Market Power and Welfare in Asymmetric Divisible Good Auctions*

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Abstract

We analyze a divisible good uniform-price auction with two groups with a finite number of identical bidders. When an equilibrium exists it is unique and the relative market power of a group increases with the precision of its private information and decreases with(its transaction costs. Consistent with the empirical evidence, we find that an increase in the transaction cost of a group of bidders induces a strategic response of the other group according to which they diminish their reaction to private information and submit steeper schedules. The "stronger" group (with more precision of private information, lower transaction costs and/or more oligopsonistic) has more market power and has to receive a higher subsidy to behave competitively. The deadweight loss increases with the quantity auctioned and with the degree of payoff and informational asymmetry when the strong group values the asset at least as the weak group.

Keywords: demand/supply schedule competition, private information, liquidity auctions, Treasury auctions, electricity auctions

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1 Introduction

Divisible good auctions are common in many markets such as government bonds, liquidity (refinancing operations), electricity, and emission markets.[1] In those auctions both market power and asymmetries among the participants are important. Even in large markets, asymmetries may make market power relevant. However, the difficulties in dealing with asymmetric bidders with market power competing in demand or supply schedules, compounded by the presence of private information, have been hampering theoretical work in this area. The present paper contributes to fill the gap in analyzing asymmetric auctions.

Treasury auctions have bidders with significant market shares. This is particularly so in systems where there is a primary dealership system with participation in the auctions limited to a reduced number of bidders (this occurs, for example, in 29 out of 39 countries in the survey of Arnone and Iden (2003)). A prime example are U.S. Treasury auctions (which are uniform-pricing).[2] In these auctions, it should be noted that the top five bidders typically purchase close to one-half of US Treasury issues (Malvey and Archibald (1998)). Experimental work finds substantial demand reduction in uniform-price auctions (see Kagel and Levin (2001) and Engelbrecht-Wiggans et al. (2006), among others).

Armantier and Sbaï (2006) test whether bidders in French Treasury auctions are symmetric. The authors conclude that participants in French Treasury auctions may be divided into two distinct groups, differentiated by their level of risk aversion and the quality of their information about the value of the security to be sold. One small group consists of large financial institutions, which possess better information and are willing to take more risks. Kastl (2011) also finds evidence of two differentiated groups of bidders in (uniform price) Czech Treasury auctions. Other papers that also report asymmetries between bidders in Treasury auctions are Umlauf (1993) in Mexico, Bjonnes (2001) in Norway, and Hortaçsu and McAdams (2010) in Turkey, among others.

Bindseil et al. (2009) and Cassola et al. (2013) also find that heterogeneity between bidders in liquidity auctions is relevant. Cassola et al. (2013), analyzing the evolution of bidding data in the European Central bank's weekly refinancing operations before and during the early part of the financial crisis, show that the impact of the 2007 subprime market crisis was heterogeneous among European banks. Moreover, the authors conclude that the significant shift in

[1] See Lopomo et al. (2011) for examples of such auctions.

[2] The reduced number of primary dealers makes the U.S. Treasury market imperfectly competitive (Bikhchandani and Huang (1993)). Uniform-price auctions have been and are used often in Treasury, liquidity and electricity auctions, among others. See Brenner et al. (2009) for Treasury auctions, with the US a leading example since November 1998.

bidding behavior after August 9, 2007 may reflect a change in their cost of obtaining short-term funding in the interbank market and/or a strategic response to other bidders. Concretely, the authors find that one third of bidders did not experience any change in their costs of short-term funds from alternative sources and the change in their bidding behavior was simply a strategic response: they increased their bids so as to best-respond to the higher bids of their rivals.[3]

Market concentration is high in other markets, such as wholesale electricity. This issue has attracted attention of academics and policymakers. A number of empirical studies have concluded that sellers have exercised significant market power in wholesale electricity markets (see Green and Newbery (1992), Wolfram (1998), Borenstein et al. (2002), and Joskow and Kahn (2002), among others). [4] Most, but not all, wholesale electricity markets use a uniform-price auction which tends to be preferred to a pay-as-you-bid auction (see Cramton and Stoft (2006, 2007)). In several of those markets (such as in California or Australia) generating companies bid to sell power and wholesale customers bid to buy power. In addition, in these markets asymmetries are prevalent. For example, in wholesale electricity markets some generators have a high proportion of nuclear technology, with very flat marginal costs, while others have a high proportion of fossil fuel technologies with steep marginal costs. Holmberg and Wolak (2015) argue specifically that in wholesale electricity markets there is asymmetric information in the production costs of suppliers. Evidence of the impact of cost heterogeneity on bidding in wholesale electricity markets is provided in Bustos-Salvagno (2015) and Crawford et al. (2007).

This paper makes progress within the linear-Gaussian family of models by incorporating bidders' asymmetries in terms of payoffs and information. We present a model of a uniform price auction where asymmetric strategic bidders compete in demand schedules for an inelastic supply (supply schedule competition for an inelastic demand is easily accommodated). Bidders may differ in their valuations, transaction costs and/or the precision of their private information.[5] For simplicity, and with empirical foundation, heterogeneity is reduced to two groups. Agents are identical within each group. In this setting, we analyze under which conditions a linear equilibrium with symmetric treatment of agents of the same group exists, i.e., equilibria such

[3] Bidder asymmetry has also been found in procurement markets such as school milk (Porter and Zona (1999) and Pesendorfer (2000)) and public works (Bajari (1998)).

[4] In January 2007, the European Commission asserted that "...at the wholesale level, gas and electricity markets remain national in scope, and generally maintain the high level of concentration of the pre-liberalization period. This gives scope for exercising market power..." (Inquiry pursuant to Article 17 of Regulation (EC) No 1/2003 into the European gas and electricity sectors (Final Report), Brussels, 10.1.2007).

[5] One reason behind the differences in private information among bidders maybe the presence of both dealers and direct bidders in auctions (such as in U.S. Treasury auctions). Dealers aggregate the information of clients and bid with a higher precision of information (see the evidence of Hortaçsu and Kastl (2012) for Canadian Treasury auctions and Boyarchenko et al. (2015) for a theoretical model).

that the demand functions are linear and identical among individuals of the same type. We show that when equilibrium exists it is unique and we derive comparative statics results.

In particular, our analysis shows that the number of individuals of a group, the transactions costs, the correlation of values and the precision of private information affect the sensitivity of traders' demands to private information and prices. More correlated values induce traders to react less to the private signal and the price. We also find that the relative market power of a group increases with the precision of its private information and decreases with its transaction costs. For example, an increase in the transaction cost of a group of bidders induces a strategic response of the other group according to which they diminish their reaction to private information and submit steeper schedules. This is consistent with the empirical findings of Cassola et al. (2013) in the post-crisis liquidity auctions in Europe.

If a group of traders is "stronger" (with more precision in private information, lower transaction costs and/or more oligopolistic/oligopsonistic) then they react more to the private signal and the price. This may help explain the finding by Hortaçsu and Puller (2008) in the Texas balacing market, where there is no accounting for private information on costs, that small firms use steeper schedules than predicted by theory.[6]

We find also that with asymmetric groups bid shading may turn into a bid premium. Expected revenue in the auction where the expected values between groups differ need not be decreasing in the transaction costs of bidders, the noise in their signals, or the correlation of values. This contrasts with the result when groups are symmetric. We bound the expected revenue of the auction between the revenues of auctions with symmetric extremal identical groups.

We consider large markets and find that when there is a small group and a large group, then the oligopsonistic group commands a higher degree of market power but the large group does not behave competitively, retaining some market power. We also prove that the equilibrium under imperfect competition converges to a price-taking equilibrium in the limit as the number of traders of both groups becomes large.

Finally, we provide a welfare analysis. We characterize the deadweight loss at the equilibrium and show how a subsidy scheme may induce an efficient allocation. We find that if there is a group with more precision in private information, with lower transaction costs or/and less

[6] It is worth noting that the linear supply function model has been used extensively in the estimation of market power in wholesale electricity auctions. Holmberg et al. (2013) provide a foundation for the continuous approach as an approximation to the discrete supply bids in a spot market. Brandts et al. (2013) find in their experimental work that observed behavior is consistent with the supply function model rather than with a discrete multi-unit auction model. Ciarreta and Espinosa (2010) provide empirical support for the smooth supply model over the discrete-bid auction model with Spanish data.

numerous, then this group should receive a higher subsidy. This is so since the "stronger" group will behave more strategically and has to be compensated more to become competitive. In addition, we highlight how the heterogeneity of bidders documented in the empirical papers, be it informational or in terms of preferences or size of groups, may increase deadweight losses. In particular, we find that the deadweight loss increases with the quantity auctioned and payoff and informational asymmetry when the strong group values the asset at least as much as the weak group.

This paper is related to the received literature on divisible good auctions. Results in symmetric pure common value models have been obtained by Wilson (1979), Back and Zender (1993), and Wang and Zender (2002), among others.[7]

Results in interdependent values models with symmetric bidders are obtained by Vives (2011, 2014) and Ausubel at al. (2014), among others. Vives (2011), focusing on the tractable family of linear-Gaussian models, shows how private information yields more market power than full information levels. Bergemann et al. (2015) generalize the information structure in Vives (2011) keeping the symmetry assumption. Rostek and Weretka (2012) partially relaxes the symmetry assumption in Vives (2011) and replaces it with a weaker "equicommonality" assumption on the matrix correlation among the agents' values. This assumption states that the sum of correlations in each column of this matrix (or, equivalently, in each row) is the same, and that the variances of all traders' values are also the same. Unlike our model, Rostek and Weretka's model maintains the symmetry assumption of the precision of private signals and transaction costs. As a result, the equilibrium derived in their paper is still symmetric, with all traders using identical strategies.[8]

Despite the importance of bidder asymmetry, results in multiunit auctions have been difficult to obtain. This is the reason why most of the papers that deal with this issue focus on auctions for a single item. In sealed-bid first price single-unit auctions an equilibrium exists under quite general conditions (Lebrun (1996), Maskin and Riley (2000a) and Athey (2001), Reny and Zamir (2004)) and uniqueness is explored in Lebrun (1999) and Maskin and Riley (2003). Maskin and Riley (2000b) study asymmetric auctions and Cantillon (2008) shows that the seller's expected

[7] Wilson (1979) compares a uniform-price auction for a divisible good with an auction in which the good is treated as an indivisible good. This author finds that the price can be significantly lower if bidders are allowed to submit bid schedules rather than a single bid price. Back and Zender (1993), extending Wilson (1979), compare a uniform-price auction with a discriminatory auction. It is shown that there exist equilibria in which the seller's revenue can be much lower than the revenue obtained in the discriminatory auction. Wang and Zender (2002) show that if there is supply uncertainty and bidders are risk averse, there may exist equilibria of a uniform-price auction that provide higher expected revenue than the revenue obtained in a discriminatory auction.

[8] Ausubel at al. (2014) find that in symmetric auctions with decreasing linear marginal utility, the seller's revenue is larger in a discriminatory auction than in a uniform-price auction.

revenue is lower the more asymmetric bidders are. Under complete information progress has been made in linear divisible good auction models by characterizing linear supply function equilibria (e.g., Akgün (2004) and Anderson and Hu (2006)). One exception incorporating incomplete information is Kyle (1989). This author considers a Gaussian model of a divisible good double auction where some bidders are privately informed and others are uninformed.

The remainder of this paper is organized as follows. Section 2 outlines the model. Section 3 characterizes the equilibrium, analyzes its existence and uniqueness and derives comparative statics results. Large markets are dealt with in Section 4 and the welfare analysis is developed in Section 5. Concluding remarks are presented in Section 6 and the proofs are gathered in the Appendix.

2 The model

A finite number of traders face an inelastic supply for a risky asset. Let Q denote the aggregate quantity supplied in the market. In this market there are buyers of two types: type 1 and type 2. Suppose that there are n_i traders of type i, $i = 1, 2$. The profits of a representative trader of type i, trader h, when the price of the asset is p, are given by

$$\pi_h = (\theta_i - p)\, x_h - \frac{\lambda_i}{2} x_h^2.$$

Thus, for any trader of type i, the marginal benefit of buying x_h units of the asset is $\theta_i - \lambda_i x_h$, where θ_i denotes the valuation of the asset and the parameter $\lambda_i > 0$ is an adjustment for transaction costs, opportunity costs or proxy for risk aversion. Traders maximize expected profits and submit demand schedules, and an auctioneer selects a price that clears the market. The case of supply schedule competition for an inelastic demand is easily accommodated by considering negative demands ($x < 0$) and an inelastic demand $Q < 0$. In this case a producer of type i has a quadratic production cost $-\theta_i x_i + \frac{\lambda_i}{2} x_i^2$.

We assume that θ_i is normally distributed with mean $\overline{\theta}_i$ and variance σ_θ^2, $i = 1, 2$. Moreover, θ_1 and θ_2 can be correlated, with correlation coefficient $\rho \in [0, 1]$. Thus, $cov(\theta_1, \theta_2) = \rho\sigma_\theta^2$.[9] All traders of type i receive the same noisy signal $s_i = \theta_i + \varepsilon_i$, where ε_i is normally distributed, with null mean and variance $\sigma_{\varepsilon_i}^2$. Error terms in the signals are uncorrelated across groups ($cov(\varepsilon_1, \varepsilon_2) = 0$) and with the valuations of the asset ($cov(\varepsilon_i, \theta_j) = 0$, i, $j = 1, 2$).

In our model two traders of distinct types may differ in several aspects:

1) different ex-ante willingness to possess the asset $\left(\overline{\theta}_1 \neq \overline{\theta}_2\right)$,

[9]Notice that the value of ρ will depend of the type of security. In this sense, Bindseil et al. (2009) argue that the common value component is less important in a central bank repo auction than in a T-bill auction.

2) different transaction costs $(\lambda_1 \neq \lambda_2)$, or

3) different precisions of private information $(\sigma^2_{\varepsilon_1} \neq \sigma^2_{\varepsilon_2})$.

Applications of this model are Treasury auctions and liquidity auctions. For Treasury auctions, θ_i is the private value of the securities to bidder i which incorporates the resale value as well as idiosyncratic elements such as different liquidity needs between bidders of the two groups. The private information in this context stems from different expectations about θ (for instance, bidders have different forecasts of inflation, and securities are denominated in nominal terms). For the case of liquidity auctions, θ_i is the price or interest rate that group i may command in the secondary interbank market (which is OTC). Here λ_i reflects the structure of a counterparty's pool of collateral in a repo auction. A bidder bank prefers to offer illiquid collateral to the central bank in exchange for funds, but as allotment increases, the bidder must offer more liquid types of collateral, which have a higher opportunity cost.

3 Equilibrium

Denote by X_i the strategy of a bidder of type i, $i = 1, 2$, a mapping from signal space to the space of demand functions. Thus, $X_i(s_i, \cdot)$ is the demand function of a bidder of type i corresponding to a given signal s_i. In a Bayesian equilibrium, given his signal s_i, each bidder chooses a demand function to maximize his conditional profit, taking as given the strategies of other traders. We will restrict attention to anonymous linear Bayesian equilibria where strategies are identical among the traders of the same type, "equilibrium" for short.

Definition. *An equilibrium is a linear Bayesian equilibrium such that the demand functions for traders of type i, $i = 1, 2$, are identical and equal to*

$$X_i(s_i, p) = b_i + a_i s_i - c_i p,$$

where b_i, a_i and c_i are constants.

3.1 Equilibrium characterization

Consider a trader of type i. Given linear strategies of rivals and market clearing (i.e., $(n_i - 1)X_i(s_i, p) + x_i + n_j X_j(s_j, p) = Q$, where $j = 1, 2$ and $j \neq i$), this trader faces a residual inverse supply

$$p = I_i + d_i x_i,$$

where the intercept $I_i = ((n_i - 1)(b_i + a_i s_i) + n_j (b_j + a_j s_j) - Q) / ((n_i - 1) c_i + n_j c_j)$ reveals s_j (together with s_i and when $a_j \neq 0$) and the slope $d_i = ((n_i - 1) c_i + n_j c_j)^{-1}$ is an index of the market power of the trader.[10] Hence, this trader's information set (s_i, p) is informationally equivalent to (s_i, I_i). Therefore, this bidder chooses x_i to maximize

$$\mathbb{E}\left[\pi_i | s_i, p\right] = \left(\mathbb{E}\left[\theta_i | s_i, I_i\right] - I_i - d_i x_i\right) x_i - \frac{\lambda_i}{2} x_i^2.$$

The F.O.C. is given by

$$\mathbb{E}\left[\theta_i | s_i, I_i\right] - I_i - 2d_i x_i - \lambda_i x_i = 0$$

or, equivalently,

$$X_i\left(s_i, p\right) = \frac{\mathbb{E}\left[\theta_i | s_i, p\right] - p}{d_i + \lambda_i}. \tag{1}$$

The S.O.C. that guarantees a maximum is $2d_i + \lambda_i > 0$. Using the expression of I_i and provided that $a_j \neq 0$, (s_i, p) is informationally equivalent to (s_1, s_2). Hence, since $\mathbb{E}\left[\theta_i | s_i, p\right] = \mathbb{E}\left[\theta_i | s_i, I_i\right]$, we have that

$$\mathbb{E}\left[\theta_i | s_i, p\right] = \mathbb{E}\left[\theta_i | s_1, s_2\right]. \tag{2}$$

From Gaussian distribution theory,

$$\mathbb{E}\left[\theta_i | s_i, s_j\right] = \overline{\theta}_i + \Xi_i \left(s_i - \overline{\theta}_i\right) + \Psi_i \left(s_j - \overline{\theta}_j\right), \tag{3}$$

where

$$\Xi_i = \frac{1 - \rho^2 + \widehat{\sigma}_{\varepsilon_j}^2}{\left(1 + \widehat{\sigma}_{\varepsilon_i}^2\right)\left(1 + \widehat{\sigma}_{\varepsilon_j}^2\right) - \rho^2} \text{ and } \Psi_i = \frac{\rho \widehat{\sigma}_{\varepsilon_i}^2}{\left(1 + \widehat{\sigma}_{\varepsilon_i}^2\right)\left(1 + \widehat{\sigma}_{\varepsilon_j}^2\right) - \rho^2},$$

with $\widehat{\sigma}_{\varepsilon_i}^2 = \sigma_{\varepsilon_i}^2 / \sigma_\theta^2$ and $\widehat{\sigma}_{\varepsilon_j}^2 = \sigma_{\varepsilon_j}^2 / \sigma_\theta^2$. Notice that (3) implies that

1) the private signal s_i is useful in the prediction of θ_i whenever $1 - \rho^2 + \widehat{\sigma}_{\varepsilon_j}^2 \neq 0$, i.e., the liquidation values are not perfectly correlated ($\rho \neq 1$) or traders of type j are imperfectly informed about θ_j ($\sigma_{\varepsilon_j}^2 \neq 0$), and

2) the private signal s_j is useful in the prediction of θ_i, $i, j = 1, 2$, $i \neq j$, whenever $\rho \widehat{\sigma}_{\varepsilon_i}^2 \neq 0$, i.e., when the private liquidation values are correlated ($\rho \neq 0$) and traders of type i are imperfectly informed about θ_i ($\sigma_{\varepsilon_i}^2 \neq 0$).

The next proposition summarizes the previous results, shows the relationship between a_i and c_i in equilibrium and the positiveness of these coefficients.

Proposition 1. *Let $\rho < 1$. In equilibrium, the demand function of a trader of type i, $i = 1, 2$, is given by*

$$X_i\left(s_i, p\right) = \frac{\mathbb{E}\left[\theta_i | s_i, p\right] - p}{d_i + \lambda_i},$$

[10] We assume that $\left(n_i - 1\right) c_i + n_j c_j \neq 0$.

with $d_i + \lambda_i > 0$, $d_i = ((n_i - 1) c_i + n_j c_j)^{-1}$, *and* $a_i = \Delta_i c_i > 0$, $\Delta_i = \left(1 + (1+\rho)^{-1} \widehat{\sigma}^2_{\varepsilon_i}\right)^{-1}$, $i = 1, 2$.[11]

The equilibrium demand function depends on $\mathbb{E}[\theta_i|s_i, p]$. Concerning the price coefficient (see Lemma A1 in the Appendix), $c_i = \left(1 - \Psi_i (n_i c_i + n_j c_j)(n_j a_j)^{-1}\right) / (d_i + \lambda_i)$, in Expression (11) the term $\Psi_i (n_i c_i + n_j c_j)(n_j a_j)^{-1}$ is the information-sensitivity weight of the price. Notice that the more informative the price is ($\Psi_i (n_i c_i + n_j c_j)(n_j a_j)^{-1}$ higher), the lower the price coefficient will be (c_i lower). Moreover, notice that this term vanishes when $\Psi_i = 0$, i.e., when the liquidation values are uncorrelated or the private signal s_i is perfectly informative ($\rho = 0$ or $\sigma^2_{\varepsilon_i} = 0$), since in these cases the price does not convey any additional information to a trader of type i.

Concerning the case $\rho = 1$, an equilibrium does not exist. The reason of this fact is the following: if the price reveals a sufficient statistic for the common liquidation value, then no trader has an incentive to put any weight on his signal. But if traders put no weight on signals then the price cannot contain any information on the common valuation. This is the basically the Grossman-Stiglitz paradox (1980). From now on, we focus on the cases in which $\rho < 1$.

Since $a_i > 0$ and $c_i > 0$, $i = 1, 2$, in an equilibrium the higher the value of the private signal observed by a trader or the lower the price, the higher the quantity demanded by him. Therefore, in an equilibrium (2) holds and, consequently, we have that the equilibrium price is *privately revealing,* i.e., the private signal and the price allows a trader of type i to learn about θ_i as much as he would if he had access to all the information available in the market, (s_1, s_2).

When $\rho = 0$ or when both signals are perfectly informative ($\sigma^2_{\varepsilon_i} = 0$, $i = 1, 2$), the optimal demand functions are given by $X_i(s_i, p) = (\mathbb{E}[\theta_i|s_i] - p) / (d_i + \lambda_i)$, $i = 1, 2$. Hence, it follows that $c_i = (d_i + \lambda_i)^{-1}$, and using the expressions of d_i, it follows that

$$d_i = \left(\frac{n_i - 1}{d_i + \lambda_i} + \frac{n_j}{d_j + \lambda_j} \right)^{-1}, \; i, j = 1, 2, \; j \neq i.$$

It can be shown that whenever $n_1 + n_2 > 2$, then this system has a unique solution satisfying $d_i + \lambda_i > 0$, $i = 1, 2$.

Another setup where the price is also not useful in providing information is the full (shared) information framework. In fact, the equilibrium values of d_1 and d_2 when $\rho = 0$ are equal to those corresponding to the full (-shared) information setup (denoted by d_i^f, $i = 1, 2$). We have that $d_i^f < d_i$ for $i = 1, 2$. Asymmetric information induces market power over and above the full information level.

The following proposition shows under which conditions an *equilibrium* exists. In case of existence, its uniqueness is guaranteed.

[11] Notice that $2d_i + \lambda_i > 0$ implies that $d_i + \lambda_i > 0$, $i = 1, 2$.

Proposition 2. *There exists a unique equilibrium if and only if* $\overline{z}_N > \overline{z}_D$*, where* $\overline{z}_N$ *and* $\overline{z}_D$ *denote the highest root, respectively, of* $q_N(z)$ *and* $q_D(z)$*, with*

$$q_N(z) = n_2^2 \frac{\Xi_1}{\Delta_1} + n_2 \left(\frac{\Xi_1}{\Delta_1} (2n_1 - 1) - (n_1 + 1) \right) z - (n_1 - 1) \left(1 - \frac{\Xi_1}{\Delta_1} \right) n_1 z^2$$

and

$$q_D(z) = -n_2 (n_2 - 1) \left(1 - \frac{\Xi_2}{\Delta_2} \right) + n_1 \left(\frac{\Xi_2}{\Delta_2} (2n_2 - 1) - (n_2 + 1) \right) z + n_1^2 \frac{\Xi_2}{\Delta_2} z^2.$$

Let $z = \frac{c_1}{c_2}$*. If* $\lambda_i > 0$*,* $i = 1, 2$*, in equilibrium* $\overline{z}_D < z < \overline{z}_N$ *and* $\lim_{\lambda_1 \to 0} z = \overline{z}_N$ and $\lim_{\lambda_2 \to 0} z = \overline{z}_D$.

For an equilibrium to exist we need that $c_i > 0$, $i = 1, 2$. For a better understanding of the equilibrium and the condition that guarantees its existence, we develop some particular cases.

Remark 1. If $n_1 = 1$ and $n_2 = 1$, we have that $\overline{z}_N = \left(2\Delta_1 \Xi_1^{-1} - 1\right)^{-1}$ and $\overline{z}_D = 2\Delta_2 \Xi_2^{-1} - 1$. Given that $\Delta_i \Xi_i^{-1} > 1$, $i = 1, 2$, direct computations yield $\overline{z}_N < \overline{z}_D$. Applying Proposition 2, we can conclude that in this case an equilibrium does not exist. Therefore, $n_1 + n_2 \geq 3$ is a necessary condition for the existence of an equilibrium in our model. This result is in line with Kyle (1989) and Vives (2011).

We consider two particular cases of the model: a monopsony competing with a fringe and symmetric groups.

Monopsony with fringe

Corollary 1. *If* $n_2 = 1$*, the equilibrium exists if* $1 - \rho^2 > (2\rho - 1)\widehat{\sigma}^2_{\varepsilon_1}$ *and* $n_1 > \bar{n}_1\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)$*, where* $\bar{n}_1$ *increases in* $\rho, \widehat{\sigma}^2_{\varepsilon_1}$ *and* $\widehat{\sigma}^2_{\varepsilon_2}$*. If, furthermore,* $\lambda_2 = 0$ *and* $\sigma^2_{\varepsilon_2} = 0$*, then* $\bar{n}_1\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right) = \frac{\left(1+\rho+\widehat{\sigma}^2_{\varepsilon_1}\right)(1-\rho)}{1-\rho^2-(2\rho-1)\widehat{\sigma}^2_{\varepsilon_1}}$*, and* $x_2 = c_2(\theta_2 - p)$*, with* $c_2 = n_1 c_1$*.*

An equilibrium with linear demand functions exists provided there is a sufficiently competitive trading environment (n_1 high enough). In the particular case in which $\lambda_2 = 0$ and $\sigma^2_{\varepsilon_2} = 0$, the expressions of the equilibrium coefficients can be explicitly characterized (see the Appendix). Using the expressions of c_i, $i = 1, 2$, it follows that if $n_1 = \bar{n}_1$, then the equilibrium does not exist because the demand functions would be completely inelastic ($c_i = 0$, $i = 1, 2$).

Symmetric groups

Consider the following symmetric case: $n_2 = n_1 = n$, $\lambda_1 = \lambda_2 = \lambda$, and $\sigma^2_{\varepsilon_1} = \sigma^2_{\varepsilon_2} = \sigma^2_{\varepsilon}$. In this case in equilibrium $z = 1$. From Proposition 2, we know that when the equilibrium exists the value of z belongs to the interval $(\overline{z}_D, \overline{z}_N)$. Therefore, $\overline{z}_N > 1 > \overline{z}_D$, or equivalently,

$q_N(1) > 0$ and $q_D(1) > 0$. After some algebra, we have that the previous inequalities are satisfied if and only if $n > 1 + \rho\widehat{\sigma}_\varepsilon^2 \left((1-\rho)\left(1+\rho+\widehat{\sigma}_\varepsilon^2\right)\right)^{-1}$, where $\widehat{\sigma}_\varepsilon^2 = \sigma_\varepsilon^2/\sigma_\theta^2$. Therefore, the existence of equilibrium is guaranteed provided that n is high enough, or when ρ or $\widehat{\sigma}_\varepsilon^2$ are low enough.

Vives (2011) also analyzes divisible good auctions with symmetric bidders, but in his model the private signals of bidders are different among them. In his setup the condition that guarantees the existence of equilibrium is $2n > 2+M$, where $M = 2n\rho\widehat{\sigma}_\varepsilon^2\left((1-\rho)\left(1+(2n-1)\rho+\widehat{\sigma}_\varepsilon^2\right)\right)^{-1}$. Direct computations yield that the condition derived in Vives' model is more stringent than the one derived in the present setup. The reason is that in Vives (2011) the degree of asymmetric information (and induced market power) is higher since each of the $2n$ traders obtains a private signal.

The remaining of this section is devoted to show some properties that satisfy the equilibrium coefficients and to compare the equilibrium quantities.

Comparative statics

Corollary 2. *Suppose that $\sigma_{\varepsilon_1}^2 \geq \sigma_{\varepsilon_2}^2$, $\lambda_1 \geq \lambda_2$, and $n_1 \geq n_2$, with one (or more) of these inequalities strict. In equilibrium,*

a) group 2 ("strong") reacts more to information ($a_1 < a_2$, $c_1 < c_2$) and has more market power ($d_1 < d_2$);

b) the difference $d_1 + \lambda_1 - (d_2 + \lambda_2)$ is in general ambiguous. If

$$\frac{(1-\rho)\,n_1 n_2\left(1+\rho+\widehat{\sigma}_{\varepsilon_1}^2\right)}{n_2\left(1-\rho^2+\widehat{\sigma}_{\varepsilon_1}^2\right)+n_1\rho\widehat{\sigma}_{\varepsilon_1}^2}+\frac{(1-\rho)\,n_1\,(n_2-1)\left(1+\rho+\widehat{\sigma}_{\varepsilon_2}^2\right)}{n_1\left(1-\rho^2+\widehat{\sigma}_{\varepsilon_2}^2\right)+n_2\rho\widehat{\sigma}_{\varepsilon_2}^2} \leq 1, \tag{4}$$

then $d_1 + \lambda_1 < d_2 + \lambda_2$ always holds. Otherwise, $d_1 + \lambda_1 > d_2 + \lambda_2$ if and only if λ_1/λ_2 is high enough.

The first part of Corollary 2 shows that if a group of traders is less informed, has higher transaction costs or is more numerous, then it reacts less to both private signals and prices. In particular, notice that traders of group 1 with less precise private information rely more on the price for information (a higher $\Psi_1(n_1c_1+n_2c_2)(n_2a_2)^{-1}$) and this makes the overall price response $\left(c_1 = \left(1-\Psi_1(n_1c_1+n_2c_2)(n_2a_2)^{-1}\right)/(d_1+\lambda_1)\right)$ smaller. Similarly, traders of group 1 with n_1 larger put more information weight on the price (which depends more strongly on s_1).

The second part of Corollary 2 is useful to compare the allocations across groups. It indicates that the inequality $d_1 + \lambda_1 > d_2 + \lambda_2$ holds whenever the differences between groups mainly stem from the transaction costs and λ_1/λ_2 is high enough.[12] When signals are perfect ($\sigma^2_{\varepsilon_i} = 0$, $i = 1, 2$), then $d_1 + \lambda_1 > d_2 + \lambda_2$ if and only if $\lambda_1 > \lambda_2$.

Corollary 3. *Let $X_i(s_i, p) = b_i + a_i s_i - c_i p$ be the equilibrium demand function of bidders of type i, $i = 1, 2$. For $i = 1, 2$, $i \neq j$:*

a) An increase in $\overline{\theta}_i$, Q, or a decrease in $\overline{\theta}_j$, raises b_i.

b) An increase in λ_i, λ_j, $\sigma^2_{\varepsilon_i}$, $\sigma^2_{\varepsilon_j}$, or ρ makes demands less responsive to private signals and prices (lower a_i and c_i) and increases market power (d_i)

c) If $\sigma^2_{\varepsilon_i}$ and/or λ_i increase, then d_i/d_j decreases.

d) If n_i and/or n_j increase, then d_i decreases.

From Lemma A1, we know that the only equilibrium coefficient affected by the quantity offered in the auction (Q) and the prior mean of the valuations ($\overline{\theta}_i$ and $\overline{\theta}_j$) is the coefficient b_i. Corollary 3a indicates that if Q increases, all the bidders will increase their demand (higher b_1 and b_2). Moreover, if the prior mean of the valuation of group i increases, the bidders of this group demand a higher quantity of the risky asset (higher b_i). Then, the intercept of the inverse residual supply for a bidder of group j is higher with an increase in $\overline{\theta}_i$. This makes traders of group j to reduce their demand of the risky asset (lower b_j).

Corollary 3b shows how the response to private information and price varies with several parameters. If the transaction costs for a bidder increase, then the bidder is less interested in the risky asset and a_i and c_i decrease in λ_i. Moreover, an increase in the transaction costs parameter of a group also affects the behavior of the traders of the other group. If λ_i increases, then c_i decreases. Then the slope of the inverse residual supply for group j increases (higher d_j). This induces traders of group j to reduce their demand sensitivity to signals and prices (lower a_j and c_j). We see, therefore, how an increase in the transaction cost of group i (say a deterioration of their collateral in liquidity auctions) leads not only to a steeper demand for group i but also to a steeper demand for group j as a reaction. Figure 1 depicts the case of initially identical groups which differentiate with a shock that raises the willingness to pay for liquidity in a weak group (2) with higher λ and a higher impact on $\overline{\theta}$.

Figure 1 here

We also analyze how the response to private information an price varies with a change in the precision of private signals. If the private signal of bidders of type i is less precise ($\sigma^2_{\varepsilon_i}$ rises), then the demand of these bidders is less sensitive to private information and prices. A trader finds optimal to rely less on his private information when his private signal is less precise. Moreover, the reduction in the precision of the private signal makes a bidder of type i to have

[12] This is due to the fact that when $n_1 = n_2$ and $\widehat{\sigma}^2_{\varepsilon_1} = \widehat{\sigma}^2_{\varepsilon_2}$, then (4) does not hold.

more incentives to take prices into consideration when predicting θ_i and this leads to a steeper demand function slope for this bidder (lower c_i) and the same happens for a bidder of type j (in this case because of strategic complementarity in the slopes of the demand functions).[13]

We also obtain the higher the correlation among the value parameters (higher ρ), the lower the responsiveness to the private signals (lower a_i, $i = 1, 2$) and steeper inverse demand functions (lower c_i, $i = 1, 2$). To understand these results recall that when there is correlation between the value parameters ($\rho > 0$), a trader of type i learns about θ_i from prices. In fact, the larger is ρ, the price is more informative about θ_i, which makes demands less sensitive to private information. The rationale for the relationship between correlation and the sole of demand as follows. An increment in the price of the risky asset makes agents more optimistic about its liquidation value, which leads to a smaller reduction in the quantities demanded as compared to the case of uncorrelated valuations.[14]

In addition, taking into account that the equilibrium values of d_1 and d_2 when $\rho = 0$ are equal to those corresponding to the full (-shared) information setup, Corollary 3 implies that $d_i > d_i^f$, $i = 1, 2$. Thus, private information creates market power over and above the full information level.

Corollary 3c suggests that an increase in the noise in the signal of one group or in its transaction costs parameter decreases relatively its market power since d_i/d_j, $i \neq j$, decreases.

Corollary 3d provides the foreseeable result that an increase in the number of participants in the auction (higher n_i or n_j) reduces their market power.

Equilibrium quantities

Finally, we examine the equilibrium quantities. Let $t_i = \mathbb{E}[\theta_i | s_1, s_2]$, $i = 1, 2$, be the predicted values with full information (s_1, s_2). After some algebra, it follows that equilibrium quantities are functions of the vector of predicted values $t = (t_1, t_2)$:

$$x_i(t) = \underbrace{\frac{n_j (t_i - t_j)}{n_i (d_j + \lambda_j) + n_j (d_i + \lambda_i)}}_{x_i^I(t)} + \underbrace{\frac{d_j + \lambda_j}{n_i (d_j + \lambda_j) + n_j (d_i + \lambda_i)} Q}_{x_i^C(t)}, \; i = 1, 2, \; j \neq i. \qquad (5)$$

Notice that these expressions indicate that the equilibrium quantities can be decomposed into two terms: an *information* trading term and a *clearing* trading term, denoted respectively by

[13] This result (in the supply competition model) may help explain the fact that in the Texas balancing market small firms use steeper supply functions than those predicted by theory (Hortaçsu and Puller (2008)). Indeed, smaller firms may have signals of worse quality because of economies of scale in information gathering.

[14] Note that a high price conveys the good news that the private signal received by traders of the other group is high. When the valuations are positively correlated, a bidder infers that a high private signal of the others means that his own valuation is high.

$x_i^I(t)$ and $x_i^C(t)$ for the group i, $i = 1, 2$. Regarding the information trading term, notice that it vanishes when $t_1 = t_2$, whereas it has a positive (negative) value for the group with the higher (lower) value of t_i. Moreover, $n_1 x_1^I(t) + n_2 x_2^I(t) = 0$. Concerning the clearing trading term, notice that it vanishes when $Q = 0$; otherwise, it is positive for both groups and it is lower (higher) for the group with higher (lower) $d_i + \lambda_i$. In addition, $n_1 x_1^C(t) + n_2 x_2^C(t) = Q$.

Taking expectations in (5), we have

$$\mathbb{E}[x_1(t)] - \mathbb{E}[x_2(t)] = \frac{n_1 + n_2}{n_1(d_2 + \lambda_2) + n_2(d_1 + \lambda_1)} \left(\overline{\theta}_1 - \overline{\theta}_2\right) + \frac{d_2 + \lambda_2 - (d_1 + \lambda_1)}{n_1(d_2 + \lambda_2) + n_2(d_1 + \lambda_1)} Q.$$

Combining the previous expression and Corollary 2, we obtain the following remarks.

Remark 3. If Q is low enough, then $\mathbb{E}[x_1(t)] > \mathbb{E}[x_2(t)]$ whenever $\overline{\theta}_1 > \overline{\theta}_2$. By contrast, if Q is high enough, then $\mathbb{E}[x_1(t)] > \mathbb{E}[x_2(t)]$ whenever $d_2 + \lambda_2 > d_1 + \lambda_1$. Under the assumptions of Corollary 2, this inequality if satisfied provided that (4) holds or whenever λ_1/λ_2 is low enough.

Remark 4. If $Q = 0$, a the double auction case, then $\mathbb{E}[x_2(t)] < 0 < \mathbb{E}[x_1(t)]$ if and only if $\overline{\theta}_1 > \overline{\theta}_2$. Then group 1 are buyers and group 2 sellers. When $Q = 0$ it is easy to see that $b_i = \frac{\hat{\sigma}^2_{\varepsilon_i} a_i}{1-\rho^2}\left(\overline{\theta}_i - \rho\overline{\theta}_j\right), i \neq j,\ i = 1, 2$.

3.2 Bid shading, expected discount and expected revenue

In this subsection we would like to determine factors that affect the magnitudes of bid shading, expected discount and expected revenue. Let $\tilde{t} = \frac{n_1 t_1 + n_2 t_2}{n_1 + n_2}$. From the demands of bidders, it follows that $p(t) = t_i - (d_i + \lambda_i) x_i(t),\ i = 1, 2$. Hence,

$$p(t) = \tilde{t} - \frac{(d_1 + \lambda_1) n_1 x_1(t) + (d_2 + \lambda_2) n_2 x_2(t)}{n_1 + n_2}. \tag{6}$$

Bid shading

For a trader of type i, the expected marginal benefit of buying x_i units of the asset is $t_i - \lambda_i x_i$. Therefore, the average marginal benefit is given by $\tilde{t} - (\lambda_1 n_1 x_1 + \lambda_2 n_2 x_2)/(n_1 + n_2)$. The magnitude of bid shading is the difference between the average marginal valuation and the auction price, i.e., $(d_1 n_1 x_1 + d_2 n_2 x_2)/(n_1 + n_2)$. Using (5), bid shading is given by:

$$\frac{n_2 d_2(d_1 + \lambda_1) + n_1 d_1(d_2 + \lambda_2)}{(n_1 + n_2)(n_1(d_2 + \lambda_2) + n_2(d_1 + \lambda_1))} Q + \frac{(t_2 - t_1)(d_2 - d_1) n_2 n_1}{(n_1 + n_2)(n_1(d_2 + \lambda_2) + n_2(d_1 + \lambda_1))}, \tag{7}$$

Some remarks are in order:

- Bid shading increases in Q.

- When $d_1 = d_2 = d$ (as, for instance, in the symmetric case), bid shading consists of only one term (the first one) and it is equal to $dQ/(n_1 + n_2)$.

- When $d_1 \neq d_2$, the second term of (7) is negative whenever whenever the group that values more the asset ($t_1 > t_2$) has less market power ($d_1 < d_2$).

- If group 1 has higher transaction costs ($\lambda_1 > \lambda_2$), is more numerous ($n_1 > n_2$), and less informed ($\sigma^2_{\varepsilon_1} > \sigma^2_{\varepsilon_2}$), then $c_1 < c_2$, and hence, $d_1 < d_2$. If $t_1 > t_2$ the second term of (7) is negative, and both terms have opposite sign. Therefore, we have that if Q is low (zero for example) or if the difference of the predicted values of the asset is high, we obtain negative bid shading.

Expected Discount

The expected discount is defined as $\mathbb{E}\left[\tilde{t}\right] - \mathbb{E}\left[p\left(t\right)\right]$. Using (6), we have that the expected discount is equal to $\left((d_1+\lambda_1)n_1\mathbb{E}\left[x_1\left(t\right)\right] + (d_2+\lambda_2)n_2\mathbb{E}\left[x_2\left(t\right)\right]\right)/\left(n_1+n_2\right)$. After some algebra, we obtain the following expression:

$$\frac{\left(d_1+\lambda_1\right)\left(d_2+\lambda_2\right)}{n_1\left(d_2+\lambda_2\right)+n_2\left(d_1+\lambda_1\right)}Q + \frac{n_1 n_2\left(d_2+\lambda_2-d_1-\lambda_1\right)\left(\overline{\theta}_2-\overline{\theta}_1\right)}{\left(n_1+n_2\right)\left(n_1\left(d_2+\lambda_2\right)+n_2\left(d_1+\lambda_1\right)\right)}. \tag{8}$$

- When $d_1 + \lambda_1 = d_2 + \lambda_2 = d + \lambda$ (as in the symmetric case), the expected discount is $\frac{(d+\lambda)Q}{n_1+n_2}$.

- The first term is always positive provided $Q > 0$, whereas the second term is positive whenever $d_1 + \lambda_1 > d_2 + \lambda_2$ and $\overline{\theta}_1 > \overline{\theta}_2$.

- If the group 1 ex-ante values more the asset $\left(\overline{\theta}_1 > \overline{\theta}_2\right)$, is more risk averse ($\lambda_1 > \lambda_2$), is more numerous ($n_1 > n_2$) and less informed ($\sigma^2_{\varepsilon_1} > \sigma^2_{\varepsilon_2}$), Corollary 2 shows that $d_1+\lambda_1 > d_2+\lambda_2$ whenever the differences between groups mainly stem from the transaction costs (and $\frac{\lambda_1}{\lambda_2}$ is high enough). In this case, both terms are positive, and hence, we have that the expected discount is positive. On the other hand, if both groups have similar transactions costs, then the two terms in (8) have opposite signs. In particular, when Q is low, we expect a negative discount.

Expected Revenue

The expected price is given by

$$\mathbb{E}[p]=\frac{\frac{n_1}{d_1+\lambda_1}\overline{\theta}_1+\frac{n_2}{d_2+\lambda_2}\overline{\theta}_2-Q}{\frac{n_1}{d_1+\lambda_1}+\frac{n_2}{d_2+\lambda_2}}.$$

and the expected revenue for the seller is $E[p]Q$. It is worth noting that in the double auction case ($Q=0$), $\mathbb{E}[p]$ is a convex combination of $\overline{\theta}_1$ and $\overline{\theta}_2$ and for symmetric groups except possibly for the means $\mathbb{E}[p]=\left(\overline{\theta}_1+\overline{\theta}_2\right)/2$.

Corollary 4.

a) If $\overline{\theta}_1=\overline{\theta}_2$ the expected price is increasing in n_i and decreasing in λ_i, $\sigma^2_{\varepsilon_i}$, or ρ, $i=1,2$. Otherwise, if $\left|\overline{\theta}_1-\overline{\theta}_2\right|$ is large enough these results need not hold.

b) The expected revenue:

- increases in $\overline{\theta}_i$, $i=1,2$, and in Q for $E[p]>0$;

- is between the larger expected revenue of the auction in which both groups are ex-ante identical with a large number of bidders (each group with $\max\{n_1,n_2\}$), high expected valuation ($\max\left\{\overline{\theta}_1,\overline{\theta}_2\right\}$), low transaction costs ($\min\{\lambda_1,\ \lambda_2\}$) and precise signals ($\min\{\sigma^2_{\varepsilon_1},\ \sigma^2_{\varepsilon_2}\}$) and the smaller expected revenue of the auction in which both groups are ex-ante identical with the opposite characteristics (i.e., $\min\{n_1,n_2\}$, $\min\left\{\overline{\theta}_1,\overline{\theta}_2\right\}$), $\max\{\lambda_1,\ \lambda_2\}$ and $\max\{\sigma^2_{\varepsilon_1},\ \sigma^2_{\varepsilon_2}\}$).

The corollary indicates that the relationship between the expected price and λ_i, $\sigma^2_{\varepsilon_i}$, or ρ, $i=1,2$ is potentially ambiguous. For example, when $\overline{\theta}_2-\overline{\theta}_1$ is high enough then $\mathbb{E}[p]$ is decreasing in n_1. However, when $\overline{\theta}_1=\overline{\theta}_2$ or Q is high enough, the derived results are in line with the results derived in the symmetric case where $\mathbb{E}[p]=\overline{\theta}-\frac{(d+\lambda)}{2n}Q$ (see, for instance, Proposition 2 in Vives (2010)).

We are interested in understanding how ex-ante differences among bidders affect the seller's expected revenue. Suppose that the group 2 is "strong": with lower transaction cost ($\lambda_2<\lambda_1$), less numerous ($n_2<n_1$) and better informed ($\sigma^2_{\varepsilon_2}<\sigma^2_{\varepsilon_1}$). If this group values less ($\overline{\theta}_2<\overline{\theta}_1$) (more ($\overline{\theta}_2>\overline{\theta}_1$)) the asset, this reduces (rises) the expected revenue. If $\overline{\theta}_1\approx\overline{\theta}_2$, Corollary 4a suggests that the fact that the group 2 is small ($n_2<n_1$) reduces the expected revenue, while the fact that they have low transaction costs and precise signals ($\lambda_2<\lambda_1$ and $\sigma^2_{\varepsilon_2}<\sigma^2_{\varepsilon_1}$) has the opposite effect positive effect. This means the ex-ante differences in the two groups affect the seller's expected revenue ambiguously in general. However, still we can obtain result b) using result a).

4 Large markets

This section is concerned with determining whether the equilibrium under imperfect competition converges to a price-taking equilibrium in the limit as the number of traders becomes large. We examine two possible scenarios: in the first one, a single group of bidders (the group 1) is large, whereas in the second scenario both groups are large. We consider the case of an inelastic *per capita* supply of q, i.e., $Q = (n_1 + n_2)q$.

4.1 Oligopsony with competitive fringe

Proposition 3. *Let $n_1 \to \infty$ and $n_2 < \infty$. Then equilibrium exists if and only if $n_2 > \bar{n}_2\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)$, where $\bar{n}_2$ is increasing in ρ and $\widehat{\sigma}^2_{\varepsilon_1}$, and decreasing in $\widehat{\sigma}^2_{\varepsilon_2}$ whenever $(2\rho - 1)\,\widehat{\sigma}^2_{\varepsilon_1} < 1 - \rho^2$.*

- *An agent in the large group just absorbs the inelastic demand in the limit ($\lim\limits_{n_1\to\infty} b_1 = q$, $\lim\limits_{n_1\to\infty} a_1 = \lim\limits_{n_1\to\infty} c_1 = 0$), and keeps some market power ($\lim\limits_{n_1\to\infty} d_1 > 0$), while an agent in the small group commands a higher degree of market power ($\lim\limits_{n_1\to\infty} d_2 > \lim\limits_{n_1\to\infty} d_1$).*
- *The price in the limit depends only on the valuation and market power of the competitive fringe:*
$$\lim_{n_1\to\infty} p = \mathbb{E}\left[\theta_1|s_1, s_2\right] - \left(\lim_{n_1\to\infty} d_1 + \lambda_1\right) q.$$

When $n_2 = 1$, the existence condition boils down to the existence condition $(2\rho - 1)\,\widehat{\sigma}^2_{\varepsilon_1} < 1 - \rho^2$ in Corollary 1. Proposition 3 shows that when $n_2 = \bar{n}_2\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)$, the demand function for the group 2 would be completely inelastic $\left(\lim\limits_{n_1\to\infty} c_2 = 0\right)$. This is the reason why the inequality $n_2 > \bar{n}_2\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)$ is required for the existence of equilibrium. Neither the *aggregate* demand of group 1 or of group 2 are flat in the limit and both groups command some market power. We see that an agent in the large group just absorbs the inelastic supply, behaving like a "Cournot quantity setter", and keeping some market power ($\lim\limits_{n_1\to\infty} d_1 > 0$), while the small group commands a higher degree of market power ($\lim\limits_{n_1\to\infty} d_2 > \lim\limits_{n_1\to\infty} d_1$).[15]

Using the expressions of the optimal demands given in Proposition 1, the market clearing condition can be written as:
$$n_1 \frac{\mathbb{E}\left[\theta_1|s_1, p\right] - p}{d_1 + \lambda_1} + n_2 \frac{\mathbb{E}\left[\theta_2|s_2, p\right] - p}{d_2 + \lambda_2} = (n_1 + n_2)q,$$
or,
$$\frac{n_1}{n_1 + n_2} \frac{\mathbb{E}\left[\theta_1|s_1, s_2\right] - p}{d_1 + \lambda_1} + \frac{n_2}{n_1 + n_2} \frac{\mathbb{E}\left[\theta_2|s_1, s_2\right] - p}{d_2 + \lambda_2} = q.$$
Taking the limit when n_1 converges to infinity in the previous expression, it follows that
$$\frac{\mathbb{E}\left[\theta_1|s_1, s_2\right] - \lim\limits_{n_1\to\infty} p}{\lim\limits_{n_1\to\infty} d_1 + \lambda_1} = q,$$

[15] This inequality is due to the fact that $0 = \lim\limits_{n_1\to\infty} c_1 < \lim\limits_{n_1\to\infty} c_2$.

which implies that $\lim_{n_1 \to \infty} p = \mathbb{E}[\theta_1 | s_1, s_2] - \left(\lim_{n_1 \to \infty} d_1 + \lambda_1 \right) q.$

4.2 A large price-taking market

Consider now the following setup: there is a continuum of bidders $[0,1]$. Let q denote the aggregate (average) quantity supplied in the market. Suppose that a fraction μ_i of these bidders $(0 < \mu_i < 1)$ are traders of type $i, i = 1, 2$. The following proposition characterizes the equilibrium of this continuum economy and shows that this equilibrium is the limit of equilibrium of a finite economy:

Proposition 4. *Suppose that $Q = (n_1 + n_2)q$, and that n_1 and n_2 go to infinity and that $n_i/(n_1 + n_2)$ converges to μ_i, $0 < \mu_i < 1$, $i = 1, 2$. Then, the equilibrium coefficients converge to the equilibrium coefficients of the equilibrium of the continuum economy setup, which are given by*

$$\begin{aligned} b_i &= \frac{\widehat{\sigma}^2_{\varepsilon_i} \left(\rho \lambda_j q + \mu_j \left(\overline{\theta}_i - \rho \overline{\theta}_j \right) \right)}{\mu_i \rho \lambda_j \widehat{\sigma}^2_{\varepsilon_i} + \mu_j \lambda_i \left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_i} \right)}, \\ a_i &= \frac{\mu_j \left(1 - \rho^2 \right)}{\mu_i \rho \lambda_j \widehat{\sigma}^2_{\varepsilon_i} + \mu_j \lambda_i \left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_i} \right)}, \quad \textit{and} \\ c_i &= \frac{\mu_j \left(1 - \rho \right) \left(1 + \rho + \widehat{\sigma}^2_{\varepsilon_i} \right)}{\mu_i \rho \lambda_j \widehat{\sigma}^2_{\varepsilon_i} + \mu_j \lambda_i \left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_i} \right)}, \end{aligned}$$

where $i, j = 1, 2$, $i \neq j$.

5 Welfare analysis

This section focus on the welfare loss at the *equilibrium*. Initially, we provide a characterization of the *equilibrium* allocation and a characterization of the deadweight losses.

5.1 A characterization of the equilibrium and efficient allocations

Recall that $t_i = \mathbb{E}[\theta_i | s_1, s_2]$, $i = 1, 2$, i.e., the predicted values with full information (s_1, s_2) and $t = (t_1, t_2)$. The strategies in the equilibrium induce outcomes as functions of the realized vector of predicted values t and are given in (5). It is easy to see that the outcome at the equilibrium maximizes the following *distorted* benefit maximization program:[16]

$$\underset{x_1, x_2}{Max} \mathbb{E}\left[n_1 \left(\theta_1 x_1 - (d_1 + \lambda_1) \frac{x_1^2}{2} \right) + n_2 \left(\theta_2 x_2 - (d_2 + \lambda_2) \frac{x_2^2}{2} \right) \middle| t \right]$$

[16] See Lemma A3 in the Appendix.

$$s.t. \; n_1 x_1 + n_2 x_2 = Q,$$

where d_1 and d_2 are the equilibrium parameters. The efficient allocation would obtain if we set $d_1 = d_2 = 0$ and corresponds to a price taking *equilibrium* (denoted by a superscript o). The equilibrium strategy of a bidder of type i, will be of the form: $X_i^o(s_i, p) = b_i^o + a_i^o s_1 - c_i^o p, i = 1, 2$; it will arise from the maximization of the following program:

$$\max_{x_i} \left(\mathbb{E}\left[\theta_i | s_i, p\right] - p \right) x_i - \frac{\lambda_i}{2} x_i^2,$$

taking prices as given, $i = 1, 2$. The F.O.C. of the two optimization problems will yield

$$\mathbb{E}\left[\theta_i | s_i, p\right] - p - \lambda_i x_i = 0, \; i = 1, 2.$$

Identifying coefficients and solving the corresponding system of equations, it follows that there exists a unique *equilibrium* in this setup. The equilibrium coefficients coincide with those in Proposition 4 for the continuum market.

Proposition 5. *Let* $Q = (n_1 + n_2) q$ *and* $\mu_i = \frac{n_i}{n_1 + n_2}, i = 1, 2$. *There exists a unique price-taking equilibrium and the equilibrium coefficients coincide with the equilibrium coefficients of the continuous setup, whose expressions are given in the statement of Proposition 4.*

The following corollary provides some comparative statics results:

Corollary 5. *Let* $i, j = 1, 2, \; i \neq j$. *The only equilibrium coefficients affected by* Q, $\overline{\theta}_i$ *and* $\overline{\theta}_j$ *are the demand functions' intercepts (*b_i^o *which increases in* $\overline{\theta}_i$ *and* Q, *and decreases in* $\overline{\theta}_j$*). Moreover, the demands of group* i *are less sensitive to private signals and prices (*a_i *and* c_i *lower) with an increase in* λ_i, λ_j, ρ, $\sigma_{\varepsilon_i}^2$ *and* μ_i, *with a decrease in* μ_j, *and are not affected by* $\sigma_{\varepsilon_j}^2$.

Notice that under competitive behavior we derive an additional comparative statics result: the relationship between the equilibrium coefficients and the proportion of individuals of group 1. In particular, increasing the proportion μ_1 of traders of type 1 leads to an increased information component in the price for a type 1 trader $\Psi_1 \left(n_1 c_1^o + n_2 c_2^o \right) \left(n_2 a_2^o \right)^{-1}$ and a lower overall response to the price $c_1^o = \lambda_1^{-1} \left(1 - \Psi_1 \left(n_1 c_1^o + n_2 c_2^o \right) \left(n_2 a_2^o \right)^{-1} \right)$ and the opposite for a trader of type 2.

The fact that the auction outcome can be obtained as the solution to a maximization problem with a more concave objective function suggests that inefficiency may be eliminated by quadratic subsidies that compensate for the distortions. The subsidy κ_i to a trader of type i must be such that it compensates for the distortion $d_i(\kappa_i)$ taking into account the existence of the subsidy. Since the aim is to induce competitive behavior, the trader should be lead to respond c_i^o to the price. This means that the exact amount of the subsidy κ_i must be $d_i(c_1^o, c_2^o)$ since this would be the distortion when traders use the competitive linear strategy. The following proposition shows that with a proper selection of subsidies, bidders act as if they were competitive and, consequently, the equilibrium allocation is efficient.

Proposition 6. *The quadratic subsidies* $\kappa_i x_i^2/2$, $i = 1, 2$, *with* $\kappa_i = d_i(c_i^o, c_j^o) = \left((n_i - 1)\, c_i^o + n_j c_j^o\right)^{-1}$, $i = 1, 2$, $i \neq j$, *induces an efficient allocation. The per capita subsidies* κ_i, $i = 1, 2$, *increase in* ρ, $\widehat{\sigma}^2_{\varepsilon_1}$, $\widehat{\sigma}^2_{\varepsilon_2}$, λ_1 *and* λ_2, *and decrease in* n_1 *and* n_2*.*

Combining Propositions 5 and 6, we have that the expressions of optimal subsidies are given by

$$\kappa_i = \frac{1}{n_j\,(1-\rho)} \left(\frac{(n_i - 1)\left(1 + \widehat{\sigma}^2_{\varepsilon_i} + \rho\right)}{n_i \lambda_j \rho \widehat{\sigma}^2_{\varepsilon_i} + n_j \lambda_i \left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_i}\right)} + \frac{n_i \left(1 + \widehat{\sigma}^2_{\varepsilon_j} + \rho\right)}{n_i \lambda_j \left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_j}\right) + n_j \lambda_i \rho \widehat{\sigma}^2_{\varepsilon_j}} \right)^{-1},$$

$i = 1, 2$, $i \neq j$. Notice also that the expressions of the optimal subsidies in the case $\rho = 0$ (or full information) are much simpler: $\kappa_i = \left((n_i - 1)\,\lambda_i^{-1} + n_j \lambda_j^{-1}\right)^{-1}$, $i = 1, 2$.

Notice that the subsidies are decreasing in the number of traders due to the fact that the competitive behavior is already approached in the market without subsidies when the number of agents is large. Moreover, $\text{sgn}\{\kappa_1 - \kappa_2\} = \text{sgn}\{c_1^o - c_2^o\}$. Thus, $\kappa_1 < \kappa_2$ if and only if $c_1^o < c_2^o$. Therefore, this indicates that the bidders who have to receive a higher subsidy are those whose demands are more sensitive to the price. Taking into account Corollary 5, we can conclude that if there is a group with more precision in private information, with lower transaction costs or/and less numerous, then this group should receive a higher subsidy. This is so since the "stronger" group has a more pronounced strategic behavior and has to be compensated more to become competitive.

The result has policy implications. It implies, for example, that a central bank which aims at distributing liquidity efficiently among banks will relax collateral requirements (i.e., provide a larger subsidy) to the strong group. This sounds counterintuitive because the efficiency motive may conflict with the central bank function as a lender of last resort which may tend to shore up weak banks (e.g, the ECB relaxing the colateral requirements for Greek banks to avoid a meltdown of the Greek banking system). In a wholesale electricity market characterized by a small (oligopolistic) group and a fringe, a regulator that wants to improve productive efficiency should subsidize more the oligopolistic group. This could be accomplished by differential subsidies to renewable energy technology that lower the marginal cost of production.

5.2 Deadweight loss

The expected deadweight loss (denoted by $\mathbb{E}[DWL]$) at the equilibrium is the difference between expected total surplus at the efficient allocation (denoted ETS^o) and at the equilibrium (denoted by ETS). In the Appendix (see Lemma A4), it is shown that

$$\mathbb{E}[DWL] = \frac{1}{2}\lambda_1 n_1 \mathbb{E}\left[(x_1(t) - x_1^o(t))^2\right] + \frac{1}{2}\lambda_2 n_2 \mathbb{E}\left[(x_2(t) - x_2^o(t))^2\right],$$

where $(x_1^o(t), x_2^o(t))$ corresponds to the price-taking equilibrium. Using the expressions of $(x_1(t), x_2(t))$ and $(x_1^o(t), x_2^o(t))$, it follows that

$$\begin{aligned}\mathbb{E}[DWL] &= \frac{n_2 n_1 (n_2 d_1 + n_1 d_2)^2}{2(n_2\lambda_1 + n_1\lambda_2)(n_2(d_1+\lambda_1) + n_1(d_2+\lambda_2))^2}\mathbb{E}(t_1 - t_2)^2 + \\ &\quad + \frac{n_2 n_1 (n_2 d_1 + n_1 d_2)(\lambda_2 d_1 - \lambda_1 d_2)}{(n_2\lambda_1 + n_1\lambda_2)(n_2(d_1+\lambda_1) + n_1(d_2+\lambda_2))^2} Q\left(\overline{\theta}_1 - \overline{\theta}_2\right) + \\ &\quad \frac{n_2 n_1 (\lambda_1 d_2 - \lambda_2 d_1)^2}{2(n_2\lambda_1 + n_1\lambda_2)(n_2(d_1+\lambda_1) + n_1(d_2+\lambda_2))^2} Q^2,\end{aligned}$$

where $\mathbb{E}\left[(t_1 - t_2)^2\right] = \left(\overline{\theta}_1 - \overline{\theta}_2\right)^2 + (1-\rho)^2 \sigma_\theta^2 \left(2(1+\rho) + \widehat{\sigma}_{\varepsilon_1}^2 + \widehat{\sigma}_{\varepsilon_2}^2\right) / \left(\left(1 + \widehat{\sigma}_{\varepsilon_1}^2\right)\left(1 + \widehat{\sigma}_{\varepsilon_2}^2\right) - \rho^2\right)$.

The expected deadweight loss consists of three terms. The first is purely due to uncertainty and information, and is the product of two factors. The first one

$$\frac{n_2 n_1 (n_2 d_1 + n_1 d_2)^2}{2(n_2\lambda_1 + n_1\lambda_2)(n_2(d_1+\lambda_1) + n_1(d_2+\lambda_2))^2}$$

increases in d_1 and d_2. As d_1 and d_2 increase in ρ, we have that this multiplier increases in ρ. The second factor $\mathbb{E}\left[(t_1 - t_2)^2\right]$ decreases in ρ and in $\widehat{\sigma}_{\varepsilon_i}^2$, and vanishes when ρ approaches 1 or in when there is no uncertainty ($\sigma_\theta^2 = 0$) provided that $\overline{\theta}_1 = \overline{\theta}_2$. This second factor increases with the dispersion in the values of the traders, $\left(\overline{\theta}_1 - \overline{\theta}_2\right)^2$ and ρ^{-1}. Consequently, the first term of $\mathbb{E}[DWL]$ may increase or decrease in ρ. The first term is the only one present in a double auction (where $Q = 0$).

The third term derives from the absorption of Q by the traders and is increasing in the quantity offered Q as well as in the discrepancy between d_1/d_2 and λ_1/λ_2. The second term is an interaction term which is positive for $Q > 0$ if and only if $(\lambda_2 d_1 - \lambda_1 d_2)\left(\overline{\theta}_1 - \overline{\theta}_2\right) > 0$. That is, when the relative distortion between groups d_1/d_2 is large when $\left(\overline{\theta}_1 - \overline{\theta}_2\right) > 0$. When $d_1/d_2 = \lambda_1/\lambda_2$, $\mathbb{E}[DWL]$ consists of only the first term. This is due to the fact that in this case the non-informational trading term corresponding to the equilibrium with imperfect competition and the one corresponding to the competitive equilibrium coincide. Note that if we interpret the traders as producers competing to supply a fixed demand Q, the condition $d_1/d_2 = \lambda_1/\lambda_2$

means that the ratio of the production of the two types of firms is aligned with the slopes of marginal costs. This condition guarantees productive efficiency provided that $\overline{\theta}_1 = \overline{\theta}_2$ and $\rho = 1$ (and since there is a fixed demand this coincides with overall efficiency).

In addition, if group 1 has higher transaction costs ($\lambda_1 > \lambda_2$), is more numerous ($n_1 > n_2$), and less informed ($\sigma^2_{\varepsilon_1} > \sigma^2_{\varepsilon_2}$), then $d_1/d_2 < \lambda_1/\lambda_2$. Then, the third term of the expression of $\mathbb{E}[DWL]$ is not null. In addition, if group 1 ex-ante values less the asset ($\overline{\theta}_1 - \overline{\theta}_2 < 0$) then, the second term of the expression of $\mathbb{E}[DWL]$ is positive.

The deadweight loss also increases with Q, $\left|\overline{\theta}_1 - \overline{\theta}_2\right|$, and $|\lambda_2 d_1 - \lambda_1 d_2|$ when the "stronger" group values no less the asset.

With full information, when $\sigma^2_{\varepsilon_1} = \sigma^2_{\varepsilon_2} = 0$, d_1 and d_2 are independent of ρ, and hence, we can conclude that in this case $\mathbb{E}[DWL]$ decreases in ρ. Similarly, when $\rho = 0$, d_1 and d_2 are independent of $\sigma^2_{\varepsilon_1}$ and $\sigma^2_{\varepsilon_2}$, and $\mathbb{E}[DWL]$ decreases in $\sigma^2_{\varepsilon_1}$ and $\sigma^2_{\varepsilon_2}$. The following proposition summarizes some results.

Proposition 7. *The deadweight loss may be increasing or decreasing with information parameters* $(\rho, \widehat{\sigma}^2_{\varepsilon_i})$. *It increases with payoff and information asymmetry and with* Q *when the "stronger" group (say* $i = 2$*) values no less the asset (i.e.,* $\lambda_1 > \lambda_2$, $n_1 > n_2$, $\sigma^2_{\varepsilon_1} > \sigma^2_{\varepsilon_2}$ *and* $\overline{\theta}_1 \leq \overline{\theta}_2$*). With symmetric groups the deadweight loss is independent of* Q.

6 Concluding remarks

We analyze a divisible good uniform-price auction where two types of bidders compete. In each group there is a finite number of identical bidders. At the unique equilibrium the relative market power of a group increases with the precision of private information and decreases in its transaction costs. Consistent with the empirical evidence, we find that an increase in the transaction cost of a group of bidders induces a strategic response of the other group submitting steeper schedules. The "stronger" group (with more precision of private information, lower transaction costs and/or more oligopolistic) commands more market power and consequently has to receive a higher subsidy to behave competitively. The deadweight loss increases with the quantity auctioned and degree of payoff and informational asymmetry when the stronger group values no less the asset.

The results have policy implications. Consider a regulator who wants to mitigate the inefficiency arising out of market power in an industry with two groups of firms (e.g., a small oligopolistic group and a competitive fringe). The regulator has to consider the strategic effects of his intervention in one group on the behavior of the other group. We find that a regulator who wants to mitigate the inefficiency arising out of market power should subsidize more the stronger/more oligopolistic group. Our framework is amenable to study competition policy analyzing the effects of merger and industry capacity redistribution.

Appendix

Lemma A1. *Let $\rho < 1$. In equilibrium, the demand functions for all the traders of type i, $i = 1, 2$, are given by*

$$X_i(s_i, p) = \frac{\mathbb{E}[\theta_i | s_i, p] - p}{d_i + \lambda_i},$$

with $d_i + \lambda_i > 0$.[17] *The equilibrium coefficients satisfy the following system of equations:*

$$b_i = \frac{(1 - \Xi_i)\overline{\theta}_i - \Psi_i \overline{\theta}_j - \frac{\Psi_i(n_i b_i + n_j b_j - Q)}{n_j a_j}}{d_i + \lambda_i}, \tag{9}$$

$$a_i = \frac{\Xi_i - \frac{n_i a_i}{n_j a_j}\Psi_i}{d_i + \lambda_i} > 0, \text{ and} \tag{10}$$

$$c_i = \frac{1 - \frac{\Psi_i(n_i c_i + n_j c_j)}{n_j a_j}}{d_i + \lambda_i}, \tag{11}$$

where $i, j = 1, 2$, $j \neq i$.Moreover, in equilibrium $a_i > 0$, $i = 1, 2$.

Proof: Consider a trader of type i. Recall that at the beginning of Subsection 3.1 we obtain $X_i(s_i, p) = (\mathbb{E}[\theta_i | s_i, p] - p) / (d_i + \lambda_i)$ and $\mathbb{E}[\theta_i | s_i, p] = \mathbb{E}[\theta_i | s_i, s_j]$. Since we are looking for strategies of the form $X_i(s_i, p) = b_i + a_i s_i - c_i p$ and $X_j(s_j, p) = b_j + a_j s_j - c_j p$, we use these expressions in the market clearing condition, and we get

$$p = \frac{n_i(b_i + a_i s_i) + n_j(b_j + a_j s_j) - Q}{n_i c_i + n_j c_j},$$

and hence,

$$s_j = \frac{(n_i c_i + n_j c_j)p + Q - n_i(b_i + a_i s_i) - n_j b_j}{n_j a_j}.$$

Thus, from Expression (3), it follows that

$$\begin{aligned}\mathbb{E}[\theta_i | s_i, s_j] &= (1 - \Xi_i)\overline{\theta}_i - \Psi_i \overline{\theta}_j + \Psi_i\left(\frac{Q - n_i b_i - n_j b_j}{n_j a_j}\right) \\ &\quad + \left(\Xi_i - \frac{n_i a_i}{n_j a_j}\Psi_i\right)s_i + \Psi_i\left(\frac{n_i c_i + n_j c_j}{n_j a_j}\right)p.\end{aligned}$$

[17] Notice that from the S.O.C. we have that $2d_i + \lambda_i > 0$, $i = 1, 2$.These inequalities imply that $d_i + \lambda_i > 0$, $i = 1, 2$.

Therefore, the expression of the optimal demand function of a bidder of type i, given in (1), becomes

$$X_i(s_i, p) = \frac{(1-\Xi_i)\,\overline{\theta}_i - \Psi_i \overline{\theta}_j + \Psi_i\left(\frac{Q - n_i b_i - n_j b_j}{n_j a_j}\right) + \left(\Xi_i - \frac{n_i a_i}{n_j a_j}\Psi_i\right) s_i + \Psi_i\left(\frac{n_i c_i + n_j c_j}{n_j a_j}\right) p - p}{d_i + \lambda_i}.$$

Identifying coefficients, we obtain the expressions of the demand coefficients given in (9)-(11).

Finally, we show the positiveness of the coefficients a_i, $i = 1, 2$. From (10), we have

$$a_1 = \frac{\Xi_1}{\frac{n_1 \Psi_1}{n_2 a_2} + d_1 + \lambda_1} \text{ and } a_2 = \frac{\Xi_2}{\frac{n_2 \Psi_2}{n_1 a_1} + d_2 + \lambda_2}.$$

Substituting the expression of a_2 in the previous expression for a_1 and operating, we get

$$a_1 = \frac{n_2(\Xi_1\Xi_2 - \Psi_1\Psi_2)}{n_2\Xi_2(d_1 + \lambda_1) + n_1\Psi_1(d_2 + \lambda_2)}. \tag{12}$$

Using (12) in the previous expression for a_2, it follows that

$$a_2 = \frac{n_1(\Xi_1\Xi_2 - \Psi_1\Psi_2)}{n_1\Xi_1(d_2 + \lambda_2) + n_2\Psi_2(d_1 + \lambda_1)}. \tag{13}$$

Direct computations yield $\Xi_1\Xi_2 - \Psi_1\Psi_2 = (1-\rho^2) / \left(\left(1 + \widehat{\sigma}^2_{\varepsilon_1}\right)\left(1 + \widehat{\sigma}^2_{\varepsilon_2}\right) - \rho^2\right) > 0$, whenever $\rho \neq 1$. Moreover, using the positiveness of $d_i + \lambda_i$, Ξ_i and Ψ_i, $i = 1, 2$, we can conclude that in *equilibrium* the coefficients a_1 and a_2 are strictly positive. ∎

Lemma A2. *Let $z = c_1/c_2$. In equilibrium,*

$$\begin{aligned}
b_1 &= \frac{\Psi_1}{n_2}\frac{n_1\Xi_2\frac{a_1}{a_2} - n_2\Psi_2}{n_1(\Xi_1\Xi_2 - \Psi_1\Psi_2)}Q + a_1\left(\frac{\Xi_2\overline{\theta}_1 - \Psi_1\overline{\theta}_2}{\Xi_1\Xi_2 - \Psi_1\Psi_2} - \overline{\theta}_1\right), &(14)\\
a_1 &= \Delta_1 c_1, &(15)\\
c_1 &= \frac{\frac{\Xi_1}{\Delta_1} - \frac{n_1}{n_2}\left(1 - \frac{\Xi_1}{\Delta_1}\right)z - \frac{z}{(n_1-1)z + n_2}}{\lambda_1}, &(16)
\end{aligned}$$

$$\begin{aligned}
b_2 &= \frac{\Psi_2}{n_1}\frac{n_2\Xi_1\frac{a_2}{a_1} - n_1\Psi_1}{n_2(\Xi_1\Xi_2 - \Psi_1\Psi_2)}Q + a_2\left(\frac{\Xi_1\overline{\theta}_2 - \Psi_2\overline{\theta}_1}{\Xi_1\Xi_2 - \Psi_1\Psi_2} - \overline{\theta}_2\right), &(17)\\
a_2 &= \Delta_2 c_2, \text{ and} &(18)\\
c_2 &= \frac{\frac{\Xi_2}{\Delta_2} - \frac{n_2}{n_1}\left(1 - \frac{\Xi_2}{\Delta_2}\right)\frac{1}{z} - \frac{1}{n_1 z + n_2 - 1}}{\lambda_2}, &(19)
\end{aligned}$$

where $\Delta_i = (\Xi_i\Xi_j - \Psi_i\Psi_j) / (\Xi_j - \Psi_i) = \left(1 + (1+\rho)^{-1}\widehat{\sigma}^2_{\varepsilon_i}\right)^{-1}$, $i, j = 1, 2$, $j \neq i$. Moreover, z is the unique positive solution of the following equation:

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{\Xi_1}{\Delta_1} - \frac{n_1}{n_2}\left(1 - \frac{\Xi_1}{\Delta_1}\right)z - \frac{z}{(n_1-1)z + n_2}}{\frac{\Xi_2}{\Delta_2}z - \frac{n_2}{n_1}\left(1 - \frac{\Xi_2}{\Delta_2}\right) - \frac{z}{n_1 z + n_2 - 1}}. \tag{20}$$

Proof: In relation to the expressions of b's, notice that (10) implies

$$d_1+\lambda_1 = \frac{\Xi_1 - \frac{n_1a_1}{n_2a_2}\Psi_1}{a_1} \text{ and} \tag{21}$$

$$d_2+\lambda_2 = \frac{\Xi_2 - \frac{n_2a_2}{n_1a_1}\Psi_2}{a_2}. \tag{22}$$

Substituting these expressions in (9), it follows that

$$b_1 = a_1\frac{(1-\Xi_1)\,\overline{\theta}_1 - \Psi_1\overline{\theta}_2 - \frac{\Psi_1(n_1b_1+n_2b_2-Q)}{n_2a_2}}{\Xi_1 - \frac{n_1}{n_2}\frac{a_1}{a_2}\Psi_1} \text{ and}$$

$$b_2 = a_2\frac{(1-\Xi_2)\,\overline{\theta}_2 - \Psi_2\overline{\theta}_1 - \frac{\Psi_2(n_1b_1+n_2b_2-Q)}{n_1a_1}}{\Xi_2 - \frac{n_2}{n_1}\frac{a_2}{a_1}\Psi_2}.$$

Thus,

$$n_1b_1+n_2b_2 = n_1a_1\frac{(1-\Xi_1)\,\overline{\theta}_1 - \Psi_1\overline{\theta}_2 - \frac{\Psi_1(n_1b_1+n_2b_2-Q)}{n_2a_2}}{\Xi_1 - \frac{n_1}{n_2}\frac{a_1}{a_2}\Psi_1} + n_2a_2\frac{(1-\Xi_2)\,\overline{\theta}_2 - \Psi_2\overline{\theta}_1 - \frac{\Psi_2(n_1b_1+n_2b_2-Q)}{n_1a_1}}{\Xi_2 - \frac{n_2}{n_1}\frac{a_2}{a_1}\Psi_2},$$

which implies

$$\begin{aligned} n_1b_1+n_2b_2 &= \frac{\frac{n_1a_1}{n_2a_2}\Xi_2\Psi_1 - 2\Psi_1\Psi_2 + \frac{n_2a_2}{n_1a_1}\Xi_1\Psi_2}{\Xi_1\Xi_2-\Psi_1\Psi_2}Q - a_1n_1\overline{\theta}_1 - a_2n_2\overline{\theta}_2 + \\ &\quad + \frac{n_1a_1\left(\Xi_2\overline{\theta}_1 - \Psi_1\overline{\theta}_2\right) + n_2a_2\left(\Xi_1\overline{\theta}_2 - \Psi_2\overline{\theta}_1\right)}{\Xi_1\Xi_2-\Psi_1\Psi_2}. \end{aligned}$$

Hence,

$$b_1 = \frac{\Psi_1}{n_2}\frac{n_1\Xi_2\frac{a_1}{a_2} - n_2\Psi_2}{n_1\left(\Xi_1\Xi_2-\Psi_1\Psi_2\right)}Q + a_1\left(\frac{\Xi_2\overline{\theta}_1 - \Psi_1\overline{\theta}_2}{\Xi_1\Xi_2-\Psi_1\Psi_2} - \overline{\theta}_1\right) \text{ and}$$

$$b_2 = \frac{\Psi_2}{n_1}\frac{n_2\Xi_1\frac{a_2}{a_1} - n_1\Psi_1}{n_2\left(\Xi_1\Xi_2-\Psi_1\Psi_2\right)}Q + a_2\left(\frac{\Xi_1\overline{\theta}_2 - \Psi_2\overline{\theta}_1}{\Xi_1\Xi_2-\Psi_1\Psi_2} - \overline{\theta}_2\right).$$

Concerning the expressions of a's, substituting (21) and (22) in (11), it follows that

$$c_1 = a_1\frac{1 - \frac{\Psi_1(n_1c_1+n_2c_2)}{n_2a_2}}{\Xi_1 - \frac{n_1}{n_2}\frac{a_1}{a_2}\Psi_1} \text{ and} \tag{23}$$

$$c_2 = a_2\frac{1 - \frac{\Psi_2(n_1c_1+n_2c_2)}{n_1a_1}}{\Xi_2 - \frac{n_2}{n_1}\frac{a_2}{a_1}\Psi_2}. \tag{24}$$

Hence,

$$n_1c_1+n_2c_2 = n_1a_1\frac{n_2a_2 - \Psi_1\left(n_1c_1+n_2c_2\right)}{n_2a_2\Xi_1 - n_1a_1\Psi_1} + n_2a_2\frac{n_1a_1 - \Psi_2\left(n_1c_1+n_2c_2\right)}{n_1a_1\Xi_2 - n_2a_2\Psi_2},$$

which implies that

$$n_1c_1 + n_2c_2 = \frac{n_1a_1(\Xi_2 - \Psi_1) + n_2a_2(\Xi_1 - \Psi_2)}{\Xi_1\Xi_2 - \Psi_1\Psi_2}.$$

Then, substituting the previous expression in (23) and (24), we obtain (15) and (18).

In relation to the expressions of c's, using the expressions of d_1 and d_2, (15) and (18), (21) and (22) can be rewritten as

$$\begin{aligned}
\lambda_1 &= \frac{\frac{\Xi_1}{\Delta_1} - \frac{n_1}{n_2}\frac{\Psi_1}{\Delta_2}\frac{c_1}{c_2}}{c_1} - \frac{1}{(n_1-1)c_1 + n_2c_2} \text{ and} \\
\lambda_2 &= \frac{\frac{\Xi_2}{\Delta_2} - \frac{n_2}{n_1}\frac{\Psi_2}{\Delta_1}\frac{c_2}{c_1}}{c_2} - \frac{1}{n_1c_1 + (n_2-1)c_2},
\end{aligned}$$

or, since

$$\frac{\Psi_1}{\Delta_2} = 1 - \frac{\Xi_1}{\Delta_1} \text{ and } \frac{\Psi_2}{\Delta_1} = 1 - \frac{\Xi_2}{\Delta_2}, \tag{25}$$

$$\begin{aligned}
\lambda_1 &= \frac{\frac{\Xi_1}{\Delta_1} - \frac{n_1}{n_2}\left(1 - \frac{\Xi_1}{\Delta_1}\right)\frac{c_1}{c_2}}{c_1} - \frac{1}{(n_1-1)c_1 + n_2c_2} \text{ and} \\
\lambda_2 &= \frac{\frac{\Xi_2}{\Delta_2} - \frac{n_2}{n_1}\left(1 - \frac{\Xi_2}{\Delta_2}\right)\frac{c_2}{c_1}}{c_2} - \frac{1}{n_1c_1 + (n_2-1)c_2},
\end{aligned}$$

which imply (16) and (19) since $z = c_1/c_2$. Moreover, dividing the previous two equalities, (20) is obtained.

Finally, we show that (20) has a unique positive solution. After some algebra, (20) is equivalent to $p(z) = 0$, where

$$p(z) = p_3z^3 + p_2z^2 + p_1z + p_0,$$

with

$$\begin{aligned}
p_3 &= n_1^2(n_1-1)\left(n_2\frac{\Xi_2}{\Delta_2}\lambda_1 + n_1\left(1 - \frac{\Xi_1}{\Delta_1}\right)\lambda_2\right), \\
p_2 &= n_1\left((3n_2n_1 - n_1 - 2n_2 + 1)\left(n_2\frac{\Xi_2}{\Delta_2}\lambda_1 - n_1\frac{\Xi_1}{\Delta_1}\lambda_2\right) + \right. \\
&\quad \left. +\lambda_2n_1(2n_2n_1 - n_1 + 1) - (n_1-1)(n_2+1)n_2\lambda_1\right), \\
p_1 &= n_2\left((3n_2n_1 - 2n_1 - n_2 + 1)\left(n_2\frac{\Xi_2}{\Delta_2}\lambda_1 - n_1\frac{\Xi_1}{\Delta_1}\lambda_2\right) + \right. \\
&\quad \left. +\lambda_2n_1(n_2-1)(n_1+1) - (2n_2n_1 - n_2 + 1)n_2\lambda_1\right), \text{ and} \\
p_0 &= -n_2^2(n_2-1)\left(n_2\left(1 - \frac{\Xi_2}{\Delta_2}\right)\lambda_1 + n_1\frac{\Xi_1}{\Delta_1}\lambda_2\right).
\end{aligned}$$

Notice that $p(0) < 0$ and $\lim_{z\to\infty} p(z) = \infty$. Consequently, there exists $z \in (0, \infty)$ such that $p(z) = 0$. Moreover, we have that $p_2 > p_1$. This property tells us that there exists only one change of sign in the coefficients of $p(z)$. Applying the Descartes' rule, it follows that there exists a unique positive root of $p(z)$. ■

Proof of Proposition 1: This proof directly follows from Lemma A1 and Lemma A2. ■

Proof of Proposition 2: (Necessity). From Proposition 1 we know that $a_i > 0$, $i = 1, 2$. Combining this property with Expressions (15) and (18), we have that in *equilibrium* the coefficients c_1 and c_2 are strictly positive. Moreover, (16) and (19) can be rewritten as

$$c_1 = \frac{q_N(z)}{((n_1 - 1) z + n_2) n_2 \lambda_1} \text{ and } c_2 = \frac{q_D(z)}{(n_1 z + n_2 - 1) n_1 z \lambda_2},$$

where

$$q_N(z) = n_2^2 \frac{\Xi_1}{\Delta_1} + n_2 \left(\frac{\Xi_1}{\Delta_1} (2n_1 - 1) - (n_1 + 1) \right) z - (n_1 - 1) \left(1 - \frac{\Xi_1}{\Delta_1} \right) n_1 z^2$$

and

$$q_D(z) = -n_2 (n_2 - 1) \left(1 - \frac{\Xi_2}{\Delta_2} \right) + n_1 \left(\frac{\Xi_2}{\Delta_2} (2n_2 - 1) - (n_2 + 1) \right) z + n_1^2 \frac{\Xi_2}{\Delta_2} z^2.$$

Let $\overline{z}_N$ and $\overline{z}_D$ denote the highest root of $q_N(z)$ and $q_D(z)$, respectively. Notice that the positiveness of c_1 and c_2 is equivalent to $\overline{z}_N > z > \overline{z}_D$. Therefore, $\overline{z}_N > \overline{z}_D$.

(Sufficiency). Suppose that $\overline{z}_N > \overline{z}_D$. Recall that Lemma A2 shows that there exists a unique positive value of z that solves (20), which can be rewritten as $\frac{\lambda_1}{\lambda_2} = \frac{n_1(n_2 - 1 + n_1 z) q_N(z)}{(n_2 + (n_1 - 1) z) n_2 q_D(z)}$. This implies that $\overline{z}_N > z > \overline{z}_D$. Notice that these inequalities guarantee the positiveness of c_1 and c_2. Therefore, d_1 and d_2 are strictly positive, and consequently, the S.O.C. of the optimization problems are satisfied. Thus, we can conclude that whenever $\overline{z}_N > \overline{z}_D$ there exists a unique *equilibrium*. ■

Corollary 1. *If $n_2 = 1$, the equilibrium exists if $1 - \rho^2 > (2\rho - 1) \widehat{\sigma}_{\varepsilon_1}^2$ and $n_1 > \bar{n}_1 \left(\rho, \widehat{\sigma}_{\varepsilon_1}^2, \widehat{\sigma}_{\varepsilon_2}^2 \right)$ where $\bar{n}_1$ increases in $\rho, \widehat{\sigma}_{\varepsilon_1}^2$ and $\widehat{\sigma}_{\varepsilon_2}^2$. If, furthermore, $\lambda_2 = 0$ and $\sigma_{\varepsilon_2}^2 = 0$, then*

$$\bar{n}_1 \left(\rho, \widehat{\sigma}_{\varepsilon_1}^2, \widehat{\sigma}_{\varepsilon_2}^2 \right) = \frac{\left(1 + \rho + \widehat{\sigma}_{\varepsilon_1}^2 \right) (1 - \rho)}{1 - \rho^2 - (2\rho - 1) \widehat{\sigma}_{\varepsilon_1}^2}, \ x_1 = b_1 + a_1 s_1 - c_1 p \text{ and } x_2 = c_2 (\theta_2 - p),$$

where $b_1 = \rho \widehat{\sigma}_{\varepsilon_1}^2 \left(n_1 (1 - \rho) \left(1 + \rho + \widehat{\sigma}_{\varepsilon_1}^2 \right) \right)^{-1} Q + a_1 \widehat{\sigma}_{\varepsilon_1}^2 \left(1 - \rho^2 \right)^{-1} \left(\overline{\theta}_1 - \rho \overline{\theta}_2 \right)$, $a_1 = \Delta_1 c_1$, $c_1 = 2 \left(n_1 - \bar{n}_1 \left(\rho, \widehat{\sigma}_{\varepsilon_1}^2, \widehat{\sigma}_{\varepsilon_2}^2 \right) \right) \left(1 - \rho^2 - (2\rho - 1) \widehat{\sigma}_{\varepsilon_1}^2 \right) / \left(\lambda_1 \left(1 - \rho^2 + \widehat{\sigma}_{\varepsilon_1}^2 \right) (2n_1 - 1) \right)$, *and* $c_2 = n_1 c_1$.

Proof of Corollary 1: Notice that if $n_2 = 1$, then $\overline{z}_D = \left(2\Delta_2 \Xi_2^{-1} - 1 \right) / n_1$. Thus, the condition that guarantees the existence of an *equilibrium* is equivalent to $q_N \left(\left(2\Delta_2 \Xi_2^{-1} - 1 \right) / n_1 \right) > 0$. Direct computations yield that the last inequality holds if and only if

$$\left(2 - \Xi_2 \Delta_2^{-1} \right) \left(1 - \Xi_1 \Delta_1^{-1} - \Xi_2 \Delta_2^{-1} \right) + \left(\Xi_2 \Delta_2^{-1} - 2 \left(1 - \Xi_1 \Delta_1^{-1} \right) \right) n_1 > 0.$$

As $1 < \Xi_1\Delta_1^{-1} + \Xi_2\Delta_2^{-1}$, we can conclude that an *equilibrium* will exist if and only if $\Xi_2\Delta_2^{-1} > 2\left(1-\Xi_1\Delta_1^{-1}\right)$ and $n_1 > \left(2-\Xi_2\Delta_2^{-1}\right)\left(\Xi_1\Delta_1^{-1}+\Xi_2\Delta_2^{-1}-1\right)/\left(\Xi_2\Delta_2^{-1}-2\left(1-\Xi_1\Delta_1^{-1}\right)\right)$. Using the expressions of Ξ_i and Δ_i, the previous two inequalities are equivalent to $1-\rho^2 > (2\rho-1)\,\widehat{\sigma}^2_{\varepsilon_1}$ and $n_1 > \bar{n}_1\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)$, where

$$\bar{n}_1\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right) = 1 + \frac{\rho\left(1-\rho^2+\widehat{\sigma}^2_{\varepsilon_1}\right)\left((1+\rho)\left(\widehat{\sigma}^2_{\varepsilon_1}+\widehat{\sigma}^2_{\varepsilon_2}\right)+2\widehat{\sigma}^2_{\varepsilon_1}\widehat{\sigma}^2_{\varepsilon_2}\right)}{(1+\rho)\left(\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2\right)\left(1-\rho^2-(2\rho-1)\,\widehat{\sigma}^2_{\varepsilon_1}\right)}. \tag{26}$$

It can be shown that $\bar{n}_1$ increases in $\rho, \widehat{\sigma}^2_{\varepsilon_1}$ and $\widehat{\sigma}^2_{\varepsilon_2}$. In particular, when $\sigma^2_{\varepsilon_2} = 0$, then (26) can be rewritten $\bar{n}_1\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, 0\right) = 1 + \rho\widehat{\sigma}^2_{\varepsilon_1}/\left(1-\rho^2-(2\rho-1)\,\widehat{\sigma}^2_{\varepsilon_1}\right)$.

Further, in the case that one group is formed by a unique trader perfectly informed and with no transaction cost ($n_2 = 1$, $\lambda_2 = 0$ and $\sigma^2_{\varepsilon_2} = 0$). Then, $z = \overline{z}_D = 1/n_1$, $\Xi_1 = \left(1-\rho^2\right)/\left(1-\rho^2+\widehat{\sigma}^2_{\varepsilon_1}\right)$, $\Psi_1 = \rho\widehat{\sigma}^2_{\varepsilon_1}/\left(1-\rho^2+\widehat{\sigma}^2_{\varepsilon_1}\right)$, $\Xi_2 = 1$ and $\Psi_2 = 0$. Then, from Lemma A2, the coefficients of the demand functions are given by:

$$\begin{aligned}
b_1 &= \frac{\rho\widehat{\sigma}^2_{\varepsilon_1}}{n_1(1-\rho)\left(1+\rho+\widehat{\sigma}^2_{\varepsilon_1}\right)}Q + a_1\frac{\widehat{\sigma}^2_{\varepsilon_1}}{1-\rho^2}\left(\overline{\theta}_1-\rho\overline{\theta}_2\right),\\
a_1 &= \Delta_1 c_1,\\
c_1 &= \frac{2\left(n_1-\bar{n}_1\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)\right)\left(1-\rho^2-\widehat{\sigma}^2_{\varepsilon_1}(2\rho-1)\right)}{\lambda_1\left(1-\rho^2+\widehat{\sigma}^2_{\varepsilon_1}\right)(2n_1-1)}\\
b_2 &= 0, \text{ and}\\
a_2 &= c_2 = \frac{c_1}{z} = n_1 c_1. \blacksquare
\end{aligned}$$

Proof of Corollary 2: a) Suppose that $\sigma^2_{\varepsilon_1} \geq \sigma^2_{\varepsilon_2}$, $\lambda_1 \geq \lambda_2$, and $n_1 \geq n_2$. Using the expressions of Ξ_i and Δ_i, $i = 1, 2$, it is easy to see that in this case $\Xi_2\Delta_2^{-1} > \Xi_1\Delta_1^{-1}$. Next, we distinguish two cases:

Case 1: $(n_1+n_2-2)\,n_1/\left((n_1+n_2)(n_1+n_2-1)\right) \geq 1-\Xi_2\Delta_2^{-1}$. Evaluating the polynomial $p(z)$, stated in the proof of Lemma A2, at $z = 1$, we have that

$$p(1) = (n_1+n_2)(n_1+n_2-1)^2 \times$$
$$\left(n_2\left(\frac{(n_1+n_2-2)\,n_1}{(n_1+n_2)(n_1+n_2-1)}-\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)\lambda_1 - n_1\left(\frac{(n_1+n_2-2)\,n_2}{(n_1+n_2)(n_1+n_2-1)}-\left(1-\frac{\Xi_1}{\Delta_1}\right)\right)\lambda_2\right).$$

As $(n_1+n_2-2)\,n_1/\left((n_1+n_2)(n_1+n_2-1)\right) \geq 1-\Xi_2\Delta_2^{-1}$ and $\lambda_1 \geq \lambda_2$,

$$p(1) \geq (n_1+n_2)(n_1+n_2-1)^2 \times$$
$$\left(n_2\left(\frac{(n_1+n_2-2)\,n_1}{(n_1+n_2)(n_1+n_2-1)}-\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)\lambda_2 - n_1\left(\frac{(n_1+n_2-2)\,n_2}{(n_1+n_2)(n_1+n_2-1)}-\left(1-\frac{\Xi_1}{\Delta_1}\right)\right)\lambda_2\right) =$$

$$= (n_1+n_2)(n_1+n_2-1)^2\left(\left(1-\frac{\Xi_1}{\Delta_1}\right)n_1 - n_2\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)\lambda_2,$$

and since $n_1 \geq n_2$,

$$p(1) \geq (n_1+n_2)(n_1+n_2-1)^2\lambda_2 n_2\left(\frac{\Xi_2}{\Delta_2}-\frac{\Xi_1}{\Delta_1}\right) \geq 0.$$

This implies that $z \leq 1$, and therefore, $c_1 \leq c_2$. In addition, using the expressions of d_1 and d_2, we get $\operatorname{sgn}\{d_1 - d_2\} = \operatorname{sgn}\{c_1 - c_2\}$, which implies $d_1 \leq d_2$. Finally, notice that $\Delta_1 \leq \Delta_2$ whenever $\sigma^2_{\varepsilon_1} \geq \sigma^2_{\varepsilon_2}$. Hence, $a_1/a_2 = z\Delta_1/\Delta_2 \leq 1$.

Case 2: $(n_1+n_2-2)\,n_1/\left((n_1+n_2)(n_1+n_2-1)\right) < 1 - \Xi_2\Delta_2^{-1}$. Notice that

$$\frac{(n_1+n_2-2)\,n_2}{(n_1+n_2)(n_1+n_2-1)} - \left(1-\frac{\Xi_1}{\Delta_1}\right) \leq \frac{(n_1+n_2-2)\,n_1}{(n_1+n_2)(n_1+n_2-1)} - \left(1-\frac{\Xi_2}{\Delta_2}\right),$$

since $\Xi_2\Delta_2^{-1} > \Xi_1\Delta_1^{-1}$ and $n_1 \geq n_2$. Thus, in this case we have that

$$\begin{aligned} q_N(1) &= (n_1+n_2-1)(n_1+n_2)\left(\frac{(n_1+n_2-2)\,n_2}{(n_1+n_2)(n_1+n_2-1)} - \left(1-\frac{\Xi_1}{\Delta_1}\right)\right) < 0 \text{ and} \\ q_D(1) &= (n_1+n_2-1)(n_1+n_2)\left(\frac{(n_1+n_2-2)\,n_1}{(n_1+n_2)(n_1+n_2-1)} - \left(1-\frac{\Xi_2}{\Delta_2}\right)\right) < 0. \end{aligned}$$

Taking into account the shape of these polynomials, the previous two inequalities imply that $\overline{z}_D > 1 > \overline{z}_N$. However, Proposition 2 indicates that in this case there is no *equilibrium*.

b) By virtue of (21) and (22), the inequality $d_1 + \lambda_1 > d_2 + \lambda_2$ is equivalent to

$$\frac{\Xi_1 - \frac{n_1}{n_2}\frac{a_1}{a_2}\Psi_1}{a_1} > \frac{\Xi_2 - \frac{n_2}{n_1}\frac{a_2}{a_1}\Psi_2}{a_2}.$$

Using (15) and (18), and after some algebra, the previous inequality is equivalent to

$$z < \frac{\frac{\Xi_1}{\Delta_1} + \frac{n_2}{n_1}\frac{\Psi_2}{\Delta_1}}{\frac{\Xi_2}{\Delta_2} + \frac{n_1}{n_2}\frac{\Psi_1}{\Delta_2}},$$

or, from (25),

$$z < \frac{\frac{\Xi_1}{\Delta_1} + \frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)}{\frac{\Xi_2}{\Delta_2} + \frac{n_1}{n_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)}. \tag{27}$$

We distinguish two cases:

Case 1: $\dfrac{\frac{\Xi_2}{\Delta_2}+\frac{\Xi_1}{\Delta_1}-1}{\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}} \leq \dfrac{\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)}{n_1\left(\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)+(n_2-1)\left(\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}\right)}.$

Using the expressions of Ξ_i and Δ_i, we get that the previous inequality is equivalent to

$$\frac{(1-\rho)\, n_1 n_2 \left(1+\rho+\widehat{\sigma}_{\varepsilon_1}^2\right)}{n_2\left(1-\rho^2+\widehat{\sigma}_{\varepsilon_1}^2\right)+n_1\rho\widehat{\sigma}_{\varepsilon_1}^2}+\frac{(1-\rho)\, n_1\left(n_2-1\right)\left(1+\rho+\widehat{\sigma}_{\varepsilon_2}^2\right)}{n_1\left(1-\rho^2+\widehat{\sigma}_{\varepsilon_2}^2\right)+n_2\rho\widehat{\sigma}_{\varepsilon_2}^2}\leq 1.$$

Moreover, after some algebra, we have that

$$q_D\left(\frac{\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)}{\frac{\Xi_2}{\Delta_2}+\frac{n_1}{n_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)}\right)\leq 0.$$

Consequently, $\frac{\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)}{\frac{\Xi_2}{\Delta_2}+\frac{n_1}{n_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)}<\overline{z}_D<z$, which implies that in this case $d_1+\lambda_1<d_2+\lambda_2$ holds.

Case 2: $\frac{\frac{\Xi_2}{\Delta_2}+\frac{\Xi_1}{\Delta_1}-1}{\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}}>\frac{\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)}{n_1\left(\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)+(n_2-1)\left(\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}\right)}$.

In this case, taking into account that z is the unique positive solution of (20), the inequality given in (27) is equivalent to

$$\frac{\lambda_1}{\lambda_2}>\frac{\frac{\frac{\Xi_2}{\Delta_2}+\frac{\Xi_1}{\Delta_1}-1}{\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}}-\frac{\left(\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)}{(n_1-1)\left(\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)+n_2\left(\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}\right)}}{\frac{\frac{\Xi_2}{\Delta_2}+\frac{\Xi_1}{\Delta_1}-1}{\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}}-\frac{\left(\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)}{n_1\left(\frac{\Xi_1}{\Delta_1}+\frac{n_2}{n_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\right)+(n_2-1)\left(\left(1-\frac{\Xi_1}{\Delta_1}\right)\frac{n_1}{n_2}+\frac{\Xi_2}{\Delta_2}\right)}}.$$

Taking into account that $\sigma_{\varepsilon_1}^2\geq\sigma_{\varepsilon_2}^2$, $\lambda_1\geq\lambda_2$, and $n_1\geq n_2$, and after some algebra, we get that both sides of the inequality are higher than (or equal to) 1. Therefore, if the value of $\frac{\lambda_1}{\lambda_2}$ is high enough, we obtain $d_1+\lambda_1>d_2+\lambda_2$. Otherwise, the opposite inequality holds. ■

Proof of Corollary 3: In what follows we prove the following comparative statics results:

a) $\frac{\partial b_i}{\partial\overline{\theta}_i}>0$, $\frac{\partial a_i}{\partial\overline{\theta}_i}=0$ and $\frac{\partial c_i}{\partial\overline{\theta}_i}=0$,
b) $\frac{\partial b_i}{\partial\overline{\theta}_j}<0$, $\frac{\partial a_i}{\partial\overline{\theta}_j}=0$ and $\frac{\partial c_i}{\partial\overline{\theta}_j}=0$,
c) $\frac{\partial b_i}{\partial Q}>0$, $\frac{\partial a_i}{\partial Q}=0$ and $\frac{\partial c_i}{\partial Q}=0$,
d) $\frac{\partial a_i}{\partial\lambda_i}<0$ and $\frac{\partial c_i}{\partial\lambda_i}<0$,
e) $\frac{\partial a_i}{\partial\lambda_j}<0$ and $\frac{\partial c_i}{\partial\lambda_j}<0$,
f) $\frac{\partial a_i}{\partial\rho}<0$ and $\frac{\partial c_i}{\partial\rho}<0$,
g) $\frac{\partial}{\partial\sigma_{\varepsilon_i}^2}\left(\frac{d_i}{d_j}\right)<0$, $\frac{\partial}{\partial\sigma_{\varepsilon_j}^2}\left(\frac{d_i}{d_j}\right)>0$, $\frac{\partial}{\partial\lambda_i}\left(\frac{d_i}{d_j}\right)<0$, and $\frac{\partial}{\partial\lambda_j}\left(\frac{d_i}{d_j}\right)>0$,
h) $\frac{\partial a_i}{\partial\sigma_{\varepsilon_i}^2}<0$ and $\frac{\partial c_i}{\partial\sigma_{\varepsilon_i}^2}<0$,
i) $\frac{\partial a_i}{\partial\sigma_{\varepsilon_j}^2}<0$ and $\frac{\partial c_i}{\partial\sigma_{\varepsilon_j}^2}<0$, and
k) $\frac{\partial d_i}{\partial n_i}<0$ and $\frac{\partial d_j}{\partial n_i}<0$, $i,j=1,2$, $i\neq j$.

From Lemma A1, we know that the equilibrium coefficients that depend on $\overline{\theta}_i$, $\overline{\theta}_j$ and Q are b's. Using Lemma A2 and after some algebra, the results given in a), b) and c) are obtained. In what follows, without any loss of generality, let $i = 1$. First, we prove that $\frac{\partial z}{\partial \lambda_1} < 0$. Recall that from Lemma A2, we know that z is the unique positive solution of the following equation:

$$\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)} = 0, \tag{28}$$

where

$$\begin{aligned} N(z) &= \frac{\Xi_1}{\Delta_1} - \frac{n_1}{n_2}\left(1 - \frac{\Xi_1}{\Delta_1}\right) z - \frac{z}{(n_1 - 1)\, z + n_2} \text{ and} \\ D(z) &= \frac{\Xi_2}{\Delta_2} z - \frac{n_2}{n_1}\left(1 - \frac{\Xi_2}{\Delta_2}\right) - \frac{z}{n_1 z + (n_2 - 1)}, \end{aligned}$$

with

$$\frac{\Xi_1}{\Delta_1} = \frac{\left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_2}\right)\left(1 + \rho + \widehat{\sigma}^2_{\varepsilon_1}\right)}{\left(\left(1 + \widehat{\sigma}^2_{\varepsilon_1}\right)\left(1 + \widehat{\sigma}^2_{\varepsilon_2}\right) - \rho^2\right)(1 + \rho)} \text{ and } \frac{\Xi_2}{\Delta_2} = \frac{\left(1 - \rho^2 + \widehat{\sigma}^2_{\varepsilon_1}\right)\left(1 + \rho + \widehat{\sigma}^2_{\varepsilon_2}\right)}{\left(\left(1 + \widehat{\sigma}^2_{\varepsilon_1}\right)\left(1 + \widehat{\sigma}^2_{\varepsilon_2}\right) - \rho^2\right)(1 + \rho)}.$$

Applying the Implicit Function Theorem,

$$\frac{\partial z}{\partial \lambda_1} = -\frac{\frac{\partial}{\partial \lambda_1}\left(\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)}\right)}{\frac{\partial}{\partial z}\left(\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)}\right)}.$$

As

$$\frac{\partial}{\partial \lambda_1}\left(\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)}\right) = \frac{1}{\lambda_2} > 0 \text{ and } \frac{\partial}{\partial z}\left(\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)}\right) > 0$$

because of $z \in \left(\overline{z}_D, \overline{z}_N\right)$, we can conclude that $\frac{\partial z}{\partial \lambda_1} < 0$.

Next, we study the relationship between $c's$ and λ_1. Differentiating (19), we have

$$\frac{\partial c_2}{\partial \lambda_1} = \frac{\partial c_2}{\partial z}\frac{\partial z}{\partial \lambda_1} = \frac{1}{\lambda_2}\left(\frac{n_2}{n_1}\left(1 - \frac{\Xi_2}{\Delta_2}\right)\frac{1}{z^2} + \frac{n_1}{\left(n_2 + n_1 z - 1\right)^2}\right)\frac{\partial z}{\partial \lambda_1} < 0,$$

since $\frac{\partial z}{\partial \lambda_1} < 0$. Moreover, as $c_1 = zc_2$, it follows that

$$\frac{\partial c_1}{\partial \lambda_1} = \frac{\partial z}{\partial \lambda_1} c_2 + z\frac{\partial c_2}{\partial \lambda_1} < 0$$

because of the positiveness of c_2 and z, and the negativeness of $\frac{\partial z}{\partial \lambda_1}$ and $\frac{\partial c_2}{\partial \lambda_1}$. In relation to a's, from (15) and (18), direct computations yield $\frac{\partial a_1}{\partial \lambda_1} < 0$ and $\frac{\partial a_2}{\partial \lambda_1} < 0$, since $\frac{\partial c_1}{\partial \lambda_1} < 0$ and $\frac{\partial c_2}{\partial \lambda_1} < 0$.

Now, we study how the correlation coefficient ρ affects a_1. Let $y = \frac{a_1}{a_2}$. As $a_1 = \Delta_1 c_1$ and $a_2 = \Delta_2 c_2$, then $z = \frac{\Delta_2}{\Delta_1} y$. Substituting this expression in (20), and after some algebra,we have that

$$\frac{\lambda_1}{\lambda_2} y = \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}, \tag{29}$$

where

$$\widetilde{N}(y,\rho) = \frac{1-\rho^2+\widehat{\sigma}^2_{\varepsilon_2} - \frac{n_1}{n_2}\widehat{\sigma}^2_{\varepsilon_1}\rho y}{\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2} - \frac{1}{(n_1-1)\frac{1+\rho+\widehat{\sigma}^2_{\varepsilon_1}}{1+\rho} + n_2\frac{1+\rho+\widehat{\sigma}^2_{\varepsilon_2}}{1+\rho}\frac{1}{y}}$$

and

$$\widetilde{D}(y,\rho) = \frac{1-\rho^2+\widehat{\sigma}^2_{\varepsilon_1} - \frac{n_2}{n_1}\widehat{\sigma}^2_{\varepsilon_2}\rho \frac{1}{y}}{\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2} - \frac{1}{n_1\frac{1+\rho+\widehat{\sigma}^2_{\varepsilon_1}}{1+\rho}y + (n_2-1)\frac{1+\rho+\widehat{\sigma}^2_{\varepsilon_2}}{1+\rho}}.$$

Moreover, $a_1 = \widetilde{N}(y,\rho)/\lambda_1$ and $a_2 = \widetilde{D}(y,\rho)/\lambda_2$. Hence,

$$\frac{\partial a_1}{\partial \rho} = \frac{\frac{\partial}{\partial y}\widetilde{N}(y,\rho)\frac{\partial y}{\partial \rho} + \frac{\partial}{\partial \rho}\widetilde{N}(y,\rho)}{\lambda_1}.$$

Thus, in order to show $\frac{\partial a_1}{\partial \rho} < 0$, it suffices to prove that

$$\frac{\partial}{\partial y}\widetilde{N}(y,\rho)\frac{\partial y}{\partial \rho} + \frac{\partial}{\partial \rho}\widetilde{N}(y,\rho) < 0. \tag{30}$$

Direct computations yield $\frac{\partial}{\partial y}\widetilde{N}(y,\rho) < 0$. Then, (30) is equivalent to

$$\frac{\partial y}{\partial \rho} > -\frac{\frac{\partial}{\partial \rho}\widetilde{N}(y,\rho)}{\frac{\partial}{\partial y}\widetilde{N}(y,\rho)}. \tag{31}$$

Moreover, recall that y in equilibrium is the unique positive value that satisfies (29). Thus, applying the implicit function theorem, it follows that

$$\frac{\partial y}{\partial \rho} = -\frac{\frac{\partial}{\partial \rho}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right)}{\frac{\partial}{\partial y}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right)}$$

Then, (31) can be rewritten as

$$-\frac{\frac{\partial}{\partial \rho}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right)}{\frac{\partial}{\partial y}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right)} > -\frac{\frac{\partial}{\partial \rho}\widetilde{N}(y,\rho)}{\frac{\partial}{\partial y}\widetilde{N}(y,\rho)},$$

or using the fact that in equilibrium $\frac{\partial}{\partial y}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right) > 0$, (31) is satisfied if and only if

$$-\frac{\partial}{\partial \rho}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right) > -\frac{\frac{\partial}{\partial \rho}\widetilde{N}(y,\rho)}{\frac{\partial}{\partial y}\widetilde{N}(y,\rho)}\frac{\partial}{\partial y}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right). \tag{32}$$

Notice that

$$\frac{\partial}{\partial\rho}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right) = -\frac{\left(\frac{\partial}{\partial\rho}\widetilde{N}(y,\rho)\right)\widetilde{D}(y,\rho) - \widetilde{N}(y,\rho)\left(\frac{\partial}{\partial\rho}\widetilde{D}(y,\rho)\right)}{\widetilde{D}^2(y,\rho)},$$

or using (28),

$$\frac{\partial}{\partial\rho}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right) = -\frac{\frac{\partial}{\partial\rho}\widetilde{N}(y,\rho) - \frac{\lambda_1}{\lambda_2}y\frac{\partial}{\partial\rho}\widetilde{D}(y,\rho)}{\widetilde{D}(y,\rho)}.$$

Analogously,

$$\frac{\partial}{\partial y}\left(\frac{\lambda_1}{\lambda_2}y - \frac{\widetilde{N}(y,\rho)}{\widetilde{D}(y,\rho)}\right) = \frac{\lambda_1}{\lambda_2} - \frac{\left(\frac{\partial}{\partial y}\widetilde{N}(y,\rho)\right) - \frac{\lambda_1}{\lambda_2}y\left(\frac{\partial}{\partial y}\widetilde{D}(y,\rho)\right)}{\widetilde{D}(y,\rho)}.$$

Therefore, (32) is equivalent to

$$\frac{\frac{\partial}{\partial\rho}\widetilde{N}(y,\rho) - \frac{\lambda_1}{\lambda_2}y\frac{\partial}{\partial\rho}\widetilde{D}(y,\rho)}{\widetilde{D}(y,\rho)} > -\frac{\frac{\partial}{\partial\rho}\widetilde{N}(y,\rho)}{\frac{\partial}{\partial y}\widetilde{N}(y,\rho)}\left(\frac{\lambda_1}{\lambda_2} - \frac{\left(\frac{\partial}{\partial y}\widetilde{N}(y,\rho)\right) - \frac{\lambda_1}{\lambda_2}y\left(\frac{\partial}{\partial y}\widetilde{D}(y,\rho)\right)}{\widetilde{D}(y,\rho)}\right),$$

or,

$$-\frac{y\frac{\partial}{\partial\rho}\widetilde{D}(y,\rho)}{\widetilde{D}(y,\rho)} > -\frac{\frac{\partial}{\partial\rho}\widetilde{N}(y,\rho)}{\frac{\partial}{\partial y}\widetilde{N}(y,\rho)}\left(1 + \frac{y\frac{\partial}{\partial y}\widetilde{D}(y,\rho)}{\widetilde{D}(y,\rho)}\right). \tag{33}$$

Moreover, recall that $a_2 = \widetilde{D}(y,\rho)/\lambda_2$. The positiveness of a_2 tells us that $\widetilde{D}(y,\rho) > 0$. After some algebra, we have that $\frac{\partial}{\partial\rho}\widetilde{D}(y,\rho) < 0$, $\frac{\partial}{\partial\rho}\widetilde{N}(y,\rho) < 0$ and $\frac{\partial}{\partial y}\widetilde{D}(y,\rho) > 0$. Hence, we conclude that the LHS of (33) is positive, whereas RHS of (33) is negative since $\frac{\partial}{\partial y}\widetilde{N}(y,\rho) < 0$. Consequently, the fact that (33) is satisfied allows us to conclude that $\frac{\partial a_1}{\partial\rho} < 0$.

Concerning the effect of ρ on c_1, recall that

$$c_1 = \frac{a_1}{\Delta_1} = \frac{1+\rho+\widehat{\sigma}^2_{\varepsilon_1}}{1+\rho}a_1.$$

This expression tells us that c_1 is the product of two decreasing positive functions in ρ. Therefore, $\frac{\partial c_1}{\partial\rho} < 0$.

Next, we prove that $\frac{\partial}{\partial\sigma^2_{\varepsilon_1}}\left(\frac{d_1}{d_2}\right) < 0$ and $\frac{\partial}{\partial\sigma^2_{\varepsilon_2}}\left(\frac{d_1}{d_2}\right) < 0$. From the expressions of the slopes and z, it follows that $\frac{d_1}{d_2} = \frac{n_1 z + (n_2-1)}{(n_1-1)z + n_2}$. Applying the chain rule, we get $\frac{\partial}{\partial\sigma^2_{\varepsilon_i}}\left(\frac{d_1}{d_2}\right) = \frac{\partial}{\partial z}\left(\frac{d_1}{d_2}\right)\frac{\partial z}{\partial\sigma^2_{\varepsilon_i}}$. As $\frac{\partial}{\partial z}\left(\frac{d_1}{d_2}\right) > 0$, we know that the sign of $\frac{\partial}{\partial\sigma^2_{\varepsilon_i}}\left(\frac{d_1}{d_2}\right)$ is the same as the sign of $\frac{\partial z}{\partial\sigma^2_{\varepsilon_i}}$. Applying the Implicit Function Theorem,

$$\frac{\partial z}{\partial\sigma^2_{\varepsilon_i}} = -\frac{\frac{\partial}{\partial\sigma^2_{\varepsilon_i}}\left(\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)}\right)}{\frac{\partial}{\partial z}\left(\frac{\lambda_1}{\lambda_2} - \frac{N(z)}{D(z)}\right)}.$$

Direct computations yield that

$$\frac{\partial}{\partial \sigma_{\varepsilon_1}^2}\left(\frac{\lambda_1}{\lambda_2}-\frac{N(z)}{D(z)}\right)>0 \text{ and } \frac{\partial}{\partial \sigma_{\varepsilon_2}^2}\left(\frac{\lambda_1}{\lambda_2}-\frac{N(z)}{D(z)}\right)<0.$$

In addition, since

$$\frac{\partial}{\partial z}\left(\frac{\lambda_1}{\lambda_2}-\frac{N(z)}{D(z)}\right)>0,$$

we obtain $\frac{\partial z}{\partial \sigma_{\varepsilon_1}^2}<0$ and $\frac{\partial z}{\partial \sigma_{\varepsilon_2}^2}>0$, and hence, we conclude that $\frac{\partial}{\partial \sigma_{\varepsilon_1}^2}\left(\frac{d_1}{d_2}\right)<0$ and $\frac{\partial}{\partial \sigma_{\varepsilon_2}^2}\left(\frac{d_1}{d_2}\right)>0$. Moreover, the negativeness of $\frac{\partial z}{\partial \lambda_1}$ and the positiveness of $\frac{\partial z}{\partial \lambda_2}$ allows us to conclude that $\frac{\partial}{\partial \lambda_1}\left(\frac{d_1}{d_2}\right)<0$ and $\frac{\partial}{\partial \lambda_2}\left(\frac{d_1}{d_2}\right)>0$.

Now, we study how a_1 and c_1 vary with a change in $\sigma_{\varepsilon_1}^2$ and $\sigma_{\varepsilon_2}^2$. In order to do that first we analyze the effect of $\sigma_{\varepsilon_1}^2$ and $\sigma_{\varepsilon_2}^2$ on d_1 and d_2. From Proposition 1, we know that $d_i=\left(\left(n_i-1\right)c_i+n_jc_j\right)^{-1}$ and $a_i=\Delta_i c_i>0$, $i=1,2$. Therefore,

$$d_i=\left(\left(n_i-1\right)\frac{a_i}{\Delta_i}+n_j\frac{a_j}{\Delta_j}\right)^{-1},\ i,j=1,2,\ j\neq i.$$

Substituting the expressions of (12) and (13) and the expression of Δ_i given in Lemma A2, it follows that

$$d_i=\left(\frac{\left(n_i-1\right)n_j}{n_j\Upsilon_i\left(d_i+\lambda_i\right)+n_i\left(\Upsilon_i-1\right)\left(d_j+\lambda_j\right)}+\frac{n_jn_i}{n_i\Upsilon_j\left(d_j+\lambda_j\right)+n_j\left(\Upsilon_j-1\right)\left(d_i+\lambda_i\right)}\right)^{-1},$$

where $\Upsilon_i=\frac{\Xi_j}{\Xi_j-\Psi_i}=\frac{1-\rho^2+\widehat{\sigma}_{\varepsilon_i}^2}{(1-\rho)\left(1+\rho+\widehat{\sigma}_{\varepsilon_i}^2\right)}>1$, $i,j=1,2,\ j\neq i,$. From the previous expressions for d_1 and d_2, we derive the following equations that are satisfied in equilibrium:

$$F_i\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)=0,\ i=1,2,$$

where

$$F_i\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)=\frac{\left(n_i-1\right)n_jd_i}{n_j\Upsilon_i\left(d_i+\lambda_i\right)+n_i\left(\Upsilon_i-1\right)\left(d_j+\lambda_j\right)}+\frac{n_in_jd_i}{n_i\Upsilon_j\left(d_j+\lambda_j\right)+n_j\left(\Upsilon_j-1\right)\left(d_i+\lambda_i\right)}-1,$$

$i,j=1,2,\ j\neq i$. Let $DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)$ denote the following matrix:

$$\begin{pmatrix} \frac{\partial F_1}{\partial d_1}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right) & \frac{\partial F_1}{\partial d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right) \\ \frac{\partial F_2}{\partial d_1}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right) & \frac{\partial F_2}{\partial d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right) \end{pmatrix}.$$

Direct computations yield

$\frac{\partial F_i}{\partial d_i}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)=$

$$\frac{n_j(n_i-1)(n_i(\lambda_j+d_j)(\Upsilon_i-1)+\Upsilon_i\lambda_i n_j)}{(n_j\Upsilon_i(d_i+\lambda_i)+n_i(\Upsilon_i-1)(d_j+\lambda_j))^2}+\frac{n_i n_j(\lambda_i n_j(\Upsilon_j-1)+\Upsilon_j n_i(\lambda_j+d_j))}{(n_i\Upsilon_j(d_j+\lambda_j)+n_j(\Upsilon_j-1)(d_i+\lambda_i))^2}$$

and

$$\frac{\partial F_i}{\partial d_j}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)=$$

$$-\left(\frac{d_i n_i n_j(n_i-1)(\Upsilon_i-1)}{(n_j\Upsilon_i(d_i+\lambda_i)+n_i(\Upsilon_i-1)(d_j+\lambda_j))^2}+\frac{d_i n_i^2 n_j\Upsilon_j}{(n_i\Upsilon_j(d_j+\lambda_j)+n_j(\Upsilon_j-1)(d_i+\lambda_i))^2}\right),$$

$i,j=1,2$, $j\neq i$. After some tedious algebra, the determinant of $DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)$ is given by

$$\begin{aligned}
&n_1n_2^2(n_1-1)\frac{\lambda_1\Upsilon_1 n_2+\lambda_2 n_1(\Upsilon_1-1)}{(n_2\Upsilon_1(d_1+\lambda_1)+n_1(\Upsilon_1-1)(d_2+\lambda_2))^3}+\\
&n_1^2n_2(n_2-1)\frac{\lambda_1 n_2(\Upsilon_2-1)+\lambda_2\Upsilon_2 n_1}{(n_1\Upsilon_2(d_2+\lambda_2)+n_2(\Upsilon_2-1)(d_1+\lambda_1))^3}+\\
&\frac{n_1n_2}{(n_2\Upsilon_1(d_1+\lambda_1)+n_1(\Upsilon_1-1)(d_2+\lambda_2))^2(n_1\Upsilon_2(d_2+\lambda_2)+n_2(\Upsilon_2-1)(d_1+\lambda_1))^2}\times
\end{aligned}$$

$$\begin{aligned}
&((n_1n_2+(n_1-1)(n_2-1))(\lambda_1 n_2(\Upsilon_2-1)+\lambda_2\Upsilon_2 n_1)(\lambda_1\Upsilon_1 n_2+\lambda_2 n_1(\Upsilon_1-1))+\\
&\quad n_2(d_2n_1(\Upsilon_1-1)+\Upsilon_1 d_1 n_2)(\lambda_1(\Upsilon_2-1)(n_1-1)(n_2-1)+\lambda_2\Upsilon_2 n_1^2)+\\
&\quad n_1(d_1n_2(\Upsilon_2-1)+\Upsilon_2 d_2 n_1)(\lambda_1\Upsilon_1 n_2^2+\lambda_2(\Upsilon_1-1)(n_1-1)(n_2-1))).
\end{aligned}$$

Notice that all the summands in the previous expression are positive. In particular, the determinant of $DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)$ is not null and, therefore, this matrix is invertible. Hence, we can apply the Implicit Function Theorem, we have

$$\begin{pmatrix}\frac{\partial d_1}{\partial\sigma_{\varepsilon_1}^2} & \frac{\partial d_1}{\partial\sigma_{\varepsilon_2}^2}\\ \frac{\partial d_2}{\partial\sigma_{\varepsilon_1}^2} & \frac{\partial d_2}{\partial\sigma_{\varepsilon_1}^2}\end{pmatrix}=$$

$$-\left(DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\right)^{-1}\begin{pmatrix}\frac{\partial F_1}{\partial\sigma_{\varepsilon_1}^2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right) & \frac{\partial F_1}{\partial\sigma_{\varepsilon_2}^2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\\ \frac{\partial F_2}{\partial\sigma_{\varepsilon_1}^2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right) & \frac{\partial F_2}{\partial\sigma_{\varepsilon_2}^2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\end{pmatrix}. \tag{34}$$

Notice that

$$\left(DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\right)^{-1}=\begin{pmatrix}\frac{\frac{\partial F_2}{\partial d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)}{\det\left(DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\right)} & -\frac{\frac{\partial F_1}{\partial d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)}{\det\left(DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\right)}\\ -\frac{\frac{\partial F_2}{\partial d_1}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)}{\det\left(DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\right)} & \frac{\frac{\partial F_1}{\partial d_1}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)}{\det\left(DF_{d_1,d_2}\left(\sigma_{\varepsilon_1}^2,\sigma_{\varepsilon_2}^2,d_1,d_2\right)\right)}\end{pmatrix}.$$

Hence, we know that all the elements of $\left(DF_{d_1,d_2}\left(\sigma^2_{\varepsilon_1},\sigma^2_{\varepsilon_2},d_1,d_2\right)\right)^{-1}$ are positive. Moreover,

$$\begin{aligned}
\frac{\partial F_i}{\partial \sigma^2_{\varepsilon_i}}\left(\sigma^2_{\varepsilon_1},\sigma^2_{\varepsilon_2},d_1,d_2\right) &= -\frac{d_i n_j\left(n_i-1\right)\left(n_j\left(d_i+\lambda_i\right)+n_i\left(d_j+\lambda_j\right)\right)}{\left(n_j\Upsilon_i\left(d_i+\lambda_i\right)+n_i\left(\Upsilon_i-1\right)\left(d_j+\lambda_j\right)\right)^2}\frac{\rho\left(1+\rho\right)}{\left(1-\rho\right)\sigma^2_\theta\left(1+\rho+\widehat{\sigma}^2_{\varepsilon_i}\right)^2}, \text{ and}\\
\frac{\partial F_i}{\partial \sigma^2_{\varepsilon_j}}\left(\sigma^2_{\varepsilon_1},\sigma^2_{\varepsilon_2},d_1,d_2\right) &= -\frac{d_i n_i n_j\left(n_j\left(d_i+\lambda_i\right)+n_i\left(d_j+\lambda_j\right)\right)}{\left(n_i\Upsilon_j\left(d_j+\lambda_j\right)+n_j\left(\Upsilon_j-1\right)\left(d_i+\lambda_i\right)\right)^2}\frac{\rho\left(1+\rho\right)}{\left(1-\rho\right)\sigma^2_\theta\left(1+\rho+\widehat{\sigma}^2_{\varepsilon_j}\right)^2},
\end{aligned}$$

$i,j=1,2,\ j\neq i$. Using all these expressions, (34) implies that $\frac{\partial d_i}{\partial \sigma^2_{\varepsilon_i}}>0$ and $\frac{\partial d_i}{\partial \sigma^2_{\varepsilon_j}}>0$, $i,j=1,2$, $i\neq j$.

Next, we study the comparative statics of c_1 and c_2 with respect to $\sigma^2_{\varepsilon_1}$. Recall that

$$\begin{aligned}
c_1 &= \frac{a_1}{\Delta_1}=\frac{n_2}{n_2\Upsilon_1\left(d_1+\lambda_1\right)+n_1\left(\Upsilon_1-1\right)\left(d_2+\lambda_2\right)} \text{ and}\\
c_2 &= \frac{a_2}{\Delta_2}=\frac{n_1}{n_1\Upsilon_2\left(d_2+\lambda_2\right)+n_2\left(\Upsilon_2-1\right)\left(d_1+\lambda_1\right)}.
\end{aligned}$$

Using the fact that Υ_1, d_1 and d_2 are increasing in $\sigma^2_{\varepsilon_1}$ and that Υ_2 is independent of $\sigma^2_{\varepsilon_1}$, we have the denominators of the previous expressions are increasing in $\sigma^2_{\varepsilon_1}$, which allows us to conclude that c_1 and c_2 are decreasing in $\sigma^2_{\varepsilon_1}$. Combining these results with the fact that Δ_1 is decreasing in $\sigma^2_{\varepsilon_1}$ and Δ_2 is independent of $\sigma^2_{\varepsilon_1}$, it follows that a_1 and a_2 are decreasing in $\sigma^2_{\varepsilon_1}$, since $a_1=\Delta_1 c_1$ and $a_2=\Delta_2 c_2$.

Finally, concerning to h), notice that doing a similar reasoning as before we derive the following equations that are satisfied in equilibrium:

$$F_i\left(n_1,n_2,d_1,d_2\right)=0,\ \ i=1,2,$$

where

$$F_i\left(n_1,n_2,d_1,d_2\right)=\frac{\left(n_i-1\right)n_j d_i}{n_j\Upsilon_i\left(d_i+\lambda_i\right)+n_i\left(\Upsilon_i-1\right)\left(d_j+\lambda_j\right)}+\frac{n_i n_j d_i}{n_i\Upsilon_j\left(d_j+\lambda_j\right)+n_j\left(\Upsilon_j-1\right)\left(d_i+\lambda_i\right)}-1,$$

$i,j=1,2,\ j\neq i$. Hence,

$$\begin{pmatrix} \frac{\partial d_1}{\partial n_1} & \frac{\partial d_1}{\partial n_2}\\ \frac{\partial d_2}{\partial n_1} & \frac{\partial d_2}{\partial n_2}\end{pmatrix}=-\left(DF_{d_1,d_2}\left(n_1,n_2,d_1,d_2\right)\right)^{-1}\begin{pmatrix} \frac{\partial F_1}{\partial n_1}\left(n_1,n_2,d_1,d_2\right) & \frac{\partial F_1}{\partial n_2}\left(n_1,n_2,d_1,d_2\right)\\ \frac{\partial F_2}{\partial n_1}\left(n_1,n_2,d_1,d_2\right) & \frac{\partial F_2}{\partial n_2}\left(n_1,n_2,d_1,d_2\right)\end{pmatrix}.$$

In addition,

$$\frac{\partial F_i}{\partial n_i}\left(n_1,n_2,d_1,d_2\right)=$$

$$d_i n_j\frac{\left(\Upsilon_i-1\right)\left(d_j+\lambda_j\right)+\Upsilon_i n_j\left(d_i+\lambda_i\right)}{\left(n_j\Upsilon_i\left(d_i+\lambda_i\right)+n_i\left(\Upsilon_i-1\right)\left(d_j+\lambda_j\right)\right)^2}+\frac{d_i n_j^2\left(\Upsilon_j-1\right)\left(d_i+\lambda_i\right)}{\left(n_i\Upsilon_j\left(d_j+\lambda_j\right)+n_j\left(\Upsilon_j-1\right)\left(d_i+\lambda_i\right)\right)^2}, \text{ and}$$

$\frac{\partial F_i}{\partial n_j}(n_1, n_2, d_1, d_2) =$

$$\frac{d_i n_i (\Upsilon_i - 1)(n_i - 1)(d_j + \lambda_j)}{(n_j \Upsilon_i (d_i + \lambda_i) + n_i (\Upsilon_i - 1)(d_j + \lambda_j))^2} + \frac{\Upsilon_j d_i n_i^2 (d_j + \lambda_j)}{(n_i \Upsilon_j (d_j + \lambda_j) + n_j (\Upsilon_j - 1)(d_i + \lambda_i))^2},$$

$i, j = 1, 2$, $j \neq i$. Therefore, all these partial derivatives are positive. Combining this result with the positiveness of all the elements of $(DF_{d_1,d_2}(n_1, n_2, d_1, d_2))^{-1}$,[18] we can conclude that $\frac{\partial d_i}{\partial n_i} < 0$ and $\frac{\partial d_i}{\partial n_j} < 0$, $i, j = 1, 2$, $j \neq i$. ■

Proof of Corollary 4: The expression of the expected price can be rewritten as follows:

$$\mathbb{E}[p] = \frac{1}{1+\Lambda}\overline{\theta}_1 + \frac{1}{1+\frac{1}{\Lambda}}\overline{\theta}_2 - \frac{Q}{\frac{n_1}{d_1+\lambda_1} + \frac{n_2}{d_2+\lambda_2}}, \tag{35}$$

where $\Lambda = \frac{\frac{n_2}{d_2+\lambda_2}}{\frac{n_1}{d_1+\lambda_1}}$. First, suposse that $\overline{\theta}_1 = \overline{\theta}_2$. Then the expected price satisfies

$$\mathbb{E}[p] = \overline{\theta}_1 - \frac{Q}{\frac{n_1}{d_1+\lambda_1} + \frac{n_2}{d_2+\lambda_2}}.$$

From Corollary 3 we know that c_i and c_j are decreasing in λ_i, $\sigma^2_{\varepsilon_i}$ or ρ. Hence, d_i and d_j are decreasing in λ_i, $\sigma^2_{\varepsilon_i}$, or ρ. Using this property in the expression of $\mathbb{E}[p]$, it follows that $\mathbb{E}[p]$ is decreasing in these variables. If Q is high enough, then the third term in (35) dominates and we obtain the same results.

Now, we show that if $\overline{\theta}_1 \neq \overline{\theta}_2$ and Q is not very high, then the previous results may not hold. To simplify let $Q = 0$. If Λ were increasing in λ_1, then $\mathbb{E}[p]$ would be decreasing (increasing) in λ_1 whenever $\overline{\theta}_1$ is much higher (lower) than $\overline{\theta}_2$. ■

Proof of Corollary 4: a) First, suppose that $\overline{\theta}_1 = \overline{\theta}_2$. In this case

$$E[p] = \overline{\theta}_1 - \frac{Q}{\frac{n_1}{d_1+\lambda_1} + \frac{n_2}{d_2+\lambda_2}}.$$

From Corollary 3 we know that d_i and d_j decrease in n_i, and increase in $\sigma^2_{\varepsilon_i}$, λ_i and ρ. Using these results in the previous expression, we conclude that the expected price is increasing in n_i and decreasing in λ_i, $\sigma^2_{\varepsilon_i}$, and ρ.

Now, suppose that $\overline{\theta}_1 \neq \overline{\theta}_2$. The results we have just derived may not hold if $\left|\overline{\theta}_1 - \overline{\theta}_2\right|$ is large enough. For example, let us focus on the relationship between the expected price and n_1. To study this relationship, we first show that $\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}$ decreases in n_1. Recall that

$$d_2 = \left(\frac{n_1 n_2}{n_2 \Upsilon_1 (d_1 + \lambda_1) + n_1 (\Upsilon_1 - 1)(d_2 + \lambda_2)} + \frac{(n_2 - 1) n_1}{n_1 \Upsilon_2 (d_2 + \lambda_2) + n_2 (\Upsilon_2 - 1)(d_1 + \lambda_1)}\right)^{-1}.$$

[18] Notice that $(DF_{d_1,d_2}(n_1, n_2, d_1, d_2))^{-1} = \left(DF_{d_1,d_2}\left(\sigma^2_{\varepsilon_1}, \sigma^2_{\varepsilon_2}, d_1, d_2\right)\right)^{-1}$

Hence,

$$1=\frac{1}{\frac{n_2}{\Upsilon_1\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}+\Upsilon_1-1}+\frac{n_2-1}{\Upsilon_2+(\Upsilon_2-1)\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}}}+\frac{\lambda_2}{d_2+\lambda_2}.$$

The fact that d_2 decreases in n_1 implies that $\frac{\lambda_2}{d_2+\lambda_2}$ increases in n_1. Then, the previous inequality tells us that

$$\frac{n_2}{\Upsilon_1\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}+\Upsilon_1-1}+\frac{n_2-1}{\Upsilon_2+(\Upsilon_2-1)\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}}$$

increases in n_1. For this to be possible, $\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}$ needs to be decreasing in n_1.

Given that the expected price satisfies

$$E[p]=\frac{1}{1+\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}}\overline{\theta}_1+\left(1-\frac{1}{1+\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}}\right)\overline{\theta}_2-\frac{Q}{\frac{n_1}{d_1+\lambda_1}+\frac{n_2}{d_2+\lambda_2}},$$

we have that the relationship between the expected price and n_1 is ambiguous. For instance, if $\overline{\theta}_2$ is low enough, the fact that d_1, d_2 and $\frac{n_2(d_1+\lambda_1)}{n_1(d_2+\lambda_2)}$ are decreasing in n_1 allows us to conclude that the expected price increases in n_1. However, if $\overline{\theta}_2$ is large and $\overline{\theta}_1$ and Q are low enough, then the expected price decreases in n_1.

b) From the expression of the expected revenue it follows that it increases in $\overline{\theta}_i$, $i=1,2$, and in Q, whenever $Q<\frac{n_1}{d_1+\lambda_1}\overline{\theta}_1+\frac{n_2}{d_2+\lambda_2}\overline{\theta}_2$, or equivalently, $E[p]>0$.

In addition, using the expression of the expected revenue, it follows that

$$Q\left(\min\left\{\overline{\theta}_1,\overline{\theta}_2\right\}-\frac{Q}{\frac{n_1}{d_1+\lambda_1}+\frac{n_2}{d_2+\lambda_2}}\right)\leq QE[p]\leq Q\left(\max\left\{\overline{\theta}_1,\overline{\theta}_2\right\}-\frac{Q}{\frac{n_1}{d_1+\lambda_1}+\frac{n_2}{d_2+\lambda_2}}\right).$$

Notice that left-hand side (LHS) and the right-hand side (RHS) of this expression corresponds to the expected revenue in an auction where all participants have an expected valuation of $\min\left\{\overline{\theta}_1,\overline{\theta}_2\right\}$ and $\max\left\{\overline{\theta}_1,\overline{\theta}_2\right\}$, respectively. Using Corollary 4a, we know that both LHS and RHS increase in n_i and decreasing in λ_i and $\sigma^2_{\varepsilon_i}$. Hence, we obtain that $QE[p]$ is lower than the expected revenue of the symmetric auction in which both groups are ex-ante identical being large (each group with $\max\{n_1,n_2\}$ bidders), with high expected valuation ($\max\left\{\overline{\theta}_1,\overline{\theta}_2\right\}$), low transaction costs (min$\{\lambda_1,\lambda_2\}$) and precise signals (min$\{\sigma^2_{\varepsilon_1},\sigma^2_{\varepsilon_2}\}$), and larger than the expected revenue of the symmetric auction in which both groups are ex-ante identical but with the opposite characteristics (i.e., $\min\{n_1,n_2\}$, $\min\left\{\overline{\theta}_1,\overline{\theta}_2\right\}$), max$\{\lambda_1,\lambda_2\}$ and max$\{\sigma^2_{\varepsilon_1},\sigma^2_{\varepsilon_2}\}$). ■

Proof of Proposition 3: Direct computations yield

$$\overline{z}_N=\frac{n_2\left((n_1-1)\left(2\frac{\Xi_1}{\Delta_1}-1\right)-\left(2-\frac{\Xi_1}{\Delta_1}\right)+\sqrt{\left(2-\frac{\Xi_1}{\Delta_1}\right)^2+(n_1-1)\left(n_1+3-6\frac{\Xi_1}{\Delta_1}\right)}\right)}{2n_1(n_1-1)\left(1-\frac{\Xi_1}{\Delta_1}\right)}$$

and

$$\overline{z}_D = \frac{n_2 + 1 - \frac{\Xi_2}{\Delta_2}(2n_2 - 1) + \sqrt{\left(2 - \frac{\Xi_2}{\Delta_2}\right)^2 + (n_2 - 1)\left(n_2 + 3 - 6\frac{\Xi_2}{\Delta_2}\right)}}{2\frac{\Xi_2}{\Delta_2}n_1}.$$

Thus, for the case that n_1 tends to infinity, notice that $\lim_{n_1\to\infty} \overline{z}_N = \lim_{n_1\to\infty} \overline{z}_D = 0$. Furthermore, using the previous expressions and after some tedious algebra, the necessary and sufficient condition for the existence of an equilibrium (i.e., $\lim_{n_1\to\infty} \frac{\overline{z}_N}{\overline{z}_D} > 1$) is equivalent to $n_2 > \bar{n}_2\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)$, where

$$\bar{n}_2\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right) = \frac{\left((2-\rho)\,\widehat{\sigma}^2_{\varepsilon_2} + 2\left(1-\rho^2\right)\right)\widehat{\sigma}^2_{\varepsilon_1}\rho}{\left(1-\rho^2\right)\left(\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2\right)}.$$

Moreover, taking the limit in (20), it follows that $\lim_{n_1\to\infty} z = 0$ and

$$\lim_{n_1\to\infty} n_1 z = n_2 \frac{\frac{\Xi_1}{\Delta_1}}{1 - \frac{\Xi_1}{\Delta_1}}. \tag{36}$$

Using the expressions included in the statement of Lemma A2, and after some tedious algebra, we get

$$\lim_{n_1\to\infty} b_1 = q, \ \lim_{n_1\to\infty} a_1 = 0, \ \lim_{n_1\to\infty} c_1 = 0,$$

$$\begin{aligned}
\lim_{n_1\to\infty} b_2 \;&=\; \frac{\widehat{\sigma}^2_{\varepsilon_2}\left(\frac{(n_2-1)\left(1-\rho^2\right)}{\left(1-\rho^2+\widehat{\sigma}^2_{\varepsilon_2}\right)} + \frac{\left(1-\rho^2+\widehat{\sigma}^2_{\varepsilon_1}(1-2\rho)\right)}{\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2}\right)}{(1-\rho)\,\lambda_2\left(n_2(1+\rho) - \frac{\widehat{\sigma}^2_{\varepsilon_1}\rho\left(1+\rho+\widehat{\sigma}^2_{\varepsilon_2}\right)}{\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2}\right)}\left(q\rho\lambda_1 + \overline{\theta}_2 - \rho\overline{\theta}_1\right) + \\
&\quad + q\frac{\rho^2\widehat{\sigma}^2_{\varepsilon_2}\widehat{\sigma}^2_{\varepsilon_1}}{n_2\left(1-\rho^2\right)\left(\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2\right)}, \\
\lim_{n_1\to\infty} a_2 \;&=\; \Delta_2 \mathop{lim}_{n_1\to\infty} c_2, \text{ and} \\
\lim_{n_1\to\infty} c_2 \;&=\; \frac{n_2 - \bar{n}_2\left(\rho, \widehat{\sigma}^2_{\varepsilon_1}, \widehat{\sigma}^2_{\varepsilon_2}\right)}{\lambda_2\frac{1-\rho^2+\widehat{\sigma}^2_{\varepsilon_2}}{1-\rho}\left(\frac{n_2}{\left(1+\rho+\widehat{\sigma}^2_{\varepsilon_2}\right)} - \rho\frac{\widehat{\sigma}^2_{\varepsilon_1}}{(1+\rho)\left(\left(1+\widehat{\sigma}^2_{\varepsilon_1}\right)\left(1+\widehat{\sigma}^2_{\varepsilon_2}\right)-\rho^2\right)}\right)}.
\end{aligned} \tag{37}$$

Next, in relation to the expressions of d_1 and d_2, we have that

$$\lim_{n_1\to\infty} d_1 = \lim_{n_1\to\infty}\frac{1}{(n_1-1)\,c_1 + n_2 c_2} = \lim_{n_1\to\infty}\frac{1}{\left(\frac{(n_1-1)}{n_1}n_1 z + n_2\right)c_2} = \frac{1}{\left(\lim_{n_1\to\infty} n_1 z + n_2\right)\lim_{n_1\to\infty} c_2} > 0.$$

The fact that $n_1 z$ and c_2 converge to a positive finite number (see (36) and (37)) implies that d_1 does not converge to zero. A similar result is obtained with the limit of d_2. In particular, notice that

$$\begin{aligned}\lim_{n_1\to\infty} d_2 &= \lim_{n_1\to\infty}\frac{1}{n_1c_1+(n_2-1)\,c_2} = \lim_{n_1\to\infty}\frac{1}{(n_1z+n_2-1)\,c_2} = \frac{1}{\left(\lim\limits_{n_1\to\infty} n_1z+n_2-1\right)\lim\limits_{n_1\to\infty} c_2} > \\ \lim_{n_1\to\infty} d_1 &> 0.\end{aligned}$$

Finally, notice that if we consider that the small group is fully informed and the large group fully uninformed, then the equilibrium coefficients for the group 2 become:

$$\begin{aligned}\lim_{n_1\to\infty} b_2 &= 0, \text{ and}\\ \mathit{lim}_{n_1\to\infty}\, a_2 &= \mathit{lim}_{n_1\to\infty}\, c_2 = \frac{n_2-2\rho}{(n_2-\rho)\,\lambda_2}. \blacksquare\end{aligned}$$

Proof of Proposition 4: Suppose that n_1 and n_2 go to infinity and that $\frac{n_1}{n_1+n_2}$ converges to μ_1. Taking limits in the equation that characterizes z (i.e., (20)), it follows that

$$\frac{\lambda_1}{\lambda_2}=\frac{\frac{\Xi_1}{\Delta_1}-\frac{\mu_1}{\mu_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)z}{\frac{\Xi_2}{\Delta_2}z-\frac{\mu_2}{\mu_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)}.$$

Therefore,

$$z=\frac{\frac{\Xi_1}{\Delta_1}+\frac{\mu_2}{\mu_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\frac{\lambda_1}{\lambda_2}}{\frac{\mu_1}{\mu_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)+\frac{\Xi_2}{\Delta_2}\frac{\lambda_1}{\lambda_2}}.$$

Moreover, taking the limit in the expressions of the equilibrium coefficients given in Proposition 3, it follows that

$$\begin{aligned}b_i &= \frac{\Psi_i}{\mu_j}\frac{\mu_i\Xi_j\frac{a_i}{a_j}-\mu_j\Psi_j}{\mu_i\left(\Xi_i\Xi_j-\Psi_i\Psi_j\right)}q+a_i\left(\frac{\Xi_j\overline{\theta}_i-\Psi_i\overline{\theta}_j}{\Xi_i\Xi_j-\Psi_i\Psi_j}-\overline{\theta}_i\right),\\ a_i &= \Delta_i c_i,\ i,j=1,2, j\neq i,\\ c_1 &= \frac{\frac{\Xi_1}{\Delta_1}-\frac{\mu_1}{\mu_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)z}{\lambda_1},\ \text{and}\\ c_2 &= \frac{\frac{\Xi_2}{\Delta_2}-\frac{\mu_2}{\mu_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\frac{1}{z}}{\lambda_2}.\end{aligned}$$

Substituting

$$z=\frac{\frac{\Xi_1}{\Delta_1}+\frac{\mu_2}{\mu_1}\left(1-\frac{\Xi_2}{\Delta_2}\right)\frac{\lambda_1}{\lambda_2}}{\frac{\mu_1}{\mu_2}\left(1-\frac{\Xi_1}{\Delta_1}\right)+\frac{\Xi_2}{\Delta_2}\frac{\lambda_1}{\lambda_2}}$$

in the previous expressions and after some algebra, we get

$$\begin{aligned} b_i &= \frac{\lambda_j \Psi_i}{\mu_j \lambda_i \Xi_j + \mu_i \lambda_j \Psi_i} q + a_i \left(\frac{\Xi_j \overline{\theta}_i - \Psi_i \overline{\theta}_j}{\Xi_i \Xi_j - \Psi_i \Psi_j} - \overline{\theta}_i \right), \\ a_i &= \frac{\mu_j \left(\Xi_i \Xi_j - \Psi_i \Psi_j \right)}{\mu_j \lambda_i \Xi_j + \mu_i \lambda_j \Psi_i}, \text{ and} \\ c_i &= \frac{\mu_j \left(\Xi_j - \Psi_i \right)}{\mu_j \lambda_i \Xi_j + \mu_i \lambda_j \Psi_i}, \; i, j = 1, 2, j \neq i. \end{aligned}$$

Next, we derive the equilibrium in the following continuous setup: Consider now that there is a continuum of bidders $[0, 1]$. Let q denote the aggregate quantity supplied in the market. Suppose that a fraction μ_1 of these bidders are traders of type 1 and the remainder fraction, μ_2, are bidders of type 2.

Consider a trader of type i. This bidder chooses to maximize

$$\mathbb{E}\left[\pi_i | s_i, p\right] = \left(\mathbb{E}\left[\theta_i | s_i, p\right] - p\right) x_i - \frac{\lambda_1}{2} x_i^2.$$

The F.O.C. is given by

$$\mathbb{E}\left[\theta_i | s_i, p\right] - p - \lambda_i x_i = 0$$

or, equivalently,

$$X_i\left(s_i, p\right) = \frac{\mathbb{E}\left[\theta_i | s_i, p\right] - p}{\lambda_i}. \tag{38}$$

Moreover, positing linear strategies, the market clearing condition implies that

$$p = \frac{\mu_i \left(b_i + a_i s_i\right) + \mu_j \left(b_j + a_j s_j\right) - q}{\mu_i c_i + \mu_j c_j}, \tag{39}$$

provided that $\mu_i c_i + \mu_j c_j$. Using the expression of p and provided that $a_i \neq 0$, $i = 1, 2$, it follows that

$$\mathbb{E}\left[\theta_i | s_i, p\right] = \mathbb{E}\left[\theta_i | s_i, s_j\right].$$

Hence,

$$\mathbb{E}\left[\theta_i | s_i, p\right] = \overline{\theta}_i + \Xi_i \left(s_i - \overline{\theta}_i\right) + \Psi_i \left(s_j - \overline{\theta}_j\right).$$

Using (39), $s_j = \frac{q - \mu_i b_i - \mu_j b_j - \mu_i a_i s_i + p\left(\mu_i c_i + \mu_j c_j\right)}{\mu_j a_j}$. Therefore,

$$\mathbb{E}\left[\theta_i | s_i, p\right] = \overline{\theta}_i + \Xi_i \left(s_i - \overline{\theta}_i\right) + \Psi_i \left(\frac{q - \mu_i b_i - \mu_j b_j - \mu_i a_i s_i + p\left(\mu_i c_i + \mu_j c_j\right)}{\mu_j a_j} - \overline{\theta}_j \right).$$

Substituting this expression in (38), and identifying coefficients, it follows that

$$b_i = \frac{1}{\lambda_i}\left(\left(1-\Xi_i\right)\overline{\theta}_i - \Psi_i\overline{\theta}_j + \frac{\Psi_i\left(q-\left(\mu_i b_i+\mu_j b_j\right)\right)}{a_j\mu_j}\right), \tag{40}$$

$$a_i = \frac{1}{\lambda_i}\left(\Xi_i - \Psi_i\frac{\mu_i a_i}{\mu_j a_j}\right), \text{ and} \tag{41}$$

$$c_i = \frac{1}{\lambda_i}\left(1-\frac{\Psi_i}{\mu_j a_j}\left(\mu_i c_i + c_j\mu_j\right)\right), \; i,j=1,2, j\neq i. \tag{42}$$

Using (41), it follows that

$$\frac{a_i}{a_j} = \frac{\lambda_j\left(\Xi_i - \Psi_i\frac{\mu_i a_i}{\mu_j a_j}\right)}{\lambda_i\left(\Xi_j - \Psi_j\frac{\mu_j a_j}{\mu_i a_i}\right)}.$$

Hence,

$$\frac{a_i}{a_j} = \frac{\mu_j\left(\Psi_j\lambda_i\mu_j + \Xi_i\lambda_j\mu_i\right)}{\mu_i\left(\Psi_i\lambda_j\mu_i + \lambda_i\Xi_j\mu_j\right)}.$$

Then, plugging the previous expression into (41), we get

$$a_i = \mu_j\frac{\Xi_i\Xi_j - \Psi_i\Psi_j}{\mu_i\Psi_i\lambda_j + \mu_j\lambda_i\Xi_j}. \tag{43}$$

Using (40), and after some algebra, we get

$$\mu_i b_i + \mu_j b_j = \frac{\frac{\mu_i}{\lambda_i}\left(\overline{\theta}_i\left(1-\Xi_i\right) - \Psi_i\overline{\theta}_j + q\frac{\Psi_i}{\mu_j a_j}\right) + \frac{\mu_j}{\lambda_j}\left(\overline{\theta}_j\left(1-\Xi_j\right) - \Psi_j\overline{\theta}_i + q\frac{\Psi_j}{\mu_i a_i}\right)}{\frac{\Psi_i}{\lambda_i}\frac{\mu_i}{\mu_j a_j} + \frac{\Psi_j}{\lambda_j\mu_i}\frac{\mu_j}{a_i} + 1}.$$

Substituting (43) and the last expression in (40),

$$b_i = \frac{\lambda_j\Psi_i}{\mu_i\lambda_j\Psi_i + \mu_j\lambda_i\Xi_j}q + \frac{\mu_j\left(\Xi_j\Xi_i - \Psi_j\Psi_i\right)}{\mu_i\lambda_j\Psi_i + \mu_j\lambda_i\Xi_j}\left(\frac{\Xi_j\overline{\theta}_i - \Psi_i\overline{\theta}_j}{\Xi_j\Xi_i - \Psi_j\Psi_i} - \overline{\theta}_i\right).$$

Moreover, from (42), and after some algebra, we get

$$\mu_i c_i + c_j\mu_j = \frac{\frac{\mu_i}{\lambda_i} + \frac{\mu_j}{\lambda_j}}{\frac{\mu_i}{\mu_j a_j}\frac{\Psi_i}{\lambda_i} + \frac{\mu_j}{\mu_i a_i}\frac{\Psi_j}{\lambda_j} + 1}.$$

Using (43) and the last expression in (42), it follows that

$$c_i = \frac{\mu_j\left(\Xi_j - \Psi_i\right)}{\mu_j\lambda_i\Xi_j + \mu_i\lambda_j\Psi_i}.$$

Comparing the equilibrium coefficients of the limiting case with the ones of the continuous case, we can conclude that the equilibrium coefficients converge to the equilibrium coefficients of the *equilibrium* of the continuous setup Finally, taking into account the expressions of Ξ_i, Ξ_j, Ψ_i and Ψ_j, we obtained the desired expressions stated in the statement of this result. ■

Lemma A3. *The equilibrium maximizes the following distorted benefit maximization program:*

$$\underset{x_1,x_2}{Max}\mathbb{E}\left[n_1\left(\theta_1 x_1-(d_1+\lambda_1)\frac{x_1^2}{2}\right)+n_2\left(\theta_2 x_2-(d_2+\lambda_2)\frac{x_2^2}{2}\right)\middle| t\right]$$
$$s.t.\ n_1x_1+n_2x_2=Q,$$

where d_1 and d_2 are the equilibrium parameters.

Proof of Lemma A3: Notice that the Lagrangian function of is given by

$$\mathcal{L}(x_1,x_2,\mu)=n_1\left(t_1x_1-(d_1+\lambda_1)\frac{x_1^2}{2}\right)+n_2\left(t_2x_2-(d_2+\lambda_2)\frac{x_2^2}{2}\right)-\mu\left(n_1x_1+n_2x_2-Q\right),$$

where μ denotes the Lagrange multiplier. Differentiating we obtain the F.O.C.:

$$\begin{aligned} n_1\left(t_1-(d_1+\lambda_1)x_1\right)-\mu n_1 &= 0, && (44)\\ n_2\left(t_2-(d_2+\lambda_2)x_2\right)-\mu n_2 &= 0, \text{ and} && (45)\\ n_1x_1+n_2x_2 &= Q. && (46)\end{aligned}$$

From (44) and (45), it follows that

$$x_1=\frac{t_1-\mu}{d_1+\lambda_1}\text{ and }x_2=\frac{t_2-\mu}{d_2+\lambda_2}.$$

Substituting these expressions in (46) and operating we have

$$\mu=\frac{n_1\frac{t_1}{d_1+\lambda_1}+n_2\frac{t_2}{d_2+\lambda_2}-Q}{\frac{n_1}{d_1+\lambda_1}+\frac{n_2}{d_2+\lambda_2}}.$$

Then,

$$x_i=\frac{t_i-\frac{n_i\frac{t_i}{d_i+\lambda_i}+n_j\frac{t_j}{d_j+\lambda_j}}{\frac{n_i}{d_i+\lambda_i}+\frac{n_j}{d_j+\lambda_j}}}{d_i+\lambda_i}+\frac{\frac{Q}{\frac{n_i}{d_i+\lambda_i}+\frac{n_j}{d_j+\lambda_j}}}{d_i+\lambda_i},\ i=1,2,$$

i.e., the equilibrium quantities. Moreover, since the objective function is concave and the constraint is a linear equation, we conclude that the critical point is a global maximum. Hence, we have that the *equilibrium* maximizes the previous optimization program. ■

Proof of Proposition 5: Recall that in the competitive setup, the F.O.C. of the two optimization problems will yield

$$\mathbb{E}\left[\theta_i|s_i,p\right]-p-\lambda_ix_i=0,i=1,2.$$

Doing similar computations as in the proof of Lemma A1, we derive the following system of equations:[19]

$$
\begin{aligned}
b_i &= \frac{\left(1-\Xi_i\right)\overline{\theta}_i - \Psi_i\overline{\theta}_j + \Psi_i\left(\frac{Q-n_ib_i-n_jb_j}{n_ja_j}\right)}{\lambda_i}, \\
a_i &= \frac{\Xi_i - \frac{n_i}{n_j}\frac{a_i}{a_j}\Psi_i}{\lambda_i}, \text{ and} \\
c_i &= \frac{1-\Psi_i\left(\frac{n_ic_i+n_jc_j}{n_ja_j}\right)}{\lambda_i}, \ i,j=1,2, j\neq i.
\end{aligned}
$$

Taking into account that $Q=(n_i+n_j)q$ and $\mu_i=\frac{n_i}{n_i+n_j}$, we have that the previous system is identical to the system of equations given in (40)-(42). Consequently, we can conclude that the equilibrium coefficients given in the statement of Proposition 4 are the equilibrium coefficients in the competitive setup. ■

Proof of Proposition 6: Doing similar computations as in the proof of Lemma A1, we obtain that the equilibrium coefficients satisfy the following system of equations:

$$
\begin{aligned}
b_i &= \frac{\left(1-\Xi_i\right)\overline{\theta}_i - \Psi_i\overline{\theta}_j - \frac{\Psi_i(n_ib_i+n_jb_j-Q)}{n_ja_j}}{d_i+\lambda_i-d_i(c_i^o,c_j^o)}, \\
a_i &= \frac{\Xi_i - \frac{n_ia_i}{n_ja_j}\Psi_i}{d_i+\lambda_i-d_i(c_i^o,c_j^o)} > 0, \text{ and} \\
c_i &= \frac{1-\frac{\Psi_i(n_ic_i+n_jc_j)}{n_ja_j}}{d_i+\lambda_i-d_i(c_i^o,c_j^o)}, \ i,j=1,2, j\neq i.
\end{aligned}
$$

Comparing this system of equation and the one derived in the proof of Proposition 5, we obtain that the equilibrium coefficients of the price-taking *equilibrium* solves this system. Therefore, we can conclude that the quadratic subsidies $\frac{\kappa_i}{2}x_i^2$, $i=1,2$, induce an efficient allocation. ■

Lemma A4. *In equilibrium, the expected deadweight satisfies*

$$
\mathbb{E}\left[DWL\right] = \frac{1}{2}\lambda_1 n_1 \mathbb{E}\left[\left(x_1\left(t\right)-x_1^o\left(t\right)\right)^2\right] + \frac{1}{2}\lambda_2 n_2 \mathbb{E}\left[\left(x_2\left(t\right)-x_2^o\left(t\right)\right)^2\right].
$$

Proof of Lemma A4: Notice that $ETS=\mathbb{E}\left[\mathbb{E}\left[TS|t\right]\right]$, where

$$
\begin{aligned}
\mathbb{E}\left[TS|t\right] &= \mathbb{E}\left[n_1\left(\theta_1 x_1\left(t\right)-\lambda_1\frac{\left(x_1\left(t\right)\right)^2}{2}\right)+n_2\left(\theta_2 x_2\left(t\right)-\lambda_2\frac{\left(x_2\left(t\right)\right)^2}{2}\right)\middle| t\right] = \\
& n_1\left(t_1 x_1\left(t\right)-\lambda_1\frac{\left(x_1\left(t\right)\right)^2}{2}\right)+n_2\left(t_2 x_2\left(t\right)-\lambda_2\frac{\left(x_2\left(t\right)\right)^2}{2}\right).
\end{aligned}
$$

[19] To ease the notation the superscript o is omitted in this proof.

A Taylor series expansion of $\mathbb{E}[TS|t]$ around the price-taking equilibrium $(x_1^o(t), x_2^o(t))$, stopping at the second term due to the fact that $\mathbb{E}[TS|t]$ is quadratic, yields

$$\begin{aligned}\mathbb{E}[TS|t](x(t)) &= \mathbb{E}[TS|t](x^o(t)) + \nabla\mathbb{E}[TS|t](x^o(t))(x(t) - x^o(t)) + \\ &+\frac{1}{2}(x(t) - x^o(t))' D^2\mathbb{E}[TS|t](x^o(t))(x(t) - x^o(t)),\end{aligned}$$

where $\nabla\mathbb{E}[TS|t](x^o(t))$ and $D^2\mathbb{E}[TS|t](x^o(t))$ are, respectively, the gradient and the Hessian matrix of $\mathbb{E}[TS|t]$ evaluated at $x^o(t)$. Notice that we know

$$\nabla\mathbb{E}[TS|t](x^o(t)) = (n_1(t_1 - \lambda_1 x_1^o(t)), n_2(t_2 - \lambda_2 x_2^o(t))).$$

Using the expressions of $x_1^o(t)$ and $x_2^o(t)$,

$$\nabla\mathbb{E}[TS|t](x^o(t))(x(t) - x^o(t)) =$$

$$= (n_1(t_1 - \lambda_1 x_1^o(t)), n_2(t_2 - \lambda_2 x_2^o(t))) \begin{pmatrix} x_1(t) - x_1^o(t) \\ x_2(t) - x_2^o(t) \end{pmatrix} =$$

$$n_1\left(\frac{n_1\frac{t_1}{\lambda_1}+n_2\frac{t_2}{\lambda_2}}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}} - \frac{Q}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}}\right)(x_1 - x_1^o) + n_2\left(\frac{n_1\frac{t_1}{\lambda_1}+n_2\frac{t_2}{\lambda_2}}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}} - \frac{Q}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}}\right)(x_2 - x_2^o) =$$

$$\left(\frac{n_1\frac{t_1}{\lambda_1}+n_2\frac{t_2}{\lambda_2}}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}} - \frac{Q}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}}\right)(n_1(x_1 - x_1^o) + n_2(x_2 - x_2^o)) = \left(\frac{n_1\frac{t_1}{\lambda_1}+n_2\frac{t_2}{\lambda_2}}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}} - \frac{Q}{\frac{n_1}{\lambda_1}+\frac{n_2}{\lambda_2}}\right)(Q - Q) = 0.$$

In addition, $D^2\mathbb{E}[TS|t](x^o(t)) = \begin{pmatrix} -\lambda_1 n_1 & 0 \\ 0 & -\lambda_2 n_2 \end{pmatrix}$. Hence,

$$\mathbb{E}[TS|t](x(t)) - \mathbb{E}[TS|t](x^o(t)) = -\frac{1}{2}\lambda_1 n_1 (x_1(t) - x_1^o(t))^2 - \frac{1}{2}\lambda_2 n_2 (x_2(t) - x_2^o(t))^2,$$

and consequently,

$$\mathbb{E}[DWL] = \frac{1}{2}\lambda_1 n_1 \mathbb{E}\left[(x_1(t) - x_1^o(t))^2\right] + \frac{1}{2}\lambda_2 n_2 \mathbb{E}\left[(x_2(t) - x_2^o(t))^2\right]. \blacksquare$$

References

[1] Akgün, U. (2004). Mergers with supply functions. Journal of Industrial Economics 52(4), 535-546.

[2] Anderson, E., Hu, X. (2008). Finding supply function equilibria with asymmetric firms. Operations Research 56(3), 697-711.

[3] Armantier, O. and E. Sbaï (2006). Estimation and comparison of treasury auction formats when bidders are asymmetric. *Journal of Applied Econometrics* 21, 745–779.

[4] Arnone, M. and G. Iden (2003). Primary dealers in government securities: policy issues and selected countries' experience. IMF Working Paper 03/45.

[5] Athey, S. (2001). Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica* 69(4), 861–890.

[6] Ausubel, L. M., P. Cramton, M. Pycia, M. Rostek, and M. Weretka (2014). Demand reduction and inefficiency in multi-unit auctions. *The Review of Economic Studies* 81(4), 1366-1400.

[7] Back, K. and J.F. Zender (1993). Auctions of divisible goods: on the rationale for the Treasury experiment. *The Review of Financial Studies* 6, 733–764.

[8] Bajari, P. (1998). A structural econometric model of the first price sealed bid auction with asymmetric bidders. Manuscript, Stanford University.

[9] Bergemann, D., T. Heumann, and S. Morris (2015). Information and market power. Cowles Foundation working paper no. 2017.

[10] Bikhchandani, S., and C. Huang (1993). The Economics of Treasury Securities Markets. *Journal of Economic Perspectives*, 7(3): 117-134.

[11] Bindseil, U., K. Nyborg, and I. Strebulaev (2009). Bidding and performance in repo auctions: Evidence from ECB open market operations. *Journal of Money, Credit and Banking* 41(7), 1391-1421.

[12] Bjonnes, G.H. (2001). Winner's curse in discriminatory price auctions: Evidence from the Norwegian Treasury bill auctions. SIFR Research Report Series 3, Institute for Financial Research.

[13] Borenstein, S., J. Bushnell, and F. Wolak (2002). Measuring market inefficiencies in California's restructured wholesale electricity market. *American Economic Review* 92(5), 1376-1405.

[14] Boyarchenko, N., D.O. Lucca, and L. Veldkamp (2015). Intermediaries as information aggregators: an application to U.S. Treasury Auctions. Federal Reserve Bank of New York Staff Report no. 726.

[15] Brandts, J., Reynolds, S. and A. Schram. 2014. Pivotal suppliers and market power in experimental supply function competition. Economic Journal 124(579), 887-916.

[16] Brenner, M., D. Galai, and O. Sade (2009). Sovereign debt auctions: Uniform or discriminatory? *Journal of Monetary Economics* 56(2), 267-274.

[17] Bustos-Salvagno, J. (2015). Bidding behavior in the Chilean electricity market. Energy Economics 51, 288-299.

[18] Cantillon, E. (2008). The effect of bidders' asymmetries on expected revenue in auctions. *Games and Economic Behavior* 62(1), 1-25.

[19] Cassola, N., A. Hortaçsu, and J. Kastl (2013). The 2007 subprime market crisis through the lens of European Central Bank auctions for short-term funds. *Econometrica* 81(4), 1309-1345.

[20] Ciarreta, A. and M.P. Espinosa (2010). Supply Function Competition in the Spanish Wholesale Electricity Market. Energy Journal 31 (4), 137-157.

[21] Cramton, P. and S. Stoft (2006). Uniform price auctions in electricity markets. U. Maryland Working paper.

[22] Cramton, P. and S. Stoft (2007). Why we need to stick with uniform price auctions in electricity markets. Electricity Journal 20(1), 26-37.

[23] Crawford, G., J. Crespo and H. Tauchen (2007). Bidding asymmetries in multi-unit auctions: Implications of bid function equilibria in the British spot market for electricity. International Journal of Industrial Organization 25 (6), 1233-1268.

[24] Engelbrecht-Wiggans, R., J.A. List, and D. Lucking-Reiley (2006). Demand reduction in multi-unit auctions with varying numbers of bidders: Theory and field experiments. *International Economic Review* 47, 203–231.

[25] European Commission (2007). DG Competition report on energy sector inquiry, 10 January 2007, SEC (2006) 1724, Brussels, Commission of the European Communities.

[26] Green, R., and D. Newbery (1992): Competition in the British Electricity Spot Market. *Journal of Political Economy*, 100, 929–953.

[27] Grossman, S. and J. Stiglitz (1980). On the impossibility of informationally efficient markets. *American Economic Review* 70, 393-408.

[28] Holmberg, P., D. Newbery, and D. Ralph (2013). Supply function equilibria: Step functions and continuous representations. *Journal of Economic Theory* 148 (4), 1509-1551.

[29] Holmberg, P. and F. Wolak (2015). Electricity markets: Designing auctions where suppliers have uncertain costs. IFN working paper no. 1099.

[30] Hortaçsu, A. and J. Kastl (2012). Valuing dealers' informational advantage: a study of Canadian Treasury auctions. *Econometrica* 80(6), 2511-2542.

[31] Hortaçsu, A. and D. McAdams (2010). Mechanism choice and strategic bidding in divisible good auctions: An empirical analysis of the Turkish Treasury auction market. *Journal of Political Economy* 118(5), 833-865.

[32] Hortaçsu, A. and S. Puller (2008). Understanding strategic bidding in multi-unit auctions: A case study of the Texas electicity spot market. *RAND Journal of Economics* 39(1), 86-114.

[33] Joskow, P., and E. Kahn (2002). A quantitative analysis of pricing behavior in California's wholesale electricity market during summer 2000. *The Energy Journal* 23(4), 1-35.

[34] Kagel, J. H. and D. Levin (2001). Behavior in multi-unit demand auctions: Experiments with uniform price and dynamic Vickrey auctions. *Econometrica* 69, 413–454.

[35] Kastl, J. (2011). Discrete bids and empirical inference in divisible good auctions. *The Review of Economic Studies* 78(3), 974-1014.

[36] Kyle, A. (1989). Informed speculation with imperfect competition. *The Review of Economic Studies* 56, 317-356.

[37] Lebrun, B. (1996). Existence of an equilibrium in first price auctions. *Economic Theory* 7(3), 421–443.

[38] Lebrun, B. (1999). First price auctions in the asymmetric N bidder case. *International Economic Review* 40(1), 125–142.

[39] Lopomo, G., L. Marx, D. McAdams and B. Murray (2011). Carbon allowance auction design: an assessment of options for the United States. Review of Environmental Economics and Policy 5(1) 25-43.

[40] Malvey, P.F. and C.F. Archibald (1998). Uniform-price auctions: Update of the Treasury experience. US Treasury.

[41] Maskin, E. and J.G. Riley (2000a). Asymmetric auctions. *The Review of Economic Studies* 67(3), 413-438.

[42] Maskin, E. and J.G. Riley (2000b). Equilibrium in sealed high bid auctions. *The Review of Economic Studies* 67(3), 439–454.

[43] Maskin, E. and J.G. Riley (2003). Uniqueness in sealed high bid auctions. *Games and Economic Behavior* 45(2), 395–409.

[44] Pesendorfer, M. (2000). A study of collusion in first-price auctions. *The Review of Economic Studies* 67(3), 381–411.

[45] Porter, R.H. and J.D. Zona (1999). Ohio school milk markets: An analysis of bidding. *RAND Journal of Economics* 30(2), 263–288.

[46] Reny, P. and S. Zamir (2004). On the existence of pure strategy monotone equilibria in asymmetric first-price auctions. *Econometrica* 72(4), 1105-1125.

[47] Rostek, M. and M. Weretka (2012). Price inference in small markets. *Econometrica* 80(2), 687-711.

[48] Umlauf, S.R. (1993). An empirical study of the Mexican Treasury bill auction. *Journal of Financial Economics* 33, 313-340.

[49] Vives, X. (2010). Asset auctions, information, and liquidity. *Journal of the European Economic Association* 8(2-3), 467-477.

[50] Vives, X. (2011). Strategic supply function competition with private information. *Econometrica* 79(6), 1919-1966.

[51] Vives, X. (2014). On the possibility of informationally efficient markets. *Journal of the European Economic Association* 12(5), 1200-1239.

[52] Wang, J.J.D. and J.F. Zender (2002). Auctioning divisible goods. *Economic Theory* 19(4), 673–705.

[53] Wilson, R. (1979). Auctions of shares. *Quarterly Journal of Economics* 93, 675–698.

[54] Wolfram, C. (1998): "Strategic Bidding in a Multiunit Auction: An Empirical Analysis of Bids

[55] to Supply Electricity in England and Wales," Rand Journal of Economics, 29, 703–725.