Information and Learning in Markets

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Chapter 5

Strategic Traders in Financial Markets

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Plan of the Chapter

Basic static models of financial markets with asymmetric information and strategic traders.

1. Impact of large traders on market properties.
   - Traders compete simultaneously in demand schedules
   - Informed traders move first
   - Market makers move first

2. Consequences of traders using market orders and of discriminatory pricing.

Plan of the Chapter

Pros of this modelling choice:

2. Realistic in circumstances such as commodity futures markets, government security auctions, or when insiders in a firm have information about a merger prospect.
3. Avoids some of the paradoxes of strongly informationally efficient markets.

However, it raises the issue of when it is appropriate to adopt the “continuum” approximation or the assumption of a finite number of traders:

1. Check that as the market grows large strategic equilibria of finite economies converge to the competitive REE of continuum economy.
2. Find out when a competitive REE is close to the strategic equilibrium.
5.1 Competition in Demand Schedules

Is equilibrium uniqueness granted in this setup?

- Think of a uniform price auction of a nonrandom amount of shares with a finite number of bidders.

- Wilson (1979) shows
  1. Set of equilibria is very large
  2. Some of the equilibria are very collusive (even though the game is noncooperative and one-shot).


Kyle (1989): strategic version of the competitive RE model studied in Section 4.2.1.

- $n$ informed and $m$ uninformed traders ("market makers").
- Traders simultaneously submit demand schedules to an auctioneer.
- All other assumptions in the model are the same as in Section 4.2.1

Characterize the symmetric linear Bayesian equilibrium of this game (SLBE).
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Consider the candidate SLBE

$$X_I(s_i, p) = a s_i - c_I p + b_I, \quad X_U(p) = b_U - c_U p.$$  

From market clearing

$$\sum_{i=1}^{n} X_I(s_i, p) + mX_U(p) + u = 0,$$

we obtain that

$$p = \lambda \left( n b_I + m b_U + u + a \sum_{i=1}^{n} s_i \right),$$

where

$$\lambda^{-1} = n c_I + m c_U,$$

can be taken as an index of market depth.
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

The price is informationally equivalent to

\[ \hat{z} = \frac{\lambda^{-1} p - nb_I - mb_U}{na} \]

\[ = \theta + \frac{1}{n} \sum_{i=1}^{n} \epsilon_i + \frac{1}{an} u. \]

Therefore, the price precision \((\text{Var}[^\theta | p])^{-1}\):

\[ \tau = \tau_\theta + \frac{1}{(n\tau_\epsilon)^{-1} + (n^2 a^2 \tau_u)^{-1}} = \tau_\theta + \phi_U n \tau_\epsilon, \]

where

\[ \phi_U = \frac{na^2 \tau_u}{\tau_\epsilon + na^2 \tau_u}, \]

fraction of the precision of the informed traders’ revealed to the uninformed by the price.
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Letting

\[ \hat{z}_i = \frac{\lambda^{-1} p - as_i - nb_I - mb_U}{(n - 1)a} \]

\[ = \theta + \frac{1}{n - 1} \sum_{k \neq i} \epsilon_k + \frac{1}{(n - 1)a} u, \]

\[ \Rightarrow \{s_i, p\} \, \text{and} \, \{s_i, \hat{z}_i\} \, \text{are observationally equivalent (o.e.), and} \]

\[ \tau_I = (\text{Var}[\theta|s_i, p])^{-1} = \tau_\theta + \tau_\epsilon + \phi_I(n - 1)\tau_\epsilon, \]

where

\[ \phi_I = \frac{(n - 1)a^2\tau_u}{\tau_\epsilon + (n - 1)a^2\tau_u}, \]

represents the fraction of the precision of the other \( n - 1 \) informed traders revealed to a given informed trader by the price.

\[ \Rightarrow \text{When} \, \phi_I = \phi_U = 1, \text{prices are fully revealing}. \]
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Since $p$ and $\hat{z}$ are observationally equivalent (o.e.):

1. $E[\theta|p] = E[\theta|\hat{z}]$.

2. Also:

$$E[\theta|p] = \frac{\tau \theta}{\tau} \bar{\theta} + \frac{\phi U \tau \epsilon}{a \tau} (\lambda^{-1} p - nb_I - mb_U),$$

Since $\{s_i, p\}$ and $\{s_i, \hat{z}_i\}$ are o.e.

4. $E[\theta|s_i, p] = E[\theta|s_i, \hat{z}_i]$

6. Also:

$$E[\theta|s_i, p] = \frac{\tau \theta}{\tau_I} \bar{\theta} + \frac{(1 - \phi_I) \tau \epsilon}{\tau_I} s_i + \frac{\phi_I \tau \epsilon}{a \tau_I} (\lambda^{-1} p - nb_I - mb_U).$$

In equilibrium each trader $i$ optimizes against a linear residual supply curve:

$$p = p_{Ii} + \lambda_I x_i, \quad \lambda_I = \frac{1}{(n - 1) c_I + mc_U},$$

and $p_{Ii}$ is a linear function of the signals of the other informed traders and $u$. 
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Implications:

1. Conditioning on the price = conditioning on the intercept of the residual supply.
2. Profits of speculator $i$:

$$\pi_i = (\theta - p_{Ii} - \lambda_I x_i) x_i,$$

normally distributed $|\{s_i, p_{Ii}\}$. CARA expected utility maximization is equivalent to choosing $x_i$ to maximize

$$(E[\theta|p_{Ii}, s_i] - p_{Ii}) x_i - \frac{\lambda_I + \rho_I \text{Var}[\theta|p_{Ii}, s_i]}{2} x_i^2.$$

The optimal solution yields

$$x_i = \frac{E[\theta|p_{Ii}, s_i] - p_{Ii}}{2\lambda_I + \rho_I \text{Var}[\theta|p_{Ii}, s_i]}.$$
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Given that

1. \( E[\theta|p_{II}, s_i] = E[\theta|p, s_i] \)
2. \( \text{Var}[\theta|p_{II}, s_i] = \text{Var}[\theta|p, s_i] \)
3. \( p_{II} = p - \lambda_I x_i \)

we solve for \( x_i \):

\[
X_I(s_i, p) = \frac{E[\theta|s_i, p] - p}{\lambda_I + \rho_I \text{Var}[\theta|s_i, p]}, \quad \lambda_I = \frac{1}{(n - 1)c_I + mc_U}.
\]

For an uninformed trader we obtain

\[
X_U(p) = \frac{E[\theta|p] - p}{\lambda_U + \rho_U \text{Var}[\theta|p]}, \quad \lambda_U = \frac{1}{nc_I + (m - 1)c_U}.
\]
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Remark

There are 2 reasons why a trader restricts his trades:

1. Market power: $\lambda_I$ or $\lambda_U > 0$,
2. Risk aversion: $\rho_I$ or $\rho_U > 0$.

When $\lambda_I = \lambda_U = 0$, we are in a competitive model.
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Kyle (1989)’s result:

**Proposition**

A unique symmetric equilibrium exists if

1. \( n \geq 2 \) and \( m \geq 1 \), or
2. \( n \geq 3 \) and \( m = 0 \), or
3. \( n = 0 \) and \( m \geq 3 \).

If \( n = 1 \) a SLBE exists if \( m \) is large enough and if \( n + m \leq 2 \), a SLBE does not exist.

Given that noise traders have inelastic demands, for a linear equilibrium to exist there must be enough competition among informed and/or uninformed agents.
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

- **Informational incidence**

\[ \zeta \equiv \frac{\lambda a T I}{\tau \epsilon}, \]

increase in the price when the informed trader's asset valuation of
the asset increases by $1 as a result of a higher \( s_i \). In equilibrium

\[ \zeta \leq \frac{1}{2}, \]

and prices **never** transmit more than half the pooled private
information of traders.

- **Expected trading losses** of noise traders:

\[ E[(\theta - p)u] = -\lambda \sigma_u^2. \]

Noise traders loose money proportionally to their amount of trade
and market liquidity \( \lambda \).
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

At a price-taking Bayesian equilibrium:

1. Informed (uninformed) traders behave as if $\lambda_I = 0$ ($\lambda_U = 0$).
2. Equilibrium existence is ensured whenever $\sigma_u^2 > 0$ and $\rho_I > 0$.
3. The informational parameter in the competitive market $\phi^c_I$ is larger than with imperfect competition $\phi^c_I > \phi_I$. 
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

Other results

1. As \((\tau \epsilon(n - 1))^{-1}(\rho^2 I \sigma^2_u) \rightarrow 0\), \(\phi_I^c \rightarrow 1\) and prices become fully revealing at the price taking outcome.

2. With imperfect competition, as \((\tau \epsilon(n - 1))^{-1}(\rho^2 I \sigma^2_u) \rightarrow 0\), \(\zeta \rightarrow 1/2\) (and \(\phi_I\) remains below and bounded away from \(1/2\)).

3. Even if \(\phi_I, \phi_U < 1/2 \not\Rightarrow\) prices cannot become fully revealing in some circumstances (e.g. when \(\tau \epsilon \rightarrow \infty\)).

4. As \(\uparrow m, \downarrow \rho_U \Rightarrow \uparrow \phi_I, \phi_U\).

5. For a finite number \(m\) of market makers, prices “over-react”: 
\[
E[\theta - \overline{\theta}|p] = \beta (-p - \overline{\theta}), \quad \beta \in (0, 1).
\]
5.1 Competition in Demand Schedules

5.1.1 REE with Imperfect Competition

If traders are risk-neutral, closed form solutions can be obtained. For example:

1. Let $n = 1$, for $m \to \infty$, then $\zeta = 1/2$ and $\phi_U = (2\tau_\theta + \tau_\epsilon)^{-1}\tau_\theta$ and when $\tau_\epsilon \to \infty$, $\tau \to 2\tau_\theta$.

2. With $m = 0$, $n > 2$
   - $a$ is increasing in $\tau_\epsilon$ and in $\sigma_u^2$ and is independent of $\tau_\theta$.
   - $\lambda^{-1} = nC_I$ is increasing with the precision of public information $\tau_\theta$ and with $\sigma_u$.
   - Price precision is increasing in $\tau_\theta$ and $\tau_\epsilon$ (and independent of $\sigma_u^2$ because of risk neutrality).
5.1 Competition in Demand Schedules

5.1.2 Free Entry and Large Markets

Now:

- Endogenize the number of informed speculators.
- There are two stages and a countable infinity of potential traders with CARA coefficient $\rho$.
  
  1. In the first stage any trader (except noise traders) can become informed by paying a fixed amount $F > 0$.
  2. In the second stage the speculators who have decided to enter compete to make money out of the noise traders.

Free entry of uninformed speculators, even if risk averse, implies that $E[\theta|p] = p$. 
5.1 Competition in Demand Schedules
5.1.2 Free Entry and Large Markets

Equilibrium number of informed traders:

- Let $\pi_i(n)$ be the random profits of informed trader $i$ when $n$ have entered.
- $\Pi(n)$ is the certainty equivalent of profits that makes a trader indifferent between paying to be informed or remain uninformed:
  $$-\exp\{-\rho\Pi(n)\} = E[-\exp\{-\rho\pi_i(n)\}].$$
- Let $n^*(\sigma_u)$ denote the equilibrium number of informed speculators in the imperfect competition model when noise trading has standard deviation $\sigma_u$. $n^*(\sigma_u)$ is the largest $n$ such that
  $$\Pi(n) \geq F.$$
- Kyle (1989) shows that there is a unique free-entry equilibrium of the two-stage game.
5.1 Competition in Demand Schedules
5.1.2 Free Entry and Large Markets

Is the competitive rational expectations equilibrium considered in Section 4.2.1 the limit of the strategic equilibria of the growing markets? Kovalenkov and Vives (2007):

**Proposition**

Let $\rho > 0$ and $F^* \equiv (1/2\rho) \log(1 + \tau_\epsilon/\tau_\theta)$. As $\sigma_u \to \infty$ three cases appear in a free-entry equilibrium:

1. If $F \geq F^*$, then no trader chooses to become informed and prices contain no information.
2. If $0 < F < F^*$, then $n^*(\sigma_u)$ grows proportionally to $\sigma_u$. Prices are noisy indicators of the fundamental value in the limit market.
3. If $F = 0$, then all traders choose to become informed and prices are fully revealing.
5.1 Competition in Demand Schedules
5.1.2 Free Entry and Large Markets

Sketch of proof:

1. Consider the case of a fixed number of speculators $n$:

$$\Pi(n) = \frac{1}{2\rho} \log \left( 1 + \frac{(1 - \phi_I)(1 - \phi_U)\tau\epsilon}{\tau} \frac{1 - 2\zeta}{(1 - \zeta)^2} \right), \quad \Pi(n) \in [0, F^*],$$

for any $n$ and $\sigma_u$.

2. If

1. $n$ grows faster than $\sigma_u$, then $\Pi(n) \to 0$
2. $n$ grows more slowly than $\sigma_u$, then $\Pi(n) \to F^*$.

3. Suppose now that the number of speculators is endogenous.

1. If $F = 0$, then all traders become informed and prices become fully revealing.
2. If $F \geq F^*$ then it does not pay to become informed and prices are unrelated to the fundamental value.
As $n$ grows unboundedly with $\sigma_u \to \infty$, $\Pi(n) \to F$.

If $0 < F < F^*$, then $n$ must grow unboundedly as $\sigma_u \to \infty$. Therefore, in this case $n^*(\sigma_u)$ must grow at the same rate as $\sigma_u$.

Hence, for the whole range of intermediate values of $F$

- $n^*(\sigma_u)$ is proportional to $\sigma_u$.
- Thus, it is natural to consider sequences of markets where the numbers of informed speculators are proportional to $\sigma_u$. 
5.1 Competition in Demand Schedules

5.1.2 Free Entry and Large Markets

Consider a sequence of markets where noise trade $\sigma_u$ grows at the rate $n$: $u(n) = nw_0$.

- Does the competitive REE of the idealized limit economy or the one of the finite economy provide a better approximation to the true strategic equilibrium of a market with a given number of informed traders?

- Note: the results of the previous Proposition hold when we replace strategic by price-taking behavior.

- We can define for the price-taking equilibrium model the certainty equivalent of profits $\Pi^c(n)$ and the equilibrium number of informed speculators $n^*_c(\sigma_u)$.

- Again, there is a unique free-entry equilibrium of the two-stage game in the price-taking case.
5.1 Competition in Demand Schedules

5.1.3 Convergence to Price-Taking

Do price-taking equilibria provide a reasonable approximation of strategic market interaction?

1. Consider sequences of markets where the numbers of informed speculators are proportional to $\sigma_u$. At the $n$-th market there are $n$ informed agents. Let $u(n) = nu_0$ with $\text{Var}[u_0] = \sigma_{u_0}^2$.

2. Consider the following decomposition:

$$p_n - p_\infty = \underbrace{(p_n - p_n^c)}_{(1)} + \underbrace{(p_n^c - p_\infty)}_{(2)}.$$

3. (1) captures the difference between equilibrium prices for the price-taking $p_n^c$ and strategic equilibria $p_n$ in the same finite market. (2) captures the change in the competitive price from the finite market $p_n^c$ to the limit market $p_\infty$. 
5.1 Competition in Demand Schedules
5.1.3 Convergence to Price-Taking

1. \( E[(p_n - p_\infty)^2] \) inherits the order of the higher order term of \( E[(p_n - p_n^c)^2] \) or \( E[(p_n^c - p_\infty)^2] \).

2. The term \( E[(p_n - p_n^c)^2] \) corresponds to the strategic effect and the term \( E[(p_n^c - p_\infty)^2] \) to the limit effect. As the former term converges to zero faster than the latter, \( E[(p_n - p_\infty)^2] \) inherits the order of \( E[(p_n^c - p_\infty)^2] \).

3. \( \sqrt{E[(p_n - p_n^c)^2]} = \sqrt{\text{Var}[p_n - p_n^c]} \) is of the order \( 1/n \), and \( \sqrt{E[(p_n^c - p_\infty)^2]} = \sqrt{\text{Var}[p_n^c - p_\infty]} \) if of the order \( 1/\sqrt{n} \).

4. Hence, \( \sqrt{E[(p_n - p_\infty)^2]} = \sqrt{\text{Var}[p_n - p_\infty]} \) inherits the higher order i.e. \( 1/\sqrt{n} \).
5.1 Competition in Demand Schedules

5.1.3 Convergence to Price-Taking

Intuition

- The demand of an informed trader in equilibrium is given by

\[ X_I = \frac{E[\theta|s_i, p] - p}{\lambda_I + \rho \text{Var}[\theta|s_i, p]} \]

- In the competitive case \( \lambda_I = 0 \). It can be checked that \( \lambda_I \) is of the order of \( 1/n \) and this explains why market power vanishes at the rate \( 1/n \).

- In a symmetric linear equilibrium prices in a finite economy with \( n \) informed traders, (either \( p_n^c \) or \( p_n \)), are a linear function of the fundamental value \( \theta \), its expectation \( \bar{\theta} \), the base noise trading \( u_0 \), and the average error in the signals of the traders \( (\sum_{i=1}^{n} \epsilon_i)/n \).

- Prices in the limit economy depend on the same variables except \( (\sum_{i=1}^{n} \epsilon_i)/n \) (SLLN).

- The distance between \( p_n^c \) (or \( p_n \)) and the limit price \( p_\infty \) depends on \( (\sum_{i=1}^{n} \epsilon_i)/n \) and this average error term converges to 0 at a rate of \( 1/\sqrt{n} \) as \( n \) tends to infinity.
A similar result can be shown to hold for utilities:

\[
\sqrt{E \left[ \left( \frac{U(\pi^c_n)}{U(\pi_n)} - 1 \right) \right]},
\]

is of the order of \(1/n\), while

\[
\sqrt{E \left[ \left( \frac{U(\pi^c_c)}{U(\pi_\infty)} - 1 \right) \right]},
\]

and

\[
\sqrt{E \left[ \left( \frac{U(\pi_n)}{U(\pi_\infty)} - 1 \right) \right]},
\]

are of the order of \(1/\sqrt{n}\).
5.1 Competition in Demand Schedules

5.1.3 Convergence to Price-Taking

- The competitive approximation is OK even in a moderately sized market when competitive traders have incentives to be restrained in their trading (i.e. when their informationally adjusted risk bearing capacity is not very large).

- As traders become risk neutral the contrast between the competitive and the strategic cases is stark when they have to acquire the information at cost $F > 0$.
  - No traders choose to become informed in the price-taking case because of their closeness to risk neutrality (Grossman and Stiglitz (1980) paradox).
  - In the strategic case the informed traders take into account the effect of their actions on the price and therefore can restrict their trade.
5.1 Competition in Demand Schedules

5.1.4 Summary

Competition in demand schedules generates a large multiplicity of equilibria whenever supply or demand uncertainty is nonrandom or has bounded support. In the CARA-normal model a unique symmetric linear equilibrium exists:

1. A large trader has potentially two reasons to restrict his trade: risk aversion and market power.
2. Large traders are more cautious than small ones when reacting to their private information because they are aware of the price impact and the information leakage of their trades.
3. Prices tend to be less informative in the presence of large traders.
4. With free entry and costly information acquisition the number of informed traders grows in proportion with the size of the market.
5. With risk averse traders a price-taking rational expectations equilibrium is not far from its strategic counterpart as long as a moderate number of traders are present in the market.
6. When traders are risk neutral competitive and strategic equilibria are very far apart.
5.2 Informed Traders Move First

Another look at the sequential trade model with large informed traders.

- Noise and informed traders move first and competitive market makers set prices upon observing the order flow.
- We analyze the incentives of large informed traders to confuse market makers and their effect on market quality.
5.2 Informed Traders Move First

5.2.1 The Market Order Game

**Model**

- A single risky asset, with random liquidation value $\theta$, and a riskless asset, with unitary return, are traded among noise traders, a fringe of risk-averse competitive informed agents of mass $1 - \mu$, and a large risk neutral informed trader (the “insider”) of mass $\mu$ with the intermediation of competitive risk neutral market makers.

- Think in terms of the model in Section 4.3 where a mass $\mu$ of the competitive informed form a coalition.

- The rest of informed traders $1 - \mu$ remain competitive. All traders, except market makers, use market orders.

**Note that**

- If $\mu = 1$ we have the one-shot auction in Kyle (1985).
- If $\mu = 0$ we have the model in Section 4.3 with $\nu = 0$. 
5.2 Informed Traders Move First

5.2.1 The Market Order Game

**Insider**

- The insider observes $\theta$ and acts strategically.
- An insider buying $\mu y$ of the asset at price $p$ are obtains $\pi = (\theta - p)\mu y$. His initial wealth is normalized to zero.
- The insider submits a market order $\mu Y(\theta)$.

**Competitive Fringe and Noise Traders**

- Fringe of competitive informed agents indexed in $[\mu, 1]$.
- Each competitive agent receives a (private) signal of the same precision about $\theta$, is risk averse ($\rho > 0$ is the common coefficient of CARA).
- The initial wealth of informed agents is normalized to zero.
- Noise traders submit an order $u$.
- An informed trader submits a market order $X(s_i)$.

The distributional assumptions on random variables are as in Section 4.2.3.
5.2 Informed Traders Move First

5.2.1 The Market Order Game

**Market Makers**
Market makers are risk neutral and set prices efficiently conditional on
\( \omega = \int_\mu^1 X(s_i) di + \mu y + u. \) Thus, \( p = E[\theta|\omega]. \)

**Equilibrium**
- We will look for a linear (symmetric) perfect Bayesian equilibrium (PBE) of the game.

**Definition**
A PBE is a set of functions (strategies) for the insider \( \theta \mapsto y = Y(\theta), \)
each competitive informed trader \( s_i \mapsto x_i = X(s_i), \) and a price function
\( \omega \mapsto p = P(\omega), \) such that each informed trader maximizes expected
utility taking as given the (correctly anticipated) price functional and the
strategies of the other traders, and market makers set prices with a fixed
(and correct) conjecture about the strategies of the informed traders,
\( Y(\cdot) \) and \( X(\cdot): P(\omega) = E[\theta|\omega]. \)

- Assume that any observed order flow is compatible with the
  conjectured strategies of the informed traders.
Characterize the linear PBE:

- Market makers, given conjectured strategies \( X(s_i) = a(s_i - \bar{\theta}) \) and \( Y(\theta) = \alpha(\theta - \bar{\theta}) \) know that

\[
(1 - \mu)^{-1} \int_{\mu}^{1} X(s_i) di = a(\theta - \bar{\theta}) \equiv X(\theta).
\]

Hence

\[
\omega = (1 - \mu)X(\theta) + \mu Y(\theta) + u = A(\theta - \bar{\theta}) + u,
\]

where \( A = \mu\alpha + (1 - \mu)a \).

- Given the properties of normal distributions it is immediate that

\[
P(\omega) = E[\theta|\omega] = \lambda \omega + \bar{\theta}, \text{ where } \lambda = A\tau_u/\tau, \tau = \tau_\theta + A^2\tau_u.
\]
Given a conjectured price functional of the form \( P(\omega) = \lambda \omega + \bar{\theta} \), informed traders optimize.

- The insider must choose \( y \) to maximize \( E[\pi|\theta] = \mu y (\theta - E[p|\theta]) \).
- Anticipating (correctly) the average strategy for the competitive fringe \( X(\theta) = a(\theta - \bar{\theta}) \), he obtains
  \[ E[p|\theta] = \lambda ((1 - \mu)X(\theta) + \mu y) + \bar{\theta}, \]
  which yields an optimal strategy
  \[ Y(\theta) = \alpha(\theta - \bar{\theta}), \quad \alpha = \frac{\lambda^{-1} - (1 - \mu)a}{2\mu}, \]
  (if the second order condition \( \lambda > 0 \) holds).
5.2 Informed Traders Move First

5.2.1 The Market Order Game

- Competitive trader \( i \) chooses \( x_i \) to maximize
  
  \[
  E[\pi_i|s_i] - \rho \text{Var}[\pi_i|s_i]/2,
  \]
  where \( \pi_i = (\theta - p)x_i \), because of CARA utility and joint normality of \( \theta - p \) and \( s_i \).

- The optimal demand is given by
  
  \[
  X(s_i) = a(s_i - \bar{\theta}),
  \]
  where
  
  \[
  a = \frac{1}{\rho(\sigma^2_\varepsilon + \text{Var}[p])}.
  \]
Restricting attention to (Bayesian) equilibria in linear strategies it follows that there is a unique equilibrium:

**Proposition**

There is a unique linear equilibrium. It is given by

\[ Y(\theta) = \alpha(\theta - \bar{\theta}), \quad X(s_i) = a(s_i - \bar{\theta}), \quad p = \lambda \omega + \bar{\theta}, \]

and where \( \omega = A(\theta - \bar{\theta}) + u \). The parameters \( a \) and \( \alpha \) are the unique positive solution of the two-equation system:

\[
\begin{align*}
\alpha & = \frac{\lambda^{-1} - (1 - \mu)a}{2\mu} \\
a & = \frac{1}{\rho(\tau\epsilon^{-1} + \tau_\theta^{-1} - \tau^{-1})},
\end{align*}
\]

with \( \lambda = A\tau_u/\tau, \quad \tau = \tau_\theta + A^2\tau_u \), and \( A = \mu\alpha + (1 - \mu)a \).
5.2 Informed Traders Move First

5.2.1 The Market Order Game

If $\mu = 1$ we are in Kyle (1985).

- $\tau = 2\tau_\theta$ and $\alpha = \sigma_u/\sigma_{\theta}$, $\lambda = \sigma_{\theta}/2\sigma_u$.

- The expected profits of the insider equal the losses of noise traders:
  \[ \lambda \sigma_u^2 = \sigma_u \sigma_{\theta}/2. \]

- “Camouflage” effect.

- The insider is aware of the impact of his trading on the informativeness of prices and trades so that only half his information is impounded in the prices ($\tau = 2\tau_\theta$).

- The amount of noise trading does not influence the precision of prices.
5.2 Informed Traders Move First

5.2.1 The Market Order Game

Consider the general case $\mu \in (0, 1)$.

- In equilibrium $a > 0$, and $\alpha > 0$ since the expected profit of the insider equals $\mu \alpha / \tau$.
- For a given market depth, $\lambda^{-1}$, $\alpha$ is decreasing in $a$. 

5.2 Informed Traders Move First

5.2.1 The Market Order Game

- $1/\lambda$ measures the depth of the market and $\tau$ measures the informativeness of prices. Both variables are determined by $A = \mu \alpha + (1 - \mu) a$.

- $\tau$ and $A$ decrease with $\rho$, $\sigma_\varepsilon$, $\sigma_\theta$, $\sigma_u$ and $\mu$.

- $\text{Var}[p] = \tau^{-1} - \tau^{-1}$, hence all factors (except $\tau_\theta$) which increase $\tau$ also increase $\text{Var}[p]$. A higher $\sigma_\theta^2$ has a double impact on $\text{Var}[p]$: A negative indirect effect, since it decreases $\tau$, and a positive direct effect which dominates.
5.2 Informed Traders Move First

5.2.1 The Market Order Game

Volume

- Total volume traded, denoted by $TV$:

$$E[TV] = \frac{E[\int_\mu^1 |x_i| \, di] + \mu E[|y|] + E[|\omega|] + E[|u|]}{2},$$

- The behavior of the total trading volume is driven by the behavior of the volume traded by informed (competitive plus insider) agents.
- The effect of the noise trading on the (expected) trading volume is positive.
- Increases in the coefficient of risk aversion and/or the noise in the signals induce a decrease in expected total trading volume.
5.2 Informed Traders Move First

5.2.1 The Market Order Game

A Noisy Strategy for the Insider?

- Can the insider have the incentive to introduce noise in his order to hide his information?
- No:
  - As in Kyle (1985) the insider is optimizing against a fixed conjecture on the behavior of market makers (a fixed $\lambda$).
  - Given any $\lambda$ then it is optimal not to introduce noise in the order since the only effect of placing a noisy order is just to distort trade from its optimal level given $\theta$. 
5.2 Informed Traders Move First

5.2.1 The Market Order Game

**Multiasset Markets and Contagion**


5.2 Informed Traders Move First

5.2.2 Multiple or Unique Equilibrium?

- A unique linear equilibrium exists.
- However, in a version of the model with $\mu = 1$, noise trading $u$ with a discrete distribution with finite support, Biais and Rochet (1997) find a continuum of equilibria.
- Reason: the concept of PBE does not put any restriction on out of equilibrium beliefs.
- In the model we have considered noise is distributed continuously over an unbounded support, any order flow is compatible with equilibrium behavior by the insider and therefore out of equilibrium beliefs are not a concern.
- This means, in particular, that the insider optimizes against a fixed (conjectured) $\lambda$ set by the market makers.
5.2 Informed Traders Move First
5.2.2 Multiple or Unique Equilibrium?

- However, with $\mu = 1$, a variation of the model admits a unique equilibrium.

- Rochet and Vila (1994), using techniques from mechanism design, show that the equilibrium is unique globally in a modified market order game in which $(u, \theta)$ have a joint distribution on a compact support and where the insider observes the amount of noise trading before placing his order.

- In this case thus the insider has both fundamental ($\theta$) and non-fundamental ($u$) information.

- The mechanism design approach in this case tries to find the optimal organization of the market from the point of view of noise traders.

- The problem is finding a menu of contracts offered to the insider specifying for each order flow $\omega$ the price $P(\omega)$ and potential transfer to the insider $T(\omega)$ that minimize his profit.
5.2 Informed Traders Move First

5.2.2 Multiple or Unique Equilibrium?

- Let $\Pi(u, \theta) \equiv \max_\omega \{(\theta - P(\omega))(\omega - u) + T(\omega)\}$

- Contracts have to be
  - Incentive compatible: $\Pi(u, \theta) + u\theta$ must be convex.
  - Individually rational: $\Pi(u, \theta) \geq 0$.

- The result is that the equilibrium of the modified market order game minimizes, as unique solution, the expected profits of the insider subject to the incentive compatibility and individual rationality constraints.

- The modified market order game is equivalent to a demand schedule game.
Consider a version of the general model in which the insider and the informed traders submit demand schedules instead of market orders.

- Informed trader $i$ submits a demand schedule $X(s_i, p)$, contingent on the private signal.
- Similarly, the insider’s strategy is a demand function $Y(\theta, p)$.
- Noise traders’ aggregate order is $u$.
- Market makers quote prices efficiently, conditionally on $L(p)$:

$$L(p) = \int_{\mu}^{1} X(s_i, p) di + \mu Y(\theta, p) + u \Rightarrow p = E[\theta|L(\cdot)].$$
5.2 Informed Traders Move First

5.2.3 The Demand Schedule Game

Proposition

Under the assumptions, in the demand schedule game there exists a unique linear equilibrium:

\[ X(s_i, p) = a(s_i - p), \quad Y(\theta, p) = \alpha(\theta - p), \]

\[ p = \lambda z + \bar{\theta}, \quad \text{and} \quad z = A(\theta - \bar{\theta}) + u, \]

where \( a = \tau_\epsilon / \rho, \quad \alpha = (2\mu\lambda)^{-1}, \quad A = \mu\alpha + (1 - \mu)a, \quad \lambda = \lambda \tau_u / \tau, \]
\( \tau = \tau_\theta + A^2 \tau_u. \) In equilibrium we have that:

\[ \lambda = \frac{1}{2\sqrt{(\tau_\theta / \tau_u) + (1 - \mu)^2(\tau_\epsilon / \rho)^2}}. \]
5.2 Informed Traders Move First
5.2.3 The Demand Schedule Game

- When $\mu = 1$, the equilibrium is a limit case of Kyle (1989).
- The equilibrium is the same as in the modified market order game in which the insider observes the amount of noise trading before placing his order (Rochet and Vila (1994)).
- Comparing the market order game (denoted by superscript $MO$) with the demand schedule game when $\mu = 1$, we have that: $\alpha^{MO} = \alpha = \sigma_u/\sigma_\theta$, $\tau^{MO} = \tau = 2\tau_\theta$ and $\lambda^{MO} = \lambda = \sigma_\theta/2\sigma_u$, the insider makes the same expected profits in both games.
- For $\mu < 1$, informed traders react more to their private information in the demand schedule game, $a^{MO} \leq \tau_\epsilon/\rho$. 
5.2 Informed Traders Move First
5.2.3 The Demand Schedule Game

- Total expected traded volume is given by

\[ E[TV] = \sqrt{\frac{2}{\pi} \frac{\mu \sqrt{\alpha^2/\tau} + (1 - \mu) a \sqrt{\tau^{-1} + \tau^{-1}} + \sqrt{\sigma_u^2 + A^2/\tau} + \sigma_u}{2}}. \]

- Comparative static results are easily derived: e.g., when the relative size of the insider increases, market depth diminishes, the insider is more cautious and price informativeness declines.
5.2 Informed Traders Move First

5.2.4 Summary

- A large informed trader tries to hide behind liquidity traders in order to preserve his informational advantage. However, he does not add noise to his order to confuse market makers.

- Market depth and price precision decrease with the relative weight of the insider in the market.

- Multimarket extension explains contagion.

- Informed traders, strategic or competitive, react more to private signals if they use demand schedules.

- Despite the signaling nature of the game between informed and uninformed there are market microstructures for which there is a unique equilibrium.
5.3 Market Makers Move First

Consider quote-driven markets.

- Akin to a screening problem where uninformed agents move first.
- We analyze the adverse selection problem that market makers face and the implications for the bid-ask spread in a discriminatory pricing context.
- So far we have considered only uniform pricing schemes. However, in a limit order book there is price discrimination.
  - When a trader faces the book typically his order will be executed at different prices.
- To highlight the differences between uniform and discriminatory pricing we will concentrate first on the case where market makers do not face informed traders.
5.3 Market Makers Move First

5.3.1 Uniform vs. Discriminatory Pricing

Compare uniform versus discriminatory pricing when market makers face no adverse selection.

- There are \( m \) market makers who compete for the orders of noise traders as in Section 5.1.1 but \( n = 0 \).
- Suppose that market maker \( i \) posts a trading schedule \( P_i(z) \).
- Total payment to the market maker for \( x_i \) units of the stock:
  \[ T_i(x_i) = \int_0^{x_i} P_i(z) \, dz. \]
- Different units are transacted at different prices. With discriminatory pricing if a marker maker posts a demand schedule \( X_U(p) \) then each marginal unit \( z \) is transacted at a price \( P(z) = X_U^{-1}(z) \).
- Strategic incentives are altered in relation to the uniform pricing case. Market makers can bid more aggressively because when bidding for a marginal unit they don’t worry about inframarginal losses.
5.3 Market Makers Move First
5.3.1 Uniform vs. Discriminatory Pricing

- In Kyle (1989): uniform pricing and – under assumptions – there exists a unique symmetric linear equilibrium for $n = 0, m = 3$.
- Given that the price conveys no information on $\theta$:

\[
X_U(p) = \frac{E[\theta|p] - p}{\lambda_U + \rho_U \text{Var}[\theta|p]} = \frac{\bar{\theta} - p}{((m - 1)c_U)^{-1} + \rho_U \sigma_{\theta}^2}.
\]

- Expressing the above in terms of inverse demand:

\[
p = \bar{\theta} - \rho_U \sigma_{\theta}^2 y - ((m - 1)c_U)^{-1} y, \ \text{where} \ \bar{\theta} - \rho_U \sigma_{\theta}^2 y: \text{trader’s marginal valuation of the risky asset.}
\]

\[
-((m - 1)c_U)^{-1} y: \text{strategic effect, where} \ c_U = -X'_U(p) > 0.
\]
5.3 Market Makers Move First

5.3.1 Uniform vs. Discriminatory Pricing

- In equilibrium

\[ c_U = \frac{m - 2}{(m - 1)\rho_U \sigma^2}, \]

and

\[ p(y) = \bar{\theta} - (\rho_U \sigma^2_{\bar{\theta}}((m - 1)y)/(m - 2)). \]

- For a general distribution of \( u \), the equilibrium is the unique one in the linear class but there are, in fact, a continuum of non linear equilibria when \( u \) has compact support (Wang and Zender (2002)).

- In nonlinear equilibria even if traders are risk neutral \( p < \bar{\theta} \).

- Some of the equilibria are quite “collusive.” Only in the linear equilibrium we have that \( p = \bar{\theta} \) when \( \rho_U = 0 \).

- With discriminatory auctions the scope for collusion is reduced.
5.3 Market Makers Move First
5.3.1 Uniform vs. Discriminatory Pricing

- In the discriminatory case if bidders are risk neutral \( p = \bar{\theta} \) in equilibrium. Risk aversion is needed for the presence of market power in a discriminatory auction (analogy with Bertrand competition).

- Risk aversion implies that the marginal valuation of the asset is decreasing in the quantity demanded (as increasing marginal costs in an oligopoly model).

- In contrast, in the uniform-price auction bidders may enjoy market power even if they are risk neutral (as it would happen in a Cournot oligopoly model with constant marginal costs).
5.3 Market Makers Move First

5.3.1 Uniform vs. Discriminatory Pricing

- Viswanathan and Wang (2002) compare the linear uniform pricing solution above with the outcome of discriminatory pricing (in the CARA-normal model). In the latter case the symmetric equilibrium in demand schedules is:

\[ p = \bar{\theta} - \rho U \sigma_\theta^2 y + \frac{H(u)}{(m - 1)X'_U(p)}, \quad H(u) = \Lambda(u)/g(u), \]

- \( g(u) \) density function of noise trading \( u \), and \( \Lambda(u) \) solves

\[ \Lambda'(u) = \rho U \Lambda(u) \left( \frac{\bar{\theta} - p(u) - \rho U \sigma_\theta^2 / m}{m} - p(u) \right) - g(u). \]

- In this set up the strategic term is \( H(u)((m - 1)X'_U(p))^{-1} \) and affects the intercept of the demand function.

- Zero-quantity discount with discriminatory pricing due to risk averse market makers who do not know whether noise traders will want to buy/sell more than the marginal unit.
5.3 Market Makers Move First

5.3.1 Uniform vs. Discriminatory Pricing

- Viswanathan and Wang (2002) compare the linear uniform pricing solution with the discriminatory one (with mean-variance preferences) and find that in equilibrium discriminatory pricing entails a flatter demand schedule but also a zero-quantity discount.

- Discriminatory pricing intensifies competition among market makers.

- With asymmetric information about $\theta$ among market makers: only when bidders are risk neutral there is an equilibrium in which they submit flat schedules (Wang and Zender (2002)).

- Evidence that market makers have market power (Christie and Schultz (1994) and Christie et al. (1994), and Barclay et al. (1999) for NASDAQ).
5.3 Market Makers Move First

5.3.2 Market Makers Facing Adverse Selection

How do market makers may protect themselves against adverse selection when moving prior to informed traders and how competition among market makers affects the outcome.

- Copeland and Galai (1983): model in which an uninformed monopolistic market maker sets a bid and ask price and faces with some probability a trader perfectly informed about the liquidation value of the risky asset and with the complementary probability a liquidity trader.

- The market maker gains with the latter but loses with the former. This explains the spread.

- A higher probability of facing an informed trader (a more severe adverse selection problem) widens the spread.
5.3 Market Makers Move First
5.3.2 Market Makers Facing Adverse Selection

Biais, Martimort and Rochet (2000), extend Bernhardt and Hughson (1997), to study the effect of competition on spreads and welfare in a limit order book market.

- \( m \) risk neutral market makers and a single risk averse informed trader (with CARA utility with coefficient \( \rho \)) who receives a shock \( u \) to his endowment of the risky asset, which has liquidation value \( \theta \), and a signal \( s \) about \( \theta \).
- The information structure is similar to the Grossman-Stiglitz model: \( \theta = s + \epsilon \), where \( s \) and \( \epsilon \) are independent, \( \epsilon \) is normally distributed.
- The distributions of \( u \) and \( s \) have bounded supports.
5.3 Market Makers Move First
5.3.2 Market Makers Facing Adverse Selection

Structure and timing of the trading game:

1. Market makers simultaneously post trading schedules $P_i(z)$ that build up a limit order book.

2. The transfer payment to market maker $i$ for $x_i$ units:

   $$T(x_i) = \int_0^{x_i} P_i(z) \, dz.$$

With an increasing schedule $P_i(\cdot)$, a sequence of marginal prices $P_i(z)$ can be interpreted as a sequence of limit orders yielding a convex transfer $T_i(\cdot)$.

3. The informed trader, contingent on the realization of endowment shock $u$ and signal $s$, determines the vector of trades with market makers $x_i$, $i = 1, \ldots, m$.

4. $\epsilon, \theta = s + \epsilon$, are realized and consumption occurs.

5. There is discriminatory pricing as different units trade at different prices: Each additional unit is more expensive if $P_i(\cdot)$ is increasing.
5.3 Market Makers Move First

5.3.2 Market Makers Facing Adverse Selection

- The final wealth of the informed trader is

\[ W = (u + x\theta) - T(x), \]

where \( x = \sum_{i=1}^{m} x_i \) and \( T(x) = \sum_{i=1}^{m} T_i(x_i). \)

- \( W \) is normally distributed \(|\{u, s\}|\). The informed trader maximizes

\[ E[W|u, s] - \frac{\rho}{2} \text{Var}[W|u, s] = \left( us - \frac{\rho}{2} u^2 \sigma^2 \epsilon \right) + \left( vx - \frac{\rho}{2} x^2 \sigma^2 \epsilon - T(x) \right), \]

where \( v \equiv s - \rho \sigma^2 \epsilon u \) is marginal valuation for the risky asset.

- The first term is the reservation utility of the trader; the second term are gains from trade.

- Two motives for trade: a liquidity reason and an informational reason.

- Maximization of ex ante social welfare yields \( x^O(v) = E[-u|v] \).
5.3 Market Makers Move First
5.3.2 Market Makers Facing Adverse Selection

Monopolistic Case \( (m = 1) \):

- The supply schedule is discontinuous for orders around zero.
- The monopolist market maker optimizes the gains from trade minus the informational rent which needs to be given to the informed trader. This results in a trading volume and total welfare which are lower than optimal.
Oligopolistic Case ($m > 1$):

- Each market maker behaves as a monopolist facing a residual demand curve.
- There is a unique equilibrium in convex supply schedules where market makers charge positive mark-ups and make positive expected profits.
- Trading volume is larger than in the monopolistic case but lower than optimal.
- Because of adverse selection some traders are excluded from the market and there is also a strictly positive bid-ask spread for infinitesimal small trades.
- Outcome of imperfect competition disappears in a (pure) private value environment.
- With common values adverse selection reduces the aggressiveness of competition in supply schedules (winner’s curse).
5.3 Market Makers Move First

5.3.2 Market Makers Facing Adverse Selection

- With common values adverse selection reduces the aggressiveness of competition in supply schedules. Each market maker faces a residual demand with finite elasticity.

- The phenomenon is related to the winner’s curse in common value auctions: A bidder refrains from bidding aggressively because winning conveys the news that the signal the bidder has received was too optimistic (the highest signal in the pool).
5.3 Market Makers Move First

5.3.2 Market Makers Facing Adverse Selection

- Increasing the number of market makers decreases market power but does not eliminate bid-ask spread.
- The limit market has features of a monopolistically competitive equilibrium.
- This is as in Glosten (1994): in the competitive case the limit price for the marginal buy order $y$ is $P(y) = E[\theta|\ x \geq y]$ (for a sell order: $P(y) = E[\theta|\ x \leq y]$).
- For example, when facing a buy order for $y$ market makers know that the limit order at a certain price is hit when the total size of the trade $x$ is at least no smaller than the cumulated depth of the book up to that price.
5.3 Market Makers Move First
5.3.2 Market Makers Facing Adverse Selection

- It follows then from risk neutral competitive market making that
  \[ P(y) = E[\theta | x \geq y] \].

- It is worth noting that there is a spread even for small trades. The
  reason is that, under discriminatory pricing, market makers do not
  know whether the informed trader will want to buy more than the
  marginal unit.

- An infinitesimal order has a discrete impact on the price because it
  conveys a non-infinitesimal amount of information. This is why the
  bid-ask spread subsists even for very small orders.

- Under uniform pricing and perfect competition among market
  makers we would have that for an order of any size \( x \) expected
  profits are zero.

- This is akin to the order driven system studied by Kyle (1985) (see
  Section 5.2.1) where Bertrand competition among market makers
  drives their expected profits down to zero (because they move
  second and observe the same order flow).
5.3 Market Makers Move First
5.3.2 Market Makers Facing Adverse Selection

- Glosten (1989) compares perfect competition and the specialist system under uniform pricing. He finds that the specialist system stays open for larger market sizes than with perfect competition among market makers.

- In a competitive system market makers reduce liquidity and make the price schedule steeper to protect themselves against the adverse selection problem of large orders. This may induce a market breakdown for very large orders.

- Instead, the specialist (a monopolist) can cross-subsidize trades with different order sizes and is able to keep the market open for larger trades. Discriminatory pricing with the limit order book, by favoring larger orders, can accomplish the same objective even with perfect competition.
5.3 Market Makers Move First

5.3.3 Summary

- We have examined equilibria where uninformed traders move first with uniform and discriminatory pricing.

- Under uniform pricing there are typically multiple equilibria in demand schedules, under discriminatory pricing, and mild restrictions, a unique equilibrium arises.

- In the discriminatory pricing case:
  1. Risk aversion is needed for the existence of market power of market makers.
  2. Discriminatory pricing tends to intensify competition among market makers.
  3. Market makers face a basic adverse selection problem as they set prices.
  4. Strictly positive bid-ask spread even for infinitesimal trades.
  5. Market makers facing adverse selection charge positive mark-ups and make positive expected profits even if they are risk neutral and compete in supply schedules.
5.4 An Application: Welfare Analysis of Insider Trading

General environment for conducting a welfare analysis of insider trading.

- Insider trading is a particular case of informed trading involving a breach of fiduciary duty towards those on the other side of the trades or where the trader uses information over which he has no property rights.
- Example: ImClone former CEO, Samuel Waksal.
- There is also evidence that insiders do trade in advance of information release and earn excess returns (see Seyhun (1992 or 1986), Damodaran and Liu (1993) and Aboody and Lev (2000)).
5.4 An Application: Welfare Analysis of Insider Trading

Regulation:

- Leading regulations: “abstain-or-disclose” rule in the U.S. (SEC rule 10b-5 of the 1934 Act) and a prohibition against trading on inside (and precise) information in the EU (2003 Directive).

- Recently, tougher disclosure requirements have been imposed in the United States and in some European countries in order to avoid early selective disclosure of material information.

- Evidence that enforcement of insider trading laws reduces the cost of equity in a country (Bhattacharya and Daouk (2002)).

- However, welfare consequences of the regulation of insider trading are less well understood.
Received literature highlights three main related effects of insider trading:

1. Adverse selection,
2. Accelerating the resolution of uncertainty,
3. Modifying insurance and hedging opportunities.
5.4 An Application: Welfare Analysis of Insider Trading

5.4.1 Received Literature

**First effect:** Insider trading is seen as a tax which increases the bid-ask spread and reduces market depth (King and Roell (1988)).

- A potential effect is a decrease of ex ante investment before trading in the stock market occurs (Manove (1989), Ausubel (1990), and Bhattacharya and Nicodano (2001)).
- In Ausubel (1990), preventing insider trading increases the expected return of outsiders, this induces them to invest more and this may benefit insiders. The outcome is that a ban on insider trading may be Pareto superior.
- However, in this case inside information has no productive value (and is unrelated to investment).


**Second effect:** The presence of insiders tends to make prices more informative.

- Leland (1992) shows that the average investment level may be higher with insider trading because risk averse outsiders increase the demand for the risky asset associated to investment.
- Welfare analysis (subject to the usual problems in noise-trader models) implies that liquidity traders and outside investors are hurt and that insiders and owners of firms issuing shares benefit (because of a higher issuing price).
- The net effect of insider trading is ambiguous.
- Robustness issues (Repullo (1999)).
5.4 An Application: Welfare Analysis of Insider Trading

5.4.1 Received Literature

- **Third effect:** impact of insider trading on risk sharing and hedging.
  - Hirshleifer (1971): early revelation of information (before traders are able to take a hedging position) may destroy insurance possibilities.
  - Insider trading will tend to hurt then uninformed hedgers.
  - However, early revelation of information may also help insurance possibilities (Dow and Rahi (2003)).
In summary, despite the accumulation of work on the effects of insider trading, the analysis has been hampered by one or more of the following:

- Assumption of exogenous noise traders.
- Assumption of competitive behavior by “large” agents.
- Ill-defined incentives to float the firm or project.
- “Inside” information emanating from “outside” the firm or with no productive value.
5.4 An Application: Welfare Analysis of Insider Trading

5.4.2 A Framework for the Analysis

How to model the impact of insider trading on the investment and welfare of market participants when all agents are rational and aware of their position in the market?

- Risk averse entrepreneur has a project requiring investment and wants to hedge it partially by selling shares of the firm in the stock market.

- Competitive risk-averse market makers and hedgers – who have a random endowment of an asset correlated with the project of the firm.

- The entrepreneur invests and sets the number of shares to be issued at $t = 0$. He obtains information about the value of the project in the course of production at $t = 1$. The stock market opens at $t = 2$ and the entrepreneur trades together with market makers and hedgers. Neither the stock price nor private information have a chance to affect investment.
Interpretation: at $t = 2$

- A secondary market for the stock opens and the firm remains under the control of a coalition of insiders:
  - Coalition of insiders in a high-tech company learn valuable information about the effectiveness of a new drug being developed by the firm that shortly will be released to the market.

Alternative interpretation (primary market):

- Venture capitalist starting a new project and deciding to go public at $t = 0$ and with IPO at $t = 2$. 
Medrano and Vives (2004):

- Characterize linear equilibria at the market stage with and without insider trading.

- The combination of market power and adverse selection in the presence of hedgers may prevent the existence of a linear equilibrium (as in Bhattacharya and Spiegel (1991)).

- A linear equilibrium always exists when the combined risk tolerance-weighted informational advantage of the insider ($\frac{\xi}{\rho_I}$) and the hedgers ($\frac{\xi_u}{\rho_H}$) is not very high: i.e., when the main trading motive for hedgers and the entrepreneur is insurance.
Other findings:

- Investment increases in hedging effectiveness of the asset market from the point of view of the entrepreneur. This hedging effectiveness is (in general) decreasing in the precision of the information of the entrepreneur/insider.

- An insider with better information will be able to speculate more profitably but to hedge less of his investment.

- If the signal received by the entrepreneur is public knowledge (and not perfect), then an equilibrium always exists.

- As the precision of the signal increases, market depth, price volatility, and average stock price increase but investment and the expected utility of the insider, market makers, and hedgers decreases.

- Public information revelation leads to less uncertainty about the payoff of the project and to a deeper market. However, the hedging capacity of the market decreases from the entrepreneur's point of view as the public signal is more precise (Hirshleifer effect).
Should the insider be allowed to trade on his private information?

- Alternatives to not trading on the basis of private information:
  - A regime of public disclosure of the signal (PD).
  - A regime where no private information is used (NI).
Effects of an “abstain-or-disclose” rule depend on whether information is acquired for free or at a cost:

- If the entrepreneur learns the signal for free in the course of his activity ⇒ Disclosure occurs ⇒ Relevant benchmark is PD.
- If learning the signal has a cost ⇒ Entrepreneur does not learn ⇒ Relevant benchmark is NI.
5.4 An Application: Welfare Analysis of Insider Trading

5.4.3 Regulating Insider Trading

Results:

- When compared with the NI regime:
  - 1. Insider trading increases the informativeness of prices and price volatility, decreases market depth and investment and the risk premium is reduced.
  - 2. Expected utility of all traders tends to decrease.
  - 3. In general, insider trading is Pareto-inferior because of adverse selection.

- When compared with the PD regime:
  - 1. Insider trading reduces market depth, price precision and price volatility. In general, it reduces real investment, risk premia, and expected utility of the insider and speculators.
  - 2. Two negative forces potentially affect the hedging effectiveness of the stock market: adverse selection and public information revelation (Hirshleifer effect). The PD regime eliminates adverse selection but maximizes public disclosure.
  - 3. The effect on hedgers is ambiguous and depends on the precision of information of the insider.
In summary:

- If the insider has information and no obligation to disclose it, some of his information is leaked into prices and becomes public.

- This revelation of information has weaker effects than public disclosure because there is only partial revelation. However, because of adverse selection, market makers will now demand a larger premium to accommodate orders.

- The net effect on welfare and investment depends on the benchmark of comparison.

- Parallel in the literature on security design (Demange and Laroque (1995, 2002) and Rahi (1996)).
In this chapter we have examined static financial market models in the framework of rational expectations with strategic traders.

General theme of the chapter: large traders, even if they are risk neutral, refrain from trading aggressively because they are aware of their price and information impact.

Effect of strategic trading on market quality parameters depends on specific features of the market microstructure.
Summary

Other main results are as follows:

1. Prices tend to be less informative and the market more shallow in the presence of large informed traders.
2. The presence of strategic traders resolves paradoxes arising in the competitive REE paradigm.
3. With risk averse traders a price-taking rational expectations equilibrium is not far from its strategic counterpart.
4. “Camouflage” effect.
5. Risk averse informed traders react more to their private signals if they use demand schedules instead of market orders.
Summary

Despite the signaling nature of the game between informed and uninformed traders when the former move first there are market microstructures for which there is a unique equilibrium.

While uniform price auctions have typically multiple equilibria in demand schedules, under discriminatory pricing, and mild restrictions, there is a unique equilibrium.

Discriminatory pricing tends to intensify competition among market makers.

Market makers that move first and face adverse selection charge positive mark-ups and make positive expected profits.

Insider trading induces adverse selection and advances the resolution of uncertainty. Its welfare impact depends on the trade-off between the two effects.