Acquisitions, Common Ownership, and the Cournot Merger Paradox *

MIGUEL ANTÓN   JOSÉ AZAR   MIREIA GINÉ   LUCA X. LIN

September 5, 2019

Abstract

The "Cournot merger paradox" states that, in a symmetric Cournot game, all the gains from a merger between two firms are captured by non-merging rivals in the same industry. We extend the model to allow for common ownership, and show that there are two conflicting effects on the incentive to merge arising from higher common ownership. A direct effect of internalizing positive spillovers from the merger to rivals increases the incentive to merge, and an indirect one through the profit.

Keywords: Common Ownership, Mergers and Acquisitions, Synergies, Merger Paradox.

*Emails: manton@iese.edu, jazar@iese.edu, mgine@iese.edu, llin@iese.edu. IESE Business School, University of Navarra. Av Pearson, 21, 08034 Barcelona, Spain. Antón and Gine gratefully acknowledge the financial support of the Department of Economy and Knowledge of the Generalitat de Catalunya (Ref: 2017 SGR 1244). Azar gratefully acknowledges the financial support of Secretaria d'Universitats i Recerca del Departament d'Empresa I Coneixement de la Generalitat de Catalunya (Ref: 2016 BP00358).

Consider a Cournot oligopoly game with N identical firms, linear inverse demand $P = a - bQ$, and zero costs. If the firms are separately owned, the first-order condition for firm n is

$$a - 2bq_n - bq_{-n} = 0, \tag{1}$$

where q_n is the quantity produced by firm n and q_{-n} is the quantity produced by the other firms. This game has a unique equilibrium, with each firm producing $q^* = \frac{a}{b}(N+1)^{-1}$ and total quantity $Q^* = \frac{a}{b}N(N+1)^{-1}$ and price $P^* = a(N+1)^{-1}$. The equilibrium level of profits per firm is $\pi^*(N) = \frac{a^2}{b}(N+1)^{-2}$.

Suppose that a merger between two firms creates no synergies. The merger is value-increasing for the owners if and only if $\pi^*(N-1) > 2\pi^*(N)$, i.e.,

$$N^2 < 2N + 1. \tag{2}$$

This condition only holds if $N = 2$, that is, a merger to monopoly, but not for any $N > 2$. This result is known in the industrial organization literature as the Cournot merger paradox (Salant et al., 1983):

Proposition 1 (Cournot Merger Paradox – Salant, Switzer and Reynolds, 1983)**.** *In the absence of synergies, the combined value of the acquiring and target firms falls after a merger, except in the case of a merger to monopoly.*

Intuitively, the reason is that the two merging firms lose market share after they become one firm relative to the market share of both of them separately – except in the case of a merger to monopoly. The non-merging rivals, on the other hand, gain market share and, therefore, they benefit more from the merger than the two merging firms. In the symmetric Cournot case the negative market share effect is so large that, in the absence of positive synergies, the owners of the merging firms are actually worse off after the merger.

Consider instead the case in which some shareholders have common ownership. In particular, consider a situation in which firm i has a set of shareholders of measure one. The number of shares is normalized to one. A subset of shareholders, with mass $1-\lambda$, hold one share – since the group has mass $1-\lambda$, in total they hold $1-\lambda$ shares of firm i. The rest of the shares are held by a mass λ of shareholders that hold one share in each firm in the industry – since the group has mass λ, in total they hold λ shares in each firm. Assume also that each firm maximizes a weighted average of shareholder profits, with control proportional to ownership. In this example, proportional control means that each shareholder gets equal control, implying that as a group the common owners have a share λ of control at firm i, and as a group the separate owners have a share $1-\lambda$ of control.

In this case, firm i puts a weight λ on the profits of other firms relative to its own profits (this is often referred to in the literature as the Edgeworth sympathy coefficient). The equilibrium level of profits per firm with N firms in this case is

$$\pi_N^{firm} = \frac{a^2}{b} N^{-1} H(1+H)^{-2}, \tag{3}$$

where $H = N^{-1}\left[1+(N-1)\lambda\right]$ is the modified Herfindahl-Hirschman index (MHHI). The level of total industry profits with N firms is

$$\pi_N^{industry} = \frac{a^2}{b} H(1+H)^{-2}. \tag{4}$$

Consider now a potential acquisition of firm 1 by firm 2 for a negotiated acquisition price b. We assume that the acquisition brings an exogenous synergy $s \geq 0$ that accrues to the acquiring firm if the acquisition happens.

We start by extending the result of Proposition 1 characterizing the change in the profits of the merging firms and of the industry in the absence of synergies, to the case of common ownership, that is, λ>0:

Proposition 2. *Suppose $N > 2$. Then, under common ownership ($0 < \lambda \leq 1$), and in the absence* *of synergies, a merger between two firms implies: (i) a decline in the combined value of the acquiring and target firms, (ii) an increase in the combined value of the non-merging firms, (iii) an increase in the combined value of the industry (except in the case of $\lambda = 1$, where the value of the industry is constant).*

Proof. See Appendix A1. □

Corollary 1. *The return that the common owners obtain from their acquirer and target holdings is negative, but their overall portfolio return (including non-merging rivals) is positive.*

We next explore how the change in shareholders' incentives under the common ownership case may impact firms' decision to engage in a merger. The calculation of shareholder incentives is more complicated relative to the separate ownership case. Let's consider, in turn, the value for each type of shareholder if the acquisition goes through (their inside option) and if the acquisition does not go through (their outside option).

First, consider the shareholders of firm n that don't have ownership in the other firms. The value of their outside option V_1^O is equal to the profits of firm 1 when there are N firms in the market:

$$V_1^O = \pi_N^{firm}. \tag{5}$$

The value of their inside option V_1^I on the other hand is simply the payment they receive by selling their shares to the owners of firm 2 if the firm is bought:

$$V_1^I = b. \tag{6}$$

Now consider the shareholders of firm 2 that don't have ownership in the other firms. Just like for the owners of firm 1 that don't have common ownership, the value of their outside option V_2^O is equal to the profits of firm 2 when there are N firms in the market:

$$V_2^O = \pi_N^{firm}. \tag{7}$$

The value of their inside option V_2^I is the value of the profits of firm 2 when there are $N - 1$

firms in the market, minus the payment they made to acquire firm 1, plus the value of the synergy:

$$V_2^I = \pi_{N-1}^{firm} - b + s, \tag{8}$$

where H' is the MHHI with $N-1$ firms, given by $H' = (N-1)^{-1}\left[1 + (N-2)\lambda\right]$.

Consider now the shareholders with common ownership, which hold one share in each firm in the industry. The value of their outside option V_C^O is equal to the total profits of the industry when there are N firms:

$$V_C^O = N\pi_N^{firm} = \pi_N^{industry}. \tag{9}$$

The value of their inside option V_C^I is equal to the total profits in the industry when there are $N-1$ firms, plus the value of the synergy, minus the payment firm 2 makes to firm 1 for the acquisition (which they pay because they are shareholders of firm 2), plus the payment firm 2 makes to firm 1 for the acquisition (which they receive because they are shareholders of firm 1). The last two terms obviously cancel out, so their inside option value is simply the joint profits with $N-1$ firms:

$$V_C^I = (N-1)\pi_{N-1}^{firm} + s = \pi_{N-1}^{industry} + s. \tag{10}$$

Since the common shareholders have shares in all firms in the industry, they profit from the merger. The separate shareholders of the target and the acquiring firms jointly lose from the merger. A proposed merger therefore creates a conflict between the common and separate shareholders of the merging firms.

Mergers are negotiated by the management of the two firms, and each manager needs to balance the heterogeneous interests of her shareholders. Under the standard assumption that the manager of each firm maximizes a weighted average of shareholder utilities, we can construct aggregate inside and outside option values for each firm, which we denote as v_1^I, v_2^I, v_1^O, and v_2^O:

$$v_1^O = \lambda V_C^O + (1-\lambda)V_1^O = \lambda \pi_N^{industry} + (1-\lambda)\pi_N^{firm} \tag{11}$$

$$v_2^O = \lambda V_C^O + (1-\lambda)V_2^O = \lambda \pi_N^{industry} + (1-\lambda)\pi_N^{firm} \tag{12}$$

$$v_1^I = \lambda V_C^I + (1-\lambda)V_1^I = \lambda \left[\pi_{N-1}^{industry} + s\right] + (1-\lambda)b \tag{13}$$

$$v_2^I = \lambda V_C^I + (1-\lambda)V_2^I = \lambda \pi_{N-1}^{industry} + (1-\lambda)\left[\pi_{N-1}^{firm} - b\right] + s. \tag{14}$$

A Nash bargaining solution in which two firms merge exists if there is a payment b such that the inside value is greater than the outside value for both the acquiring and the target firm. The condition for the merger to be accepted by the target is that the share-weighted-average of overall portfolio value of the target's shareholders is greater with the merger than without it, i.e., $v_1^I \geq v_1^O$:

$$b \geq \frac{\lambda}{1-\lambda}\left[-\Delta\pi^{industry} - s\right] + \pi_N^{firm}, \tag{15}$$

where $\Delta\pi^{industry} = \pi_{N-1}^{industry} - \pi_N^{industry} \geq 0$ is the change in industry profits induced by the acquisition.

Similarly, the condition for the merger to be accepted by the acquirer is that $v_2^I \geq v_2^O$:

$$b \leq \frac{\lambda}{1-\lambda}\Delta\pi^{industry} + \pi_{N-1}^{firm} - \pi_N^{firm} + \frac{s}{1-\lambda}. \tag{16}$$

Thus, a merger can occur if and only if the right hand side of equation 15 (i.e., the minimum payment required by the target firm) is smaller than the right hand side of equation 16 (i.e., the maximum payment that the acquirer is willing to offer). In this case there is a payment b such that both the objective function of the target and the acquirer would increase with the acquisition. We summarize this analysis in the following proposition:

Proposition 3. *A Nash bargaining solution in which two firms in the industry agree to merge exists if* *and only if the synergy is greater than a threshold $\bar{s}$:*

$$s \geq \bar{s} = -\frac{1-\lambda}{1+\lambda}\Delta\pi^{merging} - 2\frac{\lambda}{1+\lambda}\Delta\pi^{industry}. \tag{17}$$

where $\Delta\pi^{merging} = \pi_{N-1}^{firm} - 2\pi_{N}^{firm}$.

The synergy threshold has two distinct components. The first one is the change in profits for the merging firms. From Propositions 1 and 2 we know that the combined profits of the merging firms is lower after the merger (or equal if $\lambda = 1$). The second component is the change in profits in the industry due to merger. Each term is weighted by a coefficient that depends on the degree of overlapping ownership as captured by λ[1].

For the case of separate ownership when $\lambda = 0$, all the weight is put in the first term and, therefore, the synergy threshold is positive and equal to the loss in profits by the merging firms. That is, separate owners require a positive synergy to support the merger.

For the case of common ownership, i.e. $\lambda > 0$, the overall synergy threshold will depend on the weight on the profits of the merging firms plus the weight on the profits of the industry. As λ increases and gets closer to 1, the coefficient weight shifts from the change in profits of the merging firms to the profits from the industry. Since the total profits of the industry are always higher after the merger, common owners would support the merger at a lower synergy threshold. For this reason, the second term in Equation 17 can reduce the synergy threshold.

It is important to note that the effect of λ on the synergy threshold is two-fold. It affects the coefficient weights of the two components of the synergy (we call this the *direct effect*). However, it also affects the change around the merger in both the profits of the merging firms and the industry profits (we call this the *indirect effect*).

To understand the contribution of each effect, Figure 1 shows the synergy threshold as function of λ for different values of N. When $N = 2$, the synergy required is lower than 0 for every λ, which is the case for merging to monopoly. When $N > 2$ but is also small, the combined

[1] These two coefficients are not proper weights that add up to 1, but behave in a monotonic fashion: as lambda increases the first coefficient decreases, while the second one increases.

value of the two firms after the merger drops significantly. If all shareholders are separate owners ($\lambda = 0$), they require a rather *high* synergy to compensate for the drop in the combined value of the firms. However, as we increase the presence of common owners, the synergy required drops monotonically to zero because the drop in the combined value of the two firms after the merger is compensated with the increase in value of the non-merging firms.

As we increase N, the effect of common ownership on the synergy threshold is less pronounced, because the change in both profits of the merging firms and the industry due to the merger become smaller. If N is very large (i.e. $N = 50$) then profits are small because the industry is close to perfect competition, and therefore the synergy required to compensate owners of the merging firms for their loss in profits is lower. We observe that, in this scenario, λ has a a very small effect on the synergy.

For low and mid levels of N common ownership has a strong impact on the level of synergy threshold because it changes the intensity of oligopolistic competition between the firms. However, this effect becomes small when N increases as the industry becomes close to competitive.[2]

Interestingly, for low levels of λ we actually observe a hump-shape. Since λ can affect both the weights (direct effect) and the change in profits (indirect effect), when common ownership is very low, an increase in λ actually increases the synergy threshold because the indirect effect on profits dominates the direct effect on weights. Figures 2 show the contribution of each effect to the change in the synergy threshold for N equal to 3 and 10. In sum, this model shows that the Cournot merger paradox is not *solved* with the presence of common ownership. When $N > 2$ we do need a positive synergy for the merger, so the paradox is still there. However we have shown that common ownership helps reduce the synergy required.

In summary, we illustrated how common ownership helps overcome the Cournot merger paradox.[3] While we illustrate the idea that common ownership of rival firms increases the

[2]Note that when λ is 1 then the synergy threshold is zero across all N.

[3]Perry and Porter (1985) and Deneckere and Davidson (1985) show that mergers could be rational under some conditions if firms are capacity-constrained or if they compete in prices with differentiated products

incentive to merge using the classic Cournot case, the same insight applies also to Bertrand competition and also more generally, as long as some of the gains from the merger are internalized by non-merging rival firms.

References

Deneckere, Raymond and Carl Davidson, "Incentives to form coalitions with Bertrand competition," *The RAND Journal of Economics*, 1985, pp. 473–486.

Perry, Martin K and Robert H Porter, "Oligopoly and the incentive for horizontal merger," *The American Economic Review*, 1985, *75* (1), 219–227.

Salant, Stephen W, Sheldon Switzer, and Robert J Reynolds, "Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium," *The Quarterly Journal of Economics*, 1983, *98* (2), 185–199.

Figure 1: **Synergy Threshold Needed for a Nash Bargaining Solution.** This figure shows the minimum synergy needed for the existence of a Nash bargaining solution with agreement to merge between two firms, as a function of common ownership (measured by λ) and the number of firms (N).

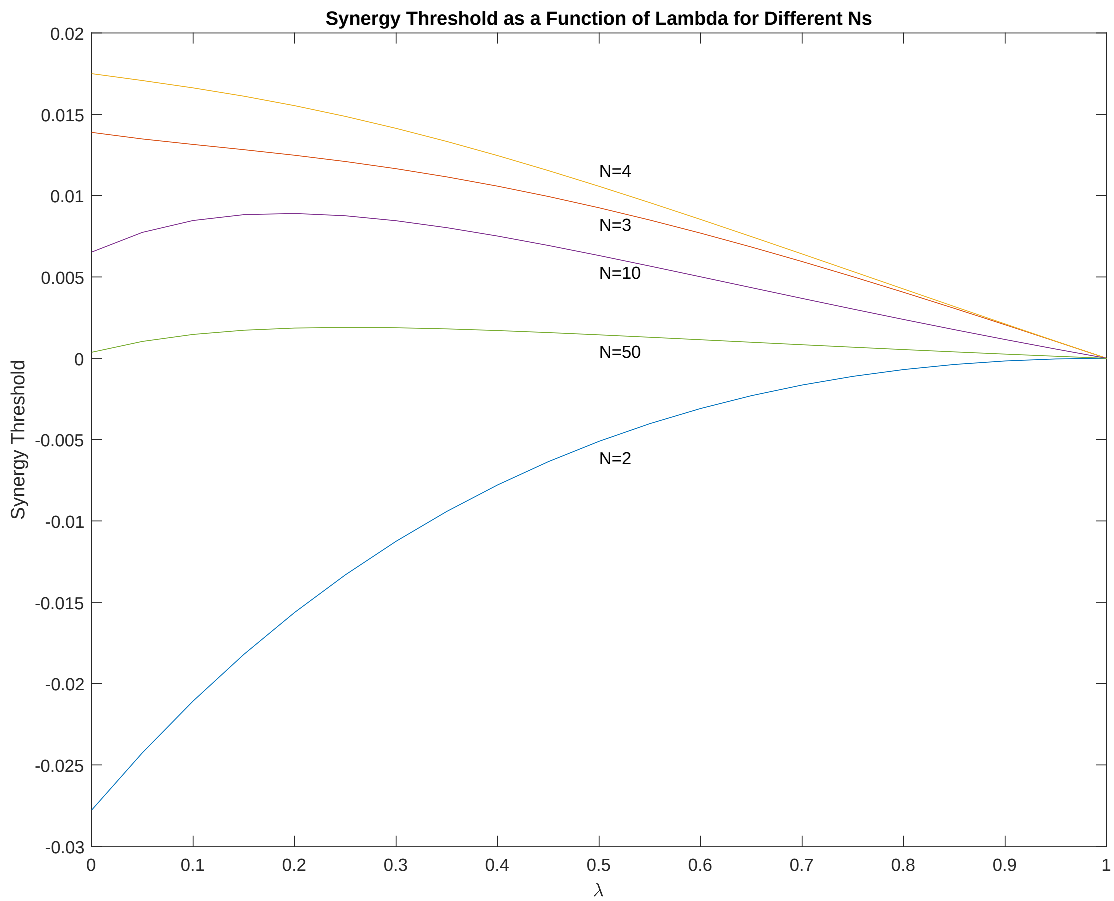
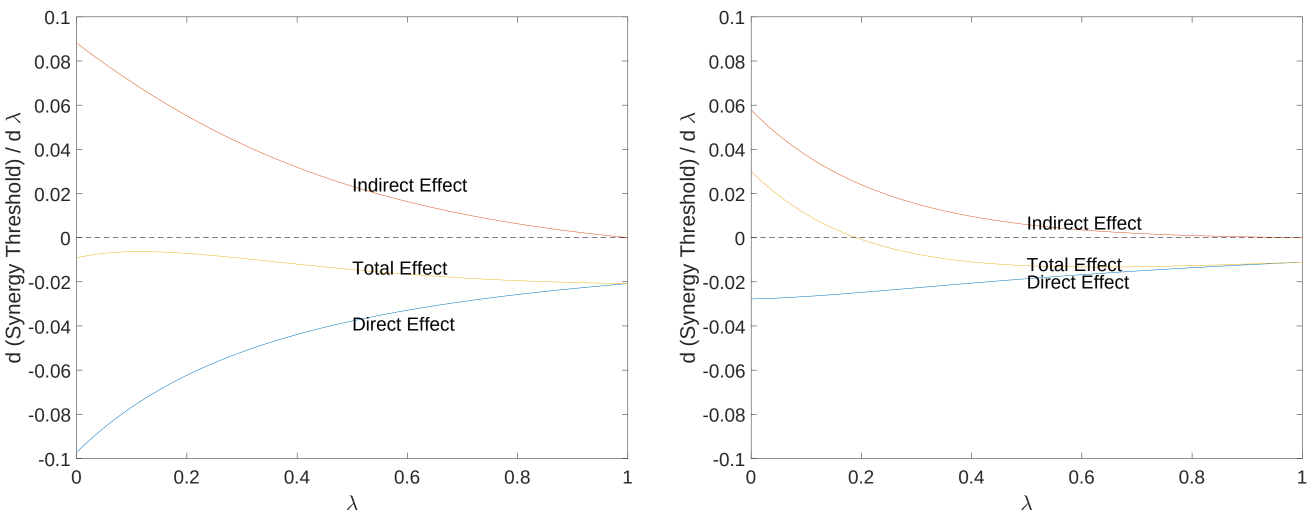


Figure 2: **Direct, Indirect, and Total Effects of λ on Synergy Threshold.** This figure shows the change in the synergy threshold as a function of common ownership (measured by λ) and the number of firms (N). The Direct Effect is the partial derivative on the weights keeping the change in profits constant. The Indirect Effect is the partial derivative on the change in profits keeping the weights constant. The derivative of the synergy threshold with respect to λ (obtained by taking derivative with respect to λ in the expression for $\bar{s}$ from equation 17) is:

$$\frac{\partial \bar{s}}{\partial \lambda} = \underbrace{\frac{2}{(1+\lambda)^2}\left(\Delta\pi^{merging} - \Delta\pi^{industry}\right)}_{\text{direct effect}} \underbrace{-\frac{1-\lambda}{1+\lambda}\frac{\partial\left(\Delta\pi^{merging}\right)}{\partial\lambda} - 2\frac{\lambda}{1+\lambda}\frac{\partial\left(\Delta\pi^{industry}\right)}{\partial\lambda}}_{\text{indirect effect}}.$$



(a) $N = 3$ (b) $N = 10$

A Appendix

A.1 Mathematical Appendix

PROOF OF PROPOSITION 2:

(i) The combined value of the two firms declines after the merger if and only if

$$2\frac{N-1}{N} > \frac{H'(1+H)^2}{H(1+H')^2}. \tag{18}$$

We have already shown that this is true if $\lambda = 0$, since in that case the expression simplifies to $N^2 > 2N+1$, which holds for $N = 2$ but not $N > 2$. To show that it also holds for $\lambda > 0$, we just need to show that $\frac{H'(1+H)^2}{H(1+H')^2}$ is non-increasing in λ. For this, we need to show that the increase in $H'/(1+H')^2$ when λ increases is lower (not strictly) than the increase in $H/(1+H)^2$ when λ increases. Since $H'/(1+H')^2$ is the same as $H/(1+H)^2$ evaluated at $N-1$ instead of N, this is the same as showing that $\frac{\partial^2 H/(1+H)^2}{\partial\lambda\partial(1/N)}$ is non-positive. We can show this to be true by taking the derivative of the log of this expression, since the sign of the two derivatives has to be the same. We start by taking the derivative with respect to λ, and then we take the derivative with respect to $1/N$:

$$\frac{\partial \log\left[H/(1+H)^2\right]}{\partial\lambda} = \frac{1-1/N}{H} - 2\frac{1-1/N}{1+H}. \tag{19}$$

$$\frac{\partial^2 \log\left[H/(1+H)^2\right]}{\partial\lambda\partial(1/N)} = \frac{-H-(1-1/N)(1-\lambda)}{H^2} - 2\frac{-(1+H)-(1-1/N)(1-\lambda)}{(1+H)^2} \tag{20}$$

$$= -\frac{1}{H^2} + \frac{4}{(1+H)^2} \tag{21}$$

$$= \frac{4H^2-(1+H)^2}{H^2(1+H)^2} \tag{22}$$

$$= \frac{3H^2-2H-1}{H^2(1+H)^2}. \tag{23}$$

The quadratic in the numerator has roots at $H = -1/3$ and $H = 1$, and therefore the expression is non-positive for all $H \in [0,1]$. Therefore, the inequality holds not just for $\lambda = 0$ but also for $\lambda > 0$.

[Note that going from (A.3) to (A.4) we use that $(1 - 1/N)(1 - \lambda) = 1 - H$.]

(iii) We need to show that $H/(1 + H)^2$ is non-decreasing in $1/N$. To see this, we can take the derivative of log $\left[H/(1 + H)^2\right]$ with respect to $1/N$:

$$\frac{\partial \log\left[H/(1+H)^2\right]}{\partial(1/N)} = \frac{1-1/N}{H} - 2\frac{1-1/N}{1+H} \tag{24}$$

$$= (1-1/N)\left[\frac{1}{H} - \frac{2}{1+H}\right] \tag{25}$$

$$= (1-1/N)\,\frac{1+H-2H}{H(1+H)} \tag{26}$$

$$= (1-1/N)\,\frac{1-H}{H(1+H)} \geq 0. \tag{27}$$

The inequality is strict except in the case of $\lambda = 1$.

(ii) This follows from (i) and (iii), since an increase in the value of the industry and a decline in the value of the merging firms implies an increase in the value of the rest of the industry firms. □