ECONOMETRICA

JOURNAL OF THE ECONOMETRIC SOCIETY

An International Society for the Advancement of Economic Theory in its Relation to Statistics and Mathematics

http://www.econometricsociety.org/

Econometrica, Vol. 79, No. 6 (November, 2011), 1919–1966

STRATEGIC SUPPLY FUNCTION COMPETITION WITH PRIVATE INFORMATION

XAVIER VIVES

IESE Business School, University of Navarra, 08034 Barcelona, Spain

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STRATEGIC SUPPLY FUNCTION COMPETITION WITH PRIVATE INFORMATION

BY XAVIER VIVES[1]

A finite number of sellers (n) compete in schedules to supply an elastic demand. The cost of each seller is random, with common and private value components, and the seller receives a private signal about it. A Bayesian supply function equilibrium is characterized: The equilibrium is privately revealing and the incentives to rely on private signals are preserved. Supply functions are steeper with higher correlation among the cost parameters. For high (positive) correlation, supply functions are downward sloping, price is above the Cournot level, and as we approach the common value case, price tends to the collusive level. As correlation becomes maximally negative, we approach the competitive outcome. With positive correlation, private information coupled with strategic behavior induces additional distortionary market power above full information levels. Efficiency can be restored with appropriate subsidy schemes or with a precise enough public signal about the common value component. As the market grows large with the number of sellers, the equilibrium becomes price-taking, bid shading is on the order of $1/n$, and the order of magnitude of welfare losses is $1/n^2$. The results extend to inelastic demand, demand uncertainty, and demand schedule competition. A range of applications in product and financial markets is presented.

KEYWORDS: Reverse auction, demand schedule competition, double auction, market power, adverse selection, competitiveness, public information, rational expectations, collusion, welfare.

1. INTRODUCTION

MANY MARKETS ARE CHARACTERIZED by traders competing in demand or supply schedules. This type of competition is very common in financial markets and some goods markets like wholesale electricity. Competition in supply functions has been used also to model bidding for government procurement contracts, management consulting, or airline pricing reservation systems, and provides a reduced form for strategic agency and trade policy models. Furthermore, the jury is still out on whether the price or quantity competition model is the better fit for different oligopolistic markets, and the supply function model

[1]I am very grateful to the editor and four referees for their extensive comments and feedback. I thank Carolina Manzano, Natalia Fabra, Juanjo Ganuza, John Moore, Jean Charles Rochet, Marzena Rostek, and participants at seminars at Columbia, Harvard–MIT theory seminar, LSE, Yale, Toulouse, Chicago, Northwestern, Princeton, and ESSET at Gerzensee for very helpful comments, Rodrigo Escudero, Jorge Paz, and Vahe Sahakyan for excellent research assistance, and the Department of Economics at MIT for its hospitality. The research in this paper received funding from the European Research Council (European Advanced Grants scheme) under the European Community's Seventh Framework Programme (FP7/2007-2013) / ERC Grant agreement 230254. Complementary support from Project Consolider-Ingenio CSD2006-00016 and project ECO2008-05155 of the Spanish Ministry of Science and Innovation at the Public–Private Sector Research Center at IESE, as well as the Barcelona GSE Research Network and the Generalitat de Catalunya, is acknowledged.

 DOI: 10.3982/ECTA8126

appears to be an attractive contender. In many of the situations depicted, private information is relevant, uncertainty has both common and private values components, and traders are potentially strategic. This is the case, for example, in dealer markets, in both Treasury and central bank liquidity auctions as well as in the reverse auctions proposed by the U.S. Treasury to extract toxic assets from the balance sheets of banks. Supply function models in the industrial organization tradition have ignored private information, while in the finance tradition, demand function models have relied on the presence of noise traders.

In this paper, I present a tractable model of strategic competition in schedules with an information structure that encompasses private and common values, avoiding the need to introduce noise traders or noisy supply as well as paradoxes associated to fully revealing equilibria and allowing a full welfare analysis. A main result is that private information with positive correlation of values generates market power over and above the full information level and this has deleterious welfare consequences. As a by-product of the analysis, I am able to explain market anomalies as well as provide policy prescriptions.

Consider a market where n sellers compete in supply functions to satisfy a downward sloping demand. The market price equates aggregate supply and demand. Each seller receives a noisy signal of the uncertain intercept of his private marginal cost, which has a nonrandom slope.[2] The modeling strategy is to consider linear-quadratic payoffs coupled with an affine information structure, which admits common and private value components, that yields a unique symmetric linear Bayesian supply function equilibrium (SFE) of the game among the n sellers. Linear equilibria are tractable, particularly in the presence of private information, have desirable properties like simplicity, and have proved to be very useful as a basis for empirical analysis. The characterization of a linear equilibrium with supply function competition when there is market power and private information needs some careful analysis to be able to model the capacity of a seller to influence the market price at the same time that the seller learns from the price. Kyle (1989) pioneered this type of analysis in a financial market context by introducing noise trading to prevent prices from being fully revealing and the market from collapsing. The present paper provides a tractable alternative to models with an aggregate exogenous shock and is based on rational traders who are heterogeneous because of idiosyncrasies that translate into their market positions.

Except in the limit cases of pure common value and maximal negative correlation, a unique SFE is found. This equilibrium is privately revealing; that is, the private information of a firm and the price provide sufficient data on the joint information in the market. This means, in particular, that each trader has

[2]We could consider as well the symmetric situation where n buyers with uncertain private valuations compete to fulfill an upward sloping supply schedule. In this paper, I stick to the seller convention until applications are developed.

incentives to rely on his private signal despite the fact that the price aggregates information about the signals of other traders in the market.

In the linear equilibrium, sellers who have private information on their positively correlated costs are more cautious when they see a price increase, since it may mean that costs are high. Sellers are even more cautious, using steeper schedules, when signals are noisier or costs parameters are more correlated. The market looks less competitive under those circumstances as reflected in increased expected price–cost margins. This is reminiscent of the winner's curse in auctions. Indeed, price has an information role in addition to its traditional capacity as an index of scarcity. In fact, when the first effect is strong enough, supply functions slope downward and prices are above the Cournot level. This is in contrast to the results of Klemperer and Meyer (1989) with symmetric information. More surprisingly perhaps, as we approach the common value case, the price tends toward the collusive level. This is because of information-induced market power at the unique linear equilibrium and not because of the existence of a vast multiplicity of equilibria. Even with constant marginal costs, there is market power in equilibrium when adverse selection is severe enough. Relaxation of competition due to adverse selection also was obtained by Biais, Martimort, and Rochet (2000) in a different pure common value environment. When costs are negatively correlated, then there is "favorable" selection and competition is intensified with respect to the full information benchmark, the competitive outcome being attained with maximal negative correlation.

Sellers at the strategic equilibrium act as if there were price-takers, but with steeper than actual marginal costs. The distortion has a full information market power component and another component induced by private information, which is increasing with the correlation of cost shocks and noise in the signals (when costs are positively correlated). Both aggregate/allocative and distributive/productive inefficiency increase with the size of the distortion, implying sales that are too low and too similar among sellers. As we approach the common value case, expected profits converge to the collusive level. Furthermore, simulations suggest that, typically, the expected deadweight loss increases as the common value case is approached and the signals are noisier. A welfare optimal allocation can be implemented by a price-taking Bayesian supply function equilibrium. I show how a quadratic subsidy that lowers the perceived slope of the sellers' marginal costs may induce price-taking behavior and restore efficiency.

This paper also considers the large market case where the number of sellers and the demand are replicated (with the number of sellers n and the size of the market as well). Then both the distortion induced by private information and bid shading are decreasing in n. Furthermore, bid shading is on the order of $1/n$, there is no efficiency loss (in the limit) in a large market, and the order of magnitude of the expected deadweight loss is $1/n^2$.[3]

The welfare evaluation of the SFE is in marked contrast to the Cournot equilibrium in the presence of private information. The reason is that the SFE aggregates information but the Cournot market does not.[4] In a large Cournot market, in general, there is a welfare loss due to private information even in the limit.

The results are shown to be robust to a number of extensions: costly information acquisition, with maintained incentives under certain conditions even when close to the common value case; inelastic demand, which makes the model basically a double auction similar to Kyle (1989) but with no need for noise traders; demand uncertainty, which makes the equilibrium noisy and shows how an increase in noise in the public statistic lessens the adverse selection problem (when there is positive correlation); and the introduction of a public signal, which may restore efficiency if it is precise enough. The model with demand uncertainty has as a limit, with an appropriate choice of parameters, the markets considered in the linear Klemperer and Meyer (1989) model and in the Kyle (1989) risk neutral informed traders model.

A leading application of the model to goods markets is to wholesale electricity. The model also admits other interpretations. The cost shock could be related to some ex post pollution or emissions damage that is assessed on the firm or it could be a random opportunity cost of serving the market that is related to revenue management dynamic considerations. At the same time, the reinterpretation of the results in terms of demand schedule competition opens up a host of applications to financial markets (e.g., to legacy loans, central bank liquidity, and Treasury auctions). Each of these applications is dealt with in Section 6.

Competition in supply or demand schedules has a long tradition in the literature. It was studied in the absence of uncertainty by Grossman (1981) and Hart (1985), who showed a great multiplicity of equilibria.[5] Similar results in a complete information setting were obtained by Wilson (1979) in a share auction model and by Bernheim and Whinston (1986) in a menu auction. Back and Zender (1993) and Kremer and Nyborg (2004) obtained related results

[3]This is also the rate of convergence to efficiency obtained in a double auction context by Cripps and Swinkels (2006). Vives (2011a) dealt with the limit continuum economy case and provided a foundation for competitive rational expectations equilibria.

[4]The welfare analysis in the supply function model contrasts with the one in models where there is no endogenous public signal such as the Cournot market in Vives (1988), the beauty contest in Morris and Shin (2002), or the general linear-quadratic setup of Angeletos and Pavan (2007).

[5]Grossman thought of firms signing implicit contracts with consumers that committed the firm to a supply function. Hart uncovered the equivalence between choosing a reaction function and a supply function.

for Treasury auctions. Some of the equilibria can be very collusive.[6] Klemperer and Meyer (1989) showed how adding uncertainty in the supply function model can reduce the range of equilibria and even pin down a unique equilibrium (linear in a linear-quadratic model) provided the uncertainty has unbounded support. In this case, the supply function equilibrium is always between the Cournot and competitive (Bertrand) outcomes.[7] The supply function models considered typically do not allow for private information.[8] Kyle (1989) introduced private information into a double auction for a risky asset of unknown liquidation value and derived a unique symmetric linear Bayesian equilibrium in demand schedules when traders have constant absolute risk aversion, there is noise trading, and uncertainty follows a Gaussian distribution.

The plan of the paper is as follows. Section 2 presents the supply function model with strategic sellers and characterizes a SFE and its comparative static properties. Section 3 performs a welfare analysis that characterizes the distortion at the SFE and deadweight losses, including welfare simulations and a comparison with Bayesian Cournot equilibria, and shows how the efficient allocation can be attained with price-taking equilibria and implemented with subsidy schemes. Section 4 studies replica markets and characterizes the convergence to price-taking behavior and the order of magnitude of deadweight losses as the market grows large. Section 5 deals with the extensions: inelastic demand, demand uncertainty, public signals, and demand schedule competition. Section 6 develops the applications. Concluding remarks, including potential policy implications, close the paper. Proofs are gathered in the Appendix and in the Supplemental Material (Vives (2011c)), which also includes details of the simulations of the model, the analysis of endogenous information acquisition, and the Bayesian Cournot model.

2. A STRATEGIC SUPPLY FUNCTION MODEL

Consider a market for a homogenous good with n sellers. Seller i faces a cost

$$C(x_i; \theta_i) = \theta_i x_i + \frac{\lambda}{2} x_i^2$$

of supplying x_i units of the good, where θ_i is a random parameter and $\lambda > 0$.[9] Demand arises from an aggregate buyer who has quasilinear preferences and

[6] Back and Zender (2001) and LiCalzi and Pavan (2005) showed how the auction can be designed to limit those collusive equilibria.

[7] This is also the result in Vives (1986), where the slope of the supply function is fixed by technological considerations.

[8] Exceptions are the empirical papers of Hortaçsu and Puller (2008) and Kühn and Machado (2004) on electricity.

[9] We could also deal easily with the case where the seller faces an adjustment cost of the form $\lambda(x_i - \hat{x}_i)^2/2$, where $\hat{x}_i$ is a target quantity for agent i.

gross surplus $U(y) = \alpha y - \beta y^2/2$, where α and β are positive parameters and y is the consumption level. This gives rise to the inverse demand $P(y) = \alpha - \beta y$. In a reverse auction, for example, the buyer presents the schedule $P(y) = \alpha - \beta y$ to the sellers who will bid to supply.[10] Total surplus (TS) is therefore given by $\text{TS} = U(\sum_i x_i) - \sum_i C(x_i, \theta_i)$.

We assume that θ_i is normally distributed (with mean $\alpha > \bar{\theta} > 0$ and variance σ_θ^2). The parameters θ_i and θ_j, $j \neq i$, are correlated with $\text{cov}[\theta_i, \theta_j] = \rho\sigma_\theta^2$ and $\rho \in [-(n-1)^{-1}, 1]$ for $j \neq i$. The average parameter $\tilde{\theta} \equiv (\sum_{i=1}^n \theta_i)/n$ is thus normally distributed with mean $\bar{\theta}$, $\text{var}[\tilde{\theta}] = (1 + (n-1)\rho)n^{-1}\sigma_\theta^2$, and $\text{cov}[\tilde{\theta}, \theta_i] = \text{var}[\tilde{\theta}]$.[11] Seller i receives a signal $s_i = \theta_i + \varepsilon_i$ with ε_i normally distributed, $E[\varepsilon_i] = 0$, and $\text{var}[\varepsilon_i] = \sigma_\varepsilon^2$. Error terms in the signals are uncorrelated among themselves and with the θ_i parameters. Ex ante, before uncertainty is realized, all sellers face the same prospects. [12]

Our information structure encompasses the cases of "common value" and of "private values." For $\rho = 1$, the θ parameters are perfectly correlated and we are in a *common value* model. When signals are perfect, $\sigma_\varepsilon^2 = 0$ for all i, and $0 < \rho < 1$, we are in a *private values* model. Agents receive idiosyncratic shocks, which are imperfectly correlated, and each agent observes his shock with no measurement error. When $\rho = 0$, the parameters are independent and we are in an *independent values* model. When $\rho < 0$, the cost parameters are negatively correlated. The case of nonnegative correlation is empirically more relevant.

2.1. *Equilibrium*

Sellers compete in supply functions. We restrict attention to symmetric linear-Bayesian supply function equilibrium (SFE for short).[13] The strategy for seller i is a price contingent schedule $X(s_i, \cdot)$. This is a map from the signal space to the space of supply functions. Given the strategies of sellers $j = 1, \ldots, n$ for given realizations of signals, market clearing implies that $p = P(\sum_{j=1}^n X(s_j, p))$. Let us assume that there is a unique market clearing

[10] We comment in Section 5.1 on how the results specialize to the case of inelastic demand and in Section 5.4 on how they can be reinterpreted for the case of demand instead of supply bids.

[11] Note that $\text{var}[\tilde{\theta}] \geq 0$ for $\rho \geq -(n-1)^{-1}$ since then $1 + (n-1)\rho \geq 0$.

[12] With normal distributions there is positive probability that prices and quantities are negative in equilibrium. This can be controlled by choice of the variances of the distributions and the parameters α, β, λ, and $\bar{\theta}$.

[13] What makes the model tractable is the combination of linear-quadratic payoffs coupled with an affine information structure (that is, a pair of prior and likelihood that yields affine conditional expectations as under the normality) that allows for the existence of linear equilibria. It is crucial that the slopes of demand and costs are not affected by uncertainty. Adding (intercept) demand uncertainty presents no problem as long as the affine information structure is kept (see Section 5.2).

price $\hat{p}(X(s_1, \cdot), \dots, X(s_n, \cdot))$ for any realizations of the signals.[14] Then profits for seller i, for any given realization of the signals, are given by

$$\pi_i(X(s_1, \cdot), \dots, X(s_n, \cdot)) = pX(s_i, p) - C(X(s_i, p)),$$

where $p = \hat{p}(X(s_1, \cdot), \dots, X(s_n, \cdot))$. This defines a game in supply functions and we want to characterize a SFE. Given linear strategies of rivals $X(s_j, p) = b - as_j + cp$, $j \neq i$, seller i faces a residual inverse demand

$$\begin{aligned} p &= \alpha - \beta \sum_{j \neq i} X(s_j, p) - \beta x_i \\ &= \alpha - \beta(n-1)(b + cp) + \beta a \sum_{j \neq i} s_j - \beta x_i. \end{aligned}$$

Provided $1 + \beta(n-1)c > 0$, it follows that $p = I_i - dx_i$, where $I_i \equiv d(\alpha\beta^{-1} - (n-1)b + a\sum_{j \neq i} s_j)$ and $d \equiv (\beta^{-1} + (n-1)c)^{-1}$. The (endogenous) parameter d is the (absolute value of the) slope of inverse residual demand for a seller and plays an important role in the characterization of equilibrium and its welfare properties. All the information provided by the price to seller i about the signals of others is subsumed in the intercept of residual demand I_i. The expression for residual demand disentangles the capacity of a seller to influence the market price (d) by learning from the price (I_i). Note that I_i is informationally equivalent to $h_i \equiv \beta a \sum_{j \neq i} s_j$. The information available to seller i is therefore $\{s_i, p\}$ or, equivalently, $\{s_i, h_i\}$. Seller i chooses x_i to maximize

$$\begin{aligned} E[\pi_i | s_i, p] &= x_i(p - E[\theta_i | s_i, p]) - \frac{\lambda}{2} x_i^2 \\ &= x_i(I_i - dx_i - E[\theta_i | s_i, p]) - \frac{\lambda}{2} x_i^2. \end{aligned}$$

The first order conditions (FOC) are $I_i - E[\theta_i | s_i, I_i] - 2dx_i - \lambda x_i = 0$ or, equivalently, $p - E[\theta_i | s_i, p] = (d + \lambda)x_i$. The second order sufficient condition for a maximum is $2d + \lambda > 0$. An equilibrium must also fulfill $1 + \beta(n-1)c > 0$. The following proposition characterizes the linear equilibrium and the following subsections describe its properties.[15]

PROPOSITION 1: *Let* $-(n-1)^{-1} < \rho < 1$ *and* $\sigma_\varepsilon^2 / \sigma_\theta^2 < \infty$.

(i) *If* $\lambda > 0$, *there is a unique SFE. It is given by the supply function* $X(s_i, p) = (p - E[\theta_i | s_i, p])/(d + \lambda)$, *where* $d = (\beta^{-1} + (n-1)c)^{-1}$, *and* $c \equiv \partial X / \partial p$ *is given by the largest solution to a quadratic equation* $g(c; M) = 0$, *where* $M \equiv$

[14] If there is no market clearing price, assume the market shuts down; if there are many market clearing prices, then the one that maximizes volume is chosen.

[15] We use the terms "increasing" or "decreasing" in the strict sense unless otherwise stated.

$\frac{n\rho\sigma_\varepsilon^2}{(1-\rho)(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2)}$. *We have that* $a \equiv -\partial X/\partial s_i > 0$, $0 < d < \beta n$, $c > -M((1+M)\beta n)^{-1}$, $1 + M > 0$, *and c decreases with* λ *and* M, *ranging from* $-(\beta n)^{-1}$ *to* ∞ *as* M *ranges from* ∞ *to* -1.

(ii) *If* $\lambda = 0$, *there is a SFE if and only if* $n - M - 2 < 0$. *Then* $c = c_0 \equiv -(n - M)/((n - M - 2)\beta n)$.

2.2. *Information Revelation*

The equilibrium price p is a linear function of, and therefore reveals, the aggregate information $\tilde{s} \equiv (\sum_i s_i)/n$.[16] The equilibrium is *privately revealing*. That is, for seller i, (s_i, p) or $(s_i, \tilde{s})$ is a sufficient statistic of the joint information in the market $s = (s_1, \dots, s_n)$ in the estimation of θ_i (see Allen (1981)). In particular, in equilibrium we have that the conditional distribution of posterior beliefs of θ_i fulfills $E[\theta_i|s_i, p]$ $(= E[\theta_i|s_i, \tilde{s}]) = E[\theta_i|s]$.[17] That is, from the price, a seller obtains the collective information of other sellers (which is relevant as long as costs are correlated) but his private signal is still useful to improve the estimation of his cost parameter (provided $\rho < 1$). This means that incentives to rely on (and purchase) private signals remain, since a private signal adds information for seller i on top of the information conveyed by the price. (Indeed, we have that $a \equiv -\partial X/\partial s_i > 0$ if $\rho < 1$.)

If the signals are costly to acquire and agents face a convex cost of acquiring precision $\tau_\varepsilon \equiv 1/\sigma_\varepsilon^2$, then it is possible to show that each seller will have an incentive to purchase some precision for any $-(n-1)^{-1} < \rho < 1$ and any n provided that the marginal cost of acquiring precision is low enough for small amounts of precision or that the prior is diffuse enough. The reason is as follows. By purchasing a signal, seller improves the information on his random cost parameter even though he learns the signals of the other sellers through the price. When the number of sellers is large or correlation is high, the improvement will be small, but if the seller can purchase a little bit of precision at a low cost, he will do it. Furthermore, the more diffuse is the prior, the higher is the marginal value of information.[18]

An equivalent formulation that highlights the aggregate and idiosyncratic components of uncertainty is to let $\vartheta_i \equiv \theta_i - \tilde{\theta}$ and note that $\theta_i = \tilde{\theta} + \vartheta_i$, where

[16]Average quantity is given by $\tilde{x} \equiv (\sum_i x_i)/n = b - a\tilde{s} + cp$. Substituting $p = \alpha - \beta n\tilde{x}$ into the inverse demand, noting that in equilibrium $1 + \beta nc > 0$, and solving for p, we obtain $p = (1 + \beta nc)^{-1}(\alpha - \beta nb + \beta na\tilde{s})$.

[17]Note that under normality, the conditional expectation is a sufficient statistic for the conditional distribution.

[18]If the marginal cost of acquiring precision is positive at zero and ρ is close to 1, then for n large enough, there is no purchase of information. However, this is not the case in the natural case of a large market where the number of buyers and sellers grow together (as in Section 4). See Section S.4 in the Supplemental Material (Vives (2011c)) for the information acquisition model, results, and proofs.

$\text{cov}[\vartheta_i, \tilde{\theta}] = 0$ and $n^{-1}\sum_{i=1}^{n}\vartheta_i = 0$.[19] Then it becomes clear that it is key to the private revealing property of the equilibrium that the same signal s_i conveys information about the idiosyncratic component η_i and an aggregate component $\tilde{\theta}$, and that the price reveals a sufficient statistic of the signals of sellers other than i.[20]

The equilibrium is in contrast to the pure common value "noise trader" model of Kyle (1989), where risk-averse traders, some with private information about the value of the risky asset, face liquidity traders. Here the collective information of traders would be revealed by the price and the market would collapse, except for the presence of noise traders. In our base model, there is no shock to the residual demand function (be it noise traders or demand uncertainty) and, consequently, in the pure common value case ($\rho = 1$ and $\sigma_\varepsilon^2 < \infty$), the equilibrium collapses.[21] Indeed, when $\rho = 1$ and $0 < \sigma_\varepsilon^2 < \infty$, a fully revealing rational expectations equilibrium is not implementable and there is no linear equilibrium. The reason should be well understood: If the price reveals the common value, then no seller has an incentive to put any weight on his signal (and the incentives to acquire information disappear as well). But if sellers put no weight on their signals, then the price cannot contain any information on the costs parameters. This is the essence of the Grossman–Stiglitz paradox (1980). The approach in our paper allows a welfare analysis to be performed since it does away with the need to introduce noise traders who do not have a well defined utility function. The private value component in the valuation of an agent in our model arises naturally in applications as we will see in Section 6.

2.3. *Private Information and Market Power*

Despite the fact that the SFE is privately revealing, it is distorted in relation to the full information supply function equilibrium where sellers share $s = (s_1, \dots, s_n)$ (denoted by a superscript f). Indeed, following a similar analysis as before, it is easy to see that $X^f(s_i, p) = (p - E[\theta_i|s])/(d^f + \lambda)$, where d^f and c^f correspond to the case $M = 0$ in Proposition 1. Whenever there is no correlation between the cost parameters ($\rho = 0$) or signals are perfect (the private values case with $\sigma_\varepsilon^2/\sigma_\theta^2 = 0$),[22] then $M = 0$, $E[\theta_i|s_i, p] = E[\theta_i|s] = E[\theta_i|s_i]$,

[19]We could also let $\hat{\vartheta}_i \equiv \theta_i - \tilde{\theta}_\infty$, where $\tilde{\theta}_\infty \equiv \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n}\theta_i$ is now the common component. Then we would have both $\text{cov}[\hat{\vartheta}_i, \tilde{\theta}_\infty] = 0$ and $\text{cov}[\hat{\vartheta}_i, \hat{\vartheta}_j] = 0$ for $i \neq j$.

[20]This latter property obtains typically in the linear-Gaussian models with uniform correlation in the parameters (see, e.g., Vives (2008, Chap. 3)). However, Rostek and Weretka (2010) showed that this need not be the case with heterogeneous correlation (when considering symmetric equilibria which depend only on average correlation).

[21]As we will see in Section 2.4, the equilibrium also collapses when $\rho = -(n-1)^{-1}$ since then there is no aggregate uncertainty ($\text{var}[\tilde{\theta}] = 0$).

[22]In this case, the equilibrium is independent of ρ and it exists even if $\rho = 1$ or $\rho = -(n-1)^{-1}$.

and seller i does not learn about θ_i from prices. In these cases, the SFE coincides with the full information equilibrium.[23]

When $\rho\sigma_\varepsilon^2 \neq 0$, the price at a SFE serves a dual role as an index of scarcity and as a conveyor of information. This can be seen from the supply function $X(s_i, p) = (p - E[\theta_i|s_i, p])/(d + \lambda)$. Indeed, a high price has a direct effect to increase the competitive supply of a seller, but also conveys news that costs for the seller are high if $\rho > 0$ (since then $E[\theta_i|s_i, p]$ is increasing in p) or low if $\rho < 0$ (since then $E[\theta_i|s_i, p]$ is decreasing in p). When $\rho\sigma_\varepsilon^2 > 0$, supply functions at the SFE are steeper than with full information $c < c^f$ (and $d > d^f$) due to adverse selection. Private information creates market power (d) over and above the full information level (d^f). When $\rho\sigma_\varepsilon^2 < 0$, supply functions are flatter ($c < c^f$) and the informational effect of the price is pro-competitive since a high price conveys the news to a seller that the costs of rivals are high and, therefore, that his own costs are low due to negative correlation. As ρ increases from $\rho = 0$, the adverse selection problem worsens, and as ρ turns negative, the adverse selection problem disappears and becomes "favorable" selection. The parameter M (a function of ρ and $\sigma_\varepsilon^2/\sigma_\theta^2$) is an index of adverse selection and the slope of the supply function becomes steeper (c decreases) with M (Proposition 1). We have that when $\sigma_\varepsilon^2 > 0$, M is increasing in ρ and $\text{sgn}\{\partial M/\partial(\sigma_\varepsilon^2/\sigma_\theta^2)\} = \text{sgn}\{M\} = \text{sgn}\{\rho\}$.

As either $|\rho|$ increases and cost parameters become more correlated or $\sigma_\varepsilon^2/\sigma_\theta^2$ increases and private signals are (relatively) less precise, the price signal becomes more relevant to estimate θ_i. More precisely, the absolute value of the weight on the information component of the price h_i in $E[\theta_i|s_i, h_i]$ increases in $|\rho|$ and $\sigma_\varepsilon^2/\sigma_\theta^2$.[24] When $\rho\sigma_\varepsilon^2 > 0$, the result is that as ρ or $\sigma_\varepsilon^2/\sigma_\theta^2$ increases then c decreases since a high price is bad news (i.e., the seller learns more from the price about its cost shock and reacts less to a price change than if the price was only an index of scarcity). When $\rho\sigma_\varepsilon^2 < 0$, as $|\rho|$ or $\sigma_\varepsilon^2/\sigma_\theta^2$ increases, then c increases since a high price is good news.[25]

As $\rho \to 1$, then $M \to \infty$ and c becomes negative. As $\rho \to 1$, we have that the equilibrium collapses in the limit. In fact, since $a \to 0$, $c \to -(\beta n)^{-1}$, and $b \to \alpha(\beta n)^{-1}$, the supply function of a seller converges to the per capita seller demand function $x = (\alpha - p)(\beta n)^{-1}$ (see Figure 1) or, equivalently, the aggregate supply function converges to the demand function. As $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$, the SFE also collapses and there is a discontinuity in the limit when $\rho \neq 0$.[26]

[23]This equilibrium is robust to the introduction of noise in the demand function as in Klemperer and Meyer (1989); see Section 5.2.

[24]See the Proof of Claim A.1 in the Appendix.

[25]From the expression for the weight a of private information in the strategy of a seller and the fact that d decreases in c, we have that a decreases in ρ and in $\sigma_\varepsilon^2/\sigma_\theta^2$ when $\rho \geq 0$.

[26]When $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$, we have that $a \to 0$ and $c \to \widehat{c}$, with $\widehat{c} = c^f$ for $\rho = 0$ and $c^f > \widehat{c}$ ($c^f < \widehat{c}$) for $\rho > 0$ ($\rho < 0$). (See Claim A.2 in the Appendix.) However, the equilibrium in the limit economy with $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$ (even when $\rho = 1$) is given by $X(p) = c^f(p - \bar{\theta})$ since $E[\theta_i|s_i, p] = \bar{\theta}$;

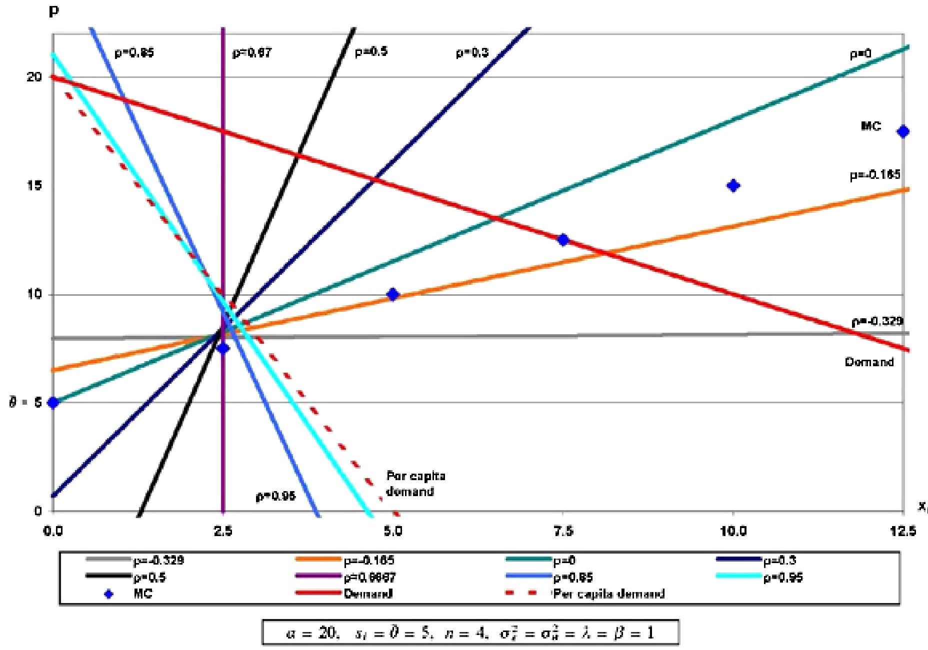

FIGURE 1.—The SFE $X(\bar{\theta}, p)$ as ρ goes from $-(n-1)^{-1}$ to 1.

There are particular parameter combinations when $\rho > 0$ for which the scarcity and informational effects balance and sellers set a zero weight ($c = 0$) on public information. In this case, sellers do not condition on the price and the model reduces to the Cournot model where sellers compete in quantities. However, in this particular case, when supply functions are allowed, not reacting to the price (public information) is optimal. Figure 1 depicts the change in the equilibrium supply function as ρ goes from $-(n-1)^{-1}$ to 1 for $s_i = \bar{\theta}$: $X(\bar{\theta}, p)$.

Constant Marginal Costs[27] *($\lambda = 0$)*

If $n - 2 \geq M$, there is no SFE (Proposition 1(ii)) and the slope of supply degenerates to competitive ($c \to \infty$) as $\lambda \to 0$. However, whenever adverse selection is important enough, $n - 2 < M$, then as $\lambda \to 0$, $c \to c_0$ and there is a SFE with slope c_0 (negative when $n < M$). (See Claim A.3 in the Appendix.) In short, for high enough adverse selection, sellers have market power even with constant returns.

therefore, it coincides with the limit of the SFE as $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$ only when $\rho = 0$. There is a discontinuity in the equilibrium correspondence when $\rho \neq 0$. This discontinuity disappears when there is noise in the demand function (see Section 5.2).

[27]This case approximates classical multiunit auction environments.

Our results are related to the winner's curse in common value auctions (Milgrom and Weber (1982)): A bidder refrains from bidding aggressively because winning conveys the news that the signal he received was too optimistic (the highest signal in the pool). To protect against the winner's curse, bidders shade their bid more the less precise are their signals (see Reece (1978)). In our model, a seller refrains from competing aggressively with its supply function because a high price conveys the bad new that costs are high, and the more so, the less precise is his signal. However, in the typical auction model, sellers bid for a unit of a good, while in our model, sellers compete in schedules to fulfill a demand for a divisible good and, therefore, the analogy works with respect to adverse selection but not necessarily with respect to market power.[28]

The results are also reminiscent of asymmetric information models where traders submit steeper schedules to protect themselves against adverse selection.[29] Biais, Martimort, and Rochet (2000), in a common value environment in a discriminatory auction, showed that adverse selection reduces the aggressiveness of competition in supply schedules of risk-neutral uninformed market makers, facing a risk-averse informed trader who is subject also to an endowment shock. At the unique equilibrium in convex supply schedules the outcome is of imperfect competition because marginal prices are increasing with the size of trade as market makers protect themselves against informed trading. The latter, combined with the optimal response of the informed agent, determines a residual demand curve with finite elasticity for every market maker. This imperfect competition result disappears in a pure private value environment where there is no asymmetric information about the value of the asset and adverse selection arises only out of the idiosyncratic endowment shock to the trader. Then marginal prices need no longer be increasing in the amount traded to reflect the informational content of trade. In both Biais, Martimort, and Rochet (2000) and our paper private information generates market power.[30] In our model in the pure private value case ($\rho = 0$) there is some market power provided that $\lambda > 0$ and it vanishes when $\lambda \to 0$.

[28]Our results are perhaps more closely related to the generalized winner's curse or "champion's plague" pointed out in Ausubel (2004) for multiunit auctions according to which, and translated into our context, the expected cost of a bidder conditional on being allocated a larger quantity is greater than with a smaller quantity.

[29]I will discuss the precise relationship with the Kyle (1989) model when I introduce noise and demand schedule competition (Section 5.4).

[30]However, their framework is very different from ours: In their paper, the competing market makers/sellers are uninformed while in ours they are privately informed; the monopsonistic informed buyer selects quantities in the posted schedules while we have a passive competitive demand; the buyer has liquidity shock while there is no noise in our demand; and the auction is discriminatory while ours is uniform. As we see in Section S.1 of the Supplemental Material, the behavior of a large market is quite different in both models.

2.4. *Competitiveness*

The competitiveness of a market is usually measured in terms of absolute and relative margins over marginal costs which are closely related to the perceived elasticity of the residual demand of a seller. For seller i, the residual demand is $\beta^{-1}(\alpha - p) - \sum_{j\neq i} X(s_j, p)$, with elasticity $\hat{\eta}_i = p/dx_i$. The (absolute value of the) slope of residual demand is $d^{-1} = \beta^{-1} + (n-1)c$. From the equilibrium FOC we have that $p - E_i[\mathrm{MC}_i] = dx_i$, where $E_i[\mathrm{MC}_i] \equiv E[\theta_i|s] + \lambda x_i$ is the (interim) expected marginal cost (MC) of seller i. In Lerner index form,

$$\frac{p - E_i[\mathrm{MC}_i]}{p} = \frac{1}{\hat{\eta}_i}.$$

A similar relation holds for the margin over average (interim) expected marginal cost $E_n[\mathrm{MC}_n] \equiv n^{-1}\sum_{i=1}^n E_i[\mathrm{MC}_i]$, $p - E_n[\mathrm{MC}_n] = d\tilde{x}$, and, correspondingly, for the aggregate (interim) Lerner index

$$\frac{p - E_n[\mathrm{MC}_n]}{p} = \frac{1}{(\beta^{-1} + (n-1)c)\beta n\eta} = \frac{d}{\beta n\eta},$$

where $\eta = p/(\beta n\tilde{x})$ is the elasticity of demand. It follows that $p = E[\tilde{\theta}|\tilde{s}] + (d + \lambda)\tilde{x}$.[31]

Three important benchmarks for rivalry are perfect competition, Cournot equilibrium and collusion. If sellers are price takers and act with full information $s = (s_1, \dots, s_n)$, then $p - E_i[\mathrm{MC}_i] = 0$ and $p - E_n[\mathrm{MC}_n] = 0$, and this corresponds to the case when $c = \infty$ and $d = 0$.

The case $c = 0$ corresponds to a Bayesian Cournot equilibrium, where seller i sets a quantity contingent only on his information $\{s_i\}$, and the aggregate (interim) Lerner index is $(n\eta)^{-1}$.[32] The supply function and the Cournot equilibrium (and allocations) coincide when $M = n(1 + \lambda\beta^{-1})^{-1}$, in which case $c = 0$ and $d = \beta$. When $c > 0$, we are in the usual case in which the supply function equilibrium has positive slope and is between the Cournot and the competitive outcomes (e.g., Klemperer and Meyer (1989) when uncertainty has unbounded support and full information, in which case $c = c^f > 0$). However, when $c < 0$, the aggregate (interim) Lerner index is larger than the Cournot level.

If sellers were to collude with full information (share the signals $s = (s_1, \dots, s_n)$ and maximize joint profits), it is easy to see that we would obtain the usual collusive (monopoly) Lerner formula

$$\frac{p - E_n[\mathrm{MC}_n]}{p} = \frac{1}{\eta}.$$

[31]Noting that $n^{-1}\sum_{i=1}^n E[\theta_i|s] = E[\tilde{\theta}|s] = E[\tilde{\theta}|\tilde{s}]$, the latter equality holding since $\tilde{s}$ is a sufficient statistic for s in relation to $\tilde{\theta}$.

[32]There is a unique Bayesian Cournot equilibrium and it is linear (see Proposition S.1 in the Supplemental Material).

What is surprising, as we will show, is that as ρ ranges from $-(n-1)^{-1}$ to 1, we have that d ranges from 0 to βn and, correspondingly, the price ranges from competitive to collusive. The following proposition states the competitiveness-related results plus a volatility result.

PROPOSITION 2: *Let $-(n-1)^{-1} < \rho < 1$ and $0 < \sigma_\varepsilon^2/\sigma_\theta^2 < \infty$. Then the following results hold at the SFE*:

(i) *The slope of equilibrium supply is steeper* (*c smaller*) *with increases in ρ, and c ranges from ∞ to $-1/\beta n$ and d ranges from 0 to βn as ρ ranges from $-(n-1)^{-1}$ to 1. When $\rho > 0$ ($\rho < 0$), c decreases* (*increases*) *with $\sigma_\varepsilon^2/\sigma_\theta^2$.*

(ii) *As ρ ranges from $-(n-1)^{-1}$ to 1 the price ranges from competitive to collusive. When $c > 0$ ($c < 0$), the price is less* (*greater*) *than the Cournot level. When $\rho > 0$ ($\rho < 0$), the price is greater* (*less*) *than the full information level.*

(iii) *The expected price $\bar{p}$ and margin $\bar{p} - E[\mathrm{MC}_n] = dE[\tilde{x}]$ are increasing in ρ and $\sigma_\varepsilon^2/\sigma_\theta^2$ (when $\rho > 0$), and with n^{-1} (for $c > 0$ when $\rho \geq 0$). They are decreasing in $\sigma_\varepsilon^2/\sigma_\theta^2$ when $\rho < 0$.*

(iv) *Price volatility* var$[p]$, *when $\rho > 0$, decreases with σ_ε^2 and increases with σ_θ^2.*

It is remarkable that sellers may approach aggregate collusive margins in a one-shot noncooperative equilibrium because of informationally induced market power. Let us recall that at the full information equilibrium (corresponding to $\rho = 0$), which indicates the pure market power distortion, the aggregate Lerner index equals $d^f/(\beta n\eta)$. As $\rho \to 1$, the private information distortion becomes more severe and sellers protect themselves by increasing the slope of their supplies, and they become less and less aggressive. Furthermore, as $\rho \to -(n-1)^{-1}$, the increased "favorable" selection implies that market power is reduced and eliminated in the limit.

The explanation of the result is as follows. The aggregate margin and output tend to the collusive level, maximal market power, because as $\rho \to 1$ and $d \to \beta n$, the aggregate supply function converges to the demand function and the market collapses. This is precisely the case in which the slope of residual demand for an individual seller is collusive since sellers tend to produce the same (as we will see in Section 3.1, despite the existence of some productive inefficiency as long as $\rho < 1$) and as $\rho \to 1$, then $\frac{1}{\hat{\eta}_i} \to \frac{1}{\eta}$ and

$$\frac{p - E_n[\mathrm{MC}_n]}{p} \to \frac{1}{\eta}.$$

This would not happen if equilibrium were to exist for $\rho = 1$. Indeed, with noisy demand (Proposition 7), equilibrium exists even if $\rho = 1$ and the aggregate margin is never fully collusive. (In a similar vein, as $\rho \to -(n-1)^{-1}$, the competitive outcome obtains as the equilibrium also collapses since $c \to \infty$ (see Figure 1), but with demand uncertainty, it does not collapse and the limit is then not fully competitive.)

REMARK 1: *It is worth noting that the distortion d may increase with n when $c < 0$.*[33] *This does not happen with full information because then c^f (d^f) is increasing* (*decreasing*) *in n* (*see Claim* A.4 *in the Appendix*); *neither will it happen when demand is replicated with the number of sellers* (*see Section* 4).

3. WELFARE ANALYSIS

To assess the welfare loss at the SFE we provide an outcome-based characterization of the equilibrium and a characterization of the deadweight losses. At the full information equilibrium sellers have market power and there is no private information. There is a welfare loss due to market power. At the SFE there is an additional welfare loss due to private-information-induced market power. I show also that the efficient outcome can be implemented with a price-taking supply function equilibrium and how subsidies can implement the efficient allocation.

3.1. *A Characterization of the SFE Outcome and Welfare*

Let $t_i \equiv E[\theta_i|s]$, $i = 1, \dots, n$, and $t = (t_1, \dots, t_n)$ be the predicted values with full information s. The strategies at a SFE, where $E[\theta_i|s_i, p] = t_i$, induce outcomes as a function of the realized vector of predicted values t: $(x_i(t))_{i=1}^n$ and $p(t)$. It is easy to see then that the outcome at the SFE maximizes a distorted surplus function with common information t,

$$\max_{(x_i)_{i=1}^n} \left\{ E[\mathrm{TS}|t] - \frac{d}{2} \sum_{i=1}^{n} x_i^2 \right\},$$

where d is the equilibrium SFE parameter (Proposition 1). That is, the market solves the surplus maximizing program with a distorted cost function which represents both higher total and marginal costs[34]

$$\hat{C}(x_i, \theta_i) \equiv C(x_i, \theta_i) + \frac{d}{2} x_i^2.$$

The result follows since the (sufficient) FOC of the distorted planning problem are

$$p - E[\theta_i|t] - (d + \lambda) x_i = 0, \quad i = 1, \dots, n,$$

[33]For example, $d(n = 3) > d(n = 2)$ with parameters $\beta = \lambda = 1$ and $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$ when ρ is close to 1. See Figure S.1a in Section S.3 of the Supplemental Material, which contains more results and details of the simulations.

[34]The SFE allocation would be obtained by price-taking sellers with distorted cost functions $\hat{C}(x_i, \theta_i)$ and full information t. However, in this case, supply functions would always be upward sloping, $x_i = (p - E[\theta_i|t])/(d + \lambda)$, since there is no informative role for the price to play.

which are identical to those of the SFE since $E[\theta_i|s_i, p] = E[\theta_i|t]$. Similarly, the full information supply function equilibrium can be obtained as the solution to a distorted planning program replacing d by d^f. It is clear that the full (shared-) information efficient allocation obtains by setting $d = 0$. The implied allocation is symmetric (since the total surplus optimization problem is strictly concave, and the sellers and information structure are symmetric).

We can consider a SFE allocation parameterized by d for a given realization of predicted values t, $(x_i(t; d))_{i=1}^n$. The deadweight loss ($E[\mathrm{DWL}|t]$) at the SFE is the difference between total surplus at $(x_i(t; d))_{i=1}^n$ and at the efficient allocation $(x_i(t; d = 0))_{i=1}^n$. The wedge $d > 0$ induces both distributive/productive and aggregate/allocative inefficiency. Distributive inefficiency refers to an inefficient distribution of sales/production of a given aggregate (average) quantity $\tilde{x}$. Sellers minimize distorted costs $\hat{C}(x_i, \theta_i)$ with $d > 0$—equivalent to a fictitious more convex technology—and the choices of individual quantities are biased toward too similar sales, $x_i - \tilde{x} = (\tilde{t} - t_i)(d + \lambda)^{-1}$, while cost minimization would require letting $d = 0$. Aggregate inefficiency refers to a distorted level of average quantity while producing in a cost-minimizing way. Note that average quantity $\tilde{x}(t; d) = (\alpha - \tilde{t})/(\beta n + \lambda + d)$ is decreasing in d. The impact of the distortion on profits is also of interest. An increase in d increases margins but also productive inefficiency with an a priori ambiguous impact on profits. The following proposition states the results.

PROPOSITION 3: *Consider an allocation parameterized by d for a given realization of predicted values t. The following outcomes result*:

(i) *Both aggregate and distributive inefficiency*, *and therefore* $E[\mathrm{DWL}|t]$, *are increasing in d*.

(ii) *Average profits increase in d for d small and decrease in d for d close to βn*.

The intuition for result (i) should be clear since increases in d, for a given realization of predicted values, reduce average output and bias individual outputs toward excessive similarity. With regard to result (ii), when the distortion is small, increasing d increases average profits by increasing margins more than productive inefficiency, while the opposite happens when the distortion is large.

Do the welfare results extend to averaging over predicted values, that is, when taking unconditional expectations? From Proposition 3, we have that for given predicted values t, both types of inefficiency increase with d, and, therefore, increase with ρ and σ_ε^2, but changes in those parameters change the probability distribution over t. The (expected) deadweight loss ($E[\mathrm{DWL}]$) at the SFE is the difference between expected total surplus (ETS) at the efficient allocation (ETS^o) and at the SFE (denoted by ETS). Let $x_i = x_i(t; d)$ and $x_i^o = x_i(t; 0)$. It can be checked that

$$E[\mathrm{DWL}] = n\big((\beta n + \lambda)E[(\tilde{x} - \tilde{x}^o)^2] + \lambda E[(u_i - u_i^o)^2]\big)/2,$$

with $u_i \equiv x_i - \tilde{x}$ and $u_i^o \equiv x_i^o - \tilde{x}^o$, where the first term corresponds to aggregate inefficiency and the second term corresponds to distributive inefficiency. The following proposition takes into account the averaging effect and characterizes deadweight losses.

PROPOSITION 4:
(i) *Expected aggregate inefficiency always increases in* ρ (*if* $\sigma_\varepsilon^2 > 0$) *while it may increase or decrease in* σ_ε^2. *If* $\rho \leq 0$, *then it decreases in* σ_ε^2.
(ii) *Expected distributive inefficiency may increase or decrease in* ρ *and in* σ_ε^2. *If* $\sigma_\varepsilon^2 = 0$, *then it decreases in* ρ; *if* $\rho \leq 0$, *then it decreases in* σ_ε^2.
(iii) *Expected profits*, *when* $\sigma_\varepsilon^2 > 0$, *converge to the collusive level as* $\rho \to 1$ *and to the competitive level when* $\rho \to -(n-1)^{-1}$. *If* $\sigma_\varepsilon^2 = 0$, *then they decrease in* ρ; *if* $\rho \leq 0$, *then they decrease in* σ_ε^2.

REMARK 2: *With respect to the welfare loss induced by full information market power*, *both expected aggregate and distributive inefficiency decrease in* σ_ε^2, *and expected aggregate* (*distributive*) *inefficiency increases* (*decreases*) *in* ρ.

The intuition for the results is as follows. (i) The aggregate inefficiency term $E[(\tilde{x} - \tilde{x}^o)^2] = ((\beta n + \lambda)^{-1} - (\beta n + \lambda + d)^{-1})^2 E[(\alpha - \tilde{t})^2]$ increases in d and in the variance of the prediction $\tilde{t} = E[\tilde{\theta}|\tilde{s}]$, var$[\tilde{t}]$. Increases in ρ increase both (if $\sigma_\varepsilon^2 > 0$), while increases in σ_ε^2 decrease var$[\tilde{t}]$, and this effect may dominate if ρ is small. (ii) The distributive inefficiency term $E[(u_i - u_i^o)^2] = (\lambda^{-1} - (\lambda + d)^{-1})^2 E[(t_i - \tilde{t})^2]$ increases in d and in $E[(t_i - \tilde{t})^2] = \text{var}[t_i - \tilde{t}]$. The nonmonotonicity with respect to ρ (if $\sigma_\varepsilon^2 > 0$) and σ_ε^2 (if $\rho > 0$) follows since var$[t_i - \tilde{t}]$ decreases in ρ (if $\sigma_\varepsilon^2 > 0$) and σ_ε^2 (if $\rho > 0$). Indeed, with $\rho = 1$ or $\sigma_\varepsilon^2 = \infty$ there would be no distributive inefficiency. The results when $\rho\sigma_\varepsilon^2 = 0$ follow since then d is independent of ρ and σ_ε^2, and only the averaging effect is present; when $\rho < 0$, then d is decreasing in σ_ε^2.[35] (iii) When $\rho \to 1$ and $\sigma_\varepsilon^2 > 0$ we know (from Proposition 2) that $d \to \beta n$ and $\tilde{x}(t, d) \to \tilde{x}^m \equiv \tilde{x}(t, \beta n)$, the average collusive output. Furthermore, as $\rho \to 1$, firms produce the same, productive inefficiency at the SFE vanishes, and expected profits converge to the collusive level. The same result holds when "collusive" replaces "competitive" and $\rho \to -(n-1)^{-1}$.

3.2. *Simulations*

In the central scenario of the simulations (with $\rho \in [0, 1)$), increases in ρ or σ_ε^2 increase the deadweight loss at the SFE ($E[\text{DWL}] \equiv \text{ETS}^o - \text{ETS}$); see Figure 2.[36] However, increasing ρ may decrease $E[\text{DWL}]$ when σ_ε^2 is small for

[35]The same applies for the welfare loss induced by standard market power since $d = d^f$ is independent of ρ and σ_ε^2.
[36]See Section S.3.1 in the Supplemental Material for details and more results of the simulations.

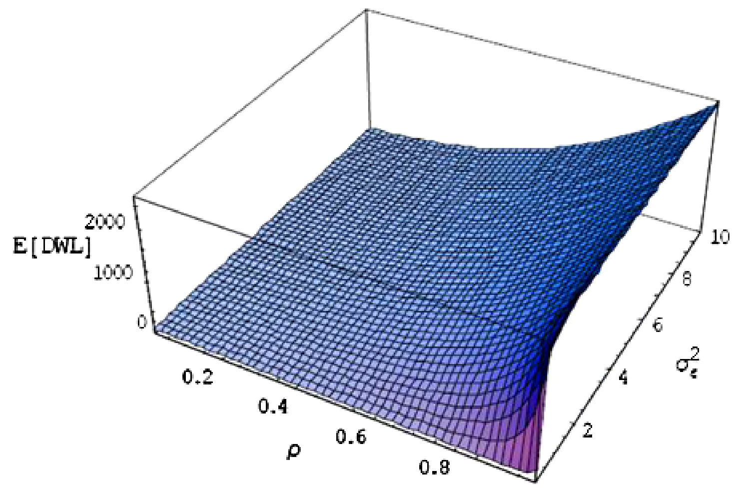

FIGURE 2.—$E[\text{DWL}] \equiv \text{ETS}^o - \text{ETS}$ as a function of ρ and σ_ε^2 (with parameters $\beta = \lambda = 1$, $\sigma_\theta^2 = 1$, $n = 4$, $\alpha = 200$, and $\bar{\theta} = 20$).

a range of ρ bounded away from 1, and increasing σ_ε^2 may decrease $E[\text{DWL}]$ when ρ is small. Furthermore, expected profits $E[\pi_i]$ increase in ρ or σ_ε^2 provided ρ or σ_ε^2 is not too close to 0; otherwise, $E[\pi_i]$ may decrease in ρ or σ_ε^2 and this will tend to be so for σ_θ^2 large.[37]

The outcome of the simulations performed suggests thus that the results of Proposition 3(i) derived for given predicted values of cost parameters extend to averaging over those values provided that ρ and σ_ε^2 are not too small. This is so since for given t, increases in ρ or σ_ε^2 increase both aggregate and distributive inefficiency. When averaging over predicted values, distributive inefficiency decreases with increases in ρ or σ_ε^2 and the effect overwhelms the impact given t when σ_ε^2 (or ρ) is small enough. With respect to expected profits, the results of Proposition 3(ii) do not extend to the ex ante situation since now increasing ρ or σ_ε^2 increases the expected margin and although it also increases the distortion d, it reduces the expected distributive inefficiency because the predictions of sellers are more aligned. However, when either ρ or σ_ε^2 is small, distributive inefficiency may weigh more.

REMARK 3: *The deadweight loss due to private-information-induced market power* $\text{ETS}^f - \text{ETS}$ (*which equals* $E[\text{DWL}]$ *minus the deadweight loss in the full information equilibrium* $\text{ETS}^o - \text{ETS}^f$) *increases in* ρ *or* σ_ε^2 ($E[\text{DWL}]$ *tends to increase with increases in* ρ *or* σ_ε^2 *while* $\text{ETS}^o - \text{ETS}^f$ *always diminishes with* σ_ε^2

[37]Recall that $E[\pi_i]$ decrease in ρ when $\sigma_\varepsilon^2 = 0$ and in σ_ε^2 when $\rho = 0$ (Proposition 4(iii)). See Figures S.2 and S.3 in the Supplemental Material for illustrations of the simulations.

and distributive inefficiency—which tends to dominate—also diminishes with ρ). $\text{ETS}^f - \text{ETS}$ *may decrease in* σ_ε^2 *when* ρ *is small*.

Supply Function versus Cournot

For σ_ε^2 or ρ small, sellers at the supply function market act with full information $c > 0$ and have less market power than in the (Bayesian) Cournot equilibrium, where sellers do not act with full information. For larger ρ and $\sigma_\varepsilon^2 > 0$, $c < 0$ and sellers in the supply function market have more market power, and this may dominate the information effect. Simulations suggest that for parameters for which $c > 0$ at the SFE, the supply function market attains a higher expected total surplus than the Cournot market and, for n not too large, the opposite happens when $c < 0$ (e.g., for ρ close to 1).[38] (See Sections S.2 and S.3.3, and Figure S.5 in the Supplemental Material.) When a supply function market is modeled, for convenience, à la Cournot a bias is introduced, overestimating the welfare loss with respect to the actual supply function mechanism on two counts when supply functions slope upward: excessive market power and lack of information aggregation. When the equilibrium supply function slopes downward, the Cournot market underestimates market power and then the Cournot market may, in principle, under- or overestimate the deadweight loss in relation to supply function competition.

3.3. *Price-Taking Equilibrium, Efficiency, and Optimal Subsidies*

If sellers acted as price-takers and had full information, the allocation would be (full information) efficient. It is easy to see that the efficient full information allocation can be implemented by a symmetric *price-taking* linear Bayesian supply function equilibrium (price-taking SFE for short, denoted with a superscript PT). This is an equilibrium where sellers do not perceive the influence that their supply decisions have on prices but still condition on their private signals and try to learn from prices. The FOC for a price-taking SFE are the same as in the Proof of Proposition 1 letting $d = 0$,

$$p = E[\theta_i|s_i, p] + \lambda x_i \quad \text{for} \quad i = 1, \dots, n,$$

yielding a supply function $X^{\text{PT}}(s_i, p) = (p - E[\theta_i|s_i, p])/\lambda$. As before, p reveals $\tilde{s}$ in equilibrium, $E[\theta_i|s_i, p] = E[\theta_i|s_i, \tilde{s}]$, and the price-taking equilibrium implements the efficient solution since sellers have full information and act competitively.

From the previous analysis, it may be conjectured that first best efficiency can be restored by a quadratic subsidy $\kappa x_i^2/2$ that "compensates" for the distortion

[38]When parameters are such that $c = 0$, the SFE and (Bayesian) Cournot allocations coincide and, therefore, they are both equally efficient.

$dx_i^2/2$ and induces sellers to act competitively.[39] The question is whether we can find a κ (with $\kappa > 0$ for a subsidy) such that $\lambda - \kappa + d(\kappa) = \lambda$, where $d(\kappa) \equiv (\beta^{-1} + (n-1)c(\lambda - \kappa))^{-1}$ is the (endogenous) distortion when the slope of marginal cost is $\lambda - \kappa$. That is, whether we can find a solution to the fixed-point equation $d(\kappa) = \kappa$. If we can find such a κ, a seller would act effectively as if he were competitive and facing a marginal cost with slope λ, and the FOC would be

$$p - E[\theta_i|s] - (d(\kappa) + \lambda - \kappa)x_i = p - E[\theta_i|s] - \lambda x_i = 0.$$

The following proposition states the results.

PROPOSITION 5: *Let* $-(n-1)^{-1} < \rho < 1$ *and* $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$. *Then the following situations exist*:

(i) *There is a unique price-taking SFE and the equilibrium implements the efficient allocation. The slope of supply is given by* $c^{\mathrm{PT}} = (\lambda^{-1} - (\beta n)^{-1}M)/(M + 1)$, *which is decreasing with* M *and* λ.

(ii) *There is an optimal quadratic subsidy* $\kappa^* x_i^2/2$, $\kappa^* = (\beta^{-1} + (n-1) \times c^{\mathrm{PT}}(\lambda))^{-1}$, *which implements price-taking behavior. Implementation need not be unique if adverse selection is severe*: $n - 2 < M$. *The optimal subsidy* κ^* *increases with* λ *and* ρ, *and it increases* (*decreases*) *with* σ_ε^2 *when* $\rho > 0$ ($\rho < 0$).

REMARK 4: *In contrast to result* (i) *at a price-taking Bayesian quantity-setting equilibrium*, *there is typically a welfare loss because of lack of information aggregation*.[40]

REMARK 5: *We have that* $c^{\mathrm{PT}} > 0$ *for* M *small* (*and negative a fortiori*) *and* $c^{\mathrm{PT}} < 0$ *for* M *large*. *The price-taking supply function coincides with the marginal cost schedule only when there is no learning from prices* (*that is*, *when* $M = 0$, *in which case both schedules boil down to* $p = E[\theta_i|s_i] + \lambda x_i$).[41] *The supply function of a seller in the price-taking equilibrium is always flatter than the supply function in the strategic equilibrium since* $d > 0$ *and* $c^{\mathrm{PT}} - c = (\lambda^{-1} - (d + \lambda)^{-1})(M + 1)^{-1} > 0$.[42]

[39] Angeletos and Pavan (2009) provided a thorough analysis of tax-subsidy schemes in quadratic continuum economies with private information and with agents using noncontingent strategies (e.g., of the Cournot type). In their model, however, there is no learning from endogenous public signals and taxes are contingent on aggregate realizations.

[40] See Section S.2 in the Supplemental Material.

[41] As in the strategic case, the supply function of a seller converges to the per capita seller demand function as $\rho \to 1$, and $c^{\mathrm{PT}} \to \infty$ as $\rho \to -(n-1)^{-1}$.

[42] Sellers respond more cautiously to their private signals when they are strategic since they take into account the price impact coming from the amount sold as well as the potential informational leakage from their actions: $a^{\mathrm{PT}} - a > 0$. By the same token, given that $d > d^f > 0$ we have that $c^{\mathrm{PT}} > c^f > c$ and $a^{\mathrm{PT}} > a^f > a$.

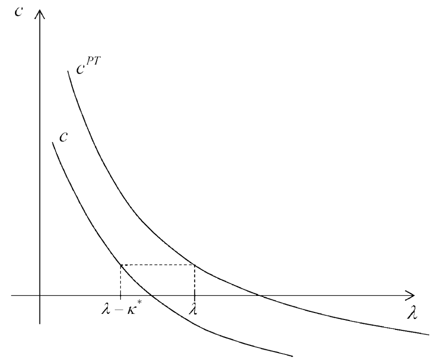

FIGURE 3A.—The equilibrium parameters c^{PT} and c as a function of λ, and the optimal subsidy κ^* when $n - M - 2 \geq 0$ and $M > 0$.

To understand result (ii), note that we have to find a κ such that $c(\lambda - \kappa) = c^{\text{PT}}(\lambda)$, and this will yield $d(\kappa) \equiv (\beta^{-1} + (n-1)c^{\text{PT}}(\lambda))^{-1}$. When $n - 2 \geq M$ and the adverse selection problem is moderate, we are in the situation depicted in Figure 3a and we can find a subsidy $\lambda - \kappa > 0$ to implement price-taking behavior. However, when $n - 2 < M$, we are in the situation depicted in Figure 3b and we may need to induce effective increasing returns: $\lambda - \kappa < 0$. However, for negative slopes of marginal costs, there are two linear equilibria (with slopes of supply c_2 and c_1; see Lemma A.1 in the Appendix). This means that we can always find an optimal quadratic subsidy, but the implementation of the efficient allocation is unique only when there is no need to induce nega-

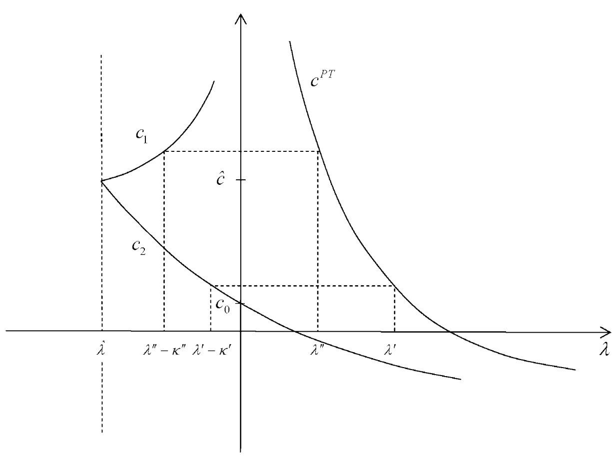

FIGURE 3B.—The equilibrium parameters c^{PT} and c as a function of λ, and the optimal subsidy κ when $n - M - 2 < 0$ (the case depicted is with $n - M > 0$). When $\lambda = \lambda'$, the optimal subsidy is κ' with $c_2(\lambda - \kappa') = c^{\text{PT}}(\lambda)$, and when $\lambda = \lambda''$, the optimal subsidy is κ'' with $c_1(\lambda - \kappa'') = c^{\text{PT}}(\lambda)$.

tive slopes of effective marginal costs. When $n-2<M$ and $c^{\mathrm{PT}}(\lambda)>c_0$, there is one equilibrium that implements price-taking behavior with $\lambda-\kappa^*<0$. For example, for λ such that $c^{\mathrm{PT}}>\hat{c}$ (λ'' in Figure 3b), we need to choose the higher c_1 equilibrium, while for lower λ with $c_0<c^{\mathrm{PT}}<\hat{c}$ (λ' in Figure 3b), we need to choose the lower c_2 equilibrium. It is worth noting that, indeed, when $\rho\to-(n-1)^{-1}$ competitive behavior is already approached in the market with no subsidy and, therefore, $\kappa^*\to 0$.

4. CONVERGENCE TO PRICE-TAKING BEHAVIOR IN LARGE MARKETS

To study whether (and if so, how fast) the inefficiency of supply function equilibria disappears in large markets, we consider replica markets where the numbers of sellers and buyers grow at the same rate n. More precisely, suppose that there are n buyers, each with quasilinear preferences and benefit function $u(x)=\alpha x-\beta x^2/2$ where x is the consumption level. This gives rise to the inverse demand $P_n(y)=\alpha-\beta y/n$, where y is total consumption. There are n sellers as before. Total surplus is therefore given by $\mathrm{TS}=nu(y/n)-\sum_i C(x_i,\theta_i)$ and per capita surplus is given by TS/n. We restrict attention in this section to the case of nonnegative correlation $\rho\in[0,1)$.

We denote with subscript n the magnitudes in the n-replica market. The results we have obtained so far, except possibly comparative statics with respect to n, hold when β is replaced by β/n. The following proposition characterizes the convergence of the SFE to a price-taking equilibrium as the market grows. (See Section S.1 in the Supplemental Material for the definitions of orders of sequences, further results, proofs, and comments.) As we have seen before, the price-taking equilibrium is first best efficient since it aggregates information. We confirm that the efficient outcome is approached as the market becomes large.

PROPOSITION 6: *Let $\rho\in[0,1)$. In the replica market, the following conditions hold*:

(i) *As the market grows large, the market price p_n at the SFE converges in mean square to the price-taking Bayesian price p_n^{PT} at the rate of $1/n$.*

(ii) *The deadweight loss at the SFE $(\mathrm{ETS}_n^o-\mathrm{ETS}_n)/n$ is of the order of $1/n^2$.*

A large market approaches efficiency in prices at a rate $1/n$, which is the same as the usual rate under complete information. This is a statement that bid shading is on the order of $1/n$. It follows from the fact that the distortion $d_n=(n\beta^{-1}+(n-1)c_n)^{-1}$ is of order $1/n$, and both equilibria aggregate information since p_n and p_n^{PT} reveal the average signal $\tilde{s}_n$. Simulations suggest that both d_n and $E[\mathrm{DWL}_n]/n$ are monotonically decreasing in n.[43]

[43]See Section S.3.2 in the Supplemental Material for further results and details of the simulations. The optimal subsidy $\kappa_n^*\equiv(\beta^{-1}n+(n-1)c_n^{\mathrm{PT}}(\lambda))^{-1}$ can also be checked to be decreasing in n.

Cripps and Swinkels (2006) obtained a parallel result in a double auction environment. They considered a generalized private value setting where bidders can be asymmetric and can demand or supply multiple units. Under some regularity conditions (and a weak requirement of "a little independence," where each player's valuation has a small idiosyncratic component), they found that as the number of players grows (say that there are n buyers and n sellers), all nontrivial equilibria of the double auction converge to the competitive outcome and inefficiency vanishes at the rate of $1/n^{2-\chi}$ for any $\chi > 0$.

As in Kyle (1989), we could ask what happens when the total amount of precision available to agents is fixed as the market grows large. Then as n tends to infinity, $\tau_\varepsilon \to 0$. In this case it is easy to see that $M \to \infty$ as $n \to \infty$ and, as when $\rho \to 1$, the supply function of a seller converges to the per capita demand, $d \to \beta n$, and the equilibrium collapses. In the limit, we approach the collusive price (as in Proposition 2) and, therefore, an extreme form of the noncompetitive limit of Kyle.

REMARK 6: *For a given $\rho > 0$ and for large enough n, we always have that the supply function equilibrium attains a higher surplus than the* (*Bayesian*) *Cournot equilibrium. This is so since as n grows, the SFE—but not the Cournot equilibrium—converges to the full information first best. In a large enough market, the Cournot model always overestimates the welfare loss since a deadweight loss remains due to private information when $\rho > 0$* (*while convergence to price-taking behavior obtains as in the supply function market*).[44]

5. EXTENSIONS

In this section we test the robustness of the results in the context of inelastic demand, demand uncertainty, public signals, and demand schedule competition.

5.1. *Inelastic Demand*

The case where an auctioneer demands q units of the good is easily accommodated by letting $\beta \to \infty$ and $\alpha/\beta \to q$. Then from the inverse demand, we obtain $y = (\alpha - p)/\beta \to q$. Let $\rho > -(n-1)^{-1}$ and $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$. It can be checked that there is a unique SFE if and only if $n - 2 - M > 0$.[45] In equilibrium we have that $c = \frac{n-2-M}{\lambda(n-1)(1+M)} > 0$ and $d = ((n-1)c)^{-1}$ is decreasing with n.

[44]Proposition 6 holds for the (Bayesian) Cournot equilibrium by replacing ETS_n^o with the expected total surplus at the price-taking (Bayesian) Cournot equilibrium. See Proposition S.2 and Figure S.7 in the Supplemental Material.

[45]The analysis is analogous to the proof of Proposition 1. Now the second order conditions (SOC) hold if $c > 0$. If $q = 0$, we are in a double auction case and there is also a no-trade equilibrium. See Vives (2010) for a presentation of the model with demand bidders facing an inelastic supply.

A necessary condition for the existence of the SFE if $\rho\sigma_\varepsilon^2 \geq 0$ is that $n \geq 3$. If $\rho\sigma_\varepsilon^2 < 0$, then $M < 0$ and there is an equilibrium for $n \geq 2$. As M increases, c decreases, and as $n-2-M \to 0$, $c \to 0$ and the SFE collapses. This is because of the combination of adverse selection and market power when demand is inelastic: The supply schedules become too inelastic to sustain a linear equilibrium as $n-2-M \to 0$. The market breaks down when traders submit vertical schedules since with vertical residual demand curves, traders sometimes like to force unbounded prices. This is similar to the double auction context of Kyle (1989) in which a linear equilibrium exists only if the number of informed traders is greater than or equal to 3 (when there are no uninformed traders). In our more general model with strategic agents facing an elastic demand function from passive buyers, the market does not break down since there is always price elasticity from demand. With inelastic demand, supply functions are always upward sloping ($c > 0$). This is as in the auction models of Kyle (1989) or Wang and Zender (2002), where *demand* schedules always have the "right" slope.

The results obtained specialize to the inelastic demand case. We highlight here the differences. With regard to Proposition 2, we have that in the range of existence $n-2-M > 0$, in part (ii), the price is always between the competitive and the Cournot price (since $c > 0$); in part (iii), expected bid shading increases with $q/n(p - E_n[\mathrm{MC}_n] = dq/n$ and d is decreasing with n); and in part (iv), var$[p]$ increases with ρ and σ_θ^2, and decreases with σ_ε^2 (since $\tilde{x} = q/n$ and, therefore, var$[p]$ = var$[E[\tilde{\theta}|\tilde{s}]]$). With regard to Propositions 3 and 4, the results for distributive inefficiency apply (there is no inefficiency in the aggregate quantity). With regard to Proposition 5, in part (i), we have that $c^{\mathrm{PT}} = (\lambda(M+1))^{-1}$ and $c \to \infty$ as $\lambda \to 0$, and in part (ii), for any $\lambda > 0$, there is always a $\kappa \in (0, \lambda)$ such that $c(\lambda - \kappa) = c^{\mathrm{PT}}(\lambda)$ with effective marginal costs with positive slope. This yields $\kappa^* = ((n-1)c^{\mathrm{PT}}(\lambda))^{-1} = \lambda(1+M)/(n-1)$. The subsidy κ^* increases with M and λ, and decreases with n.

5.2. *Demand Uncertainty*

Demand uncertainty can be incorporated easily in the model as long as it follows a Gaussian distribution and enters in an additive way: $P(y) = \alpha + u - \beta y$ with $u \sim N(0, \sigma_u^2)$ independent of the other random variables. The analysis of the equilibrium proceeds as in Section 2.1 except now the intercept of residual demand I_i is informationally equivalent to $h_i \equiv u + \beta a \sum_{j \neq i} s_j$. The following proposition characterizes the equilibrium (see Section S.5 in the Supplemental Material for a development, complete statement of results, and proofs).[46]

[46]Vives (2011b) considered the common value case with demand uncertainty in a limit large market and performed a welfare analysis.

PROPOSITION 7: *Let* $\lambda > 0$. *For any* $\rho \in [-(n-1)^{-1}, 1]$, $\sigma_u^2 > 0$, *and* $\sigma_\varepsilon^2 \geq 0$, *there exists a SFE. It is given by* $X(s_i, p) = (p - E[\theta_i|s_i, p])/(d + \lambda)$, *where* $0 < d < \beta n$. *As* $\sigma_u^2 \to 0$, *d tends to the value of d in Proposition* 1 (*where* $\sigma_u^2 = 0$), *and as* $\sigma_u^2 \to \infty$, *then* $d \to d^f$. *The equilibrium is unique if* $\rho \geq 0$ *or* $\rho < 0$ *and* $n > 3$.[47] *Then if* $\sigma_\varepsilon^2 > 0$, *d increases in* ρ, *and if* $\rho\sigma_\varepsilon^2 > 0$ (*resp.* $\rho\sigma_\varepsilon^2 < 0$), *d is decreasing* (*resp. increasing*) *with* σ_u^2.

REMARK 7: *When* $\lambda = 0$, *equilibrium exists if and only if* $n - 2 - M < 0$ (*that is*, *we need* $\rho > 0$ *and large enough*).

The properties of the equilibrium follow.

THE EQUILIBRIUM IS NOISY: It is immediate that the price is now informationally equivalent to $u + \beta n a \tilde{s}$, where, as before, $a \equiv -\partial X/\partial s_i$ and sellers learn only imperfectly from the price about the average signal $\tilde{s}$. Now we have a noisy linear equilibrium instead of a privately revealing one since on top of their private signal sellers have a noisy estimate of the average signal. The equilibrium exists even when $\rho = 1$ or $\rho = -(n-1)^{-1}$ since with uncertain demand, even for extreme values of ρ, there is aggregate uncertainty. The consequence is that the collusive and competitive cases are not attained when $\sigma_u^2 > 0$ when $\rho = 1$ or $\rho = -(n-1)^{-1}$ (respectively).

CONDITIONS FOR THE NOISE INDEPENDENCE PROPERTY—Equilibrium Independent of σ_u^2: (i) When $\rho\sigma_\varepsilon^2 = 0$ ($M = 0$), then the equilibrium does not depend on σ_u^2 and $d = d^f$ (as in Proposition 1 when $\rho\sigma_\varepsilon^2 = 0$). When $\sigma_\varepsilon^2 \to \infty$ and $\sigma_u^2 > 0$, then again the equilibrium is independent of σ_u^2 and $d \to d^f$, yielding $X(p) = c^f(p - \bar{\theta})$. This limit is also the equilibrium when $\sigma_\varepsilon^2 = \infty$. This is in contrast with the case $\sigma_u^2 = 0$, where, as we saw in footnote 26 (Claim A.2 in the Appendix), there is a discontinuity in the limit when $\sigma_\varepsilon^2 \to \infty$. In these cases, there is no relevant asymmetric information, no learning form prices, and the equilibrium is independent of the distribution of demand uncertainty (as in Klemperer and Meyer (1989)).[48]

COMPARATIVE STATICS: We have that d decreases (and c increases) in σ_u^2 when $\rho\sigma_\varepsilon^2 > 0$ because as there is more noise in the demand, the information roles of the price and the adverse selection problem are diminished. The supply function is flatter with higher σ_u^2 (a high price need not be such bad new

[47]Simulations suggest that the equilibrium is unique also when $\rho < 0$ for $n = 2, 3$.

[48]When $\rho = 1$ and $\sigma_\varepsilon^2 = 0$, we are in fact exactly in the Klemperer and Meyer (1989) case with linear and uncertain demand, symmetric quadratic costs, and no private information (and we obtain the same equilibrium with supply slope $c = c^f$).

about costs since it may come from a high demand realization).[49] This means that as σ_u^2 increases, average output increases and the expected margin over marginal cost decreases. The opposite happens when $\rho\sigma_\varepsilon^2 < 0$ since then there is favorable selection and an enhanced information role of the price is pro-competitive. As $\sigma_u^2 \to \infty$, we obtain that $d \to d^f$ since then sellers do not learn from the price and rely only on their private signals. As $\sigma_u^2 \to 0$, d converges to the value in the privately revealing equilibrium of Proposition 1.

As ρ increases and cost parameters become more correlated, d increases since the weight on the information component of the price h_i in $E[\theta_i|s_i, h_i]$ increases with ρ (see Claim S.1 in the Supplemental Material). We have that $d > d^f$ when $\rho\sigma_\varepsilon^2 > 0$ and $d < d^f$ when $\rho\sigma_\varepsilon^2 < 0$. As $\rho \to 1$, d attains its largest value $\dddot{d} < \beta n$ for given σ_ε^2, σ_θ^2, and σ_u^2, and the price falls short of the collusive level.[50] As $\rho \to -(n-1)^{-1}$, d attains its smallest value $\underset{\dots}{d} > 0$. As $\sigma_u^2 \to 0$, $\dddot{d} \to \beta n$, and $\underset{\dots}{d} \to 0$.

It is easy to see that d is nonmonotone in σ_ε^2 when $\rho > 0$.[51] For σ_ε^2 small, increases in σ_ε^2 increase the value of the price signal $u + \beta n a\tilde{s}$ (the weight on the information component of the price h_i in $E[\theta_i|s_i, h_i]$) while for σ_ε^2 large, they diminish it (since the price is not very informative as $a \to 0$ when $\sigma_\varepsilon^2 \to \infty$). This does not matter when $\sigma_u^2 = 0$, since then the price is not noisy and recovers $\tilde{s}$ as long as $a > 0$. The result is that d increases with σ_ε^2 for σ_ε^2 small and decreases with σ_ε^2 for σ_ε^2 large.

5.3. *Public Signal*

Suppose that sellers receive a public signal on $\tilde{\theta}$, $r = \tilde{\theta} + \delta$, where $\delta \sim N(0, \sigma_\delta^2)$ and $\operatorname{cov}(\tilde{\theta}, \delta) = 0$ (and δ is also independent of the rest of the random variables in the model). Then linear strategies will be of the form $X(s_i, r, p) = b - as_i - er + cp$. A similar analysis as in Section 2.1, with the information set of seller i now being $\{s_i, r, p\}$ or, equivalently, $\{s_i, r, h_i\}$ where $h_i \equiv \beta a \sum_{j\neq i} s_j$, leads to the following proposition (see Section S.6 in the Supplemental Material for a proof).

PROPOSITION 8: *Let* $-(n-1)^{-1} < \rho < 1, \sigma_\varepsilon^2/\sigma_\theta^2 < \infty$, *and* $\sigma_\delta^2 > 0$. *Then there is a unique SFE. It is given by the supply function* $X(s_i, p) = (p -$

[49] Bernhardt and Taub (2010) obtained the opposite result in a model with private information about the demand for a homogeneous product in which price is observed with noise. Then as the variance of the noise increases, the information role of the price signal is diminished, a high price signal is less likely to mean a high price, and agents rely less on the price.

[50] Note that with demand uncertainty a collusive seller recovers the value of u from the price by using a supply function. The Lerner condition in the collusive case is exactly as in Section 2.4.

[51] Indeed, for $\rho > 0$, d attains its minimum value $d = d^f$ both when $\sigma_\varepsilon^2 = 0$ and when $\sigma_\varepsilon^2 \to \infty$. Furthermore, d increases in σ_ε^2 when $\sigma_\varepsilon^2 \le \sigma_\theta^2$ and eventually it decreases as σ_ε^2 grows (see Proposition S.5(iii) in the Supplemental Material).

$E[\theta_i|s_i, r, p])/(d+\lambda)$, *where the parameters* d *and* a *are characterized as in Proposition* 1, *and* c *is the largest solution to the quadratic equation* $g(c; Q) = 0$, *where*

$$Q = \frac{\sigma_\varepsilon^2(n^2\sigma_\delta^2\rho - \sigma_\theta^2(1-\rho)(1+(n-1)\rho))}{(1-\rho)(n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))}.$$

We have that $1+Q>0$. *As* $\sigma_\delta^2 \to \infty$, *then* $Q \to M$, *and as* $\sigma_\delta^2 \to 0$, $Q \to -1$, *then* $c \to +\infty$ *and* $d \to 0$. *If* $\sigma_\varepsilon^2 > 0$, *then* Q, *and therefore* d, *increases with* σ_δ^2 *and* ρ, *and* $\operatorname{sgn}\{\partial Q/\partial\sigma_\varepsilon^2\} = \operatorname{sgn}\{Q\}$.

The equilibrium is privately revealing as in our base case, with the price measurable in the public signal r, and in $\tilde{s}$, $p = (1+\beta nc)^{-1}(\alpha - \beta nb + \beta n(er + a\tilde{s}))$ and $\tilde{x} = (\alpha - E[\tilde{\theta}|\tilde{s}, r])/(\beta n + \lambda + d)$. Now the public signal provides information on aggregate uncertainty $\tilde{\theta}$ beyond the average signal $\tilde{s} = \tilde{\theta} + \tilde{\varepsilon}$. For σ_δ^2 low enough, we have $Q < 0$ even if $\rho > 0$ and d will be reduced below the full information level d^f when sellers know $\tilde{s}$ (but not the public signal). The public signal provides additional information to the collective information of sellers.[52] When $Q < 0$, a high price is good news for the costs of a seller (the weight on the price in $E[\theta_i|s_i, r, p]$ is negative).[53] Note that $Q < 0$ when $\rho\sigma_\delta^2$ is positive and small or negative. When $\rho < 0$, there is favorable selection as in the case where there is no public signal.

The index of adverse selection is now Q instead of M. It follows that the effects of changes in ρ on c and d are the same as in the case $\sigma_\delta^2 = \infty$ since Q is increasing in ρ.[54] When $\rho \to 1$, the distortion $d \to \beta n$ (since $Q \to \infty$) and we approach collusive pricing (as in the case when there is no public signal). Similarly as in the base case, and for the same reasons, we have that as $\rho \to 1$, the linear equilibrium collapses. When $\rho \to -(n-1)^{-1}$, we also approach the competitive outcome (since then $Q \to -1$). Increases in σ_ε^2 decrease c if $\rho\sigma_\delta^2$ is large, but if $\rho\sigma_\delta^2$ is small or negative, then c increases with σ_ε^2. This is so since $\operatorname{sgn}\{\partial Q/\partial\sigma_\varepsilon^2\} = \operatorname{sgn}\{Q\}$, and when $\rho\sigma_\delta^2$ is small (but positive) or negative,

[52]If the public signal were to be just a noisy version of $\tilde{s}$, then as noise vanishes we would recover the full information distortion d^f.

[53]The coefficient of h_i in $E[\theta_i|s_i, r, h_i]$ equals $(d+\lambda)Q/\beta n$ (see Claim S.3 in the Supplemental Material). To fix the intuition, consider the case $\rho = 0$. Then for seller i the public signal r provides a noisy signal about θ_i: $rn = \theta_i + (\sum_{j\neq i}\theta_j + n\delta)$. The price provides a noisy signal about $\sum_{j\neq i}\theta_j$ and, therefore, helps reading the public signal r. When the seller sees a high price, then it infers that $\sum_{j\neq i}\theta_j$ is high and, therefore, θ_i is low, the more so the less noisy is the public signal (σ_δ^2 low). A similar intuition applies when $\rho\sigma_\delta^2 \geq 0$ is small, and this explains why then $Q < 0$ and a high price is good news. As ρ increases, eventually a high price becomes bad news.

[54]The same applies in relation to σ_θ^2 for ρ not too negative since then Q is decreasing in $\sigma_\theta^2(\operatorname{sgn}\{\partial Q/\partial\sigma_\theta^2\} = -\operatorname{sgn}\{n^2\sigma_\delta^2\rho + \sigma_\varepsilon^2(1+(n-1)\rho)\}$; see the proof of Proposition 8).

$Q < 0$ and $d < d^f$. Then increases in σ_ε^2 reduce Q and increase c since the information role of the price is enhanced and a high price is good news.[55]

As the precision of the public signal improves (σ_δ^2 decreases), the adverse selection problem diminishes and the slope of equilibrium strategies as well as the distortion are lowered. In fact, when $\sigma_\delta^2 \to 0$, the distortion is eliminated, $d \to 0$ (since $Q \to -1$). Note that in the limit as $\sigma_\delta^2 \to 0$, there is no equilibrium (indeed, $c \to +\infty$).[56]

5.4. *Demand Schedule Competition*

The model can be restated in terms of competition among buyers of an asset of unknown ex post average value $\tilde{\theta} \equiv (\sum_{i=1}^n \theta_i)/n$ and with value θ_i for buyer i. With a change of variables $z_i \equiv -x_i$, we have the results for demand competition. The (inverse) supply of the asset is given by $p = \alpha + \beta \sum_i z_i$ (with $\bar{\theta} > \alpha > 0$, $\beta > 0$), where $\sum_i z_i$ is the total quantity demanded. The equilibrium demand is $Z(s_i, p) = (E[\theta_i|s_i, p] - p)/(d + \lambda) = -b + as_i - cp$ (with the endogenous parameters as in Proposition 1). The marginal benefit of buying z_i units of the asset for buyer i is $\theta_i - \lambda z_i$, where θ_i is the value with a private component and λz_i is a transaction cost, opportunity cost, or risk aversion component.[57] The profits of buyer i are given by $\pi_i = (\theta_i - p)z_i - \lambda z_i^2/2$. A real market example is firms that purchase labor of unknown average productivity $\tilde{\theta}$ because of technological uncertainty, and face an inverse linear labor supply and quadratic adjustment costs in the labor stock.

The extensions to the supply competition model also apply here. Of particular interest is the case of supply uncertainty since it corresponds to the *noise trader model* when $\rho = 1$. Suppose that noise traders have a price-elastic demand (negative supply) $\beta^{-1}(\alpha + u - p)$. Then market clearing implies that $\beta^{-1}(\alpha + u - p) + \sum_i z_i = 0$ and, therefore, $p = \alpha + u + \beta \sum_i z_i$. It follows from Proposition 7 that increasing noise trading (σ_u^2) increases c, decreases d, and decreases the expected margin (of expected marginal benefit over price) while c and d are nonmonotone in σ_ε^2. When supply is inelastic and random according to u and $\lambda = 0$, we recover the Kyle (1989) model with $n > 2$ risk-neutral

[55]Indeed, when $\sigma_\varepsilon^2 > 0$, the absolute value of the weight on h_i in $E[\theta_i|s_i, r, h_i]$ increases in $|\rho|$ and σ_ε^2 (see Claim S.2 in Section S.6 of the Supplemental Material).

[56]When $\sigma_\delta^2 = 0$, $E[\tilde{\theta}|\tilde{s}, r] = r = \tilde{\theta}$ (since $\tilde{s} = \tilde{\theta} + \tilde{\varepsilon}$) and the candidate equilibrium price does not depend on the average signal. This implies that at a symmetric equilibrium sellers do not put any weight on their private signals (i.e., $a = 0$), but this is inconsistent since when $\sigma_\delta^2 = 0$, p reveals $\tilde{s}$ at a candidate linear equilibrium, and, therefore, $E[\theta_i|s_i, r, p] = \tilde{\theta} + \frac{\sigma_\theta^2(1-\rho)}{\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2}(s_i - \tilde{s})$ and a seller will put some weight on the difference between his private signal and the average so as to estimate θ_i (which is imperfect correlated with $\tilde{\theta}$).

[57]Note that the adjustment cost is exogenous while with constant absolute risk aversion (CARA) preferences, for example, it would be endogenous and would depend, in expectation, on the degree of risk aversion times the variance of θ conditional on the information of the trader.

informed investors. Considering noise of the form βu, we have that demand $\beta^{-1}(\alpha+\beta u-p)\to u$ as $\beta\to\infty$ and market clearing is given by $u+\sum_i z_i=0$. It follows then that in equilibrium we have an inverse of market depth (the Kyle lambda) $\partial p/\partial u=(nc)^{-1}$, exactly as in Kyle (1989), decreasing in σ_u^2, increasing in σ_θ^2, and nonmonotone in σ_ε^2.[58]

6. APPLICATIONS

In this section we provide several applications of the model: electricity markets, strategic trade policy, pollution damages, revenue management, and financial markets. We consider supply schedule competition examples first, followed by demand schedule competition cases, and we look at fit to our model, links to the results, and empirical evidence.

6.1. *Supply Schedule Competition*

Wholesale Electricity Markets

The day-ahead or spot market, which has separate auctions for each delivery period (half-hourly or hourly), and the balancing market, which secures that demand and supply match at each point in time, fit our model since they are typically organized as uniform price multiunit auctions. Supplies are discrete in the spot market but smooth in our model.[59] The continuous (linear in particular) supply approach has been widely used and empirically implemented in electricity markets.[60] In our base model, the random residual demand a firms faces is due to cost uncertainty. Demand uncertainty is a relevant factor in the wholesale market and then our extension (Section 5.2) applies.[61] Private cost information related to plant availability will be relevant when there is a day-ahead market organized as a pool where firms submit hourly or daily supply

[58]See Claim S.2 in Section S.5 of the Supplemental Material and Exercise 5.1 in Vives (2008) for the Kyle model with risk neutral investors.

[59]Holmberg, Newbery, and Ralph (2008) showed that if prices are selected from a discrete grid, where (realistically) the number of price levels is small in comparison to the number of quantity levels, then the step functions converge to continuous supply functions as the number of steps increases. This justifies the approximation of step functions with smooth supply functions. The modeling of the auction with discrete supplies leads to existence problems of equilibrium in pure strategies (see von der Fehr and Harbord (1993)).

[60]See Green and Newbery (1992), and Green (1996, 1999). For the Texas balancing market (ERCOT), see Niu, Baldick, and Zhu (2005), Hortaçsu and Puller (2008), and Sioshansi and Oren (2007).

[61]In a wholesale electricity market, the demand intercept α is a continuous function of time (load–duration characteristic) that yields the variation of demand over the time horizon considered. At any time, there is a fixed α and the market clears. In the British pool up to 2001—the first liberalized wholesale market—generators had to submit a single supply schedule for the entire day. Over this period, residual demand facing a firm may vary considerably due to demand uncertainty and plant outings.

schedules. The residual demand faced by a firm will be random (even with predictable demand) since the supply of other firms depends on plant availability, which is random. The firm may have privileged information because of technical issues, transport problems, hydro availability in the reservoirs, and the terms of supply contracts for energy inputs or imports.[62] Furthermore, in an emission rights system, future rights allocations may depend on current emissions and firms may have different private estimates of such allocation. This affects the opportunity cost of using current emission rights.

The empirical evidence points to firms bidding over marginal costs.[63] The Cournot framework has been used often but tends to predict prices that are too high given realistic estimates of the demand elasticity. Our model helps us to understand the biases introduced by taking the Cournot modeling shortcut when firms compete in supply functions (see Section 3.2). There is also evidence of information aggregation: Mansur and White (2009) showed how a centralized auction market in the eastern United States yields very important information aggregation benefits over bilateral trading to achieve an efficient allocation in a situation where differences in marginal costs and production are private information among firms. In our model, prices are revealing of average cost conditions (Proposition 1), but strategic behavior on the basis of private information prevents the achievement of an efficient allocation (Propositions 3 and 4).

We also have seen (Proposition 2) how increasing the noise in the private signal σ_ε^2 makes the slope of supply steeper (when $\rho > 0$). This result may help to explain the fact that in the Texas balancing market, small firms use steeper supply functions than predicted by theory and that such departures explain the major portion of losses in productive efficiency (Hortaçsu and Puller (2008)).[64] Indeed, smaller firms may have signals of worse quality because of economies of scale in information gathering, where residual noncontract private cost information has not been taken into account in the estimation. Consistent with our analysis, the welfare losses due to the "excess steepness" of supply functions over and above standard market power may be more important than the losses due to the latter. Finally, it is worth noting that the usual restriction

[62] The latter include constraints in take-or-pay contracts for gas, where the marginal cost of gas is zero until the constraint—typically private information to the firm—binds, or price of transmission rights in electricity imports depending on the private arrangements for the use of a congested interconnector.

[63] See, for example, Borenstein and Bushnell (1999), Borenstein, Bushnell, and Wolak (2002), Green and Newbery (1992), and Wolfram (1998).

[64] The authors explained the finding by the complexity faced by small firms in setting up the bidding (and argued that to take a linear approximation to marginal costs in the Texas electricity market is reasonable).

to upward sloping schedules in electricity markets caps the market power of sellers in the spot market.[65]

Other Interpretations of the Cost Shock

The cost shock θ_i could be related to linear ex post *pollution* or emission damage which is assessed on the firm and for which the producer has some private information. The regulator can introduce a quadratic subsidy to production to eliminate the distortion originating in private information (Proposition 5) and can alleviate the distortion by disclosing the available information on the average damage $\tilde{\theta}$ (Proposition 8).

The cost shock can also be interpreted as a random opportunity cost of serving the market which is related to dynamic considerations (e.g., *revenue management* on the face of products with expiration dates and costly capacity changes).[66] The value of a unit in a shortage situation is the opportunity cost of a sale. A high opportunity cost is an indication of high value of sales in the future. In this case, a firm would have a private assessment of the opportunity cost with which it would form its supply schedule. For example, if supply function competition provides a suitable reduced form for pricing for airline travel,[67] then taking into account the information aggregation role of price may help explain pricing patterns, which have proved difficult to explain with extant theoretical models (see, e.g., McAfee and te Velde (2006)). For example, when airlines see prices going up, they may infer, correctly, that the opportunity cost is high (i.e., that expected next period demand is high) and they reduce supply in the present period to be able to supply next period at a higher profit.

The cost shock could also be a (negative) linear *subsidy* in a *strategic trade policy* game where governments manipulate the supply function of domestic firms with tariffs and subsidies.[68] Laussel (1992) considered a market with linear demand and constant marginal costs, where firms compete in a common foreign market with the help of the domestic government imposing a quadratic

[65]The rules typically require producers to submit nondecreasing (step) function offers (although in some markets, like the Amsterdam Power Exchange, retailers may submit nonincreasing demands).

[66]Such situations are where the product—be it a hotel room, airline flight, generated electricity, or tickets for a concert—has an expiration date, and capacity is fixed well in advance and can be added only at high marginal cost.

[67]Talluri and Van Ryzin (2004, p. 523) stated: "A typical booking process proceeds as follows. An airline posts availability in each fare class to the reservation systems, stating the availability of seats in each fare class." This is indeed like a supply function.

[68]More generally, strategic agency models where an owner provides incentives to the manager to compete in the marketplace typically have a reduced form that is a supply function. This is similar to the presence of adjustment costs in certain industries that commit the firms to supply functions (e.g., internal incentives in management consulting). See Vickers (1985), Fershtman and Judd (1987), and Faulí-Oller and Giralt (1995).

export tax and a linear subsidy.[69] If the amount of subsidy is uncertain and the domestic firm receives a private noisy signal about it, we can conclude (Proposition 2 when $\rho > 0$) that increasing noise in the signals softens competition. It follows that the disclosure policy of the government toward national firms can also affect competitiveness.

6.2. *Demand Schedule Competition*

6.2.1. *Legacy Loans Auctions*

Legacy loans auctions were envisioned in the U.S. Public–Private Investment Program (PPIP; March 2009) to remove bad loans from the balance sheet of banks. Basically, the banks nominate pools of legacy loans that meet certain criteria and that they wish to sell. Approved private investors bid for the pools of loans and receive a nonrecourse loan that is collateralized by the same securities to be acquired. The winning bid for each pool is then either accepted or rejected by the bank. In terms of our model, the marginal valuation of a bidder depends on the collateral that it can post. Using the same securities as collateral for the nonrecourse loan to finance the purchase is equivalent to providing a subsidy to bidders that decreases the slope of their marginal valuation. Our model then rationalizes the subsidy scheme of the Treasury since it reduces the discount of the auction price (Proposition 5).[70]

6.2.2. *Liquidity Auctions*

In an open-market central bank operation, the (often inelastic) supply of funds is met by banks' demand bids. The marginal unit value θ_i of funds for bank i is idiosyncratic, with a common component $\tilde{\theta}$ related to the interest rate/price in the secondary interbank market, and is assessed imperfectly by the bank (for example, due to uncertainty about future liquidity needs). Banks' marginal valuations are positively correlated, declining with λ, which reflects the structure of their pool of collateral (see, e.g., Ewerhart, Cassola, and Valla (2010)). A bidder bank prefers to post illiquid collateral in exchange for funds and with an increased allotment, the bidder must offer more liquid types of collateral, which have a higher opportunity cost.

[69]The quadratic tax steepens the slope of the effective marginal cost schedule of a firm and softens competition (this determines the λ in our model), and the subsidy allows the domestic firm to capture a larger share of the profits. Grant and Quiggin (1997) studied the case in which firms are competitive. Whenever supply functions are linear, the authors find an equilibrium in tax-subsidy schedules with quadratic trade revenue taxes.

[70]The supply function model can also account for Paulson's reverse auction plan to extract toxic assets from the banks: it serves a price discovery purpose, but the Treasury would be subject to overpricing. Any information the Treasury has on average valuations should be released (Proposition 8). (See Section 5 in Vives (2010).) Ausubel and Cramton (2009) proposed combining the forward auction with a reverse auction as in the Paulson plan. See also Klemperer (2010).

The model illustrates the impact in the auction discount and in the efficiency of liquidity distribution of changes in key parameters such as those happening in a crisis situation. The more severe the information problem (a larger ρ or $\sigma_\varepsilon^2/\sigma_\theta^2$) or the more costly to part with more liquid collateral (higher λ), the steeper are demand functions, the larger is the equilibrium margin, and the larger is the inefficiency in funds allocation (and the equilibrium may break down in the inelastic case; see Propositions 1, 2, 3, and 4, and Section 5.1). The effects have been corroborated empirically.[71] The central bank can try to reduce the inefficiency in the distribution of liquidity, which can be substantial when ρ and/or $\sigma_\varepsilon^2/\sigma_\theta^2$ are large, by accepting lower quality collateral from the banks in repossession auctions. This is equivalent to provide a quadratic subsidy to the banks that effectively lowers λ. The amount of the subsidy will be increasing with ρ and $\sigma_\varepsilon^2/\sigma_\theta^2$ and decreasing with n (Proposition 6 and Section 5.1). Central banks in the crisis have enlarged acceptable collateral and some of them have increased the qualifying participants in the auctions.[72]

6.2.3. *Treasury Auctions*

The sources of private information in this context are different expectations about the future resale value of securities $\tilde{\theta}$ (for instance, bidders with different forecasts of inflation with securities denominated in nominal terms) and private values arising out of different liquidity needs due to idiosyncratic shocks.[73] There is evidence that prices in Treasury auctions that feature a discount from secondary market prices increase with the noise in the signal of the bidders (as in Proposition 2), as well as aggregate information (Proposition 1).[74] Underpricing is thought to be a serious problem in uniform price auctions.[75] There

[71]The comparative static predictions of the auction discount with respect to the expected secondary market value $E[\tilde{\theta}|\tilde{s}]$ are consistent with documented features of the European Central Bank (ECB) euro auctions. (See Ewerhart, Cassola, and Valla (2010) and Vives (2010). Note, however, that the ECB auctions in the period studied are discriminatory while our model is uniform price.) Cassola, Hortaçsu, and Kastl (2009) showed that in ECB auctions after the subprime crisis in August 2007, marginal valuations for funds of banks increased and the aggregate bid curve was steeper with increased bid shading. In our model, the level effect on valuations would be represented by an increase in $\bar{\theta}$.

[72]For example, the Federal Reserve established the Term Auction Facility (with a single-price format) to broaden the range of counterparties and the range of collateral in relation to regular open market operations.

[73]See Hortaçsu and Kastl (2008) and Bindseil, Nyborg, and Strebulaev (2005).

[74]See Cammack (1991) and Nyborg, Rydqvist, and Sundaresan (2002). Gordy (1999) argued that bidders in Treasury auctions submit demand schedules to protect against the winner's curse and he associated larger bid dispersion with increased incidence of the winner's curse. Bid dispersion in our model can be linked to the slope of the demand schedule: a steeper slope occurs with more noise in the signal.

[75]See the evidence provided by Kandel, Sarig, and Wohl (1999) and by Keloharju, Nyborg, and Rydqvist (2005). U.S. Treasury auctions are exclusively uniform price since October 1998, and only a limited number of primary dealers can submit competitive bids; in Treasury auctions in Sweden, the range of participants is from 6 to 15 (Nyborg, Rydqvist, and Sundaresan (2002)).

are also worries that in the financial crisis of 2007–2008, margins and profits of (Wall Street) dealers may have grown dramatically at the expense of the Treasury and the Federal Reserve. Underpricing and high expected profits are consistent with our results (Propositions 2 and 4).

7. CONCLUDING REMARKS

In a model with private and common value uncertainty and without noise traders, we find a unique privately revealing equilibrium where traders rely on their private signals (and where the incentives to acquire information are preserved). A main result is that private information generates market power over and above the full information level. Several testable implications derive from the analysis. An increase in the correlation of cost parameters or in the noise in private signals makes supply functions steeper and increases expected price–cost margins. The average margin may be above the Cournot level and may get closer to the collusive level as correlation increases, with no coordination of sellers. When demand is uncertain, an increase in noise decreases expected margins.

The results may help explain pricing patterns that arise in electricity markets, revenue management, and auctions. For example, ignoring private cost information with supply function competition in electricity markets may underestimate the slope of supply and the Treasury may overpay in reverse auctions for toxic assets that reveal their average value due to increased correlation of values for banks. The biases introduced by a Cournot model when competition is, in fact, in supply or demand schedules are also characterized.

With regard to welfare, at the SFE, sellers supply too little and too similar quantities, the efficient allocation can be obtained with price-taking behavior, and, typically, the expected deadweight loss is increasing in the correlation of the cost parameters and in the noise of private signals. With regard to policy, price-taking behavior may be induced with an optimal quadratic subsidy, and a precise enough public signal about the common value component may restore efficiency. The former explains, for example, how, in a financial crisis, loosening collateral requirements in central bank liquidity auctions may be part of an optimal subsidy scheme to banks or how, in the U.S. PPIP scheme for legacy loans auctions, a subsidy to bidders may be rationalized.

The model and the results have already demonstrated robustness to a number of extensions that maintain the symmetry in the model. Further work should explore the role of asymmetries in technology and information structure.[76]

[76]Rostek and Weretka (2010) presented results in the latter vein.

APPENDIX: SOME PROOFS AND CLAIMS

The proofs of Propositions 6, 7, and 8 as well as complementary material, simulations, and the analysis of information acquisition can be found in the Supplemental Material.

PROOF OF PROPOSITION 1: (i) Suppose that sellers other than i use the strategy $X(s_j, p) = b - as_j + cp$. From the market clearing equation and from the point of view of seller i (provided $1 + \beta(n-1)c > 0$), the price is informationally equivalent to $h_i \equiv \beta b(n-1) - \alpha + (1 + \beta(n-1)c)p + \beta x_i = \beta a_{j\neq i} s_j$. The pair (s_i, p) is informationally equivalent to the pair (s_i, h_i), hence $E[\theta_i|s_i, p] = E[\theta_i|s_i, h_i]$. From our Gaussian information structure

$$\begin{pmatrix}\theta_i\\ s_i\\ h_i\end{pmatrix} \sim N\left(\begin{pmatrix}\bar{\theta}\\ \bar{\theta}\\ \beta a(n-1)\bar{\theta}\end{pmatrix},\right.$$
$$\left.\begin{pmatrix}\sigma_\theta^2 & \sigma_\theta^2 & \beta a(n-1)\rho\sigma_\theta^2\\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & \beta a(n-1)\rho\sigma_\theta^2\\ \beta a(n-1)\rho\sigma_\theta^2 & \beta a(n-1)\rho\sigma_\theta^2 & \beta^2 a^2(n-1)((\sigma_\theta^2 + \sigma_\varepsilon^2) + (n-2)\rho\sigma_\theta^2)\end{pmatrix}\right)$$

and the projection theorem for normal random variables, we obtain

$$\begin{aligned}
&E[\theta_i|s_i, h_i]\\
&\quad = \bar{\theta} + \frac{\sigma_\theta^2(\sigma_\theta^2(1-\rho)(1+(n-1)\rho) + \sigma_\varepsilon^2)}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)(\sigma_\theta^2(1+(n-1)\rho) + \sigma_\varepsilon^2)}(s_i - \bar{\theta})\\
&\qquad + \frac{\sigma_\theta^2\sigma_\varepsilon^2\rho}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)(\sigma_\theta^2(1+(n-1)\rho) + \sigma_\varepsilon^2)\beta a}\\
&\qquad \times (h_i - \beta a(n-1)\bar{\theta}).
\end{aligned}$$

Using the FOC $p - E[\theta_i|s_i, p] = (d+\lambda)x_i$, $X(s_i, p) = b - as_i + cp$, and $h_i = p(1 + \beta nc) - \alpha + \beta nb - \beta as_i$, we obtain

$$\begin{aligned}
&-\frac{\sigma_\varepsilon^2(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)\bar{\theta} + (\sigma_\theta^2\sigma_\varepsilon^2\rho(\beta nb - \alpha)/\beta a)}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)}\\
&\quad - \frac{\sigma_\theta^2(\sigma_\theta^2(1-\rho)(1+(n-1)\rho) + \sigma_\varepsilon^2)}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)(\sigma_\theta^2(1+(n-1)\rho) + \sigma_\varepsilon^2)}s_i\\
&\quad + \left(1 - \frac{\sigma_\theta^2\sigma_\varepsilon^2\rho(1+\beta nc)}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)\beta a}\right)p\\
&\quad = (d+\lambda)(b - as_i + cp).
\end{aligned}$$

Identifying coefficients and letting

$$M \equiv \frac{\rho\sigma_\varepsilon^2 n}{(1-\rho)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)},$$

we obtain

$$a = \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)}(d+\lambda)^{-1},$$

$$b = \frac{1}{1+M}\left(\frac{\alpha}{\beta n}M - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2}(d+\lambda)^{-1}\bar{\theta}\right)$$

with $d = (\beta^{-1} + (n-1)c)^{-1}$ and where c is given by the equation

$$c = \left((d+\lambda)^{-1} - \frac{M(1+\beta nc)}{\beta n}\right)$$

or, equivalently, the quadratic

$$\begin{aligned} g(c; M) \equiv{}& \lambda\beta(n-1)(1+M)c^2 \\ &+ \left((\beta+\lambda)(1+M) + (n-1)\left(\frac{\lambda M}{n} - \beta\right)\right)c \\ &+ \frac{\beta+\lambda}{\beta n}M - 1. \end{aligned}$$

For $n = 1$, there is a unique solution to the quadratic equation. For $n \geq 2$ and $\lambda > 0$, the discriminant of $g(\cdot; M) = 0$ is positive and, therefore, the equation has two real roots, but only the largest root

$$\begin{aligned} c_2 ={}& \big(-(n-M+2Mn)\lambda - n\beta(M-n+2) \\ &+ \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\big) \\ &/(2n\beta\lambda(n-1)(M+1)) \end{aligned}$$

is compatible with the second order condition $2d + \lambda > 0$.

It is easily checked also that $g(-M((1+M)\beta n)^{-1}; M) < 0$ and, therefore, for the largest root, we have $c > -M((1+M)\beta n)^{-1}$ because of convexity of the parabola $g(\cdot; M)$. It follows also that $c > -M((1+M)\beta n)^{-1} > -(\beta n)^{-1}$ and, therefore, $1 + \beta nc > 0$ and $1 + \beta(n-1)c > 0$ for either $c > 0$ or $c < 0$, and $0 < d < \beta n$. Furthermore, $a > 0$ since $\rho < 1$. (Note also that $c < \lambda^{-1}$ when $\rho \geq 0$ since then $M \geq 0$ and $c = (d+\lambda)^{-1} - M((\beta n)^{-1} + c) < \lambda^{-1}$ since $0 < d$ and $c + (\beta n)^{-1} > 0$.) We show that c decreases in λ. Direct computation shows that

$$\frac{\partial c_2}{\partial \lambda} = -\big(\beta n(n-M-2)^2 + \lambda(M+n)^2 + (n-M-2) \times \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\big) / \big(2(n-1)(M+1) \times \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\lambda^2\big) < 0$$

because $M+1>0$ and the numerator in the fraction is positive since

$$(\beta n(n-M-2)^2 + \lambda(M+n)^2)^2 - \big((n-M-2) \times \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\big)^2 = 4\lambda^2(M+1)(n-1)(M+n)^2 > 0.$$

The largest root of $g(\cdot; M)=0$ decreases with M since $\partial g/\partial M > 0$ for $c > -(\beta n)^{-1}$. This is so since it can be checked that $\partial g/\partial M$ is a convex parabola in c with largest root $-(\beta n)^{-1}$. It follows that for $c > -(\beta n)^{-1}$, we have that $\partial g/\partial M > 0$. As $M \to \infty$, we have that $1+\beta nc \to 0$ and $c \to -(\beta n)^{-1}$ since otherwise, from $c = (d+\lambda)^{-1} - \frac{M(1+\beta nc)}{\beta n}$ and $0 < d < \beta n$, we would have that $c \to -\infty$, which contradicts $1+\beta nc > 0$. It follows that as $M \to \infty$, $(1+\beta(n-1)c) \to 1/n$ or $d \to \beta n$. As $M \to -1$, $-M((1+M)\beta n)^{-1} \to \infty$ and, therefore, $c \to \infty$ and $d \to 0$.

(ii) If $\lambda = 0$, we have that at the candidate SFE, $d = \beta n(n-M-2)/(M-3n)$ (and $c = c_0 \equiv -(n-M)/((n-M-2)\beta n)$). Then $d > 0$ (fulfilling the SOC) if and only if $n-M-2 < 0$. *Q.E.D.*

CLAIM A.1: *Let $\sigma_\varepsilon^2 > 0$. Then the absolute value of the weight on h_i in $E[\theta_i|s_i, h_i]$ increases in $|\rho|$ and $\sigma_\varepsilon^2/\sigma_\theta^2$.*

PROOF: The coefficient of h_i in $E[\theta_i|s_i, h_i]$ equals $(d+\lambda)M/\beta n$ and is increasing in M since d is increasing in M. It follows that when $\sigma_\varepsilon^2 > 0$, the coefficient is increasing in ρ since then M is increasing in ρ, and when $M > 0$ ($M < 0$), it is increasing (decreasing) in $\sigma_\varepsilon^2/\sigma_\theta^2$ since then M is increasing (decreasing) in $\sigma_\varepsilon^2/\sigma_\theta^2$. The result follows. *Q.E.D.*

CLAIM A.2: *When $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$ we have that $a \to 0$ and $c \to \widehat{c}$, where*

$$\widehat{c} = \big(\beta(n-2) - \lambda - 2\rho(\beta+\lambda)(n-1) + \sqrt{\beta^2(n-2-2\rho(n-1))^2 + \lambda^2 + 2n\beta\lambda}\big) / \big(2\beta\lambda(n-1)((n-1)\rho+1)\big)$$

with $\widehat{c} = c^f$ *for* $\rho = 0$ *and* $c^f > \widehat{c}$ ($c^f < \widehat{c}$) *for* $\rho > 0$ ($\rho < 0$), *where*

$$c^f = \frac{\beta(n-2) - \lambda + \sqrt{\beta^2(n-2)^2 + 2n\beta\lambda + \lambda^2}}{2\beta\lambda(n-1)}.$$

Furthermore, $\widehat{c} \to -(\beta n)^{-1}$ *for* $\rho \to 1$, $\widehat{c} \to \infty$ *for* $\rho \to -(n-1)^{-1}$, *and* $\widehat{c}$ *is decreasing in* ρ.

PROOF: Note that if $\rho\sigma_\varepsilon^2 = 0$, we have that $M = 0$ and c is given by the *positive* root of $(\beta^{-1} + (n-1)c)^{-1} + \lambda = c^{-1}$. This is

$$c^f = \frac{\beta(n-2) - \lambda + \sqrt{\beta^2(n-2)^2 + 2n\beta\lambda + \lambda^2}}{2\beta\lambda(n-1)}.$$

When $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$, it is immediate that $a \to 0$, and since $M \to \rho n/(1-\rho)$ from the expression for c_2, we obtain $c \to \widehat{c}$, where with $\widehat{c} = c^f$ for $\rho = 0$, $\widehat{c} = -(\beta n)^{-1}$ for $\rho = 1$, $\widehat{c} \to \infty$ for $\rho \to -(n-1)^{-1}$, and $\widehat{c}$ is decreasing in ρ. *Q.E.D.*

CLAIM A.3: *If* $n - M - 2 \geq 0$, *then as* $\lambda \to 0$, $c \to \infty$, *and if* $n - M - 2 < 0$, *then as* $\lambda \to 0$, $c \to c_0$.

PROOF: It is immediate from the expression for c_2 that when $n - M - 2 \geq 0$, $\lim_{\lambda\to 0} c_2 = \infty$, and when $n - M - 2 < 0$, using l'Hôpital's rule we find that

$$\begin{aligned}\lim_{\lambda\to 0} c_2 &= \lim_{\lambda\to 0}\left(\frac{(\lambda + n\beta)(M+n)^2}{\sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}}\right.\\ &\qquad \left. - (n - M + 2Mn)\right)\Big/\big(2n\beta(n-1)(M+1)\big)\\ &= -\frac{n-M}{(n-M-2)\beta n}.\end{aligned}$$

Q.E.D.

PROOF OF PROPOSITION 2:

(i) The slope c decreases with M which, when $\rho\sigma_\varepsilon^2 > 0$, increases with ρ and $\sigma_\varepsilon^2/\sigma_\theta^2$.[77] When $\rho < 0$, M decreases with $\sigma_\varepsilon^2/\sigma_\theta^2$ and c increases in $\sigma_\varepsilon^2/\sigma_\theta^2$. As ρ ranges from $-(n-1)^{-1}$ to 1, M ranges from -1 to ∞, c ranges from ∞ to $-1/\beta n$, and d ranges from 0 to βn (from Proposition 1).

[77]Note that the equilibrium depends only on the ratio $\sigma_\varepsilon^2/\sigma_\theta^2$.

(ii) In equilibrium, $p = E[\tilde{\theta}|\tilde{s}] + (d+\lambda)\tilde{x}$ and from the demand function, we obtain $\tilde{x} = (\alpha - E[\tilde{\theta}|\tilde{s}])/(\beta n + \lambda + d)$. If sellers share the signals $s = (s_1, \dots, s_n)$ and maximize joint profits, they solve

$$\max_{(x_i)_{i=1}^n} E\left[\sum_{i=1}^n \pi_i \middle| s\right],$$

where $E[\sum_{i=1}^n \pi_i|s] = \sum_{i=1}^n px_i - \sum_{i=1}^n E[\theta_i|s]x_i - \frac{\lambda}{2}\sum_{i=1}^n x_i^2$, with (sufficient) FOC

$$\alpha - 2\beta\sum_{i=1}^n x_i - E[\theta_i|s] - \lambda x_i = 0, \quad i = 1, \dots, n.$$

Adding up across sellers, we obtain

$$\frac{p - E_n[\mathrm{MC}_n]}{p} = \frac{1}{\eta}.$$

It is immediate that the average collusive output is $\tilde{x}^m = (\alpha - E[\tilde{\theta}|\tilde{s}])/(2\beta n + \lambda)$. At the SFE as $\rho \to 1$ (when $\sigma_\varepsilon^2 > 0$), we have that $d \to \beta n$, and for given $\tilde{s}$, the aggregate interim Lerner index converges to the collusive level η^{-1} and $\tilde{x} \to \tilde{x}^m$. Similarly, as $\rho \to -(n-1)^{-1}$, we have that $d \to 0$ and $\tilde{x} \to \tilde{x}^o \equiv (\alpha - E[\tilde{\theta}|\tilde{s}])/(\beta n + \lambda)$, the average competitive output.

(iii) We have that $\bar{p} - E[\mathrm{MC}_n] = dE[\tilde{x}] = d(\alpha - \bar{\theta})(\beta n + \lambda + d)^{-1}$, which is increasing in d and, therefore, in ρ or $\sigma_\varepsilon^2/\sigma_\theta^2$ (when $\rho > 0$). From Claim A.4 below, d decreases with n when $\rho \geq 0$ and $c > 0$. The same happens with the expected price since $\bar{p} = \bar{\theta} + (d+\lambda)(\alpha - \bar{\theta})(\beta n + \lambda + d)^{-1}$ is increasing in d. The opposite results for $\sigma_\varepsilon^2/\sigma_\theta^2$ when $\rho < 0$, since then d is decreasing in $\sigma_\varepsilon^2/\sigma_\theta^2$.

(iv) We have that $\mathrm{var}[\tilde{x}] = (\beta n + \lambda + d)^{-2}\mathrm{var}[E[\tilde{\theta}|\tilde{s}]]$, where $E[\tilde{\theta}|\tilde{s}] = \zeta\tilde{s} + (1-\zeta)\bar{\theta}$ and $\zeta \equiv \mathrm{var}[\tilde{\theta}]/(\mathrm{var}[\tilde{\theta}] + \sigma_\varepsilon^2 n^{-1})$. It follows that

$$\mathrm{var}[E[\tilde{\theta}|\tilde{s}]] = \zeta^2\mathrm{var}[\tilde{s}] = \zeta\,\mathrm{var}[\tilde{\theta}] = \frac{((1+(n-1)\rho)\sigma_\theta^2)^2}{((1+(n-1)\rho)\sigma_\theta^2 + \sigma_\varepsilon^2)n}$$

increases in ρ (since $\rho > -(n-1)^{-1}$) and σ_θ^2, and decreases in σ_ε^2. We conclude that price volatility $\mathrm{var}[p] = (\beta n)^2\mathrm{var}[\tilde{x}]$ decreases with σ_ε^2 and increases with σ_θ^2 when $\rho > 0$ (since d increases with $\sigma_\varepsilon^2/\sigma_\theta^2$ when $\rho > 0$). *Q.E.D.*

CLAIM A.4: *d decreases with n when $\rho \geq 0$ and $c > 0$.*

PROOF: At the equilibrium, $\partial g/\partial c > 0$ and it is possible to check that

$$\frac{\partial c}{\partial n} = -\frac{\partial g/\partial n}{\partial g/\partial c}$$

$$= -\Big(\frac{(1+\lambda d^{-1})(1+cn\beta)(Mn^{-1}(1-\rho)-\rho)M}{n(\rho(n-1)+1)}$$
$$+ c\big(Mn^{-1}(\beta+2\lambda) - \big(\beta - c\lambda(1+M(2-n^{-1}))\big)\big)\Big)$$
$$\big/\big(2c\lambda\beta(n-1)(M+1) + ((\beta+\lambda)(M+1)$$
$$+ (Mn^{-1}\lambda - \beta)(n-1))\big).$$

It follows that

$$\begin{aligned}
&\frac{\partial}{\partial n}(c(n-1)) \\
&= (n-1)\frac{\partial c}{\partial n} + c \\
&= \beta(1+\lambda d^{-1})\big((n-1)(\rho - Mn^{-1}(1-\rho))(Mn^{-1} + c\beta(M+1)) \\
&\quad + c\beta(M+1)\big)/\big((\rho(n-1)+1)\beta(\partial g/\partial c)\big) \\
&> 0
\end{aligned}$$

when $c > 0$ since $\partial g/\partial c > 0$, and when $\rho \geq 0$, we have that $M \geq 0$ and $\rho - Mn^{-1}(1-\rho) \geq 0$. Note that $d = d^f$ for $\rho = 0$, and, therefore, d^f (c^f) is decreasing (increasing) in n. *Q.E.D.*

PROOF OF PROPOSITION 3:

(i) Considering a Taylor series TS expansion (stopping at the second term due to the quadratic nature of the payoff) around the efficient allocation, which maximizes $E[\text{TS}|t]$,[78] it follows that

$$\begin{aligned}
&\text{E}[\text{DWL}|t] \\
&= n\Big(\beta n(\tilde{x}(t;d) - \tilde{x}(t;0))^2 + \lambda n^{-1}\sum_i (x_i(t;d) - x_i(t;0))^2\Big)/2.
\end{aligned}$$

To supply an average quantity $\tilde{x}$, the market solves the program

$$\min_{(x_i)_{i=1}^n}\left\{E\left[\sum_{i=1}^n \hat{C}(x_i,\theta_i)\Big| t\right] \text{ s.t. } n^{-1}\sum_{i=1}^n x_i = \tilde{x}\right\},$$

yielding $\hat{x}_i = \tilde{x} + (\tilde{t} - t_i)(d+\lambda)^{-1}$, $i = 1,\dots,n$.[79] If $\tilde{x}$ and $\tilde{x}^0$ are supplied in a cost-minimizing way, then $x_i - \tilde{x} = x_i^0 - \tilde{x}^0 = (\tilde{t} - t_i)\lambda^{-1}$ and $x_i - x_i^0 = \tilde{x} - \tilde{x}^0$.

[78]The result holds true, in fact, by comparing an efficient allocation with any another allocation that is based on weakly coarser information. See Lemma 1 in Vives (2002).

[79]Note that $E[\tilde{\theta}|s] = n^{-1}\sum_{i=1}^n E[\theta_i|s] = n^{-1}\sum_{i=1}^n t_i \equiv \tilde{t}$.

Since $\tilde{x}(t;d) = (\alpha - \tilde{t})/(\beta n + \lambda + d)$, it follows that aggregate inefficiency is given by

$$\begin{aligned} &n\big((\beta n + \lambda)(\tilde{x}(t;d) - \tilde{x}(t;0))^2\big)/2 \\ &\quad = \big(n(\beta n + \lambda)((\beta n + \lambda)^{-1} - (\beta n + \lambda + d)^{-1})^2(\alpha - \tilde{t})^2\big)/2 \end{aligned}$$

and it is increasing in d. The residual in the deadweight loss is due to distributive inefficiency. Letting $u_i \equiv x_i(t;d) - \tilde{x}(t;d)$, $u_i^o \equiv x_i(t;0) - \tilde{x}(t;0)$, and $\tilde{\sigma}_t^2 \equiv n^{-1}\sum_i (t_i - \tilde{t})^2$, and noting that $u_i = (\tilde{t} - t_i)/(\lambda + d)$ and $u_i^o = (\tilde{t} - t_i)/\lambda$, then distributive inefficiency is given by

$$\lambda \sum_i (u_i - u_i^o)^2 / 2 = n\lambda(\lambda^{-1} - (\lambda + d)^{-1})^2 \tilde{\sigma}_t^2/2$$

and it is increasing in d.

(ii) Let $\pi_i(t;d) \equiv E(\pi_i|t)$, where π_i denotes profits at the SFE, and let $\tilde{\pi}(t;d) \equiv n^{-1}\sum_i \pi_i(t;d)$. Then it can be shown that in equilibrium, $\pi_i(t;d) = (d + \frac{\lambda}{2})(x_i(t;d))^2$, where $x_i(t;d) = \tilde{x}(t;d) + (\tilde{t} - t_i)(d + \lambda)^{-1}$. It follows that

$$\tilde{\pi}(t;d) = \left(d + \frac{\lambda}{2}\right)\left((\tilde{x}(t;d))^2 + \frac{\tilde{\sigma}_t^2}{(\lambda + d)^2}\right)$$

and

$$\frac{\partial \tilde{\pi}(t;d)}{\partial d} = \frac{\beta n - d}{\beta n + d + \lambda}(\tilde{x}(t;d))^2 - \frac{d}{(\lambda + d)^3}\tilde{\sigma}_t^2.$$

This implies that $\partial\tilde{\pi}/\partial d > 0$ for d small and $\partial\tilde{\pi}/\partial d < 0$ for d close to βn. *Q.E.D.*

PROOF OF PROPOSITION 4: Let $x_i = x_i(t;d)$ and $x_i^o = x_i(t;0)$. From the Proof of Proposition 3(i), we obtain

$$\begin{aligned} E[\mathrm{DWL}] &= E[E[\mathrm{DWL}|t]] \\ &= n\big(\beta n E[(\tilde{x} - \tilde{x}^o)^2] + \lambda E[(x_i - x_i^o)^2]\big)/2 \end{aligned}$$

and the corresponding decomposition

$$E[\mathrm{DWL}] = n\big((\beta n + \lambda)E[(\tilde{x} - \tilde{x}^o)^2] + \lambda E[(u_i - u_i^o)^2]\big)/2,$$

with $u_i \equiv x_i - \tilde{x}$ and $u_i^o \equiv x_i^o - \tilde{x}^o$, where the first term corresponds to aggregate inefficiency and the second term corresponds to distributive inefficiency.

(i) We have that $E[(\tilde{x} - \tilde{x}^o)^2] = ((\beta n + \lambda)^{-1} - (\beta n + \lambda + d)^{-1})^2 E[(\alpha - \tilde{t})^2]$. We know that increases in ρ or $1/\sigma_\varepsilon^2$ increase the variance of the prediction $\tilde{t} = E[\tilde{\theta}|\tilde{s}]$ (see Proof of Proposition 2(iv)); therefore, $E[(\alpha - \tilde{t})^2]$ increases in ρ and decreases in σ_ε^2. Since d increases in ρ (for $\sigma_\varepsilon^2 > 0$), we can conclude that aggregate inefficiency increases with ρ. We also have that d increases in σ_ε^2 if $\rho > 0$ and it can be checked that aggregate inefficiency may be nonmonotonic with respect to σ_ε^2. If $\rho \leq 0$, then d is weakly decreasing in σ_ε^2 and aggregate inefficiency decreases in σ_ε^2.

(ii) We have that $E[(u_i - u_i^o)^2] = (\lambda^{-1} - (\lambda + d)^{-1})^2 E[(t_i - \tilde{t})^2]$. I claim that $E[(t_i - \tilde{t})^2]$ is decreasing in ρ and σ_ε^2 when $\rho < 1$. Noting that

$$\begin{aligned} t_i &= E[\theta_i | s_i, \tilde{s}] \\ &= \bar{\theta} + \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}(s_i - \bar{\theta}) \\ &\quad + \frac{\sigma_\varepsilon^2 \sigma_\theta^2 \rho n}{(((n-1)\rho + 1)\sigma_\theta^2 + \sigma_\varepsilon^2)(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}(\tilde{s} - \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} \tilde{t} &= \bar{\theta} + \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}(\tilde{s} - \bar{\theta}) \\ &\quad + \frac{\sigma_\varepsilon^2 \sigma_\theta^2 \rho n}{(((n-1)\rho + 1)\sigma_\theta^2 + \sigma_\varepsilon^2)(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}(\tilde{s} - \bar{\theta}), \end{aligned}$$

we obtain $t_i - \tilde{t} = \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2)}(s_i - \tilde{s})$ and

$$E[(t_i - \tilde{t})^2] = \operatorname{var}[t_i - \tilde{t}] = \left(\frac{(1-\rho)\sigma_\theta^2}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}\right)^2 \operatorname{var}[s_i - \tilde{s}].$$

Since $\operatorname{var}[s_i - \tilde{s}] = (\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)(n-1)n^{-1}$, we conclude that $E[(t_i - \tilde{t})^2] = \frac{(1-\rho)^2(n-1)\sigma_\theta^4}{n(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2)}$, which is decreasing in ρ and σ_ε^2 (and increasing in n) when $\rho < 1$. The effect of ρ (σ_ε^2) on $E[(t_i - \tilde{t})^2]$ is particularly strong when σ_ε^2 (ρ) is small (close to 0), while d increases in ρ and σ_ε^2 (when $\rho > 0$). The total effect can go either way: If $\sigma_\varepsilon^2 = 0$, then d is independent of ρ and distributive inefficiency decreases in ρ; if $\rho \leq 0$, then d is weakly decreasing in σ_ε^2 and distributive inefficiency decreases in σ_ε^2.

(iii) From the Proof of Proposition 3(ii), we have that

$$E[\pi_i(t; d)] = E[\tilde{\pi}(t; d)] = \left(d + \frac{\lambda}{2}\right)\left(\frac{E[(\alpha - \tilde{t})^2]}{(\beta n + \lambda + d)^2} + \frac{E[(t_i - \tilde{t})^2]}{(\lambda + d)^2}\right).$$

When $\rho \to 1$ and $\sigma_\varepsilon^2 > 0$, we know (from Proposition 2) that $d \to \beta n$ (and $\tilde{x}(t, d) \to \tilde{x}^m \equiv \tilde{x}(t, \beta n)$). Furthermore, as $\rho \to 1$, $E[(t_i - \tilde{t})^2] \to 0$, $x_i - \tilde{x} = (\tilde{t} - t_i)(d + \lambda)^{-1}$ tends in mean square to 0, and productive inefficiency at the SFE vanishes. (Note that $x_i^m = \tilde{x}^m + (\tilde{t} - t_i)\lambda^{-1}$ and, therefore, as $\rho \to 1$, then

$x_i \to x_i^m = \tilde{x}^m$ in mean square.) It follows that $E[\pi_i(t; d)]$ converges to the collusive level as $\rho \to 1$: $(\beta n + \frac{\lambda}{2})\frac{E[(\alpha-\tilde{t})^2]}{(2\beta n+\lambda)^2}$. When $\rho \to -(n-1)^{-1}$ and $\sigma_\varepsilon^2 > 0$, then $d \to 0$, $(\tilde{x}(t, d) \to \tilde{x}^o \equiv \tilde{x}(t, 0)$, and $x_i(t, d) \to x_i^o \equiv x_i(t, 0))$, and $E[\pi_i(t; d)]$ converges to the competitive level $E[\pi_i(t; d = 0)]$. (Note also that $\mathrm{var}[\tilde{t}] \to 0$ as $\rho \to -(n-1)^{-1}$.) When $\sigma_\varepsilon^2 = 0$, $E[\pi_i]$ is linear and decreasing in ρ since then $\mathrm{sgn}\{\partial E[\pi_i]/\partial\rho\} = \mathrm{sgn}\{((\beta n + \lambda + d)^{-2} - (\lambda + d)^{-2})\sigma_\theta^2\} < 0$. If $\rho \le 0$, then d is weakly decreasing in σ_ε^2 and an increase in σ_ε^2 leads to a decrease in both $E[(\alpha - \tilde{t})^2]$ and $E[(t_i - \tilde{t})^2]$. *Q.E.D.*

PROOF OF REMARK 2: Similarly as in the Proof of Proposition 4, we obtain

$$\mathrm{ETS}^o - \mathrm{ETS}^f = n\big((\beta n + \lambda)E[(\tilde{x}^f - \tilde{x}^o)^2] + \lambda E[(u_i^f - u_i^o)^2]\big)/2,$$

and since neither ρ nor σ_ε^2 affects d^f, it follows that both aggregate and distributive inefficiency decrease in σ_ε^2 and, therefore, the deadweight loss is decreasing in σ_ε^2, and aggregate (distributive) inefficiency increases (decreases) in ρ. *Q.E.D.*

PROOF OF PROPOSITION 5:

(i) A price-taking SFE is a Bayesian equilibrium where price-taking is imposed. The equilibrium strategy of seller i is of the form $X^{\mathrm{PT}}(s_i, p) = b^{\mathrm{PT}} - a^{\mathrm{PT}}s_i + c^{\mathrm{PT}}p$ and it arises out of the maximization of expected profits taking prices as given, but using the information contained in the price:

$$\max_{x_i}\left\{(p - E[\theta_i|s_i, p])x_i - \frac{\lambda}{2}x_i^2\right\}.$$

Following the same procedure as in the Proof of Proposition 1 but with $d = 0$, we obtain the equilibrium. In the equilibrium, we have that $c^{\mathrm{PT}} = (\lambda^{-1} - (\beta n)^{-1}M)/(M+1)$, $1 + \beta n c^{\mathrm{PT}} > 0$, $a^{\mathrm{PT}} > 0$, and $p = (1 + \beta n c^{\mathrm{PT}})^{-1}(\alpha - \beta n b^{\mathrm{PT}} + \beta n a^{\mathrm{PT}}\tilde{s})$, and so p reveals $\tilde{s}$.

(ii) For a given $\kappa < \lambda$ and induced slope of marginal cost $\lambda - \kappa > 0$, we know that $\partial c/\partial\kappa > 0$ since c is decreasing in λ (Proposition 1). It follows that $d(\kappa)$ is decreasing in κ up to $\kappa = \lambda$. However, the fixed-point equation $d(\kappa) = \kappa$ need not have a solution unless it is allowing for negative slopes of effective marginal costs. We have that $c^{\mathrm{PT}}(\lambda) = (\lambda^{-1} - (\beta n)^{-1}M)(M + 1)^{-1}$ goes from $+\infty$ to $-M((1 + M)\beta n)^{-1}$ as λ moves in the range $(0, +\infty)$. Therefore, given that $c^{\mathrm{PT}}(\lambda) > c(\lambda)$ and that both are decreasing in λ, for any $\lambda > 0$ there is always a $\kappa > 0$ such that $c(\lambda - \kappa) = c^{\mathrm{PT}}(\lambda)$ provided that the range of $c(\lambda)$ is the same as $c^{\mathrm{PT}}(\lambda)$ (see Figure 3a). This is so if and only if $n - M - 2 \ge 0$. In this case, as $\lambda \to 0$, then $c(\lambda) \to \infty$. If $n - M - 2 < 0$, then as $\lambda \to 0$, $c(\lambda) \to c_0$, where

$c_0 = -(n-M)/((n-M-2)\beta n)$. (See Claim A.3.) Then only if λ is such that $c^{\mathrm{PT}}(\lambda) \le c_0$ can we find the desired $\kappa > 0$ with $\lambda - \kappa \ge 0$. Otherwise we need to induce $\lambda - \kappa < 0$ to obtain $c(\lambda - \kappa) = c^{\mathrm{PT}}(\lambda)$. This is feasible since if $n - M - 2 < 0$ and $\hat{\lambda} < \lambda < 0$, $\hat{\lambda} \equiv -\frac{n\beta}{M+n}(M + n - 2\sqrt{(M+1)(n-1)})$, then there are two linear SFE, with slopes of supply $c_1 > c_2$ $\lim_{\lambda\to 0^-} c_1 = +\infty$, $\lim_{\lambda \to 0^-} c_2 = c_0$, and

$$\lim_{\lambda\to\hat{\lambda}} c_1 = \lim_{\lambda\to\hat{\lambda}} c_2 = \hat{c} \equiv \frac{(n-M)\sqrt{(M+1)(n-1)} + (M+n)}{\beta n\sqrt{(M+1)(n-1)}(-n+M+2)},$$

$\partial c_1/\partial\lambda > 0$ and $\partial c_2/\partial\lambda < 0$ (see Lemma A.1 below). Therefore, for $n-M-2 < 0$ and $c^{\mathrm{PT}}(\lambda) > c_0$, we can always find a $\kappa > 0$ such that $c(\lambda - \kappa) = c^{\mathrm{PT}}(\lambda)$: If $c^{\mathrm{PT}}(\lambda) > \hat{c}$, then take $\kappa > 0$ such that $c_1(\lambda - \kappa) = c^{\mathrm{PT}}(\lambda)$; if $c^{\mathrm{PT}}(\lambda) \le \hat{c}$, then take $\kappa > 0$ such that $c_2(\lambda - \kappa) = c^{\mathrm{PT}}(\lambda)$. (See Figure 3b.) The optimal subsidy is increasing in M and λ(since c^{PT} decreases with M and λ). This means, in particular, that κ^* increases with ρ and increases (decreases) with σ_ε^2 when $\rho > 0$ ($\rho < 0$). *Q.E.D.*

LEMMA A.1: *If $n - M - 2 < 0$ and $\hat{\lambda} < \lambda < 0$, where $\hat{\lambda} \equiv -n\frac{\beta}{M+n}(M + n - 2\sqrt{(M+1)(n-1)})$, then there are two roots of $g(c; M) = 0$ that fulfill the SOC, with $c_1 > c_2$, $\partial c_1/\partial\lambda > 0$, $\partial c_2/\partial\lambda < 0$, $\lim_{\lambda\to 0^-} c_1 = +\infty$, $\lim_{\lambda\to 0^-} c_2 = c_0$, and*

$$\lim_{\lambda\to\hat{\lambda}} c_1 = \lim_{\lambda\to\hat{\lambda}} c_2 = \frac{(n-M)\sqrt{(M+1)(n-1)} + (M+n)}{\beta n\sqrt{(M+1)(n-1)}(M+2-n)}.$$

When $\lambda = \hat{\lambda}$, there is only one root and it fulfills the SOC.

PROOF: Let c_1 denote the smallest root of $g(c; M) = 0$ when $\lambda > 0$. We have that

$$c_2 - c_1 = \frac{\sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}}{n\beta\lambda(n-1)(M+1)}.$$

When $\hat{\lambda} < \lambda < 0$, the discriminant is also positive, and if $n - M - 2 > 0$, it can be checked that no root fulfills the SOC, while if $n - M - 2 < 0$, both roots do (and the largest solution is c_1). When $\lambda = \hat{\lambda}$, there is only one root and it fulfills the SOC. If $n - M - 2 = 0$, then $\hat{\lambda} = 0$. If $\hat{\lambda} < \lambda < 0$ and $n - M - 2 < 0$, then from the expression for c_1, $\lim_{\lambda\to 0^-} c_1 = \infty$ (and indeed $\lim_{\lambda\to 0^-} c_2 = c_0$). Direct computation yields that

$$\lim_{\lambda\to\hat{\lambda}} c_1 = \lim_{\lambda\to\hat{\lambda}} c_2 = \frac{(n-M)\sqrt{(M+1)(n-1)} + (M+n)}{\beta n\sqrt{(M+1)(n-1)}(M+2-n)}.$$

Furthermore,

$$\begin{aligned}\frac{\partial c_1}{\partial \lambda} &= \big(\beta n(n-M-2)^2 + \lambda(M+n)^2 \\ &\quad - (n-M-2) \\ &\quad \times \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\big) \\ &\quad /\big(2(n-1)(M+1) \\ &\quad \times \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\lambda^2\big) \\ &< 0\end{aligned}$$

whenever $(n-M-2) < 0$ since

$$\begin{aligned}&\beta n(n-M-2)^2 + \lambda(M+n)^2 \\ &\quad \geq \beta n(n-M-2)^2 + \hat{\lambda}(M+n)^2 \\ &\quad = 2\beta n(-2(M+1)(n-1) + \sqrt{(M+1)(n-1)}(M+n)) \geq 0\end{aligned}$$

as $-(2(M+1)(n-1))^2 + (\sqrt{(M+1)(n-1)}(M+n))^2 = (M+1)(n-1)(n-M-2)^2 \geq 0$. *Q.E.D.*

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IESE Business School, University of Navarra, Avenida Pearson, 21, 08034 Barcelona, Spain; XVives@iese.edu.

Manuscript received September, 2008; final revision received May, 2011.