We analyze the implementation problem faced by firms when trying to collude in the face of asymmetric information about costs. Assuming that transfer payments are possible, we examine the incentive compatibility and individual rationality constraints that must be satisfied by any cartel agreement. Two scenarios are considered. Firms may or may not withdraw from the agreement after each firm’s costs become known. If no withdrawal is possible, we find that the monopoly rule is implementable when weak types of individual rationality constraints are required. This contrasts with some results in the literature. If withdrawal is possible, we find a potential conflict between different forms of individual rationality constraints, in particular, between interim and ex post constraints. This conflict disappears in industries with a large number of firms.

1. Introduction

A substantial body of work has examined the quota enforcement problem in cartels. This work is based on the seminal study of Stigler (1964) on the incentives to cheat on cartel agreements. The analysis of collusive agreements between firms has been extended using models of
The present analysis addresses another potential problem for cartel viability: the presence of asymmetric information. In his pioneering work, Roberts (1983, 1985) considers the case where side payments are not possible. Cramton and Palfrey (1990) and Kihlstrom and Vives (1989) consider the enforcement of cartels without side payments. Assuming that transfer payments are possible, we examine the incentive compatibility and individual rationality constraints that must be satisfied by any cartel agreement. These constraints are shown to have important implications for the cartel agreement.

Detection of cheating by cartel members is facilitated by the dissemination of information on individual firm behavior. Trade associations have played a role in this respect, and different antitrust suits, like Linseed Oil and American Column, have made the sharing of firm-specific data suspect in the United States. Nevertheless there are a number of cartels that do not face the enforcement problem because there are legal penalties that can be applied to defectors. Typical examples include cartelization in agricultural markets and for export purposes. Agricultural cooperatives and export associations are partially exempted from antitrust laws: the first through the Cooper-Volstead Act of 1922 and the Agricultural Marketing Act of 1937; and the second through the Webb-Pomerene Act of 1913. Examples of the agricultural type are Sunkist (California citrus growers) and the large (over 30,000 members) Associated Milk Producers (AMPI). These cooperatives or committees of cooperatives can agree on prices or regulate industry sales through contractual agreements enforced by the courts or by the federal government itself. Membership in a cooperative is voluntary. In some cases, like the milk market, intercooperative agreements to limit competition have been established through a sidepayment system.

Profit pooling arrangements in rationalization cartels, where firms agree on the market price and the distribution of production and profits, provide another instance of collusion supported by side payments condoned by public authorities. Auction markets provide still another class of cases where there is widespread evidence of collusion and where the deterrence of cheating from the cartel is relatively easy.

1. See, for example, Friedman (1971), Green and Porter (1984), and Abreu (1986).
4. See Madhavan et al. (1990) and Cave and Salant (1987).
5. See Howard (1954) for an analysis of rationalization cartels in Britain in the 1950s. See also Scherer and Ross (1990).
Cartels that face no problems with quota enforcement may still not be viable because of participation constraints. That is, it may pay for a firm to defect from the agreement or, in more technical language, the cartel agreement may not be individually rational (IR). Two scenarios can be considered following a defection. In the first (IR type I), the cartel breaks down, and unrestricted competition (Cournot, e.g.) takes place. In the second (IR type II), a defection implies duopolistic competition between the rest of the cartel (which holds together) and the defector. The second scenario provides a more stringent test of cartel agreements because it precludes the stability of a cartel with a large number of participants, as argued by Postlewaite and Roberts (1980). In the first scenario, the individual rationality of a cartel agreement follows easily. Firms, which may have different costs, are instructed to produce their cartel quota (which may be zero), and any firm must earn at least as much as what it would make under unrestricted Cournot competition. There is obviously a system of (balanced) transfer payments that accomplishes this, because monopoly profits are larger than total Cournot profits for any market.

When the firms in the industry do not know each other's costs, a collusive output allocation can be implemented if firms can be induced to reveal truthfully their costs. Indeed, in the words of Scherer and Ross (1990, p. 238), "divergent ideas about appropriate price levels and market shares" may undermine the possibility of agreement and become a central problem of cartel formations and stability. Side payments in a rationalization cartel may solve the assignment problem of market shares provided truthful cost revelation obtains. Because relative costs determine the distribution of output and profit across firms, more efficient producers being assigned larger production quotas, side payments from low-to-high-cost firms must be relied on, not only to make the collusive agreement individually rational, but also to provide the inducement for truthful revelation. If a side-payment scheme can be devised that induces firms to reveal their true marginal costs when they know that their cost reports will be used to implement the industry profit maximizing output allocation, then that allocation will be said to be incentive compatible.

Firms try to implement the monopoly outcome with the help of a cartel manager, who collects the cost reports of the firms and recommends to them production quotas and side payments. The incentive compatibility and individual rationality constraints will take a different form depending on whether or not firms are allowed to withdraw from the agreement once they receive the recommendation of the cartel manager, and costs become known by everyone.

If no withdrawal is possible, once a firm has decided to partici-
pate, then a cartel outcome is implementable if it is incentive compatible and interim individually rational. If a collusive arrangement is interim individually rational, it will be acceptable to each firm when the manager knows his or her own cost but does not yet know the cost of the other firm and therefore does not yet know exactly what treatment to expect when output is allocated and side payments are made. Incentive compatibility by itself is not a problem provided costs are independent. D'Aspremont and Gérard-Varet (1979) have shown that it is always possible to find side payments that make an ex post efficient allocation—like the monopoly outcome—(Bayesian) incentive compatible in a transferable utility world provided the types of the agents are independent.7

Nevertheless, even if the industry profit-maximizing output allocation is incentive compatible, there may be a conflict between incentive compatibility and individual rationality. Specifically, the conditions imposed on the side payments by the incentive compatibility constraint may make it impossible for them to satisfy the conditions imposed by the individual rationality constraints. A cartel allocation will assign production to the lowest-cost firms. In consequence, in order to induce truthful revelation, the higher-cost firms must receive a side payment from the lower-cost ones. The potential conflict of this incentive compatibility condition with the participation constraint (IR) should be clear: The transfer cannot be too large so as to induce efficient firms not to participate and not too small so as to induce inefficient firms to misreport.

This conflict yields impossibility results in the context of public goods8 (Laffont and Maskin, 1979) and in the context of bargaining problems (Myerson and Satterwhaite, 1983). These authors find ex post efficient mechanisms without outside subsidies incompatible with incentive compatibility and individual rationality. In this paper, assuming that the random-cost parameters are independently drawn from a known distribution with a finite number of types, we find positive results for both the small and large number of firms cases. That is, we find no conflict between incentive compatibility and individual rationality (at least in its weak type I form) in a variety of circumstances including the constant marginal cost case. Our results

7. In fact, for the special case of an $n$ firm industry with constant marginal costs that can be high or low, we can show (see the appendix) that a cartel agreement that promises an (ex post) equal share of the monopoly profits to any firm is strongly incentive compatible. This means that to reveal the true costs is for any firm a dominant strategy. A similar result for the case of two firms is obtained by Roberts (1983).

8. See also Güth and Hellwig (1986).
for large markets contrast with those obtained by Cramton and Palfrey (1990). These authors find an inconsistency between incentive compatibility and individual rationality (type I) in a market with linear demand and constant marginal costs uniformly and continuously distributed over an interval if the number of firms is large enough.

More specifically, we find that with a large number of firms (a continuum in fact), if when one firm decides not to participate, the cartel disintegrates and a competitive outcome with zero profits prevails (IR type I), the cartel is viable; if, when one firm decides not to participate, the rest of the cartel persists (IR type II), then cartelization is not viable. With a large number of firms, the second situation (IR type II) is indeed more plausible, indicating the prevalence of the usual free-rider problem. Nevertheless, instances of “predatory” behavior and price squeezes of nonmembers or defectors to a large cartel indicate the possibility of very competitive outcomes in case of non-participation. For example, AMPI was challenged by the U.S. Department of Justice on the grounds of gaining market power through predatory practices (see Madhavan et al., 1990).

When firms can withdraw from the agreement once they receive the cartel manager recommendations and costs are revealed (ex post), the incentive compatibility and interim individual rationality constraints must be modified. A firm has the possibility of defecting from the agreement ex post and obtaining the (Cournot) profits associated with the full information noncooperative game that will follow, and this is anticipated at the interim stage. Furthermore, a new individual rationality constraint must be satisfied. The second form of individual rationality is called ex post individual rationality. A collusive arrangement that is ex post individually rational will be adhered to by each of the firms even after they learn exactly how they will be treated by the arrangement, that is, even after they know the costs of the other firms as well as their own cost.

9. An instance of the breakdown of a large cartel because of free-riding nonparticipants is provided by the 1925 lemon growers’ voluntary marketing agreement. It reached a participation rate of 95% in its first season, but membership eventually declined because it was more profitable to free-ride. The cartel was terminated in the mid-1930s (Shepard, 1986).

10. This situation may not be descriptive of some legal cartels because of large penalties associated with withdrawal from the cartel. An instance where side-payments occur, via profit pooling, and where there are no legal sanctions to withdrawal, may be provided by some ocean shipping cartels (“voluntary unincorporated institutional agreements” reached by the liners and called “conferences”), and, in particular, the more informal ones (see Davies, 1983).

satisfies both forms of individual rationality (interim and \textit{ex post}) will be initially acceptable to all firms and will remain acceptable to all even after they know exactly how their own cost realization compares to that of the other firms.

For small numbers of firms, we find a potential conflict between the \textit{interim} and \textit{ex post} individual rationality constraints. In the context of a very simple model (a duopoly with constant marginal cost that can be high or low), it is shown, following Kihlstrom and Vives (1989), that the demands of incentive compatibility and individual rationality are not in conflict if there is no withdrawal possibility. Nevertheless, the individual rationality constraints when firms can withdraw from the agreement may be inconsistent when the two types of firms are too far apart, precluding the implementation of the cartel outcome.

For large numbers of firms, it turns out that, given our independence assumption on cost parameters, the conflict between \textit{interim} and \textit{ex post} individual rationality constraints disappears. This is so because at both stages, firms have the same information. Any firm knows the proportions of firms according to types when the number of firms is large through the workings of the law of large numbers. In a large market, the possibility of withdrawal does not change the incentive compatibility and individual rationality constraints that a (nonrandom) cartel arrangement must fulfil.

We model the large market case with a continuum of firms\textsuperscript{12} and examine in detail the constant marginal cost case. We require \textit{strict} incentive compatibility (i.e., the truth is the \textit{unique} best strategy for any firm) to eliminate possible artifacts of the continuum model. Collusive mechanisms that are strictly incentive compatible and individually rational (type I, i.e., a defection implies the breakdown of the cartel) are easily constructed. These results are extended to industries with general cost structures where the cartel arrangement calls for the production by only \textit{one} type of firm using mechanisms that take advantage of the fact that the distribution of types in the industry is known.

The models studied in this paper are related to the bidding models of Vickrey (1961), Wilson (1977), and Milgrom and Weber (1982). The relationship with the bidding literature is observed in Roberts (1983, 1985). While the parallels with the auction models are compelling, there is an obvious difference between the auction model and a

\textsuperscript{12} For general implementation results in economies with a continuum of agents, see Mas-Colell and Vives (1992).
model in which firms bid for the right to be a monopolist. In the former model, the winner makes a payment to the seller, and the losers receive no payment while in the latter, the winner makes payments to the other firms. It should also be noted that because the monopoly profits earned by a firm depend on the firm’s cost of production, an auction for a monopoly franchise is an example of a “private values” model. In Loeb and Magatt (1979) and Riordan and Sappington (1987), firms do bid for the right to be a monopolist. In these models, the firms’ bids are not side-payment offers. Instead, the firms bid by making price offers as in a Bertrand equilibrium. The equilibrium maximizes expected consumer surplus rather than industry profits.

The outline of the paper is as follows. Section 2 presents the model and the different constraints associated with the implementation of the cartel arrangement with and without the withdrawal provision. Section 3 illustrates the implementation problem in the context of a simple duopoly model following Kihlstrom and Vives (1989). Section 4 deals with the large market case, and concluding remarks follow. In the Appendix, it is shown that when only two types of (constant) marginal costs are possible, truthful revelation is a dominant strategy by any firm.

2. The model

We consider a market with a set of firms $T$, finite or infinite, producing a homogeneous product with inverse downward-sloping demand $p(y)$ (where $p$ is the product’s price and $y$ total industry output). Firm $t \in T$ produces according to the technology $C(y, c_t)$, where $c_t$ is a parameter known to firm $t$ but unknown to the other firms. Each firm knows that the other firms’ cost parameters can take value on the set $\Omega = \{\theta_1, \ldots, \theta_J\}$, where $\theta_1 < \theta_2 < \ldots < \theta_J$.

Let $\hat{c}$ be the random function that specifies the cost parameter of each firm in $T$, and let $\tilde{c}$, be the random variable that specifies firm $t$’s cost parameter. The $\hat{c}$’s are assumed to be i.i.d. The probability that $\hat{c}$ equals $\theta_i$ will be denoted $\mu_i$. The vector $\mu = (\mu_1, \ldots, \mu_J)$ is a complete description of the probability distribution of $\hat{c}$.

The firms attempt to collude with the aid of a central agency to whom they report their cost parameters. The role of the agency is to announce and implement a rule for allocating output and making side payments. Formally, the agency announces $< y(\cdot), s(\cdot) >$ where for each function $c: T \to \Omega$ of reported costs $y(c): T \to \mathbb{R}$ and $s(c): T \to \mathbb{R}$.

When $c$ is reported, $y_t(c)$ is the output to be produced and $s_t(c)$ the side payment to be received by firm $t$. For each $c$, we assume that the side payments must be balanced, that is, they must add up to zero.
Given cost realizations $c$ denote by $\hat{y}(c)$ a production rule that achieves the maximum total profits in the industry. It will exist under mild assumptions on demand and costs, and to simplify the exposition, suppose it is unique. The optimal production rule $\hat{y}(c)$ will be given by the intersection of the marginal revenue schedule with the industry marginal cost schedule given $c$.

For the case (which we will consider at length) of constant marginal production, costs $c_i$ will denote the marginal cost of firm $t$. In this situation, the rule $\hat{y}(\cdot)$, which would obtain the collusive outcome, would allocate all the output to the firms with the lowest marginal cost, provided there is positive mass of such firms.

If there is a (positive) mass (number) of $n$ firms with costs equal to $c = \min \{c_t : t \in T\}$ then

$$\hat{y}_t(c) = \begin{cases} y^M(c)/n & \text{if } c_t = c \\ 0 & \text{otherwise} \end{cases}$$

where $y^M(c) = \arg\max_y [p(y) - c]y$.

Our aim is to find a transfer system $s(\cdot)$ that implements the monopoly rule $\hat{y}(\cdot)$. The implementation constraints will be different depending on whether firms can withdraw from the agreement once they have received the recommendation of the cartel manager (and know each other's costs) or not. If there is no possibility of withdrawal, we are in the classic case, and a cartel agreement will be implementable if it is incentive compatible (IC) and interim individually rational (INTIR). If firms can withdraw once costs are known, then in order for the monopoly rule to be implementable, the incentive compatibility constraints must be modified (IC(W)), and two individual rationality constraints must be satisfied: INTIR(W) and EXPIR or ex post individual rationality. This is to guarantee that firms want to join the cartel and report their costs (INTIR(W)) and that they do no want to quit once they receive the recommendations (EXPIR).

Let us start with the classic case where no withdrawal from the agreement is possible. Given the mechanism $<y(\cdot), s(\cdot)>$, define the function

$$\mathcal{Q}(c, c^*) = p(\hat{y}(c^*))y_t(c^*) - C(y_t(c^*), c_t) + s_t(c^*)$$

The function $\mathcal{Q}(\cdot)$ relates firm $t$'s income (profits plus side payments) to its true cost parameter $c_t$ and the function $c^*$ of reported cost parame-
ters. The IC constraints require that telling the truth be a Bayesian equilibrium of the communication game among the firms induced by the mechanism $< y(\cdot), s(\cdot) >$. That is, it must hold that for all possible reports $c_t^* \in \Omega$ and all possible true cost parameters $c_t \in \Omega$

$$E\mathbb{O}(c_t, (c_t, \hat{c}_{-t})) \geq E\mathbb{O}(c_t^*, (c_t^*, \hat{c}_{-t}))$$

(\text{IC})

where $\hat{c}_{-t}$ is the random function $\hat{c}$ restricted to $T \setminus \{t\}$.

The profit that a firm expects to earn if it does not join the cartel depends on the cartel's ability to survive defections. One extreme case is that in which unanimous agreement is required for collusion (case I). In that case, each firm can, by simply refusing to join, prevent the cartel from forming. The other extreme (case II) arises if the cartel's formation cannot be prevented by a single firm's refusal to join. In any case, a firm decides to participate in the cartel agreement once it knows its own costs but not the costs of the rivals. The alternative to participation is Cournot-Nash competition, the defector versus the cartel in case II and unrestricted in case I. Firm $t$ will agree to participate if the expected profits (computed with the information available at this stage) from participation, $E\mathbb{O}(c_t, (c_t, \hat{c}_{-t}))$, exceed the expected profits (computed with the same information) associated with non-participation.\textsuperscript{13}

The expected profits associated with nonparticipation correspond to the Bayesian-Cournot equilibria of the duopoly (case II) or oligopoly game (case I). Let us denote them generically by $E\Pi^N(c_t, \hat{c}_{-t})$. $\Pi^N(c_t, \hat{c}_{-t})$ are the realized profits, at the (assumed unique) Bayesian-Cournot equilibrium, of a firm with costs $c_t$ when its rivals have costs $\hat{c}_{-t}$. A cartel contract that specifies the output allocation and sidepayment rules $< y(\cdot), s(\cdot) >$ is \textit{interim individually rational} if for all $c_t$ in $\Omega$

$$E\mathbb{O}(c_t, (c_t, \hat{c}_{-t})) \geq E\Pi^N(c_t, \hat{c}_{-t})$$

(INTIR)

A cartel agreement is implementable when it satisfies both IC and INTIR.

When firms have the possibility to withdraw once they receive the recommendation of the cartel manager and the costs of all firms are revealed, then a firm has to be induced to tell the truth, knowing that by lying, the maximum penalty that can be imposed has to leave the firm no worse than with the Cournot profits of the full informa-

\textsuperscript{13} Notice that, according to the usual implementation approach, when a firm refuses to participate in the cartel, rival firms do not make any inference about the costs of the firm. To take into account this possibility, an explicit extensive form game with firms making the decision whether or not to participate should be considered.
1. A DUOPOLY EXAMPLE

We illustrate the different incentive compatibility and individual rationality constraints in the context of a simple duopoly with linear demand and constant marginal cost where firms can have low or high costs. The analysis follows Kihlstrom and Vives (1989) (K–V). Costs must belong to \( \Omega = \{ \theta_l, \theta_h \} \) with \( \theta_l < \theta_h \). Demand will be given by \( p = a - \hat{y} \), where \( a > \theta_h \) and \( \hat{y} \) denotes total output. Denote by \( \Pi^M(\theta) \) the monopoly profits and by \( y^M(\theta) \) the monopoly output, associated with costs \( \theta : \Pi^M(\theta) = ((a - \theta)/2) \) and \( y^M(\theta) = (a - \theta)/2 \).

The monopoly rule \( \hat{y}(\cdot) \) assigns all production to the lowest cost firms. If the two firms have the same cost, \( \theta_l \), then the monopoly output \( y^M(\theta_l) \) is shared. In the present situation, it is enough to consider side payments that involve no transfers when both firms are of the same type and a transfer, denoted \( s \), from the low to the high type when they are of different types.

We will deal first with the classic case, where no withdrawal from the agreement is possible. Given that the rival firm reports truthfully, a low type firm will tell the truth if...
This yields an upper bound on the side payment $s$:

$$s \leq \tilde{f},$$

where

$$\tilde{f} = \Pi^M(\theta_1)(\frac{1 + \mu_2}{2}) - \left(\Pi^M(\theta_2) + [\theta_2 - \theta_1]y^M(\theta_2)\right)\frac{\mu_3}{2}.$$

A high type firm will tell the truth if

$$\mu_1 \frac{\Pi^M(\theta_1)}{2} + \mu_2(\Pi^M(\theta_1) - s) \geq \mu_1 s + \mu_2 \frac{\Pi^M(\theta_2) + (\theta_2 - \theta_1)y^M(\theta_2)}{2}.$$

This yields a lower bound on the side payment $s$:

$$s \leq \tilde{f},$$

where

$$\tilde{f} = \Pi^M(\theta_1) - [\theta_2 - \theta_1]y^M(\theta_2)\left(\frac{1 + \mu_2}{2}\right) - \Pi^M(\theta_2)\frac{\mu_3}{2}.$$

It is easily seen that $\tilde{f} \geq \tilde{f}$ and, therefore, the IC constraints require $s$ to belong to the interval $[\tilde{f}, \tilde{f}]$. The side payment must be large enough for a high type to accept not to produce and low enough for a low type not to be profitable to pretend to have high costs. If the two possible costs are far apart ($\theta_2 - \theta_1$ large), and if $\mu_2$, the probability of the lowest marginal cost, is large enough, then no side payment is necessary for incentive compatibility to obtain (Proposition 1, K–V).

The alternative to collusion at the interim stage for the firms is to play a Bayesian–Cournot game. In our linear model, there is a unique Bayesian–Cournot equilibrium yielding an expected payoff for a firm of type $\theta_1$:

$$E\Pi^N(\theta_1, \hat{\theta}) = \left(\frac{2\mu + E\hat{\theta} - 3\theta_1}{6}\right)^2,$$

where $E\hat{\theta} = \mu_1 \theta_1 + \mu_2 \theta_2$.

A low-type firm will participate in the cartel if

$$\mu_1 \frac{\Pi^M(\theta_1)}{2} + \mu_2(\Pi^M(\theta_1) - s) \geq E\Pi^N(\theta_1, \hat{\theta}).$$

This yields another upper bound on the side payments:

$$s \leq \tilde{g},$$
where
\[ \tilde{g} = \frac{1}{\mu_2} \left[ \frac{\Pi^M(\theta_2)}{2} \right] - E\Pi^N(\theta_2, \tilde{\theta}) \]

A high-type firm will participate in the cartel if
\[ \mu_1 s + \mu_2 \frac{\Pi^M(\theta_2)}{2} \geq E\Pi^N(\theta_2, \tilde{\theta}) \]

This yields another lower bound on the side payment \( s \):
\[ s \geq \tilde{g} \]

where
\[ \tilde{g} = \frac{1}{\mu_1} \left[ E\Pi^N(\theta_2, \tilde{\theta}) - \frac{\Pi^M(\theta_2)}{2} \right] \]

Again, it is easy to show that \( \tilde{g} \geq g \) and the interim individual rationality constraints (INTIR) require \( s \) to be in the interval \([\tilde{g}, g]\) (Proposition 2, K–V). The cartel arrangement will be implementable if we can find a side payment \( s \) that satisfies IC and INTIR, in other words, if the intersection of \([\tilde{g}, f]\) and \([g, g]\) is nonempty. This is shown to be the case in Proposition 6; K–V.

When the possibility of withdrawal from the agreement is introduced once the costs of the firms become known, the analysis becomes more complicated because three types of constraints must be satisfied, IC(W), INTIR(W), and EXPIR, for the cartel outcome to be implementable. Here we will just point at the possible inconsistency between the interim and the ex post individual rationality constraints that may prevent the successful implementation of the monopoly rule.

Let us consider now the ex post IR constraints and denote by
\[ \Pi^C(c_t, c_{-t}) \]
the profits of a Cournot duopolist whose marginal cost is \( c_t \) and who shares a market with a firm whose marginal cost is \( c_{-t} \). There are four ex post individual rationality constraints. If \( c_t = c_{-t} = \theta_t \), we must have
\[ \frac{\Pi^M(\theta_t)}{2} \geq \Pi^C(\theta_t, \theta_t) \]

This constraint will always be satisfied with a strict inequality. If \( c_t = c_{-t} = \theta_{-t} \), we must have
\[ \frac{\Pi^M(\theta_{-t})}{2} \geq \Pi^C(\theta_{-t}, \theta_{-t}) \].
This constraint will also always be satisfied with a strict inequality. If \( c_1 = \theta_1 \) and \( c_2 = \theta_2 \), we must have

\[
PI^M(\theta_1) - s \geq PI^C(\theta_1, \theta_2),
\]

which is equivalent to

\[
PI^M(\theta_1) - PI^C(\theta_1, \theta_2) \geq s.
\]

If \( c_1 = \theta_2 \) and \( c_2 = \theta_1 \), we must have

\[
s \geq PI^C(\theta_2, \theta_1).
\]

In summary, the ex post individual rationality constraint is satisfied if the transfer \( s \) belongs to the nonempty interval \([b, E]\), where

\[
E = \delta'(e) - C(e, \theta_1),
\]

and

\[
b = PI^C(\theta_2, \theta_1).
\]

We have already seen that if EXPIR is satisfied, then INTIR and \( INTIR(W) \) are equivalent. The problem for the implementation of the monopoly outcome comes from the potential inconsistency of INTIR and EXPIR—because if \( INTIR(W) \) and EXPIR are consistent, then INTIR and EXPIR must be consistent too. That is, the intervals, \([g, \bar{g}]\) and \([h, \bar{h}]\) may have an empty intersection. In particular, \( g > \bar{h} \) whenever \( \mu_1 > \frac{1}{2} \), and the high cost firm is close to not producing a positive output at the Cournot equilibrium when faced with a low-cost firm (i.e., \( y^C(\theta_2, \theta_1) = \frac{(\theta_2 - \theta_1)}{3} \) close to zero). This happens when \( \theta_2 - \theta_1 \) is large enough (\( \theta_2 - \theta_1 \) close to \( a - \theta_1 \)). In this case, the minimum transfer that the high-cost firm needs at the interim stage, \((s \geq g)\), the low-cost firm does not want to pay it ex post because \( \bar{h} \) is the maximum it is willing to pay. In this situation we consider \( \bar{h} = PI^M(\theta_1) - PI^C(\theta_1, \theta_2) \) is low because the high-cost firm is almost out of the market. On the other hand, the minimum transfer \( g \) the high-cost firm needs at the interim stage is large because most probably the other firm is low cost (\( \mu_1 > \frac{1}{2} \)), and, hence, the probability of sharing the high-cost monopoly profits is low.

It is interesting to notice that whenever INTIR and EXPIR are inconsistent in the implementation of the monopoly rule, no other rule (which necessarily gives less profits to the firms) will be implementable. Indeed, a production rule that yields profits

\[
\Pi(\theta) < PI^M(\theta)
\]

when the low-cost firm has cost \( \theta \) will decrease \( \bar{h} \) and increase \( g \), making the inequality \( g > \bar{h} \) hold a fortiori.
4. LARGE MARKETS

In this section we consider a large market with a continuum of firms $T = [0,1]$. Total output in the market is now given by $\hat{y}(c) = \int_0^1 y(c) dt$, and transfers are balanced: $\int_0^1 s(c) dt = 0$ for every function $c: T \rightarrow \Omega$ of reported cost parameters. We restrict attention to anonymous mechanisms, where, given the reports of the firms, the assigned production quota and side payment to any firm depend only on its report and the distribution of reported costs.

It is worth noting that in the continuum model, there is no aggregate uncertainty because costs are independent, and every firm knows that the distribution of cost parameters in the market corresponds to the proportions given by $\mu = (\mu_1, \ldots, \mu_i)$. This fact has important consequences for the implementation of the cartel outcome via an anonymous mechanism. In a large market, with independent costs, the introduction of a withdrawal possibility *ex post* does not lead to a conflict between the EXPIR and the INTIR(W) constraints.

This is clear because in this context, the latter constraints are satisfied trivially. We have

$$E\pi^N(c_i, \hat{c}_-; c_i, \mu) = \pi^e(c_i, \mu),$$

where $\pi^e(c_i, \mu)$ denote the Cournot profits of a firm of type $c_i$ when the distribution of costs in the market is $\mu$. Denote by $\mathcal{O}(c_i, (c_i^*, \mu))$ the profits of a firm of type $c_i$ when it announces $c_i^*$ and the distribution of costs in the market is $\mu$. Then INTIR(W) is

$$\max \{\mathcal{O}(c_i, (c_i, \mu)), \pi^e(c_i, \mu)\} \geq \pi^e(c_i, \mu)$$

for all $c_i$ in $\Omega$, which holds trivially. INTIR(W) will hold also even if we were to allow for random mechanisms, as we shall see later.

In fact, if we consider only deterministic mechanisms, then the introduction of a withdrawal possibility *ex post* does not matter, because, given that other firms report truthfully, a firm has the same relevant information at the interim and at the *ex post* stages. In both the firm knows the distribution of costs in the market, and this is all

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14. There are well known technical difficulties associated with the assumption of a continuum of independent random variables. There are, however, ways of justifying this assumption by demonstrating that this situation can arise in the limit as the number of random variables becomes large. See Judd (1985) for a discussion of the measure-theoretic issues involved, and Vives (1988) for a continuum model that can be obtained as the limit of finite replica economies.

15. With a random mechanism, a firm at the *ex post* stage knows the realization of the recommendation of the mechanism, while at the interim stage the firm only knows the distribution used by the mediator to randomize given the reported costs.
that matters in payoff relevant terms. In other words, the implementation of the cartel outcome is not dependent on the possibility of withdrawal from the agreement. In a large market, the constraints IC + INTIR are equivalent to IC(W) + INTIR(W) + EXPIR.

All individual rationality constraints boil down to:

\[ \mathcal{O}(c_i, (c,\mu)) \succeq IF(c_i, \mu) \text{ for all } c_i \in \Omega. \] (IR)

Furthermore IC + IR is equivalent to IC(W) + IR because IC(W) is given by \( \max\{\mathcal{O}(c_i, (c,\mu)), IF(c_i, \mu)\} \succeq \max\{\mathcal{O}(c_i, (c^*_i, \mu)), IF(c_i, \mu)\} \) for all \( c_i \) and \( c^*_i \) in \( \Omega \), and IC by

\[ \mathcal{O}(c_i, (c,\mu)) \succeq \mathcal{O}(c_i, (c^*_i, \mu)) \]

for all \( c_i \) and \( c^*_i \) in \( \Omega \).

In summary, in a large market we will say that the cartel arrangement is implementable if it satisfies IC and IR.

In the analysis of the incentive compatibility constraints in the continuum model, we will insist on the truth being a strict Bayesian equilibrium of the communication game induced by the cartel mechanism. That is, we will require for any firm to tell the truth to be a unique best response when other firms also tell the truth. We impose this requirement to make sure that incentive compatibility is preserved in the finite approximations to the continuum model. This follows because with strict incentive compatibility being satisfied in the continuum model, no firm is indifferent in the limit about whether to lie or tell the truth.

We will consider first the constant marginal cost case. With constant marginal costs, the monopoly rule is very simple, as we have seen. If there is a (positive) mass of \( n \) firms with costs equal to \( \zeta = \min\{c_i: t \in T\} \), then

\[ \hat{y}_i(c) = \begin{cases} \frac{y^M(\zeta)}{n} & \text{if } c_i = \zeta \\ 0 & \text{otherwise} \end{cases} \]

Consider now the side-payment rule

\[ s(t,c) = \begin{cases} -\left[\frac{1}{n} - 1\right]\hat{s}(\zeta), & \text{if } c_i = \zeta, \\ \hat{s}(\zeta), & \text{if } c_i > \zeta, \end{cases} \]
where \( n \) is the number of firms reporting \( c \), and \( [p(y^M(c)) - c]y^M(c) > s(c) > [p(y^M(c)) - \theta]y^M(c) \) and \( j \) is defined by \( c = \theta_j \).

When \( \langle y(\cdot), s(\cdot) \rangle \) is implemented, all nonproducing firms receive the same side payment, and all firms that do produce make the same side payment. Each nonproducer receives an amount larger than the profit a firm with the second lowest marginal cost would receive if it reported the lowest marginal cost and lower than the profit of a low cost firm.

**Proposition 1:** \( \langle y(\cdot), s(\cdot) \rangle \) is (strictly) incentive compatible.

**Proof.** When there is a continuum of firms, the probability is one that for all \( j, \mu \) of these firms have marginal costs equal to \( \theta_j \). If the output allocation and side-payment functions are \( y(\cdot) \) and \( s(\cdot) \), respectively, and if all firms report truthfully, the firms with marginal costs above the minimum receive the side payment

\[
\bar{s}(\theta_j) > [p(y^M(\theta_j)) - \theta_2]y^M(\theta_j).
\]

and are told not to produce. The \( \mu_1 \) firms with the lowest marginal cost make the side payment

\[
\left\{ \frac{1}{\mu_1} - 1 \right\} \bar{s}(\theta_1),
\]

and are assigned equal shares of the monopoly output \( y^M(\theta_1) \). Thus, each of these low cost firms produces \( y^M(\theta_1) \).

For all of the \( \mu_2 \) firms with marginal costs equal to \( \theta_2 \), the incentive compatibility condition requires that

\[
\bar{s}(\theta_j) > [p(y^M(\theta_j)) - \theta_2]y^M(\theta_j) - \left\{ \frac{1}{\mu_1} - 1 \right\} \bar{s}(\theta_1).
\]

This inequality is satisfied by definition of \( \bar{s}(\theta_1) \).

Because \( \theta_j > \theta_2 \) implies

\[
[p(y^M(\theta_j)) - \theta_2]y^M(\theta_j) > [p(y^M(\theta_2)) - \theta_2]y^M(\theta_2),
\]

inequality (1) implies that the incentive compatibility condition is satisfied for all firms with marginal costs \( \theta_j \) in excess of \( \theta_2 \). The incentive compatibility condition for the \( \mu_1 \) firms with marginal costs equal to \( \theta_1 \) is

\[
[p(y^M(\theta_1)) - \theta_1]y^M(\theta_1) - \left\{ \frac{1}{\mu_1} - 1 \right\} \bar{s}(\theta_1) > \bar{s}(\theta_1),
\]

or equivalently,
[p(y^N(\theta_i)) - \theta_i]y^N(\theta_i) > \bar{s}(\theta_i). This is satisfied again by the definition of \bar{s}(\theta_i). □

If the individual rationality constraint is satisfied when the profit earned by a defecting firm is computed assuming no collusion by other sellers, the cartel agreement is said to satisfy individual rationality type I (IR type I). If the constraint is satisfied when the profit computation assumes collusion by the other firms, the cartel agreement is individually rational type II (IR type II).

It is now easy to prove:

**Proposition 2:** \(<\hat{\gamma}(\cdot), \bar{s}(\cdot)> satisfies IR type I.\)

*Proof.* If, when a firm defects, the other firms do not collude, the defector will be one of a continuum of competitive firms. Because \(\mu_1\) firms will have the lowest possible marginal cost, the competitive price will be \(\theta_i\). If the defector is one of the \(\mu_1\) firms with the lowest possible marginal cost, it will earn zero profits, even though it produces because its marginal cost will equal the price. If the defector is one of the \((1 - \mu_1)\) firms that have marginal costs above \(\theta_i\), it will earn zero profits because it doesn't produce. Thus, when the cartel does not survive defections by a single firm, defectors will expect to earn zero profits. The type I individual rationality constraints are, therefore, satisfied by \(<\hat{\gamma}(\cdot), \bar{s}(\cdot)>\) because this output allocation-side-payment rule assures all firms a positive income. □

When discussing type II individual rationality, it is natural to introduce an assumption that guarantees that each firm is "small" relative to the market. The simplest way of accomplishing this is to assume that each firm faces a capacity constraint \(k\). We, therefore, introduce such a constraint and, furthermore, assume that

\[ k \geq \frac{p^{-1}(\theta_i)}{\mu_1} \quad (2) \]

Because

\[ p^{-1}(\theta_i) \geq y^M(\theta_i), \]

a capacity constraint \(k\) satisfying (2) does not prevent firms with marginal cost \(\theta_i\) from producing \(\frac{y^M(\theta_i)}{\mu_1}\) as required by \(\hat{\gamma}(\cdot)\) when all firms report truthfully.\(^{16}\) Propositions 1 and 2 can, therefore, be extended to

\(^{16}\) Because of the capacity constraint, \(\hat{\gamma}(c)\) must be redefined for nontruthful report functions \(c\).
the case in which firms face a capacity constraint if the constraint satisfies (2). The next proposition demonstrates that $< \hat{y}(\cdot), \hat{s}(\cdot) >$ is not individually rational when the cartel survives defections.

**Proposition 3:** If each firm faces a capacity constraint $k$ satisfying (2), then $< \hat{y}(\cdot), \hat{s}(\cdot) >$ is not IR type II.

**Proof.** If a "small" firm with marginal cost $\theta_1$ defects from the cartel, it can raise its output to the capacity level $k$ without lowering the market price. By defecting, such a firm can, therefore, earn

$$[p(y^M(\theta_1)) - \theta_1]k,$$

which is more than the amount,

$$[p(y^M(\theta_1)) - \theta_1] y^M(\theta_1) \left( \frac{1}{\mu_1} \right) \hat{s}(\theta_1),$$

that would be earned by remaining in the cartel. \qed

Our results for the constant marginal cost case contrast sharply with those obtained by Cramton and Palfrey (1987). These authors assume a finite number firms with independent (constant margins) costs drawn from a continuous distribution. They find that for a large enough number of firms, the cartel outcome is not enforceable even with individual rationality constraints of type I when the cost distribution is uniform. The source of the problem is precisely the individual rationality constraint for the firm with lowest possible marginal cost. Intuitively, when the lowest-cost firm does not have enough low-cost firms closeby (i.e., when the distribution of costs is not concentrated enough around the lowest-cost firm), it does not face enough competition when defecting from the cartel agreement, and, in consequence, it may have incentives to defect (i.e., the IR constraint is violated). This obviously cannot happen if there is a mass point at the lowest possible cost because then defection implies zero profits with very high probability when the number of firms is large. Our conjecture is that there is a continuity result when the number of firms is large (but finite), between the lack of enforceability of the cartel agreement with a uniform distribution of types (Cramton-Palfrey) and the possibility result obtained in the present paper as the distribution of costs tends to concentrate mass around the lowest possible cost.

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17. When $k$ belongs to the interval $\left( \frac{y^M(\theta_1)}{\mu_1}, \frac{y^M(\theta_2)}{\mu_1} \right)$ and the supply of the two lowest types results in a price no larger than $\theta_2(p^{-1}(\theta_2) \equiv (\mu_1 + \mu_2)k)$, then the cartel outcome, with only type I firms producing, is IC and IR if $k$ is not too large or $|\theta_2 - \theta_1|$ is small.
Up to now we have not made use of the fact that the distribution of costs \( \mu \) in the market is known by the cartel manager. For a fixed and known \( \mu \), we can easily design a mechanism to implement\( ^{18} \) the cartel outcome whenever the monopoly rule, given \( \mu \), calls for the production of only one type of firms, say, type 1 firms.

For a fixed \( \mu \) consider the production rule:

\[
\hat{y}_i(c) = \begin{cases} 
\frac{y^M(\theta_1)}{\mu_1} & \text{if } c_i = \theta_1 \\
0 & \text{otherwise}
\end{cases}
\]

otherwise where \( y^M(\theta_1) \) is the monopoly output. The side-payment rule is given by

\[
\xi_i(c) = \begin{cases} 
-\left( \frac{1-n_i}{n} \right) s & \text{if } c_i = \xi \\
\frac{1}{s} & \text{otherwise}
\end{cases}
\]

otherwise where \( n \) is the mass of firms reporting \( \xi = \min \{c_i; t \in T \} \) and

\[
\Pi^M(\theta_i) > s > \mu_1 p(\frac{y^M(\theta_1)}{\mu_1}) - C(\frac{y^M(\theta_i)}{\mu_1}, \theta_i)
\]

where \( h = \arg \min_{j \geq j \geq 2} \{ C(\frac{y^M(\theta_j)}{\mu_1}, \theta_i) \} \). Notice that the inequalities are consistent because

\[
\Pi^M(\theta_i) = \mu_1 [p(\frac{y^M(\theta_1)}{\mu_1}) - C(\frac{y^M(\theta_i)}{\mu_1}, \theta_i)]
\]

and

\[
C(\frac{y^M(\theta_1)}{\mu_1}, \theta_i) < C(\frac{y^M(\theta_i)}{\mu_1}, \theta_i)
\]

because at the monopoly allocation, only type 1 firms are candidates to produce.

**Proposition 4:** Given \( \mu \), the cartel arrangement \( \hat{y}(\cdot), \xi(\cdot) \) is strictly incentive compatible.

**Proof.** The IC constraint for a type 1 firm is

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18. In the language of mechanism theory, we are dealing then with a parametric implementation problem.
\[ \frac{\Pi^M(\theta_i)}{\mu_1} - \frac{1 - \mu_1}{\mu_1} s > s, \]

which is equivalent to \( \Pi^M(\theta_i) > s \) and, therefore, satisfied by assumption. The IC constraint for a type \( j, j \neq 1 \) firm is

\[ s > \left[ p(y^M(\theta_i)) \frac{y^M(\theta_i)}{\mu_1} - C \left( \frac{y^M(\theta_i)}{\mu_1} \theta_i \right) \right] - \frac{1 - \mu_1}{\mu_1} s, \]

which is equivalent to

\[ s > \mu_1 \left( p(y^M(\theta_i)) \frac{y^M(\theta_i)}{\mu_1} - C \left( \frac{y^M(\theta_i)}{\mu_1} \theta_i \right) \right), \]

and satisfied again by assumption because

\[ C \left( \frac{y^M(\theta_i)}{\mu_1} \theta_i \right) \geq C \left( \frac{y^M(\theta_i)}{\mu_1} \theta_i \right). \]

The proposition covers general cost structures provided that given \( \mu \), only one type of firm is called to produce at the monopoly allocation. For example, firms could have U-shaped average costs with minima in the set \( \{ \theta_1, \ldots, \theta_t \} \). If one assumes that only type I firms have to produce at the cartel outcome, strict incentive compatibility follows as earlier; type I individual rationality will be satisfied provided at the competitive equilibrium type I firms produce at minimum average cost. In this case, all firms would make zero profits at the competitive equilibrium, and consequently it would pay to join the cartel. Type II individual rationality, as before, would not be satisfied because a type I firm would make more by defecting and producing to equate the monopoly price with marginal cost. If at the competitive equilibrium, other types of firms produce, or if type I firms produce more than their efficient scale outputs, then at least type I firms will get positive profits and may have an incentive to defect.

It is possible for the monopoly rule to call for the production of only a positive proportion \( \bar{\mu}_1 < \mu_1 \) of firms of type I. In this case, a random mechanism may be devised to implement the cartel outcome. Now, as we have pointed out, the withdrawal possibility \textit{ex post}, once the recommendations are known, does matter. Nevertheless, there will be no conflict between EXPIR and INTIR(W) constraints because the latter will always be satisfied trivially:

\[ E\max\{\bar{\theta}(c_t, (c_t(\mu)), \pi^c(c_t, \mu)) \geq \pi^c(c_t, \mu) \}

for all \( c_t \) in \( \Omega \) where \( \bar{\theta} \) denotes the random payoff generated by the mediator, and the expectation is computed according to the randomizing device.
We construct a mechanism that implements the cartel rule and allows for ex post withdrawal (and, therefore, it also implements the monopoly outcome when withdrawal is not a possibility) when IR type I is considered, and at the competitive equilibrium all firms make zero profits.

Consider the following mechanism: If a firm reports $\theta_i$, then with probability $\overline{\mu}_1/\mu_1$ is instructed to produce $y^M(\theta_i)/\overline{\mu}_1$ and pay $s(1 - \overline{\mu}_1)/\mu_1$ and with probability $(1 - \overline{\mu}_1/\mu_1)$ is instructed not to produce and receives $s$. If a firm reports any other type, then it is instructed not to produce and receives $s$.

The mechanism implements (strictly) the cartel rule provided $s$ satisfies

\[
\pi^M(\theta_i) > s > \max\{\overline{\mu}_1[p(y^M(\theta_i)) \frac{y^M(\theta_i)}{\mu_1} - C(\frac{y^M(\theta_i)}{\overline{\mu}_1}, \theta_i)], 0\},
\]

where $h = \arg\min_{j \geq 2} C(y^M(\theta_j), \theta_j)$. To check it involves the routine manipulation of some inequalities.

EXPIR constraints (strictly) hold for firms that are told not to produce (they get $s > 0$) and for type 1 firms when told to produce (they get $-s(1 - \overline{\mu}_1)/\mu_1$, which is positive because $\pi^M(\theta_i) > s$).

The IC(W) constraints for type 1 firms are satisfied (strictly) because the required inequality

\[
\frac{\overline{\mu}_1}{\mu_1} \max \left\{ \frac{\pi^M(\theta_i)}{\mu} - 1 - \overline{\mu}_1, 0 \right\} + (1 - \overline{\mu}_1) \max \{s, 0\} > \max \{s, 0\},
\]

boils down to $\pi^M(\theta_i) > s$ when EXPIR constraints hold.

Similarly, the IC(W) constraint for a type $j > 1$ firm holds (strictly) because the inequality

\[
\max\{s, 0\} > \frac{\overline{\mu}_1}{\mu_1} \max\{p(y^M(\theta_i)) \frac{y^M(\theta_i)}{\mu_1} - C(\frac{y^M(\theta_i)}{\overline{\mu}_1}, \theta_i) - 1 - \overline{\mu}_1, s, 0\} + (1 - \overline{\mu}_1) \max\{s, 0\},
\]

boils down to $s > \overline{\mu}_1[p(y^M(\theta_i)) \frac{y^M(\theta_i)}{\mu_1} - C(\frac{y^M(\theta_i)}{\overline{\mu}_1}, \theta_i)]$, which is satisfied by assumption. Note that the INTIR constraints will hold strictly.

5. Concluding Remarks

We have analyzed the collusion problem under asymmetric information when firms have independently distributed costs. In the case where no withdrawal from the agreement is possible, we showed that in order to implement the cartel outcome, transfer payments can be found that insure incentive compatibility and individual rationality. In the case where withdrawal is an option, we concluded that individual rationality constraints may be a problem, particularly for small num-
bers of firms. This is because of the potential conflict between interim and ex post constraints. This conflict disappears when there is a large number of firms, and the monopoly rule is seen to be implementable, in sharp contrast to the results obtained by Cramton and Palfrey (1987). The idea that cartel implementation should be more difficult with a larger number of firms is expressed by the strong form of individual rationality constraints (type II), because, in this case, a firm has an incentive to defect from the cartel. With the weak form of individual rationality constraints (type I), no such incentives exist because defection implies unrestricted market competition.\(^{19}\)

We assumed that the cartel manager is not an independent agent. In practice, the cartel manager should be provided incentives to supervise the cartel. It may be the case also that cartel decisions have to be taken by a committee of representatives of the firms, in which case even the objective of joint profit maximization could be questioned (see Cave and Salant, 1987).

Our analysis assumes risk-neutral agents. An extension of the present analysis should address risk aversion on the part of collusive firms. This is particularly applicable in agricultural markets where firms are individual farmers: The cartel allocation may be more risky than unrestricted market competition, creating difficulties for the implementation of the monopoly rule.

Although we have shown that asymmetric information is not a serious deterrent to collusion by a profit-maximizing cartel, this should in no way be interpreted as an endorsement of collusion or a recommendation that firms should collude. The study of collusion and the conditions under which it is likely to arise have a long and distinguished tradition in the industrial organization literature. The results of theoretical studies of collusion have been the basis for subsequent empirical studies, and these theoretical and empirical investigations have been useful in providing a basis for an informed governmental approach to antitrust policy. Our work has been motivated by a desire to understand the extent to which the theoretical results of the earlier literature can be extended when the assumption of perfectly informed firms is dropped. It may be premature to argue that our results provide a basis for future antitrust policies, but it would seem obvious that such policies require an understanding of the robustness of the economic theory on which they are based.

\(^{19}\) According to some studies the success of cartelization with a large number of firms is helped by the coordinating activities of trade associations. See, for example, Hay and Kelley (1974) and Fraas and Greer (1977).
Consider the case of a finite number of firms $T = \{1, \ldots, n\}$ with constant marginal costs $c$, drawn from $\Omega = \{\theta_1, \ldots, \theta_j\}$. Total output in the market will be given by $y^T = \sum_{i=1}^{T} y_{it}$, and the balanced transfer condition will be:

$$\sum_{i=1}^{T} s_i(c) = 0 \text{ for all possible vectors of reported marginal costs } \{c_1, \ldots, c_n\}.$$

If $J = 2$, that is, $\Omega = \{\theta_1, \theta_2\}$, we will be able to show that the cartel outcome is incentive compatible in a very strong way: A firm will prefer to report its true cost irrespective of the cost realizations and reports (truthful or not) of the rivals. In other words, to tell the truth will be a dominant strategy:

$$\mathbb{D}_t(c, c^*, c_t) \succeq 0 \text{ for all } t, c \in \Omega, c^* \in \Omega.$$

for all $t$ in $T, c$ and $c^*$ in $\Omega$. Because we confine reports of firms to belong to the set of possible types $\Omega$, this is equivalent to requiring (see d’Aspremont and Gérard-Varet, 1979, Theorem 1)

$$\mathbb{D}_t(c, c) \succeq 0 \text{ for all } t, c \in \Omega,$$

for all $t, c \in \Omega$ and $c^* \in \Omega$. In this case we will say that the cartel outcome associated to $<\hat{y}(\cdot), s(\cdot)>$ is strongly incentive compatible. It is worth remarking that in this situation, the probability structure of costs (even if they are correlated) does not matter. In fact a mechanism is strongly incentive compatible if, and only if, it is Bayesian incentive compatible for any individual beliefs the players may have.

The side payments $s(\cdot)$ that will enforce the cartel rule $\hat{y}(\cdot)$ are extremely simple and appealing: They involve ex post equal sharing of the monopoly profits.

Given a cost realization $(c_1, \ldots, c_n)$, the transfers are designed so that any firm ends up with an equal share of total profits if firms report truthfully. That is, every firm should get profits equal to $\frac{H_M(c)}{n}$ where $H_M(c) = [p(y_M(c)) - c]y_M(c)$ and $c = \min \{c_1, \ldots, c_n\}$. This is accomplished with the following transfer and production assignments: If $k$ firms have reported low costs, $1 \leq k \leq n$, then a low-cost firm is told to produce $\frac{y_M(c)}{k}$, and has to pay $\frac{\hat{H}_M(c)}{k}$ and a high-cost firm is told not to produce and receives $\frac{y_M(c)}{n}$. If $k = 0$, that is, all firms report high costs, then no transfers are made, and all firms are instructed to produce $\frac{y_M(c)}{n}$. Denote by $<\hat{y}(\cdot), s(\cdot)>$ the cartel arrangement just defined. Note that the transfer is balanced.
**Proposition:** Let \( \Omega = \{\theta_1, \theta_2\} \). The cartel arrangement \( \langle y(\cdot), \delta(\cdot) \rangle \) is strongly incentive compatible.

**Proof:** We show that to report truthfully is a dominant strategy. As we know, it is sufficient to show it pays to tell the truth for any firm independent of the cost realization of the rival firms. Consider a high-cost firm \( i \), and suppose first that all other firms are also high cost. The firm will report truthfully if

\[
\frac{M(\theta_2)}{n} \geq M(\theta_1) - (\theta_2 - \theta_1)y^M(\theta_1) - (n-1) \frac{M(\theta_1)}{n}.
\]

This inequality is equivalent to \( M(\theta_2) \geq M(\theta_1) - n(\theta_2 - \theta_1)y^M(\theta_1) \), which is satisfied because

\[
M(\theta_2) > M(\theta_1) - (\theta_2 - \theta_1)y^M(\theta_1) > M(\theta_1) - n(\theta_2 - \theta_1)y^M(\theta_1).
\]

(The first inequality follows from the definition of \( M(\theta_2) \), and the second is obvious.) If \( k-1 \) other firms are low cost, then firm \( i \) will report truthfully if

\[
\frac{M(\theta_1)}{n} \geq \frac{M(\theta_1)}{k} - (\theta_2 - \theta_1)y^M(\theta_1) - \frac{n-k}{k} \frac{M(\theta_1)}{n}.
\]

This inequality is easily seen equivalent to

\[
M(\theta_1) \geq M(\theta_1) - (\theta_2 - \theta_1)y^M(\theta_1),
\]

which obviously holds strictly. Consider now a low-cost firm \( i \). If all other firms are high cost, then the firm will tell the truth if

\[
\frac{M(\theta_1)}{n} \geq \frac{M(\theta_1)}{n} + (\theta_2 - \theta_1)y^M(\theta_2),
\]

which holds by definition of \( M(\theta_1) \). If some rival has low costs, then truth telling obtains trivially because in any case, the firm will get \( \frac{M(\theta_1)}{n} = 0 \). \( \Box \)

**Remark 1.** Roberts (1983) showed strong incentive compatibility for a similar mechanism and two firms.

**Remark 2.** The transfers needed to enforce the cartel outcome presuppose that firms have reserves and can be imposed (out of equilibrium) penalties larger than their net revenues. (Although aggregate feasibility is always guaranteed because transfers are balanced.) For example, if a high-cost firm lies when all the rivals are also high cost, then the firm would make \( \frac{M(\theta_1)}{n} - (\theta_2 - \theta_1)y^M(\theta_1) \), which is clearly negative.
for $n$ large. This is due to the large payment the firm must make:

$$(1 - \frac{1}{n}) M(e, \theta_i).$$

**Remark 3.** The dominant strategy result of the proposition need not generalize to the case of more than two types of firms, $J > 2$. For example, with two firms ($n = 2$) and three types ($J = 3$), it is easily seen that, with ex post equal sharing transfers, a low-cost firm may want to pretend to be an intermediate-cost firm if the other firm is high cost. (This will happen, e.g., with linear demand when the price intercept is large enough.) Nevertheless, it is always possible, for any $n$ and $M$, and given our independence assumption, to find balanced transfers to enforce the monopoly outcome in Bayesian strategies. This follows from the general results of d'Aspremont and Gérard-Varet (1979) on Bayesian incentive compatible mechanisms with transferable utility (Theorem 6, in particular). The result applies to general profit functions for firms, satisfying appropriate regularity assumptions, and to general distributions of the firms' random parameters, as long as beliefs are independent.

**References**


