On the Possibility of Informationally Efficient Markets

A Large-Market Rational Expectations Equilibrium Model

Xavier Vives[*]

IESE Business School

December 2013

Abstract

This paper presents conditions for a resolution of the Grossman-Stiglitz paradox of informationally efficient markets. We display a market with asymmetric information where a privately revealing equilibrium obtains in a *competitive* framework and where incentives to acquire information are preserved as long as the correlation in traders' valuations is not too large. The equilibrium is efficient, and the problems associated with fully revealing rational expectations equilibria are precluded without resorting to noise traders. The model is applied to explain changes in bidding behavior in central bank liquidity auctions in the crisis period. The robustness of the results to the presence of market power is tested in a large market approximation to the competitive economy.

Keywords: adverse selection, information acquisition, double auction, multi-unit auctions, rate of convergence, market power, complementarities, liquidity and Treasury auctions

JEL Codes: D82, D84, G14, E59

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[*] This paper was delivered as the Schumpeter lecture in the 28th Congress of the EEA in Gothenburg in 2013. I am grateful to Carolina Manzano, Bruno Biais, Paul Klemperer, Jaume Ventura and to participants at presentations of early versions of the paper at ESEM in Oslo (August 2011), ESSET (July 2012), Arizona State University and Caltech (April 2012) for helpful comments. I thank also Rodrigo Escudero and Jorge Paz for excellent research assistance. The research reported in this paper has been supported by European Advanced Grant (no. 230254). Complementary support from project and ECO2011-29533 of the Spanish Ministry of Education and Science at the Public-Private Sector Research Center at IESE is acknowledged.

1. Introduction

The financial crisis has questioned the informational efficiency of the market, popularized as the “efficient market hypothesis”.[1] Indeed, commentators point at bubbles in stock markets (e.g. tech bubble) and in real estate markets as evidence of the demise of the hypothesis. A basic problem in an informationally efficient market was pointed out by Grossman and Stiglitz (1976, 1980). As stated plainly by John Kay (FT July 16, 2013): "A contradiction lies at the heart of the efficient market hypothesis: if market prices did incorporate all available information about the value of an asset, no one would have an incentive to obtain that information in the first place." The implications of the lack of market informational efficiency are important for investment decisions, accounting (mark to market), and managerial incentives. We keep relying on prices, however. When prices for CDS contracts shoot up for banks, firms, or countries we infer trouble ahead (in terms of an increase in probability of default); when interbank spreads spike we infer liquidity and solvency problems for banks. A question therefore is whether the reliance on prices is warranted and how information issues help explain behavior by market participants during a crisis.

Rational expectations models have proved to be a workhorse for the analysis of situations involving uncertainty and private information. An important aim has been to provide a workable model of Hayek’s (1945) idea that prices aggregate the dispersed information of agents in the economy, given prices’ dual role as index of scarcity and conveyors of information. However, the concept of a rational expectations equilibrium (REE) is not without problems—and this is especially true of fully revealing REE in competitive markets.[2] The concept has two main difficulties. First, the above mentioned Grossman-Stiglitz paradox: If information is costly and prices are fully revealing, and if the traders perceive that they cannot affect the informational content of prices, then at equilibrium no trader pays to gather information because all information (including his) is already freely

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[1] "For more than four decades, financial markets and the regulations that govern them were underpinned by what is known as the efficient markets hypothesis. All that changed after the financial crisis." (Norma Cohen, FT, Jan 24, 2012)

[2] The concept of a fully revealing REE is relevant since, absent other frictions, this equilibrium is Pareto optimal.

available in the prices. However, if no trader gathers information, then the prices cannot convey information; and if prices convey no information and information is sufficiently cheap then some trader will want to acquire information. In those circumstances there is no equilibrium and fully revealing equilibrium prices are logically impossible (Grossman and Stiglitz (1976, 1980); Matthews (1984), Jackson (2003) for auctions). Second, the equilibrium need not be implementable; that is, it may not be possible to find a trading mechanism (in a well-specified game) that delivers the fully revealing REE. An added problem arises if the competitive REE is defined in a finite-agent economy, since then traders realize that prices convey information but do not realize the impact of their actions on the price (this is the "schizophrenia" problem of Hellwig (1980)). These problems are typically overcome by considering noisy REE in large economies. Indeed, noise traders in competitive models have prevented trade from collapsing.[3]

This paper presents a simple, competitive, large-market model without the recourse to noise traders and in which the valuation of each trader has a common and a private value component. A key ingredient is that each trader receives a private signal which provides "bundled" information for both the common and the private value components. It shows how to obtain a privately revealing equilibrium in a well-specified game where each trader submits a demand schedule and has incentives to rely on his private signal and on the price. In a privately revealing equilibrium the price *and* the private signal of a trader are sufficient statistics for the pooled information of all traders in the market. The equilibrium is efficient, preserves incentives to acquire information provided that the common value component of the valuation is not overwhelming, and overcomes the problems of fully revealing REE without reliance on noise trading. Therefore, our equilibrium provides the conditions for a resolution of the Grossman-Stiglitz paradox. Furthermore, the Bayesian equilibrium in demand schedules obtained in the large market is not an artifact of the continuum specification for traders. We verify that the large limit market equilibrium approximates well large finite markets equilibria in which traders are strategic and have incentives to influence prices, provided that the limit equilibrium calls

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[3] See, for example, Diamond and Verrecchia (1981) and Admati (1985).

for positive information acquisition. This delivers a foundation for REE in the context of the model presented.

The model is of the linear-normal variety, as in Grossman and Stiglitz (1980), and it assumes declining marginal valuations. It is quite tractable and allows us to address the case of a good with an elastic exogenous supply as well as the case of a double auction; in addition, it enables us to characterize explicitly not only information acquisition but also rates of convergence of finite markets to the continuum limit. The model admits interpretation in terms of both financial markets and markets for goods.

We find that there is a unique linear equilibrium. In equilibrium, a high price indicates a high valuation, and this reduces responsiveness to price when there is private information. Indeed, demand schedules in this case are steeper and there is a greater extent of adverse selection in the market,[4] which increases with the correlation of valuations and the noise in the signals. If the information effect is large enough, demand schedules may be upward sloping. Demand, as long as it is downward sloping, becomes steeper also as the slope of marginal valuation is steeper and as the slope of exogenous supply is flatter. The case of a downward-sloping exogenous supply of the good allows us to capture complementarities among the agents in the market, and makes aggregate excess demand upward sloping.

If the signals are costly to acquire and if traders face a convex cost of acquiring precision, then there is an upper bound on the correlation of valuation parameters below which there are incentives to purchase some precision. This upper bound is decreasing in the precision of the prior and in the marginal cost of acquiring precision; this bound is 1—that is, perfect correlation—when the marginal cost (at zero precision) of acquiring precision is zero or when the prior is diffuse. A more diffuse prior or less correlation among valuations induces more effort to acquire information, and this effort is, in fact,

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[4] See Akerlof (1970) and Wilson (1980).

socially optimal. The parameter region where the Grossman-Stiglitz paradox is maintained increases as correlation increases and we approach the common value case.

The continuum economy is an idealization that allows solving Hellwig's trader schizophrenia problem since in the limit economy price-taking behavior is individually optimal. We check how large a market is needed for the equilibrium in the continuum economy to approximate well a finite market. The rate at which equilibria in finite replica markets (with n traders and corresponding exogenous supply) approach the equilibrium in the continuum economy is $1/\sqrt{n}$, the same rate at which the average signal of the traders tends to the limit average valuation parameter. Convergence accelerates as we approach a common value environment with better signals or with less prior uncertainty. The corresponding (per capita) welfare loss in the finite market with respect to the limit market is of the order of $1/n$, and again convergence is faster when closer to the common value case or when there is less prior uncertainty. However, the effect of noise in the signals is ambiguous here because it has opposing effects on allocative and distributive efficiency.

The convergence results extend to the endogenous information acquisition case as long as the equilibrium in the continuum economy calls for a positive purchase of information. However, when this is not the case we have situations where the limit of equilibria with positive information acquisition in finite economies as the market grows large has no information purchase but this is not an equilibrium in the continuum economy, or where an equilibrium with no information purchase in the continuum economy cannot be approximated by equilibria in finite economies. In the latter case the equilibrium in the continuum economy is an artifact of the continuum specification. The root of those results is a discontinuity in the equilibrium which obtains when one trader purchases information when the others have not in a finite market. The informed trader acquires a discrete amount of market power by purchasing a little bit of private information. This may destroy a no information purchase equilibrium. On the other hand, market power may induce information purchase in a finite market when in the continuum market there would be no equilibrium.

The model developed here can be applied to explaining how banks bid for liquidity in central bank auctions (or how bidders behave in Treasury auctions). In particular, the model can be used to simulate the impact of a financial crisis on central bank liquidity auctions. We will see how adverse selection in a context where auction prices are informative may explain the fact that aggregate bid demands became much steeper after the subprime crisis episode in 2007 or Lehman Brothers failure in 2008.

There have been several attempts to resolve the Grossman-Stiglitz paradox in a common value environment. The most popular approach is to include noise traders who make prices not fully revealing as in Grossman and Stiglitz (1980), Hellwig (1980) or Admati (1985).[5] This approach has been refined with the consideration of endowment shocks to traders (Diamond, and Verrecchia (1981), Verrecchia (1982), Ganguli and Yang (2009), Manzano and Vives (2011)), and uncertainty with larger dimension than prices (Allen (1981), Ausubel (1990)). In this approach it is found that there is strategic substitutability in information acquisition.[6]

Another line of attack has been to consider traders with market power in models with a finite number of traders. This is the case of Kyle (1989) and Jackson (1991) studying demand submission games. Kyle (1989) shows that equilibrium is well defined when noise trading is nonzero and as this noise vanishes prices do not become fully revealing because informed agents trade smaller and smaller amounts. Jackson (1991) shows the possibility of fully revealing prices with costly information acquisition, under some specific parametric assumptions, because traders realize that their actions influence prices. Kovalenkov and Vives (2013) show that risk neutral competitive traders would not enter a market and become informed (a variant of the Grossman-Stiglitz paradox). But if they are strategic, as the market grows large (parameterized by the amount of noise trading) the number of informed traders grows less than proportionately than the size of

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[5] According to Black (1986) noise trading "makes financial markets possible" and provides incentives "for people to seek out costly information which they will trade on".

[6] This explains, for example, that as noise trading vanishes the informativeness of the price is bounded above (Verrechia (1982)). The strategic substitutability result is robust in a model with endowment shocks which induce multiple equilibria (Manzano and Vives (2011)).

the market. Then prices become fully revealing because the aggregate response to private information grows even faster than noise trading and the incentives to acquire information are preserved.

Still, other work assumes non-expected utility traders: Krebs (2007) and Muendler (2007) resolve the paradox with schizophrenic agents who take into account the impact on market prices when choosing the quality of their information but not when they trade in the asset market; Condie and Ganguli (2011) and Mele and Sangiorgi (2009) consider ambiguity-averse traders.

This paper is related to the literature of information aggregation in auctions, Cournot markets, and markets in which traders compete in demand and/or supply functions. First, it is related to work on information aggregation, and on the foundations of REE in auction games, that developed from the pioneering studies of Wilson (1977) and Milgrom (1981) and have more recently been extended by Pesendorfer and Swinkels (1997). The convergence to price taking and to efficiency as double auction markets grow large has been analyzed in Wilson (1985), Satterthwaite and Williams (1989), and Rustichini, Satterthwaite, Williams (1994), and Cripps and Swinkels (2006). Our results on the model's double auction version are more closely related to Reny and Perry (2006), who present a double auction model with unit bids with a unique and privately revealing REE that is implementable as a Bayesian equilibrium, and also offer an approximation in a finite large market. Given the nature of our own model, the results presented here deal with multi-unit demands and enable characterizations of an equilibrium's comparative static properties and of information acquisition. Furthermore, the model allows us to study convergence rates and to analyze the effect of an exogenous supply of the good. Second, a parallel literature on information aggregation has developed in the context of Cournot markets (Palfrey (1985); Vives (1988)). Dynamic extensions of the models have allowed fully revealing prices, but with a temporal lag, not allowing agents to condition their demands on current prices (e.g. learning from past prices; Hellwig (1982), Dubey, Geanakoplos and Shubik (1987), Vives (1988)).[7]

Third, the paper is related to the literature on strategic competition in terms of schedules in uniform price auctions developed from the seminal work of Wilson (1979) and Kyle (1989) (see also Wang and Zender (2002)). Vives (2011a,b) considers *strategic* supply competition and provides a finite-trader counterpart to the model in this paper.[8]

The balance of the paper is organized as follows. Section 2 presents the model. Section 3 summarizes the problems with the concept of a fully revealing REE and introduces our approach and the interpretations of the model. Section 4 characterizes the equilibrium and its properties; Section 5 presents some extensions of the model, the case of inelastic supply and complementarities. Section 6 deals with information acquisition, and Section 7 considers large but finite markets. Concluding remarks close the paper and the Appendix gathers the proofs of the results.

2. The model

A continuum of traders—indexed in the unit interval $i \in [0,1]$, which is endowed with the Lebesgue measure—face a linear, downward-sloping inverse supply for a homogenous good $p = \alpha + \beta \tilde{x}$. Here $\alpha, \beta > 0$ and $\tilde{x}$ denotes aggregate quantity in our continuum economy (and also per capita quantity, since we have normalized the measure of traders to 1). We have $\tilde{x} = \int_0^1 x_i \, di$, where x_i is the individual quantity demanded by trader i. We interpret $x_i < 0$ to mean that the trader is a (net) supplier.

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[7] See also Golosov et al. (2013) for a dynamic decentralized foundation of fully revealing equilibria.

[8] In this latter paper a rate of convergence to price-taking behavior of $1/n$ is obtained, which is faster than the rate of convergence of prices to the equilibrium in the continuum economy $1/\sqrt{n}$ in the present paper. Rostek and Weretka (2012) consider competition in schedules with an asymmetric correlation structure for valuation parameters.

Traders are assumed to be risk neutral. The profits of trader i when the price is p are

$$\pi_i = (\theta_i - p)x_i - \frac{\lambda}{2}x_i^2,$$

where θ_i is a value idiosyncratic to the trader and λx_i is a marginal transaction, opportunity or limit to arbitrage cost (it could also be interpreted as a proxy for risk aversion).

We assume that θ_i is normally distributed (with mean $\overline{\theta} > \alpha$ and variance σ_θ^2). The parameters θ_i and θ_j, $j \neq i$, are correlated with correlation coefficient $\rho \in [0,1]$. We therefore have $\operatorname{cov}[\theta_i, \theta_j] = \rho\sigma_\theta^2$ for $j \neq i$. Trader i receives a signal $s_i = \theta_i + \varepsilon_i$; all signals are of the same precision, and ε_i is normally distributed with $E[\varepsilon_i] = 0$ and $\operatorname{var}[\varepsilon_i] = \sigma_\varepsilon^2$. Error terms in the signals are correlated neither with themselves nor with the θ_i parameters.

Our information structure encompasses the case of a common value and also that of private values. If $\rho = 1$, the valuation parameters are perfectly correlated and we are in a *common value* model. When $0 < \rho < 1$, we are in a *private values* model if signals are perfect and $\sigma_\varepsilon^2 = 0$ for all *i*; traders receive idiosyncratic, imperfectly correlated shocks, and each trader observes her shock with no measurement error. If $\rho = 0$, then the parameters are independent and we are in an *independent values* model. Under our assumption of normality, conditional expectations are affine.[9]

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[9] With Gaussian distributions there is positive probability that prices and quantities are negative in equilibrium. We can control for this if necessary by restricting the variances of the distributions and of the parameters α, β, λ, and $\overline{\theta}$.

Let the average valuation parameter be $\tilde{\theta} \equiv \int \theta_j \, dj$, normally distributed with mean $\overline{\theta}$ and $\operatorname{cov}\left[\tilde{\theta}, \theta_i\right] = \operatorname{var}\left[\tilde{\theta}\right] = \rho\sigma_\theta^2$ [10] The dispersion in valuations is given by $\mathbb{E}\left[\left(\theta_i - \tilde{\theta}\right)^2\right] = (1-\rho)\sigma_\theta^2$. It is maximal for $\rho = 0$ and minimal for $\rho = 1$. An equivalent formulation that highlights the aggregate and idiosyncratic components of uncertainty is to let $\eta_i \equiv \theta_i - \tilde{\theta}$ and observe that $\theta_i = \tilde{\theta} + \eta_i$, where $\operatorname{cov}\left[\eta_i, \tilde{\theta}\right] = 0$ and $\operatorname{cov}\left[\eta_i, \eta_j\right] = 0$ for $i \neq j$. It is worth emphasizing that signal $s_i = \tilde{\theta} + \eta_i + \varepsilon_i$ provides "bundled" information on the aggregate $\tilde{\theta}$ and idiosyncratic η_i components.

We adopt the convention that the average of independent and identically distributed random variables with mean zero is zero.[11] We then have $\tilde{s} \equiv \int s_i \, di = \int \theta_i \, di + \int \varepsilon_i \, di = \tilde{\theta}$ almost surely, since $\int \varepsilon_i \, di = 0$ according to our convention. Note that if $\rho = 0$ then $\tilde{\theta} = \overline{\theta}$ (a.s.).

3. Rational expectations equilibrium and the demand schedule game

In this section we begin by defining REE and expounding on its problems. We then move on to our game-theoretic approach and interpretations of the model.

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[10] This can be justified as the continuum analogue of the finite case with n traders. Then, under our assumptions, the average parameter $\tilde{\theta}_n$ is normally distributed with mean $\overline{\theta}$, $\operatorname{var}\left[\tilde{\theta}_n\right] = \left(1 + (n-1)\rho\right)\sigma_\theta^2 n^{-1}$, and $\operatorname{cov}\left[\tilde{\theta}_n, \theta_i\right] = \operatorname{var}\left[\tilde{\theta}_n\right]$. The result is obtained by letting n tend to infinity.

[11] See Vives (1988) for a justification of this convention. In any event, we will see that the equilibrium in the continuum economy is the limit of equilibria in the appropriate finite economies under the standard laws of large numbers.

3.1. Rational expectations equilibrium

A (competitive) rational expectations equilibrium is a (measurable) price function mapping the average valuation (state of the world) $\tilde{\theta}$ into prices $P\left(\tilde{\theta}\right)$ and a set of trades $x_i,\ i \in \left[0,1\right]$, such that the following two statements hold.

(1) Trader i maximizes its expected profit, $E\left[\pi_i \left| s_i, p\right.\right]$, conditional on knowing the functional relationship $P\left(\tilde{\theta}\right)$ as well as the underlying distributions of the random variables.

(2) Markets clear: $Z\left(p\right) \equiv \int_0^1 x_i\, di \ - \ \beta^{-1}\left(p-\alpha\right) = 0$.

Thus each trader optimizes while taking prices as given, as in the usual competitive equilibrium, but infers from prices the relevant information.

This equilibrium concept may be problematic. Consider the common value case ($\rho = 1$); we shall present a fully revealing REE that is not implementable. Suppose there is a competitive equilibrium of a full information market in which the traders know θ. At this equilibrium, price equals marginal benefit, $p = \theta - \lambda x_i$; therefore, individual demand is $x_i = \lambda^{-1}\left(\theta - p\right)$. The equilibrium price is given by the market-clearing condition $Z\left(p\right) = 0$ and is equal to $p = \left(\lambda\alpha + \beta\theta\right)/\left(\lambda + \beta\right)$. This allocation is also a fully revealing REE of our economy (Grossman (1981)). Indeed, looking at the price allows each trader to learn θ, which is the only relevant uncertainty, and the allocation is a REE equilibrium because traders optimize and markets clear. However, this REE has a strange property: the price is fully revealing even though a trader's demand is independent of the signals received. The question is then how has the information been incorporated into the price or what is the game and the market microstructure that yields such a result. In this case we cannot find a game that delivers as an equilibrium the fully revealing REE.[12]

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[12] If we were to insist that prices be measurable in excess demand functions, then the fully revealing REE would not exist (see Beja (1977); Anderson and Sonnenschein (1982)). However, fully revealing REE are implementable if each agent is informationally "small" or irrelevant in the sense that his private information can be predicted from the joint information of other agents (Palfrey and Srivastava (1986); Postlewaite and Schmeidler (1986); Blume and Easley (1990)).

3.2. The demand schedule game

We will restrict our attention to REE that are the outcome of a well-specified game—that is, implementable REE. The natural way to implement competitive REE in our context is to consider competition among demand functions (see Wilson (1979); Kyle (1989)) in a market where each trader is negligible.[13]

We assume that traders compete in terms of their demand functions for the exogenous supply of the good. The game's timing is as follows. At $t = 0$, random variables $(\theta_i)_{i\in[0,1]}$ are drawn but not observed. At $t = 1$, traders observe their own private signals, $(s_i)_{i\in[0,1]}$, and submit demand functions $X_i(s_i, \cdot)$ with $x_i = X_i(s_i, p)$, where p is the market price. The strategy of a trader is therefore a map from the signal space to the space of demand functions. At $t = 2$ the market clears, demands are aggregated and crossed with supply to obtain an equilibrium price,[14] and payoffs are collected. An implementable REE is associated with a Bayesian Nash equilibrium of the game in demand functions. Hereafter we discuss only the linear Bayesian demand function equilibrium (DFE).

3.3. Interpretations of the model

The model and game admit several interpretations in terms of financial markets and markets for goods as long as there are enough participants to justify the use of the continuum model assumption (this issue is dealt formally with in Section 7).

The good may be a financial asset such as central bank funds or Treasury notes, and the traders are the bidders (banks and other intermediaries) in the auction who use demand functions. In the open-market operation of central bank funds, the average valuation $\tilde{\theta}$ is related to the average price (interest rate) in the secondary interbank market which is mostly over-the-counter. The valuation θ_i for bank i reflects thus the terms that this

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[13] See Gul and Postlewaite (1992) and Mas-Colell and Vives (1993) for results on the implementation of efficient allocations in large economies.

[14] If there is no market-clearing price then assume that the market closes; if there are many market-clearing prices, choose the one that maximizes volume.

bank obtains in the secondary market as well as its liquidity needs (because of reserve requirements for example) and the bank receives an imperfect signal about its valuation. A bidder bank must offer the central bank collateral in exchange for funds, and the bidder's first preference is to offer the least liquid one. Given an increased allotment of funds, the bank must offer more liquid types of collateral at a higher opportunity cost; this implies a declining marginal valuation for the bidder with λ reflecting the structure of a counterparty's pool of collateral.[15]

In a Treasury auction, bidders will have private information related to different expectations about the future resale value $\tilde{\theta}$ of the securities (e.g., different beliefs concerning how future inflation will affect securities denominated in nominal terms) and to the idiosyncratic liquidity needs of traders.[16] We should expect that the common value component is more significant in Treasury auctions than in central bank auctions, since the main dealers buy Treasury bills primarily for resale.[17]

The good could also be an input (such as labor of uncertain productivity) whose traders are the firms that want to purchase it. Our model also accommodates the case where firms compete in supply functions to fill an exogenous demand, as in procurement auctions. In this case we assume that $\bar{\theta} < \alpha$, since θ_i is now a cost parameter and typically $x_i < 0$. For example, θ_i could be a unit ex post pollution or emission penalty to be levied on the firm and about which the producer has some private information.

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[15] See Ewerhart et al. (2010) and Cassola et al. (2013).

[16] For example, Hortaçsu and Kastl (2011) cannot reject the hypothesis that bidders in Canadian 3-month T-bill auctions have private values.

[17] See Bindseil et al. (2002).

4. Bayesian demand function equilibrium

In this section we use Proposition 1 to characterize the symmetric[18] equilibrium of the demand schedule game before discussing its properties. We then extend the range of the model in the next section to double auctions and inelastic supply and market structures with complementarities.

Proposition 1. *Let* $\rho \in (0,1)$ *and* $\sigma_\varepsilon^2 / \sigma_\theta^2 < \infty$. *Then there is a unique symmetric DFE given by*

$$X(s_i, p) = \left(E\left[\theta_i \middle| s_i, p\right] - p\right)\lambda^{-1} = b + as_i - cp\,.$$

Here

$$a = \frac{1}{\lambda(1+M)}, \quad b = -\frac{\alpha}{\beta}(1-\lambda a), \quad c = \frac{1}{\beta}\left(a(\beta+\lambda)-1\right),$$

and $M = \sigma_\varepsilon^2 \big/ \left((1-\rho)\sigma_\theta^2\right)$. *Moreover,* $a > 0$ and $-\beta^{-1} < c \le a \le \lambda^{-1}$. *Also,* c *is decreasing in M and* λ *and is increasing in* β; $Z' = -\left(c + \beta^{-1}\right) < 0$; *and the equilibrium price is given by*

$$p = \frac{\lambda\alpha + \beta\tilde{\theta}}{\lambda + \beta}\,.$$

It is worthwhile to highlight some properties of this equilibrium.

The equilibrium is, first of all, *privately revealing*.[19] The price p reveals the average parameter $\tilde{\theta}$ and, for trader i, either pair (s_i, p) or $(s_i, \tilde{\theta})$ is a sufficient statistic for the joint information in the market $(s_i, \tilde{s})$ in the estimation of θ_i. In particular, at equilibrium we have $E\left[\theta_i \middle| s_i, p\right] = E\left[\theta_i \middle| s_i, \tilde{\theta}\right]$. The privately revealing character of the equilibrium,

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[18] The symmetry requirement could be relaxed. Then the (linear and symmetric) equilibrium would be unique in the class of (linear) equilibria with uniformly bounded second moments (equivalently, in the class of equilibria with linear price functional of the type $P(\tilde{\theta})$).

[19] See Allen (1981).

which derives from the "bundled" nature of the signal s_i about the common $\tilde{\theta}$ and the idiosyncratic component η_i, implies that the incentives to acquire information are preserved under certain conditions as we shall see in Section 6.

Second, the equilibrium is *efficient*: it is a price-taking equilibrium, the price reveals $\tilde{\theta}$, and traders act with a sufficient statistic for the shared information in the economy.[20] Indeed, at equilibrium we have that price equals marginal benefit with full (shared) information: $p = E\left[\theta_i \middle| s_i, \tilde{\theta}\right] - \lambda x_i$. This would not be the case if traders had market power, since then a wedge would be introduced between price and marginal benefit (see Vives (2011a)). Neither would the equilibrium be efficient if price were noisy, since then a trader would not take into account the information externality that her trade has on other traders through the effect on the informativeness of the price (see, e.g., Amador and Weill (2010); Vives (2013)).

Let $\rho \in (0,1)$ and $\sigma_\varepsilon^2 / \sigma_\theta^2 < \infty$. When signals are perfect ($\sigma_\varepsilon^2 = 0$ and $M \equiv \sigma_\varepsilon^2 \left((1-\rho)\sigma_\theta^2\right)^{-1} = 0$), we have that $a = c = \lambda^{-1}$, $b = 0$, and $x_i = \lambda^{-1}(\theta_i - p)$. Then bidders have nothing to learn from prices, and the equilibrium is just the usual complete information competitive equilibrium (which, we remark, is independent of ρ). When $M > 0$, bidders learn from prices and the demand functions are steeper: $c < \lambda^{-1}$. Indeed, the larger is M (which is increasing in ρ and in $\sigma_\varepsilon^2 / \sigma_\theta^2$ and which is related to the degree of adverse selection), the more important is the common value component and the steeper are the demand functions (lower c). The response to a price increase is to reduce the amount demanded according to the usual scarcity effect, but this impulse is moderated (or even reversed) by an information effect, via $E\left[\theta_i \middle| s_i, p\right]$, because a high price conveys the good news that the average valuation is high. Indeed, if $\beta\lambda^{-1} = M$ then

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[20] A fully revealing REE must be ex post Pareto optimal. The reason is that it can be viewed as the competitive equilibrium of an economy with fully informed agents and so, according to the first welfare theorem, it cannot be improved on by a social planner with access to the pooled information of agents (Grossman (1981)).

$c=0$; for larger values of M, we have $c<0$ and demand is upward sloping.[21] Increases in M make the demand function (evaluated at $s_i=\overline{\theta}$) rotate around the point $(\overline{x},\overline{p})$ where $\overline{x}\equiv(\alpha-\overline{\theta})/(\beta+\lambda)$ and $\overline{p}\equiv(\lambda\alpha+\beta\overline{\theta})/(\beta+\lambda)$ are, respectively, the expected output and price. (See Figure 1.) This follows easily from the fact that $X(\overline{\theta},p)=\overline{x}-c(p-\overline{p})$.

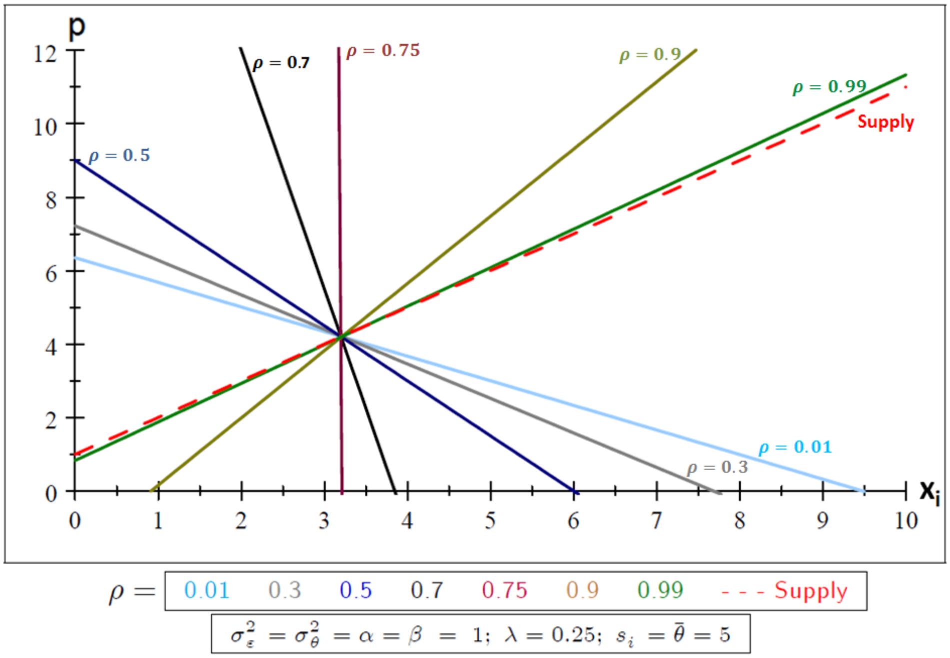

Figure 1. Equilibrium demand as a function of ρ. Demands go through the point $(\overline{x},\overline{p})=(3.2,4.2)$.

As $M\to\infty$ (be it because $\rho\to1$ or $\sigma_\varepsilon^2\to\infty$) we have that $a\to0$, $b\to-\alpha/\beta$, and $c\to-1/\beta$. Then the linear equilibrium collapses because, in the limit, traders put no weight on their private signals. If $\rho=1$ (and $0<\sigma_\varepsilon^2<\infty$) then, as stated in Section 3, there is a fully revealing REE but it is not implementable. In the common value case, the equilibrium breaks down. When signals are pure noise ($\sigma_\varepsilon^2=\infty$), the equilibrium is

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[21] C. Wilson (1980) finds an upward-sloping demand schedule in a market with asymmetric information whose quality is known only to the sellers.

$X\left(p\right)=\lambda^{-1}\left(\overline{\theta}-p\right)$ because $E\left[\theta_i\left|s_i,p\right.\right]=\overline{\theta}$ (even if $\rho=1$). However, this equilibrium is not the limit of DFE as $\sigma_\varepsilon^2\to\infty$ ($M\to\infty$).

If $\rho=0$ then there is asymmetric information but the price conveys no information on values, $c=\lambda^{-1}$, and $X\left(s_i,p\right)=\lambda^{-1}\left(E\left[\theta_i\left|s_i\right.\right]-p\right)$. Again this is not the limit of DFE as $\rho\to0$. However, it can be checked that there is no discontinuity in outcomes.

As the transaction cost λ increases, a decreases and demand becomes steeper (c decreases). As $\lambda\to0$ and limits to arbitrage disappear we have that $a,c\to\infty$ and p tends to $\tilde{\theta}$. As β decreases supply becomes flatter and equilibrium demand is less elastic until the point it becomes vertical (from $\beta=\lambda M$); beyond that point demand is upward sloping, and as $\beta\to0$ demand becomes flat at the level $p=\alpha$.[22]

In summary, demand schedules, as long as they are downward sloping, are steeper with a higher degree of adverse selection in the market (increasing with the correlation of valuations and with the noise in the signals); with a steeper slope of the marginal valuation; and with a flatter slope of exogenous supply.

5. Extensions

In this section we extend the model to some boundary cases and new interpretations: inelastic supply and double auctions and complementarities.

5.1. Inelastic supply and double auctions

The case in which an auctioneer supplies q units of the good is easily accommodated by letting $\beta\to\infty$ and $\alpha/\beta\to-q$. From the inverse supply function we obtain the average

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[22] Note that the equilibrium demand can also be written as follows: $X\left(s_i,p\right)=\left(1-\lambda a\right)\beta^{-1}\left(p-\alpha\right)+a\left(s_i-p\right)$.

quantity $y = (p - \alpha)/\beta \to q$; then $c \to a$ and $b \to (1 - \lambda a)q$. Here demand is always downward sloping, and the strategy of a trader is $X(s_i, p) = (1 - \lambda a)q + a(s_i - p)$ and $p = \tilde{\theta} - \lambda q$. The good can be in zero net supply ($q = 0$) as in a double auction, in which case $b = 0$ and $p = \tilde{\theta}$.[23] A trader is a buyer or a seller depending on whether her private signal is larger or smaller than the price.

Reny and Perry (2006) obtain a related result in a double auction with a unit mass of traders, each of whom desires at most one unit of the good. In contrast to our model, in Reny and Perry's model there is a common value for the good but the payoff of a trader depends directly on the signal he receives. This signal provides a private value component to the trader's valuation. Unlike the case for our DFE, traders in a double action with unit bids cannot condition on the market price because they submit a single bid that is contingent only on private information.[24] Nonetheless, there is a unique (and privately revealing) REE that is implementable as a Bayesian equilibrium of the double auction in symmetric increasing bidding strategies. The equilibrium is *privately revealing* because the price reveals the value of the good and this, together with the signal received by a trader, determines his payoff. The equilibrium is *efficient* because the privately revealing REE is just the competitive equilibrium when the state is known. This REE is implementable as a double auction even in a pure common value case (when the valuation of a trader is independent of his signal), in contrast to the demand competition model, owing to the double auction mechanism with bids for a single unit. At the REE both buyers and sellers are indifferent between using (or not) their private signal, so they might as well use it.

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[23] In this case there is also a no-trade equilibrium.

[24] Once traders have received their signals, they submit bids to the auctioneer. A buyer (resp., seller) indicates the maximum (minimum) price she is willing to pay for (resp., for which he is willing to sell) the desired unit. The auctioneer then uses the bids to form supply and demand schedules and finds a market-clearing price. Buyers whose bids are above the market-clearing price obtain one unit, and those with bids below the market-clearing price come away with nothing.

5.2. Complementarities

Letting $\beta < 0$ allows for complementarities. For example, if traders are suppliers ($x_i < 0$) then $\beta < 0$ means that increasing the aggregate quantity leads to price increases, a dynamic typical of network goods; conversely, if traders are demanders ($x_i > 0$) then $\beta < 0$ means that increasing the aggregate quantity lowers the price, as may occur with labor supply when the income effect dominates. We can allow negative values of β with $0 > \beta > -\lambda\alpha/\bar{\theta}$. The last inequality ensures that $E[p] > 0$ in equilibrium and implies that $\beta + \lambda > 0$ (since $\bar{\theta} - \alpha > 0$). When $\beta < 0$ we have that $-\beta^{-1} > c \geq a$ and that c increases in *M*; in other words, the exogenous supply is downward sloping and an increase in M makes demand flatter. Furthermore, excess demand is upward sloping: $Z' = -\left(c + \beta^{-1}\right) > 0$. Now the information and the scarcity effect work in the same direction, and a high price conveys the unequivocal bad news that the average valuation is low.

6. Information acquisition

Now suppose that, in a first stage of the game, private signals must be purchased at a cost, which is increasing and convex in the precision $\tau_\varepsilon \equiv 1/\sigma_\varepsilon^2$ of the signal,[25] according to a smooth function $H(\cdot)$ that satisfies $H(0) = 0$ with $H' > 0$ for $\tau_\varepsilon > 0$, and $H'' \geq 0$. Hence there are nonincreasing returns to information acquisition. At a second stage, traders receive signals according to the precision purchased and compete in demand functions. We look for subgame-perfect equilibria of the game.

Suppose that, at the first stage, all traders but i have chosen a precision $\tau_\varepsilon > 0$. Then the market equilibrium (which is unaffected by the actions of a single trader) exhibits, according to Proposition 1, the price $p = \left(\lambda\alpha + \beta\tilde{\theta}\right)/(\lambda + \beta)$, a price that reveals $\tilde{\theta}$.

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[25] For a random variable η, we use τ_η to denote $1/\sigma_\eta^2$.

This implies that the expected profit of trader i at the second stage $E\left[\pi_i\right]$ does not depend on the average precision τ_ε, because the price reveals $\tilde{\theta}$, but it does depend on his precision of information τ_{ε_i} since his private signal is used to estimate θ_i. It can be checked then that the marginal benefit of increasing τ_{ε_i} is

$$\frac{\partial E\left[\pi_i\right]}{\partial \tau_{\varepsilon_i}} = \frac{(1-\rho)^2}{2\lambda\left(\tau_\theta + (1-\rho)\tau_{\varepsilon_i}\right)^2}.$$

This marginal benefit is decreasing in ρ, τ_{ε_i}, τ_θ and λ provided that $\rho < 1$. This fact leads to the following result.

<u>Proposition 2</u>. *Let* $\rho \in [0,1)$ and $\bar{\rho} \equiv 1 - \tau_\theta \sqrt{2\lambda H'(0)}$. *There is a unique symmetric equilibrium in the two-stage game with costly information acquisition where:*

- $\tau_\varepsilon^* = 0$ *if* $\left(2\lambda\tau_\theta^2\right)^{-1} \leq H'(0)$, or equivalently $\bar{\rho} \leq 0$.
- $\tau_\varepsilon^* > 0$ *if* $H'(0) < (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$, or equivalently $\bar{\rho} > \rho \geq 0$, then τ_ε^* *is decreasing in* λ, τ_θ, *and* ρ.

Otherwise, if $\left(2\lambda\tau_\theta^2\right)^{-1} > H'(0) \geq (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$, or equivalently $\rho \geq \bar{\rho} > 0$, *there is no equilibrium.*

In short, if $\bar{\rho} \leq 0$ there is no information acquisition since the marginal benefit of acquiring information at 0 is lower than the marginal cost $\left(2\lambda\tau_\theta^2\right)^{-1} \leq H'(0)$. If $\bar{\rho} > 0$ ($\left(2\lambda\tau_\theta^2\right)^{-1} > H'(0)$) then with not too high correlation, $\rho < \bar{\rho}$, there is an equilibrium with information acquisition. However, as $\rho \to \bar{\rho}$ we have $\tau_\varepsilon^* \to 0$, and the demand function equilibrium collapses. With high correlation, $\rho \geq \bar{\rho} > 0$, we would have no information acquisition at a candidate equilibrium but in fact no equilibrium exists.

Indeed, if $\left(2\lambda\tau_\theta^2\right)^{-1} > H'(0)$ it will benefit a single trader to purchase information if the others do not and no information acquisition cannot be an equilibrium. (See Figure 2.)[26]

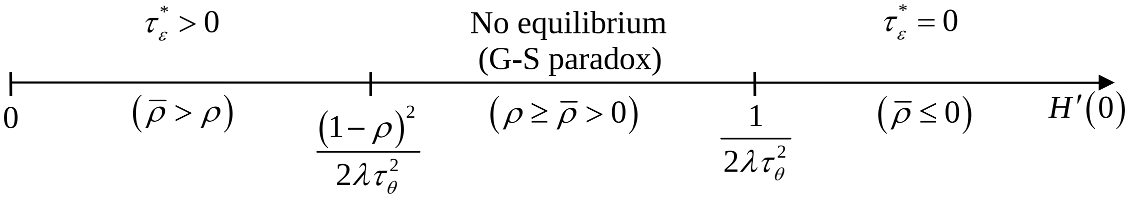

Figure 2. The range of the Grossman-Stiglitz paradox in the information acquisition game (where: $\bar{\rho} = 1 - \tau_\theta\sqrt{2\lambda H'(0)}$).

The range of parameters where there is no equilibrium, $\rho \geq 1 - \tau_\theta\sqrt{2\lambda H'(0)} > 0$, is where the Grossman-Stiglitz paradox of the impossibility of an informationally efficient market survives. Therefore, in particular, we find that when $\rho < 1$ an equilibrium exists if $H'(0) = 0$ or if the prior is diffuse (τ_θ small). In this case moving away from the pure common value case we resolve the Grossman-Stiglitz paradox. In general, a more diffuse prior, a lower cost of transacting λ, or less correlation of valuations induces more acquisition of information. In fact, as $\lambda \to 0$ we have that $\tau_\varepsilon^* \to \infty$. Hence we see that the incentives to acquire information are preserved because the equilibrium is privately revealing—as long as we are not too close to the common value case, or otherwise the marginal cost of acquiring information at zero precision is zero (and $\rho < 1$).

Remark: It can be shown (see Lemma A in the Appendix) that for $H'(0) < (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ the private and social incentives to purchase information are aligned: the marginal social and private benefits are the same and the market acquires the right amount of information $\tau_\varepsilon^* > 0$. However, when $H'(0) \geq (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ and

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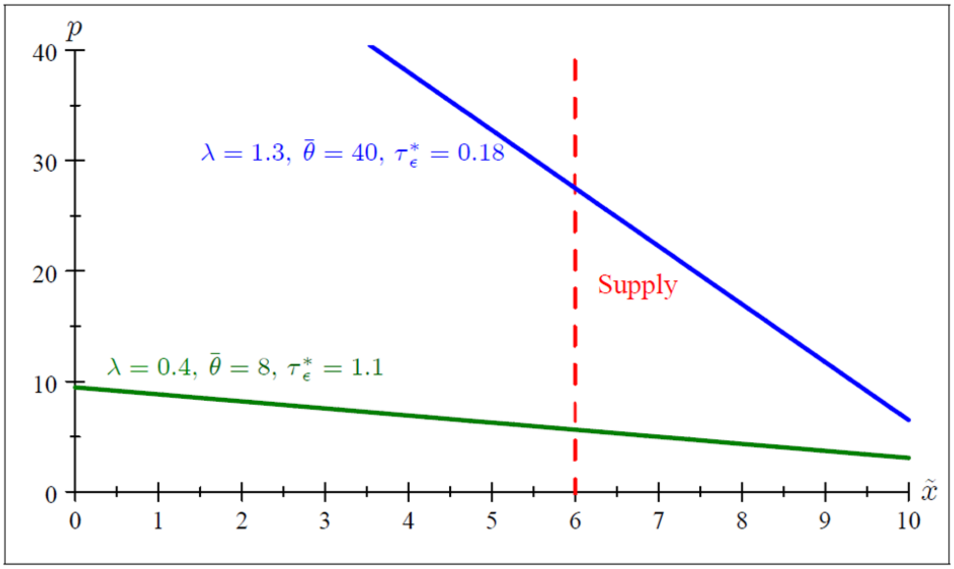
[26] We have argued for a symmetric equilibrium, but in fact, we can show that neither is there an asymmetric equilibrium in the class of trading strategies with bounded second moments.

$\rho > 0$ there is no welfare-optimal level of information purchase because of a discontinuity at $\tau_\varepsilon = 0$: We have that $E[TS]$ is decreasing in τ_ε for $\tau_\varepsilon > 0$ but $\lim_{\tau_\varepsilon \to 0^+} E[TS] = \frac{1}{2}\frac{(\bar{\theta} - \alpha)^2 + \rho\sigma_\theta^2}{\beta + \lambda}$ while $E[TS]\big|_{\tau_\varepsilon = 0} = \frac{1}{2}\frac{(\bar{\theta} - \alpha)^2}{\beta + \lambda}$ is strictly lower for $\rho > 0$. The planner would like to set τ_ε as low as possible but $\tau_\varepsilon = 0$ delivers less surplus. The discontinuity arises since the privately revealing equilibrium is efficient but discontinuous in τ_ε when $\tau_\varepsilon = 0$ for $\rho > 0$.

It is worth noting that in contrast to the classical model with noise traders where there is an upper bound in price informativeness (e.g. Verrecchia (1982)) here either the price is fully revealing or not at all. In the classical case information acquisition decisions are strategic substitutes and the more informative is the price, the less incentives there are to acquire information. In fact, the result in the Grossman-Stiglitz model is that price informativeness is constant as noise trading vanishes. In our model information acquisition decisions are strategically independent since once a positive mass of traders buy information the price is fully revealing of $\tilde{\theta}$.

It is worth remarking that the same outcome would obtain as a Nash equilibrium in a one-shot game where traders choose simultaneously the demand function and the precision of the signal. This corresponds to the case where information acquisition is covert (nonobservable). The equivalence of the games follows from the existence of a continuum of traders. In the covert information acquisition case the equilibrium exists even when $\rho = 1$ while in the sequential game if at the first stage traders where to choose a positive average precision the no equilibrium could exist at the market stage.



$\rho = 0.5;\ \tau_\theta = 0.33;\ q = 6;\ \gamma_1 = 0.2;\ \gamma_2 = 0.15$

Figure 3. Impact of a crisis. Equilibrium demands with endogenous precision when λ and $\bar{\theta}$ increase and, as a consequence, equilibrium precision τ_ε^* decreases.

Application. Consider the example of banks bidding for liquidity and the impact of a crisis. In this scenario we may expect that the correlation ρ of the values of the banks increases (equivalently, that the volatility of the average price $\tilde{\theta}$ in the secondary market for liquidity increases) and that λ also increases as it becomes more costly to supply more liquid collateral (this may correspond to a decrease in collateral quality).[27] The direct effect of an increase in ρ or λ is to make the demand schedules of the banks steeper (Proposition 1), and this effect is reinforced by the induced decrease in information precision (τ_ε^* goes down, according to Proposition 2). The effect of the crisis is thus that demand schedules are steeper and the signals noisier. (See Figure 3 where the model with inelastic supply is simulated). These effects are consistent with the empirical evidence gathered by Cassola et al (2013) when studying European Central Bank auctions and by Allen, Hortaçsu, and Kastl (2011) for Canadian liquidity auctions. These authors find, respectively, that the aggregate bid curve became steeper after the subprime

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[27] See Heider, Hoerova and Holthausen (2010).

crisis in August 2007 and after Lehman's turmoil in 2008.[28] (See Figure 4, taken from Allen et al. (2011)). In Cassola et al. (2013) it is found also that bank's bids reflect not only the opportunity costs of obtaining funds in the interbank market but also a response to other bidders. Suppose that some bidders suffer a deterioration shock to the quality of their collateral and have an increased lambda parameter but others not. Then in equilibrium the first set of bidders use steeper demand schedules and the second do as well in response to them since there is strategic complementarity in the slope of demands because of an information effect with the price being less responsive to $\tilde{\theta}$.[29]

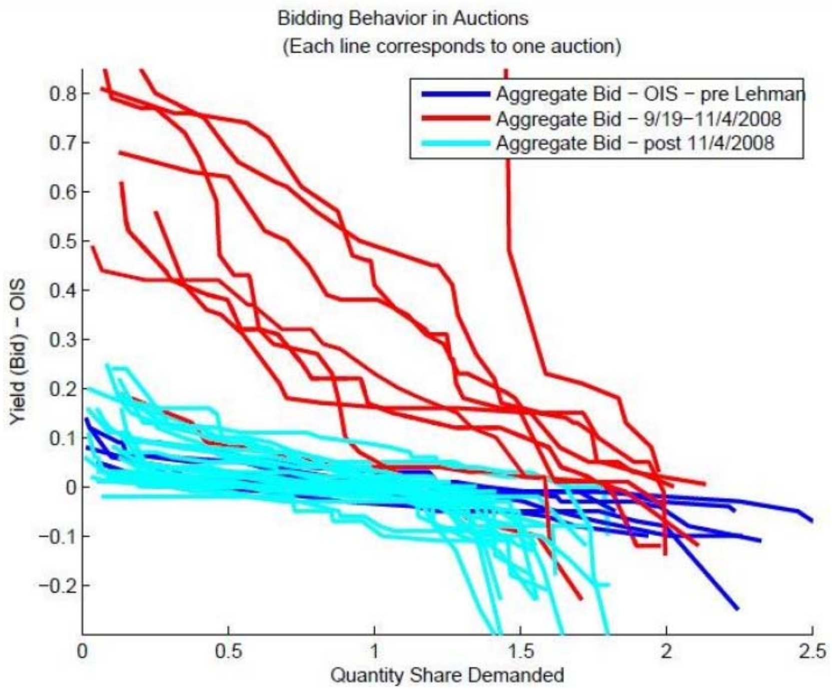

Figure 4. Aggregate demand curves in Canadian liquidity auctions (Fig. 4 in Allen et al. (2011)) and how they become steeper after the crisis of Lehman Brothers and go back to normal afterwards. The vertical axis displays the bid minus the Overnight Index Swap (OIS).[30]

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[28] Market power leading to bid shading may reinforce the steepness of the bid curve (see Vives (2011a)). It should be noted, however, that both ECB and Canadian auctions have been discriminatory, and not uniform price, so far.

[29] This can be checked in a model where traders have different λ parameters.

[30] The spread measures the difference between bids and the rates paid on overnight index swaps (OIS), instruments that are not exposed to the default risk of intermediaries.

7. Finite markets and convergence to the limit equilibrium

The question arises of whether the results obtained in the large market are simply an artifact of the continuum specification. In this section, we answer this question in the negative whenever the equilibrium calls for some information acquisition. In this case, we show, building on the results in Vives (2011a), that the equilibria in finite markets tend to the equilibrium of the continuum economy as the market grows large, which justifies our use of a continuum model to approximate the large market. Furthermore, we check how large a market has to be for the continuum approximation to be useful by computing the rate of convergence to the continuum limit. We also check in what cases the equilibrium in the continuum economy is not a good approximation of large finite markets.

7.1. Equilibrium in a finite market and convergence

Consider the following replica economy. Suppose that inverse supply is given by $P_n\left(y\right)=\alpha+\beta n^{-1}y$; here y is total quantity and n is the number of traders (buyers), each with the same benefit function as before. Increasing n will increase the number of buyers and increase the supply at the same rate. Denote with subscript n the magnitudes in the n-replica market. The information structure is the finite-trader counterpart of the structure described in Section 2. We have that $\tilde{\theta}_n\equiv\left(\sum_{i=1}^{n}\theta_i\right)\Big/n\sim N\left(\bar{\theta},\left(1+\left(n-1\right)\rho\right)n^{-1}\sigma_\theta^2\right)$ and $\operatorname{cov}\left[\tilde{\theta}_n,\theta_i\right]=\operatorname{var}\left[\tilde{\theta}_n\right]$. As n grows large the finite economy converges to the continuum economy since $n^{-1}y$ is average quantity and $\tilde{\theta}_n\to\tilde{\theta}\sim N\left(\bar{\theta},\rho\sigma_\theta^2\right)$, in mean square, and that $\operatorname{cov}\left[\tilde{\theta}_n,\tilde{\theta}\right]=\operatorname{var}\left[\tilde{\theta}\right]$.[31]

It follows from Proposition 1 in Vives (2011a) that, for $\rho\in\left[0,1\right)$, there is a unique (symmetric) DFE of the form $X_n\left(s_i,p\right)=b_n+a_ns_i-c_np$ for any n. The equilibrium is privately revealing, and the price reveals the average signal of the traders, $\tilde{s}_n$. The

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[31] See the Appendix for definitions of "in mean square" and of convergence (and rates of convergence) for random variables.

demand function of a trader is of the form $X_n(s_i, p) = \left(E\left[\theta_i \middle| s_i, p\right] - p\right)\left(d_n + \lambda\right)^{-1}$ where $d_n = \left(\beta^{-1}n + (n-1)c_n\right)^{-1}$ is the wedge or distortion introduced by market power in the presence of asymmetric information. With symmetric information (either $\tau_\varepsilon = 0$ or $\tau_\varepsilon = \infty$) then the equilibrium is independent of ρ, and it exists even if $\rho = 1$, and d_n equals the full information equilibrium level market power distortion.

Consider the case of an inelastic per capita supply of q to illustrate the derivation of equilibrium and its convergence properties as the market grows large.[32]

Suppose that traders $j \neq i$ employ linear strategies, $X(s_j, p) = b + as_j - cp$. Then the market-clearing condition, $\sum_{j\neq i} X(s_j, p) + x_i = nq$, $c \neq 0$, implies that trader i faces a residual inverse supply: $p = I_i + dx_i$, where $d = \left((n-1)c\right)^{-1}$ and $I_i = d\left((n-1)b + a\sum_{j\neq i} s_j - qn\right)$. The (endogenous) parameter d is the slope of inverse residual supply and the wedge introduced by market power. All the information that the price provides to trader i about the signals of others is contained in the intercept I_i. The information available to trader i is $\{s_i, p\}$ or, equivalently, $\{s_i, I_i\}$. Trader i chooses x_i to maximize

$$E\left[\pi_i \middle| s_i, p\right] = x_i\left(E\left[\theta_i \middle| s_i, p\right] - p\right) - \frac{\lambda}{2}x_i^2 = x_i\left(E\left[\theta_i \middle| s_i, p\right] - I_i - dx_i\right) - \frac{\lambda}{2}x_i^2 .$$

The first-order condition (FOC) is $E\left[\theta_i \middle| s_i, p\right] - p = (d + \lambda)x_i$. An equilibrium requires that $d > 0$.[33] A trader bids according to $p = E\left[\theta_i \middle| s_i, p\right] - (d + \lambda)x_i$, and competitive

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[32] See Vives (2010) for an overview of this model and its properties.

[33] The second-order sufficient condition is fulfilled when $d > 0$.

bidding obtains when $d = 0$. A buyer ($x_i > 0$) underbids, $p < E\left[\theta_i \left| s_i, p\right.\right] - \lambda x_i$; since $d > 0$, a seller ($x_i < 0$) overbids, $p > E\left[\theta_i \left| s_i, p\right.\right] - \lambda x_i$.

From the FOC and the Gaussian updating formulas for $E\left[\theta_i \left| s_i, p\right.\right]$, we immediately obtain the coefficients of the linear equilibrium strategy:

$$X_n\left(s_i, p\right) = b_n + a_n s_i - c_n p, \quad c_n = \frac{n-2-M_n}{\lambda\left(n-1\right)\left(1+M_n\right)},$$

where

$$M_n \equiv \frac{\rho\sigma_\varepsilon^2 n}{\left(1-\rho\right)\left(\sigma_\varepsilon^2 + \left(1+\left(n-1\right)\rho\right)\sigma_\theta^2\right)} \quad \text{and} \quad a_n = \frac{\left(1-\rho\right)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + \left(1-\rho\right)\sigma_\theta^2\right)}\left(d_n + \lambda\right)^{-1}$$

for $d_n = \left(\left(n-1\right)c_n\right)^{-1}$. We require $n-2-M_n > 0$ in order to guarantee the existence of an equilibrium (i.e., to obtain $d_n > 0$ and $c_n > 0$). (Observe that the inequality is always fulfilled for n large because M_n is bounded.) The reason for this requirement is that, if the inequality does not hold, then traders will seek to exploit their market power by submitting vertical schedules, and that is incompatible with the existence of equilibrium when there is no elastic exogenous supply.

The equilibrium price p_n reveals the average signal $\tilde{s}_n$; therefore, $E\left[\theta_i \left| s_i, p_n\right.\right] = E\left[\theta_i \left| s_i, \tilde{s}_n\right.\right]$ and $n^{-1}\sum_{i=1}^n E\left[\theta_i \left| s_i, \tilde{s}_n\right.\right] = E\left[\tilde{\theta}_n \left| \tilde{s}_n\right.\right]$. Averaging the FOCs, we obtain that $E\left[\tilde{\theta}_n \left| \tilde{s}_n\right.\right] - p_n = \left(d_n + \lambda\right)\tilde{x}_n = \left(d_n + \lambda\right)q$ and hence $p_n = E\left[\tilde{\theta}_n \left| \tilde{s}_n\right.\right] - \left(d_n + \lambda\right)q$.

We have that $X_n(s_i,p)\xrightarrow{n}(1-\lambda a)q+a(s_i-p)$, the trading strategy in the inelastic supply case in the limit economy.[34] Furthermore, $p_n\xrightarrow{n}p=\tilde{\theta}-\lambda q$ in mean square at the rate $1/\sqrt{n}$. This follows because $d_n\xrightarrow{n}0$ and $E\left[\tilde{\theta}_n|\tilde{s}_n\right]\xrightarrow{n}\tilde{\theta}$ in mean square (given that $\tilde{\theta}_n\xrightarrow{n}\tilde{\theta}$ and $\left(\sum_i \varepsilon_i\right)/n\xrightarrow{n}0$ in mean square, both at rate $1/\sqrt{n}$). In fact, we have $nE\left[\left(\tilde{\theta}-E\left[\tilde{\theta}_n|\tilde{s}_n\right]\right)^2\right]\xrightarrow{n}\mathrm{AV}$, where $\mathrm{AV}=(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2$ if $\rho>0$ and $\mathrm{AV}=\sigma_\theta^4\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)^{-1}$ if $\rho=0$. This means that the convergence is faster (in terms of asymptotic variance) the closer we are to the common value case, the less prior uncertainty there is, and the less noisy are the signals (if $\rho>0$).[35] The market power distortion $d_n=\left((n-1)c_n\right)^{-1}$ (i.e., the amount of over- or underbidding) is of the order $1/n$.

It is worth to compare our results with those of Reny and Perry (2006). In a finite-market counterpart of their double auction continuum model, the authors[36] prove that, with enough buyers and sellers and with a sufficiently fine grid of prices, the following statement holds: generically in the valuation functions of the traders and the fineness of the grid, there is a Bayesian equilibrium in monotonically increasing bid functions that is very close to the unique REE of the continuum economy. The main obstacle in their involved proof is that, with a finite number of traders, in the double auction the strategies of buyers and sellers are not symmetric.[37] The incentives of buyers to underbid and of

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[34] This statement is proved as follows: $c_n\xrightarrow{n}a\equiv\lambda^{-1}(M+1)^{-1}$ if $\rho>0$ (since then $M_n\xrightarrow{n}M$), and $c_n\xrightarrow{n}\lambda^{-1}$ if $\rho=0$ (since then $M_n=0$); furthermore, $a_n\xrightarrow{n}a$ because $d_n=\left((n-1)c_n\right)^{-1}\xrightarrow{n}0$. It can be checked similarly that $b_n\xrightarrow{n}(1-\lambda a)q$.

[35] If $\rho=0$ then $\tilde{\theta}=\overline{\theta}$; in this case, more noise in the signals makes $E\left[\tilde{\theta}_n|\tilde{s}_n\right]$ closer to $\overline{\theta}$, which speeds up convergence. See the Appendix for the definition of the asymptotic variance of convergence.

[36] They assume a symmetry-preserving rationing rule.

[37] The consequence is that the signal of each agent need not be affiliated with the order statistics of the bids of other agents. This failure of “single crossing” implies that standard proofs from auction theory, which rely on relationships between affiliation and order statistics with symmetric strategies, do not apply here.

sellers to overbid in order to affect the price disappear as the market grows large and price-taking behavior obtains. In our DFE, the strategy of a trader is symmetric and the trader perceives that her influence on the price is given by $d_n > 0$. A buyer underbids and a seller overbids, and the incentives to manipulate the market also disappear as n grows and $d_n \to 0$. We can in addition characterize the *rate* at which this happens (and at which convergence to the limit equilibrium obtains) and distinguish between the dissipation of market power and the averaging of noise terms.

Consider now the general case with an elastic supply. The following lemma (with proof in the Appendix) establishes that, as n grows large, the equilibria of finite markets, for given information, converge to the limit equilibrium and characterizes the convergence rates for prices and for welfare losses. Denote by ETS (resp., $n^{-1}\mathrm{ETS}_n$) the per capita expected total surplus in the continuum market (resp., in the n-replica markets).

<u>Lemma</u>. *Consider the n-replica market. Let* $\rho \in [0,1)$. *For given* $\sigma_\varepsilon^2 \geq 0$, *the symmetric DFE of the n-replica market converge to the equilibrium in the continuum economy as n tends to infinity*:

(a) $a_n \xrightarrow{n} a$, $c_n \xrightarrow{n} c$, *and* $b_n \xrightarrow{n} b$;

(b) $p_n - p \xrightarrow{n} 0$ *in mean square at rate* $1/\sqrt{n}$ *with*

$$nE\left[\left(p_n - p\right)^2\right] \xrightarrow{n} \left(\frac{\beta}{\beta+\lambda}\right)^2 \mathrm{AV},$$

where $\mathrm{AV}\left(\sigma_\varepsilon^2\right) = (1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2$ *if* $\rho > 0$ *and* $\mathrm{AV}\left(\sigma_\varepsilon^2\right) = \sigma_\theta^4\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^{-1}$ *if* $\rho = 0$;

(c) *the per capita welfare loss* $\mathrm{WL}_n \equiv \mathrm{ETS} - n^{-1}\mathrm{ETS}_n \xrightarrow{n} 0$ *at the rate* $1/n$, *and the total welfare loss*

$$n\mathrm{WL}_n \xrightarrow{n} \frac{\mathrm{AV}}{2(\beta+\lambda)} + \frac{(1-\rho)^2 \sigma_\theta^4}{2\lambda\left((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right)}.$$

In a finite n-replica market, traders have the capacity to influence prices; and the price reveals the average signal of the traders $\tilde{s}_n$, which is a noisy estimate of $\tilde{\theta}_n$. We find that, for n large, such an equilibrium is close to the equilibrium in the limit economy where traders have no market power and where the price reveals the average parameter $\tilde{\theta}$. Convergence to the equilibrium of the continuum economy occurs as $1/\sqrt{n}$, the rate at which the average signal $\tilde{s}_n$ of the traders (or the average estimate $E\left[\tilde{\theta}_n \middle| \tilde{s}_n\right]$) tends to the average parameter $\tilde{\theta}$ in the continuum economy. Convergence to price-taking behavior is faster (at the rate $1/n$, since d_n is of the order $1/n$; see Proposition 7 in Vives (2011a)), but convergence to the limit is delayed by the slower convergence of the agents' average signal. This latter convergence is faster (in terms of asymptotic variance) as we approach a common value environment (i.e., as $\rho \to 1$), when there are better signals (low σ_ε^2 for $\rho > 0$), and/or with less prior uncertainty (low σ_θ^2).

In the finite market, the per capita welfare loss (with respect to that in the limit market) is of the order of $1/n$; see part (c) of the Lemma. Here again, convergence is faster (in terms of asymptotic variance) when closer to the common value case and slower if there is more prior uncertainty. The effect of noise in the signals is ambiguous if $\rho > 0$ since an increase in σ_ε^2 will tend to raise allocative inefficiency while diminish distributive inefficiency. The explanation for those results lies in the expression for total expected welfare loss in the finite market,

$$n\mathrm{WL}_n = n\left(\left(\beta+\lambda\right)E\left[\left(\tilde{x}_n - \tilde{x}\right)^2\right] + \lambda E\left[\left(u_{in} - u_i\right)^2\right]\right)\Big/2$$

(where $u_{in} \equiv x_{in} - \tilde{x}_n$ and $u_i \equiv x_i - \tilde{x}$), which has two components on the right-hand side; the first component reflects allocative inefficiency (is the average quantity at the right level?), and the second reflects distributive inefficiency (is a given average quantity efficiently distributed among market participants?). The first term converges to $\left(\left(1-\rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2\right)\Big/\left(2\left(\beta+\lambda\right)\right)$ if $\rho > 0$, and the second term converges to $\left(1-\rho\right)^2 \sigma_\theta^4 \Big/ \left(2\lambda\left(\left(1-\rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2\right)\right)$ as $n \to \infty$. Increases in the correlation of parameters

ρ or in the precision of the prior $\tau_\theta \equiv \left(\sigma_\theta^2\right)^{-1}$ will decrease both terms; however, the first term increases with σ_ε^2 whereas the second term decreases with σ_ε^2 (since more noise in the signals aligns more individual and average quantities).[38]

The overall convergence result is driven by the rate at which the error terms in the signals vanish, which is slower than the rate of convergence to price-taking behavior.

7.2 Information acquisition in a large market

Consider the case of *covert* information acquisition where each seller does not observe the precision purchased by other sellers and look therefore at a simultaneous move game where each seller chooses its precision and the supply function $\left(\tau_{\varepsilon_i}, X_i(\cdot,\cdot)\right)_{i=1,\ldots n}$. It is possible to show a parallel result to Proposition 2 in the *n*-replica market. By Section S.4 in Vives (2011b), when $\rho < 1$ in the *n*-replica market there is a symmetric equilibrium of the game with covert information acquisition with traders buying a positive precision of information $\tau_\varepsilon^*(n) > 0$ — provided that the cost of information acquisition at zero precision is not too high (see Claim 1 in the Appendix).

The following proposition (with proof in the Appendix) characterizes convergence when information acquisition is endogenous in the case where in the limit market there is positive precision purchase (i.e. for $\rho \in [0,\bar{\rho})$ in Proposition 2). Things are more complicated in the other cases as we shall see.

Proposition 3. *Consider covert information acquisition in the* n *-replica market. Let* $\rho \in [0,\bar{\rho})$ *where* $\bar{\rho} \equiv 1 - \tau_\theta \sqrt{2\lambda H'(0)} > 0$ *. Then*

(i) There is a unique equilibrium $\tau_\varepsilon^*(n) > 0$ *for* n *large and* $\tau_\varepsilon^*(n) \to \tau_\varepsilon^* > 0$ *as* $n \to \infty$.

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[38] When $\rho = 0$, an increase in the noise of the signals reduces both terms.

(ii) Convergence to the limit equilibrium is "slow": $p_n - p \xrightarrow{n} 0$ *in mean square at rate* $1/\sqrt{n}$ *with constant of convergence* $\mathrm{AV}^* = \mathrm{AV}\left(\left(\tau_\varepsilon^*\right)^{-1}\right)$.

We have seen that whenever there is an equilibrium with $\tau_\varepsilon^* > 0$ in the continuum economy it is the limit of equilibria of finite economies. (See Figure 5a.) However, this need not be the case when the equilibrium in the continuum economy calls for $\tau_\varepsilon^* = 0$. (See Figure 5b.) Furthermore, it need not be the case either that a sequence of equilibria $\tau_\varepsilon^*(n)$ in the replica markets always converges to an equilibrium of the limit economy. (See Figure 5c.)

The reason behind those phenomena is that the equilibrium correspondence is not continuous in the precision of private information at 0 whenever the parameter correlation ρ is positive. In this case the order in which we take the limits $\tau_\varepsilon \to 0$ and $n \to \infty$ matters. Indeed, it can be checked that for a given $\tau_\varepsilon > 0$ we have that $\lim_{n\to\infty} d_n(\tau_\varepsilon) = 0$ and therefore $\lim_{\tau_\varepsilon \to 0} \lim_{n\to\infty} d_n(\tau_\varepsilon) = 0$ and there is no market power in the limit, but $\lim_{n\to\infty} \lim_{\tau_\varepsilon \to 0} d_n(\tau_\varepsilon) = \lim_{n\to\infty} d_n(0+) > 0$ except if $\beta\rho\lambda = 0$. In other words, $d_n(\tau_\varepsilon)$ is not jointly continuous in τ_ε and n at $\tau_\varepsilon = 0$.

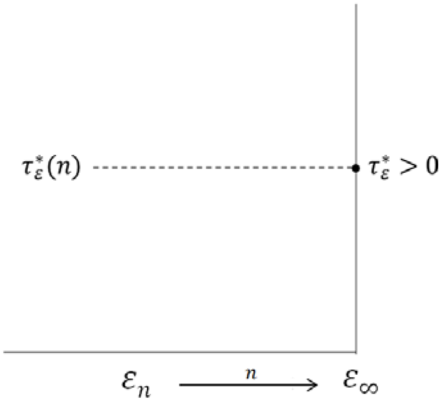

Figure 5.a. Convergence to the equilibrium in the limit market when $H'(0) < (1-\rho)^2 \left(2\lambda\tau_\theta^2\right)^{-1}$ or $\rho < \bar{\rho}$.

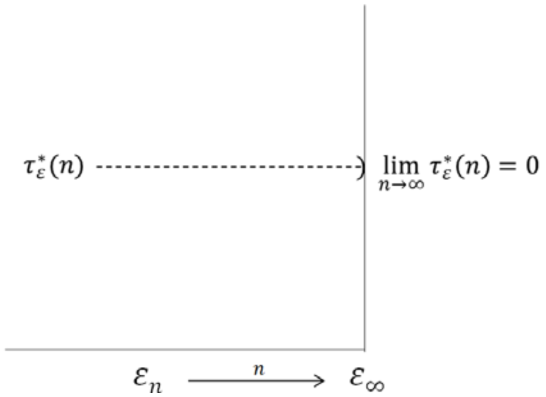

Figure 5.b. Case where the limit of equilibria of finite markets is *not* an equilibrium in the continuum market. It obtains, for example, when $H'(0) < \left(2(2\beta+\lambda)\tau_\theta^2\right)^{-1}$ and ρ is close to 1.

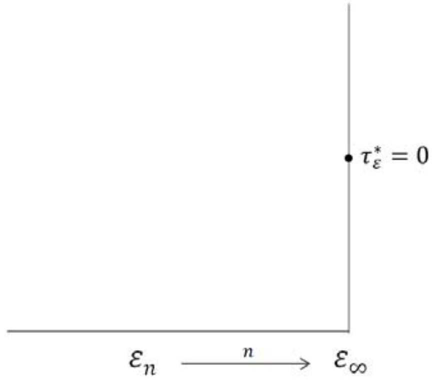

Figure 5.c. Case where the equilibrium with no information acquisition in the limit market is an artifact of the continuum specification. It obtains when $\left(2\lambda\tau_\theta^2\right)^{-1} < H'(0)$ and ρ is small.

Equilibrium in the large market may exist while its limit is not an equilibrium of the continuum market. We may have existence of equilibrium with positive purchase of information $\tau_\varepsilon^*(n) > 0$ with $\lim_{n\to\infty} \tau_\varepsilon^*(n) = 0$ when $\tau_\varepsilon^* = 0$ is not an equilibrium of the continuum economy. (See Figure 5b.)[39] For example, it can be checked that this happens for ρ close to 1 and $\left(2(2\beta+\lambda)\tau_\theta^2\right)^{-1} > H'(0)$ (recall that $\tau_\varepsilon^* = 0$ is an equilibrium of the continuum economy only when $\left(2\lambda\tau_\theta^2\right)^{-1} \le H'(0)$). In this case the region where the Grossman-Stiglitz paradox obtains for ρ close to 1 and n large is reduced from the

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[39] In technical terms this is a failure of upper-hemi continuity of the equilibrium correspondence.

nonexistence region in the continuum economy $\left(2\lambda\tau_\theta^2\right)^{-1} > H'(0) \geq (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$. (See Claim 2 in the Appendix for a general statement of the result.)

The equilibrium with $\tau_\varepsilon^* = 0$ may be an artifact of the continuum specification. We may have non-existence of equilibrium for large n in the region where there is no information acquisition equilibrium in the continuum economy ($H'(0) > \left(2\lambda\tau_\theta^2\right)^{-1}$). This happens when ρ is small (see Figure 5c and Claim 4 in the Appendix). Then $\tau_\varepsilon^* = 0$ is an equilibrium in the continuum economy but there is no equilibrium in the finite economy for large n . The equilibrium with $\tau_\varepsilon^* = 0$ is then an artifact of the continuum specification.

If a trader purchases information when other traders do not purchase information then this trader gains some market power which, importantly, does not depend on the amount of information purchased. This is so since the signal of the trader will be revealed to others traders in the equilibrium and this relevant when $\rho > 0$. This will mean that there will be a discontinuity in the marginal benefit to acquire information at 0 for this trader. If a trader purchases a little bit of information he will gain a discrete amount of market power but if he does not he has no market power. For an equilibrium with no information purchase to exist we need that expected profits with no purchase of information be larger than with any positive purchase of information when other traders do not buy information. The problem may be that the marginal return to information purchase may be negative for any positive precision (and this will be true for $H'(0) > \left(2\lambda\tau_\theta^2\right)^{-1}$), but then purchasing no precision implies a discrete change to a no information equilibrium where profits are smaller than with some information purchase. Then there cannot be an equilibrium with no purchase of information since a trader would like to be as close as possible to $\tau_\varepsilon = 0$ (since marginal profits decrease with τ_ε) but at $\tau_\varepsilon = 0$ profits discontinuously jump down. This can happen when the parameter correlation ρ is small. Then starting from a no correlation situation ($\rho = 0$) --where the profits with no information acquisition for an individual trader are just the limit of those obtained purchasing information when this purchase tends to 0- and adding some correlation (with $\rho > 0$), we have that profits increase by purchasing some information because of the gained market power by the trader. This destroys the no information equilibrium.

8. Concluding remarks

A simple large-market REE model which provides conditions to solve the paradoxes associated to fully revealing equilibria has been presented. The key to the resolution of the problems is to allow for both private and common value components in the valuations of the traders, with bundled signals about those components, together with a continuum specification which makes price-taking individually optimal. Two limitations must be taken into account. Firstly, the resolution of the Grossman-Stiglitz paradox needs, in general, a bounded common value component for traders' valuation. Second, the continuum resolution of the trader's schizophrenia problem comes at the cost of a "slow" convergence rate of equilibrium in finite markets in relation to the rate at which price-taking behavior obtains.

Appendix

Proof of Proposition 1: Trader i chooses x_i to maximize

$$E\left[\pi_i \left| s_i, p\right.\right] = x_i \left(E\left[\theta_i \left| s_i, p\right.\right] - p \right) - \frac{\lambda}{2} x_i^2 ,$$

which yields the FOC $p = E\left[\theta_i \left| s_i, p\right.\right] - \lambda x_i$. Positing linear strategies $X\left(s_i, p\right) = b + a s_i - c p$ while using the inverse supply function $p = \alpha + \beta \tilde{x}$ and our convention $\int s_i \, di = \tilde{\theta}$, we obtain (provided that $1 + \beta c \neq 0$) an expression for the price $p = \left(1 + \beta c\right)^{-1} \left(\alpha + \beta b + \beta a \tilde{\theta}\right)$. The vector $\left(\theta_i, s_i, \tilde{\theta}\right)$ is normally distributed with $E\left[\theta_i\right] = E\left[\tilde{\theta}\right] = E\left[s_i\right] = \bar{\theta}$ and with variance–covariance matrix

$$\sigma_\theta^2 \begin{pmatrix} 1 & 1 & \rho \\ 1 & \xi^{-1} & \rho \\ \rho & \rho & \rho \end{pmatrix},$$

where $\xi = \tau_\varepsilon \big/ \left(\tau_\theta + \tau_\varepsilon\right)$. It follows that $E\left[\theta_i \left| s_i, \tilde{\theta}\right.\right] = \varsigma s_i + \left(1 - \varsigma\right) \tilde{\theta}$ for $\varsigma = (1 + \sigma_\varepsilon^2 / (1 - \rho) \sigma_\theta^2)^{-1}$. Given joint normality of the stochastic variables $\left(\theta_i, s_i, \tilde{\theta}\right)$, we obtain

$$\begin{pmatrix} \theta_i \\ s_i \\ p \end{pmatrix} \sim N\left(\begin{pmatrix} \bar{\theta} \\ \bar{\theta} \\ C + D\bar{\theta} \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & D\rho\sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & D\rho\sigma_\theta^2 \\ D\rho\sigma_\theta^2 & D\rho\sigma_\theta^2 & D^2\rho\sigma_\theta^2 \end{pmatrix} \right).$$

Here $C = \left(1 + \beta c\right)^{-1} \left(\alpha + \beta b\right)$ and $D = \left(1 + \beta c\right)^{-1} \left(\beta a\right)$. If we use the projection theorem for normal random variables and assume that $\beta a \neq 0$, then

$$E\left[\theta_i \left| s_i, p\right.\right] = -\frac{C\sigma_\varepsilon^2}{D\left(\left(1 - \rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2\right)} + \frac{\left(1 - \rho\right)\sigma_\theta^2}{\left(1 - \rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2} s_i + \frac{\sigma_\varepsilon^2}{D\left(\left(1 - \rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2\right)} p .$$

Using the first-order condition, we obtain

$$\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + \left(1 - \rho\right)\sigma_\theta^2\right)} \frac{\alpha + \beta b}{\beta a} - \frac{\left(1 - \rho\right)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + \left(1 - \rho\right)\sigma_\theta^2\right)} s_i + \left(1 - \frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + \left(1 - \rho\right)\sigma_\theta^2\right)} \frac{1 + \beta c}{\beta a} \right) p$$

$$= -\lambda\left(b + a s_i - c p\right);$$

then, using the method of undetermined coefficients, we obtain the following system of equations:

$$\left.\begin{aligned}\frac{(1-\rho)\sigma_\theta^2}{\lambda\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}&=a\\ -\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}\frac{\alpha+\beta b}{\beta a}&=\lambda b\\ 1-\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}\frac{(1+\beta c)}{\beta a}&=\lambda c\end{aligned}\right\}$$

The solution to this system gives the result because for $\rho\in(0,1)$, $\sigma_\theta^2>0$, and $\sigma_\varepsilon^2<\infty$ we have that $1+\beta c=a(\beta+\lambda)>0$, and

$$\begin{aligned}a&=\frac{1}{\lambda\left(1+\left(\sigma_\varepsilon^2\big/\left((1-\rho)\sigma_\theta^2\right)\right)\right)}=\frac{1}{\lambda(1+M)},\\ b&=-\frac{\alpha}{\beta}\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}=-\frac{\alpha}{\beta}(1-\lambda a),\\ c&=\frac{\lambda^{-1}(1-\rho)\sigma_\theta^2-\beta^{-1}\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}=\frac{1}{\beta}\left(a(\beta+\lambda)-1\right).\end{aligned}$$

It is immediate that $a>0$, that $-\beta^{-1}<c\le a\le\lambda^{-1}$ for $\beta>0$, and that c decreases in M and λ but increases in β. Finally, we can use $p=(1+\beta c)^{-1}\left(\alpha+\beta b+\beta a\tilde{\theta}\right)$ together with the expressions for the equilibrium coefficients to show that $p=\left(\lambda\alpha+\beta\tilde{\theta}\right)\big/(\lambda+\beta)$ and $\tilde{x}=\left(\tilde{\theta}-\alpha\right)\big/(\beta+\lambda)$. ♦

Proof of Proposition 2: Suppose that, at the first stage, all traders but *i* have chosen a precision $\tau_\varepsilon>0$. Then the market equilibrium (which is unaffected by the actions of a single trader) exhibits, according to Proposition 1, the price $p=\left(\lambda\alpha+\beta\tilde{\theta}\right)\big/(\lambda+\beta)$, a price that reveals $\tilde{\theta}$. Trader *i* receives a signal with precision τ_{ε_i} and chooses x_i to maximize

$$\left(E\left[\theta_i\,\middle|\,s_i,\tilde{\theta}\right]-p\right)x_i-\frac{\lambda}{2}x_i^2,$$

which yields the first-order condition $E\left[\theta_i \middle| s_i, \tilde{\theta}\right] - p = \lambda x_i$. Expected profits are given by $E\left[\pi_i\right] = \frac{\lambda}{2} E\left[x_i^2\right]$, where $x_i = \left(E\left[\theta_i \middle| s_i, \tilde{\theta}\right] - p\right) \big/ \lambda$ and $p = \left(\lambda\alpha + \beta\tilde{\theta}\right) \big/ \left(\lambda + \beta\right)$. Note that $E\left[\pi_i\right]$ does not depend on τ_ε because the equilibrium reveals $\tilde{\theta}$. We remark that $E\left[\theta_i \middle| s_i, \tilde{\theta}\right] = \varsigma_i s_i + \left(1 - \varsigma_i\right)\tilde{\theta}$, where $\varsigma_i = \frac{\left(1-\rho\right)\tau_{\varepsilon_i}}{\left(1-\rho\right)\tau_{\varepsilon_i} + \tau_\theta}$ and $E\left[x_i^2\right] = \left(E\left[x_i\right]\right)^2 + \operatorname{var}\left[x_i\right]$ with $E\left[x_i\right] = \left(\bar{\theta} - \alpha\right) \big/ \left(\beta + \lambda\right)$ and

$$\begin{aligned} \operatorname{var}\left[x_i\right] &= \left(\frac{1}{\beta+\lambda}\frac{1}{\lambda}\right)^2 \operatorname{var}\left[\varsigma_i\left(\lambda+\beta\right)s_i + \left(\lambda\left(1-\varsigma_i\right) - \varsigma_i\beta\right)\tilde{\theta}\right] \\ &= \left(\frac{1}{\lambda+\beta}\frac{1}{\lambda}\right)^2 \left(\varsigma_i^2\left(\lambda+\beta\right)^2\left(\tau_{\varepsilon_i}^{-1} + \left(1-\rho\right)\tau_\theta^{-1}\right) + \lambda^2\rho\tau_\theta^{-1}\right). \end{aligned}$$

We can use this fact to obtain

$$E\left[\pi_i\right] = \frac{\lambda}{2}\left(\left(\frac{\bar{\theta}-\alpha}{\beta+\lambda}\right)^2 + \frac{\tau_{\varepsilon_i}\tau_\theta^{-1}\left(1-\rho\right)\left(\left(\beta+\lambda\right)^2 - \beta\rho\left(\beta+2\lambda\right)\right) + \rho\lambda^2}{\left(\tau_\theta + \left(1-\rho\right)\tau_{\varepsilon_i}\right)\left(\beta+\lambda\right)^2\lambda^2}\right).$$

It follows that the marginal benefit of increasing the precision of information is

$$\frac{\partial E\left[\pi_i\right]}{\partial \tau_{\varepsilon_i}} = \frac{\left(1-\rho\right)^2}{2\lambda\left(\tau_\theta + \left(1-\rho\right)\tau_{\varepsilon_i}\right)^2}.$$

Observe that this marginal benefit is decreasing in τ_{ε_i} provided that $\rho < 1$ (and thus $E\left[\pi_i\right]$ is strictly concave in τ_{ε_i}). Let

$$\phi\left(\tau_\varepsilon\right) \equiv \left.\frac{\partial E\left[\pi_i\right]}{\partial \tau_\varepsilon}\right|_{\tau_{\varepsilon_i} = \tau_\varepsilon} - H'\left(\tau_\varepsilon\right).$$

Then $\phi\left(\infty\right) < 0$ and $\phi' < 0$. We have that $\phi(0) = \left(1-\rho\right)^2\left(2\lambda\tau_\theta^2\right)^{-1} - H'(0) > 0$ if and only if $\rho < \bar{\rho} \equiv 1 - \tau_\theta\sqrt{2\lambda H'\left(0\right)}$, in which case there is a unique interior solution τ_ε^* to the equation $\phi\left(\tau_\varepsilon\right) = 0$. Note that τ_ε^* is decreasing in λ, τ_θ, and ρ because ϕ is.

For $\rho \geq \bar{\rho}$ we have that $\tau_\varepsilon = 0$ at a candidate equilibrium. When no trader other than *i* purchases information,

$$\frac{\partial E[\pi_i]}{\partial \tau_{\varepsilon_i}} = \frac{1}{2\lambda\left(\tau_\theta + \tau_{\varepsilon_i}\right)^2},$$

(the price then is $p = \left(\lambda\alpha + \beta\bar{\theta}\right) \big/ \left(\lambda + \beta\right)$ and the expression for $\partial E[\pi_i] / \partial \tau_{\varepsilon_i}$ is the same as when $\rho = 0$). It follows that $\tau_\varepsilon = 0$ is an equilibrium only if $\left(2\lambda\tau_\theta^2\right)^{-1} - H'(0) \leq 0$ (or equivalently $\bar{\rho} \leq 0$). Otherwise (i.e., if $\left(2\lambda\tau_\theta^2\right)^{-1} > H'(0) \geq (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ or $\rho \geq \bar{\rho} > 0$), it will benefit a single trader to purchase information and there is no symmetric equilibrium in the game. ♦

<u>Lemma A</u>. If $H'(0) < (1-\rho)^2 \left(2\lambda\tau_\theta^2\right)^{-1}$ the marginal social and private benefits to purchase information are the same and the market acquires the right amount of information $\tau_\varepsilon^* > 0$. When $H'(0) \geq (1-\rho)^2 \left(2\lambda\tau_\theta^2\right)^{-1}$ and $\rho > 0$ there is no welfare-optimal level of information purchase.

<u>Proof</u>: (i) Gross expected total surplus is given by

$$E[TS] = E\left[\int_0^1 \left(\theta_i x_i - \frac{\lambda}{2}x_i^2\right) di - \left(\alpha + \beta\frac{\tilde{x}}{2}\right)\tilde{x}\right],$$

which at the market allocation we have that x_i depends on $\left(s_i, \tilde{\theta}\right)$ and $\tilde{x}$ on $\tilde{\theta}$. $E[TS]$ only depends on τ_ε from $E[\theta_i x_i] - \frac{\lambda}{2}E\left[x_i^2\right]$. Some computations lead to $E[\theta_i x_i] = \bar{\theta}\left(\frac{\bar{\theta} - \alpha}{\lambda + \beta}\right) + \sigma_\theta^2 \frac{\beta(1-\rho) + \lambda(M\rho + 1)}{\lambda(\beta + \lambda)(M+1)}$ and using the expression for $E\left[x_i^2\right]$ in the proof of Proposition 2 we obtain that

$$\frac{\partial E[TS]}{\partial \tau_\varepsilon} = \frac{(1-\rho)^2}{2\lambda\left(\tau_\theta + (1-\rho)\tau_\varepsilon\right)^2},$$

which is exactly the same as the marginal profit obtained in the market solution. We have similarly that at the market equilibrium $E[TS]$ is strictly concave in τ_ε and therefore the interior solutions $\phi(\tau_\varepsilon)=0$, when $H'(0)<(1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$, will coincide with the market.

However, when $H'(0)\geq(1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ and $\rho>0$ there is no welfare-optimal level of information purchase because the maximization of $E[TS]-H(\tau_\varepsilon)$ becomes an open problem. When $H'(0)\geq(1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$, $E[TS]$ is decreasing in τ_ε for $\tau_\varepsilon>0$ but setting $\tau_\varepsilon=0$ delivers strictly less surplus than a small positive τ_ε since for $\rho>0$, $\lim_{\tau_\varepsilon\to 0^+}E[TS]=\frac{1}{2}\frac{(\overline{\theta}-\alpha)^2+\rho\sigma_\theta^2}{\beta+\lambda}>E[TS]\big|_{\tau_\varepsilon=0}=\frac{1}{2}\frac{(\overline{\theta}-\alpha)^2}{\beta+\lambda}$. ♦

<u>Measures of speed of convergence</u>. We say that the sequence (of real numbers) b_n is of the *order* n^υ (υ a real number) whenever $n^{-\upsilon}b_n\xrightarrow[n]{}k$ for some nonzero constant k.[40] The constant of convergence k is a refined measure of the speed of convergence. We say that the sequence of random variables $\{y_n\}$ converges *in mean square* to zero at the rate $1/\sqrt{n^r}$ (or that y_n is of the order $1/\sqrt{n^r}$) if $E\left[(y_n)^2\right]$ converges to zero at the rate $1/n^r$ (i.e., if $E\left[(y_n)^2\right]$ is of the order $1/n^r$). Given that $E\left[(y_n)^2\right]=\left(E[y_n]\right)^2+\operatorname{var}[y_n]$, a sequence $\{y_n\}$ such that $E[y_n]=0$ and $\operatorname{var}[y_n]$ is of order $1/n$ and converges to zero at the rate $1/\sqrt{n}$. A more refined measure of the speed of convergence for a given convergence rate is provided by the *asymptotic variance*. Suppose that $E\left[(y_n)^2\right]\xrightarrow{n}0$ at the rate $1/n^r$ and $E[y_n]=0$. Then the asymptotic variance of convergence is given by

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[40] This definition is stronger than necessary but it will suffice for our purposes.

the constant $\lim_{n\to\infty} n^r E\left[\left(y_n\right)^2\right]$. A higher asymptotic variance means that the speed of convergence is slower.

<u>Proof of Lemma:</u> (a) From the proof of Proposition 7 in Vives (2011b) we have that $c_n \xrightarrow{n} c \equiv \frac{\lambda^{-1} - \beta^{-1}M}{M+1}$, where $M = \frac{\sigma_\varepsilon^2}{(1-\rho)\sigma_\theta^2}$ if $\rho > 0$ and $c_n \xrightarrow{n} \lambda^{-1}$ if $\rho = 0$.

Furthermore, $a_n = \frac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)}\left(d_n + \lambda\right)^{-1} \xrightarrow{n} a$ because $d_n = \left(\beta^{-1}n + (n-1)c_n\right)^{-1} \xrightarrow{n} 0$. Note that $nd_n \xrightarrow{n} \left(\beta^{-1} + c\right)^{-1}$, which is equal to $\left(\beta^{-1} + \lambda^{-1}\right)^{-1}(1+M)$ if $\rho > 0$ or to $\left(\beta^{-1} + \lambda^{-1}\right)^{-1}$ if $\rho = 0$. Convergence for b_n follows similarly.

(b) From Proposition 1 in Vives (2011a) we have that $\tilde{x}_n = \left(E\left[\tilde{\theta}_n | \tilde{s}_n\right] - \alpha\right) \big/ \left(\beta + \lambda + d_n\right)$, where $\tilde{s}_n = n^{-1}\left(\sum_i s_i\right) = \tilde{\theta}_n + n^{-1}\left(\sum_i \varepsilon_i\right)$, $E\left[\tilde{\theta}_n | \tilde{s}_n\right] = \zeta_n \tilde{s}_n + (1-\zeta_n)\bar{\theta}$, and $\zeta_n \equiv \mathrm{var}\left[\tilde{\theta}_n\right] \big/ \left(\mathrm{var}\left[\tilde{\theta}_n\right] + \sigma_\varepsilon^2 n^{-1}\right)$. It follows that $\mathrm{var}\left[E\left[\tilde{\theta}_n | \tilde{s}_n\right]\right] = \zeta_n \, \mathrm{var}\left[\tilde{\theta}_n\right] = \frac{\left(\left(1+(n-1)\rho\right)\sigma_\theta^2\right)^2}{\left(\left(1+(n-1)\rho\right)\sigma_\theta^2 + \sigma_\varepsilon^2\right)n}$. Observe that $d_n \xrightarrow{n} 0$ and $E\left[\tilde{\theta}_n | \tilde{s}_n\right] \xrightarrow{n} \tilde{\theta}$ in mean square (since $\tilde{\theta}_n \xrightarrow{n} \tilde{\theta}$ and $\left(\sum_i \varepsilon_i\right) \big/ n \xrightarrow{n} 0$ in mean square, both at rate $1 \big/ \sqrt{n}$). It is immediate that $E\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n | \tilde{s}_n\right]\right)^2\right] = \frac{\sigma_\theta^2(1-\rho)\left(\rho(n-1)+1\right) + n\rho\sigma_\varepsilon^2}{n\left(\sigma_\theta^2 + \sigma_\varepsilon^2 + (n-1)\rho\sigma_\theta^2\right)}\sigma_\theta^2$ and $nE\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n | \tilde{s}_n\right]\right)^2\right] \xrightarrow{n} \mathrm{AV}$, where $\mathrm{AV} = (1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2$ if $\rho > 0$ and $\mathrm{AV} = \sigma_\theta^4\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^{-1}$ if $\rho = 0$. We have that $\tilde{x} = \left(\tilde{\theta} - \alpha\right) \big/ \left(\beta + \lambda\right)$, that $E\left[\left(\tilde{x} - \tilde{x}_n\right)^2\right] = E\left[\left(\frac{(\beta+\lambda)\left(\tilde{\theta} - E\left[\tilde{\theta}_n | \tilde{s}_n\right]\right) + d_n\left(\tilde{\theta} - \alpha\right)}{(\beta+\lambda)\left(\beta + \lambda + d_n\right)}\right)^2\right]$, and that both d_n and

$E\left[\left(\tilde{\theta}-E\left[\tilde{\theta}_n|\tilde{s}_n\right]\right)^2\right]$ are of order $1/n$; hence we obtain $nE\left[\left(\tilde{x}-\tilde{x}_n\right)^2\right]\xrightarrow{n}\mathrm{AV}/\left(\beta+\lambda\right)^2$. Therefore, $\tilde{x}-\tilde{x}_n\xrightarrow{n}0$ in mean square. The results follow since $p_n-p=\beta\left(\tilde{x}_n-\tilde{x}\right)$.

(c) Total surplus (per capita) in the continuum and in the *n*-replica markets are given, respectively, by

$\mathrm{TS}=\int_0^1\left(\theta_i x_i-\frac{\lambda}{2}x_i^2\right)di-\left(\alpha+\beta\frac{\tilde{x}}{2}\right)\tilde{x}$ and $n^{-1}\mathrm{TS}_n=n^{-1}\sum_{i=1}^{n}\left(\theta_i x_i-\frac{\lambda}{2}x_i^2\right)-\left(\alpha+\beta\frac{\tilde{x}_n}{2}\right)\tilde{x}_n$.

We can then write the expected welfare loss as

$$\mathrm{WL}_n\equiv E\left[\mathrm{TS}\right]-n^{-1}E\left[\mathrm{TS}_n\right]=\left(\left(\beta+\lambda\right)E\left[\left(\tilde{x}_n-\tilde{x}\right)^2\right]+\lambda E\left[\left(u_{in}-u_i\right)^2\right]\right)\Big/2\,,$$

where $u_{in}\equiv x_{in}-\tilde{x}_n$ and $u_i\equiv x_i-\tilde{x}$ (this follows as in the proof of Proposition 3 in Vives (2011a)). We already know from the proof of part (i)(b) that $nE\left[\left(\tilde{x}-\tilde{x}_n\right)^2\right]\xrightarrow{n}\mathrm{AV}/\left(\beta+\lambda\right)^2$.

We also know that $u_{in}=\left(\tilde{t}_n-t_{in}\right)/\left(\lambda+d_n\right)$ and $u_i=\left(\tilde{\theta}-t_i\right)/\lambda$, where $\tilde{t}_n\equiv E\left[\tilde{\theta}_n|\tilde{s}_n\right]$,

$t_{in}\equiv E[\theta_i\mid s_i,\tilde{s}_n]=$

$\bar{\theta}+\frac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)}\left(s_i-\bar{\theta}\right)+\frac{\sigma_\varepsilon^2\sigma_\theta^2\rho n}{\left(((n-1)\rho+1)\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)}\left(\tilde{s}_n-\bar{\theta}\right)$,

and $t_i\equiv E\left[\theta_i\left|s_i,\tilde{\theta}\right.\right]=\varsigma s_i+\left(1-\varsigma\right)\tilde{\theta}$ for $\varsigma=\left(1+\frac{\sigma_\varepsilon^2}{\left(1-\rho\right)\sigma_\theta^2}\right)^{-1}$. As a result,

$E\left[\left(u_{in}-u_i\right)^2\right]=\frac{1}{\left(\lambda+d_n\right)^2\lambda^2}\left(\frac{\left(1-\rho\right)\sigma_\theta^2}{\left(\sigma_\theta^2\left(1-\rho\right)+\sigma_\varepsilon^2\right)}\right)^2E\left[\left(\left(\tilde{s}_n-\tilde{\theta}\right)\lambda-\left(\tilde{\theta}-s_i\right)d_n\right)^2\right].$

Further computations yield

$$E\left[\left(\left(\tilde{s}_n-\tilde{\theta}\right)\lambda-\left(\tilde{\theta}-s_i\right)d_n\right)^2\right]=n^{-1}\left(\sigma_\theta^2\left(1-\rho\right)+\sigma_\varepsilon^2\right)\left(2\lambda d_n+\lambda^2+nd_n^2\right).$$

Therefore, $E\left[\left(u_{in}-u_i\right)^2\right]=\frac{\left(2\lambda d_n+\lambda^2+nd_n^2\right)(1-\rho)^2\sigma_\theta^4}{\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)(\lambda+d_n)^2\lambda^2 n}$; since d_n is of order $1/n$, we have $\lim_{n\to\infty} n\frac{\lambda E\left[\left(u_{in}-u_i\right)^2\right]}{2}=\frac{(1-\rho)^2\sigma_\theta^4}{2\lambda\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)}$. It follows that $n\text{WL}_n\to\frac{\text{AV}}{2(\beta+\lambda)}+\frac{(1-\rho)^2\sigma_\theta^4}{2\lambda\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)}$. ◆

<u>Proof of Proposition 3:</u>

(i) Let $E[\pi_i]$ be the expected profits of trader i when the other traders $j\neq i$ have information precision $\tau_\varepsilon>0$ and use identical strategies based on linear demand schedules with coefficients (b,a,c) . Suppose that trader i has precision τ_{ε_i} and optimizes his demand schedule $X_i(s_i,p)=\left(E\left[\theta_i\middle|s_i,p\right]-p\right)(d_i+\lambda)^{-1}$ where $d_i=\left(\beta^{-1}n+(n-1)c\right)^{-1}$. $E[\pi_i]$ are a function of $\left(b,a,c,\tau_\varepsilon,\tau_{\varepsilon_i}\right)$. If we let $d_n=\left(\beta^{-1}n+(n-1)c_n\right)^{-1}$, where c_n is the symmetric solution when $\tau_{\varepsilon_i}=\tau_\varepsilon$ (given by Proposition 1 in Vives (2011a)), then it follows from Section S.4 in Vives (2011b) that

$$\psi_n(\tau_\varepsilon)\equiv\left.\frac{\partial E[\pi_i]}{\partial\tau_{\varepsilon_i}}\right|_{\tau_{\varepsilon_i}=\tau_\varepsilon}=\frac{1}{2(2d_n+\lambda)}\frac{\left(\tau_\varepsilon(1-\rho)\left(1+\rho(n-1)\right)+\tau_\theta\right)^2}{\left(\tau_\varepsilon(1-\rho)+\tau_\theta\right)^2\left(\tau_\varepsilon\left(1+\rho(n-1)\right)+\tau_\theta\right)^2}.$$

Interior symmetric equilibria for information precision are characterized by the solution of $\psi_n(\tau_\varepsilon)-H'(\tau_\varepsilon)=0$. Observe that, since $d_n\xrightarrow{n}0$ for $\tau_\varepsilon>0$, we have then $\psi_n(\tau_\varepsilon)\xrightarrow{n}\psi_\infty(\tau_\varepsilon)\equiv\frac{(1-\rho)^2}{2\lambda\left(\tau_\theta+(1-\rho)\tau_\varepsilon\right)^2}$. We know that if $\rho\in\left[0,1-\tau_\theta\sqrt{2\lambda H'(0)}\right)$, there is a unique $\tau_\varepsilon^*>0$ which solves $\phi(\tau_\varepsilon)=\frac{(1-\rho)^2}{2\lambda\left(\tau_\theta+(1-\rho)\tau_\varepsilon\right)^2}-H'(\tau_\varepsilon)=0$. Furthermore, for n large, $\psi_n(\tau_\varepsilon)$ is strictly decreasing in τ_ε. It follows that for n large there is a unique symmetric equilibrium of the (covert) information acquisition game

$\tau_\varepsilon^*(n) > 0$, the unique solution to $\phi_n(\tau_\varepsilon) \equiv \psi_n(\tau_\varepsilon) - H'(\tau_\varepsilon) = 0$. We conclude that $\tau_\varepsilon^*(n) \to \tau_\varepsilon^* > 0$ (where $\phi(\tau_\varepsilon^*) = 0$) as $n \to \infty$.

(ii) The result follows since when at the *n*-replica we have $\tau_\varepsilon^*(n)$, then $E\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n | \tilde{s}_n\right]\right)^2\right] = \dfrac{\sigma_\theta^2 (1-\rho)(\rho(n-1)+1) + n\rho\left(\tau_\varepsilon^*(n)\right)^{-1}}{n\left(\sigma_\theta^2 + \left(\tau_\varepsilon^*(n)\right)^{-1} + (n-1)\rho\sigma_\theta^2\right)}\sigma_\theta^2$ and therefore $nE\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n | \tilde{s}_n\right]\right)^2\right] \xrightarrow{n} \mathrm{AV}^*$, where $\mathrm{AV}^* = (1-\rho)\sigma_\theta^2 + \left(\tau_\varepsilon^*\right)^{-1}$ if $\rho > 0$ and $\mathrm{AV}^* = \sigma_\theta^4\left(\sigma_\theta^2 + \left(\tau_\varepsilon^*\right)^{-1}\right)^{-1}$ if $\rho = 0$ since $\tau_\varepsilon^*(n) \to \tau_\varepsilon^* > 0$.♦

<u>Claim 1</u>. Equilibrium with positive information acquisition for a given n . Let $\psi_n(0+) \equiv \lim_{\tau_\varepsilon \to 0} \psi_n(\tau_\varepsilon) = \left(2\left(2d_n(0+) + \lambda\right)\tau_\theta^2\right)^{-1}$ where $d_n(0+) \equiv \lim_{\tau_\varepsilon \to 0} d_n(\tau_\varepsilon)$. Let $\rho < 1$. Then: (i) An interior symmetric equilibrium with $\tau_\varepsilon^*(n) > 0$ exists if $H'(0) < \psi_n(0+)$. (ii) $\lim\limits_{n\to\infty}\lim\limits_{\tau_\varepsilon \to 0} d_n(\tau_\varepsilon) = \lim\limits_{n\to\infty} d_n(0+) > 0$ except if $\beta\rho\lambda = 0$ and $\lim\limits_{\rho\to 1}\lim\limits_{n\to\infty} d_n(0+) = \lim\limits_{n\to\infty}\lim\limits_{\rho\to 1} d_n(0+) = \beta$. (iii) There is a unique value of β , $\hat{\beta} > 0,$ such that $\lim\limits_{n\to\infty}\psi_n(0+) > (1-\rho)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ if and only if $\beta < \hat{\beta}$, and $\hat{\beta} \to \infty$ as $\rho \to 1$.

<u>Proof</u>: (i) We have that $d_n(0+) = \left(n\beta^{-1} + (n-1)\widehat{c}_n\right)^{-1}$ where $\widehat{c}_n = \lim\limits_{\tau_\varepsilon \to 0} c_n$. The result follows from Proposition S3 in Vives (2011b). (ii) We obtain that

$$d_n(0+) \equiv \lim_{\tau_\varepsilon \to 0} d_n(\tau_\varepsilon) =$$

$$\frac{\beta\left(\dfrac{\rho}{1-\rho} - 1 + \dfrac{2}{n}\right) - \lambda\left(\dfrac{\rho}{1-\rho} + 1\right) + \sqrt{\beta^2\left(\dfrac{\rho}{1-\rho} - 1 + \dfrac{2}{n}\right)^2 + 2\beta\lambda\left(\dfrac{\rho}{1-\rho} + 1\right)^2 + \lambda^2\left(\dfrac{\rho}{1-\rho} + 1\right)^2}}{\dfrac{2}{1-\rho}},$$

and $\lim\limits_{n\to\infty} d_n(0+) = \dfrac{\beta(2\rho-1) - \lambda + \sqrt{\beta^2(2\rho-1)^2 + 2\beta\lambda + \lambda^2}}{2}$, which is positive except if $\beta\rho\lambda = 0$, in which case it is 0. (iii) The result follows from the expression for $\lim\limits_{n\to\infty}\psi_n(0+)$ and $\lim\limits_{n\to\infty} d_n(0+)$.♦

Claim 2: If $\left(2\left(2\lim_{n\to\infty} d_n\left(0+\right)+\lambda\right)\tau_\theta^2\right)^{-1} > H'\left(0\right) > \left(1-\rho\right)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ (and this may happen if $\beta < \hat{\beta}$), then for n large there is an equilibrium with $\tau_\varepsilon^*\left(n\right) > 0$ with $\lim_{n\to\infty}\tau_\varepsilon^*\left(n\right) = 0$ while there is no equilibrium in the continuum market. The result obtains in particular for ρ close to 1 if $\left(2\left(2\beta+\lambda\right)\tau_\theta^2\right)^{-1} > H'\left(0\right) > \left(1-\rho\right)^2\left(2\lambda\tau_\theta^2\right)^{-1}$.

Proof: Claim 1 implies that if $\left(2\left(2\lim_{n\to\infty} d_n\left(0+\right)+\lambda\right)\tau_\theta^2\right)^{-1} > H'\left(0\right)$ then $\tau_\varepsilon^*\left(n\right) > 0$ for n large and there is no equilibrium in the continuum economy since $\left(2\lambda\tau_\theta^2\right)^{-1} > H'\left(0\right) > \left(1-\rho\right)^2\left(2\lambda\tau_\theta^2\right)^{-1}$ (in particular, $\tau_\varepsilon^* = 0$ is not an equilibrium). Suppose that $\lim_{n\to\infty}\tau_\varepsilon^*\left(n\right) = \tau_\varepsilon > 0$. Since for $\tau_\varepsilon > 0$ we have that $\psi_n\left(\tau_\varepsilon\right) \xrightarrow{n} \psi_\infty\left(\tau_\varepsilon\right)$, we should have $\tau_\varepsilon = \tau_\varepsilon^*$ but we know there is no equilibrium in the continuum economy. Furthermore, for any n, as $\rho \to 1$ we have that $\hat{c}_n \to -1/\beta$, $d_n\left(0+\right) \to \beta$, and $\psi_n\left(0+\right) \to \left(2\left(2\beta+\lambda\right)\tau_\theta^2\right)^{-1}$. The result for ρ close to 1 follows. ♦

Remark: For ρ close to 1 for any number of traders (size of replica) we need the same degree of diffusion of the prior in order to have positive precision acquisition.

Equilibrium with no information acquisition in the n-replica market.

Let $E\left[\pi_{in}\right]\Big|_{\substack{\tau_{\varepsilon_j}=0 \\ j\neq i}}$ denote the expected profits of trader i when the other traders $j \neq i$ have information precision $\tau_\varepsilon = 0$ and use identical strategies based on linear demand schedules with price coefficient c. Suppose that trader i has precision $\tau_{\varepsilon_i} > 0$ and optimizes his demand schedule: $X_i\left(s_i, p\right) = \left(E\left[\theta_i \middle| s_i\right] - p\right)/\left(\lambda + d_i\right)$, which implies that $c_i \equiv \partial X_i/\partial p = 1/\left(\lambda + d_i\right)$. We have that $d_i = \left(\beta^{-1}n + \left(n-1\right)c\right)^{-1}$. It is possible to show that for given $\tau_{\varepsilon_i} > 0$, there is a unique (and asymmetric) equilibrium in demand functions for $\rho < 1$. At equilibrium d_i is the only positive root of the following cubic polynomial

$$Q(d) = Ad^3 + Bd^2 + Cd + D,$$

with

$A = n(2\beta(n-1)(\rho-1) - n\lambda)$,
$B = \left(-2(2n-3)n\lambda\beta - n^2\lambda^2 - 2(n-2)(n-1)\beta^2 + (4n\beta - 6\beta + 3n\lambda)(n-1)\beta\rho\right)$,
$C = \lambda\beta\left(-\beta\left(n^2-6n+6\right) + \lambda n(3-n) + (4n\beta - 7\beta + n\lambda)(n-1)\rho\right)$ and
$D = \lambda^2\beta^2(n-2)(\rho(n-1)+1)$.

Note that d_i is independent of τ_{ε_i} . In equilibrium, we obtain $E\left[\pi_i\right]\Big|_{\substack{\tau_{\varepsilon_j}=0\\ j\neq i}} = \frac{\lambda+2d_i}{2}E\left[\left(X_i\left(s_i,p\right)\right)^2\right]$ and it can be shown, with some work, that $\left.\frac{\partial E\left[\pi_i\right]}{\partial \tau_{\varepsilon_i}}\right|_{\substack{\tau_{\varepsilon_j}=0\\ j\neq i}} = \frac{1}{2\left(2d_i+\lambda\right)}\frac{1}{\left(\tau_{\varepsilon_i}+\tau_\theta\right)^2}$, which is decreasing in τ_{ε_i} . It follows that if $\frac{1}{2\left(2d_i+\lambda\right)}\frac{1}{\tau_\theta^2} < H'\left(0\right)$ then net expected profits decrease in τ_{ε_i}. However, if the trader chooses $\tau_{\varepsilon_i} = 0$ then there is a discontinuity since when no trader acquires information at equilibrium we have $d = d^f$ (the value for d with symmetric information, see Vives (2011a)) but $d_i \neq d^f$ and $E\left[\pi\right]|_{\tau_\varepsilon=0} = \frac{(\alpha-\overline{\theta})^2}{2\left(\beta+\lambda+d^f\right)^2}\left(\lambda+2d^f\right)$. (Recall that we assume that $H\left(0\right)=0$.) The discontinuity arises since when $\tau_{\varepsilon_i} > 0$ the price reveals the signal of trader i while there is no revelation if no one acquires information. If $E\left[\pi\right]|_{\tau_\varepsilon=0} \geq \lim_{\tau_{\varepsilon_i}\to 0} E\left[\pi_i\right]\Big|_{\substack{\tau_{\varepsilon_j}=0\\ j\neq i}}$ then there is an equilibrium with no information acquisition. But if $E\left[\pi\right]|_{\tau_\varepsilon=0} < \lim_{\tau_{\varepsilon_i}\to 0} E\left[\pi_i\right]\Big|_{\substack{\tau_{\varepsilon_j}=0\\ j\neq i}}$ then there is no such equilibrium. Trader i would like to set $\tau_{\varepsilon_i} \to 0$ but when $\tau_{\varepsilon_i} = 0$ then there is a discrete change to the no information equilibrium with strictly lower profits. In summary:

Claim 3: Equilibrium with no information acquisition for a given n. Let $\frac{1}{2\left(2d_i+\lambda\right)}\frac{1}{\tau_\theta^2}<H'(0)$. Then there is an equilibrium with no information acquisition if and only if $E[\pi]|_{\tau_\varepsilon=0}\geq \lim_{\tau_{\varepsilon_i}\to 0}E[\pi_i]\Big|_{\substack{\tau_{\varepsilon j}=0\\ j\neq i}}$.

Let us examine now the case with n large and use n subscripts for variables in the n-replica economy. We have that as n tends to infinity, if $\rho>0$, d_{in} tends to $d_{i\infty}>0$ (which is strictly increasing in ρ) and $c_n\to-\beta^{-1}$. (If $\rho=0$ we have that $d_{i\infty}=0$.) This means that even for large n trader i keeps some market power (which is increasing in ρ). We have also that $\lim_{n\to\infty}d_n^f=0$. If $\frac{1}{2\left(2d_{i\infty}+\lambda\right)}\frac{1}{\tau_\theta^2}<H'(0)$ then, for n large, there is an equilibrium with no information acquisition if and only if $\lim_{n\to\infty}E[\pi_n]|_{\tau_\varepsilon=0}\geq$ $\lim_{\tau_{\varepsilon_i}\to 0}\lim_{n\to\infty}E[\pi_{in}]\Big|_{\substack{\tau_{\varepsilon j}=0\\ j\neq i}}$.

We have that $\lim_{n\to\infty}E[\pi_n]|_{\tau_\varepsilon=0}=\frac{\lambda\left(\alpha-\overline{\theta}\right)^2}{2\left(\beta+\lambda\right)^2}$ since $\lim_{n\to\infty}d_n^f=0$. It follows that $\lim_{n\to\infty}E[\pi_n]|_{\tau_\varepsilon=0}=$ $\lim_{\tau_{\varepsilon_i}\to 0}\lim_{n\to\infty}E[\pi_{in}]\Big|_{\substack{\tau_{\varepsilon j}=0\\ j\neq i}}$ if $\alpha=\overline{\theta}$ or $\rho=0$ (recall that $d_{i\infty}=0$ for $\rho=0$), and it can be checked that $\lim_{n\to\infty}E[\pi_n]|_{\tau_\varepsilon=0}<\lim_{\tau_{\varepsilon_i}\to 0}\lim_{n\to\infty}E[\pi_{in}]\Big|_{\substack{\tau_{\varepsilon j}=0\\ j\neq i}}$ for ρ small. In summary: for n large, if $\frac{1}{2\left(2d_{i\infty}+\lambda\right)}\frac{1}{\tau_\theta^2}<H'(0)$ and ρ is low enough there is no equilibrium with no information acquisition. Furthermore, if $\frac{1}{2\lambda\tau_\theta^2}<H'(0)$ then, from Claim 1 there cannot be either an equilibrium with $\tau_\varepsilon^*(n)>0$. This is so since for n large there is an equilibrium with $\tau_\varepsilon^*(n)>0$ only if $\left(2\left(2\lim_{n\to\infty}d_n(0+)+\lambda\right)\tau_\theta^2\right)^{-1}>H'(0)$, $\lim_{n\to\infty}d_n(0+)\geq 0$. In summary:

Claim 4: If $\frac{1}{2\lambda\tau_{\theta}^{2}} < H'(0)$ and ρ is low enough there is no equilibrium with endogenous information acquisition for n large.

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