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Edgeworth and Modern Economics

Edgeworth and modern oligopoly theory

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1. Introduction

It is difficult to overemphasize the contribution of Francis Ysidro Edgeworth (1845–1926) to modern economics. His general research program was indeed ambitious: the application of mathematics and statistics to economics (or, better, to the 'moral sciences').[1] This program remains very much in the agenda of social science today. In this short paper I will concentrate on Edgeworth's contribution to oligopoly theory, discussing his ideas and the themes that have emerged from his seminal work revolving around the notion that *indeterminacy* of outcomes is the rule when there are a few agents in the market (while the outcomes become determinate in a large economy approaching perfect competition). This theme was expounded rigorously in *Mathematical Psychics* (1881) in a general equilibrium context and permeates also Edgeworth's work on oligopoly.

In section 2 Edgeworth classical duopoly analysis is discussed. As it is well-known, Edgeworth pointed out a non-existence of price equilibrium problem in the Bertrand model with increasing marginal costs and substitute products. Prices would cycle within some bounds and the extent of the indeterminacy would diminish with product differentiation and with increasing numbers of firms. In the case of complementary products he argued that a price equilibrium exists but it may not obtain for reasons related to modern criticisms of the Nash equilibrium concept.

The development of the Edgeworthian themes originating in the substitutes model in the economics literature is taken up in section 3: Bertrand–

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[1]See, for example, the entry on Edgeworth by Newman in the New Palgrave.

Edgeworth competition, the role of numbers and the degree of product differentiation in the extent of indeterminacy, and dynamics.

Section 4 presents a new tool, in honor of (in Pigou's words) 'Edgeworth, the tool maker': the theory of supermodular games. This theory gives conditions bounding the extent of indeterminacy in strategic behavior, a central concern of Edgeworth. A concluding remark ends the paper.

2. Edgeworth's ideas on oligopoly

Cournot proposed a solution to the oligopoly pricing problem with his celebrated example of the mineral spring producers. In modern terminology, firms would compete as quantity setters and the solution would be a Nash equilibrium. In consequence, prices would be determined under oligopolistic rivalry. Bertrand criticized the Cournot equilibrium and argued that the relevant choice variables for firms were prices and not quantities. In modern language, Bertrand proposed the Nash solution concept in prices. If this is the case and production costs are constant, like in Cournot's mineral water example, price will equal marginal cost, the competitive solution.

Edgeworth challenged strongly Cournot's ideas,[2] in particular the determinateness of equilibrium. As stated earlier, Edgeworth's central idea is that in situations of fewness in numbers, in oligopoly, and in contrast with monopoly or perfect competition, equilibrium is *indeterminate*.

He illustrated his theory examining price competition in a duopoly with substitute or complementary products (denote the quantities of the goods, respectively, by x and y). Positing utility functions for consumers commensurable with money (that is, linear in money), $U_i(x_i, y_i)$ for consumer i, he defines the goods to be rival if $\partial^2 U_i/\partial x_i \partial y_i < 0$ for all i (or at least on average), and complementary if the cross derivative is positive.[3] From the utility specification inverse and direct demands are derived and their properties analysed (noting, for example, that concavity of the utility function and rival goods imply gross substitutability).[4]

For *substitute* goods he considered the case of identical goods duopoly with increasing marginal costs, either in the extreme form of capacity constraints (1925), or with quadratic costs of production (1922). He concluded in both cases that prices would never reach an equilibrium position, and would oscillate, cycling indefinitely. Edgeworth also pointed out that the

[2]Edgeworth (1925), first published in 1897 See Vives (1989) for a summary.

[3]Edgeworth is credited with introducing the general functional form for the utility function. The assumption $\partial^2 U_i/\partial x_i \partial y_i < 0$ seems to have appeared for the first time in Mathematical Psychics (1881), later Auspitz and Lieben (1889) defined it to imply substitutability [see the entry on Edgeworth by Newman in The New Palgrave [Eatwell et al. (1988, p. 95)].

[4]This duopoly model constitutes a benchmark to analyze oligopolistic competition with differentiated products See, for example, Vives (1984).

'extent of indeterminateness' diminishes as the goods become more differentiated, in the limit being the firms' independent monopolies. A main insight of his analysis is that a firm will not have an incentive to supply more than its competitive supply (as determined by its marginal cost schedule) at any given price. The analysis must take into account this *voluntary trading* constraint. Indeed, in the case of capacity constraints the firm simply cannot supply more than the available capacity. The consequences is that the competitive equilibrium (which is the only possible equilibrium in pure strategies in the Edgeworth duopoly [Shubik (1959)] may be destabilized since a firm by *raising* its price can increase its profit given that the rival firm may not be able (or may not want) to supply the whole market at the competitive price. This fact together with the traditional price cutting Bertrand argument yields non-existence of a price equilibrium (in pure strategies) for plausible ranges of firms' capacities or cost specifications. His model has given rise to what now is called *Bertrand–Edgeworth competition*, where firms compete in prices but also where no firm is required to supply all the forthcoming demand at the set price.

For *complementary* goods Edgeworth thought 'at least very probable', in contrast to the case of rival products where he thought it 'certain', that 'economic equilibrium is indeterminate'. The illustration provided considers perfectly complementary products provided by (price) competing suppliers [and also the fanciful competing Nansen and Johansen 'dragging their sledge over the Arctic plains (all their dogs have died)']. The reason for the indeterminacy nevertheless is not that a position of (Nash) equilibrium (proposed by Cournot) does not exist but that this equilibrium may not obtain even if both firms are 'perfectly intelligent and aware of each other's motives'.[5] Indeed, if only one firm is perfectly intelligent and foreseeing while the other is myopic the equilibrium proposed by Edgeworth is the (future) Stackelberg equilibrium.[6] If both firms are forward looking then they may try to outsmart each other and the candidate (Nash) equilibrium may be unsettled by accident or by a player foreseeing a future move of the rival away from the equilibrium.

Edgeworth seems to be pointing at a modern criticism of Nash equilibrium: 'rationality' (utility maximization given some beliefs) and payoffs being common knowledge is not sufficient to explain Nash equilibrium behavior. The *rationalizability* literature [Bernheim (1984) and Pearce (1984)] has pointed out that Nash equilibrium is not the only outcome of common

[5] Quotations are from Edgeworth (1925).

[6] In the context of the price game with complementary products: the foreseeing firm optimizes taking into account the reaction curve of the rival, which is downward sloping [recall that price competition with complementary products is the dual of quantity competition with substitute products, see Sonnenschein (1968) and Singh and Vives (1984)].

knowledge of payoffs and rationality.[7] Equilibrium strategies are rationalizable but in general the rationalizable set is much larger. For example, in a Cournot linear demand oligopoly with more than two firms and constant marginal costs any price between the competitive and the monopoly levels is rationalizable.

In summary, Edgeworth thought that the oligopoly problem was essentially *indeterminate* and that prices would never reach an equilibrium position in markets characterized by fewness in numbers, as opposed to what happens in competitive markets. For substitute products his line of attack is to point out non-existence of price equilibria in pure strategies. For complementary products the attack can be seen more fundamental since he questions the very concept of (Nash) equilibrium.

3. The development of the Edgeworthian themes: Indeterminacy, numbers, differentiation and competitive equilibrium

In Edgeworth's rival goods model there need not exist an equilibrium in pure strategies but, as it is well known, there always exists a Nash equilibrium in mixed strategies, where firms instead of choosing prices choose probability distributions over prices.[8] In this sense, up to a probability distribution, prices are determined.

3.1. Bertrand–Edgeworth competition

There is by now a vast literature on Bertrand–Edgeworth (B–E) competition from the early contributions of Shapley (1957), Shubik (1959), Beckman (1965), Shapley and Shubik (1969), and Levitan and Shubik (1972). In B–E competition firms compete with prices realizing that competitors may not be able or may not want to supply all the forthcoming demand at the set prices. Firms announce simultaneously and independently prices, and production decisions follow the realization of the demands for the firms. There is *voluntary trading*: the quantity that each firm sells given the vector of prices is the minimum of its residual demand and its competitive supply. Obviously, a rule is needed to allocate unsatisfied demand. Two leading rationing rules have been proposed: the *proportional* (P) and the *surplus-maximizing* (SM). The first originates from Edgeworth and the second from Levitan and Shubik (1972). With the proportional rule rationing at the lowest price is

[7] In an n-player game, the sets of strategies $B_1, B_2, \ldots, B_n$ (with B_i a subset of the strategy set of player i) are rationalizable if any strategy of player i in B_i is optimal for some beliefs about the strategies played by opponents with support on the rationalizable set B_{-i} (B_{-i} denotes $(B_1, B_2, \ldots, B_n)$ except the ith component)

[8] Existence in pure strategies in the B–E model is restored if firms can commit to trade beyond the competitive supply [Dixon (1992)].

made through a queuing system. Consumers in the front of the line obtain their entire demand while others obtain nothing. Output is sold thus on a first-come–first-served basis. Resale is assumed impossible. With the surplus-maximizing rule consumers with higher willingness to pay are served first. It would arise, for example, if consumers could engage in costless arbitrage. Equilibria are typically in mixed strategies and results do depend on the rule used. For example:

(i) If firms have capacity limits, the regions of capacity space for which equilibria in pure strategies exist tends to be strictly smaller with the P rule since upward deviations from candidate equilibria are more heavily penalized with the SM rule.
(ii) Kreps and Scheinkman (1983) show using the SM rule that capacity competition followed by price competition yields Cournot outcomes. Davidson and Deneckere (1986) show that the result is not robust to a change in the rationing rule (first stage capacities tend to be larger with the P rule).

Existence conditions of mixed strategy equilibria are reported in Dasgupta and Maskin (1986). The results apply for general rationing rules if firms have capacity limits or identical convex costs. Even non-downward sloping demands can be allowed [Allen and Hellwig (1986a), Maskin (1986)].[9]

The B–E model has proved fruitful in *Industrial Organization applications* ranging from entry problems and limit pricing to price leadership [see, for example, Allen et al. (1991), Deneckere and Kovenock (1992)].

3.2. Large B–E markets

What happens in *large* B–E markets? Are their equilibria close to the competitive outcome and therefore 'determinate'?

First of all, existence of pure strategy equilibria is not restored in a large B–E market but approximate (ε) price equilibria do exist and are close to competitive equilibria if ε is small enough [Dixon (1987) in a model with convex costs]. Second, mixed strategy equilibria can be seen to be close to the competitive outcome as argued below.

To fix ideas, suppose that we have an n-firm market with concave demand $D(p)$ where each firm has capacity limit $k_n = K/n$ for some total capacity K such that $D(0) \geqq K > 0$. By fixing total capacity at the level K when the number of firms is increased each one of them is made smaller with respect to the market. With the proportional rationing scheme an equilibrium in pure strategies will exist if and only if the competitive price equals the

[9]Further characterizations of equilibria with the SM rule can be found in Kreps and Scheinkman (1983), Vives (1986), and Osborne and Pitchnik (1986); with the P rule, in Davidson and Deneckere (1986) and Allen and Hellwig (1986a, b).

monopoly price given the aggregate capacity K, p^{m}.[10] Otherwise, there is a symmetric atomless mixed strategy equilibrium in which the highest price charged is p^{m}, which is independent of n. This means that whatever the number of firms in the market, however small they are in relation to the market, the monopoly price is always named by firms. The reason is that the residual demand for a firm naming the highest price in the market has the *same* elasticity as market demand. Nevertheless the mixed-strategy equilibrium converges in distribution to the competitive price as n goes to infinity. The reason is that the incentives to undercut other firms relative to exploit monopolistically the own captive clientele increase as there are more firms in the market [Allen and Hellwig (1986a, b)]. Things are different with the SM rule. It is easily seen that if $K < D(0)$ eventually (for n large enough) all firms charge the competitive price $D^{-1}(K)$. If $K = D(0)$, then for any n there is a symmetric atomless mixed-strategy equilibrium in which the supremum of the support of the price distribution converges monotonically to zero at a rate $1/n$. The reason is that the residual demand left for a firm naming the highest price is $D(p)-(n-1)k_n$. This residual demand goes to zero as n grows since the quantity sold by the rival firms, $(n-1)K/n$ approaches the competitive level $K = D(0)$ [Vives (1986)].

We see that in both cases prices converge in distribution to the competitiveness level but with the SM rule convergence is stronger since the supports of the equilibrium price distributions also converge to the competitive price. In fact, the rate of convergence to the competitive outcome in the latter is the same as for Cournot markets under similar assumptions. Allen and Hellwig (1986a) show weak convergence of B–E equilibria to the competitive limit under very general assumptions on the demand when the P rule is used. They also show [Allen and Hellwig (1985)] that the highest competitive price or generically, any competitive price at which demand is downward sloping can always be approximated by a sequencde of B–E equilibria. Interestingly, Börgers (1992) has shown that *iterated elimination of dominated strategies* yields prices close to the competitive price for the large B–E model if the SM rule is used but not with the P rule. In order to successfully use iterated elimination of dominated strategies mixed strategies must converge not only in distribution but also in support.

3.3. *Product differentiation*

Product differentiation does not restore existence of pure strategy equilibria either as noted by Edgeworth [see also Shapley and Shubik (1969), Vives (1985), and Benassy (1989)]. Nevertheless it is true that the existence problem

[10] That is, $p^{\mathrm{m}} = \operatorname{argmax}_p \, (p \min \{K, D(p)\})$

in pure stategies is less severe the more differentiated the products are.[11] In a symmetric product differentiation model with n firms producing under increasing marginal costs Benassy (1989) shows that: (1) Given a finite n, a sufficiently large degree of substitutability σ entails non-existence. (2) For a given σ, a sufficiently large n entails existence. (3) High substitutability *and* large numbers (with n large enough to guarantee existence) imply that equilibria are close to competitive. It is worth noting that in the traditional Chamberlinian equilibrium (where the voluntary trading constraint of firms is disregarded, that is, firms are committed to supply whatever forthcoming demand at the set price) the degree of substitutability appears to be the main determinant of competitiveness (for $n \geqq 2$, a large σ implies prices close to marginal cost but for a limited degree of substitutability prices will be bounded away from marginal cost whatever large n is). The Chamberlinian equilibrium is the only candidate for a B–E equilibrium in pure strategies. A sufficient condition for this traditional equilibrium to be a B–E equilibrium is that at equilibrium competitors display an excess capacity in the aggregate larger than individual firm production (capacity is understood to be the competitive supply at the equilibrium price). In the case of CES utilities, yielding constant elasticity demand functions (η) and with constant elasticity cost functions ($\beta \geqq 1$), the inequality $n-1 \geqq (\beta-1)\ \eta$ implies the sufficient condition. For constant marginal costs ($\beta = 1$) it is always satisfied and there is no existence problem. For increasing marginal costs ($\beta > 1$) existence is guaranteed for large n and low η [see Benassy (1990)].

3.4. Dynamics

As it is well-known by now, *repeated competition* compounds the multiplicity problem in oligopoly. Essentially, any individually rational payoff (of the one-stage game) can be supported as a perfect equilibrium of a repeated game [see, for example, Friedman (1971) and Fudenberg and Maskin (1986)].

An application of the repeated game analysis in the B–E context is provided by Benoit and Krishna (1987) and Davidson and Deneckere (1990) who extend the Kreps–Scheinkman model to many periods. They consider firms choosing capacities of production first, and then competing repeatedly in prices. This allows for collusive pricing equilibria to emerge inducing capacity choices at the first stage different from the Cournot levels identified by Kreps–Scheinkman. In any case, the Cournot capacities and the Cournot

[11] Indeed, if (symmetric) capacity constraints k are added to a classical linear differentiated duopoly model with index of product differentiation γ/β, with γ/β ranging from 0, independent goods, to 1, perfect substitutes [see Singh and Vives (1984)], then it is easy to check that in the plane $(k, \gamma/\beta)$ the mixed strategy region has a cone shaped form expanding as goods become better substitutes.

price (for all market periods) constitute also an equilibrium. An interesting result of the analysis is that all equilibria of the game, except the Cournot one, involve excess capacity.

The multiplicity of equilibria in the repeated game structure carries over to fully dynamic games, in which there are linkages between the periods. Decisions about long-run variables, typically investment decisions, affect directly future demand and cost conditions. Decisions about short-term variables like prices and quantities affect also future competition in the presence of adjustment costs.

In full fledged dynamic games, it has been proposed to restrict attention to strategies that depend only on state variables (like investment) of the industry. Using this approach, Maskin and Tirole (1988) have studied alternating moves price and quantity games following an early model by Cyert and DeGroot (1970). In the pricing game they find multiple equilibria including cycles à la Edgeworth.

4. Strategic complementarity and supermodular games

As stated before (footnote 3) Edgeworth was the first to consider a utility function of the general type $U(x,y)$ and to introduce assumptions on its cross second derivatives. Indeed, the assumption $\partial^2 U/\partial x \partial y > 0$ is referred to sometimes as 'Edgeworth complementarity'. In modern terminology, when the marginal payoff of action x of an agent is increasing in the action y (which may be an action of the same agent or of another) we say that actions x and y are *strategic complements* [Bulow et al. (1983)]. The theory of supermodular games provides the theoretical framework in which to study strategic complementarities and has a bearing on the Edgeworthian indeterminacy theme: supermodular games have always equilibria in pure strategies, the equilibrium set has nice order properties, and rationalizable behavior can be confined within some bounds.

The theory of supermodular games [Topkis (1979), Vives (1985, 1990) and Milgrom and Roberts (1990)] is based on non-topological methods and exploits monotonicity properties using lattice-theoretical results [see Fudenberg and Tirole (1991, ch. 12.3) for a brief introduction to the theory].[12]

To simplify the exposition consider a smooth n-player game G where

[12] A *lattice* is a partially ordered set $(S, \geq)$ in which any two elements have a least upper bound (supremum) and a greater lower bound (infimum) in the set. For example, let $S \subset R^2, S = \{(1,0),(0,1)\}$, then S is not a lattice with the vector ordering since $(1,0)$ and $(0,1)$ have no joint upper bound in S. A lattice $(S, \geq)$ is *complete* if every non-empty subset of S has a supremum and an infimum in S

player i is described by the strategy space A_i, a subset of Euclidean space, and the twice-continuously differentiable payoff function $\pi_i: A \to R$, $A = A_1 \times \cdots \times A_n$.

The game G is (*strictly*) *supermodular* if for all i: A_i is a product of compact intervals, and the marginal profitability of any action to player i increases (strictly) with the other actions of the player and with the actions of the rivals [that is, for all a in A, $\partial^2 \pi_i / \partial a_{ih} \partial a_{ik} \geqq (>)\ 0$ for all $k \neq h$, and $\partial^2 \pi_i / \partial a_{ih} \partial a_{jk} \geqq (>)\ 0$ for all $j \neq i$ and for all h and k, where a_{ih} denotes the h action of player i].

Remark. The definition of supermodular game given is much more restrictive than necessary. See Topkis (1979) and Vives (1990) for general definitions. Note also that no concavity assumptions are made.

Let E be the set of Nash equilibria in pure strategies of the game G.

Theorem [*Topkis* (*1979*)]. *If G is a supermodular game then the set E of Nash equilibria is non-empty and a largest and a smallest equilibrium point exist.*

Remark [*Vives* (*1990*)]. If G is *strictly* supermodular, then E is a non-empty complete lattice (see footnote 12 for a definition).

Remark [*Vives* (*1990*)]. If the payoff to a player is increasing in the strategies of the other players, then the payoffs associated to the largest ($\sup E$) and smallest ($\inf E$) equilibrium points provide, respectively, an upper and a lower bound for the equilibrium payoffs for each player. These bounds can be tightened in terms of equilibrium payoffs for subsets of E if G is strictly supermodular.

Remark [*Milgrom and Roberts* (*1990*)]. The largest and the smallest equilibrium points constitute also (respectively) the largest and smallest profiles of serially undominated strategies (that is, strategies which survive after iterated elimination of strongly dominated strategies). Two interesting consequences are derived from this. First, all *rationalizable* strategies (see footnote 7 for a definition) lie between the smallest and largest equilibrium points. Second, if the game has a unique equilibrium it is *dominant solvable* [that is, the set remaining after iterated elimination of strongly dominated strategies is a singleton, see Moulin (1984)].

Supermodular theory is very general and of wide application for complex strategy spaces like those arising in Bayesian games [Vives (1990)]. Further, supermodular games have nice Cournot-tâtonnement and learning stability properties, as well as monotone comparative statics properties [see Vives (1990), Milgrom and Roberts (1990) and Krishna (1992)].

A brief explanation of the result follows.

Supermodular game theory builds on Tarski's fixed point theorem [Tarksi (1955)], which states that an increasing function[13] from a complete lattice into itself has a fixed point.

In Euclidean space 'boxes', products of compact intervals, are complete lattices. It follows then immediately from Tarski's theorem that if strategy sets are complete lattices and best reply correspondences have an increasing selection (which need not be continuous), then existence is guaranteed (just consider the best reply map and apply Tarski's theorem to a selection). What about if best replies are monotone decreasing? Unfortunately, there is no analog to Tarski's theorem for decreasing functions.[14] Nevertheless, for the case of two players we can use Tarski's theorem to show existence with decreasing best replies since the composition of two decreasing functions is increasing. Indeed, a fixed point of the composite function will yield a Nash equilibrium of the game [Vives (1990)].

In summary, provided strategy spaces are complete lattices, existence is guaranteed in general n-player games if best responses are increasing (and in two-person games even if they are decreasing). Under what conditions will best replies be monotone increasing?

Here is where supermodularity comes in. To gain a better understanding of what is involved, suppose for a moment that we are in a very nice case: strategy sets are compact intervals and the ith player best reply to a_{-i} is unique, interior and equal to $r_i(a_{-i})$. We know then that the first-order condition for profit maximization will be satisfied:

$$\frac{\partial \pi_i}{\partial a_i}(r_i(a_{-i}), a_{-i}) = 0.$$

Furthermore, if $\partial^2\pi_i/\partial a_i^2 < 0$, then r_i is continuously differentiable and

$$\frac{\partial r_i}{\partial a_j} = -\left(\frac{\partial^2 \pi_i}{\partial a_i \partial a_j}\right) \Big/ \left(\frac{\partial^2 \pi_i}{\partial a_i^2}\right), \quad j \neq i.$$

Therefore the best reply of player i will be increasing or decreasing according to the sign of $\partial^2\pi_i/\partial a_i\partial a_j$. It turns out that this result also holds when π_i is *not* quasiconcave. It is possible to show that if $\partial^2\pi_i/\partial a_i\partial a_j > 0$, $j \neq i$ (that is,

[13]Let $(S, \geqq)$ be a lattice. A function f from S to S is *increasing* if for x, y in S, $x \geqq y$ implies that $f(x) \geqq f(y)$.

[14]Think of a function f from $[0,1]$ to $[0,1]$, then the set of fixed points of the function is just the intersection of the graph of f with the 45° line. If f is decreasing f may jump down and miss the 45° line and no equilibria will exist.

when the payoff is strictly supermodular), then the best reply correspondence of player i is monotone increasing in the actions of the rivals (in the sense that *all* selections are increasing).

What does this approach have to say about *price competition* models?

First of all, it must be pointed out that the B–E game is *not* a supermodular game (indeed, if it were so equilibria in pure strategies would always exist!). In fact, a capacity constraint for firm 2 makes the best reply correspondence of firm 1 to jump *down*. Suppose we have a differentiated duopoly with linear demands for the substitute goods. When firm 2 charges a low price, the firm is capacity constrained, and then the best reply of firm 1 is to charge the monopoly price on the residual demand. As firm 2 keeps raising its price, there is a point where firm 1 is indifferent between charging the monopoly price or *lowering* its price and making firm 1 no longer capacity constrained.[15]

There is nevertheless a large class of oligopoly price games with substitute products, and dual quantity games with complementary products, which are supermodular (indeed, the typical linear oligopoly price game with substitute products and constant marginal costs is supermodular) [Vives (1990)]. The theory can be extended considering increasing transformations of the payoffs. For example, demand systems coming from CES or logic preferences yield a log-transformed game which is supermodular and for which uniqueness can be established [Milgrom and Roberts (1990)].[16]

Equilibria in pure strategies in supermodular price games will always exist and will be bounded between a largest and smallest equilibrium price vectors (which will coincide with the Pareto-best and the Pareto-worst equilibria from the point of view of firms). Typically the equilibrium set has much more order structure. For example, with two single product firms equilibrium prices are ordered [Vives (1985)]. The order structure of the equilibrium set is not only useful in welfare analysis but also in comparing Bertrand (price competition) and Cournot (quantity competition) outcomes. The fact that there is a smallest Bertrand equilibrium can be used to show that under regularity conditions all Cournot prices must be above it [Vives (1990)].

Even if equilibrium analysis is not found palatable, supermodular price games will typically narrow down possible strategic outcomes. Indeed, if the equilibrium is unique the game will be dominant solvable. If there are multiple equilibria all possible strategic outcomes will lie in the 'box' determined by the largest and the smallest price equilibrium vectors (this

[15]Similarly, in the Hotelling model there is no equilibrium in the price game if firms are located close to each other. In that case firm 1 sets a low price when firm 2 sets a low price too. As p_2 increases firm 1 raises its price up to a point where it pays to undercut firm 2's price. Firm 1 *lowers* p_1 discontinuously to price firm 2 out of the market [see D'Aspremont et al. (1979)].

[16]The key condition is that the elasticity of demand of a firm must be non-increasing in the prices of rivals See Caplin and Nalebuff (1991) for other examples.

contrasts, for example, with non-supermodular oligopoly games for which any outcome between monopoly and competitive is possible).

5. Conclusion

The Edgeworthian topics in oligopoly [indeterminacy, price dynamics, role of numbers and substitutability of products, criticism of (Nash) equilibrium] have indeed proved fruitful in modern economic analysis. Edgeworth's insights have proved also sound: indeterminacy is very much at the centerstage after the massive application of game theory methods to industrial organization problems. Our understanding of strategic competition has advanced enormously but a large region of indeterminacy in outcomes has remained. Although from the point of view of a *static equilibrium* approach pricing is 'determinate', at least up to a probability distribution (with mixed strategy equilibria), once a simple strategy space (say prices or quantities) is chosen (supposedly given a detailed knowledge of the market under study), from a *dynamic equilibrium* perspective the multiplicity of outcomes is overwhelming. The plethora of outcomes also emerges usually from a *non-equilibrium* approach (rationalizability). The theory of supermodular games provides a class of situations where this indeterminacy is bounded: equilibria in pure strategies exist, the equilibrium set has an order structure, rationalizable behavior is confined within some limits, and equilibrium outcomes have certain desirable stability conditions.

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