convergence properties of Bayesian-Cournot equilibria as the economy is replicated and conclude that large Cournot (or almost competitive) markets do not aggregate information efficiently except possibly when the production technology exhibits constant returns to scale. Even in a competitive market there is a welfare loss with respect to the first best outcome due to incomplete information in general. Nevertheless a competitive market is efficient, taking as given the decentralized private information structure of the economy. Endogenous (and costly) information acquisition is examined and seen to imply that the market outcome always falls short of the first best level with decreasing returns to scale. The results are also shown to be robust to the addition of extra rounds of competition which allows firms to use the information revealed by past prices. Explicit closed form solutions yielding comparative static results are obtained for models characterized by quadratic payoffs and affine information structures.

KEYWORDS: Information aggregation, Cournot limit, Bayesian equilibrium, competitive market, rational expectations, uncertainty, information acquisition.

1. INTRODUCTION

DO COMPETITIVE MARKETS aggregate information efficiently? This question has been answered in the affirmative by Wilson (1977) and Milgrom (1979 and 1981) in an auction context and more recently by Palfrey (1985) in a Cournot model. Wilson (1977) considers a first-price auction in which buyers have some private information about the common value of the good to be sold and shows (under certain regularity conditions) that as the number of bidders goes to infinity the maximum bid is almost surely equal to the true value. Milgrom (1979) obtains a necessary and sufficient information condition for convergence in probability to the true value. The Vickrey auction has the same type of limiting properties also (Milgrom (1981)). Palfrey (1985) considers a Cournot market with constant marginal costs where firms have private information about the uncertain demand but are otherwise identical. He shows, under mild regularity conditions on the information structure, that the random market price converges almost surely to the full information competitive price as the number of firms goes to infinity. A similar result has been obtained also by Li (1985). According to these analyses, in the limit competitive market the full information first best outcome prevails even though no firm knows the true state of demand.

All these convergence properties hold in a context where information is exogenous and agents (bidders or firms) do not observe any aggregate market statistic before making their decisions; they can only condition on their private

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information. The results obtained when the market mechanism is an auction or of the Cournot type suggest, as Palfrey puts it, "that in large enough markets, diverse private information does not drastically alter the conclusion reached by the idealized Walrasian model of pricing in a full information world" (Palfrey (1985, p. 80)). In other words there should be no welfare loss with respect to the full information first best in competitive markets with incomplete information.

The purpose of this paper is to examine the above conjecture in the context of the Cournot model. We will examine the asymptotic properties of Bayesian-Cournot equilibria under different assumptions about technology, the information acquisition process, and market observables. Two main conclusions emerge from our analysis (with the possible exception of constant returns environments): (a) there is a welfare loss with respect to the first best in large Cournot (or almost competitive) markets with incomplete information, and (b) a competitive market is second best efficient when taking as given the decentralized private information structure of the economy.

Conclusion (a) is based on a very simple idea. Consider a large Cournot or almost competitive market where firms receive private (and costless) noisy signals about the uncertain demand and have identical decreasing returns to scale technologies. A necessary condition for full information first best efficiency is that conditional on the true state of demand identical firms produce at the same marginal cost, but this is incompatible with firms having different appraisals of demand when marginal costs are increasing. In purely statistical terms there simply do not exist decentralized production strategies (equilibrium or not) which yield asymptotically efficient behavior. Therefore, there is in general a welfare loss due to incomplete information in a competitive market except if marginal costs are constant. With constant returns to scale only total output matters for welfare purposes and, again in purely statistical terms, a necessary condition for the market to be efficient is that there exist an unbiased estimator of the first best supply. Palfrey and Li's regularity assumptions on the information structure do imply the existence of such an estimator.

In general, then, a large Cournot market will not aggregate information efficiently although if firms were to pool their information the aggregate statistic would be a perfect signal and we would obtain the full information Walrasian outcome.[2] Conclusion (b) makes the point that if we insist on decentralized decision making the market does as well as possible. More precisely, the market works like a team where decentralized production rules are assigned to firms to maximize expected total surplus. This result follows easily from Radner's work on team theory (Radner (1962)).

Considering models with liner demand, quadratic costs, and affine information structure we are able to obtain closed-form solutions and to compute the welfare loss explicitly, showing that it is decreasing with the precision of the information of the firms and increasing with the basic uncertainty of demand.

[2] The incentives for information sharing in oligopoly models with uncertain demand have been analyzed by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985), and Li (1985) among others.

The inefficiency of large Cournot markets with private information is reinforced when considering endogenous acquisition of information.[3] In this case the first best outcome with decreasing returns to scale can only be attained if the cost of information is zero. Otherwise there is always a welfare loss. Nevertheless, as before, a planner could not improve upon the market if only decentralized strategies are feasible. In a competitive market with negligible agents a firm has the right (second best) incentives to acquire information. We show that the equilibrium precision of information purchased is declining in its cost and increasing in the prior variability of the uncertain demand. With constant returns to scale the analysis is more delicate. No equilibrium exists with costly information acquisition in a purely competitive market. Firms cannot make any profits at the market stage contingent on the information they receive (if they did they would expand output indefinitely). Therefore no equilibrium with positive research is possible. But if no firm does research then there are enormous incentives for a firm to buy information and consequently no equilibrium exists. Nevertheless this argument is no longer valid if firms retain some (maybe very small) monopoly power. In fact we can show that by replicating appropriately the market, so that monopoly power vanishes, equilibrium with endogenous information acquisition exists and approaches the first best outcome.

We may wonder whether the inefficiency arises because we have considered only one-shot competition in the market and therefore we have not given firms the opportunity to learn about demand conditions from prices. We claim that this is not the case: our results can be shown to be robust to the addition of extra rounds of market competition in which the information revealed by past prices can be used by firms. Although prices will be fully revealing in the competitive economy, in the interim period (firms can learn only from *past* prices) there will be a welfare loss and the incentives to purchase information will be unchanged.

2. A METHODOLOGICAL NOTE AND OUTLINE OF THE PAPER

It is our contention that the appropriate method to analyze the convergence properties of the Cournot mechanism under incomplete information is to consider replicated markets where demand is replicated at the same time that the number of firms increases since in this manner a well defined competitive limit market is obtained. This methodology is in line with the literature on large Cournot markets with complete information (see Novshek (1980)), and with the view that the continuum model is the appropriate formalization of a competitive economy (see Aumann (1964)). In the course of our analysis we will highlight the importance of the replica assumptions. Furthermore, reasoning in the limit economy it is very easy to understand the statistical reasons why a competitive market with incomplete information will not aggregate information efficiently in general and to characterize the equilibrium and its second best properties. This is what is done in Section 3.

[3] It has already been noted that costly information acquisition may change the convergence properties of the market mechanism be it auction-type (Matthews (1984)) or Cournot-type (Li, McKelvey, and Page (1985)).

The rest of the paper considers models with quadratic payoffs and affine information structure which yield computable Bayesian-Cournot equilibria and parameterize the precision of the information of firms. In Section 4 we check in the context of this model that the Bayesian (competitive) equilibrium of the limit economy is not an artifact of our continuum specification but the limit of Bayesian-Cournot equilibria of the replica markets. Furthermore closed-form solutions are derived for the competitive equilibrium and the associated welfare loss and comparative statics analysis with respect to parameters in the information structure is performed.

Section 5 deals with endogenous information acquisition. We consider a two-stage game where first firms decide how much (precision of) information to purchase and then compete in the market contingent on the chosen precisions. Again closed-form solutions are obtained for the equilibrium in the continuum economy (with decreasing returns to scale) and it is checked that this equilibrium is the limit of a sequence of unique subgame perfect equilibria of the replica markets. Comparative statics and welfare analysis of the competitive equilibrium follow easily.

In Section 6 we examine the robustness of the above results to a situation where firms can learn from past prices. A third stage where firms compete again after having observed the second stage market price is added to the model of Section 5. Concluding remarks follow.

3. AN HEURISTIC ARGUMENT IN A COMPETITIVE MARKET

Consider a competitive market with downward sloping inverse demand given by $p = P(x;\, \theta)$ where x is per capita output and θ a random parameter. Firm i receives a noisy estimate of θ, s_i. The signals received by firms are independently and identically distributed given θ and, without loss of generality, are unbiased, $E(s_i|\theta) = \theta$. There is a continuum of firms indexed by $i \in [0,1]$, each one of them with strictly convex twice-continuously differentiable variable costs given by the function $C(\cdot)$. There are no fixed costs. We will make the convention that the Strong Law of Large Numbers (S.L.L.N.) holds for our continuum economy.[4] That is, given θ, the average signal equals θ almost surely (a.s.). The arguments in this section should be viewed as heuristic; in the other sections we will deal with the continuum case as a limit of finite economies.

In this situation a production strategy for firm i is a function $x_i(\cdot)$ which associates an output to the signal received. Suppose thus that we have a market equilibrium characterized by $x_i(\cdot)$, $i \in [0,1]$; that is, $x_i(\cdot)$, $i \in [0,1]$, is a Bayesian Nash equilibrium of our continuum economy.[5] $x_i(s_i)$ must maximize $E(\pi_i|s_i) = x_i E(P(\tilde{x};\, \theta)|s_i) - C(x_i)$ where $\tilde{x}$ is average output, $\tilde{x} = \int x_i(s_i)di$. Note that the

[4] Note that we have a continuum of iid random variables according to our specification. Judd (1985) has shown that this assumption can be made consistent with measure theory.

[5] See Schmeidler (1973) for a formulation of Nash equilibrium in nonatomic games with finite strategy spaces and Mas-Colell (1986) for a reformalization of the model in terms of distributions with an application to games of incomplete information.

equilibrium must be symmetric, $x_i(s_i) = x(s_i)$ for all i, since the cost function is strictly convex and identical for all firms and signals are iid (given θ). Consequently the random variables $x(s_i)$ will be iid (given θ) and their average $\tilde{x}(\theta)$ will be nonrandom and equal[6] to $E(x(s_i)|\theta)$ according to the S.L.L.N. An interior equilibrium $x(\cdot)$ (that is, one in which $x(s_i) > 0$ a.s.) is thus characterized by $E(P(\tilde{x}; \theta)|s_i) = C'(x(s_i))$; given that firm i has received signal s_i the expected market price must equal marginal production costs.

How does the market outcome compare with the full information first best where total surplus (per capita) is maximized contingent on the true value of θ? Given θ total surplus (per capita) with average production x is $TS(x; \theta) = \int_0^x P(z; \theta)\,dz - C(x)$ and, obviously, first best production $x^0(\theta)$ is given by the unique x which solves $P(x; \theta) = C'(x)$. If firms were able to pool their private signals they could condition their production to the average signal, which equals θ a.s., and attain the first best by producing $x^0(\theta)$. Will a competitive market, where each firm can condition its production only on its private information, replicate the first best outcome?

A necessary condition for *any* (symmetric) production strategy $x(\cdot)$ to be first best optimal is that, conditional on θ, identical firms produce at the same marginal cost, namely, $C'(x(s_i)) = C'(x^0(\theta))$ a.s. but with increasing marginal costs this can happen only if $x(s_i) = x^0(\theta)$ a.s., which is basically the perfect information case. Therefore we should expect a welfare loss in a competitive market with noisy signals since a production strategy which is not based on the average signal cannot attain the first best. Proposition 1 below states the result and a more formal proof follows.

PROPOSITION 1: *In a competitive market with symmetric firms with strictly convex costs which receive private noisy signals about an uncertain demand parameter, there is a welfare loss with respect to the full information first best.*

PROOF: We show that no symmetric production strategy $x(\cdot)$, and therefore in particular no competitive production strategy, can attain the first best. The market expected total surplus (per capita) contingent on θ is given by

$$E(TS|\theta) = \int_0^{\tilde{x}(\theta)} P(z; \theta)\,dz - E\big(C\big(x(s_i)\big)|\theta\big).$$

This is strictly less than the first best $TS(x^0(\theta); \theta)$ since $TS(x^0(\theta); \theta) \geqslant TS(\tilde{x}(\theta); \theta) > E(TS|\theta)$. The first inequality being true since x^0 is the first best, the second since the cost function is strictly convex, $\tilde{x}(\theta) = E(x(s_i)|\theta)$, the signals are noisy (which means that given θ, $x(s_i)$ is still random) and in consequence $C(\tilde{x}(\theta)) < E(C(x(s_i))|\theta)$. *Q.E.D.*

In this context and in general a *necessary condition for the market outcome to be first best optimal is that marginal costs be constant*. Firms will produce then at the

[6] Equality of random variables has to be understood to hold almost surely. We will not insist on this always.

same marginal cost by assumption. This is the situation considered by Palfrey (1985). He finds that if the information structure is "regular enough" first best efficiency is achieved in the competitive limit. The intuition of the result is simple enough. Suppose that in the above described context marginal costs are zero (without loss of generality) and that inverse demand intersects the axis. Firm i will maximize $E(\pi_i|s_i) = x_i E(P(\tilde{x};\, \theta)|s_i)$ where, as before, $\tilde{x}$ is average output. Obviously in this constant returns to scale context a necessary condition for an interior equilibrium to exist is that $E(P(\tilde{x};\, \theta)|s_i) = 0$ for almost all s_i. Looking at symmetric equilibria, $x_i(s_i) = x(s_i)$ for all i, we know that average production given θ is nonrandom, $\tilde{x}(\theta)$ and therefore $E(P(\tilde{x};\, \theta)|\theta) = P((\tilde{x};\, \theta)|\theta)$. For first best efficiency we need: $E(P(\tilde{x}(\theta), \theta)|s_i) = 0$ for almost all s_i to imply that $P(\tilde{x}(\theta), \theta) = 0$ for almost all θ.

Palfrey's result states that if the signal and parameter spaces are finite and the likelihood matrix is of full rank (and demand satisfies some mild regularity conditions), then symmetric interior competitive equilibria are first best optimal. This is easily understood with a two-point support example: θ may take two values, $\underline{\theta}$ or $\bar{\theta}$, $0 < \theta \leqslant \bar{\theta}$, with equal prior probability. Firm i may receive a low ($\underline{s}$) or a high ($\bar{s}$) signal about θ with likelihood $\Pr(\bar{s}|\bar{\theta}) = \Pr(\underline{s}|\underline{\theta}) = l$, where $\frac{1}{2} \leqslant l \leqslant 1$. If $l = \frac{1}{2}$ the signal is uninformative; if $l = 1$ it is perfectly informative. Let $\bar{p}$ and $\underline{p}$ be respectively the equilibrium prices when demand is high ($\bar{\theta}$) and low ($\underline{\theta}$). Then

$$\begin{bmatrix} E(p|\bar{s}) \\ E(p|\underline{s}) \end{bmatrix} = \begin{bmatrix} l & 1-l \\ 1-l & l \end{bmatrix} \begin{bmatrix} \bar{p} \\ \underline{p} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

If the likelihood matrix is of full rank, i.e., if $l > \frac{1}{2}$ then $\bar{p} = \underline{p} = 0$ and the symmetric interior equilibrium is efficient.

In the constant returns case in order for the first best optimum to be achieved it is only necessary that total output (conditional on θ) be $x^0(\theta)$. Therefore, a necessary condition for (Bayesian) competitive equilibria to achieve the first best is that there exist some nonnegative valued function $x(\cdot)$ such that $E(x(s_i)|\theta) = x^0(\theta)$ for all θ. That is, the function $x(\cdot)$, the firm's output strategy, must be an unbiased estimate of the first best supply $x^0(\theta)$. In this case $x(\cdot)$ is a Bayesian competitive equilibrium since $P(x^0(\theta), \theta) = 0$ and $\tilde{x}(\theta) = E(x(s_i)|\theta)$. Palfrey's conditions again imply that this function exists.

We can view the above results in a purely statistical way. With decreasing returns to scale there are no decentralized (production) strategies which yield an efficient outcome if signals are noisy and therefore a competitive market will not attain the full information first best. With constant returns to scale, if there does not exist an unbiased estimator of the first best supply then again there are no decentralized strategies which yield efficiency and a competitive market will fail to attain the first best.[7]

[7]It remains an open question to characterize in general the class of information structures which insure the existence of such an unbiased estimator.

Is the outcome of the competitive market efficient in some sense in general? In fact it will be efficient given the private information structure of the economy. We show below that the competitive market works like a team where each member follows the decision rule $x(\cdot)$ that maximizes expected total surplus when there is private information. That is, the market solves the team problem with expected total surplus as objective function. The team problem with n firms is to find n decision rules $x_i(\cdot)$ which maximize

$$E\{TS(X;\theta)\} = E\left\{\int_0^X P(Z;\theta)\,dZ - \sum_{j=1}^{n} C\big(x_j(s_j)\big)\right\}$$

where $X = \sum_{j=1}^n x_j(s_j)$. (Here X and Z denote *total* output.) Radner has shown that, under some regularity conditions (concavity and differentiability of the team function among them), the decision rules $(x_1, \dots, x_n)$ are globally optimal if and only if they are person-by-person optimal, that is if $(x_1, \dots, x_n)$ cannot be improved by changing x_i for some i alone (Radner (1962, Theorem 1)). In our case TS is concave in the output of the firms since demand is downward sloping and costs are strictly convex. The optimal decision rules will be determined then by $E((\partial TS/\partial x_i)|s_i) = 0$ which is equivalent to the competitive condition $E(P(X;\theta)|s_i) = C'(x_i(s_i))$, $i = 1, \dots, n$, that we have obtained before. Proposition 2 states the result.

PROPOSITION 2: *A competitive market works like a team which chooses decentralized production rules for the firms so as to maximize expected total surplus.*

4. QUADRATIC COSTS AND AFFINE INFORMATION STRUCTURE

Consider a market with inverse demand given by $p = \theta - \beta\tilde{x}$, where $\beta > 0$, $\tilde{x}$ is per capita output, and θ a possibly random intercept. Firm i has a quadratic cost function $C(x_i) = mx_i + \lambda x_i^2$, where m is possibly random and λ is a nonnegative constant. In our model only the level of $\theta - m$ matters and therefore without loss of generality we will consider prices net of m.

Suppose now that θ is distributed according to a prior density with finite variance σ^2 and mean μ. Firm i receives a signal s_i such that $s_i = \theta + \varepsilon_i$ where ε_i is a noise term with zero mean, variance v_i and with $\operatorname{cov}(\theta, \varepsilon_i) = 0$. $1/v_i$ represents the precision of the signal. The signals received by the firms are independent conditional on θ and furthermore we assume that $E(\theta|s_i)$ is affine in s_i. These assumptions imply that $E(\theta|s_i) = (1 - t_i)\mu + t_i s_i$, where $t_i = \sigma^2/(\sigma^2 + v_i)$ and $E(s_j|s_i) = E(\theta|s_i)$, $\operatorname{cov}(s_i, s_j) = \operatorname{cov}(s_i, \theta) = \sigma^2$ for all $j \neq i$ and all i. (See Ericson (1969).) Signal s_i is more precise than signal s_i' if v_i is less than v_i', in which case the mean squared prediction error, $E\{\theta - E(\theta|s_i)\}^2$, is smaller. (See Vives (1984).) Sometimes we will refer to t_i as the precision of the information, keeping σ^2 implicitly fixed. In this case as v_i ranges from 0 to ∞, t_i ranges from 1 to 0.

The typical example of an affine information structure is for the prior and the likelihood densities to be Normal. s_i may be the average of a fixed number of independent observations from a Normal distribution with mean θ and variable variance or the average of a variable number of observations n_i from a Normal distribution with fixed variance. The precision of the information is proportional to the number of observations n_i. Other examples where the sample mean is a sufficient statistic for θ and an affine information structure obtains include cases in which the observations are conditionally independent Binomial, Negative Binomial, Poisson, Gamma or Exponential when assigned natural conjugate priors. (See DeGroot (1970).) For example, the pairs Beta-Binomial and Gamma-Poisson work. In these examples, as well as in the Normal case, s is more precise (weakly) than $s'(v \leqslant v')$ if and only if s is more informative than s' in the Blackwell sense since a more precise signal means a larger sample.

We are interested in large markets with many firms. We will study the n-firm case and its limit as n goes to infinity. It will be convenient to argue again in the limit competitive case first and then check that the competitive equilibrium is indeed the limit of a unique sequence of Bayesian-Cournot equilibria of the finite economies.

Equilibrium

There is a continuum of firms indexed by i, $i \in [0,1]$. Firm i receives a signal s_i about the uncertain demand intercept θ. The strategy of firm i is thus a function $x_i(\cdot)$ which associates an output to the signal received. We will look for Bayesian Nash equilibria of this market. We assume that the signals received by the firms are identically distributed conditional on θ. In this case $v_i = v$ and $t_i = t$ for all i.

Conditional on having received s_i firm i will maximize $E(\pi_i|s_i) = E(p|s_i)x_i - \lambda x_i^2$, where $p = \theta - \beta\tilde{x}$ and $\tilde{x}$ is average production, $\tilde{x} = \int x_i(s_i)\,di$. Given $\tilde{x}$ the optimal response of firm i is $x_i(s_i) = E(p|s_i)/2\lambda$ provided $\lambda > 0$, which will be the same for all firms and therefore equilibria will be symmetric. We have then that $x_i = x$ for all i and $E(\tilde{x}|s_i) = E(x(s_j)|s_i)$, $j \neq i$. The first order condition yields $2\lambda x(s_i) = E(\theta|s_i) - \beta E(x(s_j)|s_i)$ and it is easily checked that the unique affine function $x(\cdot)$ which satisfies it is given by $x(s_i) = a(s_i - \mu) + b\mu$ where $a = t/(2\lambda + \beta t)$ and $b = 1/(2\lambda + \beta)$. Average output given θ, $\tilde{x}(\theta)$, equals then $a(\theta - \mu) + b\mu$ since the average signal conditional on θ equals θ a.s.

When firms receive no information ($t = 0$), then $a = 0$ and production is constant at the level $b\mu$; when firms receive perfect information ($t = 1$), then $x(s_i) = as_i$. As t varies from 0 to 1 the slope of the equilibrium strategy, a, increases from 0 to b (see Figure 1). When $\lambda = 0$ the equilibrium requires that $E(p|s_i) = 0$; restricting attention to symmetric equilibria, we get that $x(s_i) = s_i/\beta$ provided that $t > 0$; that is, the above formula is valid letting $\lambda = 0$, since then $a = b = 1/\beta$. If $t = 0$ and the signals are uninformative, then $x(s_i) = \mu/\beta$. Note that in the constant returns to scale case when the signals are informative ($t > 0$), the equilibrium strategies $x(s_i) = s_i/\beta$ do not depend on t, the precision of the information.

The equilibrium we have described is not an artifact of our continuum specification; it is the limit of a unique sequence of Bayesian-Cournot equilibria

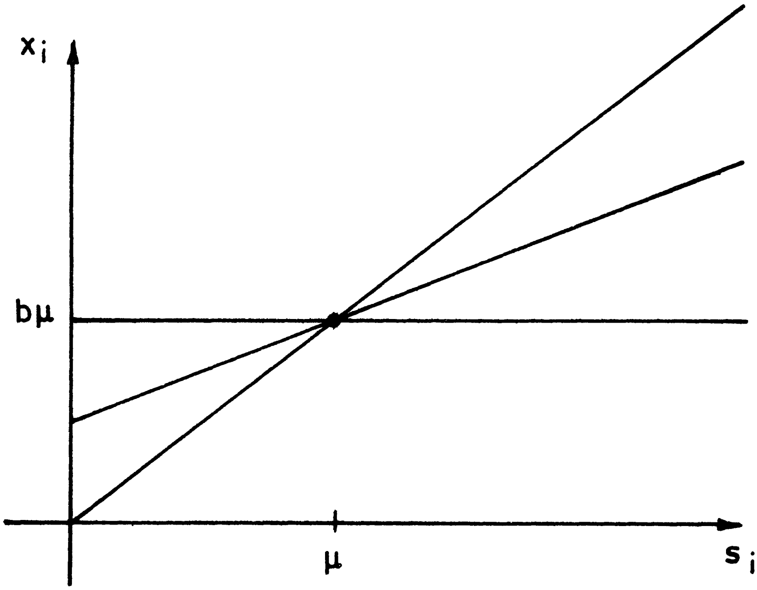

FIGURE 1.—Equilibrium strategy of firm i.

as we replicate the number of firms and consumers in the market. The n-replica market consists of n-firms facing an inverse demand given by $p = \theta - \beta X/n$ where X/n is per capita output. Proposition 3 states the result.

PROPOSITION 3: *Given* $\lambda \geq 0$ *and* $t \in [0,1]$ *the equilibrium of the continuum economy where firm i produces according to* $x(s_i) = a(s_i - \mu) + b\mu$ *with* $a = t/(2\lambda + \beta t)$ $(a = 0$ *if* $\lambda = t = 0)$ *and* $b = 1/(2\lambda + \beta)$ *is the limit of a unique sequence of Bayesian-Cournot equilibria of the n-replica markets,* $x_n(s_i) = a_n(s_i - \mu) + b_n\mu$, *where* $a_{n_n} \to a$, $b_{n_n} \to b$ *and the average output* $\Sigma x_n(s_i)/n$ *given* θ *converges almost surely to* $\tilde{x}(\theta) = a(\theta - \mu) + b\mu$.

PROOF: According to Lemma 1 in the Appendix and letting $t_i = t$ for all i there is a unique and symmetric Bayesian-Cournot equilibrium of the n-replica market given by $x_n(s_i) = a_n(s_i - \mu) + b_n\mu$ where

$$a_n = \left(2\left(\frac{\beta}{n} + \lambda\right) - \frac{\beta}{n}t\right)t/D_n(t), \qquad b_n = \left(\frac{\beta}{n} + 2\lambda\right)/D_n(1), \quad \text{and}$$

$$D_n(t) = 4\left(\frac{\beta}{n} + \lambda\right)^2 + 2\left(\frac{\beta}{n} + \lambda\right)(n-2)\frac{\beta}{n}t - \left(\frac{\beta}{n}\right)^2 t^2(n-1).$$

Therefore as n goes to infinity $a_{n_n} \to a$ and $b_{n_n} \to b$. Almost sure convergence follows from the Strong Law of Large Numbers since, given θ, the signals are iid and $E(s_i|\theta) = \theta$. Therefore, given θ, $\bar{s}_n = \Sigma_i s_i/n$ converges almost surely to θ and consequently $\Sigma_i x_n(s_i)/n = a_n(\bar{s}_n - \mu) + b_n\mu$ converges a.s. to $a(\theta - \mu) + b\mu$.

Q.E.D.

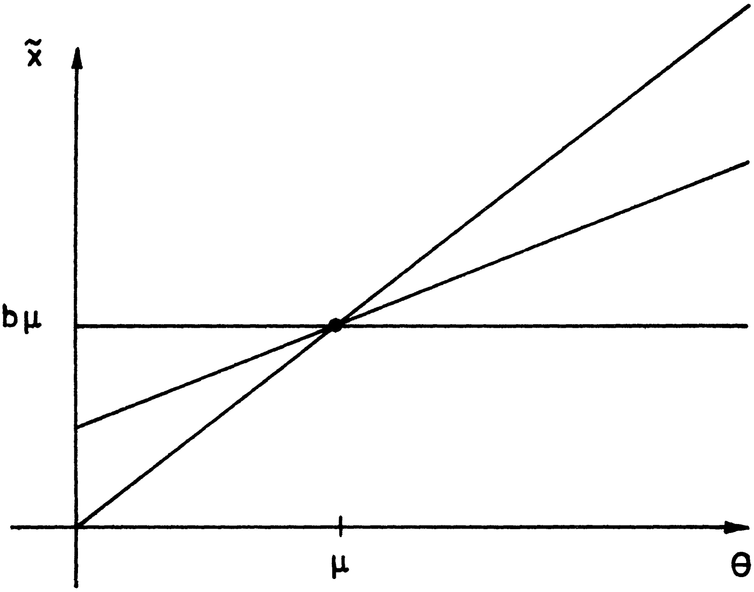

FIGURE 2.—Average production.

Welfare

Per capita total surplus with all firms producing the output $\tilde{x}$ is given by

$$TS(\tilde{x};\theta)=\theta\tilde{x}-\left(\frac{\beta}{2}+\lambda\right)\tilde{x}^2.$$

Full information first best production is therefore $x^0(\theta)=b\theta$. Average production in the competitive economy (given θ) is $\tilde{x}(\theta)=a(\theta-\mu)+b\mu$ which is equal to the full information first best production $x^0(\theta)$ only if $t=1$ (perfect information) or if $\lambda=0$ and $t>0$ (constant returns to scale). Recall that as t goes from 0 to 1 a increases from 0 to b. For $t<1$ and $\lambda>0$, as it is obvious from Figure 2, if θ is high the market underproduces and if θ is low the market overproduces with respect to the full information first best. In the constant returns to scale case average production $\tilde{x}(\theta)$ is independent of the precision of the information t and contingent on θ the market produces the right amount.

Expected total surplus at the first best is $ETS^0=E\theta^2/2$. Market expected total surplus ETS equals $(a\sigma^2+b\mu^2)/2$ and therefore the welfare loss equals $(b-a)\sigma^2/2$. The market ETS can be computed as the sum of per capita consumer surplus $\beta E(\tilde{x}(\theta))^2/2$ and per capita (or firm) profits $E\pi_i=\lambda E(x(s_i))^2$, which equal $\lambda(a^2\mathrm{Var}(s_i)+b^2\mu^2)$. Table I gives the principal magnitudes of the competitive economy.

The welfare loss with respect to the full information first best is positive when information is not perfect except if marginal costs are constant. In the previous section we have claimed that a competitive market would solve a team problem with private information and expected total surplus as objective function. Lemma

TABLE I

Equilibrium strategy of firm i:	$x(s_i)=a(s_i-\mu)+b\mu$
Expected profits of firm i:	$E\pi_i=\lambda(a^2\mathrm{Var}(s_t)+b^2\mu^2)$
Expected consumer surplus:	$ECS=\beta(a^2\sigma^2+b^2\mu^2)/2$
Expected total surplus:	$ETS=\frac{1}{2}(a\sigma^2+b\mu^2)$
Average production given θ:	$\tilde{x}(\theta)=a(\theta-\mu)+b\mu$
Full information first best production:	$x^0(\theta)=b\theta$
Welfare loss:	$WL=\frac{1}{2}(b-a)\sigma^2$

where $a=\dfrac{t}{2\lambda+\beta t}(=0 \text{ if } \lambda=t=0)$ and $b=\dfrac{1}{2\lambda+\beta}$.

1 in the Appendix shows that the n-replica market solves also a team problem but using

$$\frac{1}{n}G_n=\frac{1}{n}\left(\theta\sum_j x_j-\left(\frac{\beta}{n}+\lambda\right)\sum_j x_j^2-\frac{\beta}{2n}\sum_{i\neq j}x_ix_j\right)$$

as objective function instead of total per capita surplus

$$\frac{1}{n}TS_n=\frac{1}{n}\left(\theta\sum_j x_j-\frac{\beta}{2n}\left(\sum_j x_j\right)^2-\lambda\sum_j x_j^2\right).$$

The difference in both functions is $1/n(TS_n-G_n)=\beta/2n^2\Sigma_j x_j^2$ (noting that $(\Sigma_j x_j)^2=\Sigma_j x_j^2+\Sigma_{i\neq j}x_ix_j$). Considering square integrable sequences $(x_i)_{i=1}^{\infty}$ both TS_n/n and G_n/n converge to per capita total surplus in the continuum economy, $TS=\theta\tilde{x}-\beta\tilde{x}^2/2-\lambda\int x_j^2\,dj$, as n goes to infinity, since $\Sigma_j x_j^2/n\to_n \int x_j^2\,dj$ implies that $(TS_n-G_n)/n\to_n 0$. A competitive economy is efficient as long as the individual productive units have to rely on their private information as we have claimed in Proposition 2 above.

How does ETS change with variations in the precision of information, $1/v$, and in the basic uncertainty of demand, σ^2? One would hope that improvements in the precision of information would increase ETS (reducing the welfare loss). We will see that this is indeed the case. Increasing the basic uncertainty of demand increases ETS but ETS^0, the first best ETS, increases by more and the welfare loss is increasing in σ^2.

PROPOSITION 4: (a) *With increasing marginal costs* ($\lambda>0$) *the welfare loss increases with the basic uncertainty of demand* (σ^2) *and decreases with the precision of the information* ($1/v$); (b) *with constant marginal costs* ($\lambda=0$) *the welfare loss is zero provided that the signals are informative* ($v<\infty$ *or equivalently* $t>0$). *Otherwise the welfare loss is increasing in* σ^2.

PROOF: $WL = \frac{1}{2}(b-a)\sigma^2$. After some computations we get

$$\frac{\partial WL}{\partial \sigma^2} = \frac{1}{2}\left(b - a - \frac{2\lambda t(1-t)}{(2\lambda + \beta t)^2}\right) = \frac{1}{2b}(b^2 - a^2).$$

It is positive if and only if $b > a$. This is the case if $\lambda > 0$ and $t < 1$ or if $\lambda = 0$ and $t = 0$. If $\lambda = 0$ and $t > 0$ then $b = a$ and $WL = 0$. $WL = ETS^0 - ETS$ decreases with $1/v$ for fixed σ^2: looking at Table I we see that by increasing t we increase ETS since a is increasing in t (recall that $t = \sigma^2/(\sigma^2 + v)$) and ETS^0 stays constant. *Q.E.D.*

In case (a), why does ETS increase with the precision of information? First, per capita expected consumer surplus (ECS) increases with the precision of information (t) since consumer surplus is a convex function of average output ($CS = \beta\tilde{x}^2/2$) and increases in t increase the slope of $\tilde{x}$ (see Figure 2) and make it more variable. Second, expected profits may increase or decrease with t (see Vives (1986b) for a complete analysis) but the effect on welfare of the consumers always dominates the profit effect. On the other hand increases in σ^2 benefit both firms and consumers but have a larger effect on the first best than on the market level.

We have claimed that a welfare loss exists with incomplete information and strictly convex costs and furthermore that the inefficiency is inversely related to the precision of information. It is not difficult to imagine that the same will be true replacing quadratic costs by capacity limits; after all, a capacity limit can be interpreted as a very steep marginal cost schedule. This is formally checked in a model with constant marginal costs and capacity limits with a finite support information structure (see Vives (1986a, Section 4)). The qualitative properties of the capacity model are identical to the one analyzed in this section.

5. INFORMATION ACQUISITION

Do the inefficiencies that we found in the case in which the precision of information was given carry over when information has to be acquired at a cost? The analysis of Mathews (1984), in an auction context, and of Li, McKelvey and Page (1985), in a Cournot context, support the view that costly information acquisition changes the convergence properties of the market mechanism.

We will consider the quadratic-affine model of the last section and assume that firms can acquire additional observations at a constant cost. The precision of information of firm i, $1/v_i$, is proportional to the sample the firm obtains.[8] We will suppose then that the cost to firm i of obtaining information with precision $1/v_i$ is c/v_i where c is a positive constant. Given our information structure it will be more convenient to work with t_i. Recalling that $t_i = \sigma^2/(\sigma^2 + v_i)$, for given σ^2

[8] Li, McKelvey, and Page (1985) model information acquisition similarly. Milgrom (1981) and Milgrom and Weber (1982) consider information acquisition in auctions.

the cost of obtaining information of precision t_i will be

$$\omega(t_i) = \frac{c}{\sigma^2} \frac{t_i}{1 - t_i}:$$

to purchase information of zero precision ($t_i = 0$) is costless and to purchase perfect information ($t_i = 1$) is infinitely costly. We model information acquisition as follows. At the first stage firm i purchases information with precision t_i (or $1/v_i$). At the second stage firms receive their private signals and compete in quantities and a Bayesian-Cournot equilibrium obtains. The t_i's are common knowledge at the second stage (in a competitive market though a firm needs only to know the average precision). We want to restrict attention to subgame perfect equilibria of the two-stage game.

Equilibrium

Suppose for a moment that firms have already purchased information, firm i having an information precision of t_i. We are in a (competitive) market with a continuum of firms indexed by $i \in [0,1]$, and we assume that the first stage choices of firms are representable by a measurable function $t:[0,1] \to [0,1]$ where $t(i) = t_i \in [0,1]$. Let $\bar{t}$ denote the average information precision in the market. Assuming that $\lambda > 0$ (increasing marginal costs) we can find a competitive equilibrium of this (continuum) economy as in Section 4. Let us postulate $x_i(s_i) = a_i(s_i - \mu) + b\mu$ as the equilibrium strategy of firm i. We have then that the expected value of average production $\tilde{x}(\tilde{x} = \int x_j(s_j)\, dj)$ given s_i is $E(\tilde{x}|s_i) = \bar{a}(E(\theta|s_i) - \mu) + b\mu$ where $\bar{a} = \int a_j dj$. We can use then the first order condition $E(p|s_i) = 2\lambda x_i(s_i)$, where $E(p|s_i) = E(\theta|s_i) - \beta E(\tilde{x}|s_i)$, to solve for the parameters a_i and b getting $a_i = t_i/(2\lambda + \beta\bar{t})$ and $b = 1/(2\lambda + \beta)$. Expected profits for firm i, $E\pi_i$, will equal $\lambda E(x_i(s_i))^2$ and will be increasing in the precision of the information of the firm and decreasing in the average precision in the market since $E(x_i(s_i))^2 = b^2\mu^2 + t_i\sigma^2/(2\lambda + \beta\bar{t})^2$.

At the first stage firms have to decide how much information to purchase. Given equilibrium expected profits $E\pi_i$ at the second stage the payoff to firm i of purchasing precision t_i (given σ^2) will be $P_i(t_i, \bar{t}) = E\pi_i - \omega(t_i)$ which equals

$$\lambda\left(\frac{t_i}{(2\lambda + \beta\bar{t})^2}\sigma^2 + \frac{1}{(2\lambda + \beta)^2}\mu^2\right) - \frac{c}{\sigma^2}\frac{t_i}{1 - t_i}.$$

Note that since we are in a competitive situation the payoff to firm i only depends on the average precision of information in the market ($\bar{t}$), which firm i cannot influence. Furthermore, the marginal benefit to a firm to acquire information will be decreasing in $\bar{t}$, the (average) amount of information purchased by the other firms. More information in the market reduces the incentive for any firm to do research. The intuitive idea is that a firm wants information to estimate the market price. When firms have better information ($\bar{t}$ high) the market price varies less since producers match better production decisions with the changing demand. Consequently for high $\bar{t}$ an individual firm has less incentive to acquire information. The first order condition (which is sufficient) will be given by

$$\frac{\partial P_i}{\partial t_i} = \frac{\lambda \sigma^2}{(2\lambda + \beta \bar{t})^2} - \frac{c}{\sigma^2} \frac{1}{(1 - t_i)^2} \leqslant 0,$$

with equality if $t_i > 0$. Since the equilibrium will be symmetric we let $t_i = \bar{t} = t$ and solve for t. We get

$$t^* = \max\left\{0, \frac{\sigma^2 - 2\sqrt{c\lambda}}{\beta\sqrt{c/\lambda} + \sigma^2}\right\},$$

which in terms of the precision $1/v^*$ is

$$\max\left\{0, \frac{\sigma^2 - 2\sqrt{c\lambda}}{\sigma^2(2\lambda + \beta)}\sqrt{\lambda/c}\right\}.$$

In the constant returns to scale case ($\lambda = 0$) there is a difficulty: no equilibrium exists for the two-stage game if information acquisition is costly.[9] This fact is easily understood. Notice first that in equilibrium it must be the case that no firm makes any profit at the market stage. With constant returns to scale, equilibrium at the second stage implies that the expected value of the market price conditional on the signal received by any firm be nonpositive. Otherwise the firm would expand indefinitely. Therefore firms cannot purchase any information in equilibrium since this would imply negative profits. Furthermore zero expenditures on research are not consistent with equilibrium either. If no firm purchases information there are enormous incentives for any firm to get some information and make unbounded profits under constant returns to scale. With no firm acquiring information the average output in the market is nonrandom and any firm with some information could make unbounded profits by shutting down operations if the expected value of the market price conditional on the received signal is nonpositive and producing an infinite amount otherwise.[10]

The fact that no equilibrium with endogenous information acquisition exists under constant returns is nevertheless particular to the continuum model. With a *finite* number (n) of firms the above argument does not hold and it is possible to show that an equilibrium exists. For n large enough the equilibrium involves positive expenditures on research which are declining in n, converging to zero as n goes to infinity. The key observation is that with a finite number of firms

[9] On this issue I acknowledge a very helpful conversation with Paul Milgrom.

[10] Notice that the nonexistence argument does not depend on our quadratic-affine specification. Our nonexistence result is reminiscent of Wilson's analysis of informational economies of scale (Wilson (1974)). Wilson finds that the compounding of constant returns in physical production with information acquisition yields unbounded returns in models where the choice of the decision variable applies the same way to the production of all units of output. Obviously, this implies nonexistence of a competitive equilibrium. Nevertheless in the model in our paper scale (production) is chosen optimally in response to information and therefore it is a "non-example" in Wilson's terminology.

expected profits can be positive in the market equilibrium and this way incentives to acquire information are not destroyed. With a large number of firms, given that a firm's rivals are all purchasing a little bit of information, the optimal response of the firm is also to acquire a little information. It is not optimal to purchase zero since the firm improves its expected profits with some information and it is not optimal to do a lot of research since information is costly and the other firms have some information.

The equilibrium we have computed for the continuum economy with decreasing returns to scale (t^*) is not an artifact of our specification. It is the limit of a unique sequence of subgame perfect equilibria (S.P.E.) of the replica economies consisting of n firms facing an inverse demand $p = \theta - \beta x/n$. In the constant returns to scale case no equilibrium exists with costly research in the continuum economy but zero average expenditure on information ($t^* = 0$) is again the limit of a unique sequence of S.P.E. of the replica economies. When the cost of research is zero, firms obtain perfect information. Proposition 5 states the result and a proof follows.

PROPOSITION 5: *In the continuum economy, each firm purchasing precision,*

$$t^* = \max\left\{0, \frac{\sigma^2 - 2\sqrt{c\lambda}}{\beta\sqrt{c/\lambda} + \sigma^2}\right\}$$

corresponds to the limit as n tends to infinity of a unique sequence of symmetric subgame perfect equilibria of the n-replica markets.

PROOF: Given $\lambda \geqslant 0$ and n firms in the market, consider the second stage where firm i receives a signal of precision t_i. From Lemma 1 in the Appendix the unique Bayesian-Cournot equilibrium of the second stage is given by $x_{in}(s_i) = a_{in}(s_i - \mu) + b_n\mu$ where

$$a_{in} = \frac{n}{\beta}\left(\frac{\gamma_{in}}{1 + \Sigma_j \gamma_{jn}}\right), \quad b_n = \frac{2\lambda + \beta/n}{2(2\lambda + \beta)(\lambda + \beta/n)},$$

and

$$\gamma_{in} = \frac{t_i\beta/n}{2(\lambda + \beta/n) - t_i\beta/n} \qquad (i = 1, \ldots, n).$$

The expected profits of firm i are given by $E\pi_{in} = (\lambda + \beta/n)E(x_{in}(s_i))^2$ and the payoff of the first stage for firm i is $P_{in}(t_1, \ldots, t_n) = E\pi_{in} - \omega(t_i)$. We check in the Appendix that

$$\left.\frac{\partial^2 P_{in}}{\partial t_i^2}\right|_{\frac{\partial P_{in}}{\partial t_i} = 0} < 0$$

(P_{in} strictly quasiconcave in t_i), and that

$$\frac{\partial^2 P_{in}}{\partial t_i \partial t_j} < 0 \quad \text{for} \quad j \neq i$$

(best responses are downward sloping). Therefore there is a unique symmetric equilibrium $t_{in}^* = t_n^*$, $i = 1, \ldots, n$, using a similar argument to Lemma 1 in Li, McKelvey, and Page (1985). Let

$$\phi_n(t) = \left.\frac{\partial P_{in}}{\partial t_i}\right|_{\substack{t_j = t,\\ j=1,\ldots,n}}.$$

The unique symmetric subgame perfect equilibrium of the two-stage game t_n^* solves in t $\phi_n(t) = 0$ provided the solution is positive and equals zero otherwise. Some computations show that

$$\phi_n(t) = \frac{2(\lambda + \beta/n)(1 + (n-1)\gamma) + (n-1)\gamma t\beta/n}{[2(\lambda + \beta/n(1 + (n-1)\gamma) - (n-1)\gamma t\beta/n]^3}\left(\lambda + \frac{\beta}{n}\right)\sigma^2$$
$$- \frac{c}{\sigma^2}\frac{1}{(1-t)^2}$$

where

$$\gamma = \frac{\beta t/n}{2(\lambda + \beta/n) - \beta t/n}.$$

The solution t_n^* will be positive if $\sigma^2 > 2\sqrt{c(\lambda + \beta/n)}$. Letting n tend to infinity one gets from $\phi_n(t) = 0$

$$\frac{c}{\lambda} = \left(\frac{1-t}{2\lambda + \beta t}\sigma^2\right)^2.$$

Solving for t we obtain

$$t = \frac{\sigma^2 - 2\sqrt{c\lambda}}{\sigma^2 + \beta\sqrt{c\lambda}}.$$

We have shown thus that t_n^* converges to

$$t^* = \max\left\{0, \frac{\sigma^2 - 2\sqrt{c\lambda}}{\sigma^2 + \beta\sqrt{c/\lambda}}\right\}$$

as n goes to infinity. *Q.E.D.*

REMARK 1: In the constant returns case it follows from the proof that firms will purchase information ($t_n^* > 0$) if n is large enough (since $\sigma^2 > 2\sqrt{c\beta/n}$ for n large enough). Furthermore it also follows easily from the expression of $\phi_n(t)$ that t_n^* will be decreasing in n for large n and will eventually converge to 0. With many firms and constant returns, all firms purchase a vanishing amount of information. The average precision in the market converges to zero as n goes to infinity.

REMARK 2: From the formula for $1/v^*$ it is immediate that the precision purchased is monotone in its cost. If c is zero, firms get perfect information

($t^* = 1$ or $v^* = 0$); as c increases t^* (or $1/v^*$) decreases monotonically until c is so high that no information is purchased. Similarly, the equilibrium precision is monotonically increasing in the prior variance of demand (σ^2). More uncertainty induces the firms to acquire better information. The relationship between the slope of marginal costs (2λ) and $1/v^*$ is not monotonic. For λ small and for λ large expected profits at the market stage are low and consequently there is low expenditure on research. $1/v^*$ peaks at intermediate values of λ. It is increasing in λ for low values of λ and decreasing for high values.

Welfare

What is the first best outcome with costly acquisition of information? It is just the full information first best. In the continuum economy we can think of a center achieving the full information first best by pooling a continuum of iid signals of zero precision for which $E(s_i|\theta) = \theta$ since (conditional on θ) the true value θ is obtained (almost surely) even if the error terms of the signals have infinite variance (this is just the statement of Kolmogorov's S.L.L.N. for our continuum economy). In terms of the replica markets as n goes to infinity the optimal expenditure on information converges to zero in per capita terms and the precision of the aggregate signal goes to infinity.[11]

In the continuum economy the first best per capita expected total surplus is given by $ETS^0 = bE\theta^2/2$ according to Table I, where $b = 1/(2\lambda + \beta)$. With decreasing returns to scale, a competitive market always falls short of this first best level unless the cost of information is zero. If the competitive market expends a positive per capita amount on information, it cannot attain first best efficiency, which involves zero average expenditure on research. If the market does not buy any information then the precision of signals is zero and again an inefficient outcome obtains. The welfare loss is given by $\frac{1}{2}\ (b-a)\sigma^2 + c/v^*$ where $a = t^*/(2\lambda + \beta t^*)$, v^* and t^* are as in Proposition 5. The welfare loss is always positive except if $c = 0$.

[11] Consider the n-replica market and suppose there is a center which purchases an aggregate signal $\bar{s}_n$ with precision $1/w_n$. Given the pooled signal $\bar{s}_n$ expected per capita total surplus (gross of information costs) conditional on $\bar{s}_n$ is $E(\theta|\bar{s}_n)x - (\beta/2 + \lambda)x^2$ where x is average production. Optimal production is given by $x^0(s_n) = E(\theta|\bar{s}_n)/(\beta + 2\lambda)$. Expected total surplus as a function of the precision of the signal $1/w_n$ is then

$$\frac{n}{2(\beta + 2\lambda)} E\big(E(\theta|\bar{s}_n)\big)^2 - \frac{c}{w_n} = \frac{n}{2(\beta + 2\lambda)}\left(T_n\sigma^2 + \mu^2\right) - \frac{c}{w_n}$$

where $T_n = \sigma^2/(\sigma^2 + w_n)$. The optimal purchase of information precision is easily seen to be

$$\frac{1}{w_n^0} = \max\left\{\left(\frac{\sqrt{n}}{\sqrt{2(\beta + 2\lambda)c}} - \frac{1}{\sigma^2}\right), 0\right\}$$

which converges to infinity as n goes to infinity (and T_n^0 converges to 1 as $n \to \infty$). Nevertheless the expenditure on information in per capita terms converges to zero as n goes to infinity since $(c/nw_n^0) \to_n 0$. In the limit per capita expenditure on information is zero and the full information first best is achieved. We can reinterpret the center in the n-replica market as collecting n signals with precision $1/v_n$ and pooling them to form $\bar{s}_n$.

With *constant returns to scale* the analysis of market optimality is more delicate. We have found that in the continuum limit no equilibrium exists but nevertheless for any finite n equilibrium does exist and involves a vanishing but positive average expenditure on information. The question is: what is the limit of per capita total expected surplus as we replicate the market? From the proof of Proposition 5 and Remark 1 it is easily checked that the total precision of information in the market goes to infinity as we replicate the economy (it is of the order of magnitude $\sqrt{n}$) while the average expenditure on research goes to zero (it is of the order $1/\sqrt{n}$). This implies that in the limit first best efficiency is obtained![12] We can gain some intuition for this result as follows: Fix a large n; we know then that every firm buys a little bit of information. If firms were price takers almost an efficient outcome would obtain according to the results in Section 4; with constant returns (and the affine information structure) the market aggregates information and average expenditure on information is small. By increasing n firms eventually become price takers; they always buy some information, and the average expenditure on research goes to zero. The limit is thus the first best outcome. Li, McKelvey, and Page (1985) consider also information acquisition with a constant return technology but they do not replicate demand when the number of firms increases. They find that the aggregate purchase of information may be positive in the competitive limit even though individual firms purchases are zero. The welfare analysis indicates that the total amount of research undertaken by the market can be either too much, if the cost of research is low, or too little, if the cost of research is high. It is worth remarking that, when examining the convergence properties of equilibria without replicating demand, the "limit" market is not well defined. Consider the certainty Cournot model with constraint marginal costs: if demand is not replicated when increasing the number of firms, individual outputs are zero in the limit but aggregate output equals the competitive output. The contrast of our results with those

[12] With constant returns to scale Lemma 1 gives us the equilibrium strategies given a common precision t:

$$x_n(s_i) = a_n(s_i - \mu) + b\mu$$

with

$$a_n = \frac{nt}{\beta(2 + n(-1)t)} \quad \text{and} \quad b = \frac{1}{2\beta}.$$

Substituting in the expression for per capita total gross surplus and taking expectations, we obtain:

$$EGS_n = \frac{n(2+n)}{2(1+n)^2\beta}\mu^2 + \frac{nt((n-1)t+3)}{((n-1)t+2)^2}\frac{\sigma^2}{2\beta}.$$

Therefore per capita expected total surplus at the market equilibrium t_n^* equals

$$ETS_n = EGS_n - \frac{c}{\sigma^2}\frac{t_n^*}{1-t_n^*}.$$

As n goes to infinity t_n^* converges to zero and nt_n^* to infinity. This implies that $ETS_n \to_n (\mu^2 + \sigma^2)/2\beta$, which is the first best level.

obtained by Li, McKelvey, and Page (1985) highlights the importance of the replica assumptions made when studying convergence issues.

With decreasing returns nevertheless the market does attain the "second best" where expected total surplus is maximized given that firms can base decisions only on their private information. The intuitive explanation is as follows. From Proposition 2 we already know that the market is second best optimal at the second stage, given the expenditures on research. At the first stage there is no private information and with negligible players (a continuum of them) each firm has the right (second best) incentives to purchase information. Proposition 6 states the result.

PROPOSITION 6: *With endogenous information acquisition a competitive market with increasing marginal costs works like a team which chooses expenditures on information and decentralized production rules to maximize total expected surplus.*

PROOF: Let $\lambda > 0$. Contingent on a certain expenditure on research to get a precision of information t we know from Proposition 2 that the market works like a team to maximize expected gross surplus (gross of the cost of information) and therefore both attain $EGS(t) = (a\sigma^2 + b\mu^2)/2$, where $a = t/(2\lambda + \beta t)$ (from Table I). We show now that the market solves the program $\operatorname{Max}_t EGS(t) - \omega(t)$. The market t^* is determined by the symmetric F.O.C. of the problem $\operatorname{Max}_{t_i} E\pi_i - \omega(t_i)$, which yields:

$$\frac{\lambda\sigma^2}{(2\lambda + \beta t)^2} - \omega'(t) \leqslant 0$$

with equality if the solution is positive. This is identical to the F.O.C. of the problem $\operatorname{Max}_t EGS(t) - \omega(t)$, which is also sufficient for a maximum to obtain. *Q.E.D.*

REMARK: According to Proposition 6 there is no room for a planner to improve on market performance taking as given decentralized decision making. This contrasts with Laffont's analysis of rational expectations equilibria under asymmetric information (Laffont (1985)). In his model, with a partially revealing rational expectations equilibrium, there is scope for public intervention since individual private research decisions do not take into account the externality effect on the informativeness of prices. The externality issue arises since in the rational expectations model agents do condition on prices.

As a corollary to Proposition 6 we obtain that the welfare loss with respect to the first best is increasing in the cost of information. This is easily seen since with increasing marginal costs, the competitive market acts as if it were solving the team program $\operatorname{Max}_v \phi(v, c) = EGS(v) - c/v$. Therefore, using the envelope condition,

$$\frac{d\phi(v^*, c)}{dc} = \frac{\partial \phi}{\partial c}(v^*, c).$$

This equals $-1/v^*$, and the net expected total surplus of the market decreases with the cost of information as long as $v^* < \infty$. Proposition 7 summarizes the market performance with respect to the first best with decreasing returns.

PROPOSITION 7: *With endogenous and costly information acquisition a competitive market with decreasing returns always falls short of first best efficiency. Furthermore, the welfare loss is increasing in the cost of information as long as the market expenditure on research is positive.*

It may be thought that the inefficiency arises because we have modelled the market period as a one-shot affair and firms cannot learn about demand conditions from prices. We argue below that this is not the case.

6. REPEATED COMPETITION AND INFORMATION REVELATION THROUGH PRICES

Suppose now that we add another market period to our game in the continuum economy with increasing marginal costs. Firms purchase information at the first stage, compete in quantities contingent on the received signals at the second, observe the market price, and compete again in the last period. We claim that each firm purchasing precision t^* at the first stage, producing $x(s_i) = a(s_i - \mu) + b\mu$ at the second where $a = t^*/(2\lambda + \beta t^*)$ (both as in the two-stage game), inferring θ from the market price, and producing $x = b\theta$ at the third stage is an equilibrium path for the three-stage game.

To prove our claim suppose that firms have already chosen their precision of the information, t_i, $i \in [0,1]$. Firm i at stage two will follow the strategy $x_i(s_i) = a_i(s_i - \mu) + b\mu$ as in our previous two-stage game since in the continuum economy the firm cannot affect the market price through its quantity choice. The market price conditional on θ will be then $p = \theta - \beta\tilde{x}(\theta)$ where $\tilde{x}(\theta)$ is the average output, $\tilde{x}(\theta) = \bar{a}(\theta - \mu) + b\mu$ with $\bar{a} = \int a_i di = \bar{t}/(2\lambda + \beta\bar{t})$ and $\bar{t} = \int t_i di$.[13] Therefore $p = \theta(1 - \beta\bar{a}) - \beta(b - \bar{a})\mu$ and provided that $1 - \beta\bar{a} \neq 0$, θ can be obtained by observing the market price: $\theta = (p + \beta(b - \bar{a})\mu)/(1 - \beta\bar{a})$. $\bar{a}\beta$ is always less than 1 except if $\lambda = 0$. When θ is revealed a full information competitive equilibrium obtains at the last stage, $x = b\theta$ and $p = 2\lambda b\theta$.

The incentives to purchase information in the three-stage model are identical to those in the two-stage case since, given the precision of the information of the

[13]To obtain the result use the fact that conditional on θ, $\int t_i s_i = \bar{t}\theta$ by analogy to the fact that given θ,

$$\bar{s}_n \equiv \frac{1}{n}\sum_i \frac{t_i}{\bar{t}_n} s_i$$

converges almost surely to θ as n goes to infinity since $E(\bar{s}_n|\theta) = \theta$ and it is easily seen that $\mathrm{Var}(\bar{s}_n|\theta) \to_n 0$ since $\mathrm{Var}(\bar{s}_n|\theta)) \leq \sigma^2/4n$.

firms at the second stage, the same equilibrium as in the two-stage case obtains and the equilibrium at the third stage does not depend on previous information purchases. We have thus the same equilibrium behavior as before at the first two stages followed by a full information competitive equilibrium since the market price is fully revealing.

It is worth noting that at the last stage a fully revealing rational expectations equilibrium prevails but nevertheless firms have an incentive to purchase information since this affects expected profits at the second stage before the market price is observed (see Dubey, Geanakoplos, and Shubik (1982) for an elaboration of this idea in a strategic market game context where they show that Nash equilibria of the continuum economy are fully revealing generically).[14] According to our model in large markets where firms learn from past prices the number of firms who choose to acquire costly information need not be small. In fact in our symmetric equilibrium all firms purchase precision t^*. We see thus that the market inefficiency prevails when firms can learn from observed prices since their information content becomes only available the next period of economic activity.

7. CONCLUDING REMARKS

We have examined information aggregation in large Cournot markets where firms receive private signals about the uncertain demand and argued that the appropriate approach is to consider replica markets since that yields a well defined limit competitive market. Bayesian-Cournot equilibria are very easily characterized in the continuum market and closed form solutions can be obtained.

Our main contention is that large Cournot markets in general do not aggregate information efficiently. In a competitive market where firms receive private noisy signals there will be a welfare loss due to the incomplete information except if the technology exhibits constant returns. The market will underproduce if the true state of demand is high and will overproduce if it is low. Our analysis indicates that the full information competitive (or Walrasian) model need not be a good approximation of a large Cournot market with incomplete information if there are decreasing returns to scale and if the precision of the information of the firms is not close to perfect.

With decreasing returns to scale, if the information decision is endogenous and costly, then a further reason for market inefficiency is added and the welfare loss was shown to be increasing in the cost of information. With constant returns to scale, some subtle existence issues arise which underscore the importance of the replica assumptions made for the welfare analysis of markets with endogenous acquisition of information. The incentives to purchase information in a competitive market were also shown to be invariant to adding another market period where firms compete after having observed the previous market price (which turns out to be fully revealing).

[14] On this and related issues, see also Blume and Easley (1983), Grossman and Stiglitz (1980), Hellwig (1982), and Kihlstrom and Postlewaite (1983).

Although a competitive market will not attain in general the (full information) first best, we have argued that the market allocation is indeed efficient if we insist on decentralized decision making given the private information structure of the economy. A competitive market under incomplete information works like a team where the common payoff of the decision makers is expected total surplus.

Many extensions and refinements of the analysis presented here could be made. Our models have to be taken as a first step towards an understanding of competitive markets with incomplete information. Let us mention a few possible developments. (a) It would be interesting to examine a class of models where the S.L.L.N. does not hold, due to the correlation pattern of the signals, for example. In a related vein Rob (1987) has developed a model of private information with persistent randomness in the limit. (b) Another instance where the information aggregation issue is of relevance is a situation where firms are negligible but they still have some monopoly power, that is, a monopolistic competition world. This is explored in Vives (1987). (c) The inefficiency of competitive markets in aggregating information will have consequences for the welfare analysis of the long and medium run decisions of firms as capacity investment, technological choice, R&D, and advertising. Some results are reported in Vives (1986b).

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APPENDIX

LEMMA 1: *Consider an n-replica market with inverse demand and costs given by $p = \theta - \beta X/n$, $C(x_i) = \lambda x_i^2 (\lambda > 0)$ and let the information structure be affine, firm i having information precision $t_i \in [0,1]$. Then the Bayesian-Cournot equilibria of the game are in one-to-one correspondence with the solution to the team problem with objective function $E\{G_n(x;\theta)\}$ where*

$$G_n(x;\theta) = \theta \sum_j x_j - \left(\frac{\beta}{n} + \lambda\right) \sum_j x_j^2 - \frac{\beta}{2n} \sum_{i \neq j} x_i x_j$$

and there is a unique Bayesian equilibrium with firm i producing according to

$$x_{in}(s_i) = a_{in}(s_i - \mu) + b_n \mu$$

where

$$a_{in} = \frac{n}{\beta}\left(\frac{\gamma_{in}}{1 + \sum_j \gamma_{jn}}\right), \quad b_n = \frac{2\lambda + \beta/n}{2(2\lambda + \beta)(\lambda + \beta/n)}, \quad \text{and} \quad \gamma_{in} = \frac{t_i \beta/n}{2(\lambda + \beta/n) - t_i \beta/n}$$

for $i = 1, \ldots, n$.

PROOF: We first check, as in Basar and Ho (1974), that the Bayesian-Cournot equilibria of the game are in one-to-one correspondence to person-by-person optimization of the team function G_n. This is clear since $G_n(x)$ can be written as $\pi_i(x)+f_i((x_j)_{j\neq i})$ where

$$f_i\big((x_j)_{j\neq i}\big)=\theta\sum_{j\neq i}x_j-\left(\frac{\beta}{n}+\lambda\right)\sum_{j\neq i}x_j^2-\frac{\beta}{2n}\sum_{\substack{k\neq j\\ k,j\neq i}}x_kx_j.$$

The same outcome obtains by solving $\max_{x_i}E(\pi_i|s_i)$ or $\max_{x_i}E(G_n|s_i)$ since $f_i((x_j)_{j\neq i})$ does not involve x_i. Person-by-person optimization is equivalent to global optimization of the team function according to Theorem 4 in Radner (1962) since the random term does not affect the coefficients of the quadratic terms and it is easily checked that the team function is strictly concave in x. Furthermore, with the affine information structure, conditional expectations $E(\theta|s_i)$ are affine in s_i. Theorem 5 in Radner (1962) implies that there is a unique Bayesian equilibrium and this equilibrium involves strategies affine in the signals. To check that the candidate strategies form an equilibrium, note that expected profits of firm i conditional on receiving signal s_i and assuming firm j, $j\neq i$, uses strategy $x_j(\cdot)$, are

$$E(\pi_i|s_i)=x_i\left(E(\theta|s_i)-\frac{\beta}{n}\sum_{j\neq i}E\big(x_j(s_j)|s_i\big)-\left(\frac{\beta}{n}+\lambda\right)x_i\right).$$

First order conditions (F.O.C.) yield then

$$2\left(\frac{\beta}{n}+\lambda\right)x_i(s_i)=E(\theta|s_i)-\frac{\beta}{n}\sum_{j\neq i}E\big(x_j(s_i)|s_i\big)\quad\text{for}\quad i=1,\ldots,n.$$

By plugging in the candidate equilibrium strategy it is easily checked that they satisfy the F.O.C. (which are also sufficient given our structure). *Q.E.D.*

COMPLEMENT TO THE PROOF OF PROPOSITION 5: We check that

$$\left.\frac{\partial^2P_{in}}{\partial t_i^2}\right|_{\frac{\partial P_{in}}{\partial t_i}=0}<0$$

and that

$$\left(\frac{\partial^2P_{in}}{\partial t_i\partial t_j}\right)<0\qquad\text{for } j\neq i,\ i=1,\ldots,n.$$

$$E\pi_{in}=\left(\frac{\beta}{n}+\lambda\right)E\big(x_{in}(s_i)\big)^2=\left(b_n^2\mu^2+\left(\frac{n}{\beta}\right)^2\sigma^2F_n(t)\right)\left(\frac{\beta}{n}+\lambda\right)\quad\text{where}$$

$$F_n(t)=\frac{1}{t_i}\left(\frac{\gamma_{in}}{1+\Sigma\gamma_{jn}}\right)^2$$

using the fact that $x_{in}(s_i)=a_{in}(s_i-\mu)+b_n\mu$ where a_{in}, b_n and γ_{in} are as in Lemma 1. To ease notation we drop all the n subscripts since n is fixed. It is easily checked that

$$F(t)=\frac{t_i}{\big(\phi(1+\Delta_i)-\Delta_it_i\big)^2}\quad\text{where}$$

$$\phi=\frac{2(\lambda+\beta/n)}{\beta/n}\quad\text{and}\quad\Delta_i=\sum_{j\neq i}\gamma_j.$$

Now,

$$P_i(t) = E\pi_i - \frac{c}{\sigma^2}\frac{t_i}{1-t_i},$$

$$\frac{\partial P_i}{\partial t_i} = \left(\frac{\beta}{n}+\lambda\right)\left(\frac{\beta}{n}\right)^2\sigma^2\frac{\partial F}{\partial t_i} - \frac{c}{\sigma^2}\frac{1}{(1-t_i)^2},$$

and

$$\frac{\partial F}{\partial t_i} = \frac{\phi(1+\Delta_i)+\Delta_i t_i}{(\phi(1+\Delta_i)-\Delta_i t_i)^3},$$

$$\frac{\partial^2 P_i}{\partial t_i^2} = \frac{n\sigma^2}{\beta}\frac{\phi\Delta_i(2\phi(1+\Delta_i)+\Delta_i t_i)}{(\phi(1+\Delta_i)-\Delta_i t_i)^4} - \frac{c}{\sigma^2}\frac{2}{(1-t_i)^3},$$

and

$$\left.\frac{\partial^2 P_i}{\partial t_i^2}\right|_{\frac{\partial P_i}{\partial t_i}=0}$$

$$= \frac{2c}{\sigma^2}\frac{1}{(1-t_i)^2}\left(\frac{\Delta_i(2\phi(1+\Delta_i)+\Delta_i t_i)}{(\phi(1+\Delta_i)-\Delta_i t_i)(\phi(1+\Delta_i)+\Delta_i t_i)} - \frac{1}{1-t_i}\right),$$

which is negative since

$$\Delta_i(1-t_i)(2\phi(1+\Delta_i)+\Delta_i t_i) - \left(\phi^2(1+\Delta_i)^2 - \Delta_i^2 t_i^2\right)$$

$$= 2\phi(1-t_i)(1+\Delta_i)\Delta_i + \Delta_i^2(1-t_i)t_i + \Delta_i^2 t_i^2 - \phi(1+\Delta_i)^2$$

which is less than

$$2\phi(1-t_i)\left(\Delta_i+\Delta_i^2\right) + \Delta_i^2 t_i - 2\phi\left(1+2\Delta_i+\Delta_i^2\right)$$

$$= \Delta_i^2(2\phi - 2\phi t_i + t_i - 2\phi) + \Delta_i(2\phi(1-t_i) - 4\phi) - 2\phi < 0.$$

$$\frac{\partial^2 P_i}{\partial t_i \partial t_j} = \left(\frac{\beta}{n}+\lambda\right)\left(\frac{n}{\beta}\right)^2\sigma^2\frac{\partial^2 F}{\partial t_i\partial t_j},$$

$$\frac{\partial^2 F}{\partial t_i \partial t_j} = \frac{\partial}{\partial \Delta_i}\left(\frac{\partial F}{\partial t_i}\right)\frac{\partial \Delta_i}{\partial t_j},$$

$\frac{\partial \Delta_i}{\partial t_j} > 0$ from the definition of Δ_i and γ_j, and

$$\frac{\partial}{\partial \Delta_i}\left(\frac{\partial F}{\partial t_i}\right) = \frac{(\phi+t_i)(\phi(1+\Delta_i)-\Delta_i t_i) - 3(\phi-t_i)(\phi(1+\Delta_i)+\Delta_i t_i)}{(\phi(1+\Delta_i)-\Delta_i t_i)^4}$$

which is negative since

$$\phi + t_i < \phi + (2\phi - 3t_i) = 3(\phi - t_i) \quad \text{and} \quad \phi > 2, \quad t_i \leqslant 1.$$

We conclude that

$$\left(\frac{\partial^2 P_i}{\partial t_i \partial t_j} \right) < 0. \qquad Q.E.D.$$

REFERENCES

AUMANN, R. (1964): "Markets With a Continuum of Traders," *Econometrica*, 32, 39–50.

BASAR, T., AND V. C. HO (1974): "Informational Properties of the Nash Solutions to Two Stochastic Nonzero-Sum Games," *Journal of Economic Theory*, 7, 370–387.

BLACKWELL, D. (1951): "The Comparison of Experiments," in *Proceedings of Second Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley: U.C. Press, pp. 93–102.

BLUME, L. AND D. EASLEY (1983): "On the Game Theoretic Foundations of Market Equilibrium with Asymmetric Information," D.P. CREST C-55, University of Michigan.

CLARKE, R. (1983): "Collusion and the Incentive for Information Sharing," *Bell Journal of Economics*, 14, 383–394.

DEGROOT, M. (1970): *Optimal Statistical Decisions*. New York: McGraw-Hill.

DUBEY, P., J. GEANAKOPLOS, AND M. SHUBIK (1982): "Revelation of Information in Strategic Market Games: A Critique of Rational Expectations," Cowles Foundation Discussion Paper No. 634, Yale University.

ERICSON, W. (1969): "A Note on the Posterior Mean of a Population Mean," *Journal of the Royal Statistical Society*, 31, 332–334.

GAL-OR, E. (1985): "Information Sharing in Oligopoly," *Econometrica*, 53, 329–343.

GROSSMAN, S., AND J. STIGLITZ (1980): "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70, 393–409.

HELLWIG, M. (1982): "Rational Expectations Equilibrium with Conditioning on Past Prices: A Mean-Variance Example," *Journal of Economic Theory*, 26, 279–312.

JUDD, K. (1985): "The Law of Large Numbers with a Continuum of IID Random Variables," *Journal of Economic Theory*, 35, 19–25.

KIHLSTROM, R. (1984): "A Bayesian Exposition of Blackwell's Theorem on the Comparison of Experiments," in *Bayesian Models of Economic Theory*, ed. by M. Boyer and R. Kihlstrom. Elsevier Science Publishers, 13–31.

KIHLSTROM, R., AND A. POSTLEWAITE (1983): "Equilibrium in a Securities Market with a Dominant Trader Possessing Inside Information," CARESS Working Paper No. 83-05, University of Pennsylvania.

LAFFONT, J-J. (1985): "On the Welfare Analysis of Rational Expectations Equilibria with Asymmetric Information," *Econometrica*, 53, 1–29.

LI, L. (1985): "Cournot Oligopoly with Information Sharing," *Rand Journal*, Winter, 521–536.

LI, L., R. MCKELVEY, AND T. PAGE (1985): "Optimal Research for Cournot Oligopolists," Caltec Working Paper.

MAS-COLELL, A. (1986): "On a Theorem of Schmeidler," *Journal of Mathematical Economics*, 13, 201–206.

MATTHEWS, S. (1984): "Information Acquisition in Discriminatory Auctions," in *Bayesian Models of Economic Theory*, ed. by M. Boyer and R. Kihlstrom. Elsevier, 181–207.

MILGROM, P. (1979): "A Convergence Theorem for Competitive Bidding and Differential Information," *Econometrica*, 47, 679–688.

——— (1981): "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, 49, 921–943.

MILGROM, P., AND R. WEBER (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

NOVSHEK, W. (1980): "Cournot Equilibrium with Free Entry," *Review of Economic Studies*, 47, 473–486.

NOVSHEK, W., AND H. SONNENSCHEIN (1982): "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release," *Bell Journal of Economics*, 13, 214–218.

PALFREY, T. (1985): "Uncertainty Resolution Private Information Aggregation, and the Cournot Competitive Limit," *Review of Economic Studies*, 52, 69–83.

RADNER, R. (1962): "Team Decision Problems," *Annals of Mathematical Statistics*, 33, 857–881.

ROB, R. (1987): "Persistent Randomness in Model of Entry with Private Information," mimeo, University of Pennsylvania.

SCHMEIDLER, D. (1973): "Equilibrium Points of Nonatomic Games," *Journal of Statistical Physics*, 7, 295–300.

VIVES, X. (1984): "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, 34, 71–94.

——— (1986a): "Aggregation of Information in Large Cournot Markets," Caress W.P. #86-07, University of Pennsylvania.

——— (1986b): "Investment in Flexibility in a Competitive Market with Incomplete Information," Caress W.P. 86.08, University of Pennsylvania.

——— (1987): "Information Transmission in Large Markets," mimeo, Universitat Autònoma de Barcelona.

WILSON, R. (1974): "Informational Economies of Scale," *Bell Journal of Economics*, 184–195.

——— (1977): "A Bidding Model of Perfect Competition," *Review of Economic Studies*, 44, 511–518.