ASSET AUCTIONS, INFORMATION, AND LIQUIDITY

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Abstract
A model is presented of a uniform price auction where bidders compete in demand schedules; the model allows for common and private values in the absence of exogenous noise. It is shown how private information yields more market power than the levels seen with full information. Results obtained here are broadly consistent with evidence from asset auctions, may help explain the response of central banks to the crisis, and suggest potential improvements in the auction formats of asset auctions. (JEL: D44, D82, G14, E58)

1. Introduction

The global financial crisis stemming from subprime mortgage loans has posed a host of questions to regulators, treasuries, and central banks. One question concerns the effectiveness of auctions in efficiently providing liquidity to financial institutions or removing toxic assets from the balance sheets of banks. There are also worries that, during the present financial crisis, margins and profits of (Wall Street) dealers have grown dramatically at the expense of the Treasury and the Fed.[1]

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1. See "Wall St. Profits from Fed Role" and "Wall St. Gets its Cut from the Fed and Treasury," *Financial Times,* 3 August 2009.

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Treasury auctions move a large volume of resources and are believed to be subject to underpricing.[2] For central banks, open-market operations are a crucial instrument for providing liquidity to the financial system. The European Central Bank (ECB) typically conducts weekly repo auctions (main refinancing operations), and the U.S. Federal Reserve holds auctions on a daily basis. Central banks have reacted to the challenge posed by the financial crisis by expanding the range of acceptable collateral in refinancing operations and by changing typical auction formats. The ECB, for example, changed the variable-rate auction tender format to a full-allotment, fixed-rate tender format after the collapse of Lehman Brothers. There is also an ongoing debate on how transparent central banks should be with the information they have on the banking system, especially in a crisis situation. With regard to the auctions of toxic assets, the question is how to avoid overpayment by the government while efficiently extracting those assets from the banks in trouble.

In this paper we present a model of a uniform price auction that allows for both common and private value components; the model is based on the one developed in Vives (2009), to which the reader is referred for a full development and proofs of the results presented here. The model in this paper does not incorporate exogenous noise and highlights how market power is greater with private information than in the standard case with full information. Our model's results are broadly consistent with evidence from asset auctions; moreover, they help explain the response of central banks to the crisis and also suggest potential improvements in the auction formats of asset auctions.

The balance of the paper is organized as follows. Section 2 presents our model and comparative statics results. Section 3 provides a welfare analysis and explains how subsidy schemes may induce an efficient allocation. Section 4 derives some conclusions for central bank and Treasury auctions, and Section 5 Extends the results to reverse auctions and comments on tools for removing toxic assets. Proofs of the statements in this paper can all be derived from Vives (2009).

2. A Model of Uniform Price Auctions

Consider a uniform price auction of k units of an asset with uncertain ex-post value. The marginal benefit of buying x_i units of the asset for bidder $i = 1, \ldots, n$ is $\theta_i - \lambda x_i$, where $\theta_i \sim N(\bar{\theta}, \sigma_\theta^2)$ with $\bar{\theta} > 0$, $\sigma_\theta^2 > 0$, and $\mathrm{cov}[\theta_i, \theta_j] = \rho\sigma_\theta^2$ for $j \neq i$ with $\rho \in [0, 1]$. The parameter $\lambda > 0$ is an adjustment for transaction costs,

2. More generally, it is known that uniform price auctions are prone to underpricing and demand reduction (e.g., Ausubel and Cramton 2002). Goswami, Noe, and Rebello (1996) confirm in experiments that subjects can reach underpricing equilibria with preplay communication. See Keloharju, Nyborg, and Rydqvist (2005) for evidence on Finnish Treasury auctions, Kandel, Sarig, and Wohl (1999) for IPO (initial public offering) auctions in Israel, and Tenorio (1997) for foreign currency auctions in Zambia.

opportunity costs, or risk aversion. Bidder i receives a private signal $s_i = \theta_i + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$, $\text{cov}[\varepsilon_i, \varepsilon_j] = 0$ for $j \neq i$, and $\text{cov}[\varepsilon_i, \theta_j] = 0$ for all j and i. It follows that $\tilde{\theta} \equiv (\sum_{i=1}^n \theta_i)/n \sim N(\bar{\theta}, (1 + (n-1)\rho)\sigma_\theta^2/n)$ and $\text{cov}[\tilde{\theta}, \theta_i] = \text{var}[\tilde{\theta}]$. Therefore, the valuation for a bidder can be decomposed into a common and an idiosyncratic component: $\theta_i = \tilde{\theta} + \eta_i$, where $\text{cov}[\eta_i, \tilde{\theta}] = 0$. The model allows for a pure *common value* ($\rho = 1$), pure *private values* ($\sigma_\varepsilon^2 = 0$), and *independent values* ($\rho = 0$).

The profits of bidder i are given by $\pi_i = (\theta_i - p)x_i - \lambda x_i^2/2$, where p is the stop-out auction price. Bidders maximize expected profits and submit demand schedules, and the auctioneer selects a price that clears the market.

Examples include a Treasury auction and an open-market central bank operation (to provide liquidity for the banking system). In an open-market operation, funds offered by the central bank are bid on by each bank on the basis of their individual demand functions. The average valuation $\tilde{\theta}$ may be related to the interest rate and/or to prices in the secondary interbank market, and λ reflects the structure of a counterparty's pool of collateral. The bidder bank prefers offering the central bank illiquid collateral in exchange for funds, but with an increased allotment the bidder must offer more liquid types of collateral at a higher opportunity cost. This explains the declining marginal valuation.[3] The marginal value θ_i for funds of bank i is idiosyncratic; it is assessed imperfectly by bank i (because, for example, of uncertainty about future liquidity needs), yet it is correlated with the values of other banks. Interpreting the model for the case of Treasury auctions, the sources of private information could include different expectations about the future resale value $\tilde{\theta}$ of the securities (for instance, bidders have different beliefs regarding future inflation, and securities are denominated in nominal terms). As before, bidders may have different liquidity needs as a consequence of idiosyncratic shocks.[4]

The strategy for bidder i is a price-contingent schedule $X(s_i, \cdot)$, a map from the signal space to the space of demand functions. Given the strategies of bidders $X(s_j, \cdot)$, $j = 1, \ldots, n$, for given realizations of signals, market clearing implies that $\sum_{j=1}^n X(s_j, p) = k$. Let us assume that there is a unique market-clearing price $\hat{p}(X(s_1, \cdot), \ldots, X(s_n, \cdot))$ for any realization of the signals.[5] Bidder i's profits for any such realization are given by

3. As argued by Ewerhart, Cassola, and Valla (2010).

4. Hortaçsu and Kastl (2008) cannot reject the hypothesis that bidders in Canadian 3-month T-bill auctions have private values. Bindseil, Nyborg, and Strebulaev (2005) argue that the common value component in T-bill auctions is more important than in central bank auctions because the primary dealers buy T-bills mostly for resale.

5. If there is no market-clearing price then we assume that the market shuts down; if there is more than one such price then the largest one is chosen. An alternative would be to set the stop-out price as the highest price at which aggregate excess demand is nonnegative or, if there is no such price, to set $p = 0$ (implying that the reserve price is 0). See Wang and Zender (2002).

$$\pi_i(X(s_1,\cdot),\ldots,X(s_n,\cdot)) = (\theta_i - p)X(s_i,p) - \lambda(X(s_i,p))^2/2,$$

where $p = \hat{p}(X(s_1,\cdot),\ldots,X(s_n,\cdot))$. This defines a game in demand functions, and hereafter we restrict our attention to symmetric linear Bayesian demand function equilibria (LBDFE). If the linear strategies of rivals are given by $X(s_j,p) = b + as_j - cp$, $j \neq i$, then bidder i (provided $c > 0$) faces a residual inverse supply $p = I_i + ((n-1)c)^{-1}x_i$, where

$$I_i = ((n-1)c)^{-1}\left((n-1)b + a\sum_{j\neq i} s_j - k\right).$$

All the information that the price provides to bidder i about the signals of others is contained in the intercept I_i. The information available to bidder i is therefore $\{s_i, p\}$ or $\{s_i, I_i\}$. Bidder i chooses x_i to maximize

$$\begin{aligned} E[\pi_i \mid s_i, p] &= x_i(E[\theta_i \mid s_i, p] - p) - \frac{\lambda}{2}x_i^2 \\ &= x_i(E[\theta_i \mid s_i, p] - I_i - ((n-1)c)^{-1}x_i) - \frac{\lambda}{2}x_i^2. \end{aligned}$$

The first-order condition is[6]

$$E[\theta_i \mid s_i, I_i] - I_i - 2((n-1)c)^{-1}x_i - \lambda x_i = 0$$

or, equivalently,

$$E[\theta_i \mid s_i, p] - p = (d+\lambda)x_i,$$

where $d \equiv ((n-1)c)^{-1}$. An equilibrium also requires that $a > 0$.

Proposition 1. *Let $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$. Then there is a unique symmetric LBDFE if and only if $n - 2 - M > 0$, where*

$$M \equiv \frac{\rho\sigma_\varepsilon^2 n}{(1-\rho)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)}.$$

This equilibrium is given by

$$X(s_i,p) = (E[\theta_i \mid s_i, p] - p)/(d+\lambda) = b + as_j - cp,$$

where

$$c = \frac{n-2-M}{\lambda(n-1)(1+M)}, \quad d = ((n-1)c)^{-1}, \quad a = \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)}(d+\lambda)^{-1}.$$

6. The second-order sufficient condition is fulfilled when $c > 0$.

In equilibrium we have that $1/\lambda(1+M) > c > 0$, $a > 0$, *c decreases with M and with λ, and d is decreasing in n.*

From the market-clearing condition we obtain $p = (b + a\tilde{s})c^{-1} - k/n$, where $\tilde{s} \equiv (\sum_i s_i)/n = \tilde{\theta} + (\sum_i \varepsilon_i)/n$; therefore, the price p reveals the aggregate information $\tilde{s}$. The equilibrium is *privately revealing*—in other words, for bidder i, either (s_i, p) or $(s_i, \tilde{s})$ is a sufficient statistic for evaluating θ_i, the joint information in the market $s = (s_1, \ldots, s_n)$. In particular, and given the normality of random variables, in equilibrium we have that

$$t_i \equiv E[\theta_i \mid s_i, p] = E[\theta_i \mid s_i, \tilde{s}] = E[\theta_i \mid s].$$

Suppose that there is no correlation ($\rho = 0$) between the value parameters or that signals are perfect ($\sigma_\varepsilon^2/\sigma_\theta^2 = 0$).[7] Then $M = 0$, $E[\theta_i \mid s_i, p] = E[\theta_i \mid s] = E[\theta_i \mid s_i]$, and bidder i does not learn about θ_i from prices. In this case the LBDFE coincides with the full-information equilibrium (denoted by superscript f and for which $c^f = (d^f + \lambda)^{-1} > c$). For example, if $\sigma_\varepsilon^2 = 0$ then $X(\theta_i, p) = c^f(\theta_i - p)$. Otherwise, if $\rho > 0$ or $\sigma_\varepsilon^2/\sigma_\theta^2 > 0$, then bidders learn from prices and demand functions are steeper: $c < c^f$ (and $d > d^f$). Indeed, the larger is M (which is increasing in ρ and in $\sigma_\varepsilon^2/\sigma_\theta^2$), the more that price serves an information role of the common value component and the steeper are the demand functions (lower c). The response to a price increase is to reduce the amount demanded, but moderately because a high price conveys the good news that the average valuation is high. Likewise, a bidder refrains from competing aggressively with his demand function because a low price conveys the bad news that valuations are low.

Private information yields market power that exceeds the full-information level. For ρ or $\sigma_\varepsilon^2/\sigma_\theta^2$ high enough, the linear equilibrium collapses as M increases: $n - 2 - M \to 0$ and $c \to 0$ (in which case $d \to \infty$ and bidder i's demand approaches zero). This happens owing to the combination of adverse selection and market power: The demand schedules become too inelastic to sustain an equilibrium.[8] The market tends to collapse when the common value element is more important (ρ high), signals are noisy (σ_ε^2 high), and/or prior uncertainty is low (σ_θ^2 low). This means, in particular, that a large enough prior precision (a raise in $1/\sigma_\theta^2$) may cause the market to collapse unless σ_ε^2 increases also.[9] This is akin

7. In this case the equilibrium is independent of ρ.

8. We do not examine the potential existence of nonlinear equilibria. It is worth noting that, in Bhattacharya and Spiegel (1991), if the linear equilibrium fails to exist then there is no other equilibrium except a degenerate, no-trade one.

9. As $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$ (in which case $M \to \rho n/(1-\rho)$), the equilibrium in Proposition 1 also collapses, even if $n - 2 > \rho n/(1-\rho)$, because $a \to 0$. However, in the limit $\sigma_\varepsilon^2/\sigma_\theta^2 \to \infty$ there is another linear equilibrium (even when $\rho = 1$) in which $X(p) = c^f(\bar{\theta} - p)$ (because $E[\theta_i \mid s_i, p] = \bar{\theta}$).

to asymmetric information models in which traders submit steeper schedules so as to protect themselves against adverse selection (Kyle 1989; Biais, Martimort, and Rochet 2000; Wang and Zender 2002). Indeed, the phenomenon is similar to the so-called winner's curse in common value auctions (Milgrom and Weber 1982): The more that bidders shade their bid to protect against the winner's curse, the less precise their signals are. In Kyle (1989) and also in Wang and Zender (2002), a linear equilibrium exists only if the number of informed traders is no less than three (when there are no uninformed traders). We also need $n \geq 3$ in our model to obtain a linear equilibrium.

Comparative statics results are easily derived. We have that

$$E[\tilde{\theta} \mid \tilde{s}] = E[\tilde{\theta} \mid s] = n^{-1} \sum_{i=1}^{n} E[\theta_i \mid s] = n^{-1} \sum_{i=1}^{n} t_i \equiv \tilde{t}.^{10}$$

Bid shading is directly related to the equilibrium parameter d. From the demand function of bidder i we have that $p = t_i - (d + \lambda)x_i$, and a price-taking bidder would submit the schedule (with $d = 0$) that coincides with her marginal valuation. From market clearing and the equilibrium bids it is immediate that $p = \tilde{t} - (d + \lambda)k/n$. Let the amount of bid shading be dk/n, the difference between the auction price and the average marginal valuation $\tilde{t} - \lambda k/n$. Because d is of order $1/n$, shading must be of order $1/n^2$. If the amount to be auctioned grows with n (say, $k_n = kn$), then bid shading is of order $1/n$. The following proposition summarizes the comparative statics results.

Proposition 2. *At the LBDFE, with $n - 2 - M > 0$ and $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$, the following statements hold.*

(i) The slope of equilibrium demand is steeper (c is smaller) with increases in ρ, $\sigma_\varepsilon^2/\sigma_\theta^2$, and λ.

(ii) The amount of bid shading is increasing—and the expected price $E[p] = \bar{\theta} - (d + \lambda)q/n$ is decreasing—in ρ, $\sigma_\varepsilon^2/\sigma_\theta^2$, k, and $1/n$.

(iii) Price volatility $\mathrm{var}[p] = \xi \mathrm{var}[\tilde{\theta}]$ *decreases with σ_ε^2 and increases with ρ and σ_θ^2.*[11]

In the auction literature, the "linkage principle" states that, on average, providing bidders with more information about a good's value increases the revenue of the seller. This dynamic has been associated with mitigating the winner's curse (Milgrom and Weber 1982). In our auction model, increased prior precision (higher $1/\sigma_\theta^2$) enhances the informational role of the price, making bidders

10. The first equality holds because $\tilde{s}$ is a sufficient statistic for s in relation to $\tilde{\theta}$.
11. Note that $\mathrm{var}[p] = \mathrm{var}[\tilde{t}] = \mathrm{var}[E[\tilde{\theta} \mid \tilde{s}]] = \xi^2 \mathrm{var}[\tilde{s}] = \xi \mathrm{var}[\tilde{\theta}]$, where $E[\tilde{\theta} \mid \tilde{s}] = \xi \tilde{s} + (1-\xi)\bar{\theta}$ and $\xi \equiv \mathrm{var}[\tilde{\theta}]/(\mathrm{var}[\tilde{\theta}] + \sigma_\varepsilon^2/n)$.

more cautious (see Perry and Reny [1999] for a discussion of the failure of the linkage principle in multi-object auctions).

The results have implications for the liquidity auctions of central banks. In the asset auction there is a discount with respect to the expectation of the secondary-market or average value $E[\tilde{\theta} \mid \tilde{s}]$, because $p = E[\tilde{\theta} \mid \tilde{s}] - (d + \lambda)k/n$. As the volatility of fundamentals σ_θ^2 increases, the discount decreases. In periods with high liquidity of collateral (low λ), bid schedules are very flat. Increasing the size (k) of the auction or providing more public information (higher $1/\sigma_\theta^2$) leads to an increased discount. All of these effects are documented features of the ECB euro auctions (Ewerhart, Cassola, and Valla 2010).[12]

The results also illustrate the impact of a crisis situation. The more severe the information problem (a larger ρ or $\sigma_\varepsilon^2/\sigma_\theta^2$) or the more costly it is to put up more liquid collateral (higher λ), the steeper are demand functions, the larger are the equilibrium margin and the amount of bid shading, and the more inefficiently the funds are allocated (in the extreme, the linear equilibrium may break down). Cassola, Hortaçsu, and Kastl (2009) study ECB auctions and show that, after the subprime crisis in August 2007, marginal valuations for funds of banks increased and the aggregate bid curve was steeper; there was also increased bid shading with evidence of strategic effects.

3. Welfare

The strategies at a LBDFE induce quantity and price outcomes as a function of the realized vector s of signals or, equivalently, of predicted values t, $(x_i(t))_{i=1}^n$, and $p(t)$. The auction outcome solves the following distorted benefit maximization program:

$$\mathrm{Max}_{(x_i)_{i=1}^n}\ E\left[\sum_{i=1}^n (\theta_i x_i - (d+\lambda)x_i^2/2) \,\middle|\, t\right]$$
$$\text{subj. to: } \sum_{i=1}^n x_i = k,$$

which yields $x_i(t) = \frac{t_i - \tilde{t}}{d+\lambda} + \frac{k}{n}, i = 1, \ldots, n$. The efficient allocation would obtain if we set $d = 0$. Then it can be checked that the total optimized benefit

$$B(k; t, d) \equiv E\left[\sum_{i=1}^n (\theta_i x_i(t) - \lambda(x_i(t))^2/2) \,\middle|\, t\right] = \sum_{i=1}^n (t_i x_i(t) - \lambda(x_i(t))^2/2)$$

12. However, the ECB auctions in the period examined were discriminatory whereas ours is a uniform-price model.

is decreasing in d. Given that $d > d^f > 0$ and that d^f is independent of ρ and σ_ε^2 (and is decreasing in n), our next proposition follows immediately.

PROPOSITION 3. *For a given realization of predicted values t, inefficiency is increasing in d. As a consequence, for given t: as ρ or σ_ε^2 increases, the inefficiency due to information-induced market power $(d - d^f)$ increases; and as n increases, both the inefficiency due to standard market power (d^f) and overall efficiency (d) decrease.*

REMARK. The comparative statics of the expected deadweight loss (DWL) at the LBDFE with respect to ρ or σ_ε^2 must also take into account the averaging over predicted values. It can be checked that $\mathrm{DWL} = (n\lambda E[(x_i - x_i^c)^2])/2$ and that

$$E[(x_i - x_i^c)^2] = (\lambda^{-1} - (\lambda + d)^{-1})^2 E[(t_i - \tilde{t})^2].$$

According to simulations, $E[(t_i - \tilde{t})^2]$ is decreasing in ρ and σ_ε^2. The reason is that an increase in either parameter tends to align t_i and $\tilde{t}$ probabilistically. The result is that DWL may increase or decrease in ρ and in σ_ε^2. If $\sigma_\varepsilon^2 = 0$ (respectively $\rho = 0$), then d is independent of ρ (respectively σ_ε^2) and inefficiency decreases in ρ (respectively σ_ε^2).

Provided that $\rho < 1$ and $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$, the efficient full-information allocation is implemented by a symmetric price-taking LBDFE (denoted by a superscript c, for "competitive," on the coefficients). The equilibrium strategy of bidder i will be of the form $X^c(s_i, p) = b^c + a^c s_i - c^c p$; it will arise from the maximization of expected profits, taking prices as given but using the information contained in the price:

$$\max_{x_i} \ (E[\theta_i \mid s_i, p] - p)\, x_i - \frac{\lambda}{2} x_i^2.$$

This optimization will yield the following system of first-order conditions: $p = E[\theta_i \mid s_i, p] - \lambda x_i$ for $i = 1, \ldots, n$. If $c^c > 0$ then, as in the strategic case, p reveals $\tilde{s}$, $E[\theta_i \mid s_i, p] = E[\theta_i \mid s_i, \tilde{s}]$, and the price-taking LBDFE implements the efficient solution; in equilibrium, $p = \tilde{t} - \lambda k/n$. In summary, if $\rho < 1$ and $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$, then there is a unique symmetric price-taking LBDFE and it implements the efficient allocation. The situation is as in Proposition 1 with $d = 0$ and $c^c = 1/(\lambda(M + 1))$.

The price-taking demand function will coincide with the marginal benefit schedule only when there is no learning from prices (i.e., only if $M = 0$). A bidder's demand function is always flatter in the price-taking equilibrium than in the strategic equilibrium: $c^c - c = (\lambda(n - 1))^{-1} > 0$. Furthermore, bidders are more cautious in responding to their private signals in the strategic case:

$$\mathrm{sgn}\{a^c - a\} = \mathrm{sgn}\{\lambda^{-1} - (d + \lambda)^{-1}\} > 0.$$

By the same token, given that $d > d^f > 0$, we have $c^c > c^f > c$ and $a^c > a^f > a$.

That the auction outcome can be obtained as the solution to a distorted planning problem with a more concave objective suggests that inefficiency may be eliminated by a quadratic subsidy $\kappa x_i^2/2$ that compensates for the distortion $dx_i^2/2$.[13] The question is whether we can find a $\kappa > 0$ such that $\lambda - \kappa + d(\kappa) = \lambda$ or $d(\kappa) = \kappa$, where $d(\kappa) \equiv ((n-1)c(\lambda - \kappa))^{-1}$ is the (endogenous) distortion when the slope of marginal benefits is $\lambda - \kappa$. In this case a bidder would, in effect, act as if he were competitive and facing a marginal benefit with slope λ. Our question is answered by the following proposition.

PROPOSITION 4. *Let $\rho < 1$. Then a quadratic subsidy $\kappa^* x_i^2/2$ with $\kappa^* = ((n-1)c^c(\lambda))^{-1} = \lambda(1+M)/(n-1)$ induces an efficient allocation because then bidders "act competitively." The subsidy κ^* increases with ρ, $\sigma_\varepsilon^2/\sigma_\theta^2$, and λ, and it decreases with n.*

4. Implications for Central Bank and Treasury Auctions

A central bank has two main objectives in the liquidity auctions. The first is to inject the right amount of money so that the short-term rate stays close to its target level; the second is to provide appropriate liquidity to the banks.[14] A *fixed-quantity* auction exactly controls the aggregate amount of money injected and has a "price discovery" purpose of eliciting the values for liquidity of the banks. However, there is an inefficiency in the distribution of liquidity that can be substantial when ρ and/or $\sigma_\varepsilon^2/\sigma_\theta^2$ are large—which may be precisely the case in a crisis situation. In contrast, a *fixed-price* tender only indirectly controls the amount of money injected and does not feature price discovery, but it does eliminate any inefficiency in the distribution of liquidity because the banks bid competitively. In a crisis situation, controlling the total amount of liquidity takes a back seat to making enough liquidity available to the banks, and this fact explains why the fixed-rate tender may be preferable in this case. Since the collapse of Lehman Brothers, the ECB is accepting the banks' demands in full at a fixed rate rather than following the usual auction procedure, where banks bid for money and thereby set the interest rate. More generally, it may be a good idea to introduce some elasticity in the supply schedule of the central bank, because the fixed-price tenders are one (horizontal) extreme and the fixed-quantity auctions are another (vertical) extreme. The analysis in Vives (2009) suggests that an optimal demand schedule for the central bank should be more elastic when the information problem is more severe.[15]

13. Given the work of Angeletos and Pavan (2009), who develop a similar approach, it would be worth exploring whether efficiency could be implemented by a linear state-contingent tax.

14. See Ayuso and Repullo (2003) for a model of the ECB's open-market operations.

15. LiCalzi and Pavan (2005) study a case with no asymmetric information.

Another way to reduce inefficiency in the distribution of liquidity is to lower λ by accepting lower-quality collateral from the banks in the repo auctions. This is what most central banks have done in response to the crisis, and it is equivalent to a quadratic subsidy (increasing with ρ and $\sigma_\varepsilon^2/\sigma_\theta^2$ and decreasing with n) in our framework.

A similar analysis applies to Treasury auctions.

5. Reverse Auctions

Our model can readily accommodate supply bids for an inelastic demand. The model should be reinterpreted with a change of variables in which the supply of bidder i is $y_i \equiv -x_i$.

In the initial Paulson plan of October 2008, reverse auctions were suggested as mechanisms to extract toxic assets from banks' balance sheets. In subsequent plans, auctions have been center stage as a means of removing legacy loans.[16] The Federal Reserve is also considering reverse auctions to mop up excess liquidity in a post-crisis scenario.[17]

In a reverse auction, the buyer (say, the U.S. Treasury) announces an amount of a certain class of securities (say, residential mortgage-backed securities based on California property of a certain face value, vintage, and type) that it seeks to buy. Those securities are in the hands of multiple banks, and the Treasury wants to buy a certain proportion of them. The marginal value of the security to a bank reflects not only the intrinsic value (to the bank) of the security but also the liquidity needs of the bank (both are correlated across institutions). The bank has an imperfect estimate of security values. It will first sell the worst securities—that is, the ones whose underlying mortgages are believed to have the lowest probability of repayment—and will sell better securities only when necessary. As a result, the marginal cost of selling securities is increasing. The parameter λ reflects the "quality heterogeneity" of the bank's securities (the larger is λ, the more quickly the bank's portfolio improves as the lemons are sold). In a crisis situation, λ will tend to be higher because the quality heterogeneity of the securities will increase.

The Treasury is uninformed about the value of the securities, and the reverse auction will serve the price discovery purpose of eliciting the average value $\tilde{\theta}$. Banks that value the security less or that have greater liquidity needs will sell more. Yet because the values are highly correlated, competition will be softened and the Treasury will pay much more than the competitive price for the securities. Information that is released by the Treasury would aggravate the distortion. In the extreme, with few sellers and high adverse selection, the market may collapse.

16. Emergency Economic Stabilization Act of 2008.

17. See Ausubel and Cramton (2008) and Klemperer (2009) for proposals on how to design the auctions.

As mentioned previously, the Treasury may benefit by setting an elastic demand schedule to control market power and avoid a market breakdown.

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