Imperfect competition, risk taking, and regulation in banking

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Abstract

We assess the welfare implications of banking competition under various deposit insurance regimes in a model of imperfect competition with social failure costs and where banks are subject to limited liability. We study the links between competition for deposits and risk taking incentives, and conclude that the welfare performance of the market and the appropriateness of alternative regulatory measures depend on the degree of rivalry and the deposit insurance regime. Specifically, when competition is intense and the social failure costs high, deposit rates are excessive both in a free market and with risk-based insurance. If insurance premiums are insensitive to risk then the same is true even if there is no social cost of failure. We find also that in an uninsured market with nonobservable portfolio risk or with flat-premium deposit insurance deposit regulation (rate regulation or deposit limits) and direct asset restrictions are complementary tools to improve welfare. In an uninsured market with observable portfolio risk or with risk-based insurance deposit regulation may be a sufficient instrument to improve welfare. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

We study the impact of banks’ market power on risk taking incentives in the presence of limited liability and a social cost of failure. We find that the degree of rivalry of banks and the insurance regime are crucial determinants of the welfare performance of the market. Our model provides a framework to assess the effect of deposit regulation and asset restrictions in different insurance regimes.

Competition has traditionally been considered a source of excessive risk taking in banking and in consequence regulation has tried to control it. Rate regulation, entry restrictions, and charter limitations of banks (including the separation of commercial and investment banking) have been used by regulators to limit competition. Deposit rate regulation was established in the US during the 1930s and in Europe at different times. In fact, rates have remained regulated in most countries until recently. Sometimes governments (specially in Europe) have even encouraged collusive agreements among banks (Baltensperger and Dermine, 1987; Vives, 1991). Other regulatory facilities like the lender of last resort and deposit insurance have been widely implemented in order to prevent runs and instability in the banking system. Regulatory measures provided a long period of stability of the banking system (from the 1940s to the 1970s).

The deregulation wave which followed from mutual fund competition for deposits in the US scrapped restrictions on rate setting. Indeed, by 1983 all depository institutions in the US could freely compete in rates offered to customers. In Europe rate setting is now mostly liberalized (with some exceptions like demand deposits in France). Further, the need to better diversify the portfolios of banks has prompted a move towards less specialization in the sector. For example, savings institutions have been able to start acting like banks and compete directly with them. In general, the regulatory changes have promoted competition by decreasing geographical and activity restrictions and thus reducing entry barriers.

The large increase in bank failures in the US in the 1980s (including the Savings and Loans’ crisis) has prompted a debate over what has gone wrong in

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1 Basic regulation in the US is contained in the Pepper–McFadden Act (1927) and in the Banking Act (1933) (Glass–Steagall), the latter separating commercial from investment banking. In Europe universal banks (able to hold equity positions) were allowed in different countries following the German model. In some countries savings banks traditionally specialized in channeling savings into mortgage loans.

2 See Gual and Neven (1992).

3 However, recent regulation in the US (1991) requires S&L to hold at least 70% of assets in residential mortgages.
banking regulation. In this debate, rate regulation has been set aside as a regulatory tool, the emphasis being replaced by the focus on reforming deposit insurance (moving towards a risk-based system and limiting its coverage) and emphasizing capital requirements to control risk exposure. Despite the recent move towards competition in the banking sector it is clear that there are trade-offs involved, namely, a potential increase in failure rates, and that the we still need to understand better the effects of different regulatory tools.

A challenge to build a regulatory theory for the banking sector is to determine the specificity or uniqueness of banks with respect to other firms. Crucial features are the large weight of debt in banks’ capital structure and the wide dispersion among small investors of this debt (deposits). The large amount of debt increases the risk of failure (or insolvency) while the dispersion on small investors limits their ability to monitor the activities of the bank. Further, the social cost of failure of a bank is perceived to be large. This social cost includes the costs of financial distress and economic distress (Berger et al., 1995). The former are typically born by the bank’s creditors and shareholders and hence internalized in their decisions. However, some of the costs are completely external such as disruption of the payment system – interrupting the clearing process, inducing perhaps a failure in interbank settlements, and contagion effects – the failure of a bank carries bad news for another bank with a similar

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4 The deregulation hearings in the US Congress in 1979 to eliminate deposit rate ceilings cite evidence of studies of banking in the 1920s, before deposit insurance was established, by Cox and Benston which conclude that there was no correlation in this period between bank risk and the rates set. This evidence has been challenged by Rolnick (1987).

5 Indeed, flat-premium and complete insurance is now seen by some practitioners, at least in the US, as the main cause of excessive risk taking both on the asset and liability sides of banks balance sheet, and it has even been claimed, when arguing to eliminate rate regulation, that unregulated deposit rates and bank risk are unrelated. For example, deposit insurance ‘subsidizes uneconomical banking practices and destroys the market’s ability properly to price deposit and loan rates’ and encourages ‘deposit rates that are too high and lending rates that are too low – given the level of risk – just to win business’ (Euromoney, p. 33, US Banking, February 1991). A central concern of regulators is to limit the risk of failure of banks in order to protect depositors (or keep under control the cost of deposit insurance schemes) and avoid the external costs associated to a failing institution. The Basle Accords impose constraints on the loans that exceed 10% of the bank’s capital and prevent banks from lending to a single borrower more than 25% of the bank’s capital. Likewise, the US 1991 FDICIA allows risky activities only to well-capitalized banks. When a bank solvency level is below a certain limit it cannot expand its assets. Larger decreases in solvency may trigger the need to recapitalize or even rates ceilings may be imposed. See Dewatripont and Tirole (1994) for a summary of recent US regulation.

6 See Guttentag and Herring (1987). Some costs are only partially internalized like the loss of informational capital and destruction of long-term relationships of borrowers of the bank – who have to find other lines of credit to continue their business. This effect has been emphasized by Bernanke (1983) in his study of the Depression of the 1930s.
portfolio and can trigger its failure. In summary, in banking there is an important probability of failure, with a potentially severe moral hazard problem, and failure has associated a large social cost, typically of a systemic nature. The fact that governments are prepared to pay large amounts to bail out banks as opposed to simply liquidating them may be interpreted as evidence that the external cost of failure is perceived to be high.

In this paper we study the links between competition for deposits and risk taking in the banking sector. A main purpose of the analysis is to understand whether ‘excessive’ competition for deposits exists. To this end we explore how rate setting behavior depends on the risk position of banks both with and without deposit insurance (including both flat-premium and risk-based versions of insurance), what are the derived incentives for risk taking on the asset side, and how incentives change with rate regulation, or deposit limits (equivalent to capital requirements in the short run, when capital is fixed).

The model we present, although based on modern financial intermediation theory, is very simple and abstracts from several potentially important features of banking competition. We start by taking the need of financial intermediation between lenders and borrowers for granted because banks take advantage of their superiority in minimizing incentive transaction costs in monitoring loans (as delegated monitors like in Diamond (1984) or Krasa and Villamil (1992)). We focus on the roles of limited liability, imperfect competition among institutions, and social costs of failure in evaluating welfare under various regulatory regimes. First, we believe that imperfect competition is an important, although somewhat neglected, aspect of banking. Indeed, recent empirical research on European banking highlights the importance of imperfect competition and market power in the sector (Neven and Röller, 1994). Second, the consideration of limited liability and the existence of large perceived social costs of failure of financial institutions seem relevant elements in explaining the specificity of banks and correspondingly in evaluating regulatory measures.

Limited liability is introduced with the standard debt contract offered to depositors (not contingent on the realized portfolio of the bank). The portfolio of the bank is not perfectly diversified and in case of failure there is a social cost not born by the bank: a social cost of failure. Friction in the deposit market is

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7 First, we do not consider two-sided competition (as in Yanelle (1989, 1991), for example)) and restrict attention to deposit rivalry. We therefore ignore the effects of competition for investment projects (see Gennette (1990), for example). Second, we abstract from the consequences of diversification-based or size related economies of scale (in contrast, for example, to Matutes and Vives (1996) and Cerasi and Daltung (1996)). Likewise, we leave out liquidity and informational problems leading to bank runs (see Bryant, 1980; Diamond and Dybvig, 1983). Third, we disregard all aspects concerning informational problems of the regulator vis-à-vis the management of financial institutions (see Dewatripont and Tirole, 1993, 1994; Bensaid et al., 1993).
introduced with a standard product differentiation model. Banks choose the risk of their investment portfolio and the deposit rate offered, and investors choose how much to deposit in each bank.

We consider two cases, one where the risk of banks’ portfolio is observable by investors and the other where it is not. The former can be interpreted as if depositors were sufficiently sophisticated to realize how deposit rates and investments determine the probabilities of failure of banks and hence the expected return from deposits. This may not be a bad assumption, for example, in the market for certificates of deposit. Indeed, there is evidence that uninsured depositors do take into account the risk position of banks and penalize riskier institutions. In the latter case depositors are not well informed and have expectations about banks’ investments (which in turn determine expected failure probabilities), but in equilibrium depositors expectations are fulfilled.

The model allows us to disentangle the roles that limited liability, deposit insurance (both with flat and risk-based premia), and (imperfect) competition for deposits play in determining risk taking incentives when the level of risk depends on choices which affect both the asset and the liability side of the balance sheet. Likewise we clarify the role of deposit and asset regulation both in the case of insured and uninsured deposits and how does it depend on how informed depositors are about banks’ investment portfolio. Key parameters in our analysis are the intensity of competition and the level of the social cost of failure.

Our main findings are as follows. First, an uninsured market yields excessive rates when the failure costs are high and competition is intense. The higher the degree of competition, the larger the set of failure costs for which the market outcome delivers excessive rates (and when banks are perfectly competitive rates are excessive for any positive failure cost) (Proposition 2). Furthermore, deposit regulation is a sufficient instrument to maximize welfare when risk is observable (Proposition 3), but when it is not, investment restrictions may be needed to complement deposit controls (Proposition 4). Second, if competition is intense, flat premium deposit insurance tends to augment the aggressiveness of banks (Proposition 5). In addition, flat premiums induce excessive rates even when there is no social cost of failure and deposit regulation needs to be complemented with investment restrictions (Proposition 7). Third, risk-based insurance eliminates limited liability and generates lower equilibrium deposit and failure rates, and higher welfare, than in the case with no insurance (Proposition 8). However, even with risk-based premiums deposit rates may be too high and welfare may be improved by introducing deposit limits or rate regulation.

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8 See Rolnick (1987) for evidence from the 1920s and Hannan and Hanweck (1988) for evidence on certificate of deposits.

9 Bhattacharya (1982) and Smith (1984) also provide a rationale for interest rate restrictions.
Section 2 introduces the model. Section 3 examines competition in a free market. Section 4 explores the welfare implications of deposit regulation in an uninsured market. Section 5 introduces alternative types of deposit insurance. Concluding remarks close the paper.

2. The model

Banks are risk neutral and choose the level of risk in their investment portfolio of risky entrepreneurial projects. All portfolios are imperfectly diversified and share the same expected return per unit of funds invested. More specifically, denote by $\tilde{R}_i$ the (random) return of a unit of funds invested in entrepreneurial projects by bank $i$. $\tilde{R}_i$ is distributed according to a distribution function $G_i$ which is of class $C^2$ with positive density on the interval $(\theta, \bar{\theta})$. The return will be nonnegative if the worst it can happen is to lose the investment. Let the corresponding density function be $g_i = g(R_i, \gamma_i)$, where $\gamma_i \in [\underline{\gamma}, \bar{\gamma}]$ is an index of the risk of bank $i$'s investments. Higher values of $\gamma_i$ are associated with mean preserving spreads over $G_i$ so that $E(\tilde{R}_i) = \bar{R}$ for all $G_i$. Banks can also invest at a risk-free rate $a$ with $a(R_M)$. Thus, bank $i$ chooses $\gamma_i$ and hence chooses its level of asset risk but not the expected return per dollar invested. For example, banks have available i.i.d. normally distributed projects with mean $R$ and variance $\sigma^2$. Projects are completely divisible. By choosing $n$ of them, a bank has a mean return per unit invested normally distributed with mean $\bar{R}$ and variance $\sigma^2/n$. The parameter $\gamma$ corresponds then to $1/n$ and thus by choosing $n$ the bank can control the risk of the investment. Note that we are abstracting here from the effect of the size of the bank on diversification. That is to say, the extent to which a bank can diversify its portfolio does not depend on the asset size, since it can choose as many projects as desired whatever its size. Therefore we are assuming away diversification-based economies of scale.10

The following example satisfies the assumptions made: $\tilde{R}_i = \bar{R} + 2(1/2 - \tilde{X}_i)$ where $\tilde{X}_i$ follows a symmetric beta distribution on the interval $[0, 1]$ with parameters $(1 + (1/\gamma_i), 1 + (1/\gamma_i)]$. We have then that the support of $\tilde{R}_i$ is $[\bar{R} - 1, \bar{R} + 1], E[\tilde{R}_i] = \bar{R}$ and $\text{Var}[\tilde{R}_i] = 1/(3 + 2/\gamma_i)$. A larger $\gamma_i$ represents a mean-preserving spread of the distribution of returns. Maximum risk $\bar{\gamma}$ will correspond to a uniform distribution for $\bar{\gamma} = \infty$.

We assume that banks have no capital to invest but they can raise money from depositors/investors who cannot invest directly in entrepreneurial projects (for example, it is too costly for investors, small in relation to the size of any

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10 Diversification-based economies of scale are dealt with in Matutes and Vives (1996).
project, due to monitoring problems). To attract deposits banks offer a standard debt contract. That is, bank $i$ offers a rate $r_i$, which will be paid in case the bank does not go bankrupt. If the bank cannot fulfill its commitments, depositors receive whatever funds are left. We assume that in this case there is a social cost of failure $K$ (which is not internalized by the bank) and is related to external effects as described in the introduction.

Banks are differentiated and retain some monopoly power (we may think that banks offer different complementary and convenience services such as branch networks, ATM and credit card facilities, differential access to consumer loans and mortgages or to international operations). Depositors are risk neutral and supply elastically to bank $i$, $i = A, B$, according to a linear schedule:

$$S_i = a + b \Phi_1^i(r_i) - c \Phi_2^i(r_i),$$

where $\Phi_1^i(r_i)$ is the (common) assessment of depositors of the expected return of a unit deposited in bank $i$. This equals the actual expected return $\Phi_1^i(r_i; \gamma_i)$ if $\gamma_i$ is observable. We have thus: $\Phi_1^i(r_i; \gamma_i) = E\tilde{r}_i$ (with $\tilde{r}_i = \min\{r_i, \tilde{R}_i\}$ and $\tilde{R}_i$ is the random unit return of bank $i$). That is, $\Phi_1^i = r_i^i + \int_0^{\gamma_i} R_i g_i(R_i; \gamma_i) dR_i$ where $p_i = \int_0^\gamma g_i(R_i; \gamma_i) dR_i$ (obviously, $\Phi_1^i \leq r_i^i$). If $\gamma_i$ is not observable then $\Phi_1^i(r_i) = \Phi_1^i(r_i; \gamma_i^A)$ where $\gamma_i^A$ is the assessment of the asset risk position of bank $i$.

The supply functions of deposits can be thought of as coming from a representative investor with a utility function linear in income (or a continuum of identical depositors):

$$U = \tilde{r}_A S_A + \tilde{r}_B S_B - T(S_A, S_B),$$

with

$$T = \sigma(S_A + S_B) + (S_A^2 + S_B^2) + 2\lambda S_A S_B)/2.$$

Parameters are such that $\tilde{R} > \sigma > 0$, and $1 \geq \lambda \geq 0$. The parameter $\lambda$ is an index of differentiation of banks. When $\lambda = 1$ banks are not differentiated and when $\lambda = 0$ the banks are independent monopolies. The representative investor maximizes expected utility and this yields the inverse supplies $\Phi_1^i(r_i) = \sigma + S_i + \lambda S_j$, $i, j = A, B$. Inverting this system direct supplies are obtained with $a = -\sigma/(1 + \lambda)$, $b = 1/(1 - \lambda^2)$ and $c = \lambda/(1 - \lambda^2)$. Note that $b \geq c \geq 0$. The parameter $\sigma$ is to be interpreted as the reservation value of depositors (the risk-free rate). Indeed, when banks offer the same expected return supply is $S = a + (b - c)\Phi_1^i(r) = (\Phi_1^i(r) - \sigma)/(1 + \lambda)$, which is positive if and only if $\Phi_1^i(r) > \sigma$. Note that the representative depositor likes variety, that is, prefers to deposit in both banks. This can be interpreted literally (for example, because

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one bank offers better credit card service while the other better access to consumer loans) but it need not. In fact, similar results to the ones derived in the paper can be obtained in the context of a spatial model with a population of heterogeneous consumers.\textsuperscript{12}

Bank \( i \) declares bankruptcy when revenues cannot cover payment obligations: \( \bar{R}_i < r_i \). In this case the bank is left with no income. Thus, expected profits of bank \( i \) can be written as (operational costs are assumed to be zero for simplicity):

\[
\pi_i \equiv S_i m_i
\]

where

\[
m_i = \int_{r_i}^{\bar{R}} (R_i - r_i) g_i (R_i) dR_i,
\]

and where \( S_i \) is the deposit supply attracted by the bank. Obviously, \( m_i \geq \bar{R} - r_i \). And since \( \bar{R} \) is larger than the risk-free rate, it follows that banks will always invest all deposits in risky loans.

Notice that \( \Phi_i + m_i = \bar{R}, \) \( \partial m_i / \partial r_i = - \partial \Phi_i / \partial r_i = - p_i \), and \( \partial p_i / \partial r_i = - g_i (r_i) \). Since \( m_i \) is convex and \( \Phi_i \) concave in \( R_i \), and a larger \( \gamma_i \) represents increasing risk (a mean preserving spread of \( G_i \)), for \( r_i \) in \( (h, \bar{h}) \), \( m_i \) is strictly increasing, and \( \Phi_i \) strictly decreasing, in \( \gamma_i \). Thus, an increase in risk in the investment portfolio of bank \( i \) increases the expected margin of the bank and decreases the expected return to the depositor. Obviously, for a given assessment \( \gamma_i^e \), \( \Phi^e_i (r_i) = \Phi_i (r_i; \gamma_i^e) \) is independent of the actual level of risk \( \gamma_i \).

We will consider two alternative games. In the first one, portfolio risk is observable and can be committed to by the bank before investors deposit. In the second one, portfolio risk is not observable and thus it can be interpreted as if banks were unable to commit to portfolio choices ex ante, i.e., before investors deposit, and hence there is a moral hazard problem. More specifically, the timing of the former game is as reflected in Fig. 1a: Banks choose the risk of their investment portfolio \( (\gamma_i) \) and set deposit rates \( (r_i) \). In turn, depositors upon observing both the risk and rates offered choose how much to supply. Next, returns \( \bar{R}_i \) are obtained and payments to depositors are made if the bank does not go bankrupt.

When banks cannot commit ex ante to the portfolio risk (see Fig. 1b) depositors are endowed with a common assessment of the risk of each bank \( \gamma_i^e \). Then

\textsuperscript{12}See Anderson et al. (1992) for relations between discrete choice, address and representative consumer models of product differentiation.
banks set rates and in turn investors deposit in each bank. Banks choose the risk of their investment portfolio ($c_i$), and the rest of the game proceeds as in the previous case. In equilibrium expectations are fulfilled: $\gamma_i^e = \gamma_i$.

3. Free market competition

In this section banks compete freely with no regulation. We first study the case where portfolio risk is observable. We examine the effect of the asset risk position of banks on the deposit rates set, and then explore the effect of competition for deposits on banks’ incentives to take asset risk. Finally, we consider the case with unobservable portfolio risk (moral hazard).

3.1. Observable portfolio risk

We characterize first rate setting for given asset risks. Since asset risk positions $\gamma_i$ are observable we have: $\Phi_i^e(r_i) = \Phi_i(r_i; \gamma_i)$. The Appendix shows (Lemma A.1) that the rates set by banks, $r_i$ and $r_j$, are strategic complements and that $r_i$ and $\gamma_i$ are strategic complements (substitutes). That is, let $BR_i(r_j)$ be the best response of bank $i$ to the rate set by bank $j$, then $BR_i(r_j)$ is increasing in $r_j$, shifts outwards with $\gamma_i$, and inwards with $\gamma_j$.

That deposit rates are strategic complements is not surprising. The reason why an increase in the risk position of one bank shifts outwards its best response and shifts inwards the best response of the rival deserves more attention. For given rates, an increase in, say, $\gamma_i$ increases the expected margin of bank $A$ due to the effect of limited liability and decreases its deposits since it decreases the expected return to depositors. Both effects increase the marginal profitability of a rate increase and make the bank more aggressive. On the contrary, bank $B$ increases its deposits and as a result becomes softer: since it has to pay the rate to a larger number of deposits, the marginal profitability of a rate increase goes down.

In fact, given the rival’s deposit rate, by choosing asset risk a bank cannot increase expected profits when there is no moral hazard. Indeed, Lemma A.2 in
the Appendix shows that, given the rival’s rate, the market share and expected margin of a bank which maximizes expected profits is independent of its asset risk position. The intuition for the result is that for a given rate of a rival, a bank cannot increase its expected profits by taking more risk since depositors punish the increase in risk by patronizing less the bank. More precisely, by increasing its asset risk a bank increases its margin \( m \) (due to limited liability) but it decreases deposits (since depositors care about expected return \( \Phi \)). When the bank optimizes the two effects exactly cancel each other out (since \( m + \Phi = \bar{R} \)) and expected profits remain constant. That is, uninsured depositors discipline bank risk taking when asset risk is observable.

The following proposition characterizes the impact of portfolio risk on deposit rates, market shares and profits.

**Proposition 1.** For given observable asset risks bank competition yields a unique equilibrium. The equilibrium is symmetric in terms of margins, attraction of deposits and expected profits irrespective of the asset risk positions of banks. The margin is given by \( m(r_i; \gamma_i) = (\bar{R} - \alpha)(1 - \lambda)/(2 - \lambda) \). The rate of bank \( i, r_i, \) is increasing in the portfolio risk of \( i, \gamma_i, \) and is independent of \( \gamma_j \). This results in the bank with a more risky portfolio setting a higher deposit rate than its rival.

**Proof.** In equilibrium, \( bm_i - S_i = 0 \) and \( bm_j - S_j = 0 \) with \( S_i = a + b \Phi_i(r_i) - c \Phi_j(r_j) \). Subtracting the first equation from the second we obtain: \( b(m_i - m_j) = (b + c)(\Phi_i - \Phi_j(r_j)) \). Using the fact that \( m_i + \Phi_i = \bar{R} \) it follows that \( (2b + c)(\Phi_i(r_i) - \Phi_j(r_j)) = 0 \). This can be true only for \( \Phi_i(r_i) = \Phi_j(r_j) \). Hence, the equilibrium is characterized by \( bm - S = 0 \) with \( S = a + (b - c)\Phi \). Since \( m + \Phi = \bar{R} \), the expression for \( m \) follows. In equilibrium \( m = m(r_i; \gamma_i) \) for \( i = A, B \). The equilibrium \( r_i \) increases in \( \gamma_i \) since \( m = (\bar{R} - \alpha)(1 - \lambda)/(2 - \lambda) \) and \( m(r_i; \gamma_i) \) is increasing in \( \gamma_i \) and decreasing in \( r_i \).

The predictions of our simple model are thus consistent with the empirical findings which uncover a pattern of riskier banks paying higher rates for uninsured deposits (see, e.g., Rolnick, 1987; Hannan and Hanweck, 1988).

**Remark.** It can be checked that equilibrium rates imply an expected return for depositors \( \Phi > \alpha \). The margin of banks increases with \( \bar{R} \) and decreases with \( \alpha \). Further, the less differentiated banks are (\( \lambda \) higher and the more competition there is) the smaller the margin is and consequently the higher the rates offered are. Indeed, when banks are not differentiated (\( \lambda = 1 \)) and competition is maximal then \( m = 0 \) and therefore the rate offered by banks equals \( \bar{R} \). Market power moderates deposit rates.

According to the proposition, a bank does not manage to attract more depositors or to increase its profits in equilibrium by increasing its asset risk.
position. In terms of best response functions, an increase in $\gamma_A$ shifts $A$’s best response outwards and $B$’s best response inwards. The outcome is such in equilibrium $r_B$ remains constant (while $r_A$ increases as in Fig. 2). That is, bank $A$ could affect the equilibrium rate set by bank $B$ only by influencing the expected return bank $A$ offers to depositors but this is impossible (see Lemma A.2 in the Appendix for a proof).

From Proposition 1 it follows that when banks’ portfolio risk is observable, any asset risk choice is compatible with equilibrium.$^{13}$

3.2. Unobservable portfolio risk

Let us now consider the game with moral hazard (the portfolio risk position of a bank is unobservable). In this case, depositors anticipate the asset risk positions according to a given prior $\gamma_i^e$, and banks have incentives to undertake maximum risk. Indeed, these priors determine banks market shares of deposits for given rates, but expected margins are an increasing function of actual asset risk choices due to limited liability. That is, by setting a rate $r_i^e$ bank $i$ cannot

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$^{13}$This holds even in the sequential move case in which asset risk is chosen first and bank $i$ can condition on both $\gamma_i$ and $\gamma_j$ when deciding the deposit rate.
credibly signal that it will choose $\gamma_i < \bar{\gamma}$, since ex post the profit maximizing choice is $\bar{\gamma}$. In this context, the only equilibrium is for banks to choose maximal asset risk. In other words, the only ‘rational’ priors (i.e. consistent with banks’ choices) correspond to maximal asset risk. In summary, the conjunction of limited liability and moral hazard leads banks to undertake maximum asset risk.

4. Welfare, the intensity of competition, and regulation

Let us state first the welfare measures in our market. Expected gross surplus is given by consumer surplus plus expected profits. That is: $GS = CS + \pi_A + \pi_B$ with $CS = \Phi_A(r_A)S_A + \Phi_B(r_B)S_B - T(S_A, S_B)$ and $\pi_i = m_iS_i$. Hence, $GS = (S_A + S_B)\bar{R} - T(S_A, S_B)$. The deadweight loss is given by $F = ((1 - p_A) + (1 - p_B))K$ and corresponds to the expected social cost of failure.\(^{14}\) Finally, expected total surplus is given by $TS = GS - F$.

4.1. Welfare-optimal policies

Before examining the scope for welfare enhancing regulation it is helpful to study welfare-optimal rate setting for given symmetric asset risks $\gamma_i = \gamma$. In order to do so, Proposition 2 characterizes the welfare optimal rates and investigates how do they compare to market rates (with observable asset risk). It is assumed that the hazard rate of the distribution of returns $G, H(r) = g(r)/p(r)$, is nondecreasing in $r$.

Proposition 2. Let asset risks be symmetric and observable, $\gamma_i = \gamma$, let $\bar{\gamma} < \gamma$, and suppose that the hazard rate $H$ is nondecreasing. Then

(i) it is optimal to disintermediate when $K \geq \bar{K}(\lambda)$, where $\bar{K}(\lambda)$ is a decreasing function of $\lambda$ in the interval $[0, 1]$. Otherwise,

(ii) the welfare-optimal rate $r^*$ satisfies: $m(r^*; \gamma) = (1 + \lambda)K H(r^*)$. It is decreasing with $K$ and increasing with the degree of differentiation $1/\lambda$.

(iii) For a low degree of differentiation ($\lambda$ close to 1), market rates are higher (lower) than optimal for $K$ large (small). As differentiation is reduced the region of excessive market rates increases and for no differentiation rates are excessive for all positive $K$. For a high degree of differentiation ($\lambda$ close to 0), it is possible that (when intermediation is optimal) market rates are always too low.

Proof. See the Appendix.

Remark. When there is no social cost of failure and no differentiation then the market rate is optimal. The sources of inefficiency in our model are the external

\(^{14}\) Note that in our formulation no contagion effects are modelled. It may be plausible also to assume that the failure of both banks at the same time is costlier than $2K$. In this case the expected social cost of failure, for given probabilities of failure, increases.
cost of failure $K$ and market power $1/\lambda$. These are countervailing forces. That is, the external failure cost tends to generate excessive rates while market power tends yield rates below the optimal level. Hence, when there is a social cost of failure some degree of market power is necessary to approximate market rates to the optimal ones. The larger $K$, the larger $1/\lambda$ must be to equate market and optimal rates.\(^\text{15}\)

The rationale behind Proposition 2 is straightforward. For a large enough cost of failure $K$ intermediation is not optimal. The intermediation constraint is relaxed with higher differentiation (see Fig. 3). That is, for lower $\lambda$ intermediation is optimal for larger $K$'s. This is so since, other things equal, $\text{TS}$ increases the more differentiated the banks are. Further, the fact that market rates will tend to be too low or too high depending on the level of $K$ is easily understood.

An increase in the deposit rate of bank $A$ has three external effects, namely on bank $B$, investors, and social failure cost. The effect on the rival’s profits is negative $\frac{\partial \pi_B}{\partial r_A} = -cm_Bp_A < 0$. The effect on consumer surplus is positive: $\frac{\partial CS}{\partial r_A} = p_A S_A$. The effect on the social cost of failure is positive and equal to $g(r_A)K$. The aggregate effect on welfare is given by $\frac{\partial TS}{\partial r_A} = \frac{\partial \pi_A}{\partial r_A} + \frac{\partial \pi_B}{\partial r_A} + \frac{\partial CS}{\partial r_A} - gK$, which evaluated at the market solution $bm = S$ yields $pm(b - c) - gK$ or $p(m(b - c) - HK)$. Since $pm(b - c) = 0$, the external effect on consumer surplus dominates the effect on rivals’ profits. This classical imperfect competition effect must be set against the effect on the social cost of failure. The result is that the market will tend to set too low (high) a rate for $K$ low (high). In fact, at the market solution $r^\prime$, $\frac{\partial TS}{\partial r_A}$ will be positive or negative according to whether $m(r^\prime)(b - c) - H(r^\prime)K$ is positive or negative. Given that $(b - c) = (1 + \lambda)^{-1}$, the imperfect competition effect is larger when differentiation is high ($\lambda$ low). Therefore, high differentiation makes more likely that market rates are low relative to the optimal ones.

With returns distributed as a beta function, there is a critical $\gamma$ such that for $\gamma$ above the critical point the situation is as in Fig. 3b. The critical $\gamma$ decreases with the mean of the returns.

Let us now turn to the optimal solution when $\gamma$ is also a choice variable.

\(^{15}\) Along the schedule $TS^\prime = 0$ the (optimal) rate $r^*$ is independent of $\lambda$ and $K$. Indeed, $TS^\prime = 0$ can be rewritten as

$$K = \frac{(\bar{R} - \lambda - m^0)((\bar{R} - \lambda/2) + m^0)}{(1 - p^\prime)(1 + \lambda)}$$

and we know that $m^\prime = (1 + \lambda)KH(r^\prime)$. Combining both equations we obtain that

$$(\bar{R} - \lambda - m^0)((\bar{R} - \lambda/2) + m^0)(1 - p^\prime)H(r^\prime) = m^\prime.$$  

This is an equation in $r^\prime$ which does not involve $K$ or $\lambda$. 


Lemma 1. Let $0 < \gamma$, suppose that the hazard rate $H$ is nondecreasing in $r$ for any $\gamma$, and that the distribution of returns is symmetric. Let $y = \int_{R}^{\lambda} R g(R, \gamma) dR / \bar{R}$, then if

$$
\frac{(2y - 1)\bar{R}}{4g(\bar{R}, \gamma)} < 2 \left[ \bar{R} \left( \frac{3}{2} - y \right) - \gamma \right] \left[ y\bar{R} - \frac{\alpha}{2} \right],
$$

there exist $K_2(\bar{R}, \gamma) < K_3(\bar{R}, \gamma)$ such that for $K_3 > K > K_2$, the welfare-optimal policy is $\gamma$ and $r^0(\gamma)$, and for $K > K_3$ it is optimal to disintermediate.

Proof. See the Appendix.

Remark. Although the range of parameters for which the welfare-optimal policy is $\gamma = \gamma$ is very large (the lemma only gives sufficient conditions) it is indeed
possible that it is optimal to set \( \gamma = \bar{\gamma} \). This tends to happen for very small values of \( K \), and large values of \( \alpha \) relative to \( \bar{R} \) since then it is worth to increase the expected failure cost by setting a rate above \( \bar{R} \) (and hence to minimize the cost of failure \( \bar{\gamma} \) is optimal) to induce a larger amount of deposits. For example, it happens with beta distributed returns with \( \bar{\gamma} = \infty \) and \( \gamma = 1/30 \), for \( R = 0.0001 \), \( \alpha = 1 \), \( \bar{R} = 1.2 \), and \( \lambda = 0.05 \).

### 4.2. Rate or deposit limits under observable portfolio risk

The question arises of how can the optimal solution be implemented. We focus on the case where \( K \) is not so large so as to have optimal desintermediation and consider as potential regulatory instruments, deposit limits (or rate ceilings) and asset restrictions. It is worth noting that a deposit limit \( S \leq \bar{S} \) can be interpreted as a capital requirement. Indeed, suppose that the capital of the bank is given \( (k) \) and that a capital requirement \( \delta \) is imposed. That is, \( k/A = \delta \), where \( A \) is the banks’ investment in risky assets. In our case \( A = S + k \) since all will be invested in risky assets. This is equivalent to \( S \leq (1 - \delta)k/\delta \).

Let us explore first how rate regulation affects banks’ choice of asset risk. The following lemma does so for the case where the \( \gamma \)'s are observable.

**Lemma 2.** When the risk of the portfolio of banks is observable, a rate ceiling at a level below the market rate at the minimal level of risk, induces banks to choose the lowest level of asset risk.

**Proof.** As shown in the proof of Lemma A.1 the expected profit is a concave function of the rate set. From Lemma A.2 we know that the optimal rate set by a bank increases with its level of risk while its expected profit remains constant. Suppose that a rate ceiling below the market rate corresponding to the lowest risk level is imposed. Then the bank by choosing a higher level of risk would diminish its expected profits since optimal rates increase with risk but the rate ceiling is binding. See Fig. 4 for an illustration.

**Remark.** A similar argument shows that a rate floor at a level above the market rate at the maximal level of risk, induces banks to choose the maximal level of asset risk.

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\(^{16}\)When returns are beta-distributed with \( \bar{\gamma} \) corresponding to the uniform distribution \( \bar{\gamma} = \infty \) and \( \alpha = 1 \), the inequality

\[
\frac{(2\gamma - 1)\bar{R}}{4\gamma(\bar{R}, \bar{\gamma})} < 2 \left[ \bar{R} \left( \frac{3}{2} - \gamma \right) - \alpha \right] \left[ \gamma \bar{R} - \frac{\alpha}{2} \right]
\]

holds if \( \bar{R} = 1.5 \). With a stricter limit on the maximum risk the range of \( \bar{R} \geq 1.5 \) for which the inequality holds is larger. For example, when \( \bar{\gamma} = 1/5 \) the critical \( \bar{R} \) is 1.23 and when \( \bar{\gamma} = 1/10 \), it is 1.15.
Note that even if the asset risk position of banks is observable it need not be verifiable for regulatory purposes. That is, well-informed investors may base their decisions on soft information but regulatory actions need to be based on hard information. This verifiability problem explains why typically it is not possible to control directly the risk of the portfolio of a bank.

Remark. With a deposit limit below the market level banks will be indifferent about the level of asset risk taken. The reason is that when the deposit limit is binding, by definition, banks cannot augment their deposit attraction, and hence will be indifferent between pairs of \((r, \gamma)\) that yield identical profits and an amount of deposits equal to the deposit ceiling.

When portfolio risk is observable, rate regulation induces safe investments through a channel which relates the deposit rate and the level of asset risk that a bank would optimally choose. As we have seen, asset risk and deposit rates are strategic complements in the sense that the optimal deposit rate increases with the level of risk. Therefore, once the incentives to take risk are accounted for, it is plausible that there is scope for regulatory policy (deposit limits or rate ceilings) to improve welfare when \(K\) is relatively large.\(^{17}\) Indeed, with banks choosing the

\(^{17}\)Note that even if the asset risk position of banks is observable it need not be verifiable for regulatory purposes. That is, well-informed investors may base their decisions on soft information but regulatory actions need to be based on hard information. This verifiability problem explains why typically it is not possible to control directly the risk of the portfolio of a bank.
least asset risk, market rates are too high. Then, according to Lemma 2, the welfare optimum \( y = \gamma \) and \( r = r^0(\gamma) \). The above premises will hold, under the assumptions of Lemma 1, whenever \( K \) is between \( K_2 \) and \( K_3 \) and when \( \lambda \) is close enough to 1 (according to Proposition 2); that is, for an intermediate range of the social cost of failure and for a high degree of competition (low degree of differentiation) in the market. The latter insures that market rates are too high from the welfare point of view. On the other hand, for a low degree of competition (high degree of differentiation) it may happen that market rates are always too low. In summary:

Proposition 3. Under regularity conditions (as stated in Lemma 1), for a large social cost of failure, but not so large as to make intermediation suboptimal, it is optimal to impose a rate ceiling corresponding to the optimal rate for minimal asset risk whenever competition is intense.\(^{18}\)

4.3. Unobservable portfolio risk and asset restrictions

Suppose now that the asset positions of banks are not observable (that is, there is a moral hazard problem). Then assuming maximum risk is optimal with or without rate or deposit regulation since, as argued above, with limited liability this always increases the expected margin and does not hurt deposit supply because the risk assessment of depositors is given. As a result both deposit or rate regulation and direct asset restrictions\(^{19}\) (which in terms of our model can be understood as narrowing the potential range of the feasible risk positions \([c_1, c_N]\)) may be called for when the social cost of failure is large. In consequence:

Proposition 4. When the asset risk position of banks is not observable, competition is intense, and the social cost of failure is large, deposit (or rate) regulation may need to be combined with direct asset restrictions.

It is worth noting that the informational requirements necessary to implement rate or deposit regulation are very strong. Indeed, the regulator must know the level of product differentiation in the market (the \( \lambda \) parameter), the reservation value \( a \), the distribution of investment risk, the failure cost \( K \), as well as the functional dependence of all the magnitudes.

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\(^{18}\) With a deposit limit at the level \( \bar{S} = S(r^0(\gamma)) \) banks would be indifferent about the level of asset risk assumed.

\(^{19}\) In practice asset restrictions relate to capital–asset ratios, with risk-weighted measures of assets, and limitations on 'large exposures' and the concentration of risks, all of this as in the Basle Accords. Furthermore, there are direct asset restrictions like limitations to the activity of universal banking which can go as far as proposing a narrow bank which invests essentially only in risk-free assets.
5. Deposit insurance

Deposit insurance is widespread in developed economies. The two main rationales for deposit insurance are the prevention of systemic crisis and the protection of small investors. In our model we have abstracted from economies of scale (diversification economies and size effects) and our investors are risk neutral. Therefore, the main motives for deposit insurance are absent in our model. However, if we modify the model to include a minimal size requirement for banks (a very crude way of modelling economies of scale) then multiple equilibria emerge and we introduce the first rationale for deposit insurance in the model. Indeed, assume that each bank needs a minimum deposit size (market share) to be able to invest. Then, the expected payoff of depositing in a bank, say bank A, depends on whether other depositors have also chosen bank A. In fact, for given posted deposit rates a coordination game on depositors is induced. The game has multiple equilibria some involving no banking activity or only one bank active (showing the fragility of the industry). For instance, if each depositor expects that neither bank will attract the minimum amount of deposits necessary to invest, it is best not to deposit since the bank will fail for sure. In this game deposit insurance would force a symmetric interior equilibrium. This is a clear social benefit of deposit insurance.

In this section we explore the costs as well as other potential benefits of deposit insurance and assess the welfare trade-offs which arise. In particular, we focus on how insurance modifies banks’ incentives to set deposit rates and choose asset risk. We deal both with the usual case of insurance premiums independent of risk (Section 5.1) and with risk-based deposit insurance (Section 5.2). Finally, in Section 5.3 we explore optimal regulation in an insured context and compare both forms of insurance with the market outcome.

First, notice that as long as there is full insurance, depositors do not face any risk and hence deposit supplies are given by $S_i = a + br_i - cr_i$, $i = A, B$ (in particular, they are independent of the asset risk of either bank). From the point of view of depositors it does not matter whether asset risk is observable or not. Hence, $r_j$ and $\gamma_i$ are no longer strategic substitutes (they are unrelated). The impact of $\gamma_i$ on $r_i$ depends on the effect of $\gamma_i$ on the expected margin, and hence it depends on the insurance design.

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20 See also Matutes and Vives (1996). It is worth pointing out that ‘fragility’ in banking (i.e., the multiplicity of equilibria with some bad outcomes) is due to the coordination problem of depositors. Different equilibria arise because of self-fulfilling expectations not because of competition. The coordination problem may arise with a monopoly bank.
5.1. Flat-premium deposit insurance

Consider the following insurance design. If a bank fails, the insurance fund pays depositors the rates set by that bank. Banks contribute to the financing of the scheme with a percentage of their profits. That is, if banks do not fail, they pay the fund a tax proportional to their profits. With this type of insurance and a tax rate \( \tau \), the expected profits of bank \( i \) are given by \( \pi_i = (1 - \tau) m_i S_i \). The tax rate \( \tau \) can be thought as an insurance premium.\(^{21}\)

Let us start examining the impact of insurance on rate setting. We will assume in this section that expected profits of bank \( i \) with insurance \( \pi_i = (1 - \tau) m_i S_i \) are quasiconcave in \( r_i \). (This holds with beta-distributed returns, for example.)

Lemma A.3 in the Appendix shows that the rates \( r_i \) and \( r_j \) offered by banks are strategic complements. Furthermore, if the banks have the same asset risk positions and \( \alpha \) is small relative to \( r_i \), given \( r_j \) the best response of bank \( i \) with deposit insurance is to offer a larger rate than without.

Deposit insurance makes banks more aggressive when they have the same level of risk and the reservation value of depositors is not too large. The reason is as follows. With insurance depositors obtain the posted rates even if banks go bankrupt and hence a rate increase attracts more depositors than in the case without insurance. That is, the supply of deposits becomes more elastic to the interest rate; indeed, with insurance the elasticity of supply \((\epsilon_i)\) is \( b r_i / S_i \), without it is \( b p_i r_i / S_i \). Note that in both cases for optimal rates \( m_i / p_i r_i = 1/\epsilon_i \). Hence, the more elastic the supply of deposits is, the bigger the incentives to set a high rate are. This is so provided the introduction of insurance does not imply a dramatic increase in deposit attraction for given rates, since in such case the rate must be paid to a much larger number of infra-marginal deposits which would increase the cost of increasing the rate and act as a countervailing force. The assumptions in Lemma A.3 guarantee that this is not the case.\(^{22}\)

\(^{21}\)Indeed, consider the mechanism where, once deposits are collected, banks pay a percentage \( \beta \) (a premium) to the insurance fund and invest the rest. In this case, the expected profit of bank \( i \) is given by

\[
\pi_i = (1 - \beta) m_i \int_{r_i/(1 - \beta)}^{\infty} \left( \tilde{R}_i - \frac{r_i}{1 - \beta} \right) \Phi(R_i) dR_i = (1 - \beta) m_i S_i \left[ E \left( \frac{\tilde{R}_i}{R_i} \geq \frac{r_i}{1 - \beta} \right) - \frac{r_i}{1 - \beta} \right].
\]

By redefining the interest rates as \( r_i/(1 - \beta) \) this alternative mechanism is equivalent to the previous one. That is, the tax rate \( \tau \) can be thought as an insurance premium and the rates obtained in the original model multiplied by \( 1 - \tau \) yield the deposit rates set by banks.

\(^{22}\)This is why we cannot assure that best response functions shift outwards in an asymmetric situation. Indeed, the riskier bank may have incentives to become less aggressive since the increase in market share due to insurance increases with its level of risk. Given rates change in bank \( A’s \) deposits when insurance is introduced has the same sign as \( b[r_A - \Phi(r_A)] - c[(r_B - \Phi(r_B))]. \) This expression is increasing with \( \gamma_A \) since \( \Phi(r_A) \) is decreasing with \( \gamma_A \).
In consequence

**Proposition 5.** Suppose that the banks’ portfolio is equally risky and that banks have some monopoly power $\lambda < 1$. Then, for $\rho$ small relative to the uninsured market rate, deposit insurance with flat premiums leads to a deposit rate strictly larger than the market equilibrium.

**Remark.** Under our assumptions we cannot insure that the equilibrium with insurance is unique. However, whenever there is a symmetric equilibrium this equilibrium will be the unique one (that is, there will be no asymmetric equilibria) since the game that banks play is supermodular (according to Lemma A.3, see Vives (1990)). In the case of beta-distributed returns it can be checked that there is a unique symmetric equilibrium. A sufficient condition for uniqueness of a symmetric equilibrium is that $E(\bar{R}|\bar{R} \geq r)$ be concave or convex for $r$ in the appropriate range. This condition is fulfilled with a constant hazard rate for example.

**Remark.** Simulations show that when returns follow a beta distribution, and for a given degree of differentiation, the insured market yields higher rates than the free market when the mean of returns is above a certain critical point, i.e., when the returns to the investment are large. This critical point decreases with $\lambda$ and increases with $\gamma$, since both increase bank profitability for given rates.

**Remark.** In our model differences in rate setting with and without insurance arise only because of imperfect competition. Indeed, when there is no differentiation ($\lambda = 1$) the equilibrium rate is $\bar{\theta}$ both with and without insurance. It is easily seen that with insurance (similarly as without insurance, see Proposition 1) as $\lambda$ increases the equilibrium rate increases (to $\bar{\theta}$ as $\lambda$ tends to 1). This implies that when competition is intense ($\lambda$ close to 1) the equilibrium rate necessarily exceeds $\bar{R}$.

**Remark.** Flat premiums cannot be fair when competition is intense. Indeed, suppose that there is a tax rate $\tau$ which is ex post fair at the symmetric equilibrium $r^n$. We have then: $\tau \pi = (1 - p)[(r^n - E[\bar{R}|\bar{R} < r^n])S$. By definition, the left-hand side of this equation equals $\tau mS$, and the right-hand side $mS - (\bar{R} - r^n)S$. It follows that if a fair premium exists it must satisfy $\tau = 1 - (\bar{R} - r^n)/m(r^n)$. However, the previous remark implies that for $\lambda$ close to 1, $r^n > \bar{R}$ and therefore the premium should exceed 1.

Let us address now the issue of the choice of asset risk. Notice first that, contrary to the uninsured case, and for given rates of the rival, a bank can

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23 Indeed, with insurance and symmetric $\gamma$’s the equilibrium rate $r^n$ is characterized by $bm - ps = 0$. This is equivalent to $E(\bar{R}|\bar{R} \geq r) = r - (r - \rho)(1 - \lambda) = 0$. It follows, given that the SOC hold and that $r^n > \rho$, that $dr^n/d\lambda > 0$. It also follows that as $\lambda$ tends to 1, $r^n$ tends to $\bar{\theta}$. 
improve its expected profits by taking more asset risk even when it is observable. Indeed, consider a monopoly bank \((c = 0)\). Insurance implies that higher levels of risk do not have an impact on the supply of deposits while the expected margin increases (since the margin is a convex function of actual returns, its expectation shifts up with a mean preserving spread). Formally, \(\frac{\partial \pi_i}{\partial \gamma_i} = (1 - \tau)S_i(\hat{\gamma}_i) > 0\). The same argument applies when banks compete.\(^{24}\)

Hence:

*With flat-premium deposit insurance banks will take the maximum asset risk position irrespective of whether asset risk is observable or not.*

5.2. Risk-based insurance

How would the results obtained change if deposit insurance were to be risk-based? With fair and risk-based premiums bank \(i\) is confronted with a tax/premium schedule contingent on its asset risk position and deposit rate:

\[
\tau_i(r_i; \gamma_i) = 1 - (\bar{R} - r_i)/m_i
\]

\(^{24}\)However, when the asset risk positions can be observed and the banks’ game is sequential (asset risk being chosen first and then deposit rates), competition introduces a strategic effect since a change in the asset risk position of a bank affects its best response function. The risk position of a bank does not have an impact on the best response of its rival when there is insurance (since deposit supply is independent of risk). As a result, the equilibrium rate set by a bank increases with the risk of the rival if and only if the rival’s best response function shifts outwards with its own level of risk. The best response function of bank \(i\) necessarily shifts outwards with \(c_i\), evaluated at the equilibrium rate, its probability of failure increases with \(c_i\). More generally, the best response function of bank \(i\) shifts outwards with its level of risk if and only if

\[
\frac{1}{m_i} \frac{\partial m_i}{\partial \gamma_i} - \frac{1}{p_i} \frac{\partial p_i}{\partial \gamma_i} > 0.
\]

Therefore, the impact of a higher risk on the profit maximizing rate is ambiguous. Consider expected profits as a function of rates and risk positions: \(\pi_i(r_i, r_B; \gamma_i, \gamma_B)\). In equilibrium \(\tau_i^*(\gamma_i, \gamma_B)\).

\[\frac{d\pi_A}{d\gamma_A} = \frac{\partial \pi_A}{\partial \gamma_A} + \frac{\partial \pi_A}{\partial r_B} \frac{\partial r_B}{\partial \gamma_A}.
\]

We know that \(\frac{\partial \pi_A}{\partial \gamma_A} > 0\) and \(\frac{\partial \pi_A}{\partial r_B} = -(1 - \tau)c m_A < 0\). If \(\frac{\partial r_B}{\partial \gamma_A} < 0\) then the increase of risk of bank \(A\) increases the profits of both. With \(\frac{\partial r_B}{\partial \gamma_A} > 0\), the profits of bank \(A\) may go up or down while those of the rival decrease with increases in \(\gamma_A\). Incentives for maximal risk taking will exist when either \(\frac{\partial r_B}{\partial \gamma_A} < 0\) or else the direct effect of a higher risk on expected profits exceeds the impact of the strategic effect.
so that in expected terms the tax on the margin of the bank equals the cost of deposit insurance. The insurer must observe the asset risk position \( \gamma_i \) of the bank. The net expected profit of bank \( i \) is given by

\[
\pi_i = (1 - \tau_i)S_i m_i = (\bar{R} - r_i)S_i.
\]

It follows that risk-based premiums are such that the expected profits of a bank are independent of the asset risk taken. The increase in the expected margin due to a higher investment risk is exactly compensated by an increase in the premium paid. That is, fair risk-based premiums eliminate limited liability, i.e., banks behave as if their profits were \( \tilde{\pi}_i = (\bar{R}_i - r_i)S_i \). The consequence is that banks have reduced incentives to be aggressive in setting rates.

As before, with an appropriate reinterpretation of the deposit rate the tax can be thought of as an insurance premium. However, with respect to risk taking incentives the issue of whether the premium is paid before the choice of asset risk is taken becomes crucial. Indeed, the same arguments as in the previous sections indicate that when the choice of asset risk takes place once the premiums are paid, banks would have incentives to choose maximal asset risk.

The characterization of equilibria with risk-based deposit insurance is given in the following proposition.

**Proposition 6.** With fair and risk-based deposit insurance there is a unique equilibrium. The equilibrium is symmetric in terms of rates, attraction of deposits and expected profits irrespective of the asset risk positions of banks. The margin is given by \( \bar{R} - r'''' = (\bar{R} - \bar{x})(1 - \lambda)/(2 - \lambda) \). When premiums are paid before asset risk is determined risk taking is maximal. Otherwise, banks are indifferent as to the level of risk they take.

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\[^{25}\text{Then bank } i \text{ must pay a premium of } \beta_i(r_i) \text{ per unit deposited such that}
\]

\[
\beta_i = (1 - \beta_i) \int_0^{r_i/(1 - \beta_i)} \frac{r_i}{1 - \beta_i} - R_i \, g(R_i)dR_i.
\]

The expected profit is given by

\[
\pi_i(1 - \beta_i)S_i \int_0^{r_i/(1 - \beta_i)} \left( R_i - \frac{r_i}{1 - \beta_i} \right) g(R_i)dR_i = (\bar{R} - r_i)S_i.
\]

Hence, once again, there is no qualitative difference in the competitive behavior induced by the two mechanisms.

\[^{26}\text{Hence fair risk based premiums would have the form of } \beta_i(r_i, \bar{\gamma}) \text{ such that}
\]

\[
\beta_i = (1 - \beta_i) \int_0^{r_i/(1 - \beta_i)} \frac{r_i}{1 - \beta_i} - R_i \, g(R_i, \bar{\gamma})dR_i,
\]
Proof. The result follows immediately from the FOC of profit maximization of bank $i$: $b(R - r_i) - S_i = 0$ with $S_i = a + br_i - cr_i$. With respect to risk taking incentives, when the premium can be conditioned on asset risk the conclusion follows from the fact that the profit function is independent of $\gamma$. When asset risk is chosen once the premiums are paid, the result follows from $m$ being convex in $\gamma$. □

Remark. The ‘margin’ is equal to $(R - z)(1 - \lambda)/(2 - \lambda)$ in both the free market case and with risk-based deposit insurance. Rates need not be equal in both cases since in a free market they depend on the asset risk position of banks.

Remark. With risk based deposit insurance the equilibrium rates are the same independently on whether the premium is contingent on the choice of asset risk.

In the next section we investigate how private incentives compare with those of a social planner and explore welfare-optimal regulation.

5.3. Welfare and comparison of regimes

Let us first investigate the welfare-optimal outcome when insurance is required to satisfy budget balance. In this case the expression for total surplus is exactly as in Section 4 for given deposit supplies. The reason is that the full cost of insurance must be paid. Indeed, although now investor surplus is given by $CS = r_A S_A + r_B S_B - T(S_A, S_B)$, the effects on CS and on the cost of insurance $P$ collapse into $CS - P = \Phi_A(r_A)S_A + \Phi_B(r_B)S_B - T(S_A, S_B)$ because the cost of insuring deposits of bank $i$ is $P_i = (m_i - ((R - r_i))S_i$ and $P = P_A + P_B$. Obviously, as in the uninsured case, $GS = CS - P + \pi_A + \pi_B = (S_A + S_B)R - T(S_A, S_B)$. The only difference is that supply now depends on the posted rates: $S_i = a + br_i - cr_i$.

Lemma 3. Assume that $G$ is symmetric and that $0 < \alpha$. Then, with budget-balanced deposit insurance there is $\hat{K} > 0$ such that

(i) If $K < \hat{K}$ it is optimal to set $\gamma = \gamma$ and the optimal rate $\tilde{r}^o(\gamma)$ is such that $\bar{R} - \tilde{r}^o = Kg(\tilde{r}^o, \gamma)(1 + \lambda)$. The optimal rate is always lower than $\bar{R}$, and decreases with $K$ and $\lambda$.

(ii) If $K \geq \hat{K}$ disintermediation is optimal.

Proof. Given the structure of the problem we can restrict attention to symmetric solutions. In this case total surplus is given by

$$TS = 2 \left( S(\bar{R} - z) - \frac{1 + \lambda}{2} S^2 - (1 - p)K \right)$$
where $S = a + (b - c)r = (r - \pi)/(1 + \lambda)$. Optimizing over $r$ yields $r^o(\gamma)$. For any gamma the implied interest rate is lower than $\bar{R}$ provided that $K > 0$. (This fact together with the assumption that $G$ is symmetric implies that $TS$ is quasiconcave in $r$ for any $\gamma$). It follows then again from the symmetry of $G$ that $\gamma^o = \text{argmin}(1 - p)K$. Comparative statics follow immediately. The proposed solution yields higher welfare than disintermediation when it is associated with a positive surplus: $TS^o = TS(r^o(\gamma)), \gamma > 0$. When $K = 0$, $TS^o$ is certainly positive. Due to the envelope theorem, $dTS/dK = \partial TS/\partial K = -2(1 - p)K < 0$. For $K$ large enough $TS$ will be negative at the proposed rate $r^o$ since to sustain intermediation $r \geq \bar{\pi}$ and in this case the probability of failure is bounded away from zero. It follows that there exist $\hat{K}$, a function of the parameters $\lambda, \pi, \bar{R}$ and $\gamma$, such that $TS^o = 0$, and when $K$ exceeds $\hat{K}$ disintermediation yields higher surplus. □

**Remark.** For a given asset risk position market rates with flat premiums will be excessive when competition is intense ($\lambda$ close to 1), even without a social cost of failure. This is so since in the case considered market rates necessarily exceed $\bar{R}$ and the optimal rate is always below it. This holds, indeed, for the equilibrium asset risk position in the market solution ($\gamma = \gamma$) and for the optimal asset risk position ($\gamma = \gamma$). However, for a high degree of market power and $K$ low market rates can be too low. Indeed, if $K = 0$ then $r^o = (\gamma - \bar{R})$ and $r^o(\gamma) < \bar{R}$ if $\lambda = 0$ and $\bar{\pi}$ is low ($\pi < E(\bar{R})|\bar{R} < \bar{R}; \gamma$)).

To understand the result, for a given asset risk position, decompose an increase in the deposit rate of bank $A$ in four external effects. Apart from the three effects already mentioned before (on bank $B$, investors, and social failure cost), we have to add the effect on the cost of insurance for both bank $A$ and bank $B$. As before, the effect on the rival’s profits is negative, $\partial \pi_B/\partial r_A = -cm_B < 0$ and the effect on the social cost of failure is positive and equal to $g(r_A)K$. The effect on net (of insurance cost) consumer surplus, $CS - P$, $\partial(CS - P)/\partial r_A = mc + (b - c)(\bar{R} - r^o)$ when evaluated at the market solution $r^o$ (which fulfills $bm = pS$). In consequence, the aggregate effect on welfare at the market solution is given by

$$\frac{\partial TS}{\partial r_A} = \frac{\partial \pi_A}{\partial r_A} + \frac{\partial \pi_B}{\partial r_A} + \frac{\partial(CS - P)}{\partial r_A} - gK = (b - c)(\bar{R} - r^o) - gK.$$ 

Even when $K = 0$ we have that $\partial TS/\partial r_A < 0$ whenever $\bar{R} - r^o < 0$.

Therefore, with intense competition excessive rates overextend the market and seem to call for deposit limits (or rate ceilings) to improve welfare when

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27 Evaluating the FOC that determines $r^o(\gamma)$ at $\bar{R}$ we obtain the term $E(\bar{R})|\bar{R} > \bar{R}; \gamma) = 2\bar{R} + \bar{\pi}$, which is negative for symmetric distributions when $\pi < E(\bar{R})|\bar{R} < \bar{R}; \gamma)$. 

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there are flat premiums. Furthermore, according to Lemma 3 when the distribution of returns is symmetric, the level of asset risk which maximizes welfare (i.e., minimizes the probability of failure) is $\gamma$. However, as we argued above, with flat premiums banks undertake maximum risk because depositors’ return is assured, and this argument is not affected by deposit or rate regulation. Hence we have the following proposition:

**Proposition 7.** If competition is intense, rates are too high with flat-premium deposit insurance (even with no social cost of failure). If $G$ is symmetric, banks are induced to undertake maximal asset risk while it is optimal to undertake the least risk position. As a result, deposit limits (rate ceilings) and asset restrictions are complementary policies in order to improve welfare.

This result suggests that deposit limits (or, more in general, capital requirements) need to be complemented with asset restrictions (trying to lower $\gamma$) to approach the welfare-optimal solution when competition is intense. When competition is weak and $K$ low, however, rivalry may need to be promoted (for example, by easing entry in the industry).

We next examine how rates, the supply of deposits and failure costs compare depending on the prevailing regime for given asset risk positions. It is convenient to denote the equilibrium rates and the supply of deposits under free market, flat-premium insurance, and risk-based insurance respectively by $r'_i$, $r''_i$, and $S'$, $S''_i$, and $S'''$.

**Proposition 8.** Suppose the asset risk positions are given. Then the supply of deposits is larger with flat-premium deposit insurance than in the other regimes (for which it is equal). Risk-based insurance induces banks to set lower deposit rates than either with no insurance or with flat premiums. Correspondingly, failure rates are smallest with risk-based insurance. Total surplus will be always higher with risk-based insurance than without insurance.

**Proof.** First of all, note that with risk-based insurance the margin $\bar{R} - r''$ and deposit supply $S'''$ are exactly the same as with uninsured market competition. That is $m(r'_i) = \bar{R} - r'''$ and $S' = S'''$. Obviously, $r'_i > r'''$ since $m_i(r) > \bar{R} - r$. It is also immediate that $r''_i > r'''$ since the marginal profit of a rate increase is always larger with flat-premium deposit insurance due to the effect of limited liability: $bm_i - p_i S_i > b(\bar{R} - r_i) - S_i$. Average supply with flat-premium insurance equals $a + (b - c)(r''_i + r''_d)/2$. This is larger than $S' = S'''$ since $S''' = a + (b - c)r'''$ and $r''_i > r'''$. Now, the ranking of deposit rates implies the desired ranking in failure rates. Total surplus is higher with risk-based insurance than without insurance because $S' = S'''$ and failure rates are lower in the former case. □
Remark. Given identical asset risks and for \( \alpha \) not too large relative to the uninsured market rate \( r' \) we have that \( r'' > r' > r''' \). It follows also that failure costs accord to the inverse ranking of deposit rates.

Remark. Assuming that in the uninsured case portfolio risk is unobservable (in which case banks choose \( \gamma \) ) the proposition holds true when banks choose \( \gamma \). Indeed, in this case we have that both \( r''(\gamma) \) and \( r'(\gamma) \) are larger than \( r''' \) (because the argument for rate comparison in the proof of the proposition does not depend on the asset risk positions of the banks). Similarly, \((S'_A + S'_B)/2 > S' = S'''\). Failure rates are lowest with risk-based insurance since, for the same \( \gamma \) they increase with the deposit rate, banks choose \( \gamma \) both in the uninsured and in the flat-premium case, and if banks in the risk-based insurance regime choose \( \gamma \leq \tilde{\gamma} \) then the probability of failure decreases since \( r'' < \tilde{R} \) and the distribution of returns is symmetric.

Remark. It follows similarly that for \( \alpha \) small relative to the uninsured market rate \( r' \) we have that \( r''(\tilde{\gamma}) > r'(\tilde{\gamma}) \geq r'(\gamma) > r''' \). That is, rate aggressiveness (by regime and in decreasing order) is: flat-premium deposit insurance, uninsured market with unobservable \( \gamma \), uninsured market with observable \( \gamma \) and risk-based deposit insurance. Note also that both when portfolio risk is observable and with risk-based insurance (with premiums paid after asset risk is chosen) banks are indifferent about the level of portfolio risk chosen.

However, even with risk-based premiums welfare can be improved by introducing deposit limits (or rate regulation). The welfare-optimal rate is given in Lemma 3. For \( K \) small the market rate, \( r''' \), is too low (indeed, for \( K = 0 \), the optimal rate is \( \tilde{R} \) while \( r''' \) is lower than \( \tilde{R} \)). For \( K \) large \( r''' \) is too high. Indeed, in our framework with imperfect competition and a external cost of failure there is no presumption that risk-based deposit insurance should implement the optimal solution (even taking the need for deposit insurance for granted). In contrast, with no differentiation (\( \lambda = 1 \)) and no social cost of failure (\( K = 0 \)) the optimal allocation is attained with or without insurance.

It should be noticed that the rationale for rate regulation in our model is different from Smith (1984), where with adverse selection a perfectly competitive equilibrium may fail to exist even when there is insurance.

As already argued, for given capital, we can interpret deposit limits as capital requirements. It follows that capital requirements can improve welfare even when insurance is fairly priced. This is in contrast with the complete markets model of Rochet (1992) where capital requirements are irrelevant with risk-based insurance premiums. Rochet’s model does not include imperfect competition or a social cost of failure.
6. Concluding remarks

In this paper we have examined the consequences of imperfect competition for deposits on risk taking of banking firms subject to limited liability. Market outcomes under different regimes have been compared with welfare-optimal outcomes taking into account that there is a social cost of failure which banks do not internalize. A synthesis of the results obtained follows.

First, in a world without deposit insurance, when competition is intense banks tend to set deposit rates too high when the (social) cost of failure is high. This is so since they do not internalize the cost of failure. When the cost of failure is low rates are too low for the usual imperfect competition reasons. As competition becomes more intense the critical social failure cost above which rates are too high is lowered. With perfect competition rates are excessive whenever the social cost of failure is positive.

Moreover, when the portfolio risk is observable introducing an appropriate rate ceiling is welfare-optimal when the social cost of failure is high and competition is intense since it induces minimal risk taking. Indeed, when there are no moral hazard problems, a bank will not want to take risk on the asset side if it is constrained by a rate ceiling, since asset and liability risk are strategic complements. When there is moral hazard (unobservable portfolio risk), limited liability yields maximal asset risk. In this case, both deposit limits (or rate ceilings) and direct asset restrictions may be called for when the social cost of failure is large and competition intense.

Second, introducing flat-premium deposit insurance tends to make banks more aggressive competitors (since investors do not have incentives to punish increases in risk by the banks) and induces banks to undertake maximal asset risk positions. When competition is intense, rates are too high and the deposit supply excessive from a welfare point of view even if there is no social failure cost. Notice that deposit (or rate) regulation in the presence of deposit insurance leaves increases on asset risk unchecked since the deposit attraction of banks is not affected by them while limited liability implies that the expected margin increases with risk. As a result, deposit (rate) regulation and direct asset restrictions are complementary policies with flat-premium deposit insurance.

Finally, introducing fair and risk-based deposit insurance makes banks fully liable and decreases incentives to take risk on the deposit side (and, if the choice of asset risk is made before insurance premiums are paid, makes them indifferent to risk taking on the asset side). For given portfolio risks, risk-based insurance always dominates in welfare terms uninsured competition. However, rate regulation may still be needed to improve welfare.

In summary, maximal risk taking incentives exist with flat-premium deposit insurance and minimal with risk-based insurance. Risk taking on the asset side
is implied by limited liability and the presence of moral hazard without insurance or flat-premium deposit insurance.\(^{28}\)

In terms of policy implications we have seen that both in an uninsured market with unobservable asset risk positions and in an insured market with flat premiums, deposit limits (as a crude approximation to capital requirements) and direct asset restrictions are complementary regulatory tools when competition is intense and the social cost of failure large. Given that these premises seem to hold in the present conditions it follows that capital requirements alone will not suffice to keep risk taking under control. This is the more so when we have shown that as competition becomes more intense the critical social failure cost above which rates are too high is lowered. Nevertheless, if it is feasible to introduce risk-based insurance premiums deposit regulation (or capital requirements) may be a sufficient instrument to improve welfare. Altogether our analysis may provide a partial rationale for the measures adopted in the US 1991 FDICIA.

We have examined above the scope for rate regulation. Nevertheless, we want to emphasize here that the informational requirements to implement rate regulation are very high indeed and furthermore, as is well known, rate regulation has other costs not contemplated in the present paper, among them, the induced tendency to over invest in services, excess entry, and the possibility of regulatory capture (see, for example, Vives, 1991).

We would like to end the paper with a note of caution. The model presented in the paper sheds light on relevant aspects of bank competition but by no means is a comprehensive model. Indeed, consideration of competition on the asset side, of imperfect assessment of the risk position of banks, and a complete analysis of the effect of capital requirements (see, for example, Rochet, 1992) would be most welcomed extensions. Further, a dynamic analysis of the effect of competition on risk taking seems also necessary. This would allow to formalize the notion that increased competition may encourage risk taking by decreasing the charter value of banks (see, for example, Keeley, 1990; Suárez, 1995). In fact, this could be introduced in the model as a private cost of bankruptcy, which, most likely, would tend to make banks more conservative in assuming risks. This consideration, obviously, does not invalidate our analysis of the effects of banks not internalizing the social cost of failure. Another possible extension is the consideration of entry and the study of structural regulation (first steps in this direction have been taken by Besanko and Thakor (1992) and Gehrig (1995)).

\(^{28}\) The results should be contrasted with Daltung (1994) who, in a model with multiple claim-holders of a bank’s assets, finds that flat premiums need not aggravate the banks’ incentive to take risk.
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Appendix A

Lemma A.1. Let $BR_i(r_j)$ be the best response of bank $i$ to the rate set by bank $j$, then $BR_i(r_j)$ is increasing in $r_j$, shifts outwards with $\gamma_i$, and inwards with $\gamma_j$.

Proof. The first-order condition of bank $i$ is

$$\frac{\partial \pi_i}{\partial r_i} = p_i(bm_i - S_i) = 0$$

Whenever $\frac{\partial \pi_i}{\partial r_i} = 0$, $\frac{\partial^2 \pi_i}{\partial r_i^2} = -2bp_i^2 < 0$ and $\frac{\partial^2 \pi_i}{\partial r_i \partial r_j} = cp_i p_j > 0$. Therefore, $BR_i(r_j)$ is continuously differentiable with slope $cp_i/2bp_i > 0$. At an optimum rate, $\frac{\partial \pi_i}{\partial r_i}$ is increasing in $\gamma_i$ and decreasing in $\gamma_j$. This is so since the margin is a convex function of the actual return its expectation shifts up with the risk taken by the bank (since it is a mean preserving spread); i.e., $m_i$ shifts up with $\gamma_i$. Similarly, $\Phi_i$ decreases with $\gamma_i$ and therefore $S_i$ decreases with $\gamma_i$ and increases with $\gamma_j$.

Lemma A.2. For a given rate of bank $B$, as $\gamma_A$ increases the best response of bank $A$ is to set a higher rate so that its market share and expected margin (and consequently expected profits) remain constant.

Proof. We have that

$$m_A + S_A b^{-1} \equiv R + (a - c\Phi_B(r_B))b^{-1},$$

and from the FOC of bank $A$, $m_A = S_A b^{-1}$. It follows that, at an optimum rate,

$$S_A = \frac{\bar{R}b + a - c\Phi_B(r_B)}{2},$$
which is independent of the risk taken by bank $A$. The expected margin and profits of $A$ are also constant, given the rate set by bank $B$. Now, since $m_A$ is increasing in $\gamma_A$, when $\gamma_A$ increases in order for $m_A$ to remain constant $r_A$ must increase.

**Lemma A.3.** The rates $r_i$ and $r_j$ offered by banks are strategic complements. If the banks have the same asset risk positions and $x$ is small relative to $r_i$, given $r_j$ the best response of bank $i$ with deposit insurance is to offer a larger rate than without.

**Proof.** Banks take parametrically the insurance premium. Therefore, the marginal profit of a rate change for bank $i$ is $\partial \pi_i / \partial r_i = (1 - \tau) (bm_i - p_i S_i)$. Second-order conditions require $(\partial^2 \pi_i / \partial r_i^2) = (1 - \tau) p_i (S_i H_i - 2b) > 0)$. Simulations show that evaluated at the first-order condition they hold when returns are beta-distributed. Best replies are upward sloping since $\partial^2 \pi_i / \partial r_i^2 > 0$. If banks have the same asset risk $\gamma$ and $x$ is small it is possible to show that $p_i (a + br_i - cr_j) < a + b \Phi(r_j) - c \Phi(r_j)$. Note that $p_i (a + br_i - cr_j) - (a + b \Phi(r_j) - c \Phi(r_j)) = a(p_i - 1) + (b - c)(p_i r_i - \Phi(r_j)) + (c(p_i - pr_j) - \Phi(r_j)) + \Phi(r_j))$. It can be checked that for equal asset risks: $p_i r_i - pr_j < \Phi(r_i) - \Phi(r_j)$. Further, $a(p_i - 1) + (b - c)(p_i r_i - \Phi(r_j)) \leq 0$ if only and if $x \leq E(\tilde{R}_i | \tilde{R}_j < r_i)$. This is true for a small relative to $r_i$. A sufficient condition for the inequality to hold for all $r_i \geq x$ is that $a, \tilde{\theta}$. It follows then that $bm_i - p_i (a + br_i - cr_j) > bm_i - (a + b \Phi(r_i) - c \Phi(r_j))$ and the best response of bank $i$ with deposit insurance is larger than without. \qed

**Proof of Proposition 2.** First of all, it is clear that for $\gamma_i = \gamma$ an optimal solution will be symmetric: $r_i = r$. Total surplus with $\gamma_i = \gamma$ and $r_i = r$ is given by

$$TS = 2 \left( S(\tilde{R} - x) - \frac{1 + \lambda}{2} S^2 - (1 - p)K \right)$$

and

$$S = \max(0, (\Phi(r) - a)/(1 + \lambda)).$$

We start by proving (ii).

(ii) Optimizing over $r$ yields $m(r; \gamma) = (1 + \lambda)KH(r)$ (the problem is concave provided $H$ is nondecreasing). Comparative statics of the solution $r^*(\lambda, K)$ with respect to $K$ and $\lambda$ are immediate. Optimal deposits are given by $S^o = \max \{0, (\tilde{R} - x)(1 - \lambda)^{-1} - KH(r^o)\}$. Disintermediation yields 0 surplus and therefore intermediation is optimal whenever $TS^o = TS(r^o) \geq 0$.

(i) The equation $TS(r^o, \lambda, K) = 0$ defines a downward sloping schedule $\tilde{K}(\lambda)$ since (using the envelope theorem) $\partial TS^o / \partial K = -2(1 - p) < 0$, and $\partial TS^o / \partial \lambda = -S^2 < 0$. The schedule $\tilde{K}(\lambda)$ is well-defined for all $\lambda$ in $[0, 1]$ since for a given $\lambda$ there is always a $K$ large enough for which it is optimal to disintermediate. This is so because with returns distributed on $(\tilde{\theta}, \tilde{\theta})$ the
probability of failure is bounded away from zero whenever there is intermediation since then \( r^o > \gamma \) (indeed, a necessary condition for intermediation is that \( \Phi(r^o) > \gamma \)).

(iii) The market solution \( r' \) is independent of \( K \): \( m(r') = (\bar{R} - \gamma)(1 - \lambda)/(2 - \lambda) \). We have that \( r^o(\lambda, K) = r'(\lambda) \) for \( (\bar{R} - \gamma)(1 - \lambda)/(2 - \lambda) = KH(r^o) \). This schedule is such that when \( \lambda = 0 \), \( K = \bar{K} \) (defined by \( \bar{K} = (\bar{R} - \gamma)/2KH(r^o(0, \bar{K})) \)). Furthermore, \( TS^o(1, 0) > 0 \) (since there is no failure cost) and \( TS^o(1, \bar{K}) < 0 \) (since for \( K = \bar{K} \) and \( \lambda = 1 \), \( S^o = 0 \) and consequently \( TS^o < 0 \)). It can be checked that this defines a downward sloping schedule in \((\lambda, K)\) space. Indeed, let \( \Phi = (\bar{R} - \gamma)(1 - \lambda) - (2 - \lambda)KH(r^o)(1 + \lambda) \). Then

\[
d\lambda/dK = - \frac{\partial \Phi}{\partial K} / \frac{\partial \Phi}{\partial \lambda} = - \frac{(1 + \lambda)(2 - \lambda) \left( KH' \frac{\partial r^o}{\partial K} + H \right)}{(\bar{R} - \gamma) + (1 - 2\lambda)KH + K(2 - \lambda)(1 + \lambda)H' \frac{\partial r^o}{\partial \lambda}}.
\]

Substituting in the numerator the expression for \( \delta r^o/\delta K = -H(1 + \lambda)/(p + KH'(1 + \lambda)) \) we obtain that \( KH' \frac{\partial r^o}{\partial K} + H = H(1 - (KH(1 + \lambda)/(p + KH'(1 + \lambda))) > 0 \). Substituting in the denominator the expression for \( \delta r^o/\delta \lambda = -HK(p + KH'(1 + \lambda)) \) we obtain that \( \bar{R} - \bar{z} + (1 - 2\lambda)KH + K(2 - \lambda)(1 + \lambda)H' \frac{\partial r^o}{\partial \lambda} = \bar{R} - \bar{z} + KH(1 - 2\lambda - (2 - \lambda)(KH(1 + \lambda)/(p + KH'(1 + \lambda)))) > \bar{R} - \bar{z} - KH > 0 \) for \( K < (\bar{R} - \bar{z})/2H \) (which holds since at \( \lambda = 0 \), \( \bar{K} = (\bar{R} - \gamma)/2KH(0, \bar{K}) \)). Two possibilities may arise depending on whether the schedules \( TS^o = 0 \) and \( r' = r^o \) intersect or not (see Fig. 3). If they do not (Fig. 3a) then schedule \( r' = r^o \) bounds the regions of excessive and insufficient market rates. If they do (Fig. 3b) then for \( \lambda \) small and \( K \) small market rates are too low but for \( K \) large disintermediation is optimal. In this case whenever intermediation is optimal market rates are too low. For example, for a constant hazard rate \( H \) the boundary between the regions of excessive or defective market rates is given by \( K = (\bar{R} - \bar{z})(\beta - \lambda)/(2\beta - \lambda)(\beta + \lambda)H \). For \( \alpha = 1 \) the schedules intersect for \( H \) lower than 0.2 but do not for \( H \) larger than 0.3. \( \Box \)

**Proof of Lemma 1.** First of all, and as before, given the structure of the problem it is clear that the optimal solution must be symmetric: \( \gamma_i = \gamma \) and \( r_i = r \). We have that \( TS = 2S(\bar{R} - \bar{z}) - ((1 + \lambda)/2)S^2 - (1 - p)K \). Let \( r^o(\gamma) \) denote the optimal rate for a given \( \gamma \). Let \( TS^o(\gamma) = TS(r^o(\gamma), \gamma) \). Using the envelope theorem we have that

\[
\delta TS^o/\delta \gamma = 2((\bar{R} - \bar{z}) - (1 + \lambda)S) \left( \frac{\partial \Phi}{\partial \gamma} \right) - K \left( \frac{\partial (1 - p)}{\partial \gamma} \right).
\]
We know that $\partial \Phi / \partial \gamma < 0$ (a mean preserving spread diminishes the expected return to depositors) and, given that the distribution of returns is symmetric, $\partial (1 - p) / \partial \gamma > 0$ if and only if $r^0 < \bar{R}$. We claim that $r^0(\gamma) < \bar{R}$ for all $\gamma$ if $K > K_2(\lambda, \gamma)$. Indeed, the optimal rate is such that

$$r = \frac{1}{p} \int_{\bar{R}}^{\infty} \bar{R} dG - K(1 + \lambda)H = s(r).$$

If $H$ is increasing then $s'(r^0) = -K(1 + \lambda)H'(r^0)/p(r^0)$ is nonincreasing. Also,

$$s(\bar{R}) = 2 \int_{\bar{R}}^{\infty} \bar{R} dG - KH(1 + \lambda).$$

Now, notice that for any $\gamma s(\bar{R}) < \bar{R}$ when

$$K > \frac{2\int_{\bar{R}}^{\infty} \bar{R} dG(\gamma) - \bar{R}}{4(1 + \lambda)g(\bar{R}, \gamma)} = K_2,$$

since $g(\bar{R}, \gamma)$ decreases and $\int_{\bar{R}}^{\infty} \bar{R} dG(\gamma)$ increases in $\gamma$. We show now that when

$$\frac{(2y - 1)\bar{R}}{4g(\bar{R}, \gamma)} < 2 \left[ \bar{R} \left( \frac{3}{2} - y \right) - \alpha \right] \left[ y\bar{R} - \frac{\alpha}{2} \right],$$

$TS'(\gamma) > 0$ for $K = K^2(\lambda, \gamma)$. It is enough to show that $TS'(\gamma) > 0$ since for $K = K_2$, $\partial TS' / \partial \gamma < 0$ for all $\gamma$. When $K = K_2$ and $\gamma = \gamma$ we have that $r^0(K_2) = \bar{R}$ and $p = 1/2$. It follows that $(\bar{R} - \alpha - m^0) = \bar{R}(\frac{3}{2} - y) - \alpha$, and

$$\left( \frac{\bar{R} - \alpha}{2} + m^0 \right) = -\frac{\alpha}{2} + y\bar{R},$$

where

$$y = \int_{\bar{R}}^{\infty} Rg(R, \gamma) dR / \bar{R}.$$

$TS'(\gamma) > 0$ if and only if

$$\frac{(\bar{R} - \alpha - m^0)(\bar{R} - \alpha/2 + m^0)}{(1 - p^0)(1 + \lambda)} > K.$$

We need, thus,

$$K_2 < \frac{2 \left[ \bar{R} \left( \frac{3}{2} - y \right) - \alpha \right] \left[ y\bar{R} - \frac{\alpha}{2} \right]}{(1 + \lambda)}$$

or

$$\frac{(2y - 1)\bar{R}}{4g(\bar{R}, \gamma)} < 2 \left[ \bar{R} \left( \frac{3}{2} - y \right) - \alpha \right] \left[ y\bar{R} - \frac{\alpha}{2} \right].$$
Similarly as in Lemma 2 for $K$ large enough $TS^0(\gamma)$ will become zero. There is a unique such $K( = K_3(\lambda, \tilde{\rho}))$ for any $\lambda$ given that $TS^0$ is decreasing with $K$. □

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