Short-Term Investment and the Informational Efficiency of the Market

Xavier Vives
Institut d'Anàlisi Econòmica (CSIC)

A dynamic finite-horizon market for a risky asset with a continuum of risk-averse heterogeneously informed investors and a risk-neutral competitive market-making sector is examined. The article analyzes the effect of investors' horizons on the information content of prices. It is shown that short horizons enhance or reduce accumulated price informativeness depending on the temporal pattern of private information arrival. With concentrated arrival of information, short horizons reduce final price informativeness; with diffuse arrival of information, short horizons enhance it. In the process a closed-form solution to the dynamic equilibrium with long-term investors is derived.

In this article I study dynamic trading in a world where privately informed speculators are *risk averse* and have *short horizons*, and where the market is *informationally* (semi-strong) *efficient* due to the presence of a competitive risk-neutral market-making sector.

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Attention has been drawn recently to the effects of investors' short horizons in financial markets. This short-termism may come about, for example, because of investors' liquidity needs, because of incentive reasons related to the evaluation of the performance of money managers, or because of difficulties associated with financing long-term investment in the presence of capital market imperfections.[1] The compounding of noise trading and short-investment horizons of risk-averse traders may imply "excess" volatility and a substantial divergence of asset prices from fundamental values in the presence of systematic misperceptions [De Long et al. (1990)], or multiple "bootstrap" equilibria with different self-fulfilling expectations [Pagano (1989) and Dennert (1992)].[2] With endogenous information acquisition [as pointed out by Froot, Scharfstein, and Stein (1992)], herding by short-term rational speculators may induce investment in information which is even unrelated to fundamental values (and explain chartism, for example).

The views that emerge from this work contrast with the classical market efficiency hypothesis, which considers volatility the result of information being incorporated into prices. Still, there appears to be a widespread, although certainly not universal, belief in the (at least) semi-strong informational efficiency of financial markets. The present article derives the consequences for the informational quality of prices of short investment horizons in a market which is semi-strong efficient. What are the effects of rational speculators' short horizons and risk aversion in an informationally efficient market? How will the degree of information incorporated in prices, volatility, and departures from fundamental values be affected?

The information content of prices has been much studied in financial markets. Most of the work has considered static models.[3] Dynamic models with asymmetric information have proved difficult to analyze in the presence of risk aversion. In fact, leading papers addressing dynamic trading [such as Kyle (1985) and Glosten and Milgrom (1985)], as well as recent work,[4] assume risk-neutral agents or myopic agents. A basic technical difficulty in characterizing equilibria is the simulta-

[1] For elaborations of the two latter reasons see, respectively, Holmstrom and Ricart i Costa (1986) and Shleifer and Vishny (1990).

[2] A paper by Bhushan et al. (1992) tries to test the consistency of some theoretical models which incorporate short horizons and noise trading with reported estimates of excess volatility.

[3] Consider, for example, Kihlstrom and Mirman (1975), Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985).

[4] Diamond and Verrecchia (1987), Easley and O'Hara (1987) and Admati and Pfleiderer (1988) are examples of analyses with risk-neutral agents. Singleton (1987) deals with myopic risk-averse agents.

neous presence of far-sighted (long-term) agents and risk aversion. Wang (1993) and Gennotte and Kyle (1992) present results in continuous time models. However, these models can only deal with nested information structures (that is, where the information sets of the speculators can be completely ordered); for example, with two classes of agents, either informed with each agent receiving the same signal, or uninformed [as in the Grossman and Stiglitz (1980) model]. Some partial results in two-period models are reported by Grundy and McNichols (1989) and Brown and Jennings (1989). In the present article a complete closed-form solution to an N-period dynamic model with long-term agents will be found and provided as a benchmark of comparison. This may be of independent interest since, to the best of my knowledge, this is the first such model to provide a closed-form solution for a non-nested information structure [in particular, for the standard case in which each informed agent has a little piece of information about the fundamentals, as in Hellwig (1980) or Diamond and Verrecchia (1981)].

The article compares two models of trading ("short-term" and "long-term") in a dynamic model with asymmetric information. Trading of a risky asset and a riskless asset happens in a sequence of N periods. There are three types of traders: Risk-averse informed speculators, risk-neutral market makers, and noise traders. Informed traders receive (potentially) a new private signal every period about the liquidation value v of the risky asset. There is a continuum of both informed speculators and market makers, and both can condition their demands on the price (that is, they submit demand functions or generalized limit orders). Market makers compete for the total order flow and set the price equal to the expected value of the risky asset conditional on all public information, including the order flow. Noise traders are modeled as having a random i.i.d. demand. All random variables are normally distributed and informed agents display constant absolute risk aversion (CARA) utility functions.

In the short-term model, informed traders have a short horizon and maximize the (expected) utility of the short-run return. This can be interpreted either as traders being myopic or as being a new batch, or generation, of informed traders every period (agent i of generation n inheriting the private information of another agent of generation $n-1$) who take a position in the risky asset when "young" and liquidate it when "old" in the next period. In the long-term model informed speculators have long horizons and maximize the (expected) utility of consumption in the final period. In both models the quality of the information a speculator has at any point in time is the same. That is, agent i in period n has the same precision of private information independently of whether his horizon is long or short.

The equilibria of both models are characterized in the article. The following results hold for every temporal pattern of precisions of private signals. Long-term agents want to trade more intensely in any given period the more risk tolerant they are and the more precise the period signal. Short-term traders are less responsive to information, in terms of desired positions, because at any period they have information about the fundamental value v but care about the next period's price (since this is their return to holding the asset), which is only a garbled signal of v. For short-term traders, the more precise prices are in the estimation of the fundamental value the more responsive will be their desired positions to information. Generations closer to the end of the horizon and the realization of the fundamental value want to hold larger positions (in terms of trading intensity), since prices are closer to the fundamental value and also since, potentially, they have more precise private information. The net trading intensity of informed agents (at any period n, the difference between the trading intensity of generation n and generation $n-1$) is therefore always positive. For the last generation, which faces the prospect of the liquidation of the asset, the desired trading intensity is the same in both the short-term and the long-term cases. This means that in both cases the aggregate net trading intensity across periods is equal. The short-term and the long-term cases do differ in the temporal distribution of net trading intensities.

The temporal pattern of the precision of private signals nevertheless has important consequences for the effect of short horizons on price informativeness. In this article I consider two leading examples. In the first, there is concentrated information arrival and private signals are received only at the beginning (that is, the precision of subsequent private signals is zero). In the second, there is a constant flow of information and the precision of private signals received by agents is the same every period. In the first case, short horizons impair the informativeness of the price in period N; in the second case, they enhance it. The conclusion, therefore, is that whether short horizons increase or decrease the informational content of prices depends on the pattern of private information arrival. Why is this the case?

The central idea is that the precision incorporated in prices is a convex function of the net trading intensities (indeed, it is a linear function of the sum of the squares of net trading intensities) and therefore works like an inequality index, taking minimum values for uniform distributions of trading intensities and growing with the concentration of net trading intensities in certain periods. The maximum value is attained by concentrating all the trading intensity in one period, while the minimum spreads it equally among all the periods. With concentrated information arrival, long-term traders also concentrate their trading in the first period (when they receive information), while short-term traders spread it over all the periods. The result is a higher price precision for the long-term case. With a constant flow of information, long-term traders spread their net trading intensity equally among the periods (since they receive a new private signal of equal precision every period) while short-term speculators' net trading intensities are not equally distributed (since their trading intensities depend on the evolution of price precisions, which change over time). The result is a lower price precision in the long-term case. Therefore, concentrated patterns of information arrival before the event date will tend to favor superior final price informativeness with long-term traders, while diffuse patterns of information arrival will favor superior final price informativeness with short-term traders. In the first case (concentrated information arrival) price precision is larger with long-term traders for any period, while in the second (diffuse information arrival) price precision is larger with short-term traders only for periods close to the end of the horizon.

The case of once-and-for-all information arrival is also of interest because it replicates the *static rational expectations* benchmark with far-sighted agents. Indeed, at the unique (linear) equilibrium of the long-term model the optimal strategy of informed agents is to trade the risky asset in the first period and hold the position until the realization of its liquidation value according to the static rational expectations intensity of trade. Informed speculators trade only in the first period and noise trading is absorbed by the competitive market-making sector at stable prices in subsequent trading periods.[5] With long-term agents prices respond immediately to the arrival of information and are stable thereafter given the presence of the competitive market-making sector. Risk-averse informed agents do not have incentives to trade after the first period since no new information is forthcoming. Static rational expectations models are usually thought of as appropriate reduced forms of a dynamic process by which information is incorporated and revealed by prices. According to the analysis in the article, this reasoning can be substantiated provided there is a competitive risk-neutral market-making sector and informed agents have a long horizon. Then informed agents use a *buy and hold* strategy with a trading intensity equal to the static case of the typical large-market noisy rational expectations model, like Hellwig (1980) or Diamond and Verrecchia (1981) for example. Nevertheless, if informed specu-

[5] This is in contrast to Kyle (1985), in which a large speculator has a strategic incentive to spread trading in order to exploit his informational advantage. Our model has small agents interacting in a competitive setting.

lators have short horizons the static rational expectations benchmark will overestimate the degree of price informativeness.

The article proceeds as follows. In Section 1 the static rational expectations benchmark model is presented. Section 2 presents price dynamics with a risk-neutral competitive market-making sector. Equilibria with short horizons are characterized in Section 3. Section 4 considers long-term speculators, and Section 5 compares the informational efficiency consequences of short versus long horizons. Concluding remarks follow.

1. The Static Rational Expectations Benchmark

This section presents a version of the standard large-market noisy rational expectations model, as studied by Hellwig (1980), Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), and Admati (1985), for example, with the addition of a competitive risk-neutral sector of market makers.

A single risky asset, with random fundamental value v, and a riskless asset (with unitary return) are traded in a market with risk-averse informed agents, and noise traders, with the intermediation of risk-neutral competitive market makers.

There is a continuum of informed agents, indexed in the interval $[0, 1]$, who maximize the (CARA) utility of the return from buying x_i units of the risky asset at price p: $\pi_i = (v - p)x_i$. Agent i is endowed with a small piece of information (a private signal s_{i1}) about the ex-post liquidation value v. Utility is given by $U(\pi_i) = -\exp\{-\rho\pi_i\}$, where ρ is the (positive) coefficient of constant absolute risk aversion. The initial wealth of informed agents is normalized to zero.

It is well known that in a large market, (competitive) noisy rational expectations equilibria are implementable, allowing agents to use demand schedules as strategies.[6] Traders submit demand schedules or generalized limit orders contingent on their information, and when optimizing take into account the (equilibrium) functional relationship of prices with the random variables in the environment. Agent i's strategy is a mapping from his private information to the space of demand functions (correspondences more generally). Let $X_i(s_{i1}, \cdot)$ be the demand schedule chosen by agent i when he has received signal s_{i1}. When the price is p the desired position of the agent is then $X_i(s_{i1}, p)$.

Noise traders' demand depends on the random variable u_1. Competitive risk-neutral market makers observe the noisy limit book sched-

[6] Also, using this approach Kyle (1989) and Jackson (1991) study rational expectations equilibria with imperfect competition.

ule $L(p) = \int_0^1 X_i(s_{i1}, p)\, di + u_1$ and set the price efficiently: $p = E(v \mid L(\cdot))$.

It is assumed that all random variables are normally distributed: v with mean $\bar{v}$ and variance σ_v^2; s_{i1} with mean $\bar{v}$ and variance $\sigma_v^2 + \sigma_{\varepsilon_1}^2$, and u_1 with zero mean and variance σ_u^2. The parameters v and u are uncorrelated. Conditional on v, signals are uncorrelated across agents. The signal s_{i1} can be thought of as $s_i = v + \varepsilon_{i1}$, where v and ε_{i1} are uncorrelated and errors are also uncorrelated across agents. Error terms and noise traders' demands are also uncorrelated. The convention will be made that given v, the average signal $\int_0^1 s_{i1}\, di = v$ almost surely (a.s.) (*i.e.*, $\int_0^1 \varepsilon_{i1}\, di = 0$).[7] The distributional assumptions made are common knowledge among the agents in the economy. Denote the precision of random variable x (that is, $(\sigma_x^2)^{-1}$) by τ_x.

I restrict my attention to *linear* equilibria. In equilibrium, then, prices are normally distributed. The profits of agent i with a position x_i are $\pi_i = (v-p)x_i$, yielding a utility $U(\pi_i) = -\exp\{-\rho\pi_i\}$. As is well known, a maximization of the CARA utility function with information $\{s_{i1}, p\}$, $E(U(\pi_i) \mid s_{i1}, p)$, is equivalent to maximizing

$$E(\pi_i \mid s_{i1}, p) - \rho\, \mathrm{Var}(\pi_i \mid s_{i1}, p)/2.$$

This yields

$$x_i = \frac{E\{(v-p) \mid s_{i1}, p\}}{\rho\, \mathrm{Var}\{(v-p) \mid s_{i1}, p\}}.$$

Because of the assumed symmetric ex ante information structure, demand functions and equilibria will be symmetric. Suppose then that $x_i = X(s_{i1}, p) = a\, s_{i1} + \zeta(p)$, where a is the trading intensity of informed speculators and $\zeta(\cdot)$ is a linear function. The noisy limit order book schedule is given by (using the convention that the average signal equals v a.s.):

$$L(p) = \int_0^1 X(s_{i1}, p)\, di + u_1 = z + \zeta(p), \text{ with } z = av + u_1.$$

The competitive market making sector observes the linear *function* of p, the book $L(\cdot) = z + \zeta(\cdot)$, and sets price efficiently: $p = E(v \mid L(\cdot)) = E(v \mid z)$. Notice that what is informative about v is the intercept z of the limit order schedule $L(\cdot)$. Indeed, the random variable z, the intercept of the limit order schedule, is observationally equivalent to the market price and can be thought of as representing the new information contained in the market price.

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[7] See Admati (1985) for a justification of this convention in the context of a financial market rational expectations model and Vives (1988) in the context of a Cournot model.

The efficient pricing (zero expected profit) condition can be justified with Bertrand competition among risk-neutral market makers who have symmetric information, each one of them observing the limit order book. Alternatively, we could assume that there is a continuum of risk-neutral uninformed market makers who submit limit orders to a central clearing mechanism jointly with informed agents. Prices are set by a Walrasian auctioneer to equate the aggregate excess demand (coming from informed traders, market makers, and noise traders) to zero. In this case, in equilibrium, necessarily $E(v \mid p) = p$ since otherwise market makers would like to take unbounded positions. This condition, and the associated trades of market makers, can be obtained as the risk-neutral limit of markets with risk-averse uniformed agents. [Indeed, in Theorem 6.1 of Kyle (1989) it is shown that as the aggregate risk-bearing capacity of uniformed speculators or market makers grows without bound, in the limit prices are unbiased in the sense that $E(v \mid p) = p$.] Now, in a linear equilibrium, p will be a linear function of z (see Proposition 1.1 below) and therefore $E(v \mid p) = p$ is equivalent to $E(v \mid z) = p$.

Market makers take the counterpart of the limit order book and clear the market. From standard normal theory it follows that $p = (\tau_v \bar{v} + \tau_u a z)/\tau$, where $\tau = \tau_v + \tau_u a^2$ denotes the precision of the market price in the estimation of v ($\tau \equiv [\mathrm{Var}(v \mid p)]^{-1}$). Letting $\lambda = \tau_u a/\tau$, $p = \lambda z + (1 - \lambda a)\bar{v}$. The competitive market-making sector determines the slope λ of the market-supply schedule (with respect to the quantity u_1 traded by noise traders). As in Kyle (1989), λ^{-1} can be thought of as an index of the depth of the market. Indeed, a flat schedule ($\lambda = 0$) indicates an infinitely deep market. An increase in the trading intensity a, holding the price precision constant, induces market makers to put more weight on z in setting p, decreasing market depth. An increase in the precision incorporated in prices, τ, holding a constant, has the opposite effect.

It is immediate from normal distribution theory that $E(v \mid s_{i1}, p) = (\tau_{\varepsilon_1} s_{i1} + \tau p)/(\tau_{\varepsilon_1} + \tau)$ and $(\mathrm{Var}(v \mid s_{i1}, p))^{-1} = \tau_{\varepsilon_1} + \tau$. It follows then that $E(v \mid s_{i1}, p) - p = \tau_{\varepsilon_1}(\tau_{\varepsilon_1} + \tau)^{-1}(s_{i1} - p)$, and therefore informed agents' CARA demands are $x_i = \rho^{-1}\tau_{\varepsilon_1}(s_{i1} - p)$.

The equilibrium obtained is a modification of Hellwig's limit equilibrium (as the number of players goes to infinity) [Hellwig (1980, Proposition 5.1 and 5.2)] when there is a competitive risk-neutral market-making sector. Proposition 1.1 gives the result.

Proposition 1.1. *In the static model there is a unique linear equilibrium. It is symmetric and given by:*

$$X(s_{i1}, p) = a(s_{i1} - p),$$

with $a = \rho^{-1}\tau_{\varepsilon_1}$*, and*

$$p = \lambda z + (1 - \lambda a)\bar{v},$$

with $z = av + u_1$*,* $\lambda = \tau_u a/\tau$*, and* $\tau = \tau_v + \tau_u a^2$*.*

Corollary 2.1. *Total volatility* $\mathrm{Var}\, p + \mathrm{Var}(v \mid p)$ *is constant and equal to* σ_v^2*. The expected volume traded by informed agents* $E|x| = (2/\pi)^{1/2} a\sqrt{\tau^{-1}}$*.*

In equilibrium, informed agent i buys or sells according to whether s_i, her private estimate of v, is larger or smaller than the market estimate, p. Informed agents trade more intensely (higher a) if the precision of the signals (τ_{ε_1}) is higher or risk aversion (ρ) is lower. *Trading intensity* is independent of the amount of noise trading. This is so since an increase in noise $(\tau_u)^{-1}$ increases risk [by increasing $\mathrm{Var}(v \mid s_{i1}, p)$] but also increases the conditional expected return $[E(v \mid s_{i1}, p) - p]$, because it makes private information more valuable. The two effects exactly offset each other.

The depth of the market λ^{-1} is increasing in noise trading $(\tau_u)^{-1}$ and nonmonotonic in ρ and τ_{ε_1}. In equilibrium, and depending on parameter values, the depth of the market may be increasing in risk tolerance and the precision of information of informed agents.[8] The explanation is that these changes increase the trading intensity of informed agents, which tends to decrease market depth, but this may be more than compensated by the induced increase in the precision of prices.[9]

Total volatility is constant and equal to the ex ante volatility of v as a direct consequence of efficient pricing: $p = E(v \mid p)$. Ex-ante price volatility $\mathrm{Var}\, p = \sigma_v^2 - \mathrm{Var}(v \mid p) = (\tau_v)^{-1} - \tau^{-1}$ is increasing in the precision incorporated in prices τ.[10] Prices are more volatile if they are more informative. An increase in noise trading reduces directly the precision of prices τ (and price volatility) even though the trading intensity of informed agents is not affected. An increase

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[8] In equilibrium, market depth λ^{-1} *is* increasing in a provided $\tau_v - \tau_u a^2 < 0$. An increase in the precision of private information, leading to an increase in a, will imply a larger λ^{-1} if noise trading is small (τ_u large) and the levels of risk tolerance and precision of private information are large (which imply a large).

[9] This explanation is therefore different from the one in Subrahmanyam (1991), where the phenomenon is attributed to an increase in the degree of competition among a finite number of insiders.

[10] Indeed, it is well known that $\sigma_v^2 = E\{\mathrm{Var}(v \mid p)\} + \mathrm{Var}\{E(v \mid p)\}$, and therefore $\mathrm{Var}\, p = (\tau_v)^{-1} - \tau^{-1}$ (recall that (v, p) are jointly normally distributed and consequently $\mathrm{Var}(v \mid p)$ is nonrandom).

in risk aversion or in the noisiness of private information induces a decrease in τ via a decreased trading intensity.

Expected trading volume is indeed increasing with noise trading $(\tau_u)^{-1}$. In fact, as is usual in this type of model, there is trade because of the presence of noise traders and because informed agents have better information than risk-neutral market makers. Indeed, the expected trade of informed speculators goes to zero as noise trading vanishes, since then the precision incorporated into prices tends to infinity and therefore the information advantage of informed agents disappears. Similarly, when the precision of information of informed agents (τ_{ε_1}) tends to zero so does their trade intensity (and their expected trade). In contrast, when the precision of information of informed agents tends to infinity so does their trade intensity, market depth, and price precision, with the result that the expected trade of informed agents tends to $(2/\pi)^{1/2}\sigma_u$.[11]

2. Dynamic Trading and Efficient Pricing

Consider trading over N periods. At period $N+1$ the risky asset is liquidated and the value v (equal to p_{N+1} by definition) collected. Apart from informed agents, as before, there are noise traders at any period. Noise traders' demands follow an independently identically normally distributed process $\{u_t\}_{t=1}^{N}$ (independent of all other random variables in the model). Since we are interested in studying the effects of the investment horizons of informed traders, we will endow agents with the same private information at any point in time in the case of both a long and a short horizon. At any period there is a continuum of informed traders (indexed in the unit interval) who (potentially) receive a private signal. Informed trader i in period t receives a normally distributed signal $s_{it} = v + \varepsilon_{it}$, where, as before, v and ε_{it} are uncorrelated, and errors are also uncorrelated across agents and periods (and with noise trade). The precision of the signals τ_{ε_t} is the same across agents in the same period but may be different across periods. We assume that $\tau_{\varepsilon_1} > 0$ since otherwise there would not be trade in this period and we could then forget about period 1.

In period n agent i will have the vector of private signals $s_i^n = (s_{i1}, \ldots, s_{in})$ available. If agents are long-lived and have a long horizon this is obvious. If they have a short horizon then we can assume that they are long-lived but myopic, and that they do not forget infor-

[11] Market makers trade according to $Y = -d(p - \bar{v})$, with $d = \lambda^{-1} - a$. Market makers lean against the wind by selling when prices rise above their expected level $(p - \bar{v} > 0)$ and buying when prices decline $(p - \bar{v} < 0)$ since $d > 0$ $(1 - \lambda a > 0)$. When τ_ε goes to infinity market makers are pushed out of the market: λa tends to 1 and d tends to zero.

mation. Alternatively, a different generation of informed agents comes to market every period, each member of generation t inheriting the private signal of a member of generation $t-1$. In any case, agent i in period n has the same private information independent of whether he has a long or a short horizon. It is immediate from normal theory that a sufficient statistic for s_i^n in the estimation of v is the weighted signal $\tilde{s}_{in} = (\sum_{t=1}^{n} \tau_{\varepsilon_t})^{-1} \sum_{t=1}^{n} \tau_{\varepsilon_t} s_{it}$.

I will show later that in any linear equilibrium the strategy of agent i in period n will depend on $\tilde{s}_{in}$ and on public information (this follows since the private and public are conditionally independent). Informed agent i in period n submits a demand schedule (a limit order) $X_{in}(\tilde{s}_{in}, p^{n-1}, \cdot)$, indicating the position desired at every price p_n, contingent on the information available [the sufficient statistic for the private information and the sequence of past prices $p^{n-1} = \{p_1, \ldots, p_{n-1}\}$]. As before we restrict attention to *linear* equilibria. To obtain the net aggregate demand schedule we have to subtract the aggregate position in period $n-1$, $\int_0^1 X_{in-1}\, di$, from $\int_0^1 X_{in}\, di$. The competitive market-making sector observes the aggregate limit order book (including the demand of noise traders) $L_n(\cdot)$:

$$L_n(\cdot) = \int_0^1 X_{in}\, di - \int_0^1 X_{in-1}\, di + u_n.$$

As in the static case, equilibria will be shown to be symmetric. Consider a candidate symmetric linear equilibrium: $X_n(\tilde{s}_{in}, p^{n-1}, p_n) = a_n \tilde{s}_{in} + \zeta_n(p^n)$, where a_n is the sensitivity to private information (or trading intensity) and $\zeta_n(\cdot)$ is a linear function of past and current prices. The noisy limit order book at stage n is then

$$L_n(\cdot) = z_n + \zeta_n(p^n) - \zeta_{n-1}(p^{n-1}),$$

where $z_n = \Delta a_n v + u_n$, and $\Delta a_n \equiv a_n - a_{n-1}$ (with $a_0 = 0$) represents the net trading intensity of informed agents in period n. Notice that we have used the convention that the average signal every period, $\int_0^1 s_{it}\, di$, equals v a.s. and have taken for granted the linearity properties of integrals to obtain that, for any n, $\int_0^1 \tilde{s}_{in}\, di = v$ (a.s).

The competitive market-making sector sets p_n conditional on past public information and on z_n, the new information in the aggregate limit order schedule $L_n(\cdot)$. Past public information is summarized in the sequence $z^{n-1} = \{z_1, \ldots, z_{n-1}\}$ of informational additions from the limit order books, which is easily seen to be observationally equivalent to the sequence of prices $p^{n-1} = \{p_1, \ldots, p_{n-1}\}$. The random variables $z_n = \Delta a_n v + u_n$ can be thought of as the new information in the current price filtered from the net aggregate action of informed agents. The current price depends on the last period price (which summarizes the information contained in past prices) and on the new information z_n. The informativeness of p_n (or precision as an estimator of v, $\tau_n \equiv [\mathrm{Var}(v \mid p_n)]^{-1}$) depends on the sum of the squares of the net trading intensities of informed agents $\sum_{t=1}^{n}(\Delta a_t)^2$. That is, the informativeness of prices is determined by the intensity of *net* (as opposed to *gross*) trades by informed agents.

The following proposition characterizes the evolution of prices at any linear equilibrium. Let $p_0 = \bar{v}$.

Proposition 2.1. *At any linear equilibrium,* $p_{N+1} = v$, *and for* $n = 1, \ldots, N$

$$p_n = E(v \mid z^n) = \lambda_n z_n + (1 - \lambda_n \Delta a_n) p_{n-1},$$

$$\textit{with } \lambda_n = \tau_u \Delta a_n / \tau_n,\ z_n = \Delta a_n v + u_n,\ \textit{and } \tau_n = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_t)^2.$$

Proof. We check first that prices $\{p^n\}$ do not add information to the random variables $\{z^n\}$.

Claim. In equilibrium the vectors of random variables, $\{p^n\}$ and $\{z^n\}$, where $z_t = \Delta a_t v + u_t$, $t = 1, \ldots, n$, are observationally equivalent.

Proof. By induction. For $n = 1$, $p_1 = E(v \mid z_1)$ as in the static model, which implies that z_1 is observationally equivalent to p_1, since at equilibrium the parameters of the linear function $E(v \mid z_1)$ are known. Now, if $\{z^{n-1}\}$ and $\{p^{n-1}\}$ are observationally equivalent, then $\{z^n\}$ and $\{p^n\}$ also are observationally equivalent. This follows immediately since from the limit order schedule market makers infer z_n and set $p_n = E(v \mid z_n, z^{n-1})$. (Notice that in case $\Delta a_n = 0$ the weight put by market makers on z_n would be zero and $p_n = E(v \mid z^{n-1}) = p_{n-1}$.) ■

The price set by the competitive market-making sector is then $p_n = E(v \mid z^n)$. From standard normal theory p_n equals a convex combination of $p_{n-1} = E(v \mid z^{n-1})$ and $z_n/\Delta a_n = v + (\Delta a_n)^{-1} u_n$, with weights according to their relative precisions. The result follows: $p_n = \lambda_n \Delta a_n (z_n/\Delta a_n) + (1 - \lambda_n \Delta a_n) p_{n-1}$. An explicit expression for p_n is

$$p_n = \left(\tau_v \bar{v} + \tau_u \sum_{t=1}^{n} \Delta a_t z_t \right) / \tau_n, \text{ where } \tau_n = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_t)^2. \quad ■$$

Remark 2.1. Because of normality, $p_n = E(v \mid z^n)$ is a sufficient statistic for the information $\{p^n\}$ or $\{z^n\}$ in the estimation of v. In consequence, $p_n = E(v \mid p_n)$ and the desired position of informed agent i in period n can be written as $X_{\text{in}}(\tilde{s}_{\text{in}}, p_n)$. Notice that, given the existence of a competitive risk-neutral market-making sector, the market is (semi-strong) efficient, prices follow a martingale, and therefore historical prices or "technical analysis" [in contrast with Brown and Jennings (1989), for example] are superfluous for decision making. It is worth remarking also that (long-term or short-term) agents need not know about past prices since the current price is a sufficient statistic for all public information.

Remark 2.2. The martingale property of prices has immediate consequences for price volatility. The unconditional volatility of price p_n, $\operatorname{Var} p_n$, equals $(\tau_v)^{-1} - (\tau_n)^{-1}$, and therefore is nondecreasing in n (price informativeness, τ_n, is also nondecreasing in n). As more information is incorporated in prices ex ante volatility increases. The conditional volatility $\operatorname{Var}(p_n \mid p_{n-1})$ equals $(\tau_{n-1})^{-1} - (\tau_n)^{-1}$, and total volatility is constant: $\sum_{t=1}^{N+1} \operatorname{Var}(p_n \mid p_{n-1}) = (\tau_v)^{-1}$. Indeed, in an efficient market trading behavior affects the distribution of volatility over time, or the resolution of uncertainty, but not its total magnitude.

Let us now consider in turn equilibria with short and long term investment horizons.

3. Short-Term Investment

In this section informed traders will be assumed to maximize the utility of the short-run return. At period t, let $\pi_{it} = (p_{t+1} - p_t)x_{it}$ denote the (short-run) profits derived by agent i from buying x_{it} units of the risky asset at price p_t and selling it next period at price p_{t+1}. At stage t a strategy for agent i is a function that maps his private information $\tilde{s}_{\text{in}}$ into a demand schedule $X_t(\tilde{s}_{\text{in}}, \cdot)$. Recall that we can think of different interpretations of the short-term investment case formalizing the idea of risk-averse speculators with a short horizon. First of all, agents can be long-lived but myopic, and they do not forget information. Second, a different generation of informed agents comes to market every period, each member of generation t inheriting the private information of a member of generation $t-1$, taking a position, and liquidating it the next period.

The market solution will be taken to be the Bayesian equilibria of the N-period dynamic game (a sequence of Bayesian equilibria of the one-shot games with the defined short-run payoffs). Attention will be restricted to equilibria in linear strategies. Propositions 3.1 and 3.2 show existence and characterize equilibria for a fixed horizon N. To save on notation I will not index by N the parameters characterizing the equilibrium until Section 5. Recall that $\tau_n = \tau_v + \tau_u \sum_{t=1}^{n}(\Delta a_t)^2$.

Proposition 3.1. *Linear equilibria of the dynamic N-period trading game with short investment horizons exist and are characterized (implicitly) for* $n = 1, \ldots, N-1$, *by*

$$X_n(\tilde{s}_{\text{in}}, p_n) = a_n(\tilde{s}_{\text{in}} - p_n),$$

with $a_n = \rho^{-1}((\sum_{t=1}^{n} \tau_{\varepsilon_t})^{-1} + (\tau_{n+1})^{-1})^{-1}$. *For* $n = N$, $a_N = \rho^{-1}(\sum_{t=1}^{N} \tau_{\varepsilon_t})$.

Proof. This follows from three claims.

Claim 1. In the estimation of $v - p_n$, $\tilde{s}_{\text{in}} - p_n$ is sufficient with respect to the information $\{s_i^n, p^n\}$ (or $\{s_i^n, z^n\}$). Furthermore, $E((v - p_n) \mid \tilde{s}_{\text{in}}, p_n) = (\sum_{t=1}^{n} \tau_{\varepsilon_t}(\sum_{t=1}^{n} \tau_{\varepsilon_t} + \tau_n)^{-1}(\tilde{s}_{\text{in}} - p_n)$.

Proof. It is a consequence of the projection theorem for normal random variables that $E\{(v - p_n) \mid s_i^n, z^n\}$ is sufficient for $v - p_n$ with respect to $\{s_i^n, z^n\}$. It is immediate from Proposition 2.1 that $E(v \mid s_i^n, z^n) = E(v \mid \tilde{s}_{\text{in}}, p_n) = ((\sum_{t=1}^{n} \tau_{\varepsilon_t})\tilde{s}_{\text{in}} + \tau_n p_n)/\tau_{\text{in}}$, with $\tau_{\text{in}} = (\text{Var}(v \mid \tilde{s}_{\text{in}}, p_n))^{-1} = \sum_{t=1}^{n} \tau_{\varepsilon_t} + \tau_n$ (as usual, the weights to private information and to public information in $E(v \mid \tilde{s}_{\text{in}}, p_n)$ correspond to their relative precisions ($\sum_{t=1}^{n} \tau_{\varepsilon_t}/\tau_{\text{in}}$ and τ_n/τ_{in}, respectively).[12]

Therefore,

$$\begin{aligned} E\{(v - p_n) \mid \tilde{s}_{\text{in}}, p_n\} &= E(v \mid \tilde{s}_{\text{in}}, p_n) - p_n \\ &= \left(\left(\sum_{t=1}^{n} \tau_{\varepsilon_t}\right)\tilde{s}_{\text{in}} + \tau_n p_n\right)/\tau_{\text{in}} - p_n. \end{aligned}$$

The result follows since

$$\tau_n(\tau_{\text{in}})^{-1} - 1 = -\left(\sum_{t=1}^{n} \tau_{\varepsilon_t}\right)(\tau_{\text{in}})^{-1}. \qquad \blacksquare$$

Claim 2. In equilibrium, $X_n(\tilde{s}_{\text{in}}, p_n) = a_n(\tilde{s}_{\text{in}} - p_n)$, with

$$a_n = 1/\left(\rho\left(\left(\sum_{t=1}^{n} \tau_{\varepsilon_t}\right)^{-1} + (\tau_{n+1})^{-1}\right)\right).$$

For $n = N$, $a_N = \rho^{-1}(\sum_{t=1}^{n} \tau_{\varepsilon_t})$.

[12] Note that the error terms in the signals $\{s_i^n\}$ are independent of the noise trading process $\{u^n\}$ and therefore in the estimation of v with the information $\{s_i^n, p^n\}$ it is sufficient to consider $(\tilde{s}_{\text{in}}, p_n)$ where $\tilde{s}_{\text{in}}$ is sufficient statistic for $\{s_i^n\}$ and p_n is a sufficient statistic for $\{p^n\}$.

Proof. At a linear equilibrium and given our assumptions, prices are normally distributed. Maximization of a CARA utility function by agent i yields then at stage n

$$X_{ni} = \frac{E\{(p_{n+1} - p_n) \mid \tilde{s}_{\text{in}}, p_n\}}{\rho \operatorname{Var}\{(p_{n+1} - p_n) \mid \tilde{s}_{\text{in}}, p_n\}},$$

where $p_{n+1} - p_n = \lambda_{n+1}(\Delta a_{n+1}(v - p_n) + u_{n+1})$ from Proposition 2.1. (Note that the expression is independent of i and therefore the equilibria will be symmetric.) Further, we have that

$$E\{(p_{n+1} - p_n) \mid \tilde{s}_{\text{in}}, p_n\} = \lambda_{n+1}\Delta a_{n+1} E\{(v - p_n) \mid \tilde{s}_{\text{in}}, p_n\},$$

and

$$\operatorname{Var}\{(p_{n+1} - p_n) \mid \tilde{s}_{\text{in}}, p_n\} = (\lambda_{n+1})^2((\Delta a_{n+1})^2 \operatorname{Var}\{v \mid \tilde{s}_{\text{in}}, p_n\} + \sigma_u^2).$$

The desired result for $n < N$ follows from Claim 1 since it is immediate that $a_n = ((\sum_{t=1}^{n} \tau_{\varepsilon_t})\tau_{n+1})/\rho((\sum_{t=1}^{n} \tau_{\varepsilon_t}) + \tau_{n+1})$. For $n = N$, $p_{N+1} = v$ and $(\tau_{N+1})^{-1} = 0$. Therefore $a_N = \rho^{-1}(\sum_{t=1}^{N} \tau_{\varepsilon_t})$. ■

Claim 3. A linear equilibrium exists.

Proof. From Claim 2, for any $n < N$, $a_n = g_n(a^{n+1})$, where g_n is continuous and $a^{n+1} = (a_1, \ldots, a_{n+1})$, a_n in the compact interval $[\underline{a}, \bar{a}] = [(\rho^{-1}\tau_{\varepsilon_1}^{-1} + \tau_v^{-1})^{-1}, \rho^{-1}(\sum_{t=1}^{n} \tau_{\varepsilon_t})]$. Recall that by convention $\tau_{\varepsilon_1} > 0$. Let then $G : \mathbf{A} \to \mathbf{A}$, be defined by the component functions in the natural way, with $\mathbf{A} = [\underline{a}, \bar{a}]^N$. $\mathbf{A}$ is compact and G is continuous and therefore the existence of a fixed point follows from Brouwer's fixed-point theorem. ■

Two leading examples of our information structure are the case of concentrated arrival of information, in which traders receive private information only in the first period (that is, $\tau_{\varepsilon_t} = 0$ for $t = 2, \ldots, N$), and the case of a constant flow of information, in which the precision of signals is the same in all periods ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$ for all t). In the former case, $a_n = \rho^{-1}((\tau_{\varepsilon_1})^{-1} + (\tau_{n+1})^{-1})^{-1}$ and $a_N = \rho^{-1}\tau_{\varepsilon_1} = a$, the static trading intensity. In the latter $a_n = \rho^{-1}((n\tau_{\varepsilon_1})^{-1} + (\tau_{n+1})^{-1})^{-1}$ and $a_N = N\rho^{-1}\tau_{\varepsilon_1} = Na$. That is, the final trading intensity equals N times the static trading intensity.

Remark 3.1. If short-run agents had access only to the current signal (that is, in period n agent i has access only to s_{in}) then the equilibrium is modified to $a_n = 1/(\rho((\tau_{\varepsilon_n})^{-1} + (\tau_{n+1})^{-1}))$.

Remark 3.2. The *uniqueness* of the linear equilibrium is not asserted (no attempt is made to prove it since none of the results hinge on the uniqueness issue). For the $N = 2$ case uniqueness is easily established (at least for the case $\tau_{\varepsilon_t} = 0$ for $t = 2, \ldots, N$) and simulations also support the uniqueness conjecture for larger N.

Proposition 3.2. *In every equilibrium:*

(i) The trading intensity of generation n informed agents a_n is strictly increasing in n (and is always in the interval $[a', a_N]$, with $a' > \rho^{-1}(\tau_{\varepsilon_1}^{-1} + \tau_v^{-1})^{-1}$). In consequence, the net trading intensity Δa_n is always positive.

(ii) The informativeness and the unconditional volatility of prices are strictly increasing in n.

(iii) Net trading by informed agents in period n is $\Delta x_n = x_n - x_{n-1} = \Delta a_n(v - p_n) - a_{n-1}\Delta p_n$.

Proof.

(i) The least possible value for a_n is the unique a' that solves the implicit cubic equation $a' = \rho^{-1}(\tau_{\varepsilon_1}^{-1} + \tau_1^{-1})^{-1}$, where $\tau_1 = \tau_v + \tau_u(a')^2$ [a' is the stationary trading intensity if prices were to remain at the period 1 level and no more private information was forthcoming (that is, if $a_t = a'$, for all t)]. I show now by backwards recursion that Δa_n is strictly positive for any n. First, from Proposition 3.1, $a_N > a_{N-1}$ since $(\tau_N)^{-1} > (\tau_{N+1})^{-1} = 0$. Now, $\Delta a_n > 0$ if $\Delta a_{n+1} > 0$ again from Proposition 3.1 because $\tau_n = \tau_v + \tau_u \sum_{t=1}^{n}(\Delta a_t)^2$.

(ii) The precision τ_n is strictly increasing in n since $\Delta a_n > 0$ for all n. The result for $\mathrm{Var}\, p_n$ follows.

(iii) Denote by $x_n = \int_0^1 X_n(\tilde{s}_{\text{in}}, p_n)\, di$. According to our convention $\int_0^1 \tilde{s}_{\text{in}}\, di = v$ and therefore $x_n = a_n(v - p_n)$. It follows that $\Delta x_n = x_n - x_{n-1} = \Delta a_n(v - p_n) - a_{n-1}\Delta p_n$. Note that the martingale property of prices implies that $(v - p_n)$ and Δp_n are orthogonal. ■

Equilibria share the feature with the static model that agent i buys or sells in period n according to whether his private estimate of v, $\tilde{s}_{\text{in}}$, is larger or smaller than the current market price p_n. As we know, the current price p_n is a sufficient statistic of all past plus current prices due to competitive market making. The competitive market making sector in period n observes the noisy limit order schedule $L_n(\cdot) : L_n(p_n) = z_n + a_{n-1}p_{n-1} - a_n p_n$, and sets p_n.[13]

[13] Market makers trade according to $Y_n = -(\Delta x_n + u_n) = -d_n(p_n - p_{n-1})$, with $d_n = (\lambda_n)^{-1} - a_n$.

The trade intensity of generation n of informed agents (a_n) depends positively, ceteris paribus, on three magnitudes: the degree of risk tolerance ρ^{-1}, the total precision of signals $\sum_{t=1}^{n} \tau_{\varepsilon_t}$, and the precision of prices in period $n+1$ in the estimation of the fundamental value v. The first two effects are as in the static model. The third effect means that the closer p_{n+1} is to v, the more intensely informed speculators want to trade. In the last period (N), $p_{N+1} = v$ and then trading intensity reaches its maximum value. The third effect is easily understood. Short-term speculators have information about v but can not hold the asset until v is realized. The return of generation n agents is $(p_{n+1} - p_n)x_{in}$. Traders can anticipate the information in p_n, and the closer p_{n+1} is to v (that is the more informative is p_{n+1} about v) the better they can predict p_{n+1} (since they receive private signals about v). As n increases so does the precision of prices as estimators of v and consequently the desired trading intensity of informed agents a_n. In other words, the expected return to informed trading $E\{p_{n+1} - p_n \mid \tilde{s}_{in}, p_n\}$ is increasing relative to the risk faced, $\mathrm{Var}\{p_{n+1} \mid \tilde{s}_{in}, p_n\}$ as n increases.

In the case of concentrated information arrival, trading intensities are always less than the static level, a, which is reached in the last period. The trading intensity a_N equals the static level a because in the static model the trading intensity does not depend on the precision of prices $a = \rho^{-1}\tau_{\varepsilon_1}$. The result that the trading intensity of speculation a_n is increasing with n (small at first and catching up close to the liquidation date), even when private signals are only received in the first period, is reminiscent of a result of Dow and Gorton (1993). The authors consider an infinite-horizon model with a stock which yields random dividends in each period. Some agents may receive information in advance about the dividends of a certain period (the event date). Short-term informed traders are risk neutral and face transactions costs. The outcome is that short-term traders, when far away from the event, will not act on their information since short-term trading reduces the profitability of speculation.

It is worth emphasizing that noise trading does affect the intensity of trade via the precision of prices. For example, with $N = 2$ (and $\tau_{\varepsilon_2} = 0$) an increase in noise trading decreases the trading intensity a_1 of the first generation in the unique equilibrium. Instead of a camouflage effect [as in Kyle (1985)], an increase in noise trading increases the noise in the first period return p_2 (with respect to v) and the response of first period risk-averse informed speculators is a decreased trading intensity.

Proposition 3.1 characterizes implicitly the equilibrium but does not give a closed-form solution. Simulations conducted for a range of parameter values for the two leading examples yield some insights into the dynamics of the market.[14]

In both cases a unique equilibrium always has been found and, consistent with Proposition 3.2, a_n is increasing in n. The pattern that emerges from the simulations in the case $\tau_{\varepsilon_t} = 0$ for $t = 2, \ldots, N$, is the following: Δa_n is increasing in n (starting from $n = 2$ since $\Delta a_1 = a_1$), but at first the increase is very small and is noticeable only when close to the end of the horizon N. The conditional volatility of prices is found to be increasing with n, slowly at first and faster when close to the end of the horizon (it may decrease from period N to the liquidation stage $N + 1$). In the first rounds of trade the precision of prices is almost flat and informed agents only vary their positions slightly from period to period. When closer to the end of the horizon, the net trading intensity of informed agents increases and contiguous generations want to have increasingly different positions in the risky asset, since the precision of prices is increasing.[15] Market depth is nonmonotonic in n. It increases first due to the reduced net trading in the second period (in the first period only the first generation trades); it flattens out (at a high level) later since net trading by informed agents is very small and decreases close to the end of the horizon due to increased net informed trading activity.

In the constant flow of information case ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$, for all t) the emerging patterns are more complex.[16] The temporal evolution of trading intensities a_n is typically concave and then convex and correspondingly the net trading intensities Δa_n evolve according to a U-shaped form with mild decreases at the beginning and sharp increases close to the end of the horizon. For intermediate values of the parameters $(\rho, \sigma^2_{\varepsilon_1}, \sigma^2_u)$, the conditional volatility and market depth are nonmonotonic, with a pattern of decreasing-increasing-decreasing or U-shaped for the first, and increasing-decreasing-increasing or inverted U-shaped for the second. Otherwise, that is, for low or high values of the parameters $(\rho, \sigma^2_{\varepsilon_1}, \sigma^2_u)$, the conditional volatility is decreasing and market depth increasing in n. Notice that market depth $[(\lambda_n)^{-1} = \tau_n/\tau_u \Delta a_n]$ must be increasing in n for low n since then

[14] Simulations have been performed systematically in the following ranges of parameter values: ρ in [1, 4], σ^2_u in [.05, 2], $\sigma^2_{\varepsilon_1}$ in [.1, 2], letting $\sigma^2_v = 1$. The length of the horizon has been considered up to $N = 25$. The program Mathematica was used to run the simulations, and in all of them equilibrium has turned out to be unique (that is, starting the simulation with different initial conditions to find a solution to the equilibrium parameter equations leads to the same equilibrium).

[15] Market makers lean against the wind in general (d_n positive), although at the end of some simulations d_N may be negative (market depth has decreased and trading by the last generation of informed agents is intense).

[16] Simulations have been extended for this case to cover $\rho = .1$ and $\rho = 40$, $\sigma^2_u = .005$ and $\sigma^2_u = 40$, and $\sigma^2_{\varepsilon_1} = .05$, and $\sigma^2_{\varepsilon_1} = 40$.

Δa_n is decreasing and τ_n is increasing always in n. For larger n, Δa_n increases and may give rise to nonmonotonic patterns. Similarly, the conditional volatility $[\mathrm{Var}(p_n \mid p_{n-1}) = (\tau_{n-1})^{-1} - (\tau_n)^{-1} = \tau_u(\Delta a_n)^2/\tau_{n-1}\tau_n]$ must be decreasing for low n and nonmonotonic for larger n. Figure 1 shows a pattern for a specific case (simulated for the values $N = 25$, $\sigma_v^2 = 1$, $\rho = 2.5$, $\sigma_{\varepsilon_1}^2 = 1.05$, $\sigma_u^2 = .05$; the short-term case is represented by the continuous lines).

The qualitative simulation results for $n = 1, \ldots, N$, as well as the analytical results obtained, are summarized in Table 1, which also compares the equilibrium with short-term traders with the long-term case, treated in the next section.

4. Long-Term Investment

Trading with a short investment horizon contrasts with trading by long-term agents. Long-term informed agents maximize the expectation of the utility of final wealth. The final wealth of informed agent i is $W_{iN} = \sum_{t=1}^{N-1}(p_{t+1} - p_t)X_{it} + (v - p_N)X_{iN} = \sum_{t=1}^{N} \pi_{it}$. Informed agent i wants to maximize $E\{U(W_{iN})\} = -E\exp\{-\rho W_{iN}\}$. The following proposition characterizes the equilibrium.

Proposition 4.1. *With long-term informed agents there is a unique linear equilibrium:*

$$X_n(\tilde{s}_{\mathrm{in}}, p_n) = a_n(\tilde{s}_{\mathrm{in}} - p_n),$$

$$\textit{with } a_n = \rho^{-1}\left(\sum_{t=1}^{n} \tau_{\varepsilon_t}\right).$$

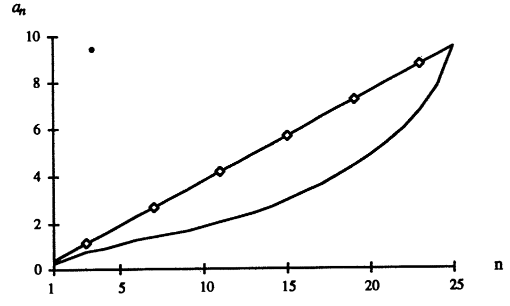
Corollary. $\Delta a_n = \rho^{-1}\tau_{\varepsilon_n}$.

Proof. See Appendix. ■

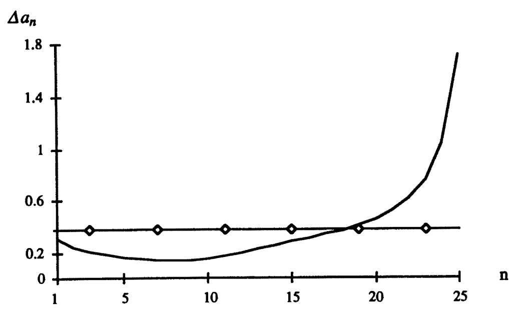
Remark 4.1. The long-term trading intensity at any period is always larger than in the short-term case. That is,

$$\rho^{-1}\left(\sum_{t=1}^{n} \tau_{\varepsilon_t}\right) > \rho^{-1}\left(\left(\sum_{t=1}^{n} \tau_{\varepsilon_t}\right)^{-1} + (\tau_{n+1})^{-1}\right)^{-1}$$

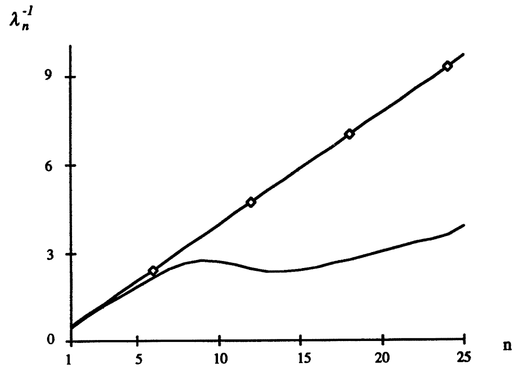
for any $n < N$, since then $(\tau_{n+1})^{-1} > 0$. Indeed, short-term trading makes agents less responsive to private information since risk-averse short-run agents in period n have information about v but care about p_{n+1}, which is a garbled signal of v because of noise trading. Only when $n = N$, $p_{N+1} = v$ and the two intensities coincide: $a_N = \rho^{-1}(\sum_{t=1}^{n} \tau_{\varepsilon_t})$. Note that the final trading intensity a_N is also the (across periods) aggregate net trading intensity: $\sum_{t=1}^{N} \Delta a_t = a_N$. Therefore in



(1A)

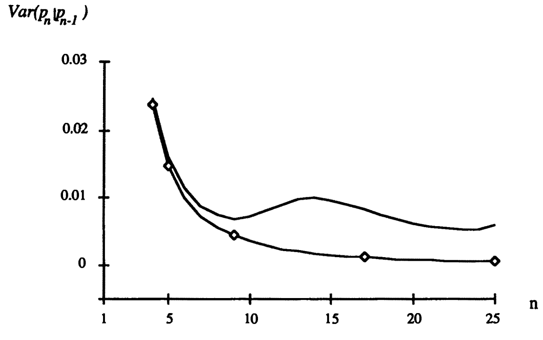


(1B)

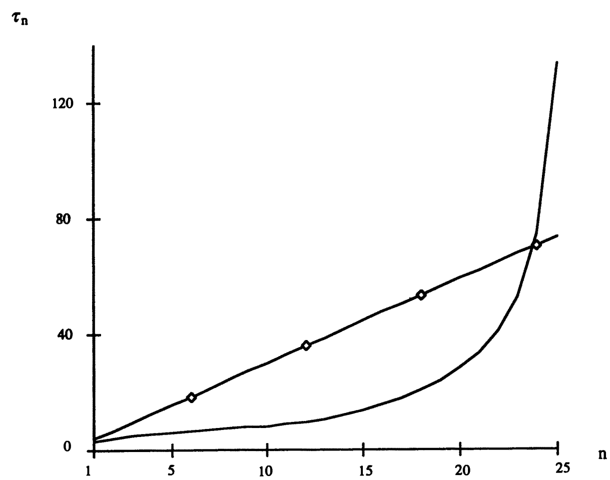


(1C)

Figure 1
The temporal evolution of short-term (with continuous lines) and long-term (with dotted lines) trading magnitudes in the case of a constant flow of information: trading intensity (1A), net trading intensity (1B), market depth (1C), conditional price volatility (1D), and price precision (1E). The parameter values of the simulation are $\rho = 2.5$; $\sigma_\varepsilon^2 = 1.05$; $\sigma_u^2 = 0.05$; $\sigma_v^2 = 1$; $N = 25$.



(1D)



(1E)

Figure 1
Continued.

terms of net trading intensities the long-term and the short-term cases differ in the temporal distribution of the same aggregate. Further, it is clear that as n increases and agents receive more information the price precision ($\tau_n = \tau_v + \tau_u \rho^{-2} \sum_{t=1}^{n} \tau_{\varepsilon_t}^2$) and ex ante volatility (Var p_n) will increase. Both will increase more with lower risk aversion or noise trading, and with higher precision of information.

Remark 4.2. The net trading intensity at period n, Δa_n, depends directly on the precision of period n signals. The more precise they are the higher is Δa_n. If the precision is constant (the constant flow

Table 1
Short-term and long-term trading (temporal evolution of magnitudes $n = 1, \ldots, N$)

		Short-term		Long-term	
		$\tau_{\varepsilon_t} = 0,\ t \geq 2$	$\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$, all t	$\tau_{\varepsilon_t} = 0,\ t \geq 2$	$\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$, all t
Trading intensity	(a_n)	Increasing	Increasing	Constant (static level)	Increasing (linearly)
Net trading intensity	(Δa_n)	Increasing ($n \geq 2$)	U-shaped	Zero ($n \geq 2$)	Constant
Market depth	(λ_n^{-1})	Inverted U-shaped	Nonmonotonic or Increasing	Static level ($n = 1$) or ∞ ($n \geq 2$)	Increasing (linearly)
Volatility	$(\mathrm{Var}\, p_n)$	Increasing	Increasing	Constant (static level)	Increasing
Conditional volatility	$(\mathrm{Var}(p_n \mid p_{n-1}))$	Increasing	Nonmonotonic or Decreasing	Static level ($n = 1$) or Zero ($n \geq 2$)	Decreasing
Price precision	(τ_n)	Increasing	Increasing	Constant (static level)	Increasing (linearly)

of information case), then, for any n, Δa_n is constant and equal to the static trading intensity $a = \rho^{-1}\tau_{\varepsilon_1}$. That is, given the constant flow of private information into the market informed traders desire to modify their positions in a constant way. The consequence is that a constant amount of information is incorporated into the prices, $\tau_n = \tau_v + \tau_u n a^2$, the price precision growing linearly with n. This contrasts with the short-term case in which the net trading intensities Δa_n, even when the precision of private information is constant across periods, vary in different periods. Further, market depth, $(\lambda_n)^{-1} = \tau_n/\tau_u \Delta a_n$, will also increase linearly with n, since τ_n grows linearly with n and Δa_n is constant, and the conditional volatility, $\text{Var}(p_n \mid p_{n-1}) = \tau_u(\Delta a_n)^2/\tau_{n-1}\tau_n$, will decrease with n.

Remark 4.3. If private information is received only in the first period then $\Delta a_n = 0$ for $n \geq 2$ and there is no informed trading after the first period. Indeed, then the equilibrium strategy for informed agents is to take a position in period 1, with a trading intensity equal to the static one, and hold to it until the end (a buy-and-hold strategy). Market makers stabilize prices at the first period level p_1 absorbing noise trading in periods $n = 2, \dots, N$. Risk-averse informed speculators do not learn anything new after the first period and therefore keep the same position until the liquidation of the risky asset. Formally, $X_n(s_{i1}, p_n) = a(s_{i1} - p_n)$, $p_n = p_1 = \lambda_1 z_1 + (1 - \lambda_1 a_1)\bar{v}$, with $z_1 = a_1 v + u_1$, $\lambda_1 = \tau_u a/\tau_1$, $\tau_1 = \tau_v + \tau_u a^2$ for $n = 2, \dots, N$. The unique (linear) equilibrium replicates the static benchmark. This provides a foundation for the static rational expectations model: adding more rounds of trade is superfluous. The fact that the static equilibrium is also a long-run equilibrium should not be surprising. After all, long-term traders condition on prices, are risk averse and, after the first round, no new private or public information is forthcoming with market makers absorbing noise trade. If we compare with the short-term trading case, the long-term (and static) trading intensity a is spread among the N periods in terms of the net trading intensities of short-run agents: $\sum_{t=1}^{N} \Delta a_t = a$.

It is worth remarking that the existence of a competitive market-making sector is crucial for the result. Indeed, according to Brown and Jennings (1989), in a two-period version of the model without a competitive market-making sector, no general existence results can be proved (equilibrium is guaranteed to exist provided second period prices are fully revealing). Further, in a related model, Grundy and McNichols (1989) show that equilibria in which second-period prices add information about the fundamental value coexist with equilibria in which they are noisy measures of first-period prices.

5. Short Horizons and Price Informativeness

In this section I compare the degree of information incorporated in prices when informed traders have short and long horizons. In the short-horizon case a new generation of one-period lived informed traders comes to market (and inherits the private information of an agent of the previous generation) every period or, alternatively, traders are myopic. In the long-horizon case informed traders live for the whole span (N periods). In both cases, there is a constant (unit) mass of informed traders every period, and they have the same precision of private information.

Price informativeness in period N is given by

$$\tau_N = \tau_v + \tau_u \sum_{t=1}^{N} (\Delta a_t)^2$$

and the total net trading intensity is $\sum_{t=1}^{N} \Delta a_t = a_N$ [equal to $\rho^{-1}(\sum_{t=1}^{N} \tau_{\varepsilon_t})$] in both the long-term and the short-term cases. In consequence, τ_N will be larger or smaller in the short-term versus the long-term case according to the relative temporal distribution of the (positive) net trading intensities, Δa_t. Indeed, τ_N will be smaller the more equally distributed are the increments Δa_t, the minimum being reached for equal increments every period. In other words, τ_N can be thought of as an inequality index of the variables Δa_t (which together add up to a_N). The precision τ_N is larger the more unequally distributed are the increments Δa_t.[17] We will comment also on relative price precisions for n periods before the last ($n < N$).

Consider the two leading examples. First, the case of once-and-for-all information reception (that is, $\tau_{\varepsilon_t} = 0$ for $t = 2, \ldots, N$); second, the case of a constant flow of information ($\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$, for all t).

5.1 Concentrated Arrival of Information

In this case with long-term agents there is informed trading only in the first period (see Remark 4.3), and equilibrium informativeness of prices τ corresponds to the static equilibrium level. Short horizons imply that informed agents will trade in every period. Will this imply that more information ends up being incorporated in prices with short-run agents who trade on the basis of their private information?

[17] For example, one inequality index is the Herfindahl (H) concentration index of an industry with n firms; it is equal to the sum of the squares of the market shares of the firms. Concentration is maximal in the monopoly case and diminishes with the number of firms and with more equal shares. In the model of this article net trading intensities play the role of market shares.

I will show now that, independent of the length of the horizon N, the information revealed by prices with short-term traders will be bounded above away from the static (long-term) case τ. This is easy to understand since the long-term case involves the maximum inequality in the temporal distribution of the net-trading intensities: informed trade is concentrated in the first period.

Proposition 5.1. *With $\tau_{\varepsilon_t} = 0$ for $t = 2, \ldots, N$, the precision of the final price p_N, τ_N, with short-term traders is bounded above and away (uniformly in N) from the precision of the static (long-term) rational expectations price τ.*

Proof. Fix N, then $\tau_N^{(N)} = \tau_v + \tau_u A_N^{(N)}$. Now, $A_N^{(N)} = \sum_{n=1}^{N}(\Delta a_n^{(N)})^2 = (a_1^{(N)})^2 + \sum_{n=2}^{N}(\Delta a_t^{(N)})^2 \leq (a_1^{(N)})^2 + (a - a_1^{(N)})^2 = a^2 - 2a_1^{(N)}(a - a_1^{(N)}) \leq a^2 - 2a(a - a_1^{(2)}) < a^2$. The first inequality holds since $\sum_{n=2}^{N} \Delta a_n^{(N)} = a - a_1^{(N)}$; the second since $a_1^{(N)}$ is in the interval $[a', a_1^{(2)}]$ [the lower bound comes from Proposition 3.2 (i); the upper bound is obtained since $a_1^{(N)}$ is increasing in $a_2^{(N)}$, and $a_2^{(N)}$ reaches its maximum possible value a when $N = 2$]; the last inequality follows since $a'(a - a_1^{(2)})$ is positive. ∎

Remark 5.1. The result also holds for any period before the last, since in the long-term case the price precision is constant from the first period, while in the short-term case it is increasing up to its value at $n = N$. In short, the informativeness of prices is higher in the long-term case period by period.

The formal intuition for the result is very simple. The precision of prices depends on the sum of the squares of the net trading intensity of informed agents $(\Delta a_n^{(N)})^2 (\tau_N^{(N)} = \tau_v + \tau_u \sum_{n=1}^{N}(\Delta a_n^{(N)})^2)$. The aggregate net trading intensity of informed agents across periods equals the static or long-run intensity of trade, that is, $\sum_{n=1}^{N} \Delta a_n^{(N)} = a$. It follows then $\sum_{n=1}^{N}(\Delta a_n^{(N)})^2 < a^2$, since $\Delta a_n^{(N)}$ is positive. The accumulated price precision will have to be smaller than the long-term precision. In summary, in order to make prices informative it is better to trade once (as with long-term traders) than to spread net trades over more periods.

Let us review the chain of facts which explains the result. First of all, the precision of prices depends on the *net* trading intensities of informed agents $(\Delta a_n^{(N)})$. Second, short-term traders desire to trade more intensely as n increases (a_n is increasing in n), but always below the static intensity a, since for larger n the next-period price (the short-run benefit) is closer to the fundamental value v, about which they have information. This means that $\Delta a_n^{(N)} > 0$ for all n. Third, in the last period short-term traders desire to trade according to the static trading intensity, $a_N = a$, because trading intensity when facing the liquidation period is independent of the precision incorporated in prices (and therefore it does not matter whether or not there is price information from the past). This means that $\sum_{n=1}^{N} \Delta a_n^{(N)} = a$. Finally, long-term traders use a buy-and-hold strategy, concentrating their trade in the first period according to the static trading intensity a, because they are risk averse and after the first period no new information (public or private) is forthcoming about v.

In essence, risk-averse short-term traders hold back on trading since they have information about v, but their return is the next-period price, which is a noisy measure of v. Long-term traders, in contrast, can wait until the liquidation period and trade only when they receive information.

By analogy with an inequality index, the more periods there are, and the more even trading intensities with short investment horizons are, the worse is price precision. If the long-term intensity is spread among more trading periods, the accumulated precision of prices (depending on the sum of the squares of the net trading intensities) will tend to diminish. For example, and only for illustrative purposes, if the static trading intensity were to be spread evenly among the N periods then precision ("concentration") attains its lowest possible level, a^2/N, and is decreasing in N.

The simulations performed with short-term traders show indeed that the price precision tends to decrease with an increase in the horizon [that is, $\sum_{n=1}^{N}(\Delta a_n^{(N)})^2$ is decreasing in N]. Increasing the horizon actually hurts the precision incorporated in prices with short-term traders.

Increasing the number of trading periods with short-term traders does not help information revelation through prices and, in the context of our model, does not matter with long-term traders. With short-term traders prices do not converge to the fundamental value as N tends to infinity (indeed, their informativeness is bounded above by the static rational expectations precision). Nevertheless, it is possible to show that in a semi-strong informationally efficient market convergence of prices to the underlying value of the asset as the number of periods N becomes infinite does occur provided trades are notional until the fundamental value v is realized (that is, traders' orders before the realization of v are not executed and can be modified; trade happens according to the outstanding orders when v is realized at some random time). In Vives (1992) an *information tâtonnement* is considered in which *theoretical* prices are quoted by a competitive market-making sector with the purpose of revealing the joint information of informed agents (that is, with a price discovery purpose). The information tâtonnement is seen to reveal the fundamental value at "normal" rate of $1/N^{1/2}$. The basic reason is that in the information tâtonnement the precision of prices depends on the (accumulated) gross trade intensities of informed speculators and not on the (accumulated) net incremental trade intensities, which are necessarily bounded.

A corollary to Proposition 5.1 is that total volatility up to period N with short-term traders $[\sum_{t=1}^{N} \mathrm{Var}(p_t \mid p_{t-1}) = (\tau_v)^{-1} - (\tau_N)^{-1}]$ is bounded above and away from the long-term case $((\tau_v)^{-1} - \tau^{-1})$. This reflects that, up to period N, less information has been incorporated into prices with short-term traders.

A comparison of the temporal evolution, for $n = 1, \ldots, N$, of some magnitudes of interest in the long-term and short-term cases is provided in Table 1. An interesting result is that the *distribution* of total volatility over time is quite different in the two cases. With long horizons information is incorporated in prices in the first period and therefore prices are more volatile at the beginning and less at the end than with short horizons. Indeed, conditional volatility has a U-shaped temporal evolution (including the liquidation period N to $N+1$). With short-term traders (conditional) volatility slowly grows over time [with the only possible exception the liquidation period (from N to $N+1$) according to the simulations].

5.2 Constant Flow of Information

Let us now consider the case in which informed traders receive signals of equal precision every period. In this case, with long-term traders, the net trading intensities are equal across periods to the static trading intensity a; meanwhile, with short-term traders, net trading intensities differ across periods. The outcome is that the result of Proposition 5.1 is reversed: long-term trading induces a lower final-price precision.

Proposition 5.2. *With $\tau_{\varepsilon_t} = \tau_{\varepsilon_1}$, for all t, the price precision of the final price with short-term traders is bounded below and away (uniformly in N) from the long-term case.*

Proof. Fix N. The minimum of $\tau_N^{(N)} = \tau_v + \tau_u \sum_{n=1}^{N} (\Delta a_n^{(N)})^2$ subject to $\sum_{n=1}^{N} \Delta a_n^{(N)} = Na$ is attained by setting $\Delta a_n^{(N)} = a$, which is precisely the long-term solution. Now, let $\Delta a_n^{(N)}$ denote the short-term solution. Then

$$\tau_N^{(N)}\ (\text{short-term}) - \tau_N^{(N)}\ (\text{long-term})$$

$$= \tau_u \left(\sum_{n=1}^{N} (\Delta a_n^{(N)})^2 - N a^2 \right)$$

$$= \tau_u \left((a_1^{(N)})^2 - a^2 + \sum_{n=2}^{N} (\Delta a_n^{(N)})^2 - (N-1) a^2 \right),$$

which is bounded away from zero since $a_1^{(N)} \leq a_1^{(2)} < a$. The first inequality follows from the expressions: $a_1^{(N)} = \rho^{-1}((\tau_{\varepsilon_1})^{-1} + (\tau_2^{(N)})^{-1})^{-1}$ and $\tau_2^{(N)} = \tau_v + \tau_u((a_1^{(N)})^2 + (a_2^{(N)} - a_1^{(N)})^2)$, noting that $a_1^{(N)}$ is increasing in $a_2^{(N)}$ and $a_2^{(N)}$ reaches its maximum possible value when $N = 2$ (equal to $2a$). The second inequality is obvious. ■

Again the formal intuition of the result is very simple. In the present case the long-term solution of equal increments $\Delta a_n^{(N)}$ minimizes the price precision. The chain of facts which explains the result follows. As before, the precision of prices depends on $\Delta a_n^{(N)}$, and $\Delta a_n^{(N)} > 0$ for all n and for both the short-term and the long-term cases. Now, in the last period both short-term and long-term traders desire to trade according to the intensity, $a_N = \rho^{-1}(\sum_{t=1}^{N} \tau_{\varepsilon_t})$, because the trading intensity when facing the liquidation period is independent of the precision incorporated in prices. This means that $\sum_{n=1}^{N} \Delta a_n^{(N)} = \rho^{-1}(\sum_{t=1}^{N} \tau_{\varepsilon_t})$ in both cases. Finally, long-term traders spread their net trade according to the constant flow of information, while short-term traders also take into account the differences in the precision of prices; this results in an uneven temporal distribution of net trading intensities.

In the present situation, increasing the number of trading periods does help the precision of the final price τ_N, since with long-term traders τ_N increases linearly with N. Indeed, as N grows without bound prices converge to the fundamental value v. This should not be surprising given that traders receive a constant flow of information.

As before, a corollary to Proposition 5.2 is that total volatility, up to period N, in the short-term case is bounded below and away from the long-term case. This reflects that, up to period N, less information has been incorporated into prices with long-run traders.

Table 1 expounds some qualitative comparisons of the short-term and the long-term regimes (including both analytical and simulation results as I have pointed out before). In the latter, trading intensities a_n grow linearly with n, while in the former, first increase in a concave, then in a convex way and are always below the long-term benchmark. The result is that net trading intensities Δa_n are first below and at the end above the long-term constant net trading intensity. In fact, both solutions are close for low or high values of the parameter vector

$(\rho, \sigma_{\varepsilon_1}^2)$. For low values of $(\rho, \sigma_{\varepsilon_1}^2)$ trading intensities are very high in both cases and the first-period prices almost reveal v. For low values of σ_u^2 trading intensities will also be very close in both cases, since prices are very informative about v. For high values of $(\rho, \sigma_{\varepsilon_1}^2)$ trading intensities are very low in both cases. For high values of σ_u^2 prices contain very little information about v in any case. The outcome, supported by the simulations, is that for both low and high values of the vector $(\rho, \sigma_{\varepsilon_1}^2, \sigma_u^2)$ short-term and long-term equilibria are close in terms of the temporal pattern of price precision, market depth, and conditional volatility. Indeed, for these parameter constellations price precision and market depth increase linearly in n and conditional volatility is decreasing in n in the two cases. The simulations show that as the values of the vector $(\rho, \sigma_{\varepsilon_1}^2, \sigma_u^2)$ increase, market depth turns from linear to nonmonotonic (increase–decrease–increase pattern) to inverted U-shaped to linear again; and conditional volatility turns from decreasing to nonmonotonic (decrease–increase–decrease pattern) to U-shaped to decreasing again.[18]

Figure 1 displays one instance of the intermediate case (with the values $N = 25$, $\sigma_v^2 = 1$, $\rho = 2.5$, $\sigma_{\varepsilon_1}^2 = 1.05$, $\sigma_u^2 = .05$; the dotted lines represent the long-term case). Conditional volatility in the long-term case, and relative to the short-term case, is larger first and smaller later (although this does not show up in Figure 1D, before period 4 the dotted line is above the continuous one). This is typically the case, according to the simulations, for reasonable values of the parameters.

Remark 5.2. The fact that (according to the simulations) short-term net trading intensities are first below and then above the constant long-term case, together with the result in Proposition 5.2 that the long-term price precision ends up (for $n = N$) above the short-term one imply that price precisions with short-term traders are first below and then above that with long-term traders. In fact, for reasonable values of the parameters, it is the case that (as in Figure 1E) only for the very last periods is the short-term price precision above the long-term one.

It is worth remarking that the departure of price precision of the short-term case from the long-term one can be substantial for plausible parameter ranges. The simulations show that the difference (τ_N(short-term) $-$ τ_N(long-term)) has an inverted U-shaped dependence on the value of any parameter of $(\rho, \sigma_{\varepsilon_1}^2, \sigma_n^2)$ (fitting the other two), and that the ratio (τ_N(short-term)$-\tau_N$(long-term))/τ_N (long-term) is either

[18] A sequence of figures describing the evolution of a_n, Δa_n, $(\lambda_n)^{-1}$, and $\mathrm{Var}(p_n/p_{n-1})$ as the vector $(\rho, \sigma_{\varepsilon_1}^2, \sigma_u^2)$ increases from low to high values (with patterns at both extremes similar to the long-term case) is available on request.

increasing or has an inverted U-shaped dependence on any parameter of $(\rho, \sigma_{\varepsilon_1}^2, \sigma_u^2)$ (fixing the other two). The ratio can reach high values for plausible parameter constellations. For example, it is more than 5 for the values $(N = 25, \sigma_v^2 = 1, \rho = 2.5, \sigma_{\varepsilon_1}^2 = 1.05, \sigma_u^2 = 1.025)$. (It is .8 for the example in Figure 1.)

The analysis of the two leading examples makes clear that in general the result is ambiguous and depends on the temporal pattern of private information reception, with concentrated and constant flow being polar cases. By continuity, and with respect to long-term trading, lumpy patterns of private information arrival (close to the pure concentrated case) will imply short horizons decrease the information content of prices, while diffuse patterns (close to the constant flow case) will imply short horizons increase the information content of prices close to the end of the horizon. The ambiguity arises from the fact that information precision depends on the distribution of net trading intensities, being lower for more evenly distributed intensities. With concentrated information arrival long-term trading provides the most unequal distribution and therefore maximum price informativeness. With a constant flow of information long-term trading provides the most equal distribution and therefore minimum final price informativeness. Nevertheless, it is worth pointing out that if short-term agents did not have access to the private signals of previous periods then in the short-term case the final trading intensity would be $a_N = a$ while in the long-term case it would be $a_N = Na$. The result of Proposition 5.1 holds here a fortiori — the long-term precision is strictly larger than the short-term one: $\tau_v + N\tau_u a^2 > \tau_v + \tau_u a^2 > \tau_v + \tau_u \sum_{n=1}^{N}(\Delta a_n^{(N)})^2$, since $\sum_{n=1}^{N} \Delta a_n^{(N)} = a$.

A question may arise about the robustness of the results to the introduction of discretionary liquidity traders. For example, in the case of concentrated private information arrival with long-run agents the market is infinitely deep after the first period, and therefore liquidity trading would tend to be concentrated in those periods. Nevertheless, I conjecture that the result for this case (Proposition 5.1) holds. Here is the argument. Suppose that in every period there is some proportion of nondiscretionary liquidity traders which have to trade in the period for exogenous reasons [as in Admati and Pfleiderer (1988), for example], but that there also exist discretionary liquidity traders who can choose in which period to trade. Then in the long-term case the price will be more informative than in the benchmark case with no discretionary liquidity trading, since the nature of the equilibrium will be unchanged and in the first period only the liquidity agents with exogenous constraints will trade, with the others shifting trade to later periods. Now, with short-term traders, for whatever temporal pattern of liquidity trading, the final precision incorporated in prices must be less than the long-term one since in the long-run case noise trading is at its minimum in the first period.

6. Concluding Remarks

This article presented a competitive model of asset pricing with private information in a (semi-strong) informationally efficient market. The effect of short investment horizons on the informativeness of prices was addressed. In the process I have characterized in closed-form the equilibrium of a dynamic market with risk-averse long-term traders with non-nested information sets. We have seen how short horizons impinge negatively or positively on the information content of of prices depending on the temporal pattern of information arrival (concentrated versus diffuse). A basic fact is that short-term trading does not change the across-periods aggregate net trading intensity of informed speculators. It just spreads the long-term trading intensity among the different trading periods leading to a diminished (increased) final price informativeness when long-term speculators concentrate (spread evenly) their trading with once-and-for-all (diffuse) information reception. We can also compare price precisions with long-term and short-term traders for any period. With concentrated arrival of information price precision is uniformly higher with long-term traders. With a constant flow of information the price precision is higher at the beginning and lower at the end with long-term traders. Obviously, if stock prices guide production decisions[19] the information content of prices and the departure of prices from fundamental values has welfare implications: the horizons of traders will matter in terms of welfare losses at the economy level.

Different possible extensions are left for future research. A main question is how robust the results will be to changes in the model presented. We have already indicated a possible extension to incorporate discretionary liquidity traders. Further, the work of Dow and Gorton (1993) in an infinite horizon context suggests that some of the results of the present paper should be robust to stationary versions of the model. Other extensions include the consideration of markets with both short-term and long-term traders, endogenous information acquisition, correlated noise trading, multisecurities markets[20], and the effects of public signals.

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[19] See, for instance, Myers and Majluf (1984) and Leland (1992) for accounts of how the informational quality of prices affects the allocation of resources.

[20] See Admati (1985) and Caballe and Krishnan (1992) for, respectively, competitive and strategic static models.

Appendix

Proof of Proposition 4.1. The result is derived by backwards recursion. Denote by X_{it} the position in the risky asset of informed agent i at period t. The final wealth of agent i is $W_{iN} = \sum_{t=1}^{N-1}(p_{t+1}-p_t)X_{it}+(v-p_N)X_{iN} = \sum_{t=1}^{N}\pi_{it}$. Informed agent i wants to maximize the expected value of the utility of final wealth: $E\{U(W_{iN})\} = -E\exp\{-\rho W_{iN}\}$.

As before, given linear strategies and market efficiency, the information of trader i in period, $N\{s_i^N, p^N\}$, can be summarized in $\{\tilde{s}_{iN}, p_N\}$. At stage N trader i chooses X_{iN} to maximize

$$-\exp\left\{-\rho\sum_{t=1}^{N-1}\pi_{it}\right\}E\{\exp\{-\rho\pi_{iN}\} \mid \tilde{s}_{iN}, p_N\}.$$

It follows then, as in the short horizon model, that $X_{iN}(\tilde{s}_{\text{in}}, p_N) = a_N(\tilde{s}_{\text{in}} - p_N)$, $a_N = \rho^{-1}(\sum_{t=1}^{N}\tau_{\varepsilon_t})$. Substituting the optimal period N strategy in the last period profit π_{iN} and letting $\tau_{iN} = \sum_{t=1}^{N}\tau_{\varepsilon_t} + \tau_N$, we obtain

$$E\{\exp\{-\rho\pi_{iN}\} \mid \tilde{s}_{iN}, p_N\} = \exp\left\{-\frac{1}{2}\rho^2 a_N^2(\tau_{iN})^{-1}(\tilde{s}_{iN} - p_N)^2\right\}.$$

At stage $N-1$ speculator i chooses X_{iN-1} to maximize

$$-\exp\left\{-\rho\sum_{t=1}^{N-2}\pi_{it}\right\}$$
$$\times\ E\{E\{\exp\{-\rho\pi_{iN-1}\}\exp\{-\rho\pi_{iN} \mid \tilde{s}_{iN}, p_N\} \mid s_i^{N-1}, p^{N-1}\}$$

or, since $\pi_{iN-1} = (p_N - p_{N-1})X_{iN-1}$,

$$-\exp\left\{-\rho\sum_{t=1}^{N-2}\pi_{it}\right\}$$
$$\times\ E\{\exp\{-\rho\pi_{iN-1}\}E\{\exp\{-\rho\pi_{iN}\} \mid \tilde{s}_{iN}, p_N\} \mid s_i^{N-1}, p^{N-1}\}.$$

Equivalently,

$$-\exp\left\{-\rho\sum_{t=1}^{N-2}\pi_{it}\right\}E\left\{\exp\{-\rho\phi_{iN-1}\} \mid s_i^{N-1}, p^{N-1}\right\},$$

where $\phi_{iN-1} = (p_N - p_{N-1})X_{iN-1} + \frac{1}{2}\rho a_N^2(\tau_{iN})^{-1}(\tilde{s}_{iN} - p_N)^2$.

Similar arguments as before establish that a sufficient statistic for $\{s_i^{N-1}, p^{N-1}\}$ in the estimation of $p_N - p_{N-1}$ and $\tilde{s}_{iN} - p_N$ is $\{\tilde{s}_{iN-1}, p_{N-1}\}$. The term ϕ_{iN-1} is a quadratic form of the bivariate (column) vector $Z = (\tilde{s}_{iN} - p_N - \mu_1, p_N - \mu_2)$ which is normally distributed (conditional on $\{\tilde{s}_{iN-1}, p_{N-1}\}$) with zero mean and variance–covariance matrix $\sum$ (that is, $\mu_1 = E(\tilde{s}_{iN} - p_N \mid \tilde{s}_{iN-1}, p_{N-1})$ and $\mu_2 = E(p_N \mid \tilde{s}_{iN-1}, p_{N-1})$):

$$\phi_{iN-1} = c + b'Z + Z'AZ$$

where $c = (\mu_2 - p_{N-1})x_{iN-1} + \frac{1}{2}\rho a_N^2(\tau_{iN})^{-1}(\mu_1)^2$; b equals the (column) vector $(\rho a_N^2(\tau_{iN})^{-1}\mu_1, x_{N-1,i})$; and A is a 2×2 matrix with $a_{11} = \frac{1}{2}\rho a_N^2(\tau_{iN})^{-1}$ and the rest zeros. It follows then (see, for example, Danthine and Moresi, 1992, p. 16) that, since $\sum$ can be checked to be non-singular and $\sum^{-1} + 2\rho A$ is positive definite $-E\{\exp\{-\rho\phi_{iN-1}\} \mid \tilde{s}_{iN-1}, p_{N-1}\}$ equals the constant $(\det \sum)^{-1/2}(\det(\sum^{-1} + 2\rho A))^{-1/2}$ times

$$-\exp\left\{-\rho\left\{c - \frac{1}{2}\rho b'\left(\sum^{-1} + 2\rho A\right)^{-1} b\right\}\right\} \tag{1}$$

The FOC to maximize (1) with respect to x_{iN-1} yields — denoting the elements of $H = (\sum^{-1} + 2\rho A)^{-1}$ by h_{ij} —

$$\mu_2 - p_{N-1} - \rho(h_{22}x_{iN-1} + h_{12}b_1) = 0, \text{ or}$$

$$x_{iN-1} = \frac{\mu_2 - p_{N-1}}{\rho h_{22}} - \frac{h_{12}}{h_{22}}\rho a_N^2(\tau_{iN})^{-1}\mu_1.$$

Now, standard (but tedious) normal calculations yield the following:

$$\mu_1 = \left(\tau_{N-1}\tau_{iN}\left(\sum_{t=1}^{N-1}\tau_{\varepsilon_t}\right)\right)\left(\tau_N\tau_{iN-1}\left(\sum_{t=1}^{N}\tau_{\varepsilon_t}\right)\right)^{-1} \times (\tilde{s}_{iN-1} - p_{N-1}),$$

$$\mu_2 - p_{N-1} = \left(\Delta\tau_N\left(\sum_{t=1}^{N-1}\tau_{\varepsilon_t}\right)\right)(\tau_N\tau_{iN-1})^{-1}(\tilde{s}_{iN-1} - p_{N-1}),$$

$$h_{22} = \left(\sum_{t=1}^{N}\tau_{\varepsilon_t}\right)^2(\tau_{\varepsilon_N})^{-1}\left(\det\left(\sum^{-1} + 2\rho A\right)\right)^{-1},$$

$$h_{12} = -\left(\sum_{t=1}^{N}\tau_{\varepsilon_t}\right)\left(\sum_{t=1}^{N-1}\tau_{\varepsilon_t}\right)(\tau_{\varepsilon_N})^{-1}\left(\det\left(\sum^{-1} + 2\rho A\right)\right)^{-1},$$

and

$$\det\left(\sum^{-1} + 2\rho A\right) = \left(\sum_{t=1}^{N}\tau_{\varepsilon_t}\right)^2\left(\tau_N\tau_{iN-1} - \tau_{N-1}\left(\sum_{t=1}^{N-1}\tau_{\varepsilon_t}\right)\right)(\Delta\tau_N\tau_{\varepsilon_N})^{-1}.$$

Plugging these values into the expression for x_{iN-1} we obtain, after simplifying,

$$x_{iN-1} = a_{N-1}(\tilde{s}_{iN-1} - p_{N-1}), \text{ with } a_{N-1} = \rho^{-1}\left(\sum_{t=1}^{N-1} \tau_{\varepsilon_t}\right).$$

Substituting the optimal period $N-1$ strategy into the profit $\pi_{N-1,i}$ and after some (again tedious) computations, we obtain

$$E\{\exp\{-\rho(\pi_{iN-1} + \pi_{Ni})\} \mid \tilde{s}_{iN-1}, p_{N-1}\}$$
$$= \exp\left\{-\frac{1}{2}\rho^2 a_{N-1}^2(\tau_{iN-1})^{-1}(\tilde{s}_{iN-1} - p_{N-1})^2\right\}.$$

At stage $N-2$ speculator i chooses X_{iN-2}, to maximize

$$-\exp\left\{-\rho\sum_{t=1}^{N-3}\pi_{it}\right\}$$

times

$$E\{E\{\exp\{-\rho\pi_{iN-2}\}\exp\{-\rho(\pi_{iN-1}+\pi_{iN})\} \mid \tilde{s}_{iN-1}, p_{N-1}\} \mid s_i^{N-2}, p^{N-2}\}$$

or

$$-\exp\left\{-\rho\sum_{t=1}^{N-3}\pi_{it}\right\}$$

times

$$E\{\exp\{-\rho\pi_{iN-2}\}E\{\exp\{-\rho(\pi_{iN-1}+\pi_{iN})\} \mid \tilde{s}_{iN-1}, p_{N-1}\} \mid s_i^{N-2}, p^{N-2}\}.$$

Equivalently,

$$-\exp\left\{-\rho\sum_{t=1}^{N-3}\pi_{it}\right\} E\{\exp\{-\rho\phi_{iN-2}\} \mid s_i^{N-2}, p^{N-2}\},$$

where $\phi_{iN-2} = (p_{N-1}-p_{N-2})X_{iN-2}+\frac{1}{2}\rho a_{N-1}^2(\tau_{iN-1})^{-1}(\tilde{s}_{iN-1}-p_{N-1})^2$.

This is exactly of the same form as obtained in the recursion from N to $N-1$ and therefore the optimal strategy will be $x_{iN-2} = a_{N-2}(\tilde{s}_{iN-2} - p_{N-2})$, with $a_{N-2} = \rho^{-1}(\sum_{t=1}^{N-2}\tau_{\varepsilon_t})$. The result follows.

Note that if private information is received only in the first period then $a_{N-1} = a$ and $p_{N-1} = p_N$ since $\Delta a_N = 0$ and therefore $\pi_{iN-1} = 0$. At this point the maximization problem of speculator i at period $N-2$ looks exactly the same as the problem at period $N-1$, and the optimal position will be $X_{iN-2}(s_{i1}, p_{N-2}) = a(s_{i1} - p_{N-2})$.

By backwards recursion we obtain $X_{i1}(s_{i1}, p_1) = a(s_{i1} - p_1)$ at period 1. The market price is stationary from period 1 on. At periods $n = 2, \ldots, N$, noise trading is absorbed by market makers at the period 1 price p_1.

Indeed, when setting prices in periods $n = 2, \ldots,$ market makers obtain no information from the limit order book, since $z_n = u_n$ and therefore $p_n = E(v/z_1) = p_1$. Both market makers and noise traders break even in periods $n = 2, \ldots, N$, since the market is infinitely deep ($\lambda_n = 0$). Obviously, noise traders lose money in period 1 when informed agents trade.

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