Preempting Speculative Currency Attacks

Robust Predictions in a Global Game with Multiple Equilibria*

George-Marios Angeletos
MIT and NBER

Alessandro Pavan
Northwestern University

April 29, 2011

Abstract

How do economic fundamentals, policy preferences, and market information shape optimal policy and devaluation outcomes during speculative currency crises? We address these questions in a global-game model where speculators lack common knowledge regarding the policy maker's commitment to defending the currency. Although the signaling role of preemptive policy measures sustains multiple equilibria, a number of tight predictions emerge that are robust across *all* equilibria. These predictions would not have been possible under common knowledge. They are obtained via a novel procedure of iterated deletion on non-equilibrium strategies that embodies the contagion effects of incomplete information while also controlling for the endogeneity of information that originates in the signaling role of policy interventions.

*This paper grew out of prior joint work with Christian Hellwig and would not have been written without our earlier collaboration; we are grateful for his contribution during the early stages of the project. Previous versions circulated under the title "Robust Predictions in Global Games with Multiple Equilibria: Defensive Policies Against Currency Attacks." For useful comments and suggestions, we thank the editor, the referees, Olivier Blanchard, Ricardo Caballero, Eddie Dekel, Kiminori Matsuyama, Stephen Morris, Elie Tamer, Ivan Werning and seminar participants at the University of Milan (Bicocca), MIT, Northwestern, Salerno, Toronto, the North American Winter Meeting of the Econometric Society, the Stony Brook Workshop on Global Games, and the 7th Conference on Macroeconomic Dynamics. *Email addresses:* angelet@mit.edu, alepavan@northwestern.edu.

1 Introduction

This paper investigates the equilibrium properties of policies that seek to preempt self-fulfilling speculative attacks. In the context of currency crises (Obstfeld, 1996), think of policies that seek to discourage speculation either by increasing the cost of short-selling the domestic currency, for example by raising domestic interest rates and imposing taxes on capital outflows, or by signaling the government's commitment to defend the domestic currency. In the context of sovereign debt runs (Calvo, 1988), think of policy reforms that improve future fiscal conditions or otherwise signal the government's commitment to sound fiscal and monetary policies. Are such measures effective in easing speculative pressures? And how does their desirability and effectiveness depend on the policy maker's evaluation of the relevant trade-offs and thereby on the underlying economic fundamentals?

One can easily conjecture intuitive answers to these questions. For example, one may expect that this kind of preemptive policy measures can only help alleviate the risk of a crisis, or that better fundamentals and stronger commitment on the policy maker's side may imply both a lower need to take such preemptive measures and a higher probability of avoiding a crisis.

These intuitions, however, break apart once one starts recognizing the self-fulfilling and signaling aspects of the phenomena under consideration. For example, to the extent that the self-fulfilling aspect is conductive to multiple equilibria, one may not rule out the possibility that this kind of policies end up igniting a self-fulfilling attack in situations where, perhaps paradoxically, the attack would have not taken place if it were not for the policy maker's attempt to preempt it. Alternatively, to the extent that the policy maker has private information about his commitment to defending the domestic currency (or to paying its debt), the same preemptive measures may back-fire by signaling that the policy maker is, perhaps, *too* eager to preempt a crisis.

This discussion underscores the intrinsic difficulties in making tight predictions and conducting policy analysis in environments with commitment problems and self-fulfilling expectations. Indeed, the canonical models of currency crises and debt runs (Obstfeld, 1996; Calvo, 1988) are ridden with so many equilibria that one often has to base policy analysis on ad hoc equilibrium selections, or perhaps to abstract altogether from the central role that coordination plays in these phenomena.[1]

Recent advances in global games have suggested that these difficulties may be overcome by considering perturbations of these models that introduce dispersed private information (Morris and Shin, 1998, 2001). The uncertainty that the agents then face about one another's beliefs and actions poses limits on their ability to coordinate on multiple self-fulfilling outcomes. In certain cases, these limits are so powerful that a unique equilibrium is selected, facilitating a precise policy analysis.

Nonetheless, when it comes to the questions raised above, previous work has shown that the signaling aspect of policy interventions can open back the door to multiple equilibria (Angeletos, Hellwig and Pavan, 2006). This is because the decision of the policy maker to take costly policy

[1] The latter is the route taken, for example, by Drazen (2000)

measures reveals information about his own commitment to defending the currency (or paying the sovereign debt). This information, in turn, can have an anchoring effect on the beliefs of the speculators, helping them coordinate on multiple self-fulfilling reactions to the same policy choices. As the policy maker anticipates such a possibility, his choices end up being driven, at least in part, by self-fulfilling expectations.

Recognizing this multiplicity is an important lesson on its own right—one that warns about how self-fulfilling market expectations can limit a policy maker's ability to tame speculative pressures and escape a crisis. Nonetheless, this does not mean that one should go back to complete-information models, where almost anything often goes. The contribution of this paper is twofold: (i) it shows that the incompleteness of information in this class of models may retain significant selection power, despite the aforementioned multiplicity; and (ii) it identifies a number of novel predictions that are possible only when one abandons the unrealistic assumption of complete information.

Model preview and key predictions. To fix ideas, we consider a model of speculative currency crises, although a re-interpretation of the results in the context of sovereign debt crises is easily possible. The model features a large number of speculators deciding whether or not to attack a currency. As in Obstfeld (1996), self-fulfilling attacks are possible because the policy maker has limited commitment: if the policy maker was committed to defending the peg at all costs, a speculative attack would never materialize. As in Morris and Shin (1998), the speculators have incomplete information about the policy maker's preferences, his commitment to defend the peg, or a variety of economic fundamentals that may ultimately determine the policy maker's decision of whether or not to defend the currency.

The policy maker may attempt to preempt a crisis by taking costly policy measures before a speculative attack builds up. These measures are intended to discourage speculation, either by increasing the cost of short-selling the domestic currency (for example, through a raise in domestic interest rates) or by improving the underlying fundamentals (for example, through reforms that are conductive to more sound fiscal and monetary policies). However, these interventions also signal the policy maker's ultimate commitment to defending the peg and hence may be driven, at least in part, by self-fulfilling market expectations and result in multiple reactions by market participants.

The first part of the paper provides a complete characterization of the set of equilibrium outcomes. The possibility of multiple equilibria builds on previous work (Angeletos, Hellwig and Pavan, 2006). The key contribution here is in providing a characterization of *all* possible equilibrium outcomes. This characterization is achieved through a novel procedure of iterated deletion of strategies that cannot be part of an equilibrium. This procedure shares certain similarities with the procedure of iterated deletion of dominated strategies in standard global games: powerful contagion effects emerge across different types of speculators and different types of the policy maker because, and only because, of the incompleteness of information. At the same time, there is a important difference between the setting considered in this paper and standard global games. In standard global games, the players' beliefs about the underlying "state of nature" are completely exogenous. In the context of currency crises, this means that the speculators' beliefs about the policy maker's commitment to defending the peg are exogenous. By contrast, in our setting, because of signaling, the speculators' beliefs are endogenous: they are a function of the strategy of the policy maker. As a result, our procedure must take into account, not only the contagion effects that emerge in the coordination game among the speculators (the receivers) for a given strategy of the policy maker (the sender), but also the endogeneity of the information sent by the policy maker through the choice of costly preemptive policies. This explains both the complexity of the theoretical exercise and, on the benefit side, the reason why this procedure delivers tight restrictions, not only on devaluation outcomes, but also on the shape of the preemptive policies that are sustained in equilibrium.

The second part of the paper then uses the aforementioned restrictions to identify a number of predictions that are *robust* in the sense that they hold true across *all* possible equilibrium selections. To fix the language, we henceforth use the term policy maker's "type" as a short-cut for a combination of the policy maker's primitive preferences over the relevant trade-offs and for the information that the policy maker may possess about a variety of economic fundamentals that may be relevant in evaluating such trade-offs. Following the pertinent literature, we thus use the term "type" and "fundamentals" interchangeably. With this broader interpretation in mind, we thus allow the policy maker's "type" to determine not only the policy maker's willingness and ability to defend the peg but also the shadow value of the domestic currency and hence the costs of speculation. Our analysis then delivers the following predictions.

- The policy choice is non-monotone in types/fundamentals: preemptive policy measures signal that the policy maker's commitment is neither too weak nor too strong.
- The devaluation outcome is monotone in types/fundamentals: the peg is abandoned if and only if the policy maker's commitment to defending the peg and, more generally, the quality of the underlying economic fundamentals, are sufficiently low.
- Different equilibria can be indexed by the level of policy necessary for preempting an attack. This can be interpreted as an index of the "aggressiveness" of market expectations: the higher this level, the larger the set of types/fundamentals for which the peg is eventually abandoned.
- For given aggressiveness, an increase in the precision of the speculators' information tends to reduce the need for preemptive policy measures, in the sense that it reduces the set of types/fundamentals for which such measures are taken.
- The option to intervene can be harmful only for sufficiently strong types. However, in any equilibrium in which some strong type is worse off, some weak type is necessarily better off. Hence, although the option to intervene leads to multiple equilibria, either the policy maker is better off no matter his type, or low types are better off at the expense of high types.

These predictions can be useful both for the policy maker and for the econometrician (or other outside observer). For the policy maker, the aforementioned predictions give him a better understanding of the effectiveness and desirability of the aforementioned policies, as well as of the signaling role of these policies. For the econometrician, the aforementioned predictions provide empirical restrictions that, at least in principle, can help him estimate and test the model

Contrast to complete information. Given the structure of the underlying environment, the aforementioned predictions seem reasonable. Yet, essentially *none* of these predictions is shared by the complete-information version of the model. Under complete information, the devaluation outcome need not be monotone in either the policy maker's type or the underlying fundamentals; intervention can occur for any arbitrary subset of the critical region (i.e., the region of fundamentals/types for which the peg is maintained if no speculator attacks but is abandoned if all speculators attack); and the value of the option to intervene can be negative for all types in the critical region.

This observation highlights the key role that incomplete information plays in our model: even though it does not pin down a unique equilibrium, it puts significant restrictions on the mapping from primitives to outcomes, leading to predictions that would have been impossible with complete information. This is best illustrated in the limit as the noise in the speculators' information vanishes: even though multiplicity survives for *any* level of noise, the limit of the set of incomplete-information outcomes is a zero-measure subset of the set of common-knowledge outcomes.

Related literature. The global-games approach to equilibrium selection was pioneered by Carlsson and van Damme (1993) and was recently extended by Morris and Shin (2003) and Frankel, Morris and Pauzner (2003). By now, global games have been used in a variety of applications, including currency crises (Morris and Shin, 1998; Corsetti, Dasgupta, Morris and Shin, 2004; Guimaraes and Morris, 2006), bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), debt crises (Corsetti, Guimaraes and Roubini, 2006, Zwart, 2007), investment spillovers (Chamley, 1999; Dasgupta 2006), and liquidity crashes (Morris and Shin, 2004).

Our approach builds on this literature, but differentiates from it in an important dimension. Whereas most of this literature treats the information structure as exogenous and conveniently chooses this structure so as to obtain a unique equilibrium, we think that the endogeneity of information is, in many cases of interest, an integral part of the phenomenon under consideration. In particular, abstracting from the signaling role of policy intervention does not seem an appealing option within the context of speculative currency crises. Indeed, the origin of speculative pressures rests precisely on the policy maker's lack of commitment, and preemptive policy measures are bound to convey information about this commitment. In this respect, we view the multiplicity that originates in the signaling role of policy interventions as an important prediction by itself. At the same, we show that global-game models can deliver tight and useful predictions even when they fail to deliver uniqueness—thereby underscoring a broader methodological message that may extend well beyond the particular exercise conducted in this paper.

Complementary in this regard is Chassang (2010). He finds that global games retain significant selection power in a dynamic setting with exit even if they do not lead to uniqueness. However, in his model, multiplicity is due to repeated play, not to the endogeneity of information.

Finally, the paper departs from several strands of the literature that studies policy in the context of self-fulfilling crises. In common-knowledge models of this kind, policy analysis is by and large restricted to identifying policies that remove the "bad" equilibrium (e.g. Cooper and John, 1988; Zettelmeyer, 2000; Jeanne and Wyplosz, 2001). More recently, Morris and Shin (2006b) and Corsetti, Guimaraes and Roubini (2006) use a global game approach to study how IMF interventions can have a catalytic effect on crises, while also potentially exacerbating the moral-hazard problem for the governments of the countries in risk of a crisis. These papers, however, abstract from the signaling effects that are at the heart of our approach. Zwart (2007) examines a model in which IMF interventions convey information but in which the policy is uniquely determined because the IMF's incentives depend only on the country's fundamentals and not on the size of the attack. Finally, Drazen (2000) and Drazen and Hubrich (2005) discuss signaling effects of policy interventions in currency crises but model the market as a single large player, thus abstracting from the coordination and self-fulfilling aspects of crises that are at the core of our analysis.

Layout. The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the set of equilibrium outcomes. Section 4 identifies predictions about policy and devaluation outcomes. Section 5 studies the equilibrium value of the option to intervene. Section 6 contrasts the multiplicity of the incomplete-information game with that of its common-knowledge counterpart. Section 7 concludes. All proofs are in the Appendix.

2 Model set-up

The economy is populated by a policy maker, who seeks to sustain a currency peg, and a continuum of speculators, who must choose whether to attack the peg (short-sell the domestic currency) or abstain from attacking.[2] The latter are indexed by i and distributed uniformly over $[0, 1]$.

The policy maker. As in the pertinent literature, the policy maker is assumed to have limited commitment in sustaining the peg and, more generally, in sustaining sound fiscal and monetary policies. It is this limited commitment that opens the door to the possibility of a speculative attack.

In addition, the policy maker controls two sets of policy instruments. Some are *preemptive*: they can be used early on in an attempt to discourage the speculators from attacking. Think, for example, of reforms that improve the economic fundamentals and raise the shadow value of the domestic currency, or policies that raise domestic interests rates, or impose taxes on capital outflows. Other policy instruments are *defensive*: if preemption fails and a speculative attack materializes,

[2] Alternatively, one can think of the speculators' decision as a choice regarding the fraction of their wealth to be held in each of the two currencies. Under risk neutrality, the two interpretations are equivalent.

the policy maker may still be able to defend the peg against the ongoing attack, for example by depleting central bank reserves, borrowing reserves from the IMF, or shutting down certain markets.

Hereafter, we proxy the preemptive policies by a variable $r \in [\underline{r}, +\infty)$, which the policy maker chooses before the speculators move. Letting $A \in [0,1]$ denote the size of the attack (the fraction of speculators attacking the currency), we then let the policy maker's payoff be given by

$$U = \begin{cases} W(\theta, r, A) & \text{if the peg is maintained} \\ L(\theta, r) & \text{otherwise} \end{cases}$$

where $\theta \in \mathbb{R}$ is an exogenous variable and where W and L are bounded and continuously differentiable functions, with $W_r, W_A, L_r < 0$ and $W_\theta > L_\theta \geq 0$.[3] The negative dependence of W and L on r captures the costs of preemptive policies. The negative dependence of W on A captures the costs of defensive measures, while the independence of L from A captures the idea that these costs are foregone if the policy maker opts to abandon the peg. Finally, the positive dependence of the differential $W - L$ on θ identifies a higher θ with a higher value of maintaining the peg.

The variable θ is private information to the policy maker and indexes his willingness, ability, or commitment to defending the peg. For example, following Obstfeld (1996), one can think of θ as the value that the policy maker assigns to reputation: abandoning the peg is costly because it destroys reputation.[4] More generally, θ determines how the policy maker evaluates all the relevant trade-offs in the choice of both preemptive and defensive policies. It can thus be thought of as a proxy for the *combination* of the policy maker's primitive preferences and of the information that the policy maker possesses about a variety of exogenous variables that may influence his decision to engage in preemptive and defensive policies. With this broader interpretation in mind, we henceforth refer to θ interchangeably as the "type" of the policy maker and as the underlying "fundamentals".

The speculators. The payoff from not attacking is normalized to zero. The devaluation premium that a speculator enjoys in case he attacks and the peg is abandoned is given by $Z(\theta, r)$, while the borrowing, transaction, and other costs that the speculator has to bear in order to attack are given by $Q(r)$, where Z and Q are bounded and continuously differentiable functions, with $Z_\theta, Z_r \leq 0$, and $Q_r > 0$. The dependence of Z on θ allows us to capture the possibility that the shadow value of the peg increases with the quality of the fundamentals. The dependence of Z and Q on r, on the other hand, captures the possibility that the policy maker be able to manipulate the speculators' incentives either by undertaking reforms that improve the shadow value of the currency

[3] Throughout, for any (real-valued) function $f : \mathbb{R}^n \to \mathbb{R}$, we denote by f_x the partial derivative of f with respect to the argument x and by f_{xy} the cross derivative of f with respect to the arguments x and y.

[4] In richer models, the value of reputation can be modeled as the outcome of repeated interaction between the policy maker and the rest of the economy. This, however, is beyond the scope of this paper. Instead, following Obstfeld (1996), Morris and Shin (2001) and others, we opt to treat θ as exogenous. The focus of our analysis is on how the value that the policy maker attaches to commitment, or reputation, impacts the possibility of a speculative attack in the present—not on how this value is ultimately determined by considerations about the future.

or by raising domestic interest rates and imposing taxes on capital-flows. To economize on notation, and without any loss of generality, we henceforth let $Q(r) = r$.

Additional assumptions. Let $V(\theta, r, A) \equiv W(\theta, r, A) - L(\theta, r)$ denote the net value of defending the peg relative to abandoning it. To ease the exposition, and without serious loss of generality, we assume that r enters symmetrically into W and L, so that V is independent of r.[5] Accordingly, we henceforth express V as $V(\theta, A)$. Note that $V_\theta > 0 > V_A$ by our assumptions that $W_\theta > L_\theta$ and $W_A < 0$. We next introduce dominance regions by assuming that there exist thresholds $\underline{\theta}$ and $\bar{\theta}$, with $\underline{\theta} < \bar{\theta}$, such that $V(\underline{\theta}, 0) = V(\bar{\theta}, 1) = 0$ and $Z(\theta, r) > r$ for all (θ, r) such that $\theta \leq \bar{\theta}$ and r is not strictly dominated for type θ.[6] These assumptions identify the interval $[\underline{\theta}, \bar{\theta}]$ with the "critical region" where multiple equilibria are possible under complete information about θ; for $\theta < \underline{\theta}$, the peg is instead abandoned even if the measure of speculators attacking is negligible, whereas for $\theta > \bar{\theta}$ the peg survives no matter the size of the attack.

We finally assume that the policy maker's payoff satisfies the following monotonicity and limit conditions, for all $r > \underline{r}$. First, $L(\underline{r}, \theta) > L(r, \theta)$ for all θ. Second, the difference $W(\theta, r, 0) - L(\underline{r}, \theta)$ is strictly increasing in θ over $[\underline{\theta}, +\infty)$. Third, $\lim_{\theta \to +\infty}[W(\theta, r, 0) - W(\theta, \underline{r}, 1)] < 0$. The first condition identifies $\underline{r}$ with the "cost-minimizing" policy that the policy maker would choose if he were not interested in maintaining the peg; this will also coincide with the policy sustained in any of the pooling equilibria. The second condition is an increasing-difference condition akin to the role played by the single crossing condition in signaling games: whenever setting $\underline{r}$ leads to devaluation while raising the policy at r spares the currency from an attack, then higher types have a stronger incentive to raise the policy than lower types. The third condition guarantees that intervention is never optimal for extremely high types. This condition plays a similar role in our setting as to that of dominance regions in standard global games: it permits us to initiate a contagion argument that compresses the set of possible equilibrium outcomes, thus facilitating predictions.[7] Lastly, we assume that, for any $\theta \geq \underline{\theta}$, $\lim_{r \to +\infty} W(\theta, r, 0) < L(\theta, \underline{r})$ which guarantees that it is too costly for the policy maker to raise r to a level that would make it dominant for the speculators not to attack. Although quite realistic, this assumption can be relaxed without any serious impact on the results.[8]

[5] Relaxing this assumption does not affect the essence of the results. However, it complicates the exposition by making the "critical range" of θ for which multiple equilibria are possible under complete information change with r. We thus opted to make the assumption that V is independent of r.

[6] Note that r is not strictly dominated for $\theta \leq \bar{\theta}$ if and only if $r \leq \rho(\theta)$ where $\rho(\theta)$ is the unique solution to $W(\theta, \rho(\theta), 0) = L(\underline{r}, \theta)$.

[7] As in standard global games, such a contagion argument is possible even if the possibility of being in the dominance regions is arbitrarily remote. When applied to our setting, this means that the threshold above which interventions become dominated can be assumed to diverge to infinity. In any event, the role of this limit assumption is only to guarantee that sufficiently strong types do not intervene, either because they expect the market to recognize that they are strong or because part of being "strong" means being committed to the least distortionary policies ($r = \underline{r}$). These properties seem natural and, as explained in due course, they obtain also under alternative reasonable refinements.

[8] This is because, along any equilibrium, it is always the case that raising the policy deters attacking, irrespectively

Timing and information. The game evolves through two phases. In the first phase, the policy maker attempts to preempt a speculative attack by raising r. In the second phase, speculators decide whether or not to attack and the policy maker decides whether or not to defend the currency against the attack. More specifically, in the first phase, the policy maker sets the policy r after learning θ. In the second phase, speculators decide simultaneously whether or not to attack, after observing the policy r, and after receiving private signals $x_i = \theta + \sigma\xi_i$ about θ; the scalar $\sigma \in (0, \infty)$ parameterizes the quality of the speculators' information, while ξ_i is noise, i.i.d. across speculators and independent of θ, with a continuous probability density function ψ strictly positive and differentiable over the entire real line, with corresponding cumulative distribution function Ψ.[9] The common prior about θ is uniform over the entire real line.[10] After observing the mass of speculators who decided to attack, the policy maker then decides whether or not to incur the cost of defending the peg.

Remarks. A delicate aspect of our specification of the policy maker's payoff is the assumption that L is independent of A. As it will become clear in due course, our predictions appear to hinge only on the assumption that the costs of an attack are smaller when the policy maker opts to abandon than when the policy maker opts to defend. This property seems natural given that, if a policy maker abandons, he is likely to do so well before the attack has had the chance to build all the force that it would have had should the policy maker have opted for keep defending.[11] Modeling the precise dynamics of the decision to stop defending is beyond the scope of this paper. Furthermore, because the analysis becomes significantly less tractable once we allow L to depend on A, we opt here for the more extreme assumption of imposing that L is independent of A.

Apart from this restriction, all other properties of the policy maker's payoff seem quite natural. Furthermore, the above specification is flexible enough to facilitate multiple interpretations of what the variables θ and r may stand for. For example, consider the following special case: $L(\theta, r) = \beta l(\theta) - C(r, \theta)$ and $W(\theta, r, A) = w(\theta) - C(r, \theta) - K(A, \theta)$. In this case, one can interpret C as the short-run cost of a temporary increase in domestic interest rates or taxes on capital flaws, K as the cost of depleting reserves or otherwise defending the peg against an attack of size A, β as the discount factor, and $w - l$ as the reputational, or continuation, value from maintaining the peg. Alternatively, depending on how one decomposes W and L between "current" and "future" values, one may interpret r as a reform that "improves the fundamentals". For example, one could let $L(\theta, r) = -C(r, \theta) + \beta B(r, \theta)$ and $W(\theta, r, A) = L(\theta, r) + \Delta(\theta, A)$, in which case C is interpreted as the present cost of the reform, B as its future benefit, Δ as the net benefit of defending the peg against a realized attack of size A, and θ as a generic exogenous fundamental that affects all the aforementioned costs and benefits.

of the impact of r on the speculators' payoff.

[9] To simplify the exposition, throughout we let $\Psi(-\infty) = 0$ and $\Psi(+\infty) = 1$.

[10] As in the rest of the literature, this improper prior is used only for convenience; the results generalize to any bounded smooth prior as long as σ, the noise in the speculators' private signals, is small enough.

[11] See also Drazen (2000) for a similar assumption and a more detailed justification.

While fairly rich in some dimensions, our framework remains highly stylized in other. For example, it does not consider the possibility that certain speculators may be "large" either in their measure or in their information and hence able to manipulate the behavior of other "smaller" speculators (Corsetti, Dasgupta, Morris and Shin, 2004). It also abstracts from many dynamic considerations that are likely to be important in currency crises such as the possibility that the speculators learn from failed past attacks (Chamley, 2004; Angeletos, Hellwig, and Pavan, 2007), or the possibility that the policy maker himself may learn from the dynamics of the attack and chooses optimally the time at which to abandon the peg (Goldstein, Ozdenoren, and Yuan, 2011). While we do expect certain features of the results described below to be affected by such dimensions, we do not expect the qualitative essence of our predictions to be overturned by their introduction.

3 Equilibrium characterization

This section characterizes the equilibrium set. We first use a procedure of iterated deletion of strategies that cannot be part of an equilibrium to obtain tighter and tighter bounds on the equilibrium set. These bounds, which are presented in subsection 3.2 and formally derived in subsection 3.3, play a central role in our analysis: they contain the equilibrium set; they rule out a large set of strategy profiles, including many of those that could have been equilibrium profiles under complete information; and they drive the core predictions we present in Section 4. Yet, these bounds need not always be the sharpest: in general, we cannot rule out the possibility that the equilibrium set is strictly smaller than the set identified by these bounds. We eliminate this ambiguity in subsection 3.4 for the case that the policy maker's payoff satisfies a natural single-crossing property.

3.1 Equilibrium definition

Our solution concept is Perfect Bayesian Equilibrium. All the analysis in this section refers to the case where $\sigma > 0$. To simplify the notation, we drop the dependence of the equilibrium variables on σ whenever highlighting such dependence is unnecessary.

Let $r(\theta)$ denote the policy chosen by type θ, $\mu(\theta|x,r)$ the cumulative distribution function of the posterior belief of a speculator who receives a signal x and observes a policy r, $a(x,r)$ the action of that speculator, and $A(\theta,r)$ the corresponding aggregate size of attack. Clearly, the peg is maintained if and only if $V(\theta,A) \geq 0$. The devaluation outcome and the policy maker's payoff can thus be expressed as follows:

$$D(\theta,A) = \begin{cases} 0 & \text{if } V(\theta,A) \geq 0 \\ 1 & \text{otherwise} \end{cases} \qquad \text{and} \qquad U(\theta,r,A) = L(\theta,r) + \max\{0, V(\theta,A)\}$$

The equilibrium definition can then be stated as follows.

Definition. *An equilibrium consists of a strategy for the policy maker,* $r : \mathbb{R} \to [\underline{r}, +\infty)$*, a posterior belief for the speculators,* $\mu : \mathbb{R}^2 \times [\underline{r}, +\infty) \to [0,1]$*, a strategy for the speculators,* $a : \mathbb{R} \times [\underline{r}, +\infty) \to \{0,1\}$*, and an aggregate attack function,* $A : \mathbb{R} \times [\underline{r}, +\infty) \to [0,1]$*, such that*

$$r(\theta) \in \arg\max_{r} U(\theta, r, A(\theta, r)) \; \forall \theta \tag{1}$$

$$\mu(\theta|x, r) \text{ is obtained from Bayes' rule using } r(\cdot) \text{ for any } x \in \mathbb{R} \text{ and any } r \in r(\mathbb{R}) \tag{2}$$

$$a(x,r) \in \arg\max_{a \in \{0,1\}} a \left[\int_{-\infty}^{+\infty} Z(\theta, r) D(\theta, A(\theta, r)) d\mu(\theta|x, r) - r \right] \; \forall (x, r) \tag{3}$$

$$A(\theta, r) = \int_{-\infty}^{+\infty} a(x, r) \tfrac{1}{\sigma} \psi\left(\tfrac{x-\theta}{\sigma}\right) dx \; \forall (\theta, r) \tag{4}$$

where $r(\mathbb{R}) \equiv \{r : r = r(\theta), \theta \in \mathbb{R}\}$ *is the set of policy interventions that are played in equilibrium. The equilibrium devaluation outcome is* $D(\theta) \equiv D(\theta, A(\theta, r(\theta)))$*.*

Conditions (1) and (3) require that the policy maker's and the speculators' actions be sequentially rational. Condition (4) requires that the aggregate size of attack is the one that obtains by aggregating the strategy of the speculators. Finally, condition (2) requires that, on the equilibrium path, the speculators' beliefs be pinned down by Bayes' rule.

3.2 Bounds

Before we can state our characterization results, we need to introduce some additional notation. Let $\mathcal{E}(\sigma)$ denote the set of all possible equilibria in the game with quality of information σ. Next, for any $s \geq \underline{r}$, let $\mathcal{E}(s; \sigma)$ denote the set of equilibria in which $r(\theta) \in \{\underline{r}, s\}$ for all θ, meaning that the policy takes either the cost-minimizing value $\underline{r}$ or the value s. For any (θ_1, θ_2) with $\theta_2 \geq \theta_1$, let $X(\theta_1, \theta_2; \sigma)$ be the unique solution to

$$\frac{\int_{-\infty}^{\theta_1} Z\left(\tilde{\theta}, \underline{r}\right) \frac{1}{\sigma} \psi\left(\frac{x-\tilde{\theta}}{\sigma}\right) d\tilde{\theta}}{1 - \Psi(\frac{x-\theta_1}{\sigma}) + \Psi(\frac{x-\theta_2}{\sigma})} = \underline{r}. \tag{5}$$

This threshold identifies the value of the signal x at which a speculator who believes that $\theta \notin [\theta_1, \theta_2]$ and that devaluation occurs if and only if $\theta < \theta_1$ is indifferent between attacking and not attacking. Finally, let

$$B(\theta_1, \theta_2; \sigma) \equiv \Psi\left(\frac{X(\theta_1, \theta_2; \sigma) - \theta_2}{\sigma}\right) \tag{6}$$

denote the aggregate size of attack that obtains when the policy maker's type is θ_2 and speculators attack if and only if $x < X(\theta_1, \theta_2; \sigma)$.

With this notation at hand, we can now state our first two results, which identify key properties of the equilibrium set that hold true either for all $\sigma > 0$ (Proposition 1) or for $\sigma > 0$ small enough (Proposition 2).

Proposition 1 (necessary conditions). *The following properties are true for any $\sigma > 0$:*

(i) $\mathcal{E}(\sigma) = \cup_{s \geq \underline{r}} \mathcal{E}(s;\sigma)$.

(ii) $\mathcal{E}(\underline{r};\sigma) \neq \varnothing$.

(iii) *Any equilibrium in* $\mathcal{E}(\underline{r};\sigma)$ *is such that*

$$r(\theta) = \underline{r} \quad \text{for all } \theta \quad \text{and} \quad D(\theta) = \begin{cases} 1 & \text{for } \theta < \theta^{\#}(\sigma) \\ 0 & \text{for } \theta > \theta^{\#}(\sigma) \end{cases},$$

where $\theta^{\#}(\sigma)$ *is the unique solution to*

$$W\left(\theta^{\#}(\sigma), \underline{r}, B(\theta^{\#}(\sigma), \theta^{\#}(\sigma);\sigma)\right) = L\left(\theta^{\#}(\sigma), \underline{r}\right). \tag{7}$$

(iv) *For any* $s > \underline{r}$, $\mathcal{E}(s;\sigma) \neq \varnothing$ *only if there exists a pair of thresholds* (θ_s^*, θ_s'') *with* $\theta_s'' \geq \theta_s^*$ *that solves the following two equations:*

$$W(\theta_s^*, s, 0) = L(\theta_s^*, \underline{r}) \tag{8}$$

$$W(\theta_s'', s, 0) = W(\theta_s'', \underline{r}, B(\theta_s^*, \theta_s'';\sigma)) \tag{9}$$

(v) *Take any* $s > \underline{r}$, *suppose* $\mathcal{E}(s;\sigma) \neq \varnothing$, *and let* $\theta_s^{**}(\sigma)$ *denote the highest solution to (9). Any equilibrium in* $\mathcal{E}(s;\sigma)$ *is such that*

$$r(\theta) = s \quad \text{only if } \theta \in [\theta_s^*, \theta_s^{**}(\sigma))] \quad \text{and} \quad D(\theta) = \begin{cases} 1 & \text{for } \theta < \min\{\theta_s^*, \theta^{\#}(\sigma)\} \\ 0 & \text{for } \theta > \theta_s^*. \end{cases}$$

Furthermore, $\theta_s^* < \theta^{\#}(\sigma)$ *if and only if* $s < r^{\#}(\sigma)$, *where* $r^{\#}(\sigma)$ *is the unique solution to*

$$W\left(\theta^{\#}(\sigma), r^{\#}(\sigma), 0\right) = L\left(\theta^{\#}(\sigma), \underline{r}\right).$$

Part (i) establishes that, in any equilibrium, either the policy is left at $\underline{r}$ by all θ, or it is raised at the same level $s > \underline{r}$ by all types who raise the policy above $\underline{r}$.

Parts (ii) and (iii) refer to the set of equilibria in which all types pool on $r = \underline{r}$. In particular, part (ii) establishes that this set is non-empty, while Part (iii) identifies a unique threshold $\theta^{\#}$ such that, in any such equilibrium, the peg is abandoned if and only if $\theta < \theta^{\#}$.[12] As shown in Condition (7), this threshold is defined by the policy maker's indifference between abandoning the peg and defending it against an attack of size $A = B(\theta^{\#}, \theta^{\#};\sigma)$, which in turn is the attack that obtains when speculators expect the policy maker never to raise the policy and the peg to be abandoned if and only if $\theta < \theta^{\#}$.[13]

[12]To be precise, the policy maker is indifferent between abandoning and not abandoning when $\theta = \theta^{\#}$. While our formal results take care of such indifferences/indeterminacies, our informal discussion often ignores them for the sake of expositional simplicity. Also, our discussion in the main text suppresses the dependence of $\theta^{\#}$, $r^{\#}$, and θ^{**} on σ to simplify the exposition, although this dependence remains explicit in the statement of the formal results.

[13]Not surprisingly, this threshold coincides with the one that obtains in a variation of our game that restricts the policy to $r \in \{\underline{r}\}$ for all θ, which is essentially the game studied in Morris and Shin (1998).

Next, consider the subset of equilibria in which some type raises the policy at s, for some $s > \underline{r}$. Part (iv) identifies necessary conditions for such an equilibrium to exist. Part (v) in turn establishes that, in any such equilibrium, there exists a pair of thresholds θ_s^* and θ_s^{**} such that (a) the policy is raised at s only if $\theta \in [\theta_s^*, \theta_s^{**}]$, (b) devaluation never occurs for $\theta > \theta_s^*$ and (c) devaluation always occurs for $\theta < \min\{\theta_s^*, \theta^{\#}\}$. As evident from Condition (8), θ_s^* is defined by the policy maker's indifference between leaving the policy at $\underline{r}$ and then devaluing, on the one hand, and raising the policy at s and eliminating the risk of an attack, on the other hand. As evident from Condition (9), θ_s^{**} is in turn defined by the policy maker's indifference between preempting a speculative attack by raising the policy at s, on the one hand, and not raising the policy and then incurring the cost of defending against an attack of size $A = B(\theta^{\#}, \theta^{\#}; \sigma)$, on the other hand. Finally, the function X defined in condition (5) identifies a tight upper bound on the set of signals for which a speculator may attack when he observes no intervention and expects intervention to occur only for $\theta \in [\theta_s^*, \theta_s^{**}]$.

When $\theta_s^* \leq \theta^{\#}$, which is the case if and only if s is low enough, then any equilibrium in $\mathcal{E}(s; \sigma)$ shares the same devaluation outcome: the peg is abandoned if and only if $\theta < \theta_s^*$. When instead $\theta_s^* > \theta^{\#}$, then the results in the proposition do not rule out the possibility that certain types in $[\theta^{\#}, \theta_s^*]$ retain the peg even without raising the policy at s. As the next proposition makes clear, this possibility disappears when the noise in the speculators' signals is small.

Proposition 2 (necessary conditions for small σ). *For any $\varepsilon > 0$, there exists $\bar{\sigma} > 0$ such that the following results hold for any $\sigma < \bar{\sigma}$:*

(i) *$\mathcal{E}(s; \sigma) \neq \varnothing$ only if $s \leq r^{\#}(\sigma) + \varepsilon$.*

(ii) *whenever $\mathcal{E}(s; \sigma) \neq \varnothing$ then $\theta_s^* \leq \theta^{\#}(\sigma) + \varepsilon$*

(iii) *whenever $\mathcal{E}(s; \sigma) \neq \varnothing$ and $s \geq \underline{r} + \varepsilon$ then $\theta_s^{**}(\sigma) \leq \theta_s^* + \varepsilon$.*

Part (i) establishes that the possibility of observing equilibrium interventions above $r^{\#}$ vanishes as the quality of information becomes high enough: in the limit as $\sigma \to 0^+$, there is no equilibrium in which the policy is raised above $r^{\#}$. Part (ii) in turn establishes that the region of fundamentals for which the devaluation outcome is potentially indeterminate disappears as the noise vanishes: as $\sigma \to 0^+$, any equilibrium in which the policy is raised at some s by some type θ is such that devaluation occurs *if and only if* $\theta \leq \theta_s^*$, with $\theta_s^* \leq \theta^{\#}$. Finally, part (iii) establishes that, as the noise vanishes, then for any $s \in (\underline{r}, r^{\#}]$, the range of fundamentals $[\theta_s^*, \theta_s^{**}]$ for which intervention is possible also vanishes. To understand this last result, note that, for $\theta = \theta_s^*$, the opportunity cost of raising the policy at s is devaluing, which explains why θ_s^* is independent of σ. Instead, for $\theta = \theta_s^{**}$, the opportunity cost is facing a positive attack whose measure however vanishes as $\sigma \to 0^+$; by implication, in the limit as $\sigma \to 0^+$, for any $\theta > \theta_s^*$ the cost of raising the policy at s outweights the cost of setting $r = \underline{r}$ and facing a small attack, which in turn implies that θ_s^{**} must necessarily converge to θ_s^* as $\sigma \to 0^+$.

3.3 Characterization procedure

We now expand on the key arguments behind Proposition 1. As anticipated, these arguments deal with the interaction of two forces: certain contagion effects that emerge because, and only because, of the incompleteness of information and that are akin to those in standard global games; and the endogeneity of information originating in the signaling role of policy interventions.

Our first argument considers the set of equilibria in which all types pool on $\underline{r}$ and establishes parts (ii) and (iii) of the proposition.

Lemma 1. *$\mathcal{E}(\underline{r};\sigma) \neq \varnothing$. Furthermore, there exist a unique threshold $\theta^{\#}(\sigma) \in (\underline{\theta},\bar{\theta})$ such that, in any equilibrium in $\mathcal{E}(\underline{r};\sigma)$, $D(\theta) = 1$ if $\theta < \theta^{\#}(\sigma)$ and $D(\theta) = 0$ if $\theta > \theta^{\#}(\sigma)$.*

Because in any equilibrium in which all types pool at $\underline{r}$ the observation of $\underline{r}$ conveys no information, the continuation game that follows the observation of $\underline{r}$ is essentially a standard global game. The lemma then follows from arguments similar to those in Morris and Shin (1998, 2003), adapted to the environment considered here. (See the Appendix for details.)

The next two arguments focus on equilibria in which some types seek to preempt a speculative attack by raising the policy above $\underline{r}$.

Lemma 2. *In any equilibrium in which some type intervenes, there exists a single $s > \underline{r}$ such that $r(\theta) = s$ whenever $r(\theta) \neq \underline{r}$. Furthermore, for that s, $A(\theta, s) = 0$ for all θ.*

Lemma 3. *For any $s > \underline{r}$, $\mathcal{E}(s;\sigma) \neq \varnothing$ only if equation (8) admits a solution. Furthermore, whenever $\mathcal{E}(s;\sigma) \neq \varnothing$, any equilibrium in $\mathcal{E}(s;\sigma)$ is such that $D(\theta) = 0$ for all $\theta > \theta_s^*$, where θ_s^* is the unique solution to (8).*

The key insights behind these two lemmas are simple. First, a type will find it optimal to intervene only if he expects the intervention to reduce the size of the attack. Second, if a certain type avoids devaluation by raising r, then any higher type must also be avoiding devaluation in equilibrium—for any higher type can always "imitate" any lower type and do at least as well.

These properties are intuitive. Clearly, there must be *some* benefit, in the form of a reduction in the size of the attack, to justify the cost of preemptive policy intervention. Our model captures this benefit in a stark way, by guaranteeing that *no* attack takes place, and the peg is maintained with probability one, following any (equilibrium) intervention. This is because we have ruled out any source of aggregate uncertainty in the size of the attack and in the devaluation outcome beyond the one introduced by the random type of the policy maker. In the absence of such additional aggregate uncertainty, equilibrium policy interventions necessarily signal to the market that devaluation will not occur (for, otherwise, the policy maker would be better off by not intervening and then devaluing). This in turn implies that, in equilibrium, any type who intervenes does so by selecting the least costly policy among those that are conductive to no devaluation, as stated in Lemma 2.

Additional aggregate uncertainty could have obtained if we had allowed for shocks to fundamentals or unpredictable shifts in the "sentiment" of some irrational speculators to occur after the policy maker has set the policy. While the introduction of such additional uncertainty may deliver a smoother relationship between the probability the peg is retained and the level of policy intervention, we do not expect our results to be unduly sensitive to our choice of abstracting from such additional uncertainty. For example, it is easy to show that our results are robust to the introduction of an exogenous random event that triggers devaluation independently of r, which one could then interpret either as the result of unfavorable changes in the fundamentals or as the impact of a severe attack by "noise traders" or "irrational speculators". Clearly, the same remains true if the probability of this "exogenous" event is decreasing in r. While it could be worthwhile to extend our analysis to more general, and more realistic, sources of aggregate uncertainty, this is a challenging task that we decided to bypass in the present paper.

One should also not misinterpret Lemma 2 as saying that there is a single level of policy intervention played in equilibrium. Rather, as established in Propositions 3 and 4 below, there is a continuum of policy levels $s > \underline{r}$ that can be played in equilibrium. Furthermore, the same type θ may be playing different r in different equilibria, just as the same r may be played by different θ. Lemma 2 thus has very little positive content on its own right. It nevertheless helps us index the equilibrium set in a convenient way: any equilibrium that does not belong to $\mathcal{E}(\underline{r};\sigma)$ necessarily belongs to $\mathcal{E}(s;\sigma)$ for some $s > \underline{r}$, thus establishing part (i) of Proposition 1.

Finally, Lemma 3 guarantees that the threshold θ_s^* is an upper bound for the set of types who abandon the peg across all equilibria in $\mathcal{E}(s;\sigma)$. This is because types above this threshold could always maintain the peg by raising the policy at s and then face no attack. If they don't do so in equilibrium, it is because they expect an even higher payoff by saving on the cost of policy interventions and instead paying the cost of defending the peg against an attack of small size. Clearly, θ_s^* is also a lower bound on the set of types who possibly raise the policy at s. For any type below this threshold, raising the policy at $r = s$ is a dominated strategy.

Moving on, the next lemma helps us identify the threshold θ_s^{**} as an upper bound to the set of types who potentially raise the policy at $r = s$.

Lemma 4. *Take any $s > \underline{r}$ for which equation (8) admits a solution and denote by θ_s^* its unique solution. A necessary condition for $\mathcal{E}(s;\sigma) \neq \varnothing$ is that equation (9) admits a solution $\theta_s'' \geq \theta_s^*$. Furthermore, any equilibrium in $\mathcal{E}(s;\sigma)$ is such that $r(\theta) = s$ only if $\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]$ where $\theta_s^{**}(\sigma)$ is the* highest *solution to equation (9).*

The proof of this lemma is based on an argument of iterated deletion of strategies that cannot be part of an equilibrium. This argument is reminiscent of the contagion effects that are paramount in the global-games literature. In our setting, it encapsulates a contagion effect from very high types, for whom raising the policy is dominated, to lower types, who are spared from the need to raise the policy thanks, and only thanks, to the incompleteness and dispersion of information among the speculators.

In particular, the fact that both raising the policy and devaluing are dominated strategies for sufficiently high types implies that speculators, on their part, find it iteratively dominant not to attack for sufficiently high signals, conditional on observing no policy intervention.[14] The dispersion of information then initiates a contagion effect that triggers speculators not to attack for lower and lower signals and hence spares the policy maker from the need to raise the policy for lower and lower θ. In the limit, this contagion effect converges to θ_s^{**}, guaranteeing that all types above this threshold are able to maintain the peg without the need for costly preemptive policy interventions. Underscoring the power of this contagion effect, the limit threshold θ_s^{**} is arbitrarily close to θ_s^* when σ is small enough, meaning that almost all types for whom policy intervention is not dominated succeed to maintain the peg without the need for costly preemptive policies. This is despite the fact that the aforementioned contagion effect is initiated with types that can be arbitrarily high.

The preceding lemma used an iteration "from above" to identify a necessary condition for existence of equilibria in which the policy is raised at $r = s > \underline{r}$ and identified an upper bound θ_s^{**} for the set of types who possibly raise the policy at $r = s$. The next lemma uses an iteration "from below" to rule out equilibria in which the peg is maintained for $\theta < \min\{\theta_s^*, \theta^\#\}$.

Lemma 5. *Take any $s > \underline{r}$ and suppose that $\mathcal{E}(s;\sigma) \neq \varnothing$. Then any equilibrium in $\mathcal{E}(s;\sigma)$ is such that $D(\theta) = 1$ for any $\theta < \min\{\theta_s^*, \theta^\#(\sigma)\}$. Furthermore, $\theta_s^* > \theta^\#(\sigma)$ if and only if $s > r^\#(\sigma)$, where $r^\#(\sigma)$ is the unique solution to*

$$W\left(\theta^\#(\sigma), r^\#(\sigma), 0\right) = L(\theta^\#(\sigma), \underline{r}).$$

As illustrated in the Appendix, this result also originates in a contagion effect that is present because, and only because, of the dispersion of information. In particular, the fact that it is dominated for sufficiently low types either to raise the policy or to maintain the peg implies that speculators find it iteratively dominant to attack for sufficiently low signals as long as they do not observe policy intervention. The dispersion of information then initiates a contagion effect such that, conditional on seeing no intervention, speculators find it iteratively dominant to attack for

[14] That raising the policy to $r = s$ is dominated for arbitrarily high types follows from the assumption that $\lim_{\theta\to+\infty}[W(\theta, s, 0) - L(\theta, \underline{r})] < 0$. Without an assumption of this sort, there may exist equilibria in which the policy maker intervenes even for arbitrarily high types. These equilibria are sustained either by the assumption that the cost of intervention vanishes for sufficiently high types, or by the speculators threatening to attack no matter how favorable their signal is when the policy maker fails to intervene. We find either property implausible. Also note that equilibria in which the speculators attack no matter their signal when $r < s$ are not robust to the following perturbation. Pick any $K > \bar{\theta}$ and any $\delta > 0$ and suppose that with probability δ types $\theta > K$ are forced to set $\underline{r}$ and assume that this event is not observed by the speculators. The aforementioned equilibria are not robust to this perturbation, no matter how unlikely this event is (i.e. no matter δ) and no matter how big K is. Instead of invoking such a refinement, we prefer to impose the aforementioned limit condition.

higher and higher signals, making devaluation iteratively dominant for higher and higher θ. In the limit, this contagion effect guarantees that all types below $\min\{\theta_s^*, \theta^\#\}$ necessarily devalue in any equilibrium in which the policy is raised at s. This last result is obtained by comparing the speculators' incentives to attack after observing $\underline{r}$ with the corresponding incentives when they expect $r(\theta) = \underline{r}$ for all θ. Because the observation of $\underline{r}$ is most informative of devaluation when all types who devalue set $r = \underline{r}$, while some of the types who maintain the peg raise the policy above $\underline{r}$, the size of attack when setting $r = \underline{r}$ is necessarily larger in any of the equilibria in which some types are expected to raise the policy at $r = s$ than in the pooling equilibria where all types are expected to set $r = \underline{r}$. Hence any type $\theta < \theta^\#$ who does not raise the policy at $r = s$ necessarily devalues in equilibrium. Because raising the policy at $r = s$ is dominated for all $\theta < \theta_s^*$, this implies that $D(\theta) = 1$ *for any* $\theta < \min\{\theta_s^*, \theta^\#\}$.[15]

The combination of Lemmas 1 through 5 establishes the results in Proposition 1. The proof of Proposition 2, which is more technical, is relegated to the Appendix. The difficulty in proving that proposition comes from the need to establish uniform convergence results that permit us to verify that the claims in parts (i)-(iii) in the proposition hold uniformly across all possible equilibria.

3.4 Complete characterization

Away from the limit where $\sigma \to 0^+$, the results in Propositions 1 and 2 leave open the possibility of equilibria in which the policy is raised only for a subset of the interval $[\theta_s^*, \theta_s^{**}]$ and for which devaluation may occur also for a subset of $[\theta^\#, \theta_s^*]$ in case that $\mathcal{E}(s, \sigma) \neq$ for $s > r^\#$. This vagueness does not hinder any of the predictions that we will make in the subsequent sections. However, it can be dispensed with under a single-crossing assumption on the policy maker's payoff.

Single-Crossing Condition (SCC). *Let*

$$\Delta W(\theta; r, A) \equiv W(\theta, r, 0) - W(\theta, \underline{r}, A)$$

For any $A > 0$ and $r > \underline{r}$, either $\Delta W(\theta; r, A) \leq 0$ for all $\theta \geq \underline{\theta}$, or there exists a $\theta^+(r, A) > \underline{\theta}$ such that $\Delta W(\theta; r, A) < 0$ if and only if $\theta > \theta^+(A, r)$. That is, $\Delta W(\theta; r, A)$ changes sign at most once as θ increases from $\underline{\theta}$ to $+\infty$.

This condition has a simple interpretation. Note that $\Delta W(\theta; r, A)$ identifies the payoff differential between defending the peg by raising the policy to $r > \underline{r}$ and facing no attack and defending it by leaving the policy at $\underline{r}$ and suffering the cost of an attack of size A, when the policy maker's type is θ. The condition then imposes that, for any given potential size of attack A and any policy

[15] That $\theta_s^* < \theta^\#$ if and only if $s < r^\#$ follows from the monotonicity of $W(\theta, s, 0) - L(\theta, \underline{r})$ in θ along with $W_r < 0$.

$r > \underline{r}$, higher types prefer defending the peg by suffering the cost of the attack while lower types by raising the policy.[16]

As mentioned above, this property is not essential for our key results. Nonetheless, it permits us to complement the above predictions with the following results.

Proposition 3 (**complete characterization**). *Suppose SCC holds. Then, the following are true for any $\sigma > 0$:*

(i) *$\mathcal{E}(s;\sigma) \neq \varnothing$ if and only if $s \leq r^{\#}(\sigma)$.*

(ii) *For any $s \in (\underline{r}, r^{\#}(\sigma)]$, there exists an equilibrium in $\mathcal{E}(s;\sigma)$ in which $r(\theta) = s$ for all $\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]$, where $\theta_s^{**}(\sigma)$ is the highest solution to (9).*

These results complement Propositions 1 and 2 in three ways. First, the preceding results ruled out equilibria with $s > r^{\#}$ only in the case where σ is small enough; part (i) of the above proposition rules them out for all σ. Second, the preceding results left open the possibility that some equilibria with $s \leq r^{\#}$ fail to exist; part (i) guarantees that such equilibria always exist. Finally, the preceding results identified θ_s^* and θ_s^{**} as, respectively, lower and upper bounds of the range of policy intervention, but left open the possibility that the range of policy intervention is strictly smaller than the interval $[\theta_s^*, \theta_s^{**}]$; part (ii) shows that these bounds are sharp in the sense that there always exists an equilibrium in which the set of policy intervention is exactly equal to this interval.

The aforementioned results rest on SCC. The role played by SCC is simple: it guarantees that, whenever the speculators follow a strategy that is decreasing in their signals, then, conditional on the peg been maintained, the policy maker finds it optimal to raise the policy if and only if θ is not too high. This in turn helps constructing equilibria in which the policy maker raises the policy if and only if θ is between the thresholds θ_s^* and θ_s^{**} of Proposition 1, thus identifying θ_s^* and θ_s^{**} as sharp bounds for the set of types who possibly raise the policy at $r = s$ in equilibrium.

The following proposition then further sharpens our characterization for the case that the distribution of the noise in the speculators' signals is log-concave—a restriction that helps guaranteeing that, along any equilibrium, the speculators' posteriors about θ are monotone in their signals.

Proposition 4 (monotonicity of speculators' behavior). ***(i)*** *Suppose the noise distribution ψ is log-concave. Then, for any $\sigma > 0$, any $s \leq r^{\#}(\sigma)$, $\mathcal{E}(s;\sigma) \neq \varnothing$.*

(ii) *Suppose SCC holds and ψ is log-concave. Then for any $\sigma > 0$, any $s \in (\underline{r}, r^{\#}(\sigma)]$, any equilibrium in $\mathcal{E}(s;\sigma)$ is such that $r(\theta) = s$ for all $\theta \in (\theta_s^*, \theta_s^{**}(\sigma))$.*

[16]For example, when $W(\theta, r, A) = -C(r, \theta) - K(A, \theta) + \beta w(\theta)$, with $C(\underline{r}, \theta) = K(0, \theta) = 0$ all θ, this condition is satisfied if $K_\theta(A, \theta) < C_\theta(r, \theta)$ meaning that the cost of defending against an attack of size A decreases with θ faster than the cost of raising the policy at r, in which case the function $\Delta W(\theta; r, A)$ is nonincreasing in θ. More generally, SCC only requires that the net benefit of reducing the size of the attack changes sign once, thus reinforcing the assumption we made in the model set-up that $\Delta W(\theta, r, A)$ be negative in the limit as θ grows large enough.

Part (i) in the above proposition establishes that log-concavity of ψ is, by itself, another sufficient condition for the existence of equilibria in which the policy maker raises the policy as s, for any $s \leq r^{\#}$.[17] Part (ii) then establishes that the combination of log-concavity of ψ with SSC suffice for ruling out equilibria in which the policy is raised for only a subset of the interval $[\theta_s^*, \theta_s^{**}]$, which further sharpens the interpretation of θ_s^{**}.

As mentioned already, none of the properties in Propositions 3 and 4 are strictly needed: the key predictions we will make in the subsequent analysis rest only on the bounds identified by Propositions 1 and 2. However, the appeal of Propositions 3 and 4 is that they provide sharper interpretations of these bounds, thereby easing the exposition.[18]

Remarks. The equilibrium definition we have used rules out mixed strategies for either the policy maker or the speculators; it also imposes symmetry on the speculators' strategies. However, from the arguments in the proofs of Lemmas 1-5, it should be clear that none of the conditions identified in these lemmas depends on these restrictions. Indeed, the policy maker can find it optimal to randomize over r, or over D, only for a zero-measure subset of θ; because this does not have any effect on the speculators' posterior beliefs about policy and devaluation outcomes, it cannot affect their best-responses. Similarly, for any r, the speculators can find it optimal to randomize over their decision to attack, or to play asymmetrically, only for a zero-measure subset of their signal space; because this does not have any effect on the aggregate size of attack, it does not impact the policy maker's incentives. Propositions 1 through 4 thus identify properties of *all* equilibrium outcomes, including those sustained by mixed-strategy or asymmetric-strategy profiles.

4 Predictions about policies and devaluation outcomes

We now show how the equilibrium properties identified in the previous section lead to a set of concrete testable predictions about policy choices and devaluation outcomes for an outside observer, say an econometrician, who does not need to know which particular equilibrium is played. We start by describing the uncertainty about the equilibrium being played by means of a generic distribution

[17] The role of log-concavity is to guarantee that, irrespective of the shape of the policy r in the region $[\theta_s^*, \theta_s^{**}]$, the probability that each speculator assigns to devaluation when observing no intervention is necessarily decreasing in the signal x. This in turn implies that the size of the attack $A(\theta, \underline{r})$ that the policy maker expects when he does not intervene is necessarily decreasing in θ. As we show in the Appendix, under such monotonicities, one can always construct equilibria in which intervention occurs for a (possibly non-connected) subset of $[\theta_s^*, \theta_s^{**}]$.

[18] We also note that the equilibria constructed in the Appendix to establish the existence results in Propositions 3 and 4 have the property that each speculator attacks if and only if $(x, r) < (x_s^*, s)$, for some x_s^*. These strategies are particularly simple and permit one to interpret s as the level of policy at which the speculators switch from "aggressive" to "lenient" behavior. Although other strategies can also be part of an equilibrium, the aforementioned ones—with the corresponding beliefs described in the Appendix—have the appealing property of passing the intuitive criterion test of Cho and Kreps (1987) and can be obtained as the limit to perturbations that introduce full-support noise in policy observations so that beliefs are always pinned down by Bayes' rule.

over the set of all possible equilibria and then examine the implied distribution of equilibrium outcomes. Next, we consider an alternative approach that looks at the bounds on these distributions of equilibrium outcomes, as identified by the theory.

4.1 Predictions for arbitrary equilibrium selections

For simplicity, hereafter we assume that the policy maker's payoff satisfies the single-crossing condition SCC. Recall that the role of this condition is to guarantee existence of equilibria in which the policy is raised at $r = s$, for any $s \in [\underline{r}, r^{\#}]$, while also guaranteeing that no equilibrium exists in which the policy is raised above $r^{\#}$. As explained above, these properties hold more generally: first, recall that equilibria in which all types pool on $\underline{r}$ exist irrespective of whether or not such assumption holds; next, recall that, for any $s \in (\underline{r}, r^{\#}]$, equilibrium existence can be easily guaranteed by assuming that the noise distribution is log-concave; lastly, from Proposition 2, recall that, for σ small enough, essentially all equilibrium satisfy $s \le r^{\#}$, irrespective of whether or not SCC holds. Therefore, the predictions below are not limited to cases where SCC holds.

Given part (i) of Proposition 1, we then have that, for any $\sigma > 0$, the equilibrium set is given by $\mathcal{E}(\sigma) = \cup_{s\in[\underline{r}, r^{\#}]}\mathcal{E}(s;\sigma)$, where $\mathcal{E}(s;\sigma)$ denotes the set of equilibria in which the range of the policy is $\{\underline{r}, s\}$. Because all equilibria within the set $\mathcal{E}\left(s;\sigma\right)$ are characterized by the same devaluation outcomes (up to a zero-measure set of types) and by the same bounds θ_s^* and θ_s^{**} on the set of types who possibly raise the policy, given the type of predictions we are interested in, from the econometrician's viewpoint, any distribution over outcomes generated by a random selection over the equilibrium set $\mathcal{E}\left(\sigma\right)$ can be replicated by a random variable $\tilde{s}$ with cumulative distribution function F and support $Supp[F] \subseteq [\underline{r}, r^{\#}]$ such that one of the pooling equilibria is played when $s = \underline{r}$, while one of the semi-separating equilibria with range $\{\underline{r}, s\}$ is played when $s \in (\underline{r}, r^{\#}]$. For any σ, we then denote by $\mathcal{F}(\sigma)$ the set of the c.d.f.s. with support $Supp[F] \subseteq [\underline{r}, r^{\#}]$ that describe the possible beliefs that the external observer may have about the equilibrium being played.[19] Importantly, note that, because θ is the policy maker's *private* information, the equilibrium being played cannot be a function of θ. This also means that the external observer cannot expect the realization of the random variable $\tilde{s}$ with distribution F to depend of θ.

Now, for any $\sigma > 0$ and any $s \in [\underline{r}, r^{\#}]$, let $D_s\left(\theta;\sigma\right)$ denote the devaluation outcome in any of the equilibria in $\mathcal{E}\left(s;\sigma\right)$, while, for any $s \in (\underline{r}, r^{\#}]$, let $\Delta_s \equiv \theta_s^{**} - \theta_s^*$ denote the (Lebesgue) measure of the set of types who potentially raise the policy at $r = s$ in any of the equilibria in $\mathcal{E}\left(s;\sigma\right)$.[20] Recall that there is no equilibrium in which some type outside the interval $[\theta_s^*, \theta_s^{**}]$ raises the policy at s, while there is an equilibrium in which all types in $[\theta_s^*, \theta_s^{**}]$ raise the policy at $r = s$. Given the

[19] As a technical restriction, we assume that $\mathcal{F}(\sigma)$ is compact with respect to the metric $d(\cdot)$ defined, for any pair $F_1, F_2 \in \mathcal{F}$, by $d(F_1, F_2) \equiv \sup\left\{|F_1(A) - F_2(A)| : A \in \Sigma\right\}$, where Σ is the Borel sigma algebra associated with the interval $[\underline{r}, \rho(\bar{\theta})]$.

[20] Recall that θ_s^* is independent of σ.

uniform prior, Δ_s can thus also be read as (a rescaling of a sharp) bound on the probability that the policy is raised at $r = s$ across all equilibria in $\mathcal{E}(s;\sigma)$.

Next, let $I_{premise}$ denote the indicator function assuming value one if *premise* is true and zero otherwise. For any selection (equivalently, for any belief) $F \in \mathcal{F}(\sigma)$, and any $r > \underline{r}$, then let

$$D(\theta; F, \sigma) \equiv \int D_s(\theta;\sigma) dF(s) \qquad \text{and} \qquad P(r, \theta; F, \sigma) \equiv \int_{s \geq r} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s)$$

denote, respectively, the probability that type θ abandons the peg and the (maximal) probability that type θ raises the policy at or above r, when the selection is F.[21] For any $r > \underline{r}$, any $F \in \mathcal{F}(\sigma)$, then let $\Delta(r; F, \sigma)$ denote the expected Lebesgue measure of the set of types who raise the policy at or above r. Once again, $\Delta(r; F, \sigma)$ can also be read as a rescaling of the probability that the policy is raised at or above r, when the selection is F. Given two selections $F, F' \in \mathcal{F}(\sigma)$, then let $F' \gg F$ if and only if $F'(s) \leq F(s)$ for all s, with strict inequality for $s \in (\underline{r}, r^{\#})$ and equality for $s \in \{\underline{r}, r^{\#}\}$.

The key predictions about devaluation and policy outcomes that our theory delivers to an outside observer who is (potentially) uncertain about which equilibrium is played can then be summarized as follows.[22]

Proposition 5 (random selections)**.** *Equilibrium policies and devaluation outcomes satisfy the following properties.*

(i) Non-monotonic policy. *For any $\sigma > 0$, any $r > \underline{r}$ and any $F \in \mathcal{F}(\sigma)$, there exist thresholds $\theta^{\circ}(r; F, \sigma)$ and $\theta^{\circ\circ}(r; F, \sigma)$, with $\underline{\theta} < \theta^{\circ\circ}(r; F, \sigma) \leq \theta^{\circ\circ}(r; F, \sigma)$, such that $P(r, \theta; F, \sigma) > 0$ only if $r \leq r^{\#}(\sigma)$ and $\theta \in [\theta^{\circ}, \theta^{\circ\circ}]$.*

(ii) Monotonic devaluation outcome. *For any $\sigma > 0$ and any $F \in \mathcal{F}(\sigma)$, $D(\theta; F, \sigma)$ is nonincreasing in θ, with $D(\theta; F, \sigma) = 1$ for $\theta \leq \underline{\theta}$ and $D(\theta; F, \sigma) = 0$ for $\theta > \theta^{\#}(\sigma)$.*

(iii) Impact of "aggressiveness". *For any $\sigma > 0$ and any $F, F' \in \mathcal{F}(\sigma)$, $F' \gg F$ implies $D(\theta; F', \sigma) > D(\theta; F, \sigma)$ for any $\theta \in (\underline{\theta}, \theta^{\#}(\sigma))$ such that (i) $D(\theta; F, \sigma) < 1$ and (ii) both F and F' are continuous at $s = \rho(\theta)$.*[23] *Moreover, if $F, F' \in \mathcal{F}(\sigma)$ are such that $F'(r) = F(r)$ and $F'(s) < F(s)$ for all $s \in (r, r^{\#}(\sigma))$, then $\Delta(r; F', \sigma) < \Delta(r; F, \sigma)$.*

(iv) Impact of noise. *Take any F such that Supp$[F] \subset (\underline{r}, \lim_{\sigma \to 0^+} r^{\#}(\sigma))$. For any $r > \underline{r}$, $\lim_{\sigma \to 0^+} \Delta(r; F, \sigma) = 0$ whereas, for any θ, any $\sigma, \sigma' > 0$, any $F \in \mathcal{F}(\sigma) \cap \mathcal{F}(\sigma')$, $D(\theta; F, \sigma) = D(\theta; F, \sigma')$. Finally, for any $\sigma, \sigma' > 0$, $\sigma' > \sigma > 0$ implies $\Delta(r; F, \sigma') \geq \Delta(r; F, \sigma)$ for all $r \in (\underline{r}, \min\{r^{\#}(\sigma), r^{\#}(\sigma')\})$ all $F \in \mathcal{F}(\sigma) \cap \mathcal{F}(\sigma')$ (with strict inequality if $F(s) < 1$ for $s < r$).*

[21] Recall that any equilibrium in $\mathcal{E}(s;\sigma)$ is such that $r(\theta) = s$ only if $\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]$ so that $P(r, \theta; F, \sigma)$ is an upper bound on the probability that type θ raises the policy at or above r.

[22] The predictions that the theory delivers to an observer who knows which equilibrium is played can be read by looking at the special case where the distribution F has a measure-1 mass point at a particular $s \in [\underline{r}, r^{\#}(\sigma)]$.

[23] That is, for any θ for which both F and F' have no mass point at $s = \rho(\theta)$, where recall that $\rho(\theta)$ is implicitly defined by $W(\theta, \rho(\theta), 0) = L(\theta, \underline{r})$.

Fix the quality of information $\sigma > 0$. Parts (i) and (ii) say that, for any selection $F \in \mathcal{F}(\sigma)$, the probability of observing the policy maker raising the policy above r is positive only if $r \leq r^{\#}$ and θ is intermediate, whereas the probability of observing him devalue is nonincreasing in θ and equal to zero for all $\theta > \theta^{\#}$.

Property (iii), on the other hand, can be interpreted as the impact of the "aggressiveness"of market expectations: the higher the level of policy intervention s at which the speculators switch to lenient behavior (i.e., refrain from attacking), the higher the cost of policy intervention necessary to prevent an attack, and the smaller the set of types who find it do so in equilibrium. Of course, since the level of "aggressiveness" (equivalently, the distribution F) is unobserved, this prediction is hard to test. Yet, it could help the econometrician identify, or estimate, the underlying equilibrium selection, F, from observed data.

Part (iv) says that, holding constant the econometrician's beliefs F about which equilibrium is played, an increase in the quality of information has no effect on the probability of devaluation, for any θ, whereas it reduces the probability of observing a policy above r, for any $r > \underline{r}$, with such a probability vanishing in the limit, when information becomes infinitely precise.

Remark. One frequent criticism of common-knowledge models of crises, such as Obstfeld (1986, 1996) and Calvo (1986), is that they document the existence of a critical region of fundamentals over which there are multiple equilibria, but say little about the relation between fundamentals and equilibrium outcomes. For example, the probability of devaluation in Obstfeld need not be monotonic in the strength of the currency. In contrast, the result in part (ii) of Proposition 5 delivers a monotonic relation between fundamentals (and hence the policy maker's type) and the devaluation outcome, while at the same time allowing for some "randomness" in this relation corresponding to the econometrician's uncertainty about the equilibrium selection. In other words, our theory delivers a concrete testable prediction that takes the form of a (non-linear) relationship between the fundamentals and the devaluation outcome.

This is akin to the predictions one often gets from unique-equilibrium models, except for the following feature. In unique-equilibrium models, the theory typically restricts the residual in the relation between the dependent and the independent variables to be zero; the randomness is then superimposed by the econometrician on the basis of the presumption that there is measurement error or omitted variables that nonetheless do not bias the results. Here, instead, the theory *itself* allows for a random residual: the residual simply captures the econometrician's uncertainty over the equilibrium being played.

The result in part (iv), on the other hand, is interesting because it suggests that the precision of information need not be important for whether or not the peg is maintained, but it may be crucial for whether or not the peg is maintained with or without policy intervention. Note, however, that this result presumes that the econometrician's beliefs F do not vary with σ. Because the model imposes no relation between F and σ, this is possible, although not necessary.

4.2 Bounds across all selections

As anticipated above, our theory also delivers useful predictions to an econometrician interested in testing, or estimating, our model, but who is not willing to assume any particular distribution over the equilibrium being played. This can be done by considering bounds on the probability of devaluation and on the probability of intervention *across all possible equilibria* and then investigating how these bounds change with the fundamentals θ, and/or the quality of information σ.

To illustrate, let $D(\theta_1, \theta_2; F, \sigma) \equiv \frac{1}{\theta_2-\theta_1}\int_{\theta_1}^{\theta_2} D(\theta; F, \sigma)d\theta$ denote the probability of devaluation conditional on the event that $\theta \in [\theta_1, \theta_2]$, for given selection $F \in \mathcal{F}(\sigma)$, and $\bar{D}(\theta_1, \theta_2; \sigma) \equiv \sup_{F\in\mathcal{F}(\sigma)} D(\theta_1, \theta_2; F, \sigma)$ and $\underline{D}(\theta_1, \theta_2; \sigma) \equiv \inf_{F\in\mathcal{F}(\sigma)} D(\theta_1, \theta_2; F, \sigma)$, the corresponding bounds across all selections. Similarly, let $P(r, \theta_1, \theta_2; F, \sigma) \equiv \frac{1}{\theta_2-\theta_1}\int_{\theta_1}^{\theta_2} P(r, \theta; F, \sigma)d\theta$ denote the probability that the policy is raised at or above r, conditional on the event that $\theta \in [\theta_1, \theta_2]$, for given selection $F \in \mathcal{F}(\sigma)$, and $\bar{P}(r, \theta_1, \theta_2; \sigma) \equiv \sup_{F\in\mathcal{F}(\sigma)} P(r, \theta_1, \theta_2; F, \sigma)$ and $\underline{P}(r, \theta_1, \theta_2; \sigma) \equiv \inf_{F\in\mathcal{F}(\sigma)} P(r, \theta_1, \theta_2; F, \sigma) = 0$ the corresponding bounds. Clearly, $\bar{D}(\theta_1, \theta_2; \sigma) \geq \underline{D}(\theta_1, \theta_2; \sigma)$ and $\bar{P}(r, \theta_1, \theta_2; \sigma) \geq \underline{P}(r, \theta_1, \theta_2; \sigma)$, with strict inequalities when $\underline{\theta} \leq \theta_1 < \theta_2 \leq \theta^{\#}$ and $\underline{r} < r < r^{\#}$. That these bounds do not coincide over a subset of the critical region reflects the equilibrium indeterminacy. The next proposition examines how these bounds depend on the fundamentals θ and the quality of information σ.

Proposition 6 (bounds). *(i) Fix $\sigma > 0$ and $r \in (\underline{r}, r^{\#}(\sigma))$. The bounds $\underline{D}(\theta_1, \theta_2; \sigma)$ and $\bar{D}(\theta_1, \theta_2; \sigma)$ are nonincreasing in (θ_1, θ_2), while the bounds $\underline{P}(r, \theta_1, \theta_2; \sigma)$ and $\bar{P}(r, \theta_1, \theta_2; \sigma)$ are nonmonotone in (θ_1, θ_2), in the partial-order sense.*

(ii) Fix (θ_1, θ_2). The bounds $\underline{D}(\theta_1, \theta_2; \sigma)$and $\underline{P}(r, \theta_1, \theta_2; \sigma)$ are independent of σ. For any (θ_1, θ_2) any $\sigma, \sigma' > 0$, $\theta_2 < \min\{\theta^{\#}(\sigma'), \theta^{\#}(\sigma)\}$ or $\theta_1 > \max\{\theta^{\#}(\sigma'), \theta^{\#}(\sigma)\}$ imply $\bar{D}(\theta_1, \theta_2; \sigma) = \bar{D}(\theta_1, \theta_2; \sigma')$. In contrast, $\lim_{\sigma\to 0^+} \bar{P}(r, \theta_1, \theta_2; \sigma) = 0$ for any $r > \underline{r}$ and any $\theta_1, \theta_2 \in \mathbb{R}$. In the special case where $Z(\theta, \underline{r}) = z > \underline{r}$ for all θ, then the bound $\bar{D}(\theta_1, \theta_2; \sigma)$ is independent of σ whereas the bound $\bar{P}(r, \theta_1, \theta_2; \sigma)$ is a nondecreasing function of σ.

The results for how the bounds are affected by the fundamentals are an immediate implication of Proposition 1. Thus consider the effect of information on the bounds. While more precise information need not affect the bounds on the probability of devaluation, it affects the bounds on the probability of intervention. In particular, in the limit as $\sigma \to 0$, the probability of observing policy interventions above r, for any $r > \underline{r}$, vanishes for all measurable sets of θ, whereas the probability of devaluation can take any value for any subset of $(\underline{\theta}, \hat{\theta})$, where $\hat{\theta} \equiv \lim_{\sigma\to 0^+} \theta^{\#}$.

These properties are particularly sharp in the case where the devaluation premium is independent θ. In this case, both the lower and and the upper bound on the probability of devaluation are independent of σ, whereas the upper bound on the probability of policy interventions is a decreasing function of the quality of information σ^{-1} and vanishes in the limit as $\sigma \to 0^+$. More generally, what the theory predicts is that policy choices are essentially uniquely determined in the limit, whereas the devaluation outcomes remain largely indeterminate. We will return to these predictions and contrast them to their counterparts under common knowledge in Section 6.

5 Predictions about payoffs

We now turn to the predictions that the model delivers for the payoff of the policy maker. In contrast to predictions about policy choices and devaluation outcomes, predictions about payoffs need not be directly testable (the econometrician may not be able to directly observe the policy maker's payoff). Nevertheless, these predictions are important for their policy implications. For example, they permit one to study the ex-ante value that the policy maker may attach to the option to intervene once θ is realized.

For any $s \in [\underline{r}, r^{\#}]$, let $U_s(\theta;\sigma)$ denote the *lowest* payoff that type θ obtains across all the equilibria in $\mathcal{E}(s;\sigma)$. Next, consider the variant of our model in which r is exogenously fixed at $\underline{r}$ for all θ, interpret this as the game in which the option to intervene is absent, and let $\tilde{U}(\theta;\sigma)$ denote the payoff that type θ obtains in the *unique* equilibrium of this game. Clearly, when $s \in \{\underline{r}, r^{\#}(\sigma)\}$, any equilibrium in $\mathcal{E}(s;\sigma)$, is such that $U_s(\theta;\sigma) = \tilde{U}(\theta;\sigma)$ for all θ.[24] Thus consider equilibria in which $s \in (\underline{r}, r^{\#})$.

Proposition 7 (payoffs)**.** *For any $s \in (\underline{r}, r^{\#}(\sigma))$, either $U_s(\theta;\sigma) \geq \tilde{U}(\theta;\sigma)$ for all θ, with strict inequality for some θ, or there exists a threshold $\hat{\theta}_s(\sigma) > \theta^{\#}(\sigma)$ such that $U_s(\theta;\sigma) < \tilde{U}(\theta;\sigma)$ only if $\theta > \hat{\theta}_s(\sigma)$, in which case necessarily $U_s(\theta;\sigma) \geq \tilde{U}(\theta;\sigma)$ for all $\theta \leq \hat{\theta}_s(\sigma)$ (with strict inequality if $\theta \in (\theta_s^*, \hat{\theta}_s(\sigma))$). Moreover, σ small enough ensures that the first case holds.*

To understand this result, note first that types below $\theta^{\#}$ cannot be worse off with the option to intervene: without this option, they would necessarily devalue, whereas with this option they are spared by devaluation in some equilibria. The same is true for types above $\theta^{\#}$ but sufficiently close to it. For these types, the cost of policy intervention is lower than the cost of the attack that they would have otherwise faced absent the option to intervene. Finally, types sufficiently higher than $\theta^{\#}$ can be worse off with the option to intervene only if the size of the attack that they face when they opt for not raising the policy exceeds the one they would have faced absent the option to intervene. In general, this may be possible for some equilibria. However, this possibility vanishes as the precision of information increases. This is because the devaluation threshold θ_s^* in any of the equilibria with intervention is necessarily lower than $\theta^{\#}$, which together with the fact that $\theta_s^{**} \to \theta_s^*$ as $\sigma \to 0$ guarantees that the size of the attack is also lower, as long as σ is small enough.

The results in Proposition 7 extend to random equilibrium selections. Indeed, fix an arbitrary set of types $[\theta_1, \theta_2] \subset \mathbb{R}$ and an arbitrary selection $F \in \mathcal{F}(\sigma)$ and consider the implied probability

[24] The result is immediate when $s = \underline{r}$. For $s = r^{\#}(\sigma)$, recall that, in this case, $\theta_s^* = \theta_s^{**}(\sigma) = \theta^{\#}(\sigma)$ and $X_s(\theta_s^*, \theta_s^{**}(\sigma);\sigma) = x^{\#}(\sigma)$.

that, conditional on $\theta \in [\theta_1, \theta_2]$, the policy maker is strictly worse off with the option to intervene. This probability is zero either for all σ, or at least for σ small enough. Notwithstanding the fact that, in general, the selection F may also depend on σ, this property suggests that the risk of being worse off with the option to intervene vanishes as market information becomes highly precise.

We can also accommodate the case that the selection F changes with σ by considering bounds on equilibrium payoffs across all possible equilibria. Let $\overline{U}(\theta;\sigma)$ and $\underline{U}(\theta;\sigma)$ denote, respectively, the supremum and infimum of the set of equilibrium payoffs that type θ can obtain in the game with the option to intervene, when the quality of information is σ. The following proposition characterizes the relation between these bounds and the payoff obtained in the game in which the option to intervene is absent.

Proposition 8 (payoff bounds)**.** *$\overline{U}(\theta;\sigma) > \tilde{U}(\theta;\sigma)$ for all finite $\theta > \underline{\theta}$, with $\lim_{\theta\to+\infty} |\overline{U}(\theta;\sigma) - \tilde{U}(\theta;\sigma)| = 0$. On the other hand, there exists a threshold $\hat{\theta}(\sigma) \geq \theta^{\#}(\sigma)$ such that $\underline{U}(\theta;\sigma) < \tilde{U}(\theta;\sigma)$ only if $\theta > \hat{\theta}(\sigma)$. Finally, $\lim_{\sigma\to 0^+} \underline{U}(\theta;\sigma) = \lim_{\sigma\to 0^+} \tilde{U}(\theta;\sigma)$ for all θ.*

Now imagine that, before knowing his type, the policy maker decides whether to maintain or to give up the option to intervene after learning θ. The aforementioned results suggest that, in general, the policy maker need not be able to ensure that he will be better off with the option to intervene no matter the realized θ: he may get "trapped" in an equilibrium in which he is worse off when θ turns out to be sufficiently high. Even then, however, the policy maker is better off for low θ. Therefore, the option to intervene either is beneficial for all θ, or it implements a form of ex-ante insurance across types.

6 Contrast to common knowledge

We now contrast the predictions that the theory delivers for the incomplete-information game with those for its common-knowledge counterpart. We further show that, while multiplicity obtains in our model for any level of noise, the set of equilibrium outcomes becomes smaller and smaller (in an appropriate sense) as the quality of market information improves—but it explodes when the noise is zero. The purpose of these exercises is two-fold: (i) to highlight that the selection power of global games has significant bite also in multiple-equilibria settings like ours; and (ii) to establish that the predictions that we have identified, albeit intuitive, would not have been possible with complete information.

Proposition 9 (common knowledge)**.** *Consider the game with $\sigma = 0$.*

(i) *A policy $r(\cdot)$ can be part of a subgame-perfect equilibrium if and only if $r(\theta) \leq \rho(\theta)$ for $\theta \in [\underline{\theta}, \bar{\theta}]$ and $r(\theta) = \underline{r}$ for $\theta \notin [\underline{\theta}, \bar{\theta}]$.*

(ii) *A devaluation outcome $D(\cdot)$ can be part of a subgame-perfect equilibrium if and only if $D(\theta) = 1$ for $\theta < \underline{\theta}$, $D(\theta) \in \{0,1\}$ for $\theta \in [\underline{\theta}, \bar{\theta}]$, and $D(\theta) = 0$ for $\theta > \bar{\theta}$.*

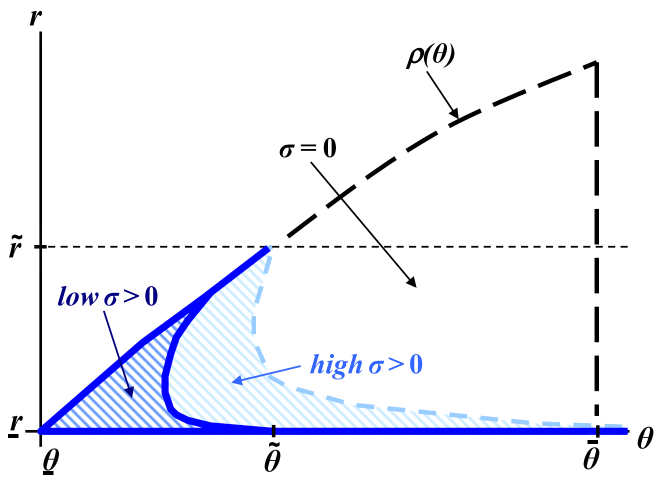

Figure 1: This figure illustrates the set of pairs (θ, r) that can be observed as equilibrium outcomes in our model ($\sigma > 0$) versus the corresponding set for the common-knowledge variant ($\sigma = 0$).

This result contrasts sharply with the results in Propositions 1-5. None of the predictions in the game with incomplete information are valid in the game with common knowledge. In particular, the policy can now take any shape in the critical region $[\underline{\theta}, \bar{\theta}]$. Similarly, the probability of devaluation can take any value within the critical region and need not be monotone in θ. In essence, "almost anything goes" within the critical region under complete information. In particular, the only policy choices and devaluation outcomes that are ruled out by equilibrium reasoning under complete-information are those that are ruled out by strict dominance. A similar "anything-goes" result holds if one looks at the policy maker's payoff.

The contrast between the complete- and incomplete-information versions of our model is most evident in the limit as $\sigma \to 0^+$. Let $\mathcal{G}(\sigma)$ denote the set of pairs (θ, r) such that, in the game with noise $\sigma \geq 0$, there is an equilibrium in which type θ sets the policy at r. We then have the following result.

Proposition 10 (limit outcomes)**.** *Let $\theta^{\#}(0^+) \equiv \lim_{\sigma \to 0^+} \theta^{\#}(\sigma)$. Under complete information,*

$$\mathcal{G}(0) = \{(\theta, r) : \textit{either } \theta \in [\underline{\theta}, \bar{\theta}] \textit{ and } \underline{r} \leq r \leq \rho(\theta), \textit{ or } \theta \notin [\underline{\theta}, \bar{\theta}] \textit{ and } r = \underline{r}\}.$$

In contrast, under incomplete information,

$$\lim_{\sigma \to 0^+} \mathcal{G}(\sigma) = \{(\theta, r) : \textit{ either } \theta \in [\underline{\theta}, \theta^{\#}(0^+)] \textit{ and } r \in \{\underline{r}, \rho(\theta)\}, \textit{ or } \theta \notin [\underline{\theta}, \theta^{\#}(0^+)] \textit{ and } r = \underline{r}\},$$

which is a zero-measure subset of $\mathcal{G}(0)$.

This result is illustrated in Figure 1 for the case where $Z(\theta, r) = z$ and $C(r, \theta) = r$ for all θ. The common-knowledge set, $\mathcal{G}(0)$, is given by the large triangular area. The incomplete-information set, $\mathcal{G}(\sigma)$ for $\sigma > 0$, is given by the dashed blue area. In this case, as long as $\sigma > 0$, the lower σ, the smaller the set of policies that can be played by any given θ, and hence the smaller the dashed area in Figure 1 (i.e., $\sigma' > \sigma > 0$ implies $\mathcal{G}(\sigma') \supset \mathcal{G}(\sigma)$). The monotonicity of $\mathcal{G}(\sigma)$ in σ does not necessarily hold in the case where the devaluation premium depends θ. However, as the proposition makes clear, what is true more generally is that, in the limit, as the noise in information vanishes, the set of policies that can be sustained in equilibrium for any given θ is a zero measure subset of the set of policies that can be sustained under common knowledge. More precisely, the set $\mathcal{G}(\sigma)$ converges to the boundary points of the set of policies that would have been possible under complete information for any $\theta \leq \hat{\theta}$, and to the cost-minimizing policy $\underline{r}$ for $\theta > \hat{\theta}$, where $\hat{\theta} \equiv \lim_{\sigma \to 0^+} \theta^\#$.

7 Concluding Remarks

This paper studied the equilibrium properties of preemptive policies against speculative currency attacks. Previous work studied the same issue only by abstracting from either the self-fulfilling nature of such attacks or the signaling role of such policies (or both). Our analysis, instead, focused precisely on the interaction of these two features. In our view, these two features are central to these phenomena: speculative pressures can be self-fulfilling only insofar the policy maker lacks perfect commitment; but then policy measures that seek to preempt such pressures are bound to convey information regarding the policy maker's commitment to defend the peg.

The combination of these features naturally sustains multiple equilibria. Nevertheless, this multiplicity is severely restrained once the conventional assumption of common knowledge is removed. We were thus able to reach the following predictions: (i) devaluation is monotone in the policy maker's type, namely in his willingness and ability to maintain the peg and, through this, in the underlying economic fundamentals; (ii) costly preemptive policy measures are selected only when the policy maker's type is intermediate; (iii) such measures are, on average, associated with smaller attacks; (iv) fewer such measures take place when market information about the policy maker's type is more precise; and (v) although the option to engage in such preemptive policies is conductive to multiple equilibria, the policy maker is typically better off with this option than without it.

These predictions have been established under specific payoff and information assumptions. Nonetheless, we expect them to hold more generally as long as, of course, certain natural monotonicities in incentives are preserved. Furthermore, because the formal analysis rested on the coordination and signaling aspects that are endemic, not only to currency crises, but also to debt crises and bank runs, we expect variants of the results to apply to these contexts as well.

Confronting the aforementioned predictions with the data seems a natural next step for future research. The underlying equilibrium multiplicity makes this task particularly challenging. Yet, recent advances in structural estimation of multiple-equilibria models (e.g., Tamer, 2003, Ciliberto and Tamer, 2009, Grieco, 2010) may help in this direction.

Last but not least, our results also contain a broader methodological message. The approach followed in most recent applications of global games is to use incomplete information as a tool to select a unique equilibrium in coordination settings that admit multiple equilibria under common knowledge—to assume certain exogenous information structures that ensure uniqueness, without investigating what determines information in the first place. For certain questions, however, understanding the endogeneity of information is essential for understanding the phenomenon under examination. This often brings back multiple equilibria. The broader methodological contribution of this paper is to illustrate that this multiplicity may be very different from the one that obtains with complete information and, more importantly, that this multiplicity need not preclude concrete, intuitive, and testable predictions—predictions that would have *not* been possible under complete information. In this paper, we illustrated these points within the context of preemptive policies against speculative currency attacks; similar points, however, are likely to be relevant also for other global-game applications where information is endogenous.[25]

[25]These include *learning* in dynamic settings (Angeletos, Hellwig and Pavan, 2007), aggregation of information through *prices* (Angeletos and Werning, 2006; Hellwig, Mukherji, and Tsyvinski, 2006; Morris and Shin, 2006a; Ozdenoren and Yuan, 2006; Tarashev, 2006), and manipulation of information through *propaganda* (Edmond, 2006). In Angeletos, Hellwig and Pavan (2007), for example, learning sustains multiplicity but all equilibria share the prediction that dynamics alternate between phases of tranquility, in which no attack is possible, and phases of distress, in which an attack is possible but does not necessarily take place.

Appendix: Proofs

Proof of Lemma 1. The proof is in two steps. Step 1 considers the continuation game that follows the observation of $\underline{r}$. Because the observation of $\underline{r}$ conveys no information, this game is essentially a standard global game (Morris and Shin, 1998, 2003). The proof below shows that this game admits a unique continuation equilibrium in monotone strategies. Standard results from global games (based on iterated deletion of strictly dominated strategies) then imply that this equilibrium is the unique equilibrium of the continuation game. Step 2 completes the proof by showing existence of a strategy profile for the speculators, along with a system of supporting beliefs, such that, given this profile, no type of the policy maker finds it optimal to raise the policy above $\underline{r}$, thus establishing that $\mathcal{E}(\underline{r};\sigma) \neq \varnothing$.

Step 1. Consider the continuation game that follows the observation of $\underline{r}$. Clearly any monotone continuation equilibrium must be characterized by thresholds $x^{\#}(\sigma)$ and $\theta^{\#}(\sigma)$ such that all speculators attack if $x < x^{\#}(\sigma)$ and not attack if $x > x^{\#}(\sigma)$ in which case devaluation occurs if $\theta < \theta^{\#}(\sigma)$ and does not occur if $\theta > \theta^{\#}(\sigma)$. Hereafter, we show that such a continuation equilibrium exists and is unique. To simplify the notation, we momentarily drop the dependence of the thresholds $x^{\#}(\sigma)$ and $\theta^{\#}(\sigma)$ on σ.

First, note that a speculator with signal x who expects devaluation to occur if $\theta < \theta^{\#}$ and not to occur if $\theta > \theta^{\#}$ finds it optimal to attack if and only if

$$\int_{-\infty}^{\theta^{\#}} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{x-\tilde{\theta}}{\sigma}\right) d\tilde{\theta} - \underline{r} \geq 0. \tag{10}$$

Because $Z(\cdot, \underline{r})$ is nonincreasing in θ, the left-hand-side of (10) is continuously strictly decreasing in x.[26] Furthermore, it is strictly positive for x small enough and strictly negative for x large enough. It follows that the inequality in (10) holds if and only if $x \leq x^{\#}$ where $x^{\#}$ is the unique solution to

$$\int_{-\infty}^{\theta^{\#}} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{x^{\#}-\tilde{\theta}}{\sigma}\right) d\tilde{\theta} = \underline{r} \tag{11}$$

Next, consider the decision to abandon the peg. Because $A(\theta, \underline{r}) = \Psi((x^{\#} - \theta)/\sigma)$ is decreasing in θ, a policy maker who faces an attack of size $A(\theta, \underline{r})$ finds it optimal to abandon the peg when $\theta < \theta^{\#}$ and maintain it when $\theta > \theta^{\#}$ where the threshold $\theta^{\#}$ is the unique solution to

$$V\left(\theta^{\#}, \Psi\left(\tfrac{x^{\#}-\theta^{\#})}{\sigma}\right)\right) = 0. \tag{12}$$

Thus any monotone equilibrium is identified by a solution $(x^{\#}, \theta^{\#})$ to conditions (11) and (12). Below, we show that a solution to these conditions exists and is unique.

[26] Note that, in any pooling equilibrium, after observing $r = \underline{r}$, the speculators' posteriors beliefs that $\theta < \theta^{\#}(\sigma)$ are given by $1 - \Psi(x - \theta^{\#}(\sigma))/\sigma)$. These beliefs can be ranked according to FOSD, implying that, given any nonincreasing function $g(\theta)$, the expected value of g given x is decreasing in x.

To this aim, let $\theta^{\#}(x^{\#})$ be the implicit function defined by (12). By the Implicit Function Theorem,

$$\frac{d\theta^{\#}(x^{\#})}{dx^{\#}} = \frac{-V_A\left(\theta^{\#}, \Psi\left(\frac{x^{\#}-\theta^{\#}}{\sigma}\right)\right)\psi\left(\frac{x^{\#}-\theta^{\#}}{\sigma}\right)}{-V_A\left(\theta^{\#}, \Psi\left(\frac{x^{\#}-\theta^{\#}}{\sigma}\right)\right)\psi\left(\frac{x^{\#}-\theta^{\#}}{\sigma}\right) + \sigma V_\theta\left(\theta^{\#}, \Psi\left(\frac{x^{\#}-\theta^{\#}}{\sigma}\right)\right)} \in (0,1).$$

Next, let $LHS(x^{\#})$ denote the function of $x^{\#}$ that is defined by the left-hand side of (11) once we replace $\theta^{\#}$ with $\theta^{\#}(x^{\#})$. Differentiating with respect to $x^{\#}$ gives the following expression:

$$\begin{aligned} \frac{\partial LHS(x^{\#})}{\partial x^{\#}} &= \frac{d\theta^{\#}(x^{\#})}{dx^{\#}} Z\left(\theta^{\#}(x^{\#}), \underline{r}\right)\frac{1}{\sigma}\psi\left(\tfrac{x^{\#}-\theta^{\#}(x^{\#})}{\sigma}\right) + \\ &\quad + \int_{-\infty}^{\theta^{\#}(x^{\#})} Z(\tilde{\theta}, \underline{r})\frac{1}{\sigma^2}\psi'\left(\tfrac{x^{\#}-\tilde{\theta}}{\sigma}\right) d\tilde{\theta}. \end{aligned} \tag{13}$$

Integrating by parts, the last term in (13) can be rewritten as

$$\begin{aligned} &-Z\left(\theta^{\#}(x^{\#}), \underline{r}\right)\frac{1}{\sigma}\psi\left(\tfrac{x^{\#}-\theta^{\#}(x^{\#})}{\sigma}\right) + \lim_{\tilde{\theta}\to-\infty} Z(\tilde{\theta}, \underline{r})\frac{1}{\sigma}\psi\left(\tfrac{x^{\#}-\tilde{\theta}}{\sigma}\right) \\ &+ \int_{-\infty}^{\theta^{\#}(x^{\#})} Z_\theta\left(\tilde{\theta}, \underline{r}\right)\frac{1}{\sigma}\psi\left(\tfrac{x^{\#}-\tilde{\theta}}{\sigma}\right) d\tilde{\theta}. \end{aligned}$$

It follows that (13) reduces to

$$\begin{aligned} \frac{\partial LHS(x^{\#})}{\partial x^{\#}} &= \left(\frac{d\theta^{\#}(x^{\#})}{dx^{\#}} - 1\right) Z\left(\theta^{\#}(x^{\#}), \underline{r}\right)\frac{1}{\sigma}\psi\left(\tfrac{x^{\#}-\theta^{\#}(x^{\#})}{\sigma}\right) \\ &\quad + \int_{-\infty}^{\theta^{\#}(x^{\#})} Z_\theta\left(\tilde{\theta}, \underline{r}\right)\frac{1}{\sigma}\psi\left(\tfrac{x^{\#}-\tilde{\theta}}{\sigma}\right) d\tilde{\theta} \end{aligned}$$

which is clearly negative. Next note that

$$\lim_{x^{\#}\to-\infty} LHS(x^{\#}) > \underline{r} > 0 = \lim_{x^{\#}\to+\infty} LHS(x^{\#})$$

By the Intermediate Value Theorem, it then follows that there exists a unique $x^{\#}$ such that $LHS(x^{\#}) = 0$, which establishes existence and uniqueness of a monotone continuation equilibrium.

Step 2. Now consider the following strategy profile for the speculators: $a(x,r) = 1$ if and only if $x < x^{\#}$. Given this profile, because $A(\theta, r)$ is independent of r, it is immediate that each type of the policy maker finds it optimal to set $r(\theta) = \underline{r}$ and then abandon if $V(\theta, A) < 0$ and retain the peg if $V(\theta, A) > 0$. In what follows, it then suffices to show that the above strategy profile for the speculators can be supported by an appropriate system of beliefs. Because, for any r, $D(\theta, r, A(\theta, r)) = 1$ if and only if $\theta < \theta^{\#}$, a speculator finds it optimal to follow the equilibrium strategy if and only if, for any r, his beliefs satisfy

$$\int_{-\infty}^{\theta^{\#}} Z(\tilde{\theta}, \underline{r}) d\mu(\tilde{\theta}|x, r) \geq \underline{r} \text{ if } x < x^{\#} \text{ and } \int_{-\infty}^{\theta^{\#}} Z(\tilde{\theta}, \underline{r}) d\mu(\tilde{\theta}|x, r) \leq r \text{ if } x \geq x^{\#} \tag{14}$$

where $\mu(\theta|x,r)$ denotes the posterior c.d.f. given x and r. When $r = \underline{r}$, Bayes' rule imposes that $\mu(\theta|x,\underline{r}) = 1 - \Psi(\frac{x-\theta}{\sigma})$; that these beliefs satisfy (14) follows directly from the definition of $x^{\#}$. When, instead, $r > \underline{r}$, a deviation is detected whatever x. There then exists an arbitrarily large set of (out-of-equilibrium) beliefs that satisfy (14). *Q.E.D.*

Proof of Lemma 2. Because the policy maker faces no uncertainty about the size of the attack, raising the policy above $\underline{r}$ and then devaluing is always dominated by leaving the policy at $\underline{r}$ and then devaluing. It follows that any type who in equilibrium raises the policy above $\underline{r}$ must be spared from devaluation, for otherwise he would be strictly better off by setting $r = \underline{r}$. By implication, the observation of any equilibrium policy $r > \underline{r}$ necessarily signals to the speculators that the peg will be maintained and thus induces each speculator not to attack no matter his signal x. But then any type of the policy maker can always save on the cost of intervention by setting the lowest $r > \underline{r}$ among those that are played in equilibrium. Hence in any equilibrium in which some type intervenes, there exists a single $s > \underline{r}$ such that $r(\theta) = s$ whenever $r(\theta) \neq \underline{r}$. Furthermore, no speculator attacks following the observation of $r = s$ which means that $A(\theta, s) = 0$ for all θ. *Q.E.D.*

Proof of Lemma 3. That $\mathcal{E}(s;\sigma) = \varnothing$ for any $s > \underline{r}$ for which (8) admits no solution follows directly from the fact that, in this case, $W(\theta, s, 0) < L(\theta, \underline{r},)$ for all θ : the net payoff that each type θ obtains by raising the policy at s and facing no attack is strictly less than the payoff that the same type obtains by leaving the policy at $\underline{r}$ and then devaluing. Thus consider s for which (8) admits a solution. The assumption that $W(\theta, s, 0) - L(\theta, \underline{r},)$ is strictly increasing in θ then implies that such a solution is unique and that any type $\theta > \theta_s^*$, by setting $r = s$, can guarantee himself a payoff strictly higher than the payoff that the same type can obtain by setting $r = \underline{r}$ and devaluing (recall that, from Lemma 2, $A(\theta, s) = 0$ for any θ). But then, in equilibrium, no type above θ_s^* abandons the peg. *Q.E.D.*

Proof of Lemma 4. Consider the sequence $\{\theta^n\}_{n=0}^{\infty}$ constructed as follows. First, let

$$\theta^0 \equiv \inf\left\{\theta \geq \theta_s^* : W(\theta, s, 0) < W(\theta, \underline{r}, 1)\right\}.$$

That $\theta^0 < +\infty$ follows from the assumption that $\lim_{\theta \to +\infty}\{W(\theta, s, 0) - W(\theta, \underline{r}, 1)\} < 0$, which simply imposes that raising the policy is dominated for sufficiently high types. Next, for any $n \geq 1$, let

$$\theta^n \equiv \inf\{\theta \geq \theta_s^* : g\left(\theta; \theta_s^*, \theta^{n-1}, \sigma\right) < 0\}.$$

where, for any $\theta \geq \underline{\theta}$, any (θ_s^*, θ') with $\theta' \geq \theta_s^*$, and any $\sigma > 0$, the function g is defined by

$$g\left(\theta; \theta_s^*, \theta', \sigma\right) \equiv U(\theta, s, 0) - U(\theta, \underline{r}, \bar{A}(\theta; \theta_s^*, \theta'))$$

where $\bar{A}(\theta;\theta_s^*,\theta') \equiv \Psi\left(\frac{X(\theta_s^*,\theta';\sigma)-\theta}{\sigma}\right)$ and where $X(\theta_s^*,\theta';\sigma)$ is the unique solution to (5). Equivalently,

$$g\left(\theta;\theta_s^*,\theta',\sigma\right) = W\left(\theta,s,0\right) - \max\left\{L(\theta,\underline{r}), W\left(\theta,\underline{r},\Psi\left(\tfrac{X(\theta_s^*,\theta';\sigma)-\theta}{\sigma}\right)\right)\right\}.$$

The function $g\left(\theta;\theta^*,\theta',\sigma\right)$ thus identifies the differential between the payoff that type θ obtains by raising the policy at $r=s$, facing no attack and then retaining the peg, and the payoff that the same type obtains by leaving the policy at $r=\underline{r}$, facing an attack of size $\bar{A}(\theta;\theta_s^*,\theta')$—that is, the attack implied by the speculators play according to the threshold strategy with cutoff $X(\theta_s^*,\theta';\sigma)$—and then optimally choosing whether or not to devalue.

This sequence has a simple interpretation. In any equilibrium in which the range of the policy is $r(\mathbb{R}) = \{\underline{r}, s\}$, no type $\theta \notin [\theta_s^*,\theta^0]$ raises the policy at $r=s$. Given so, a speculator who expects $D\left(\theta\right)=1$ if and only if $\theta<\theta_s^*$ and $r\left(\theta\right)=\underline{r}$ if and only if $\theta \notin [\theta_s^*,\theta^0]$ finds it optimal to attack when observing $\underline{r}$ if and only if $x \leq X(\theta_s^*,\theta^0;\sigma)$. To see this, note that the devaluation premium $Z\left(\theta,\underline{r}\right)$ is decreasing in θ. Furthermore, for any $\theta \leq \theta_s^*$, the probability that a speculator with signal x assigns to the event that the policy maker's type is less than θ conditional on the fact that this type is not in (θ_s^*,θ^0) is given by

$$\frac{1-\Psi\left(\frac{x-\theta}{\sigma}\right)}{1-\Psi(\frac{x-\theta_s^*}{\sigma})+\Psi(\frac{x-\theta^0}{\sigma})},$$

which is decreasing in x. It follows that the expected payoff from attacking, which is given by

$$\frac{\int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}\psi\left(\frac{x-\tilde{\theta}}{\sigma}\right)d\tilde{\theta}}{1-\Psi(\frac{x-\theta_s^*}{\sigma})+\Psi(\frac{x-\theta^0}{\sigma})} - \underline{r} \tag{15}$$

is decreasing in x.[27] Together with the fact that the expected payoff from attacking is positive for sufficiently small x and negative for sufficiently high x, this implies existence and uniqueness of a threshold $X(\theta_s^*,\theta^0;\sigma)$ such that any speculator who expects $D\left(\theta\right)=1$ if and only if $\theta<\theta_s^*$ and $r\left(\theta\right)=\underline{r}$ if and only if $\theta \notin [\theta_s^*,\theta^0]$ finds it optimal to attack when observing $\underline{r}$ if and only if $x \leq X(\theta_s^*,\theta^0;\sigma)$. But then, by implication, a speculator who expects $D\left(\theta\right)=0$ for all $\theta \geq \theta_s^*$ (but possibly also for some $\theta<\theta_s^*$) and $r\left(\theta\right)=\underline{r}$ for all $\theta \notin [\theta_s^*,\theta^0]$ (but possibly also for some $\theta \in [\theta_s^*,\theta^0]$) never finds it optimal to attack for $x > X(\theta_s^*,\theta^0;\sigma)$. To see this, note that when the peg is maintained also for some $\theta<\theta_s^*$, the payoff that a speculator expects from attacking when he observes $\underline{r}$ is smaller than when devaluation occurs for *all* $\theta<\theta_s^*$. Similarly, when the policy maker sets the policy at $\underline{r}$ also for some $\theta \in [\theta_s^*,\theta^0]$, the observation of $\underline{r}$ is less informative of devaluation than when $r(\theta)=s$ for *all* $\theta \in [\theta_s^*,\theta^0]$. Hence, the incentives to attack after observing

[27] To see this, note that, given any two absolutely continuous c.d.f.s F_1 and F_2 with $F_1(\theta) \leq F_2(\theta)$ for all $\theta<\theta_s^*$, and any non-increasing differentiable positive function $h:\mathbb{R}\to\mathbb{R}_+$, $\int_{-\infty}^{\theta_s^*} h(\tilde{\theta})dF_1(\tilde{\theta}) \leq \int_{-\infty}^{\theta_s^*} h(\tilde{\theta})dF_2(\tilde{\theta})$. Interpreting h as the devaluation premium Z and F_1 and F_2 as the speculator's posterior for two different signals x_1 and x_2, with $x_1 \geq x_2$, gives the result.

$\underline{r}$ are maximal when $D(\theta) = 1$ for all $\theta < \theta_s^*$ and $r(\theta) = r^*$ for all $\theta \in [\theta_s^*, \theta^0]$, which explains why no speculator ever finds it optimal to attack for $x > X(\theta_s^*, \theta^0; \sigma)$. Knowing this, a policy maker who expects no speculator to attack for $x > X(\theta_s^*, \theta^0; \sigma)$ never finds it optimal to raise the policy at $r = s$ for any $\theta > \theta^1$. Knowing this, no speculator finds it optimal to attack for any $x > X(\theta_s^*, \theta^1; \sigma)$ when observing $\underline{r}$, and so on.

Below, we conclude the proof by establishing that the sequence $\{\theta^n\}_{n=0}^{\infty}$ is nonincreasing. Because it is also bounded from below by θ_s^*, it has to converge. Clearly, the limit is the highest θ_s'' that solves equation (9) if such a solution exists; otherwise, the limit is θ_s^*. In the latter case, by the definition of the sequence $\{\theta^n\}_{n=0}^{\infty}$, we then have that no type above θ_s^* is willing to raise the policy at $r = s$. Together with the fact that $W(\theta, s, 0) < L(\theta, \underline{r})$ for any $\theta < \theta_s^*$, meaning that no type below θ_s^* is also willing to raise the policy, then this means that $\mathcal{E}(s; \sigma) = \varnothing$.

To see that the sequence $\{\theta^n\}_{n=0}^{\infty}$ defined above is nonincreasing, fix any $s > \underline{r}$ for which equation (8) admits a solution and let θ_s^* denote its unique solution. Towards a contradiction, suppose that there exists an $n \geq 1$ such that $\theta^n > \theta^{n-1}$. Without loss of generality, then let $n \geq 1$ be the first step in the sequence for which $\theta^n > \theta^{n-1}$ (i.e., $\theta^j \leq \theta^{j-1}$ for all $j \leq n-1$). By the definition of θ^{n-1}, then for any $\theta > \theta^{n-1}$,

$$g\left(\theta; \theta_s^*, \theta^{n-2}, \sigma\right) = W(\theta, s, 0) - \max\left\{L(\theta, \underline{r}), W\left(\theta, \underline{r}, \Psi\left(\tfrac{X(\theta_s^*, \theta^{n-2}) - \theta}{\sigma}\right)\right)\right\} < 0.$$

Because $W(\theta, s, 0) > L(\theta, \underline{r})$ for any $\theta > \theta_s^*$, this means that necessarily

$$W\left(\theta, \underline{r}, \Psi\left(\tfrac{X(\theta_s^*, \theta^{n-2}) - \theta}{\sigma}\right)\right) > L(\theta, \underline{r})$$

for any $\theta > \theta^{n-1}$. Because $\theta^{n-1} \leq \theta^{n-2}$ and because $X(\theta_s^*, \cdot; \sigma)$ is increasing, this in turn implies that, for any $\theta > \theta^{n-1}$

$$g\left(\theta; \theta^*, \theta^{n-1}, \sigma\right) = W(\theta, s, 0) - \max\left\{L(\theta, \underline{r}), W\left(\theta, \underline{r}, \Psi\left(\tfrac{X(\theta_s^*, \theta^{n-1}) - \theta}{\sigma}\right)\right)\right\} < 0.$$

By the definition of θ^n, this means that $\theta^n \leq \theta^{n-1}$, which proves that the sequence $\{\theta^n\}_{n=0}^{\infty}$ is nonincreasing. *Q.E.D.*

Proof of Lemma 5. The result is established by comparing the speculators' incentives to attack after observing $\underline{r}$ with the corresponding incentives when they expect $r(\theta) = \underline{r}$ for all θ.

Let $\{\theta_n, x_n\}_{n=0}^{\infty}$ be the following sequence. First, let $\theta_0 \equiv \underline{\theta}$ and let x_0 be implicitly defined by

$$\int_{-\infty}^{\underline{\theta}} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{x_0 - \tilde{\theta}}{\sigma}\right) d\tilde{\theta} = \underline{r}.$$

Next, for any $n \geq 1$, let $\theta_n \equiv \min\{\theta_s^*, \theta_n'\}$, where θ_n' solves $V(\theta_n', \Psi(\frac{x_{n-1} - \theta_n'}{\sigma})) = 0$, and let x_n be implicitly defined by

$$\int_{-\infty}^{\theta_n'} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{x_n - \tilde{\theta}}{\sigma}\right) d\tilde{\theta} = \underline{r}.$$

This sequence also has a simple interpretation. A speculator who observes $r = \underline{r}$ and believes that $r(\theta) = \underline{r}$ for all θ and that no other speculator attacks (in which case devaluation occurs for $\theta < \underline{\theta}$ and does not occur for $\theta > \underline{\theta}$), finds it optimal to attack if and only if $x \leq x_0$. By implication, a speculator who expects no other speculator to attack and $r(\theta) = \underline{r}$ for all $\theta < \theta_s^*$ (but possibly $r(\theta) > \underline{r}$ for some $\theta > \theta_s^*$) necessarily finds it optimal to attack for any $x < x_0$. This simply follows from the fact that the observation of $\underline{r}$ is most informative of devaluation when all types who devalue set $r = \underline{r}$, while some of the types who maintain the peg raise the policy above $\underline{r}$. However, if all speculators attack whenever $x < x_0$, the peg is abandoned for all $\theta < \theta_1$. This in turn implies that there exists an $x_1 > x_0$ such that a speculator who expects all other speculators to attack if $x < x_0$ (and hence the peg to be abandoned for all $\theta < \theta_1$) and who believes that $r(\theta) = \underline{r}$ for all θ, necessarily finds it optimal to attack for all $x < x_1$. By implication, a speculator who expects $r(\theta) = \underline{r}$ for all $\theta < \theta_s^*$ but possibly $r(\theta) > \underline{r}$ for some $\theta > \theta_s^*$, necessarily finds it optimal to attack for any $x < x_1$, and so on.

Because $\{\theta_n\}_{n=0}^{\infty}$ is increasing and bounded from above it necessarily converges. Note that V and Ψ are continuous and that the unique solution (x, θ') to $V(\theta', \Psi(\frac{x'-\theta'}{\sigma})) = 0$ and

$$\int_{-\infty}^{\theta'} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{x-\tilde{\theta}}{\sigma}\right) d\tilde{\theta} = \underline{r}$$

is attained at $\theta' = \theta^{\#}(\sigma)$ and $x = x^{\#}(\sigma)$ (by uniqueness of the monotone equilibrium established in Lemma 1). It follows that $\lim_{n\to+\infty} \theta_n = \theta_s^*$ if $\theta_s^* \leq \theta^{\#}(\sigma)$ and $\lim_{n\to+\infty} \theta_n = \theta^{\#}(\sigma)$ otherwise. By implication, $D(\theta) = 1$ for all $\theta < \min\{\theta_s^*, \theta^{\#}(\sigma)\}$. That $\theta_s^* > \theta^{\#}(\sigma)$ if and only if $s > r^{\#}(\sigma)$ follows from the fact that, for any θ, $W(\theta, \cdot, 0) - L(\theta, \underline{r})$ is continuous and strictly decreasing in s, while for any $s > \underline{r}$, $W(\cdot, s, 0) - L(\cdot, \underline{r})$ is continuous, strictly increasing in θ, and is negative for $\theta = \underline{\theta}$. *Q.E.D.*

Proof of Proposition 2. Fix $\varepsilon > 0$ and let

$$\theta^{\#}(0^+) \equiv \lim_{\sigma\to 0^+} \theta^{\#}(\sigma) \text{ and } r^{\#}(0^+) \equiv \lim_{\sigma\to 0^+} r^{\#}(\sigma) = \lim_{\sigma\to 0^+} \rho(\theta^{\#}(\sigma))$$

where, for any $\theta \geq \underline{\theta}$, $\rho(\theta)$ is implicitly defined by $W(\theta, \rho(\theta), 0) = L(\theta, \underline{r})$.

Part (i). Clearly, irrespective of σ, $\mathcal{E}(s;\sigma) = \varnothing$ whenever $W(\theta, s, 0) = L(\theta, \underline{r})$ admits no solution. Thus assume that a solution to the above equation exists and denote by θ_s^* its unique solution. We establish the result in three steps.

Step 1. By the continuity of the $\rho(\theta)$ function in θ and by the continuity of the $\theta^{\#}(\sigma)$ function in σ, there exist $\delta > 0$ and $\hat{\sigma} > 0$ such that the following is true for any $\sigma < \hat{\sigma}$: $\theta_s^* > \theta^{\#}(0^+) + \delta$ whenever $s > r^{\#}(\sigma) + \varepsilon$. To see this, fix $\varepsilon > 0$, and take $\varepsilon' < \varepsilon/2$. By continuity of $\rho(\theta)$ and $\theta^{\#}(\sigma)$ and the fact that $r^{\#}(\sigma) = \rho(\theta^{\#}(\sigma))$, there exists $\hat{\sigma} > 0$ such that, whenever $\sigma < \hat{\sigma}$, $|r^{\#}(\sigma) - r^{\#}(0^+)| < \varepsilon'$. Hence, for any $\sigma < \hat{\sigma}$, $s > r^{\#}(\sigma) + \varepsilon$ implies that $s > r^{\#}(0^+) - \varepsilon' + \varepsilon > r^{\#}(0^+) + \varepsilon'$. Furthermore, by the continuity of $\rho(\theta)$, there exists $\delta > 0$ such that $|\rho(\theta) - r^{\#}(0^+)| < \varepsilon'$ whenever $|\theta - \theta^{\#}(0^+)| < \delta$.

By the monotonicity of $\rho(\theta)$, we then have that $\rho(\theta) < s$ for any $\theta < \theta^{\#}(0^+) + \delta$ which means that $\theta_s^* \geq \theta^{\#}(0^+) + \delta$.

Step 2. Next note that there exist $0 < \sigma' < \hat{\sigma}$ and $\theta^{\max} > \underline{\theta}$ such that, for any $\sigma < \sigma'$, any $s > r^{\#}(\sigma) + \varepsilon$, any A, any $\theta > \theta^{\max}$, $W(\theta, s, 0) < W(\theta, \underline{r}, A)$. This follows from the limit condition that, for any $s > \underline{r}$, $\lim_{\theta \to +\infty}[W(\theta, s, 0) - W(\theta, \underline{r}, 1)] < 0$. Then let

$$Q \equiv \{(\theta^*, \theta) : \theta^* \in [\theta^{\#}(0^+) + \delta, \theta^{\max}], \theta \in [\theta^*, \theta^{\max}]\}$$

and, for any (θ^*, θ), $\theta \geq \theta^*$ let

$$G(\theta; \theta^*, \sigma) \equiv W(\theta, \rho(\theta^*), 0) - W\left(\theta, \underline{r}, B(\theta, \theta^*; \sigma)\right)$$

where

$$B(\theta, \theta^*; \sigma) \equiv \Psi\left(\frac{X(\theta^*, \theta; \sigma) - \theta}{\sigma}\right) \tag{16}$$

Note that, for any σ, the function $G(\cdot; \cdot, \sigma)$ is continuous over Q. Below, we establish that, for any $(\theta^*, \theta) \in Q$, $G(\theta; \theta^*, \cdot)$ is increasing in σ with $\lim_{\sigma \to 0^+} G(\theta; \theta^*, \sigma) < 0$. To see this, first note that, from (5), $B = B(\theta, \theta^*; \sigma)$ is implicitly defined by

$$\int_{-\infty}^{\theta^*} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{X(\theta^*, \theta; \sigma) - \tilde{\theta}}{\sigma}\right) d\tilde{\theta} - \underline{r}\left[1 - \Psi\left(\frac{X(\theta^*, \theta; \sigma) - \theta^*}{\sigma}\right) + B\right] = 0.$$

Integrating by parts and using (16) we then have that B is implicitly defined by

$$\begin{aligned}&\left(Z\left(\theta^*, \underline{r}\right) - \underline{r}\right)\left(1 - \Psi\left(\Psi^{-1}(B) + \frac{\theta - \theta^*}{\sigma}\right)\right) \\ &- \int_{-\infty}^{\theta^*} \frac{dZ(\tilde{\theta}, \underline{r})}{d\tilde{\theta}}\left(1 - \Psi\left(\Psi^{-1}(B) + \frac{\theta - \tilde{\theta}}{\sigma}\right)\right) d\tilde{\theta} = \underline{r}B.\end{aligned} \tag{17}$$

One can then easily verify that, for any $\theta > \theta^*$, $\lim_{\sigma \to 0^+} B(\theta, \theta^*; \sigma) = 0$ while for $\theta = \theta^*$, $\lim_{\sigma \to 0^+} B(\theta, \theta^*; \sigma) = \left(Z\left(\theta^*, \underline{r}\right) - \underline{r}\right)/Z(\theta^*, \underline{r})$. This in turn implies that, for any $(\theta^*, \theta) \in Q$, $\theta > \theta^*$, $\lim_{\sigma \to 0^+} G(\theta, \theta^*; \sigma) < 0$, whereas, for any $(\theta^*, \theta) \in Q$ such that $\theta = \theta^*$,

$$\lim_{\sigma \to 0^+} G(\theta, \theta^*; \sigma) = L(\theta^*, \underline{r}) - W\left(\theta^*, \underline{r}, \left(Z\left(\theta^*\right) - \underline{r}\right)/Z(\theta^*)\right) = -V\left(\theta^*, \left(Z\left(\theta^*\right) - \underline{r}\right)/Z(\theta^*)\right) < 0$$

where the last inequality follows from the fact that the function $V\left(\theta^*, \left(Z\left(\theta^*\right) - \underline{r}\right)/Z(\theta^*)\right)$ is strictly increasing in θ^* and equal to zero at $\theta^* = \theta^{\#}(0^+)$ (this follows from Lemma 1).

Next, using the implicit function theorem, note that

$$\begin{aligned}\frac{\partial B(\theta; \theta^*, \sigma)}{\partial \sigma} &= -\frac{\left(Z\left(\theta^*, \underline{r}\right) - \underline{r}\right)\psi\left(\Psi^{-1}(B) + \frac{\theta - \theta^*}{\sigma}\right)\left(\frac{\theta - \theta^*}{\sigma^2}\right) - \int_{-\infty}^{\theta^*} \frac{dZ(\tilde{\theta}, \underline{r})}{d\tilde{\theta}} \psi\left(\Psi^{-1}(B) + \frac{\theta - \tilde{\theta}}{\sigma}\right)\left(\frac{\theta - \tilde{\theta}}{\sigma^2}\right) d\tilde{\theta}}{\left[\begin{array}{c} -\left(Z\left(\theta^*, \underline{r}\right) - \underline{r}\right)\psi\left(\Psi^{-1}(B) + \frac{\theta - \theta^*}{\sigma}\right)\frac{d\Psi^{-1}(B)}{dB} \\ + \int_{-\infty}^{\theta^*} \frac{dZ(\tilde{\theta}, \underline{r})}{d\tilde{\theta}} \psi\left(\Psi^{-1}(B) + \frac{\theta - \tilde{\theta}}{\sigma}\right)\frac{d\Psi^{-1}(B)}{dB} d\tilde{\theta} - \underline{r} \end{array}\right]} \\ &> 0\end{aligned}$$

This establishes that, for any $(\theta^*, \theta) \in Q$, $G(\theta; \theta^*, \cdot)$ is increasing in σ with $\lim_{\sigma \to 0^+} G(\theta; \theta^*, \sigma) < 0$.

Step 3. From Dini's theorem, we then conclude that there exists a $\sigma_1 < \sigma'$ such that, for any $\sigma < \sigma_1$, $G(\theta; \theta^*, \sigma) < 0$ for any $(\theta^*, \theta) \in Q$. Furthermore, for any $\sigma < \sigma_1$, $G(\theta; \theta^*, \sigma) < 0$ for any (θ^*, θ) such that either (a) $\theta^* \in [\theta^{\#}(0^+) + \delta, \theta^{\max}]$ and $\theta > \theta^{\max}$, or (b) $\theta^* > \theta^{\max}$ and $\theta \geq \theta^*$. Together with the property established in step 1 that $\theta^*_s > \theta^{\#}(0^+) + \delta$ whenever $\sigma < \sigma_1$ and $s > r^{\#}(\sigma) + \varepsilon$, this means that for any $\sigma < \sigma_1$, any $s > r^{\#}(\sigma) + \varepsilon$, there is no solution to the system of equations given by (8) and (9). From Proposition (1) this means that, for any $\sigma < \sigma_1$, $\mathcal{E}(s; \sigma) = \varnothing$ for any $s > r^{\#}(\sigma) + \varepsilon$.

Part (ii). Let $\varepsilon > 0$ be the same as in part (i). By the continuity and strict monotonicity of the $\rho(\theta)$ function, together with the continuity of the $\theta^{\#}(\sigma)$ function, there exist $\sigma' > 0$ and $\varepsilon' \in (0, \varepsilon)$ such that, for any $\sigma < \sigma'$, $\theta_s > \theta^{\#}(\sigma) + \varepsilon$ implies that $s > r^{\#}(\sigma) + \varepsilon'$. The result in part (i) then implies that there exists a $\sigma_2 \in (0, \sigma')$ such that, for any $\sigma < \sigma_2$, $\mathcal{E}(s; \sigma) \neq \varnothing$ only if $\theta^*_s \leq \theta^{\#}(\sigma) + \varepsilon$.

Part (iii). Take the same $\varepsilon > 0$ as in parts (i) and (ii) and let $r = \underline{r} + \varepsilon$. By the limit condition that $\lim_{\theta \to +\infty}[W(\theta, r, 0) - W(\theta, \underline{r}, 1)] < 0$, there exists a $\theta^{\max}$ such that, for $\theta > \theta^{\max}$, any A, any $s \geq \underline{r} + \varepsilon$, $W(\theta, s, 0) < W(\theta, \underline{r}, A)$. This means that, for any $s \geq \underline{r} + \varepsilon$, if a solution (θ^*_s, θ) to (8) and (9) exists, then necessarily $\theta^*_s, \theta \leq \theta^{\max}$. In turn, this also means that, for any $s \geq \underline{r} + \varepsilon$, $\mathcal{E}(s; \sigma) \neq \varnothing$ only if $s \leq \rho(\theta^{\max})$. Thus assume $\rho(\theta^{\max}) \geq \underline{r} + \varepsilon$. For any $s \in [\underline{r} + \varepsilon, \rho(\theta^{\max})]$, then let θ^*_s be the unique solution to (8) and take any arbitrary $\hat{\theta} > \theta^*_s$. Because $\lim_{\sigma \to 0^+} B(\hat{\theta}, \theta^*_s; \sigma) = 0$, and because $B(\cdot, \theta^*_s; \sigma)$ is decreasing in θ (this can be seen from (17)), we then have that, for any $s \in [\underline{r} + \varepsilon, \rho(\theta^{\max})]$, either (9) admits no solution for σ small enough, in which case $\mathcal{E}(s; \sigma) \neq \varnothing$, or, if it does, then necessarily its highest solution $\theta^{**}(\sigma)$ must converge to θ^*_s as $\sigma \to 0$. For any $s \in [\underline{r} + \varepsilon, \rho(\theta^{\max})]$ any σ, then let $\Delta_s(\sigma) = \theta^{**}(\sigma) - \theta^*_s$ if (9) admits a solution and $\Delta_s(\sigma) = 0$ otherwise. Because for any θ^*, θ, $\theta \geq \theta^*$, $B(\theta; \theta^*, \sigma)$ is increasing in σ, and because at the highest solution to equation (9), the function $G(\theta; \theta^*_s, \sigma)$ is necessarily decreasing in in θ – this follows again from the fact that $\lim_{\theta \to +\infty}[W(\theta, s, 0) - W(\theta, \underline{r}, 1)] < 0$ – we then have that, for any $s \in [\underline{r} + \varepsilon, \rho(\theta^{\max})]$, $\Delta_s(\sigma)$ is increasing in σ with $\lim_{\sigma \to 0^+} \Delta(\sigma) = 0$. Because, for any $\sigma > 0$, $\Delta_s(\sigma)$ is clearly continuous in s over $[\underline{r} + \varepsilon, \rho(\theta^{\max})]$, from Dini's theorem we then have that there exists a $\sigma_3 > 0$ such that, for $\sigma < \sigma_3$, any $s \in [\underline{r} + \varepsilon, \rho(\theta^{\max})]$, either $\mathcal{E}(s; \sigma) = \varnothing$, or $\theta^{**}_s(\sigma) \leq \theta^*_s + \varepsilon$, which establish the result.

Combining the proofs of parts (i)-(iii), the result in the proposition then follows by letting $\bar{\sigma} = \min\{\sigma_1, \sigma_2, \sigma_3\}$. *Q.E.D.*

Proof of Proposition 3. We start by establishing the following preliminary result.

Lemma A2. *For any $\sigma > 0$ any $s \in (\underline{r}, r^{\#}(\sigma)]$, the system of equations given by (8) and (9) admits at least one solution (θ^*_s, θ''_s) and is such that $\theta''_s > \theta^*_s$ if $s < r^{\#}(\sigma)$.*

Proof of Lemma A2. Fix $\sigma > 0$. By definition, $r^{\#}(\sigma)$ solves $W\left(\theta^{\#}(\sigma), r^{\#}(\sigma), 0\right) = L\left(\theta^{\#}(\sigma), \underline{r}\right)$, where $\theta^{\#}(\sigma) \in (\underline{\theta}, \bar{\theta})$ is the unique threshold that corresponds to the pooling equilibria of parts (ii) and (iii) in Proposition (1). Because, for any θ, $W(\theta, \cdot, 0)$ is continuous and strictly decreasing in s and because, for any s, $W(\cdot, s, 0) - L(\cdot, \underline{r})$ is continuous and strictly increasing in θ, we have that, for any $s \in (\underline{r}, r^{\#}(\sigma)]$ equation (8) always admits a solution θ_s^*. Such a solution is unique and strictly lower than $\theta^{\#}(\sigma)$ if $s < r^{\#}(\sigma)$. Now fix both $\sigma > 0$ and $s \in (\underline{r}, r^{\#}(\sigma)]$. Note that the function

$$G(\theta; \theta_s^*, \sigma) \equiv W(\theta, s, 0) - W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta; \sigma) - \theta}{\sigma}\right)\right)$$

is continuous in θ over $[\theta_s^*, +\infty)$ with

$$\begin{aligned} G(\theta_s^*; \theta_s^*, \sigma) &= L(\theta_s^*, \underline{r}) - W\left(\theta_s^*, \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta_s^*; \sigma) - \theta_s^*}{\sigma}\right)\right) \\ &= -V\left(\theta_s^*, \Psi\left(\frac{X(\theta_s^*, \theta_s^*; \sigma) - \theta_s^*}{\sigma}\right)\right) \end{aligned}$$

and $\lim_{\theta \to +\infty} G(\theta; \theta_s^*, \sigma) < 0$. Next, consider the function $\tilde{V}(\theta) \equiv V\left(\theta, \Psi\left(\frac{X(\theta,\theta;\sigma)-\theta}{\sigma}\right)\right)$ and note that $X(\theta, \theta; \sigma)$ coincides with the unique solution to

$$\int_{-\infty}^{\theta} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma} \psi\left(\tfrac{x-\tilde{\theta}}{\sigma}\right) d\tilde{\theta} = \underline{r}$$

From the results established in the proof of Lemma 1, one can then verify that $\tilde{V}(\theta)$ is negative for $\theta < \theta^{\#}(\sigma)$ and positive for $\theta > \theta^{\#}(\sigma)$. Because $\theta_s^* \leq \theta^{\#}(\sigma)$ for any $s \in (\underline{r}, r^{\#}(\sigma)]$, we then have that $G(\theta_s^*; \theta_s^*, \sigma) \geq 0$ whenever $s \in (\underline{r}, r^{\#}(\sigma)]$, with strict inequality for $s < r^{\#}(\sigma)$. By the Intermediate Value Theorem, it then follows that a solution $\theta_s'' \geq \theta_s^*$ to equation (9) always exists when $s \in (\underline{r}, r^{\#}(\sigma)]$ and is strictly higher than θ_s^* if $s < r^{\#}(\sigma)$. ■

Next, we show that, when SCC holds, then for any $\sigma > 0$, any $s > r^{\#}(\sigma)$, $\mathcal{E}(s; \sigma) = \varnothing$. To see this, note that, when $s > r^{\#}(\sigma)$, then either (8) admits no solution, or, if it does, then its unique solution θ_s^* satisfies

$$V\left(\theta_s^*, \Psi\left(\frac{X(\theta_s^*, \theta_s^*; \sigma) - \theta_s^*}{\sigma}\right)\right) > 0$$

which means that $G(\theta_s^*; \theta_s^*, \sigma) < 0$. This in turn implies that, for any $\theta \geq \theta_s^*$.

$$\begin{aligned} G(\theta; \theta_s^*, \sigma) &\equiv W(\theta, s, 0) - W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta; \sigma) - \theta}{\sigma}\right)\right) \\ &= W(\theta, s, 0) - W(\theta, \underline{r}, B(\theta_s^*, \theta; \sigma)) \\ &< W(\theta, s, 0) - W(\theta, \underline{r}, B(\theta_s^*, \theta_s^*; \sigma)) \\ &< 0 \end{aligned}$$

where the first inequality follows from the fact that

$$B(\theta_s^*, \theta; \sigma) \equiv \Psi\left(\frac{X(\theta_s^*, \theta; \sigma) - \theta}{\sigma}\right)$$

is decreasing in θ (as shown in the proof of Proposition 2), while the second inequality follows from SCC. We conclude that equation (9) admits no solution for $s > r^{\#}(\sigma)$. From Proposition 1), this means that $\mathcal{E}(s;\sigma) = \varnothing$.

Thus consider $s \leq r^{\#}(\sigma)$. That $\mathcal{E}(\underline{r};\sigma) \neq \varnothing$ follows from Lemma 1. For any $s \in (\underline{r}, r^{\#}(\sigma)]$, the result that $\mathcal{E}(s;\sigma) \neq \varnothing$, as well as the existence of equilibria satisfying the properties of part (ii) in the proposition follows from Lemma A3 below.

Lemma A3. *Assume SCC holds. For any $\sigma > 0$, any $s \in (\underline{r}, r^{\#}(\sigma)]$, there exists an equilibrium in which $r(\theta) = s$ if $\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]$, $r(\theta) = \underline{r}$ otherwise, $a(x,r) = 1$ if and only if $(x,r) < (x_s^*, s)$, and $D(\theta) = 1$ if and only if $\theta < \theta_s^*$, where $x_s^* = X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)$.*

Proof of Lemma A3. From Lemma A2, for any $\sigma > 0$ any $s \in (\underline{r}, r^{\#}(\sigma)]$, a solution to (8) and (9) always exists. Then let θ_s^* be the unique solution to (8) and $\theta_s^{**}(\sigma)$ the highest solution to (9). We then have that, for any $\theta \in [\theta_s^*, \theta_s^{**}(\sigma))$

$$\begin{aligned}
& W(\theta, s, 0) - W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) - \theta}{\sigma}\right)\right) \\
> \; & W(\theta, s, 0) - W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) - \theta_s^{**}(\sigma)}{\sigma}\right)\right) \\
\geq \; & W(\theta_s^{**}(\sigma), s, 0) - W\left(\theta_s^{**}(\sigma), \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) - \theta_s^{**}(\sigma)}{\sigma}\right)\right) \\
= \; & 0
\end{aligned}$$

where the first inequality follows from the fact that W is decreasing in A, the second inequality from SCC, and the last equality from the definition of $\theta_s^{**}(\sigma)$. Similar arguments imply that for any $\theta > \theta_s^{**}(\sigma)$

$$W(\theta, s, 0) - W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) - \theta}{\sigma}\right)\right) < 0.$$

We are now ready to establish the result in the lemma. For simplicity, in the equilibria constructed below, we let $D(\theta, r, A) = 0$ when $V(\theta, A) = 0$, meaning that, when indifferent, the policy maker maintains the peg.

Because $A(\theta, r) = A(\theta, \underline{r}) = \Psi(\frac{x_s^* - \theta}{\sigma})$ for any $r < s$ and $A(\theta, r) = A(\theta, s) = 0$ for any $r \geq s$, the policy maker strictly prefers $\underline{r}$ to any $r \in (\underline{r}, s)$ and s to any $r > s$. Furthermore, $\underline{r}$ is dominant for any $\theta \leq \underline{\theta}$. For $\theta > \underline{\theta}$, on the other hand, the payoff from raising the policy at $r = s$ is $W(\theta, s, 0)$, while the payoff from leaving the policy at $r = \underline{r}$ is $\max\{L(\theta, \underline{r}), W(\theta, \underline{r}, A(\theta, \underline{r}))\}$. From the definition of the thresholds θ_s^* and $\theta_s^{**}(\sigma)$ and the properties established above, we then have that raising the policy at $r = s$ is optimal if and only if $\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]$, which establishes the optimality of the policy maker's strategy.

Next, consider the speculators. When $r < s$, $D(\theta, r, A(\theta, r)) = 1$ if and only if $\theta < \hat{\theta}$, where $\hat{\theta}$ is the unique solution to

$$V(\hat{\theta}, A(\hat{\theta}, \underline{r})) = 0.$$

When, instead, $r \geq s$, $D(\theta, r, A(\theta, r)) = 1$ if and only if $\theta < \underline{\theta}$. A speculator thus finds it optimal to follow the equilibrium strategy if and only if his beliefs satisfy the following two conditions:

$$\begin{array}{ll} \text{when } r < s, & \int_{-\infty}^{\hat{\theta}} Z(\tilde{\theta}, \underline{r}) d\mu(\tilde{\theta}|x, r) \geq r \text{ if } x < x_s^* \\ & \text{and } \int_{-\infty}^{\hat{\theta}} Z(\tilde{\theta}, \underline{r}) d\mu(\tilde{\theta}|x, r) \leq r \text{ if } x \geq x_s^*; \end{array} \tag{18}$$

$$\text{when } r \geq s, \int_{-\infty}^{\underline{\theta}} Z(\tilde{\theta}, \underline{r}) d\mu(\tilde{\theta}|x, r) \leq r \text{ for all } x. \tag{19}$$

Beliefs are pinned down by Bayes' rule when either $r = \underline{r}$ or $r = s$. In the first case ($r = \underline{r}$), for any $\theta \leq \theta_s^*$

$$\mu(\theta|x, \underline{r}) = \frac{1 - \Psi\left(\frac{x-\theta}{\sigma}\right)}{1 - \Psi(\frac{x-\theta_s^*}{\sigma}) + \Psi(\frac{x-\theta_s^{**}(\sigma)}{\sigma})},$$

while for any $\theta \in (\theta_s^*, \hat{\theta})$,

$$\mu(\theta|x, \underline{r}) = \frac{1 - \Psi\left(\frac{x-\theta_s^*}{\sigma}\right)}{1 - \Psi(\frac{x-\theta_s^*}{\sigma}) + \Psi(\frac{x-\theta_s^{**}(\sigma)}{\sigma})}.$$

These beliefs are clearly decreasing in x. Along with the fact that $Z(\cdot, \underline{r})$ is nonincreasing and the definition of x_s^*, it then follows that condition (18) is satisfied when $r = \underline{r}$ (the arguments are the same as in the proof of Lemma 4 in the main text). In the second case ($r = s$), $\mu(\underline{\theta}|x, s) = 0$, in which case condition (19) is clearly satisfied. Finally, whenever $r \notin \{\underline{r}, s\}$, there exist an arbitrarily large set of out-of-equilibrium beliefs that satisfy (18) and (19).

Combining the optimality of the speculators' strategies with the optimality of the policy maker's strategy gives the result. ■ *Q.E.D.*

Proof of Proposition 4. Part (i) follows from Lemma A4 below, whose proof is similar to that of Proposition 5 in Angeletos, Hellwig and Pavan (2006), adapted to the payoff structure considered here. Part (ii), on the other hand, follows from Lemma A5.

Lemma A4. *Assume ψ is log-concave. For any $\sigma > 0$, any $s \in (\underline{r}, r^{\#}(\sigma)]$, there exists a nonempty set of types $\Theta_s(\sigma) \subset [\theta_s^*, \theta_s^{**}(\sigma)]$, with* $\inf \Theta_s(\sigma) = \theta_s^*$, *a threshold x_s^*, and an equilibrium in which*

$$r(\theta) = \begin{cases} s & \text{if } \theta \in \Theta_s(\sigma) \\ \underline{r} & \text{otherwise} \end{cases} \qquad a(x, r) = 1 \text{ iff } (x, r) < (x_s^*, s) \text{ and } D(\theta) = \begin{cases} 1 & \text{if } \theta < \theta_s^* \\ 0 & \text{otherwise} \end{cases}$$

Proof of Lemma A4. The proof is in three steps. Steps 1 and 2 construct the set $\Theta_s(\sigma)$ and establish properties that are useful for step 3. Step 3 shows that there exists a system of beliefs that support the proposed strategies as part of an equilibrium.

Step 1. Fix $s \in (\underline{r}, r^{\#}(\sigma)]$ and let θ_s^* denote the unique solution to (8). Next, let $S : \mathbb{R} \to 2^{\mathbb{R}}$ and $m : \mathbb{R}^2 \to [0,1]$ denote, respectively, the correspondence and the function defined as follows:

$$\begin{aligned} S(x) &\equiv \left\{\theta : W(\theta, s, 0) \geq \max\left\{W\left(\theta, \underline{r}, \Psi\left(\tfrac{x-\theta}{\sigma}\right)\right), L(\theta, \underline{r})\right\}\right\}, \\ m(x, x') &\equiv \frac{\int_{-\infty}^{\theta_s^*} Z(\tilde{\theta}, \underline{r}) \frac{1}{\sigma}\psi(\frac{x'-\tilde{\theta}}{\sigma})d\tilde{\theta}}{1 - \int_{S(x)} \frac{1}{\sigma}\psi(\frac{x'-\tilde{\theta}}{\sigma})d\tilde{\theta}} - \underline{r} \end{aligned}$$

The set $S(x)$ represents the set of types who prefer raising the policy at $r = s$ and then facing no attack to leaving the policy at $\underline{r}$, facing an attack of size $\Psi\left(\frac{x-\theta}{\sigma}\right)$, and then optimally choosing whether or not to devalue. By definition, θ_s^* denotes the lowest type who prefers raising the policy at $r = s$ and face no attack to leaving the policy at $r = \underline{r}$ and then devaluing. It follows that $\theta_s^* \leq \inf S(x)$ for any x. In turn, $m(x, x')$ is the expected payoff from attacking for a speculator with signal x' when he observes $\underline{r}$ and believes that the peg will be abandoned if and only if $\theta < \theta_s^*$ and that the policy is $r(\theta) = \underline{r}$ if and only if $\theta \notin S(x)$.

Step 2 below shows that, when ψ is log-concave, then for any x, the function $m(x, \cdot)$ is nonincreasing in x'. It then uses this property to show that either (i) there exists an $x_s^* \in \mathbb{R}$ such that $m(x_s^*, x_s^*) = 0$, or (ii) $m(x, x) > 0$ for all x, in which case we let $x_s^* = +\infty$. In either case, the triple $(x_s^*, \theta_s^*, \Theta_s(\sigma))$ with $\Theta_s(\sigma) = S(x_s^*)$ identifies an equilibrium for the fictitious game in which the policy maker is restricted to set $r \in \{\underline{r}, s\}$ and the speculators are restricted not to attack when $r = s$. Step 3 concludes the proof by showing that the same triple identifies an equilibrium for the unrestricted game.

Step 2. Note that $S(x)$ is continuous in x, with $S(x_1) \subseteq S(x_2)$ for any $x_1 \leq x_2$ (this follows from the fact that the policy maker's payoff from not raising the policy declines with the aggressiveness of the speculators' behavior). Also note that $m(x, x')$ is continuous in (x, x') and nondecreasing in x (by the monotonicity of $S(x)$). Below we show that, when ψ is log-concave, $m(x, x')$ is also nonincreasing in x'. To see this, note that, for any $\theta' \leq \theta_s^*$, the probability that a speculator with signal x' assigns to $\tilde{\theta} < \theta'$ when observing $r = \underline{r}$ is $\mu(\theta'|x', \underline{r}) = (1 + 1/M(x'))^{-1}$, where

$$M(x') \equiv \frac{1 - \Psi(\frac{x'-\theta'}{\sigma})}{\int_{\theta'}^{\infty}\left[1 - I_x\left(\tilde{\theta}\right)\right]\frac{1}{\sigma}\psi(\frac{x'-\tilde{\theta}}{\sigma})d\tilde{\theta}}$$

with $I_x(\tilde{\theta}) = 1$ when $\tilde{\theta} \in S(x)$ and $I_x(\tilde{\theta}) = 0$ otherwise. It follows that $\mu(\theta'|x', \underline{r})$ is decreasing in x' if $d \ln M(x')/dx' < 0$ or, equivalently, if

$$\frac{\int_{-\infty}^{\theta'} \frac{1}{\sigma^2}\psi'\left(\frac{x'-\tilde{\theta}}{\sigma}\right)d\tilde{\theta}}{\int_{-\infty}^{\theta'} \frac{1}{\sigma}\psi\left(\frac{x'-\tilde{\theta}}{\sigma}\right)d\tilde{\theta}} - \frac{\int_{\theta'}^{\infty}\left[1 - I_x\left(\tilde{\theta}\right)\right]\frac{1}{\sigma^2}\psi'\left(\frac{x'-\tilde{\theta}}{\sigma}\right)d\tilde{\theta}}{\int_{\theta'}^{\infty}\left[1 - I_x\left(\tilde{\theta}\right)\right]\frac{1}{\sigma}\psi\left(\frac{x'-\tilde{\theta}}{\sigma}\right)d\tilde{\theta}} < 0. \tag{20}$$

Using the fact that $I_x\left(\tilde{\theta}\right) = 0$ for all $\tilde{\theta} \leq \theta'$, (20) is equivalent to

$$\mathbb{E}_{\tilde{\theta}}\left[\left.\frac{\psi'\left(\frac{x'-\tilde{\theta}}{\sigma}\right)}{\psi\left(\frac{x'-\tilde{\theta}}{\sigma}\right)}\right| \tilde{\theta} \leq \theta', x', \underline{r}\right] - \mathbb{E}_{\tilde{\theta}}\left[\left.\frac{\psi'\left(\frac{x'-\tilde{\theta}}{\sigma}\right)}{\psi\left(\frac{x'-\tilde{\theta}}{\sigma}\right)}\right| \tilde{\theta} > \theta', x', \underline{r}\right] < 0,$$

which holds true when ψ'/ψ is decreasing, i.e. when ψ is log-concave. That $Z(\cdot,\underline{r})$ is nonincreasing together with the fact that $\mu(\theta'|x',\underline{r})$ is decreasing in x' for any $\theta' \leq \theta_s^*$ then implies that the expected devaluation premium

$$m(x,x') = \frac{\int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}\psi\left(\frac{x'-\tilde{\theta}}{\sigma}\right)d\tilde{\theta}}{1-\Psi(\frac{x'-\theta_s^*}{\sigma})+\int_{\theta_s^*}^{\infty}\left[1-I_x\left(\tilde{\theta}\right)\right]\frac{1}{\sigma}\psi(\frac{x'-\tilde{\theta}}{\sigma})d\tilde{\theta}} - \underline{r}$$

is nonincreasing in x' (the argument is the same as in the proof of Lemma 4). For future reference, also note that $\lim_{x'\to+\infty} m(x,x') < 0$.

Having established that the expected payoff from attacking $m(x,x')$ is continuous in (x,x'), nondecreasing in x and nonincreasing in x', next note that

$$S(x) \subseteq \bar{S} \equiv \{\theta : W(\theta,s,0) \geq \max\{W(\theta,\underline{r},1), L(\theta,\underline{r})\}\}$$

which in turn implies that, for any $(x,x') \in \mathbb{R}^2$,

$$\frac{\int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}1\psi(\frac{x'-\tilde{\theta}}{\sigma})d\tilde{\theta}}{1-\int_{\bar{S}}\frac{1}{\sigma}\psi(\frac{x'-\tilde{\theta}}{\sigma})d\tilde{\theta}} - \underline{r} \geq m(x,x') \geq \int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}\psi\left(\frac{x'-\tilde{\theta}}{\sigma}\right)d\tilde{\theta} - \underline{r}.$$

It follows that, for any x, $m(x,x') \geq 0$ for all $x' \leq \hat{x}$, where $\hat{x} \in \mathbb{R}$ is the unique solution to

$$\int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}\psi\left(\frac{\hat{x}-\tilde{\theta}}{\sigma}\right)d\tilde{\theta} = \underline{r}$$

Now define the sequence $\{x_k\}_{k=0}^{\infty}$, with $x_k \in \mathbb{R}\cup\{+\infty\}$, as follows. For $k=0$, let $x_0 \equiv \hat{x}$. For $k \geq 1$, let x_k be the solution to $m(x_{k-1},x_k) = 0$ if $x_{k-1} < +\infty$; if, instead, $x_{k-1} = +\infty$, let $x_k \equiv \inf\{x' : m(x_{k-1},x') \leq 0\}$ if $\{x' : m(x_{k-1},x') \leq 0\} \neq \varnothing$ and $x_k \equiv +\infty$ otherwise. The fact that

$$m(\hat{x},\hat{x}) \geq \int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}\psi\left(\frac{\hat{x}-\tilde{\theta}}{\sigma}\right)d\tilde{\theta} - \underline{r} = 0$$

together with the continuity and monotonicities of m, ensures that this sequence is well defined and nondecreasing. It follows that either $\lim_{k\to\infty} x_k \in [\hat{x},+\infty)$, or $\lim_{k\to\infty} x_k = +\infty$. In the former case, let $x_s^* = \lim_{k\to\infty} x_k$ and $\Theta_s(\sigma) = S(x_s^*)$; in the latter, let $x_s^* = +\infty$ and $\Theta_s(\sigma) = S(\infty) \equiv \bar{S}$.

Finally, to see that $\inf\Theta_s(\sigma) = \theta_s^*$ note that the threshold $\hat{x}$ defined above coincides with $X(\theta_s^*,\theta_s^*;\sigma)$. Because $\theta_s^* \leq \theta^{\#}(\sigma)$, from the results in the proof of Lemma A2 above, we then have that $\tilde{V}(\theta_s^*) \equiv V(\theta_s^*,\Psi((X(\theta_s^*,\theta_s^*;\sigma)-\theta_s^*)/\sigma)) < 0$. Because $x_s^* \geq \hat{x}$, we then have that $V(\theta_s^*,\Psi((x_s^*-\theta_s^*)/\sigma)) \leq 0$. Because $V(\theta,A)$ is increasing in θ and decreasing in A and because $A(\theta,\underline{r}) = \Psi(\frac{x_s^*-\theta}{\sigma})$ is decreasing in θ, this in turn implies that there exits $\hat{\theta} \in [\theta_s^*,\theta_s^{**}(\sigma)]$ such that $V(\hat{\theta},\Psi((x_s^*-\hat{\theta})/\sigma)) \leq 0$ if and only if $\theta \leq \hat{\theta}$. It follows that necessarily $r(\theta) = s$ for all $\theta \in [\theta_s^*,\hat{\theta}]$, thus establishing that $\inf\Theta_s(\sigma) = \theta_s^*$.

We conclude that the triple $(x_s^*,\theta_s^*,\Theta_s(\sigma))$ identifies an equilibrium for the fictitious game in which the policy maker is restricted to set $r \in \{\underline{r},s\}$ and the speculators are restricted not to attack when $r = s$.

Step 3. We now show how the triple $(x_s^*, \theta_s^*, \Theta_s(\sigma))$ of Step 2 also identifies an equilibrium for the unrestricted game. The proof here parallels that of Lemma A3. Below we simply show existence of beliefs for the speculators satisfying conditions (18) and (19).

When $r = \underline{r}$, beliefs are pinned down by Bayes rule and such that, for any $\theta \leq \theta_s^*$,

$$\mu(\theta|x, \underline{r}) = \frac{1 - \Psi(\frac{x-\theta}{\sigma})}{1 - \int_{\Theta_s(\sigma)} \frac{1}{\sigma}\psi(\frac{x-\tilde{\theta}}{\sigma})d\tilde{\theta}},$$

while for any $\theta \in (\theta_s^*, \hat{\theta})$,[28]

$$\mu(\theta|x, \underline{r}) = \frac{1 - \Psi(\frac{x-\theta_s^*}{\sigma})}{1 - \int_{\Theta_s(\sigma)} \frac{1}{\sigma}\psi(\frac{x-\tilde{\theta}}{\sigma})d\tilde{\theta}}.$$

As shown above, these beliefs are decreasing in x. By the definition of x_s^*, it then follows that condition (18) is satisfied when $r = \underline{r}$. Next consider $r = s$. Again, in this case beliefs are pinned down by Bayes rule and such that $\mu(\underline{\theta}|x, s) = 0$, in which case condition (19) is clearly satisfied. Finally, whenever $r \notin \{\underline{r}, s\}$, there exist an arbitrarily large set of out-of-equilibrium beliefs that satisfy (18) and (19). ■

Lemma A5. *Suppose SCC holds and ψ is log-concave. Then for any $\sigma > 0$, any $s \in (\underline{r}, r^{\#}(\sigma)]$, any equilibrium in $\mathcal{E}(s;\sigma)$ is such that $r(\theta) = s$ for all $\theta \in (\theta_s^*, \theta_s^{**}(\sigma))$.*

Proof of Lemma A5. The result for $s = r^{\#}(\sigma)$ follows directly from the fact that, when SCC holds, then $\theta_s^* = \theta^{\#}(\sigma) = \theta_s^{**}(\sigma)$. That $\theta_s^* = \theta^{\#}(\sigma)$ is immediate. To see that $\theta_s^{**}(\sigma) = \theta^{\#}(\sigma)$, recall that, by definition,

$$W\left(\theta^{\#}(\sigma), \underline{r}, \Psi\left(\frac{X(\theta^{\#}(\sigma), \theta^{\#}(\sigma); \sigma) - \theta^{\#}(\sigma)}{\sigma}\right)\right) = L(\theta^{\#}(\sigma), \underline{r}) = W\left(\theta^{\#}(\sigma), r^{\#}(\sigma), 0\right)$$

Under SCC, this means that, for any $\theta > \theta^{\#}(\sigma)$, $W\left(\theta, r^{\#}(\sigma), 0\right) < W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta^{\#}(\sigma),\theta^{\#}(\sigma);\sigma)-\theta^{\#}(\sigma)}{\sigma}\right)\right)$. Because $\Psi\left(\frac{X(\theta^{\#}(\sigma),\theta^{\#}(\sigma);\sigma)-\theta^{\#}(\sigma)}{\sigma}\right) \geq \Psi\left(\frac{X(\theta^{\#}(\sigma),\theta;\sigma)-\theta}{\sigma}\right)$, as shown in the proof of Proposition 3, this means that, for any $\theta > \theta^{\#}(\sigma)$, $W(\theta, s, 0) < W\left(\theta, \underline{r}, \Psi\left(\frac{X(\theta^{\#}(\sigma),\theta;\sigma)-\theta}{\sigma}\right)\right)$, which implies that $\theta_s^{**}(\sigma) = \theta_s^* = \theta^{\#}(\sigma)$ for $s = r^{\#}(\sigma)$.

Thus consider $s \in (\underline{r}, r^{\#}(\sigma))$. From the proof of Lemma 4, $a(x, \underline{r}) = 0$ for all $x > X(\theta_s^*, \theta_s^{**}(\sigma); \sigma)$, while from the proof of Lemma 5, $a(x, \underline{r}) = 1$ for all $x < X(\theta_s^*, \theta_s^*; \sigma)$. It follows that $\Psi\left(\frac{X(\theta_s^*,\theta_s^{**}(\sigma))-\theta}{\sigma}\right) \geq A(\theta, \underline{r}) \geq \Psi\left(\frac{X(\theta_s^*,\theta_s^*)-\theta}{\sigma}\right)$ for all θ. By the fact that $\tilde{V}(\theta) \equiv V\left(\theta, \Psi\left(\frac{X(\theta,\theta;\sigma)-\theta}{\sigma}\right)\right) < 0$ for $\theta < \theta^{\#}(\sigma)$, we then have that $V\left(\theta_s^*, \Psi\left(\frac{X(\theta_s^*,\theta_s^*(\sigma))-\theta_s^*}{\sigma}\right)\right) < 0$, while by the fact that $\theta_s^{**}(\sigma)$ solves equation (9), we have that $V\left(\theta_s^{**}(s), \Psi\left(\frac{X(\theta_s^*,\theta_s^{**}(\sigma))-\theta_s^{**}(\sigma)}{\sigma}\right)\right) > 0$. Combining, we have that

$$V(\theta_s^*, A(\theta_s^*, \underline{r})) < 0 < V(\theta_s^{**}(\sigma), A(\theta_s^{**}(\sigma), \underline{r})) \tag{21a}$$

[28] Recall that all types $\theta \in (\theta_s^*, \hat{\theta})$ necessarily raise the policy to $r = s$.

Now the fact that the noise distribution ψ is log-concave implies that, after observing $r = \underline{r}$, irrespective of the shape of the equilibrium policy $r(\theta)$ in the region $[\theta_s^*, \theta_s^{**}(\sigma)]$ of possible intervention, the aggregate attack $A(\theta, \underline{r})$ is monotone in θ — see the proof of Lemma A4. Condition (21a), together with the monotonicity of $A(\theta, \underline{r})$ in θ and the property that $V_\theta > 0 > V_A$ then ensures that there exists a unique $\hat{\theta}_s \in [\theta_s^*, \theta_s^{**}(\sigma))$ such that $V(\theta, A(\theta, \underline{r})) < 0$ if and only if $\theta < \hat{\theta}_s$. Now let $\theta_s^{\sup} \equiv \sup\{\theta : r(\theta) = s\}$. Clearly, $\theta_s^{\sup} \geq \hat{\theta}_s$; if $\theta^{\sup_s} < \hat{\theta}_s$, then types $\theta \in (\theta_s^{\sup}, \hat{\theta}_s)$ would be better off by raising the policy at $r = s$ and avoiding an attack rather than leaving the policy at $\underline{r}$ and then devaluing. But then $\theta_s^{\sup}$ must solve the indifference condition

$$W(\theta_s^{\sup}, s, 0) = W(\theta_s^{\sup}, \underline{r}, A(\theta_s^{\sup}, \underline{r}))$$

Now let

$$h(\theta) \equiv W(\theta, s, 0) - W(\theta, \underline{r}, A(\theta, \underline{r}))$$

Note that SCC, along with the monotonicity of $A(\cdot, \underline{r})$, implies that h changes sign only once. It follows that all $\theta \in (\theta_s^*, \theta_s^{\sup})$ necessarily raise the policy at $r = s$. But then necessarily

$$A(\theta, \underline{r}) = \Psi\left(\frac{X(\theta_s^*, \theta_s^{\sup}; \sigma) - \theta}{\sigma}\right)$$

This means that $\theta_s^{\sup}$ must solve (9). Now recall that $\theta_s^{**}(\sigma)$ is the highest solution to (9). That $\theta_s^{\sup} = \theta_s^{**}(\sigma)$ in turn follows from the fact that, under SCC, the function $G(\theta; \theta_s^*, \sigma)$ defined above in the proof of Proposition 3 is strictly decreasing in θ which implies that the solution to (9) is unique. ■ *Q.E.D.*

Proof of Proposition 5. Most of the results in the proposition follow from Propositions 1-3 along with properties (1)-(2) below. To save on notation, hereafter we let $\theta_{\underline{r}}^*(\sigma) \equiv \theta_{\underline{r}}^{**}(\sigma) \equiv \theta^{\#}(\sigma)$ denote the unique thresholds corresponding to any of the pooling equilibria of $\mathcal{E}(\underline{r}; \sigma)$.

Property 1. *For any $\sigma > 0$ any $s, s' \in (\underline{r}, r^{\#}(\sigma)]$, $s > s'$ implies that $\Delta_s(\sigma) < \Delta_{s'}(\sigma)$.*

To see this, note that, for any $s \in (\underline{r}, r^{\#}(\sigma)]$, $\Delta_s(\sigma) \equiv \theta_s^{**}(\sigma) - \theta_s^*$ is the highest solution to $\hat{G}(\Delta; s, \sigma) = 0$, where

$$\hat{G}(\Delta; s, \sigma) \equiv G(\theta_s^* + \Delta; \theta_s^*, \sigma) \equiv w(\theta_s^* + \Delta, s, 0) - w\left(\theta_s^* + \Delta, \underline{r}, \Psi\left(\tfrac{X(\theta_s^*, \theta_s^* + \Delta; \sigma) - (\theta_s^* + \Delta)}{\sigma}\right)\right)$$

The property that $\lim_{\Delta \to +\infty} \hat{G}(\Delta; s, \sigma) < 0$ implies that $\hat{G}(\cdot; s, \sigma)$ must be locally strictly decreasing in Δ at $\Delta = \Delta_s(\sigma)$; that is,

$$\frac{\partial \hat{G}(\Delta_s(\sigma); s, \sigma)}{\partial \Delta} < 0.$$

Furthermore,

$$\begin{aligned} \frac{\partial \hat{G}(\Delta_s(\sigma); s, \sigma)}{\partial s} &= \left[W_\theta(\theta_s^* + \Delta_s(\sigma), s, 0) - W_\theta\left(\theta_s^* + \Delta_s(\sigma), \underline{r}, \hat{B}(\Delta_s(\sigma); \theta_s^*, \sigma)\right)\right] \times \frac{d\theta_s^*}{ds} \qquad (22) \\ &\quad + W_r(\theta_s^* + \Delta_s(\sigma), s, 0) - W_A\left(\theta_s^* + \Delta_s(\sigma), \underline{r}, \hat{B}(\Delta_s(\sigma); s, \sigma)\right) \frac{\partial \hat{B}(\Delta_s(\sigma); s, \sigma)}{\partial s} \\ &< 0 \end{aligned}$$

where $\hat{B}(\Delta; s,\sigma) \equiv \Psi\left(\frac{X(\theta_s^*,\theta_s^*+\Delta;\sigma)-(\theta_s^*+\Delta)}{\sigma}\right)$. That the first two terms of (22) are nonpositive follows from SCC along with the property that θ_s^* is strictly increasing in s and that W is decreasing in r. To see that the third term of (22) is also negative, recall that $W_A < 0$ and that $\hat{B}(\Delta; s,\sigma)$ is implicitly defined by

$$\int_{-\infty}^{\theta_s^*} Z(\tilde{\theta},\underline{r})\frac{1}{\sigma}\psi\left(\frac{X(\theta_s^*\theta_s^*+\Delta;\sigma)-\tilde{\theta}}{\sigma}\right)d\tilde{\theta} - \underline{r}\left[1-\Psi\left(\frac{X(\theta_s^*\theta_s^*+\Delta;\sigma)-\theta_s^*}{\sigma}\right)+\hat{B}(\Delta;s,\sigma)\right]=0$$

Integrating by parts and using the definition of $\hat{B}(\Delta; s,\sigma)$ we then have that $\hat{B}(\Delta; s,\sigma)$ is implicitly defined by

$$\begin{aligned}&\left(Z\left(\theta_s^*,\underline{r}\right)-\underline{r}\right)\left(1-\Psi\left(\Psi^{-1}(\hat{B})+\frac{\Delta}{\sigma}\right)\right)\\ &-\int_{-\infty}^{\theta_s^*}\frac{\partial Z(\tilde{\theta},\underline{r})}{\partial\tilde{\theta}}\left(1-\Psi\left(\Psi^{-1}(\hat{B})+\frac{\theta_s^*+\Delta-\tilde{\theta}}{\sigma}\right)\right)d\tilde{\theta}=\underline{r}\hat{B}\end{aligned} \tag{23}$$

from which we obtain that

$$\begin{aligned}\frac{\partial\hat{B}(\Delta;s,\sigma)}{\partial s} &= -\frac{\int_{-\infty}^{\theta_s^*}\frac{\partial Z(\tilde{\theta},\underline{r})}{\partial\tilde{\theta}}\frac{1}{\sigma}\psi\left(\Psi^{-1}(\hat{B})+\frac{\theta_s^*+\Delta-\tilde{\theta}}{\sigma}\right)\frac{d\theta_s^*}{ds}d\tilde{\theta}}{\left[\begin{array}{c}-\left(Z\left(\theta_s^*\right)-\underline{r}\right)\psi\left(\Psi^{-1}(\hat{B})+\frac{\Delta}{\sigma}\right)\frac{d\Psi^{-1}(\hat{B})}{dx}\\ +\int_{-\infty}^{\theta_s^*}\frac{\partial Z(\tilde{\theta},\underline{r})}{\partial\tilde{\theta}}\psi\left(\Psi^{-1}(\hat{B})+\frac{\theta_s^*+\Delta-\tilde{\theta}}{\sigma}\right)\frac{d\Psi^{-1}(\hat{B})}{dx}d\tilde{\theta}-\underline{r}\end{array}\right]}\\ &< 0.\end{aligned}$$

From the Implicit Function Theorem, we then have that, for any $\sigma > 0$, any $s \in (\underline{r}, r^{\#}(\sigma))$, $\partial\Delta_s(\sigma)/\partial s < 0$, which implies the result.

Property 2. *For any $\sigma,\sigma' > 0$, any $s \in (\underline{r}, \min\{r^{\#}(\sigma), r^{\#}(\sigma')\})$, $\sigma' > \sigma$ implies $\Delta_s(\sigma') > \Delta_s(\sigma)$.*

This follows from the proof of Proposition 2.

Given the aforementioned properties, the results in the proposition can be established as follows. First, note that, for any $\sigma > 0$, any $F \in \mathcal{F}(\sigma)$, any θ, any $r > \underline{r}$,

$$P(r,\theta;F,\sigma)=\begin{cases}\int_{s\in[r,r^{\#}(\sigma)]} I_{\{\theta\in[\theta_s^*,\theta_s^{**}(\sigma)]\}}dF(s) \text{ if } r\in(\underline{r},r^{\#}(\sigma)]\\ 0 \text{ if } r>r^{\#}(\sigma)\end{cases}$$

$$\Delta(r;F,\sigma)=\begin{cases}\int_{s\in[r,r^{\#}(\sigma)]}\Delta_s(\sigma)dF\left(s\right) \text{ if } r\in(\underline{r},r^{\#}(\sigma)]\\ 0 \text{ if } r>r^{\#}(\sigma).\end{cases}$$

Part (i). Fix $\sigma > 0$ and $F \in \mathcal{F}(\sigma)$, take any $r \in (\underline{r}, r^{\#}(\sigma)]$ and let $\theta^{\circ}(r;F,\sigma) = \theta_r^*$ and $\theta^{\circ\circ}(r;F,\sigma) = \sup\{\theta_s^{**}(\sigma) : s \in [r, r^{\#}(\sigma)]\}$. From Proposition 1, we then have that $P(r,\theta;F,\sigma) > 0$ only if $\theta \in [\theta^{\circ}(r;F,\sigma), \theta^{\circ\circ}(r;F,\sigma)]$.

Part (ii). Again, fix $\sigma > 0$ and take any $F \in \mathcal{F}(\sigma)$. That $D(\theta;F,\sigma)$ is nonincreasing in θ, with $D(\theta;F,\sigma) = 1$ for $\theta < \underline{\theta}$ and $D(\theta;F,\sigma) = 0$ for $\theta > \theta^{\#}(\sigma)$ follows directly from Proposition 1.

Part (iii). Fix $\sigma > 0$ and take any pair $F, F' \in \mathcal{F}(\sigma)$. For any $\theta \in (\underline{\theta}, \theta^{\#}(\sigma))$,

$$\begin{aligned} D(\theta; F', \sigma) &= F'(\underline{r}) + 1 - F'(\rho(\theta)) + [F'(\rho(\theta)) - \lim_{s \to \rho(\theta)^-} F'(s)] \Pr(r(\theta) = \underline{r} | \theta, s = \rho(\theta)) \\ D(\theta; F, \sigma) &= F(\underline{r}) + 1 - F(\rho(\theta)) + [F(\rho(\theta)) - \lim_{s \to \rho(\theta)^-} F(s)] \Pr(r(\theta) = \underline{r} | \theta, s = \rho(\theta)) \end{aligned}$$

Because $F'(s) = F(s)$ for $s \in \{\underline{r}, r^{\#}(\sigma)\}$ and $F'(s) < F(s)$ for all $s \in (\underline{r}, r^{\#})$, then $D(\theta; F', \sigma) > D(\theta; F, \sigma)$, unless $D(\theta; F, \sigma) = 1$, or both[29] F and F' have a mass point at $s = \rho(\theta)$ and $r(\theta) = \underline{r}$ in some of the equilibria in $\mathcal{E}(\rho(\theta); \sigma)$, in which case

$$[F(\rho(\theta)) - \lim_{s \to \rho(\theta)^-} F(s)] \Pr(r(\theta) = \underline{r} | \theta, s = \rho(\theta)) > 0.$$

Next, consider the probability of intervention. From Property 1 above, $\Delta_s(\sigma)$ is a positive, strictly decreasing, and differentiable function of s, for any $s \in (\underline{r}, r^{\#}(\sigma))$. Now fix $r \in (\underline{r}, r^{\#}(\sigma))$. That $F(r) = F'(r)$ along with $F'(s) < F(s)$ for all $s \in (r, r^{\#}(\sigma))$ then imply that

$$\begin{aligned} \Delta(r; F, \sigma) - \Delta(r, F', \sigma) &= \int_{s \in [r, r^{\#}(\sigma)]} \Delta_s(\sigma) dF(s) - \int_{s \in [r, r^{\#}(\sigma)]} \Delta_s(\sigma) dF'(s) \\ &= -\Delta_r(\sigma)[F(r) - F'(r)] + \\ &\quad - \int_{s \in [r, r^{\#}(\sigma)]} \frac{d\Delta_s(\sigma)}{ds} [F(s) - F'(s)] ds \\ &> 0. \end{aligned}$$

Part (iv). Take any c.d.f. F with support $Supp[F]$ such that $F(s) = 0$ for all $s \le r_1$ and $F(s) = 1$ for all $s \ge r_2$ for some $r_1, r_2 \in \mathbb{R}$ with $\underline{r} < r_1 < r_2 < \lim_{\sigma \to 0^+} r^{\#}(\sigma)$. Note that this implies that there exists $\bar{\sigma} > 0$, such that, for any $\sigma < \bar{\sigma}$, $Supp[F] \subset [\underline{r}, r^{\#}(\sigma)]$. That, for any $r > \underline{r}$, $\lim_{\sigma \to 0^+} \Delta(r, F, \sigma) = 0$ follows from the Dominated Convergence Theorem. To see this, let $H : \mathbb{R} \to \mathbb{R}$ be the function whose domain is $Supp[F]$ and that is given by

$$H(s) = \sup\{\theta : W(\theta, s, 0) - W(\theta, \underline{r}, 1) \ge 0\} - \theta_s^*$$

for all $s \in Supp[F]$. It is immediate that $H(s) \ge 0$, that $H(\cdot)$ is strictly decreasing and that $\int H(s) dF(s) < \infty$. Furthermore, for any $\sigma < \bar{\sigma}$ and any $s \in Supp[F]$, $\Delta_s(\sigma) \le H(s)$. These properties, together with the result in Proposition 2, then imply that

$$\lim_{\sigma \to 0^+} \Delta(r, F, \sigma) = \lim_{\sigma \to 0^+} \int_{s \ge r} \Delta_s(\sigma) dF(s) = \int_{s \ge r} \lim_{\sigma \to 0^+} \Delta_s(\sigma) dF(s) = 0.$$

[29] If F' has no mass point at $s = \rho(\theta)$, then

$$\begin{aligned} & D(\theta; F', \sigma) - D(\theta; F, \sigma) = -F'(\rho(\theta)) + F(\rho(\theta)) \\ & -[F(\rho(\theta)) - \lim_{s \to \rho(\theta)^-} F(s)] \Pr(r(\theta) = \underline{r} | \theta, s = \rho(\theta)) \\ & \ge -F'(\rho(\theta)) + \lim_{s \to \rho(\theta)^-} F(s) = \lim_{s \to \rho(\theta)^-} [F(s) - F'(s)] > 0. \end{aligned}$$

That, for any θ, any $\sigma, \sigma' > 0$, any $F \in \mathcal{F}(\sigma) \cap \mathcal{F}(\sigma')$, $D(\theta; F, \sigma) = D(\theta; F, \sigma')$ follows from Proposition 1.

Lastly, that $\sigma' > \sigma > 0$ implies $\Delta(r, F, \sigma') \geq \Delta(r, F, \sigma)$ for all $r \in (\underline{r}, \min\{r^{\#}(\sigma), r^{\#}(\sigma')\})$ all $F \in \mathcal{F}(\sigma) \cap \mathcal{F}(\sigma')$, with strict inequality if $F(s) < 1$ for $s < r$ (i.e., unless F assigns measure one to $s < r$), follows from Property 2 above. *Q.E.D.*

Proof of Proposition 6. Part (i) To see that $\underline{D}(\theta_1, \theta_2; \sigma)$ and $\bar{D}(\theta_1, \theta_2; \sigma)$ are both nonincreasing in (θ_1, θ_2), in the weak-order sense, consider any pair (θ_1, θ_2), (θ_1', θ_2') such that $\theta_1 \leq \theta_1'$ and $\theta_2 \leq \theta_2'$. Clearly, the distribution of $\tilde{\theta}$ conditional on the event that $\theta \in (\theta_1', \theta_2')$ first-order-stochastically dominates the distribution of $\tilde{\theta}$ conditional on the event that $\theta \in (\theta_1, \theta_2)$. Along with the fact that, for any $F \in \mathcal{F}(\sigma)$, $D(\cdot; F, \sigma)$ is nonincreasing in θ, this means that, for any $F \in \mathcal{F}(\sigma)$, $D(\theta_1, \theta_2; F, \sigma) \geq D(\theta_1', \theta_2'; F, \sigma)$. Standard envelope arguments, then imply that the same monotonicities apply to $\underline{D}(\theta_1, \theta_2; \sigma)$ and $\bar{D}(\theta_1, \theta_2; \sigma$. The result for $\underline{P}(r, \theta_1, \theta_2; \sigma)$ and $\bar{P}(r, \theta_1, \theta_2; \sigma)$ follows directly from Proposition 5.

Part (ii). That $\underline{P}(r, \theta_1, \theta_2; \sigma)$ is independent of σ is immediate. That also $\underline{D}(\theta_1, \theta_2; \sigma)$ is independent of σ follows from the fact that, for any $\sigma > 0$, $\lim_{s \to \underline{r}^+} \theta_s^* = \underline{\theta}$ along with the fact that $D_s(\theta; \sigma) = 0$ for any $\theta > \theta_s^*$ and any equilibrium in $\mathcal{E}(s; \sigma)$. That, for any (θ_1, θ_2), any $\sigma, \sigma' > 0$, $\theta_2 < \min\{\theta^{\#}(\sigma'), \theta^{\#}(\sigma)\}$ or $\theta_1 > \max\{\theta^{\#}(\sigma'), \theta^{\#}(\sigma)\}$ imply $\bar{D}(\theta_1, \theta_2; \sigma) = \bar{D}(\theta_1, \theta_2; \sigma')$ follows from the fact that, for any $\sigma > 0$, any θ, $D_s(\theta; \sigma) \leq D_{\underline{r}}(\theta; \sigma)$ together with the fact that $D_{\underline{r}}(\theta; \sigma) = 1$ for all $\theta < \theta^{\#}(\sigma)$ while $D_{\underline{r}}(\theta; \sigma) = 0$ for all $\theta > \theta^{\#}(\sigma)$.

Next, consider the claim that $\lim_{\sigma \to 0^+} \bar{P}(r, \theta_1, \theta_2; \sigma) = 0$ for any $r > \underline{r}$ and any $\theta_1, \theta_2 \in \mathbb{R}$. Clearly, the result is true if $r > r^{\#}(0^+) \equiv \lim_{\sigma \to 0^+} r^{\#}(\sigma)$. Thus take $r \in (\underline{r}, r^{\#}(0^+)]$. Let $\mathcal{F}$ denote an arbitrary set of c.d.f.s F with support $Supp[F] \subset [\underline{r}, \rho(\bar{\theta})]$ with the following properties: (i) $\mathcal{F}(\sigma) \subset \mathcal{F}$ for any $\sigma > 0$; (ii) $\mathcal{F}$ is compact with respect to the metric $d(\cdot)$ defined, for any pair $F_1, F_2 \in \mathcal{F}$, by $d(F_1, F_2) \equiv \sup\{|F_1(A) - F_2(A)| : A \in \Sigma\}$, where Σ is the Borel sigma algebra associated with the interval $[\underline{r}, \rho(\bar{\theta})]$. For any $\sigma > 0$, any c.d.f. $F \in \mathcal{F}$, any $\theta_1 < \theta_2$, and any $r > \underline{r}$, then let

$$\hat{P}(r, \theta_1, \theta_2; F, \sigma) \equiv \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \int_{s \in [r, \rho(\bar{\theta})]} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s) \, d\theta$$

with the convention that, given any $s \in [r, \rho(\bar{\theta})]$, $I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} = 0$ if equation (9) admits no solution $\theta_s'' \geq \theta_s^*$. Note that

$$\hat{P}(r, \theta_1, \theta_2; F, \sigma) = P(r, \theta_1, \theta_2; F, \sigma) \equiv \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} P(r, \theta; F, \sigma) d\theta$$

if $F \in \mathcal{F}(\sigma)$, that is, if, given $\sigma > 0$, one restricts F to have support $Supp[F] \subset [r, r^{\#}(\sigma)]$.

By Proposition 2, for any $s \leq r^{\#}(0^+)$, $\lim_{\sigma \to 0^+} \Delta_s(\sigma) = 0$. This implies that, for any $\theta \in [\theta_1, \theta_2]$, any $s \in [\underline{r}, \rho(\bar{\theta})]$ with $\theta_s^* \neq \theta$,

$$\lim_{\sigma \to 0^+} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} = 0.$$

By the Lebesgue Dominated Convergence Theorem, we then have that, for any $\theta \in [\theta_1, \theta_2]$

$$\lim_{\sigma \to 0^+} \int_{s \in [r, \rho(\bar{\theta})]} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s) = \int_{s \in [r, \rho(\bar{\theta})]} \lim_{\sigma \to 0^+} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s)$$

where the last integral is equal to zero, unless F has a mass point at an s such that $\theta_s^* = \theta$. It follows that

$$\lim_{\sigma \to 0^+} \int_{\theta_1}^{\theta_2} \left\{ \int_{s \in [r, \rho(\bar{\theta})]} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s) \right\} d\theta = \int_{\theta_1}^{\theta_2} \left\{ \lim_{\sigma \to 0^+} \int_{s \in [r, \rho(\bar{\theta})]} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s) \right\} d\theta = 0, \tag{24}$$

where the first equality is again by the Dominated Convergence Theorem, while the second equality follows from the following property. Given any c.d.f. F with support $Supp[F] \subset [\underline{r}, \rho(\bar{\theta})]$, there does not exist a Lebesgue positive-measure set $E \subset [\theta_1, \theta_2]$ such that, for any $\theta \in E$, $\theta_s^* = \theta$ with strictly positive probability. Formally, the set

$$S \equiv \left\{ s \in [\underline{r}, \rho(\bar{\theta})] : F(s) > \lim_{x \to s^-} F(x) \right\}$$

has zero Lebesgue measure, which in turn implies that the set

$$\Theta^+ \equiv \left\{ \theta \in [\theta_1, \theta_2] : \theta_s^* = \theta \text{ with } F(s) > \lim_{x \to s^-} F(x) \right\}$$

also has zero Lebesgue measure. This means that the set of points $\theta \in [\theta_1, \theta_2]$ such that

$$\lim_{\sigma \to 0^+} \int_{s \in [r, \rho(\bar{\theta})]} I_{\{\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]\}} dF(s) > 0$$

has zero Lebesgue measure, which implies the result in (24).

Having established that, for any c.d.f. $F \in \mathcal{F}$, $\lim_{\sigma \to 0^+} \hat{P}(r, \theta_1, \theta_2; F, \sigma) = 0$, we now show that this property also implies that

$$\lim_{\sigma \to 0^+} \left\{ \sup_{F \in \mathcal{F}(\sigma)} P(r, \theta_1, \theta_2; F, \sigma) \right\} = 0.$$

First note that, by definition, for any $\sigma > 0$,

$$\sup_{F \in \mathcal{F}(\sigma)} P(r, \theta_1, \theta_2; F, \sigma) \leq \sup_{F \in \mathcal{F}} \hat{P}(r, \theta_1, \theta_2; F, \sigma)$$

which implies that

$$\lim_{\sigma \to 0^+} \left\{ \sup_{F \in \mathcal{F}(\sigma)} P(r, \theta_1, \theta_2; F, \sigma) \right\} \leq \lim_{\sigma \to 0^+} \left\{ \sup_{F \in \mathcal{F}} \hat{P}(r, \theta_1, \theta_2; F, \sigma) \right\}. \tag{25}$$

To establish the result, it thus suffices to show that the right hand side of (25) is zero. This is established as follows. First, note that, by assumption, $\mathcal{F}$ is compact in the metric $d(F_1, F_2) \equiv$

$\sup\{|F_1(A)-F_2(A)| : A\in\Sigma\}$. Because it is metric, then $\mathcal{F}$ is also Hausdorff. Next, note that, for any $\bar{\sigma}>0$, the function family $\left\{\hat{P}(r,\theta_1,\theta_2;\cdot,\sigma)\right\}_{\sigma\in(0,\bar{\sigma}]}$ with domain $\mathcal{F}$ and range in $[0,1]$ is uniform equicontinuous in the metric $d(\cdot)$ defined above. This means that for any $\varepsilon>0$, there exists a $\delta>0$ (which may depend on ε only) such that for any $\sigma\in(0,\bar{\sigma}]$ (i.e., for any family member $\hat{P}(r,\theta_1,\theta_2;\cdot,\sigma)$), any $F_1,F_2\in\mathcal{F}$ such that $d(F_1,F_2)<\delta$,

$$|\hat{P}(r,\theta_1,\theta_2;F_1,\sigma)-\hat{P}(r,\theta_1,\theta_2;F_2,\sigma)|<\varepsilon.$$

To see that this is true, note that

$$\begin{aligned}
&|\hat{P}(r,\theta_1,\theta_2;F_1,\sigma)-\hat{P}(r,\theta_1,\theta_2;F_2,\sigma)| \\
&=\tfrac{1}{\theta_2-\theta_1}\left|\int_{\theta_1}^{\theta_2}\left[\int_{s\in[r,\rho(\bar{\theta})]}I_{\{\theta\in[\theta_s^*,\theta_s^{**}(\sigma)]\}}dF_1(s)-\int_{s\in[r,\rho(\bar{\theta})]}I_{\{\theta\in[\theta_s^*,\theta_s^{**}(\sigma)]\}}dF_2(s)\right]d\theta\right| \\
&=\tfrac{1}{\theta_2-\theta_1}\left|\int_{\theta_1}^{\theta_2}\left[F_1(A(\theta,\sigma))-F_2(A(\theta,\sigma))\right]d\theta\right|
\end{aligned}$$

where $A(\theta,\sigma)\equiv\left\{s\in[r,\rho(\bar{\theta})] : \theta\in[\theta_s^*,\theta_s^{**}(\sigma)]\right\}$. It is then easy to see that, for any $\sigma\in(0,\bar{\sigma}]$, the result follows by letting $\delta=\varepsilon$. It is also easy to see that the function family $\left\{\hat{P}(r,\theta_1,\theta_2;\cdot,\sigma)\right\}_{\sigma\in(0,\bar{\sigma}]}$ is uniformly bounded, i.e., that there exists $M>0$ such that $|\hat{P}(r,\theta_1,\theta_2;F,\sigma)|<M$ all $F\in\mathcal{F}$, all $\sigma\in(0,\bar{\sigma}]$. From the Ascoli-Arzela Theorem, any sequence of equicontinuous, uniformly bounded, functions defined on a compact Haurdorff space has a uniformly convergent subsequence (i.e., a subsequence that is convergent in the sup-norm). This implies that $\lim_{\sigma\to0^+}\left\{\sup_{F\in\mathcal{F}}\hat{P}(r,\theta_1,\theta_2;F,\sigma)\right\}=0$.

Lastly, we show that when $Z(\theta,\underline{r})=z>\underline{r}$ for all θ, then the bound $\bar{D}$ is independent of σ whereas the bound $\bar{P}(r,\theta_1,\theta_2;\sigma)$ is a nondecreasing function of σ. The first property follows directly from the fact that, in this case, $\theta^{\#}(\sigma)$ and hence $r^{\#}(\sigma)$ are independent of $\sigma>0$, together with the fact that, for any $\theta>\theta^{\#}$, $D_s(\theta;\sigma)=0$ for all s, while for any $\theta<\theta^{\#}$ $D_{\underline{r}}(\theta,\sigma)=1$ for all $\sigma>0$. The second property follows from the fact that, given any distribution F with support[30] $Supp[F]\subset[\underline{r},r^{\#}]$,

$$P(r,\theta_1,\theta_2;F,\sigma)=\frac{1}{\theta_2-\theta_1}\int_{\theta_1}^{\theta_2}\int_{s\in[r,r^{\#}]}I_{\{\theta\in[\theta_s^*,\theta_s^{**}(\sigma)]\}}dF(s)\,d\theta$$

That $P(r,\theta_1,\theta_2;F,\sigma)$ is increasing in σ then follows from the fact that $\theta_s^{**}(\sigma)$ is increasing in σ. By the envelope theorem, that each $P(r,\theta_1,\theta_2;F,\sigma)$ is weakly increasing in σ, then implies that the upper bound $\bar{P}(r,\theta_1,\theta_2;\sigma)$ is also weakly increasing in σ, which establishes the result. *Q.E.D.*

Proof of Proposition 7. For any θ, let

$$\Delta\Pi_s(\theta;\sigma)\equiv W(\theta,s,0)-\max\left\{W\left(\theta,\underline{r},\Psi\left((x^{\#}(\sigma)-\theta)/\sigma\right)\right);L(\theta,\underline{r})\right\}$$

denote the difference between the payoff $W(\theta,s,0)$ that type θ obtains by raising the policy at $r=s$ and then maintaining the peg in the game in which policy interventions are possible and

[30]Recall that in this case $\mathcal{F}(\sigma)=\mathcal{F}(\sigma')$ all $\sigma,\sigma'>0$.

the equilibrium payoff $\tilde{U}(\theta;\sigma) = \max\left\{W\left(\theta, \underline{r}, \Psi\left((x^{\#}(\sigma)-\theta)/\sigma\right)\right); L(\theta,\underline{r})\right\}$ that the same type obtains in the game in which the option to intervene is absent. The fact that $s < r^{\#}(\sigma)$ implies that $\Delta\Pi_s(\theta;\sigma) = W(\theta, s, 0) - L(\theta,\underline{r}) > 0$, for any type $\theta \in (\theta_s^*, \theta^{\#}(\sigma)]$; indeed any such type, by raising the policy at $r = s$, can guarantee himself a payoff $W(\theta, s, 0)$ that is strictly higher than the payoff $\tilde{U}(\theta;\sigma) = L(\theta,\underline{r})$ that the same type would obtain absent the possibility to intervene. Furthermore, the fact that $\Delta\Pi_s(\theta;\sigma)$ is continuous over $[\theta^{\#}(\sigma), +\infty)$ together with SCC and the limit condition that $\lim_{\theta\to+\infty} W(\theta, s, 0) - W(\theta,\underline{r},1) < 0$, imply that there exists a unique $\hat{\theta}_s(\sigma) > \theta^{\#}(\sigma)$ such that $\Delta\Pi_s(\theta;\sigma) > 0$ for $\theta \in (\theta^{\#}(\sigma), \hat{\theta}_s(\sigma))$ and $\Delta\Pi_s(\theta;\sigma) < 0$ for $\theta > \hat{\theta}_s(\sigma)$. Hence, no matter which particular equilibrium in $\mathcal{E}(s;\sigma)$ is played, any type $\theta \in (\theta_s^*, \hat{\theta}_s(\sigma))$ is necessarily strictly better off with the option to intervene, whereas any type $\theta \leq \theta_s^*$ is just as well off.

Next note that any type $\theta > \hat{\theta}_s(\sigma)$ can be strictly worse off with the option to intervene only if the attack he expects when leaving the policy at $r = \underline{r}$ is strictly greater than the attack he would have faced without the option to intervene, which is possible if and only if $X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) > x^{\#}(\sigma)$.[31] However, σ small enough ensures that $X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) < x^{\#}(\sigma)$ and hence that the policy maker is always better off with the option to intervene, no matter his type. This follows from the fact that, when $\sigma \to 0^+$, $\theta_s^{**}(\sigma) \to \theta_s^*$ and $X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) \to \theta_s^*$ in the game with policy intervention,[32] whereas $x^{\#}(\sigma) \to \theta^{\#}(\sigma)$ in the game in which the option to intervene is absent. Together with the fact that $\theta_s^* < \theta^{\#}(\sigma)$ when $s \in (\underline{r}, r^{\#}(\sigma))$ then gives the result. *Q.E.D.*

Proof of Proposition 8. First, consider the supremum of the equilibrium payoffs. For any $\theta > \underline{\theta}$, the highest *feasible* payoff is $W(\theta,\underline{r},0)$, the payoff enjoyed when retaining the peg without facing any attack and without incurring any cost of policy intervention. This payoff can be approximated arbitrarily well in the game in which intervention is possible (it suffices to take any equilibrium in which s is sufficiently close to $\underline{r}$) whereas in the game in which the option to intervene is absent $W(\theta,\underline{r},0)$ can be approximated only by taking the limit as $\theta \to +\infty$. This establishes the first part of the proposition.

Next consider the infimum of the equilibrium payoffs. As explained in the proof of Proposition 7, type θ can be worse off with the option to intervene only if there exists an $s \in (\underline{r}, r^{\#}(\sigma))$ such that $X(\theta_s^*, \theta_s^{**}(\sigma);\sigma) > x^{\#}(\sigma)$; recall that this means that there exists an equilibrium in which the attack that type θ faces when not raising the policy above $\underline{r}$ is larger than in the game without the option to intervene. Now suppose that such an s exists. From the result in Proposition 7, in this case any type $\theta < \hat{\theta}_s(\sigma)$ is still weakly better off (strictly for $\theta \in (\theta_s^*, \hat{\theta}_s(\sigma))$). However, by taking the

[31] Recall from the analysis in Section 3, that (i) in any equilibrium in $\mathcal{E}(s;\sigma)$ no speculator attacks when, after observing $r = \underline{r}$, he receives a signal $x > X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)$, and (ii) that there always exists one equilibrium in $\mathcal{E}(s;\sigma)$ such that, after observing $r = \underline{r}$, each speculator attacks if and only if he receives a signal $x < X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)$.

[32] The first property follows directly from Proposition 2. To see that the second property also holds, note that, if this were not true, then the size of attack $A(\theta_s^{**}(\sigma),\underline{r})$ at $\theta = \theta_s^{**}(\sigma)$ would converge to either zero or one, making it impossible for type $\theta_s^{**}(\sigma)$ to be indifferent between raising the policy at $r = s$ and setting $r = \underline{r}$.

equilibrium in $\mathcal{E}(s;\sigma)$ in which all speculators attack when $r=\underline{r}$ if and only if $x<X(\theta_s^*,\theta_s^{**}(\sigma);\sigma)$ ensures that, for all $\theta>\hat{\theta}_s(\sigma)$, the payoff

$$\max\{W(\theta,s,0);W(\theta,\underline{r},\Psi((X(\theta_s^*,\theta_s^{**}(\sigma);\sigma)-\theta)/\sigma))\}$$

that type θ obtains with the option to intervene is strictly lower than his payoff in the game in which the option to intervene is absent.[33] This means that there exists $\hat{\theta}(\sigma)$ such that $\underline{U}(\theta;\sigma)<\tilde{U}(\theta;\sigma)$ only if $\theta>\hat{\theta}(\sigma)$. That $\hat{\theta}(\sigma)\geq\theta^{\#}(\sigma)$ is immediate given that there is no equilibrium in which a type $\theta<\theta^{\#}(\sigma)$ can be made worse off.

Finally, to see why, for any θ, the difference between $\underline{U}(\theta;\sigma)$ and $\tilde{U}(\theta;\sigma)$ vanishes as $\sigma\to0^+$, note that, for any $\theta\leq\theta^{\#}(0^+)\equiv\lim_{\sigma\to0^+}\theta^{\#}(\sigma)$, this difference is clearly zero because the lower bound is simply the payoff obtained in any equilibrium in which type θ is forced to abandon the peg. For types $\theta>\theta^{\#}(0^+)$, on the other hand, the result follows from the fact that for any $s\in(\underline{r},r^{\#}(0^+))$, $\lim_{\sigma\to0^+}\Delta_s(\sigma)=0$ and

$$\lim_{\sigma\to0^+}X(\theta_s^*,\theta_s^{**}(\sigma);\sigma)=\theta_s^*<\lim_{\sigma\to0^+}x^{\#}(\sigma)$$

This implies that the lowest bound on the payoff for each type $\theta>\theta^{\#}(0^+)$ is attained under any of the pooling equilibria, which is clearly the same payoff as in the game in which the option to intervene is absent. *Q.E.D.*

Proof of Proposition 9. For $\theta<\underline{\theta}$, it is dominant for the policy maker to set $\underline{r}$ and abandon the peg. Similarly, for $\theta>\bar{\theta}$, it is dominant for the policy maker to maintain the peg, in which case it is iteratively dominant for the speculators not to attack, irrespective of r, which in turn implies that it is iteratively dominant for the policy maker to set $r=\underline{r}$. Finally, take any $\theta\in[\underline{\theta},\bar{\theta}]$. Clearly, there is no subgame-perfect equilibrium in which $r(\theta)>\rho(\theta)$. On the other hand, for any $r\leq\rho(\theta)$, the assumption that $Z(\theta,r)\geq r$ implies that the continuation game among the speculators is a coordination game with two (extreme) continuation equilibria, one where nobody attacks and the peg is maintained and another one where all speculators attack and the peg is abandoned. This implies that, for any $r'\leq\rho(\theta)$, there exists a subgame-perfect equilibrium in which the speculators attack if and only if $r<r'$, the policy maker abandons the peg if and only if $V(\theta,A)\leq0$ and the policy is set at $r(\theta)=r'$. Both parts (i) and (ii) in the proposition follow from these properties. *Q.E.D.*

Proof of Proposition 10. The characterization of $\mathcal{G}(0)$ follows directly from Proposition 9. Thus consider $\mathcal{G}(\sigma)$ for $\sigma>0$ and note that this set is given by

$$\mathcal{G}(\sigma)=\{(\theta,r):\text{ either }r=\underline{r}\text{ and }\theta\in\mathbb{R},\text{ or }r\in(\underline{r},r^{\#}(\sigma)]\text{ and }\theta_r^*\leq\theta\leq\theta_r^{**}(\sigma)\}.$$

[33] To see this, note that if $W(\theta,s,0)>W(\theta,\underline{r},\Psi((X(\theta_s^*,\theta_s^{**}(\sigma);\sigma)-\theta)/\sigma))$ the result follows from the definition of $\hat{\theta}_s(\sigma)$. If, instead, the inequality is reversed, the result follows from the fact that, by not raising the policy, type θ faces an attack $\Psi((X(\theta_s^*,\theta_s^{**}(\sigma);\sigma)-\theta)/\sigma)$ greater than the attack $\Psi\left((x^{\#}(\sigma)-\theta)/\sigma\right)$ he would have faced without the option to intervene.

The results in Proposition 2 in turn imply that, for any $\varepsilon > 0$ small enough, there exists $\bar{\sigma} > 0$ such that for any $\sigma \in (0, \bar{\sigma})$,

$$\mathcal{G}(\sigma) \subset \left\{ \begin{array}{l} (\theta, r)\text{: either } r = \underline{r} \text{ and } \theta \in \mathbb{R}, \\ \text{or } r \in (\underline{r}, \underline{r} + \varepsilon] \text{ and } \theta > \theta^*_r, \\ \text{or } r \in [\underline{r} + \varepsilon, r^{\#}(0^+) + \varepsilon] \text{ and } \theta \in [\theta^*_r, \theta^*_r + \varepsilon] \end{array} \right\}$$

This property, together with the fact that, for all $\sigma > 0$,

$$\mathcal{G}(\sigma) \supset \{(\theta, r) : \text{ either } r = \underline{r} \text{ and } \theta \in \mathbb{R}, \text{ or } r \in (\underline{r}, r^{\#}(\sigma)] \text{ and } \theta = \theta^*_r = \rho^{-1}(r)\},$$

establishes the result.

References

[1] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2006) "Signaling in a Global Game: Coordination and Policy Traps," *Journal of Political Economy* 114(3), 452-484.

[2] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2007) "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks," *Econometrica*, 75(3), 711-756.

[3] Angeletos, George-Marios, and Iván Werning (2006), "Crises and Prices: Information Aggregation, Multiplicity and Volatility," *American Economic Review* 96, 1721-1737.

[4] Atkeson, Andrew (2001): "Discussion on Morris and Shin," *NBER Macroeconomics Annual 2000.*

[5] Calvo, Guillermo (1988) "Servicing the Public Debt: the Role of Expectations." *American Economic Review* 78, 647-661.

[6] Carlsson, Hans, and Eric van Damme (1993), "Global Games and Equilibrium Selection," *Econometrica* 61, 989-1018.

[7] Ciliberto, F. and E. Tamer, 2009, "Market Structure and Multiple Equilibria in Airline Markets," *Econometrica*, 77, 1791-1828.

[8] Chamley, Christophe (1999), "Coordinating Regime Switches," *Quarterly Journal of Economics* 114, 869-905.

[9] Chamley, Christophe (2003), "Dynamic Speculative Attacks," *American Economic Review*, 93(3), 603-621.

[10] Chassang, Sylvain (2010), "Fear of Miscoordination and the Robustness of Cooperation in Dynamic Global Games with Exit," forthcoming in *Econometrica.*

[11] Cho, In-Koo, and David Kreps (1987) "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics* 102, 179-222.

[12] Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris and Hyun Song Shin (2004) "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders" *The Review of Economic Studies*, 71(1), 87-114.

[13] Corsetti, Giancarlo, Bernardo Guimaraes, and Nouriel Roubini (2006), "International Lending of Last Resort and Moral Hazard: A Model of IMF's Catalytic Finance," *Journal of Monetary Economics* 53, 441-471.

[14] Dasgupta, Amil (2006): "Coordination and Delay in Global Games" *Journal of Economic Theory*, forthcoming.

[15] Edmond, Chris (2006): "Information Manipulation, Coordination and Regime Change," working paper, NYU Stern School of Business.

[16] Drazen, Allan (2000), "Interest-rate and Borrowing Defense Against Speculative Attack," *Carnegie-Rochester Conference Series on Public Policy* 53, 303-248.

[17] Drazen, Allan and Stefan Hubrich (2005), "A Simple Test of the Effect of Exchange Rate Defense," working paper, University of Maryland.

[18] Frankel, David, Stephen Morris, and Ady Pauzner (2003), "Equilibrium Selection in Global Games with Strategic Complementarities," *Journal of Economic Theory*, 108, 1-44.

[19] Goldstein, Itay, and Ady Pauzner (2005), "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance* 60, 1293-1328.

[20] Goldstein, Itay, Emre Ozdenoren, an Kathy Yuan, (2011), "Learning and Complementarities in Speculative Attacks", the *Review of Economic Studies*, vol. 78(1), 263-292.

[21] Grieco, Paul (2010), "Discrete Games with Flexible Information Structures: An Application to Local Grocery Markets," working paper, Pennsylvania State University.

[22] Guimaraes, Bernardo, and Stephen Morris (2006), "Risk and Wealth in a Model of Self-fulfilling Currency Attacks, "working paper, LSE and Yale University.

[23] Hellwig, Christian, Arijit Mukherji, and Aleh Tsyvinski (2006), "Self-Fulfilling Currency Crises: the Role of Interest Rates," *American Economic Review,* 96, 1770-1788.

[24] Jeanne, Olivier, and Charles Wyplosz (2001), “The International Lender of Last Resort: How Large is Large Enough?” working paper #8381, NBER.

[25] Kraay, Aart (2001), “Do High Interest Rates Defend Currencies During Speculative Attack?”, World Bank mimeo.

[26] Morris, Stephen, and Hyun Song Shin (1998), “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks,” *American Economic Review* 88, 587-597.

[27] Morris, Stephen, and Hyun Song Shin (2001), “Rethinking Multiple Equilibria in Macroeconomics,” *NBER Macroeconomics Annual 2000.*

[28] Morris, Stephen, and Hyun Song Shin (2003), “Global Games—Theory and Applications,” in *Advances in Economics and Econometrics*, 8th World Congress of the Econometric Society (M. Dewatripont, L. Hansen, and S. Turnovsky, eds.), Cambridge University Press.

[29] Morris, Stephen, and Hyun Song Shin (2004), “Liquidity Black Holes,” *Review of Finance* 8, 1-18.

[30] Morris, Stephen, and Hyun Song Shin (2006a), “Endogenous Public Signals and Coordination,” working paper, Princeton University.

[31] Morris, Stephen, and Hyun Song Shin (2006b), “Catalytic Finance,” *Journal of International Economics* 70, 161-177.

[32] Obstfeld, Maurice (1996), “Models of Currency Crises with Self-Fulfilling Features,” *European Economic Review* 40, 1037-47.

[33] Ozdenoren, Emre, and Kathy Yuan (2006), “Feedback Effects and Asset Prices,” forthcoming in *Journal of Finance.*

[34] Rochet, Jean-Charles, and Xavier Vives (2004), “Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?” *Journal of the European Economic Association* 2-6, 1116-1147.

[35] Tarashev, Nikola, (2007), “Currency Crises and the Informational Role of Interest Rates,” *Journal of the European Economic Association*, 5(4), 1-36.

[36] Tamer, Elie (2003), “Incomplete Simultaneous Discrete Response Model with Multiple Equilibria,” *Review of Economic Studies*, 147–165.

[37] Zettelmeyer, Jeromin (2000), “Can Official Crisis Lending be Counterproductive in the Short Run?” *Economic Notes* 29, 12-29.

[38] Zwart, Sanne (2007) "The mixed blessing of IMF intervention: Signalling versus liquidity support," *Journal of Financial Intermediation,* 3, 149–174.