The Market for OTC Derivatives*

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Abstract

We develop a model of equilibrium entry, trade, and price formation in over-the-counter (OTC) markets. Banks trade derivatives to share an aggregate risk subject to two trading frictions: they must pay a fixed entry cost, and they must limit the size of the positions taken by their traders because of risk-management concerns. Although all banks in our model are endowed with access to the same trading technology, some large banks endogenously arise as "dealers," trading mainly to provide intermediation services, while medium sized banks endogenously participate as "customers" mainly to share risks. We use the model to address positive questions regarding the growth in OTC markets as trading frictions decline, and normative questions of how regulation of entry impacts welfare.

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1 Introduction

Several stylized observations regarding trading patterns in over-the-counter (OTC) markets for derivatives have recently received considerable attention from policy makers and the public alike. First, the large volume of bilateral trades at varied prices creates an intricate liability structure between participating banks. Second, for the banks participating in these markets, the gross volume of trade greatly exceeds the net volume. This difference between gross and net volume is particularly striking for a few large banks, who have very large gross positions but very small net positions. Because these large banks hold nearly offsetting long and short positions, they can be viewed as intermediaries or "dealers." In contrast, medium sized banks act as "customers": they trade mostly in one direction, either long or short, so their gross and net volume are close to each other. Lastly, these OTC markets are segmented: most smaller banks do not participate at all. Clearly, these trading patterns differ from the Walrasian market benchmark, in which all agents participate, gross and net positions always coincide, and trade occurs at a common price.

In this paper, we offer a parsimonious equilibrium model of entry, trade, and price formation in an OTC market. Trade in our model is subject to two key trading frictions. First, a fixed entry cost must be paid by participating banks; second, traders within each bank face limits on the position they can take with any counterparty. The first friction is motivated by the observation that trade in OTC derivatives markets requires specialized capital and expertise. The second friction is motivated by the observation that, within banks, a standard risk management practice is to put limits on traders' positions. We show that, based on these two trading frictions alone, our model can rationalize the observed market patterns: namely, the intricate liability structure, the discrepancies between gross and net positions, the dispersion of prices, and banks' participation patterns. In particular, although all banks in our model have access to the same trading technology, some large banks endogenously participate as "dealers," trading to provide intermediation services, mainly to profit from price dispersion, while medium-sized banks endogenously participate as "customers," mainly to share risks. Lastly, because of the fixed cost, small banks do not participate. We use the model to address positive questions regarding the growth in OTC markets as trading frictions decline, and normative questions of how regulation of entry impacts welfare.

We consider a theoretical financial system composed of a continuum of financial institutions we call banks. A bank is viewed as a coalition of many risk-averse agents, called traders. Banks' coalitions have heterogeneous sizes and heterogeneous endowments of a

non-tradable risky loan portfolio, creating heterogeneous exposures to an aggregate risk factor. Since traders in banks are risk averse, they will attempt to equalize these exposures. While, in our model, banks cannot alter their exposure directly by trading their loan endowment, they can do it synthetically by trading derivatives "swap" contracts. Specifically, conditional on their size and initial exposure to aggregate risk, banks first choose whether to pay a fixed cost in order to enter into an OTC market for swaps. Next, traders from participating banks trade swaps bilaterally, subject to a trade size limit. Finally, banks consolidate their positions internally, and loans and swap contracts pay off. We first characterize the equilibrium bilateral and aggregate trade volumes, as well as pricing, conditional on entry patterns. After this analysis, we study entry.

Consider first the market equilibrium after entry. Conditional on banks' entry decisions, the traders in each participating bank establish bilateral trading relationships almost surely with traders from every other bank. When the traders from two banks match, they bargain over the terms of a swap contract: the direction, the size, and the price. Gains from trade are determined by the expected post-trade risk exposures of the traders' respective banks. The direction of the contract which each trader executes depends on whether their counterparty's bank expects a larger or smaller post-trade exposure to the aggregate risk factor than their own bank. The size of the contract is constrained by a trade size limit. Lastly, the price is set to split the gains from trade equally between the two traders. Thus, within a bank, some traders execute long contracts, and some enter short contracts, at dispersed prices that depend on the post-trade exposures of their different counterparties. At the end of the period, the swap portfolio of a participating bank is made up of the swap contracts of all its traders. In equilibrium, traders thus share risk within their banks, and banks share aggregate risk amongst each other through the OTC derivatives market.

All the bilateral relationships between traders of different banks are constrained by a trade size limit: a trader cannot sign more than a given amount of swap contracts with her counterparty, either long or short. We assume that these trading limits are allocated to the traders in a bank before information about the risk exposure of each trader's counterparty is revealed. The restriction that banks cannot reallocate trading limits across traders once trade has begun is what effectively limits risk sharing.

Next, consider entry. Banks' decisions to enter the market are driven by two motivations, the strength of which are determined by their sizes and initial risk exposures. One motivation to enter is to share aggregate risk. This motivation is strong when a bank's initial risk exposure is significantly higher or lower than the market-wide average. A second motivation for a bank to enter is to capture the trading profits due to

the equilibrium price dispersion across different bilateral trades. These motivations result in entry patterns corroborated by empirical evidence: small-sized banks cannot spread the fixed entry cost over many traders, and choose not to enter. Medium-sized banks only find it optimal to enter the market if their gains from trading in the OTC market are large enough, which we show occurs when their initial risk exposure is significantly higher or lower than the market-wide average. They use the OTC market to take a large net position, either short or long, and in this sense act as customers. Finally, large-sized banks are willing to enter the swap market irrespective of their initial risk exposure. If their initial risk exposure is significantly higher or lower than the market-wide average, they enter as customers for the same reason as the medium-sized banks do. If their initial exposure is near the market-wide average, they do not desire much change in their risk exposure, and therefore do not have incentives to enter as customers. Nevertheless, they enter for a different reason: their size allows them to conduct sufficiently many trades to profit from equilibrium price dispersion while covering the fixed cost of entry. Such banks do not change their exposure much and have many offsetting long and short positions. In this sense, large banks with average risk exposures emerge endogenously as dealers who provide intermediation services in the OTC market, have large gross position but small net positions.

To offer theoretical insights into the growth of OTC markets for derivatives over the last decade, we ask how trading patterns change in response to a decrease in trading frictions. We contrast two possible scenarios: a decline in the entry cost representing more sophisticated trading technologies, or a relaxation of trading limits representing improvements in operational risk management practices. Our results indicate that in both cases, reducing frictions causes the market to grow, in the sense of increasing gross notional outstanding per capita. But predictions differ markedly in other dimensions. For example, when trading technologies improve (entry cost falls), the market grows through an increase in intermediation activity, and the average net-to-gross notional ratio decreases. When risk-management technologies improve (trading limits are relaxed), the market can grow through an increase in customer-to-customer trades. As a result, the net-to-gross notional ratio can increase. Thus, according to the model, the evolution of the net-to-gross notional ratio can help distinguish a decrease in frictions due to an improvement in trading versus risk-management technologies.

Finally, we also ask how a planner or policy maker might improve the OTC market structure. In our model, large banks choose to enter as dealers, provide intermediation by taking offsetting long and short positions, and make profits thanks to equilibrium price dispersion. One policy question is whether the private incentives to provide intermediation

that arise endogenously in response to the trading frictions in the OTC market lead to socially efficient entry. We show that large banks enter too much, and in this sense the market is too concentrated: a social planner could improve welfare by removing some larger dealer banks from the market and encouraging smaller banks to enter.

The paper proceeds as follows. Section 2 surveys the literature. To offer some empirical context for the market structure arising in our model, Section 3 presents stylized facts characterizing the OTC market for Credit Default Swaps (CDS). Section 4 presents the economic environment, Section 5 solves for equilibrium trading conditional on entry patterns, and Section 6 studies equilibrium entry decisions, showing that our model rationalizes the banks' participation patterns observed in the data. Finally, Section 7 analyzes efficiency and Section 8 concludes. Proofs not given in the text are gathered in the appendix.

2 Related Literature

Our main contribution relative to the literature on OTC markets is to develop a model that is sufficiently tractable to analyze endogenous entry, explain empirical patterns of participation across banks of different sizes, and address normative issues regarding the size and composition of the market. In particular, by allowing the system of bilateral trades and price dispersion to arise endogenously as a result of market entry, and by studying the costs and benefits of a structure in which certain banks play a more important role in intermediating trade, we are able to study the costs and benefits of a more concentrated market structure.[1]

Several recent papers consider ideas related to the role of the market structure in determining trading outcomes in OTC markets. Duffie and Zhu (2011) use a framework similar to that in Eisenberg and Noe (2001) to show that a central clearing party for CDS only may not reduce counterparty risk because such a narrow clearinghouse could reduce cross contract class netting benefits. Babus (2009) studies how the formation of long-term lending relationships allows agents to economize on costly collateral, demonstrating how star-shaped networks arise endogenously in the corresponding network formation game. Gofman (2011) emphasizes the role of the bargaining friction in determining whether trading outcomes are efficient in an exogenously specified OTC trading system represented by a graph.

[1]The costs of concentration have been a key concern to regulators of OTC derivatives markets. See, for example, the quarterly reports from the Office of the Comptroller of the currency at www.occ.treas.gov/topics/capital-markets/financial-markets/trading/derivatives/derivatives-quarterly-report.html, as well as ECB (2009), and Terzi and Ulucay (2011).

The effects of the trading structure on trading outcomes has been studied in the literature on systemic risk.[2] Allen and Gale (2000) develop a theory of contagion in a circular system, which they use to consider systemic risk in interbank lending markets. This framework has been employed by Zawadowski (2013) to consider counterparty risk in OTC markets. Eisenberg and Noe (2001) also study systemic risk, but use lattice theory to consider the fragility of a financial system in which liabilities are taken as given (see also the recent work of Elliott, Golub, and Jackson, 2012). Our work differs from the papers discussed above by considering entry.

One of the most commonly employed frictions used to study OTC markets, following Duffie, Gârleanu, and Pedersen (2005), is the search friction.[3] Kiefer (2010) offers an early analysis of CDS pricing within this framework. Our paper is most closely related to the earlier work by Afonso and Lagos (2012), who develop a different search model in order to explain trading dynamics in the Federal Funds Market. While their focus is on the dynamics of reserve balances, they also consider the importance of intermediation by banks in the reallocation of reserves over the course of the day. We differ from this literature in that we develop a different model, in the spirit of Shi (1997), with a trading friction that is conceptually different from search. We gain tractability by collapsing all trading dynamics into a single multilateral trading session. This allows us to study entry.

[2]See Stulz (2011) for a discussion of the potential for systemic risk in CDS markets.

[3]Duffie, Gârleanu, and Pedersen (2002) apply the paradigm to markets for borrowing stocks. Vayanos and Wang (2007), Weill (2008), and Vayanos and Weill (2008) study cross-sectional asset pricing in multi-asset extensions of the original model. Lagos and Rocheteau (2009) and Gârleanu (2009) demonstrate that relaxing the asset holding constraint can have an important impact on asset prices and on the entry decisions of intermediaries. Afonso (2011) studies the externalities involved in investors' entry decisions. Weill (2007) and Lagos, Rocheteau, and Weill (2011) study the liquidity provision of dealers in response to liquidity shocks. Biais, Hombert, and Weill (2010) use this paradigm to analyze equilibrium in limit order markets. Duffie and Strulovici (2012) study limited capital mobility between markets and its impact on asset prices. Pagnotta and Philippon (2011) analyze exchanges competing to offer trading speed. He and Milbradt (2012) study the interaction between default risk and OTC market liquidity. Information diffusion is addressed by Duffie and Manso (2007), Golosov, Lorenzoni, and Tsyvinski (2008), Duffie, Malamud, and Manso (2009), and recently by Babus and Kondor (2013) using different techniques. Adverse selection is studied by Guerrieri and Shimer (2012) and Chang (2012). Trejos and Wright (2012) offer a unified analysis of asset pricing in finance and monetary models of bilateral trades. Bolton, Santos, and Scheinkman (2012) show how OTC markets may inefficiently "cream-skim" assets from organized exchange.

3 CDS Stylized Facts

We will focus our analysis on the market for Credit Default Swap (CDS).[4] This market is large: in the third quarter of 2011, the top twenty-five U.S. bank holding companies participating in over-the-counter (OTC) derivatives markets had $13.58 trillion in assets, and held almost twice as much, or $22.28 trillion, in credit derivatives notional. Most of these credit derivatives are CDS contracts. In what follows, we collect data from the Office of the Comptroller of the Currency for the top 25 bank holding companies in derivatives, from these Bank Holding Companies' FR Y-9C filings, and from the Depository Trust & Clearing Corporation (DTCC) Trade Information Warehouse. We use these data to document the following stylized facts which characterize the market for CDS.[5]

3.1 Dealers, customers, and non participants

Based on their patterns of trades in the CDS market, banks can be divided into three categories: dealers, customers, and non participants.

Dealers. A few large banks, commonly referred to as "dealers," concentrate most of the market, measured in terms of gross notional in credit derivatives outstanding. As shown in Figure 1, in the U.S., over ninety-five percent of the gross notional in credit derivatives is consistently held by only five bank holding companies.[6] Figure 2 reveals that these gross positions remain large after controlling for bank size: namely, graphing gross notional to trading assets across banks ranked by trading assets shows that the top 25 bank holding companies in derivatives trade disproportionately more than others.

Dealer banks appear to be motivated by trading or intermediation activity as opposed to hedging underlying exposure. To document that this activity is not motivated by hedging, consider first that, for these banks, there is significant netting between long and short contracts, as measured by their ratio of net to gross notional. Figure 3 plots net

[4] As will become clear later, our theoretical analysis applies more generally to OTC trading of any derivatives contracts, in which counterparties make a fixed-for-floating exchange of cash flow streams, and in which the floating stream is exposed to aggregate risk. This includes, for examples, interest rate swaps, CDS of sovereign entities, CDS indices and, to the extent that default risk is correlated across firms, CDS on single firms.

[5] For other recent studies of CDS markets, see Bolton and Oehmke (2011b), Bolton and Oehmke (2011a), Oehmke and Zawadowski (2013) for theoretical studies, and Arora, Gandhi, and Longstaff (2012), Chen et al. (2011), Saretto and Tookes (2011), and Shachar (2012) for empirical analyses. Bolton and Oehmke (2013) review the growing literature on the market for CDS.

[6] The International Swaps and Derivatives Association (2010) shows that concentration is less dramatic in an expanded data set including non-U.S. financial institutions: for CDS, about ninety percent of gross notional is held by fourteen global dealer banks.

to gross notionals for the top 25 bank holding companies in derivatives and shows that this fraction is on average close to zero for the largest dealer banks, which appear on the right-hand side.[7] For example, JP Morgan's net to gross notional ratio is -0.1%.

Another piece of evidence is derived from regulatory reporting. Namely, starting in the first quarter of 2009, and implemented to a greater extent in the second quarter of 2009, bank holding companies' FR Y-9C filings report the notional of purchased credit derivatives that are recognized as a guarantee for regulatory capital purposes. Figure 4 shows the fraction of purchased credit derivatives from Q2 2009 to Q4 2011 that could be counted as a guarantee for regulatory purposes for the largest 12 vs. the rest of the top 25 bank holding companies in derivatives. It reveals that, for the top 12 bank holding companies in derivatives, less than 0.5% of purchased credit derivatives could be recognized as a guarantee. This is consistent with dealer banks simply trading CDS to earn spreads on intermediation volume.

Customers. In general middle-sized and smaller banks tend to be customer banks (though not all customers are small). Their trading patterns are opposite of those of dealer banks. Figure 2 shows that middle-sized banks have smaller gross notional outstanding, and Figure 3 shows that their net-to-gross notional ratio is often close to 100%. For instance, Bank of New York Mellon has a net-to-gross notional ratio close to 100%, meaning that nearly all its CDS positions are going in the same direction. Turning to regulatory reporting, Figure 4 shows that, for the 13 smaller bank holding companies amongst the top 25, almost 40% of purchased credit derivatives were recognized as a guarantee. This is consistent with the notion that customer banks are more likely to use purchased CDS to change their credit exposure and to hedge.

Non-participants. Many banks do not participate at all in the CDS market. The Federal Reserve Bank of Chicago lists about 14,000 U.S. bank holding companies, while Chen et al. (2011) report that only about 900 bank holding companies worldwide trade in CDS.

[7] We report statistics for netting of long and short positions multilaterally and across contracts. While ISDA master agreements account only for bilateral netting, the aggregate data we can access does not allow us to disentangle bilateral relationships. Even adhering to ISDA's strict definition of netting, we can verify from the 10-Q's of individual firms that large banks indeed enjoy large netting benefits. See, for example, the excerpts from the 10-Q's for Bank of America or Goldman Sachs in the Financial Times Alphaville at ftalphaville.ft.com/blog/2011/12/21/808181/do-you-believe-in-netting-part-1/.

3.2 Prices and bilateral trade patterns

Two further stylized facts are of interest to us.

Price Dispersion. Prices vary by counterparty. This is apparently true since pricing data from Markit are composite quotes from multiple sources. Arora, Gandhi, and Longstaff (2012) use heterogeneity in quotes from multiple dealers to a single customer to assess the extent to which counterparty risk is priced. Interestingly, they find that little of the price dispersion is explained by counterparty risk. This finding is consistent with the notion that spreads are driven by other bank's characteristics, such as their credit exposures and their outside trading options. Shachar (2012) also uses data on individual trades, but studies the impact of dealers' post-trade credit risk exposures on their ability to provide liquidity. This evidence is consistent with the banks' preferences and pricing in our model; banks price each contract based on their pre-trade risk exposure combined with any additional default risk arising from the rest of their portfolio. Further indirect evidence of price dispersion is provided by Mayordomo, Pena, and Schwartz (2010).

Network of Trades. Finally, banks are connected by a complex liability structure. The OTC market for CDS is an opaque market in which the liability linkages are private information. Following the financial crisis, the Depository Trust & Clearing Corporation (DTCC) expanded its collection of data on the stock of OTC swaps outstanding and has began releasing aggregate data on gross and net notional outstanding for CDS on a large number of underlying entities. Two features of these data are of note.

First, these data confirm that banks (financial institutions aggregated up to the holding company level) take gross positions far in excess of their net positions, even when narrowing down to CDS on individual reference entities. In February 2013, looking across CDS on the top 1000 single name reference entities by gross notionals, the ratio of aggregate net to aggregate gross exposure is 7%. Similarly, for the 370 CDS on indices for which there are 10 contracts or more outstanding, the ratio of aggregate net to aggregate gross exposure is 13%.[8]

Second, the DTCC also offers aggregated data on the gross notionals signed between institutions they define as dealers and institutions they define as customers. In February 2013, dealer-to-dealer trades accounted for roughly 60% of all CDS gross notional outstanding, customer-to-dealer trade for roughly 40%, and customer-to-customer trades for

[8]The data from which we computed these statistics are in Table 6 and 7 available on the DTCC website.

a very small fraction, less than 1%.[9]

As these facts clearly indicate, trading patterns in this market differ from what would be predicted by a traditional Walrasian model. In Walrasian markets, gross exposures and net exposures coincide, there is no price dispersion, and there is no explicit network of trade.

4 The economic environment

This section presents the economic environment.

4.1 Preferences and endowments

The economy is populated by a unit continuum of risk-averse agents, called traders. Traders have utility functions with identical constant absolute risk aversion and are endowed with a technology to make payments by producing a storable consumption good at unit marginal cost.[10] To model the financial system, we take a novel approach in the literature: we assume that traders are organized into large coalitions called banks. As will become clear, this assumption allows us to model banks' gross and net exposures even when we collapse all trade dynamics into a single multilateral trading session.

Banks are heterogeneous along two dimensions: their sizes, denoted by S, which we identify with the number of traders in the coalition, and their per capita endowment of some non-tradeable risky loan portfolio, denoted by ω. We interpret this endowment as arising from the bank's lending activity to households and corporations, which we do not model explicitly here. Although the distribution of (S, ω) in the economy at large is exogenous, we will endogenize its distribution amongst banks that choose to participate in the OTC market by studying entry.

Banks' sizes are cross-sectionally distributed according to the continuous density $\varphi(S)$ over the support $[\underline{S}, \infty)$, $\underline{S} \geq 0$. The density has thin enough tails, in that $\lim_{S\to\infty} S^3\varphi(S)$ exists and is finite. Because the economy-wide number of traders is one, we must have $\int_{\underline{S}}^{\infty} S\varphi(S)\, dS = 1$.

Banks' per capita endowments of loan portfolio are cross-sectionally distributed according to a uniform distribution over $[0, 1]$. The per capita payoff for the bank from

[9] See Table 1 on the same DTCC website. Figure 5 plots the time series of the composition of trade by counterparty type in the CDX market, using data from the DTCC for October of 2008 to the present.

[10] Precisely, if an agent consumes C and produces H, his utility is $U\{C - H\} = -\frac{1}{\alpha}e^{-\alpha(C-H)}$, for some coefficient of absolute risk aversion $\alpha > 0$. Given no wealth effect, an equivalent interpretation is that the agent has a large endowment of storable consumption good that she uses to make payments.

its illiquid loan portfolio is the size of the portfolio, ω, times each loan's payoff $1 - D$, where 1 represents the face value of the loans and D represent default risk. We assume that $D \in [0, 1]$ is a (non-trivial) random variable with a twice continuously differentiable moment generating function, identical for all loans. Thus, ω represents a bank's pre-trade, per-capita exposure to an aggregate default risk factor.

To clarify the economic forces at play, we assume that banks' sizes and per-capita endowments are independently distributed in the economy at large. As a result, we will argue that the correlation between bank's size and per-capita endowment that arises in the OTC market equilibrium is purely endogenous. Even with no presumed correlation between size and per-capita endowment ex-ante, our model with entry costs and trading limits helps rationalize banks' participation and trading patterns, as described above in Section 3.[11]

4.2 Two key OTC market frictions

If banks could trade their loan endowments directly in a competitive market, they would find it optimal to share risk fully by equalizing their exposures to the aggregate default risk factor.[12] In our model, we prevent such full risk sharing by assuming that banks cannot trade their loan endowments directly, but we allow partial risk sharing by letting banks enter an OTC derivatives market to trade swap contracts, resembling CDS. The timing of entry and trade is as follows. First, each bank chooses whether to participate in the OTC market. Next, traders from active banks meet in the OTC market. Finally, banks consolidate the positions of their traders and all payoffs from loan portfolios and swap contracts are realized.

Trade in the OTC market is impaired by two frictions: *fixed entry costs* and *trading limits*.

First, when a bank chooses to participate in the OTC market, it has to pay a fixed entry cost. In practice, mass processing of OTC derivatives order tickets requires a large infrastructure, and trading in derivatives requires specialized expertise which is costly to acquire. In our model, the fixed cost implies that smaller banks will not enter and trade. As we discuss below, a bank that chooses to participate will endogenously take on the

[11] Nevertheless, our model is flexible enough to handle more general joint distributions of size and per capita endowments. For example, in an earlier version of the paper, we provided a characterization of the post-entry equilibrium when larger banks have more neutral pre-trade exposures than smaller banks, for example through greater internal diversification.

[12] One could add a speculative trading motive by assuming that banks have different beliefs about the mean loss-upon-default, $\mathbb{E}[D]$. When D is normally distributed, the model of trading in the OTC market would remain as tractable as the one we discuss here.

trading patterns of a dealer, or that of a customer, depending on its risk exposure relative to that of other participants.

Second, when traders from participating banks meet and sign CDS, their banks subject them to a trading limit. This limit proxies for risk-management constraints on individual trading desk positions in practice. We do not model the microfoundations of the trading limit, however we note that, in practice, traders typically do face line limits. For example, Saita (2007) states that the traditional way to prevent excessive risk taking in a bank "has always been (apart from direct supervision...) to set notional limits, i.e., limits to the size of the positions which each desk may take."[13,14] Theoretically, one might motivate such limits as stemming from moral hazard problems, concerns about counterparty risk and allocation of scarce collateral, or from capital requirement considerations. In this sense, the trading limit is conceptually different from the traditional search friction considered in the literature. As we show below, this trading limit is what prevents participant banks from fully sharing their risk in the OTC market. It is also what leads to price dispersion in equilibrium and hence creates incentives for banks to enter and provide intermediation services.

4.3 Payoff of non-participating banks

Traders in non-participating banks consume the per-capita payoff of their loan portfolio endowment, $\omega(1-D)$, with expected utility:[15]

$$\mathbb{E}\left[U\{\omega(1-D)\}\right] \equiv -\frac{1}{\alpha}\mathbb{E}\left[e^{-\alpha\omega(1-D)}\right].$$

The corresponding certainty-equivalent payoff is:

$$\mathrm{CE}_n(\omega) = \omega - \Gamma\left[\omega\right], \qquad \text{where } \Gamma\left[\omega\right] \equiv \frac{1}{\alpha}\log\left(\mathbb{E}\left[e^{\alpha\omega D}\right]\right). \tag{1}$$

[13] In addition, measures such as DV01 or CS1% which measure positions' sensitivities to yield and credit spread changes, as well as risk weighted asset charges, are used to gauge and limit the positions of a particular desk's traders.

[14] On September 18, 2013, in a Wall Street Journal column about the London Whale scandal, Stephen R. Etherington explains the importance of risk limits for risk management: "Traders are specifically hired to take financial risk for the firm's gain. Assigning risk limits for each trader is the key control that, when aggregated with all trader limits, ensures that the firm's overall market risk remains tolerable. When traders exceed their limits, they are "going rogue" and exposing the firm to higher market risks than management intended."

[15] Given identical concave utility, this is indeed the *ex-ante* optimal allocation of risk amongst traders in the bank.

That is, $\mathrm{CE}_n(\omega)$ is equal to the face value of the loan portfolio endowment, ω, net of the certainty equivalent cost of bearing its default risk, $\Gamma[\omega]$.

Lemma 1. *The certainty equivalent cost bearing default risk,* $\Gamma[\omega]$*, is twice continuously differentiable, strictly increasing,* $\Gamma'[\omega] > 0$*, and strictly convex,* $\Gamma''[\omega] > 0$*.*

These intuitive properties follow by taking derivatives. Note that, when D is normally distributed with mean $\mathbb{E}[D]$ and variance $\mathbb{V}[D]$, $\Gamma[\omega]$ is the familiar quadratic function:

$$\Gamma[\omega] = \omega\mathbb{E}[D] + \omega^2 \frac{\alpha\mathbb{V}[D]}{2}. \tag{2}$$

The first term is the expected loss $\omega\mathbb{E}[D]$ upon default. The second term is an additional cost arising because banks are risk averse and the loss is stochastic.[16]

4.4 Payoff of participating banks

In the OTC market, each trader from every bank is matched with a trader from some other bank to bargain over a CDS contract. Matching is uniform, in the sense that all traders in the population of participating banks are equally likely to be matched. Thus, the probability of being matched with a trader from a bank whose per-capita endowment is less than ω is equal to $N(\omega)$, the fraction of such traders in the OTC market. The distribution $N(\omega)$ is the result of entry decisions that we study later. For now we just need to guess that $N(\omega)$ admits a continuous density $n(\omega)$, positive almost everywhere.

Notice that, in our model, the trading technology is the same for all traders. In particular, traders from large banks do not have any built-in technological advantage in trading. Thus, in our model, any greater tendency for traders of large banks to become intermediaries is purely endogenous.

As will become clear shortly, our model has a natural homogeneity property: two participating banks with identical per capita loan portfolio endowments, ω, have identical per capita trading behavior. As discussed formally after Proposition 1, this implies that after entry, size is no longer a state variable for the bank, and that the OTC market equilibrium will only depend on the distribution $n(\omega)$.

CDS contracts in the OTC market. When a trader from a bank of type ω (an "ω-trader") meets a trader from a bank of type $\tilde{\omega}$ (an "$\tilde{\omega}$-trader"), they bargain over the

[16] Clearly, a normal distribution does not satisfy our assumption that $D \in [0,1]$. For this reason, it implies that $\Gamma[\omega]$ is decreasing for ω negative enough. However, and as will become clear as we progress, our results only rely on strict convexity and so they continue to hold with a normally distributed D.

terms of a derivative contract resembling a CDS. The ω-trader sells $\gamma(\omega, \tilde{\omega})$ contracts to the $\tilde{\omega}$-trader, whereby she promises to make the random payment $\gamma(\omega, \tilde{\omega})D$ at the end of the period, in exchange for the fixed payment $\gamma(\omega, \tilde{\omega})R(\omega, \tilde{\omega})$. If $\gamma(\omega, \tilde{\omega}) > 0$ then the ω-trader sells insurance, and if $\gamma(\omega, \tilde{\omega}) < 0$ she buys insurance. As explained before, traders face a position limit: in any bilateral meeting, they cannot sign more than a fixed amount of contracts, k, either long or short. Taken together, the collection of CDS contracts signed by all banks $(\omega, \tilde{\omega}) \in [0, 1]^2$ must therefore satisfy:

$$\gamma(\omega, \tilde{\omega}) + \gamma(\tilde{\omega}, \omega) = 0 \tag{3}$$

$$-k \leq \gamma(\omega, \tilde{\omega}) \leq k. \tag{4}$$

Equation (3) is simply a bilateral feasibility constraint, and equation (4) is the trading limit.

Bank's per capita consumption. A trader in this economy faces two kinds of risk. The first is idiosyncratic risk over the type of counterparty they will trade with, namely the size and risk exposure of their counterparty's bank. The second is aggregate default risk. But since there is a large number of traders in each bank, traders can diversify their idiosyncratic counterparty-type risk so that they are left with only the per capita exposure to default risk. Specifically, we assume that at the end of the period, traders of bank ω get together and consolidate all of their long and short CDS positions.[17] By the law of large numbers, the per capita consumption in an active bank with per capita endowment ω and size S is:

$$-\frac{c}{S} + \omega(1 - D) + \int_0^1 \gamma(\omega, \tilde{\omega}) \Big(R(\omega, \tilde{\omega}) - D \Big) n(\tilde{\omega}) d\tilde{\omega}. \tag{5}$$

The first term is the per capita entry cost. The second term is the per capita payout of the loan portfolio endowment, after default. The third term is the per capita consolidated amount of fixed payments, $\gamma(\omega, \tilde{\omega})R(\omega, \tilde{\omega})$, and random payments, $\gamma(\omega, \tilde{\omega})D$, on the portfolio of contracts signed by all ω-traders. Note in particular that, given uniform matching, $n(\tilde{\omega})$ represents the fraction of ω-traders who met $\tilde{\omega}$-traders.

One can see that the position limit k is indeed crucial. Banks in our model almost surely trade with every other bank and thus would want to allocate all their trading

[17] As is well known from other models with "large coalition" or "large families," we could equivalently assume that traders can buy and sell CDS in two ways: i) with traders from other banks, in a bilateral OTC market subject to trade size limits and ii) with traders from the same bank, in an internal frictionless competitive market. The internal competitive market leads to full risk sharing within the bank, just as with the large coalition.

capacity to their best counterparties, thereby achieving full risk-sharing. Again, we argue that in reality risk management practices aimed to alleviate standard moral problems prevent such reallocation of trading capacity.

Our assumption that traders consolidate their CDS positions captures some realistic features of banks in practice. Within a bank, some traders will go long and some short, depending on whom they meet and trade with. Because of this, our model is able to distinguish between gross and net exposure to credit risk resulting from trades in the CDS market.

Certainty equivalent payoff. To calculate the certainty equivalent payoff, it is useful to break down the bank's per capita consumption in equation (5) into a deterministic and a random component. Namely, in bank ω, the per capita deterministic payment is:

$$-\frac{c}{S}+\omega+\int_0^1 \gamma(\omega,\tilde{\omega})R(\omega,\tilde{\omega})n(\tilde{\omega})d\tilde{\omega}. \tag{6}$$

Similarly, the per capita random payment is $-g(\omega)D$, where

$$g(\omega)\equiv\omega+\int_0^1 \gamma(\omega,\tilde{\omega})n(\tilde{\omega})d\tilde{\omega}, \tag{7}$$

is the sum of the initial exposure, ω, and of the exposure acquired in bilateral matches. The function $g(\omega)$ thus represents the bank's *post-trade* exposure to default risk.

Just as with non-participating banks in equation (1), we find that the per capita certainty equivalent payoff of a participating bank is

$$\mathrm{CE}_p(\omega,S)=-\frac{c}{S}+\omega+\int_0^1 \gamma(\omega,\tilde{\omega})R(\omega,\tilde{\omega})n(\tilde{\omega})d\tilde{\omega}-\Gamma\left[g(\omega)\right], \tag{8}$$

the per-capita deterministic payment, net of the certainty equivalent cost of bearing the exposure $g(\omega)$ to aggregate default risk.

Bargaining in the OTC market. To determine the terms of trade in a bilateral meeting, we need to specify the objective function of a trader. To that end, we follow the literature which allows risk sharing within families, such as in Lucas (1990), Andolfatto (1996), Shi (1997), Shimer (2010), and others, and assume that a trader's objective is to maximize the marginal impact of her decision on her bank's utility. This assumption means that a trader is small relative to her institution and that she does not coordinate her strategy with other traders in the same institution. One could think, for instance,

about a trading desk in which all traders work independently knowing that all risks will be pooled at the end of the day.

Precisely, when a trader signs $\gamma(\omega, \tilde{\omega})$ contracts at a price $R(\omega, \tilde{\omega})$ per contract, her marginal impact on her bank's utility is defined as:

$$\mathbb{E}\left[\Lambda(\omega, D)\gamma(\omega, \tilde{\omega})\Big(R(\omega, \tilde{\omega}) - D\Big)\right], \text{ where } \Lambda(\omega, D) \equiv \frac{U'\left\{y(S, \omega, D)\right\}}{\mathbb{E}\left[U'\left\{y(S, \omega, D)\right\}\right]},$$

and $y(S, \omega, D)$ is the bank's per capita consumption derived in equation (5).

The first term in the expectation, $\Lambda(\omega, D)$, is bank ω's stochastic discount factor. Since utility is exponential, there are no wealth effects and so $\Lambda(\omega, D)$ is invariant to deterministic changes in the level of consumption. In particular, it does not depend on the entry cost, c/S, and therefore does not depend on size.

The second term is the trader's contribution to her bank's consumption: the number of contracts signed, $\gamma(\omega, \tilde{\omega})$, multiplied by the net payment per contract, $R(\omega, \tilde{\omega}) - D$. Using the formula for the cost of risk bearing, $\Gamma\left[g(\omega)\right]$, the ω-trader's objective function can be simplified to:

$$\gamma(\omega, \tilde{\omega})\Big(R(\omega, \tilde{\omega}) - \Gamma'\left[g(\omega)\right]\Big). \tag{9}$$

Note that this can be viewed as the trader's marginal contribution to the certainty equivalent payoff (8). The expression is intuitive. If the trader sells $\gamma(\omega, \tilde{\omega})$ CDS contracts, she receives the fixed payment $R(\omega, \tilde{\omega})$ per contract but, at the same time, she expects to increase her bank's cost of risk bearing. Since the trader is small relative to her bank, she only has a marginal impact on the cost of risk bearing, equal to $\gamma(\omega, \tilde{\omega})\Gamma'\left[g(\omega)\right]$.

The objective of the other trader in the match, the $\tilde{\omega}$-trader, is similarly given by:

$$\gamma(\omega, \tilde{\omega})\Big(\Gamma'\left[g(\tilde{\omega})\right] - R(\omega, \tilde{\omega})\Big), \tag{10}$$

where we used the bilateral feasibility constraint of equation (3), stating that $\gamma(\tilde{\omega}, \omega) = -\gamma(\omega, \tilde{\omega})$. The trading surplus is therefore equal to the sum of (9) and (10):

$$\gamma(\omega, \tilde{\omega})\Big(\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\Big).$$

We assume that the terms of trade in a bilateral match between an ω-trader and an $\tilde{\omega}$-trader are determined via symmetric Nash bargaining. The first implication of Nash bargaining is that the terms of trade are (bilaterally) Pareto optimal, i.e, they must

maximize the surplus shown above. Since the marginal cost of risk bearing, $\Gamma'[x]$, is increasing, this immediately implies that:

$$\gamma(\omega,\tilde{\omega}) = \begin{cases} k & \text{if } g(\tilde{\omega}) > g(\omega) \\ [-k,k] & \text{if } g(\tilde{\omega}) = g(\omega) \\ -k & \text{if } g(\tilde{\omega}) < g(\omega). \end{cases} \tag{11}$$

This is intuitive: if the $\tilde{\omega}$-trader expects a larger post-trade exposure than the ω-trader, $g(\tilde{\omega}) > g(\omega)$, then the ω-trader sells insurance to the $\tilde{\omega}$-trader, up to the trading limit. And vice versa if $g(\tilde{\omega}) < g(\omega)$. When the post-trade exposures are the same, then any trade in $[-k,k]$ is optimal.

The second implication of Nash bargaining is that the unit price of a CDS, $R(\omega,\tilde{\omega})$, is set so that each trader receives exactly one half of the surplus. This implies that:

$$R(\omega,\tilde{\omega}) = \frac{1}{2}\Big(\Gamma'[g(\omega)] + \Gamma'[g(\tilde{\omega})]\Big). \tag{12}$$

That is, the price is halfway between the two traders' marginal cost of risk bearing. As is standard in OTC market models, prices depend on traders' "infra-marginal" characteristics in each match, instead of depending on the characteristic of a single "marginal" trader, as would be the case in a Walrasian market.

It is important to note that a trader's reservation value in a match is determined by her *post-trade* exposure, which results from the simultaneous trades of all traders in her institution. This means that, although our model is static, outside options play a key role in determining prices: if a trader chooses not to trade in a bilateral match, she still enjoys the benefits created by the trades of all other traders in her institution. This is similar to the familiar outside option of re-trading later arising in dynamic models.[18]

5 Equilibrium conditional on entry

Conditional on the distribution of traders, $n(\omega)$, generated by entry decisions, an *equilibrium* in the OTC market is made up of measurable functions $\gamma(\omega,\tilde{\omega})$, $R(\omega,\tilde{\omega})$, and $g(\omega)$ describing, respectively, CDS contracts, CDS prices, and post-trade exposures, such that:

(i) CDS contracts are feasible: $\gamma(\omega,\tilde{\omega})$ satisfies (3) and (4);

[18] As mentioned in footnote 17, allowing traders of the same bank to pool their CDS contracts is essentially equivalent to assuming that traders can exchange CDS in a competitive "intra-bank" market. In that market, the price of a CDS contract is $\Gamma'[g(\omega)]$. Thus, the outside option of a trader in a bilateral match can be viewed as the outside option of trading later in the intra-bank market.

(ii) CDS contracts are optimal: $\gamma(\omega, \tilde{\omega})$ and $R(\omega, \tilde{\omega})$ satisfies (11) and (12) given $g(\omega)$;

(iii) post-trade exposures are consistent: $g(\omega)$ satisfies (7) given $\gamma(\omega, \tilde{\omega})$ and $n(\omega)$.

In what follows, we will say that $\gamma(\omega, \tilde{\omega})$ is *the basis of an equilibrium* if the triple $\gamma(\omega, \tilde{\omega})$, $g(\omega)$, and $R(\omega, \tilde{\omega})$ is an equilibrium conditional on entry, where $g(\omega)$ is constructed from $\gamma(\omega, \tilde{\omega})$ according to (7), and $R(\omega, \tilde{\omega})$ is constructed from $g(\omega)$ according to (12).

5.1 Constrained efficiency

In order to establish the existence and uniqueness of an equilibrium, it is useful to first analyze its efficiency properties. To that end, we consider the planning problem of choosing a collection of CDS contracts, $\gamma(\omega, \tilde{\omega})$, in order to minimize the average cost of risk bearing across banks,

$$\inf \int_0^1 \Gamma\left[g(\omega)\right] n(\omega)\, d\omega, \tag{13}$$

with respect to some bounded measurable $\gamma(\omega, \tilde{\omega})$, subject to (3), (4), and (7). Given that certainty equivalents are quasi-linear, a collection of CDS contracts solves the planning problem if and only if it is Pareto optimal, in that it cannot be Pareto improved by choosing another feasible collection of CDS contracts and making consumption transfers. We then establish:

Proposition 1. *The planning problem has at least one solution. All solutions share the same post-trade risk exposure, $g(\omega)$, almost everywhere. Moreover, a collection of CDS contracts, $\gamma(\omega, \tilde{\omega})$, solves the planning problem if and only if it is the basis of an equilibrium.*

It follows from this proposition that an equilibrium exists, conditional on the distribution of traders, $n(\omega)$, generated by entry decisions. Moreover, the equilibrium post-trade exposures, $g(\omega)$, and bilateral prices, $R(\omega, \tilde{\omega})$, are uniquely determined. The proposition shows that our restriction that CDS contracts only depend on ω is without much loss of generality. Indeed, if CDS contracts were allowed to depend on any other bank characteristics, such as size, then the same efficiency result would hold: equilibrium post-trade exposures would solve a generalized planning problem in which CDS contracts are allowed to depend on these characteristics. It is then easy to show that this generalized planning problem has the same solution as (13); the planner, in other words, would find

it strictly optimal to choose post–trade exposures that only depend on ω.[19] Therefore, in any equilibrium, post-trade exposures coincide with the unique solution of (13).

5.2 Equilibrium post-trade exposures: some general results

We now establish elementary properties of the post-trade exposure function. First, we show that:

Proposition 2. *Post-trade exposures are non-decreasing and closer together than pre-trade exposures:*

$$0 \leq g(\tilde{\omega}) - g(\omega) \leq \tilde{\omega} - \omega, \text{ for all } \omega \leq \tilde{\omega}. \tag{14}$$

The proposition shows that there is partial risk sharing. For example, in the special case of full risk-sharing, then $g(\tilde{\omega}) - g(\omega) = 0$ and the inequality is trivially satisfied. With partial risk sharing, we obtain a weaker result: $g(\tilde{\omega}) - g(\omega)$ is smaller than $\tilde{\omega} - \omega$, but in general remains larger than zero.

Proposition 3. *If $g(\omega)$ is increasing at ω, then:*

$$g(\omega) = \omega + k\left[1 - 2N(\omega)\right]. \tag{15}$$

If $g(\omega)$ is flat at ω then:

$$g(\omega) = \mathbb{E}\Big[\omega \,\Big|\, \omega \in [\underline{\omega}, \overline{\omega}]\Big] + k\left[1 - N(\overline{\omega})\right] - kN(\underline{\omega}), \tag{16}$$

where the expectation is taken with respect to $n(\omega)$, conditional on $\omega \in [\underline{\omega}, \overline{\omega}]$, and where $\underline{\omega} \equiv \inf\{\tilde{\omega} : g(\tilde{\omega}) = g(\omega)\}$ and $\overline{\omega} \equiv \sup\{\tilde{\omega} : g(\tilde{\omega}) = g(\omega)\}$ are the boundary points of the flat spot surrounding ω.

The intuition for this result is the following. If $g(\omega)$ is strictly increasing at ω, then it must be that a ω-trader sells k contracts to any trader $\tilde{\omega} > \omega$, and purchases k contracts from any traders $\tilde{\omega} < \omega$. Aggregating across all traders in bank ω, the total number of contracts sold by bank ω is $k\left[1 - N(\omega)\right]$ per capita. Likewise, the total number of contracts purchased by bank ω is $kN(\omega)$ per capita. Adding all contracts sold and subtracting all contracts purchased, we obtain (15).

[19] Precisely, suppose that CDS contracts depend on the pre-trade exposure, ω, and on some other vector of characteristics denoted by x. Then, the CDS contracts $\hat{\gamma}(\omega, \tilde{\omega}) \equiv \int \gamma(\omega, x, \tilde{\omega}, \tilde{x}) n(dx \,|\, \omega) n(d\tilde{x} \,|\, \tilde{\omega})$ are feasible and generate post-trade exposures $\hat{g}(\omega) = \int g(\omega, x) n(dx \,|\, \omega)$. Because the cost of risk-bearing is convex, the planner prefers $\hat{\gamma}(\omega, \tilde{\omega})$ over $\gamma(\omega, x, \tilde{\omega}, \tilde{x})$, and strictly so if $g(\omega, x)$ varies with x.

Now consider the possibility that $g(\omega)$ is flat at ω and define $\underline{\omega}$ and $\overline{\omega}$ as in the proposition. By construction, all banks in $[\underline{\omega}, \overline{\omega}]$ have the same post-trade exposure. Therefore, $g(\omega)$ must be equal to the average post-trade exposure across all banks in $[\underline{\omega}, \overline{\omega}]$ which is given in equation (16): the average pre-trade exposure across all banks in $[\underline{\omega}, \overline{\omega}]$, plus all the contracts sold to $\tilde{\omega} > \overline{\omega}$-traders, minus all the contracts purchased from $\tilde{\omega} < \underline{\omega}$-traders. The contracts bought and sold among traders in $[\underline{\omega}, \overline{\omega}]$ do not appear since, by (3), they must net out to zero.

To derive a sufficient condition for a flat spot, differentiate equation (15): $g'(\omega) = 1 - 2kn(\omega)$. Clearly, if this derivative turns out negative, then (15) cannot hold, i.e., $g(\omega)$ cannot be increasing at ω.

Corollary 4. *If $2kn(\omega) > 1$, then $g(\omega)$ is flat at ω.*

This corollary means that, when $n(\omega)$ is large, then the post-trade exposure function is flat at ω. Intuitively, when there is a large density of traders in the OTC market with similar endowments, these traders can match very easily. As a result, they can execute so many bilateral trades amongst themselves that they manage to pool their risks fully in spite of their limited trading capacity.

A reasoning by contradiction offers a more precise intuition. Assume that $n(\omega)$ is large in some interval $[\omega_1, \omega_2]$, but that $g(\omega)$ is strictly increasing. Then, when two traders from this interval meet, it is always the case that the low-ω trader sells k CDS to the high-ω trader. In particular, ω_1 sells insurance to all traders in $(\omega_1, \omega_2]$, and ω_2 buys insurance from all traders in $[\omega_1, \omega_2)$. If there are sufficiently many traders to be met in $[\omega_1, \omega_2]$, then this can imply that $g(\omega_1) > g(\omega_2)$, contradicting the property that $g(\omega)$ be non-decreasing.

The above results also provide a heuristic method for constructing the post-trade exposure function, $g(\omega)$, induced by some particular distribution of traders, $n(\omega)$. One starts from the guess that $g(\omega)$ is equal to $\omega + k\,[1 - 2N(\omega)]$, as in equation (15). If this function turns out to be non-decreasing, then it must be equal to $g(\omega)$. Otherwise, one needs to "iron" its decreasing spots into flat spots. The levels of the flat spots are given by (16). The boundaries of the flat spots are pinned down by the continuity condition that, at each boundary point, post-trade exposures must satisfy both (15) and (16).

5.3 Example: U-shaped and symmetric distributions

To build more intuition, we solve for the equilibrium under the assumption that $n(\omega)$ is U-shaped and symmetric around $\frac{1}{2}$. That is, we assume that $n(\omega)$ is decreasing over $[0, \frac{1}{2}]$, increasing over $[\frac{1}{2}, 1]$ and satisfies $n(\omega) = n(1-\omega)$. Aside from the fact these assumptions lead to a closed form solution, this type of distribution is of special interest because,

under natural conditions, it will hold in the entry equilibrium of Section 6. An example U-shaped and symmetric $n(\omega)$ is shown in Figure 6.

5.3.1 Post-trade exposures

We focus attention on $\omega \in [0, \frac{1}{2}]$ because the construction over $[\frac{1}{2}, 1]$ is symmetric. First, since $n(\omega)$ is decreasing over $[0, \frac{1}{2}]$ it follows that $\omega + k[1 - 2N(\omega)]$ is increasing over $[0, \frac{1}{2}]$ if and only if it is increasing for $\omega = 0$, that is, if and only if $2kn(0) \leq 1$. If that condition is satisfied, then clearly $g(\omega)$ is non-decreasing and is given by equation (15). Otherwise, we guess that $g(\omega)$ is first flat over some interval $[0, \underline{\omega}]$, and then increasing over the subsequent interval $[\underline{\omega}, \frac{1}{2}]$. The boundary $\underline{\omega}$ of the flat spot must satisfy two conditions. First, the post-trade exposure must be equal to

$$g(\underline{\omega}) = \underline{\omega} + k[1 - 2N(\underline{\omega})].$$

That is, a trader just to the right of $\underline{\omega}$ must buy k contracts from all $\tilde{\omega} < \underline{\omega}$ and sell k contracts to all $\tilde{\omega} > \underline{\omega}$. The second condition is given by Proposition 3, which states that post-trade exposures in the flat spot must be equal to

$$g(\underline{\omega}) = \mathbb{E}\Big[\omega \,\Big|\, \omega \in [0, \underline{\omega}]\Big] + k[1 - N(\underline{\omega})]$$

Taking the difference between the two, we obtain:

$$H(\underline{\omega}) = 0, \qquad \text{where } H(\omega) \equiv \int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} - kN(\omega)^2.$$

If there is some $\underline{\omega} \in (0, \frac{1}{2})$ such that $H(\underline{\omega}) = 0$, then we have found the upper boundary of the flat spot. Otherwise, the post-trade exposures must be flat over the entire interval $[0, \frac{1}{2}]$. The construction is illustrated in Figure 7, and summarized below:

Proposition 5. *Suppose that the distribution of traders, $n(\omega)$, is U-shaped and symmetric around $\omega = \frac{1}{2}$. Then, there are $\underline{\omega} \in [0, \frac{1}{2}]$ and $\overline{\omega} = 1 - \underline{\omega}$ such that, for $\omega \in [0, \underline{\omega}]$ and $\omega \in [\overline{\omega}, 1]$, $g(\omega)$ is flat, and for $\omega \in [\underline{\omega}, \overline{\omega}]$, $g(\omega)$ is increasing and equal to $g(\omega) = \omega + k[1 - 2N(\omega)]$. Moreover:*

- *if $k \leq \frac{1}{2}[n(0)]^{-1}$, then $g(\omega)$ has no flat spot.*
- *if $\frac{1}{2}[n(0)]^{-1} < k < 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, then $g(\omega)$ has flat and increasing spots.*
- *if $k > 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, then $g(\omega)$ is flat everywhere and equal to $\frac{1}{2}$.*

5.3.2 CDS contracts

The post-trade exposures of Proposition 5 are implemented with the following collection of CDS contracts. For all $\omega \in [\underline{\omega}, 1-\underline{\omega}]$, the implementation is straightforward: since $g(\omega)$ is increasing, it must be the case that a ω trader buys k contracts from all $\tilde{\omega} < \omega$, and sells k contracts to all $\tilde{\omega} > \omega$. Matters are more subtle within the flat spots: indeed, when two traders $(\omega, \tilde{\omega})$ in $[0, \underline{\omega}]^2$ or $[1-\underline{\omega}, 1]^2$ meet, all trades in $[-k, k]$ leave them indifferent. Yet, they must trade in such a way that their respective institutions wind up with identical post-trade exposures, $g(\underline{\omega})$. To find bilaterally feasible contracts delivering identical post-trade exposures, we guess that, when two traders $\omega < \tilde{\omega}$ meet, the ω-trader sells to the $\tilde{\omega}$ trader a number of contracts, which we denote by $z(\tilde{\omega})$, which only depends on $\tilde{\omega}$. When the ω trader meets a trader $\tilde{\omega} > \underline{\omega}$, he must sell k contracts since, in this case, $g(\tilde{\omega}) > g(\omega)$. This guess is illustrated in Figure 8 and means that:

$$g(\underline{\omega}) = \omega - z(\omega)N(\omega) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega})n(\tilde{\omega})\, d\tilde{\omega} + k\left[1 - N(\underline{\omega})\right]$$

The first term is the initial exposure. The second term adds up all the contracts purchased from $\tilde{\omega} < \omega$; the third term adds up all the contracts sold to $\tilde{\omega} \in (\omega, \underline{\omega}]$; and the fourth term adds up all the contracts sold to $\tilde{\omega} \in (\underline{\omega}, 1]$. Taking derivatives delivers an ordinary differential equation for $z(\omega)$, which we can solve explicitly with the terminal condition $z(\underline{\omega}) = k$.

Proposition 6. *The post-trade exposures of Proposition 5 are implemented by the following CDS contracts. For all $\omega \in \left[0, \frac{1}{2}\right]$ and $\tilde{\omega} > \omega$:*

$$\begin{aligned}
&\text{if } \tilde{\omega} \leq \tfrac{1}{2} \,:\, \gamma(\omega, \tilde{\omega}) = \min\left\{k, z(\tilde{\omega})\right\};\\
&\text{if } \tilde{\omega} > \tfrac{1}{2} \,:\, \gamma(\omega, \tilde{\omega}) = \min\left\{k, z\left(\tfrac{1}{2}\right)\right\}, \text{ where } z(\omega) \equiv \frac{\int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2},
\end{aligned}$$

All other $\gamma(\omega, \tilde{\omega})$ are then uniquely determined by symmetry, $\gamma(1-\omega, 1-\tilde{\omega}) = -\gamma(\omega, \tilde{\omega})$, and bilateral feasibility, $\gamma(\omega, \tilde{\omega}) + \gamma(\tilde{\omega}, \omega) = 0$.

While bilateral feasibility puts non-trivial restrictions on CDS contracts, there can be multiple collections of CDS contracts implementing the same equilibrium. The one proposed above has, however, two appealing features: it is consistent with the intuitive notion that banks with low exposures sell to banks with high exposures, and it is continuous at the boundary of the flat spot.

5.3.3 Notionals

We now study bank trading patterns. We show in particular that, in our OTC market, traders employed by the same bank execute both long and short contracts, and as a result a bank's gross notional can greatly exceed its net notional. In the aggregate, the OTC market can have a larger trading volume than its Walrasian counterpart. For brevity we offer only a graphical analysis but a precise analytical characterization can be found in Appendix A.5.3.

Contracts sold and bought. Following some of the measurements performed by the US Office of the Comptroller of the Currency (OCC), we define the (per-capita) number of contracts sold and bought by bank ω:

$$G^+(\omega) = \int_\omega^1 \gamma(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}, \quad \text{and} \quad G^-(\omega) = -\int_0^\omega \gamma(\omega, \tilde{\omega}) n(\tilde{\omega}) d\tilde{\omega},$$

keeping in mind that, with the particular implementation proposed in Proposition 6, $\gamma(\omega, \tilde{\omega}) > 0$ for $\omega < \tilde{\omega}$, and $\gamma(\omega, \tilde{\omega}) < 0$ for $\omega > \tilde{\omega}$. The number of contracts sold, $G^+(\omega)$, shown in the left panel, is decreasing over $[0, 1]$ and equal to zero for $\omega = 1$: this reflects the fact that low-ω banks, with low pre-trade exposure to the aggregate default risk factor, have more risk bearing capacity, and hence tend to supply more insurance to others than high-ω banks, with high pre-trade exposures. Note that $G^+(\omega)$ is positive even for banks $\omega > \frac{1}{2}$: that is, even though banks with $\omega > \frac{1}{2}$ are net buyers of insurance, $g(\omega) - \omega < 0$, their traders sell insurance when they meet traders of banks $\tilde{\omega} > \omega$. Symmetrically, the number of contracts purchased, $G^-(\omega)$, shown in the right panel, is zero for $\omega = 0$, and then is increasing over $[0, 1]$. Because banks with high pre-trade exposures to aggregate default risk have less risk bearing capacity, they demand more insurance from others.

Intermediation. As shown in Figure 11, all banks with $\omega \in (0, 1)$ provide some intermediation: they simultaneously buy and sell CDS contracts since both $G^+(\omega)$ and $G^-(\omega)$ are positive. A natural measure of intermediation volume is $\min\{G^+(\omega), G^-(\omega)\}$: this represents, for each bank ω, the per-capita volume of fully offsetting CDS contracts. One sees that this volume is smallest for extreme-ω banks, and largest for middle-ω banks. Extreme-ω banks play the role of "customer banks" and do not provide much intermediation. In order to bring their exposures closer to $\frac{1}{2}$ most of their traders sign contracts in the same direction. Middle-ω banks, on the other hand, play the role of "dealer banks", who provide more intermediation: they do not need to change their exposure much, so they can use their trading capacity to take large offsetting long and short positions to

take advantage of price dispersion.

To compare our model's implication to the data of the DTCC on the volume of trading amongst customers, between customer and dealers, and amongst dealers, we must take a stand on which banks we term dealers or customers. Assume, consistent with the trading patterns implied by the model, that dealers have "middle" endowment $\omega \in [\omega^\star, 1 - \omega^\star]$ for some $\omega^\star \leq \frac{1}{2}$, and that customers are the remaining banks. These middle-ω banks have relatively higher gross exposure, lower net exposure, and make relatively more intermediation profits. The fraction of traders in dealer banks is then $n_D = 1 - 2N(\omega^\star)$ and the fraction of traders in customer banks is $n_C = 2N(\omega^\star)$. For simplicity, assume that post-trade exposures, $g(\omega)$, have no flat spots so that all trades are up to the trading limit, k. Then, with uniform matching, the distribution of exposures coincides with the distribution of customer-to-customer, customer-to-dealer, and dealer-to-dealer matches: $n_{CC} = n_C^2$, $n_{CD} = 2n_C n_D$, and $n_{DD} = n_D^2$. If we define dealers (by choosing $\omega^\star$) so that 75% of the traders belong to dealer banks, which is approximately the fraction of trading assets in the top 4 or 5 large dealer banks, then there are 50% more dealer-to-dealer than customer-to-dealer trades. This is roughly consistent with the average composition of trades in the DTCC data. As a consequence of uniform matching, the model also implies that only $1/16 = 6.25\%$ of gross notionals is accounted for by direct customer-to-customer trades. The finding that there are substantially fewer customer-to-customer trades than dealer-to-customer trades is qualitatively consistent with data. However, quantitatively, in the data, there are even fewer direct customer-to-customer trades than is implied by our model if we choose to label dealers and customers as we have done above.[20]

Gross notional. The gross notional is $G^+(\omega) + G^-(\omega)$, the total number of CDS signed by a ω-bank. One sees that the gross notional is largest and the net notional is smallest for middle-ω dealer banks. Indeed, middle-ω banks lie in an increasing spot of $g(\omega)$, so their traders always use all of their capacity limit, either selling k or purchasing $-k$. Extreme-ω customer banks, on the other hand, lie in a flat spot of $g(\omega)$, and their traders do not use all of their capacity when they meet other traders from the same flat spot. This feature of the equilibrium will be a key driver of some of the cross sectional variation across banks' trading behavior.

[20]The model has an endogenous force reducing the amount of customer-to-customer trades, coming from the fact that customers lie in a flat spot of $g(\omega)$ and do not trade to their limit k with each other. Quantitatively, however, the most that this endogenous force can do is reduce the volume of customer to customer trades by a factor of 2.

Net notional. The net notional is the difference between contracts sold and purchased by a ω-bank, $\left|G^+(\omega) - G^-(\omega)\right| = \left|g(\omega) - \omega\right|$. Clearly, it is lowest for middle-ω dealer banks, which enter the OTC market with $\omega \simeq \frac{1}{2}$ and thus do not need to change their risk exposures much.

Trading volume. Because middle-ω banks engage in intermediation activity, the OTC market can create excess trading volume. More precisely, trading volume is non-monotonic in k, and for intermediate k it is always larger than the Walrasian volume. This is interesting because excess volume can increase as frictions decline. Thus, large notionals can be evidence of improved OTC market functioning.

To understand why volume is non-monotonic in k, we first observe that, when k is small, then mechanically trading volume is small because traders have little trading capacity. When k is greater than $1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$ but less than 1, we show in the Appendix that, for *any* equilibrium set of CDS contracts, not only that of Proposition 6, the OTC market optimally circumvents frictions by creating excess trading volume, relative to its Walrasian counterpart. Note that this is in spite of the fact that the post-trade exposures are the same as in the Walrasian equilibrium (see Proposition 5). Finally, when k is large enough, one can find CDS contracts generating a volume that is arbitrarily close to the Walrasian volume.

5.3.4 Bilateral prices

As shown in equation (12), in the equilibrium of our CDS market, prices are dispersed. Thanks to bargaining, a ω-trader is able to sell CDS above her marginal cost of providing insurance, $\Gamma'[g(\omega)]$, and she is also able to buy below her marginal value, which is also equal to $\Gamma'[g(\omega)]$. This implies in particular that middle-ω banks, who do not change their expose in the OTC market, make positive profits from intermediation.

Another implication of (12) is that the average price faced by an ω-bank is increasing in its post-trade exposure, $g(\omega)$. Thus, customers with the most risky post trade positions, as measured by $g(\omega)$, face the highest prices. This is consistent with the evidence in Arora, Gandhi, and Longstaff (2012), since post-trade risk exposure is likely to be related to variables that measure default risk. In our model, the adverse pricing offered to banks with large post trade risk exposures reflects their large gains from trade and not counterparty credit risk. Thus, what appears to be variation in counterparty risk may actually be measuring variation in gains from trade.

6 Equilibrium entry decisions

In this section we study the entry of banks into the OTC market, thus endogenizing $n(\omega)$. We first characterize entry incentives and offer a general existence result. Next, we provide conditions under which the equilibrium $n(\omega)$ turns out to be U-shaped and symmetric. Lastly, we show that, with entry, our model explains qualitative empirical relationships between bank size, net notionals, gross notionals, and intermediation activity.

6.1 Entry incentives

Banks' decisions to enter the market are driven by two motivations, the strength of which is determined by their sizes and initial risk exposures. The first motivation is to hedge underlying risk exposure. The second motivation is to capture the trading profits due to the equilibrium price dispersion across different bilateral trades. This second motivation gives banks an incentive to provide intermediation services.

More formally, given any distribution of traders in the OTC market, $n(\omega)$, we can calculate the *ex-ante* value of entering the market:

$$\Delta(\omega) \equiv \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \int_0^1 \gamma(\omega,\tilde{\omega}) R(\omega,\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}, \tag{17}$$

per trader's capita. The first term is the change in exposure: it is negative if the bank is a net seller of insurance, and positive if it is a net buyer. The second term is the sum of all CDS premia collected and paid by the bank. It is positive if the bank is a net seller, and is negative if it is a net buyer.

To obtain a decomposition of entry incentives into hedging and price dispersion motives we note that, in a Walrasian market, a bank would conduct all its trades at a common price. The hedging motive corresponds to the change in exposure and the sum of all CDS premia collected or paid by the bank, if valued at this common price. The price dispersion motive corresponds to the residual trading profits associated with deviations of actual transaction prices, $R(\omega,\tilde{\omega})$, from this common price. In what follows, we provide a decomposition of entry incentives into hedging and price dispersion motives by assuming that the common price is $\Gamma'\left[g(\omega)\right]$, the bank's marginal cost of risk bearing.[21] This leads

[21] Clearly, there are many possible decompositions. In her discussion of our paper, Briana Chang argued that choosing a different hypothetical common price could offer further insights into the nature of intermediation profits. One advantage of our decomposition is to facilitate our welfare analysis of Section 7.

to the decomposition:

$$\Delta(\omega) = K(\omega) + \frac{1}{2}F(\omega)$$

where the function $K(\omega)$ is:

$$\begin{aligned} K(\omega) &\equiv \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} \\ &= \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \Gamma'\left[g(\omega)\right]\big(g(\omega) - \omega\big), \end{aligned}$$

because $\int_0^1 \gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} = g(\omega) - \omega$. The function $F(\omega)$ is:

$$\begin{aligned} F(\omega) &\equiv 2\bigg(\int_0^1 \gamma(\omega,\tilde{\omega})R(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} - \int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\bigg) \\ &= \int_0^1 \gamma(\omega,\tilde{\omega})\big(\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\big)n(\tilde{\omega})\,d\tilde{\omega} \\ &= k\int_0^1 \big|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\big|n(\tilde{\omega})\,d\tilde{\omega}, \end{aligned}$$

where the second line follows from using the formula (12) for $R(\omega,\tilde{\omega})$, and the third line from the optimality condition (11).

The function $K(\omega) \geq 0$ represents the per-capita *competitive surplus* of bank ω. Specifically, it represents the utility of entry assuming that traders in the bank have no bargaining power, i.e., the price of every contract is equal to $\Gamma'\left[g(\omega)\right]$. Notice that it is positive if $g(\omega) \neq \omega$ and zero if $g(\omega) = \omega$. In other words, $K(\omega)$ only measures incentives to hedge exposure.

The function $F(\omega)$ is also positive and measures the *frictional surplus* of bank ω. When $\Gamma'\left[g(\tilde{\omega})\right]$ is not equalized across banks, a ω-bank uses its bargaining power to sell insurance at a price higher than its marginal cost, and to buy insurance at price lower than its marginal value. The trading profits thus generated add up to $\frac{1}{2}F(\omega)$, where the frictional surplus is multiplied by $\frac{1}{2}$ because a trader only has half of the bargaining power. As long as $g(\omega)$ is not everywhere flat, $F(\omega)$ is positive even for a bank that does not change its exposure. It thus measures incentives to enter to take advantage of price dispersion, and provide intermediation services.

When D is normally distributed and $n(\omega)$ is U-shaped and symmetric, we obtain a sharper characterization of both functions:

Proposition 7 (Entry Incentives)**.** *Suppose that the distribution of active traders, $n(\omega)$, is U-shaped and symmetric, and assume that D is normally distributed. Then both the*

competitive surplus, $K(\omega)$, and the frictional surplus, $F(\omega)$, are U-shaped and symmetric around $\frac{1}{2}$.

The proposition means that extreme-ω "customer" banks, who use the OTC market to change their exposures, have the greatest incentives to enter. This occurs for two reasons. First, low-ω banks have the biggest risk-bearing capacity, and high-ω banks have the largest need to unwind their risk, so they obtain a large surplus when they trade CDS. This is measured by the competitive surplus, $K(\omega)$. Second, extreme-ω banks reap larger trading profits from bargaining, as measured by the frictional surplus, $F(\omega)$, the average gap between their marginal cost of risk bearing and that of their counterparty.[22]

Middle-ω "dealer" banks, on the other hand, do not change their exposure much and have the smallest incentive to trade in the OTC market. The only reason they enter is to earn intermediation profits.

6.2 Entry equilibrium

Keeping in mind that a bank has to pay a fixed cost c to enter the market, it will enter if and only if:

$$\Delta(\omega) - \frac{c}{S} \geq 0 \quad \Longleftrightarrow \quad S \geq \Sigma(\omega) \text{ where } \Sigma(\omega) \equiv \frac{c}{\Delta(\omega)}. \tag{18}$$

Let $\Psi(S)$ be the measure of traders in banks with sizes greater than S. For $S \geq \underline{S}$, $\Psi(S) = \int_S^\infty x\varphi(x)\,dx$, and for $S \leq \underline{S}$, $\Psi(S) = 1$. Given that sizes and per-capita endowments are independent in the banks' cross section, the *measure* of active traders with endowment ω is $\Psi\left[\Sigma(\omega)\right]$. The *distribution* of active traders is, then:

$$T\left[n\right](\omega) = \frac{\Psi\left[\Sigma(\omega)\right]}{\int_0^1 \Psi\left[\Sigma(\tilde{\omega})\right]\,d\tilde{\omega}}, \tag{19}$$

where the denominator normalizes the measure of active traders by the total measure of active traders in the OTC market, keeping in mind that ω is uniformly distributed in the banks' cross-section.

An *entry equilibrium* is a fixed point $n(\omega)$ of equation (19), where $\Sigma(\omega)$ is defined from $\Delta(\omega)$ in equation (18), and $\Delta(\omega)$ is defined from $n(\omega)$ in equation (17).

Proposition 8 (Existence)**.** *There exists a continuous function, n, positive almost everywhere and such that $\int_0^1 n(\tilde{\omega})\,d\tilde{\omega} = 1$, solving the functional equation $T[n] = n$.*

[22]The frictional surplus, $F(\omega)$, measures the average absolute distance between bank ω's marginal cost of risk bearing and that of other banks, and so it is minimized for the median bank. With a symmetric $n(\omega)$ the median bank is at $\omega = \frac{1}{2}$.

Because our proof applies the Schauder fixed point theorem, it establishes existence but not uniqueness.[23] Note also that the solution of the fixed point problem is the basis of an equilibrium with *positive* entry: given the distribution of traders $n(\omega)$, some banks have positive gains from trade and, if large enough, will find it optimal to enter.[24]

Proposition 7 showed that a U-shaped and symmetric $n(\omega)$ maps into a U-shaped and symmetric $\Delta(\omega)$ and, therefore, into a hump-shaped and symmetric threshold, $\Sigma(\omega)$. Since the function $\Psi(x)$ is decreasing, it follows that $T[n](\omega)$ is U-shaped and symmetric. Therefore, the operator T preserves the U-shaped and symmetric property. This immediately implies that:

Corollary 9. *If D is normally distributed, there exists an equilibrium $n(\omega)$ that is U-shaped and symmetric.*

In the entry equilibrium of Corollary 9, small-sized banks do not enter the OTC market because their entry cost per capita is too large. Middle-sized banks enter, but only at the extremes of the ω spectrum, where $\Delta(\omega)$ is largest. In other words, middle-sized banks tend to be customers. Large banks enter at all points of the ω spectrum: at the extreme, as customers, and in the middle, as dealers. This pattern of participation thus implies that middle-ω dealer banks are, on average, larger than extreme-ω customer banks.

6.3 Empirical implications

In this section we study some empirical implications of our model.

6.3.1 Conditional moments

In Section 3, we provided stylized facts about gross and net CDS notional in a cross-section of banks sorted by trading assets, a natural empirical measure of bank size. To derive model counterparts of these facts, we calculate population moments conditional on bank size, for several variables of interest. Precisely, let $n(\omega \mid S)$ be the distribution of traders conditional on bank size, and let $x(\omega)$ be some cross-sectional variable of interest

[23] In Appendix A.10, we establish uniqueness of a symmetric equilibrium in a simplified model with three endowment types and quadratic cost of risk bearing.

[24] A crucial step in applying the Schauder fixed point theorem is to prove that the set of distributions generated by the operator $T[n]$ is equicontinuous. Two observations deliver this property in our model. First, from Proposition 2, the post-trade exposure function, $g(\omega)$, is Liptchitz with a coefficient that does not depend on $n(\omega)$. This Liptchitz property is then inherited by $\Delta(\omega)$ and $\Psi[\Sigma(\omega)]$. The second property is that, as one varies the perceived *distribution* of traders in the OTC market, n, the *measure* of traders who choose to enter, given by the denominator of $T[n](\omega)$, remains bounded away from zero. This follows because the closure of the set of entry incentives $\Delta(\omega)$ is compact and because entry incentives cannot be uniformly zero over the ω spectrum. See the appendix for the detailed proof.

(price, notional, etc...).[25] The expectation of $x(\omega)$ with respect to the distribution of traders conditional on S is defined as:

$$\mathbb{E}_S\left[x(\omega)\right] \equiv \int_0^1 x(\omega) n(\omega \mid S)\, d\omega.$$

Proposition 10 (Cross-sectional facts)**.** *In an entry equilibrium with a U-shaped and symmetric $n(\omega)$:*

- *conditional gross notional, $\mathbb{E}_S\big[G^+(\omega) + G^-(\omega)\big]$, is non-decreasing in S;*
- *conditional net notional, $\mathbb{E}_S\big[|G^+(\omega) - G^-(\omega)|\big]$, is non-increasing in S;*
- *conditional intermediation, $\mathbb{E}_S\big[\min\{G^+(\omega), G^-(\omega)\}\big]$, is non-decreasing in S;*
- *conditional price dispersion, $\mathbb{E}_S\big[R(\omega,\tilde{\omega})^2\big] - \mathbb{E}_S\big[R(\omega,\tilde{\omega})\big]^2$, is non-increasing in S.*

These results, illustrated in Figure 12, are the model counterparts of the stylized facts we documented using US data from the OCC and bank holding companies' financial reports. Just as we normalize by trading assets in the data to control for the mechanical effect of size on various measures of notional, the proposition focuses on "per capita" quantities. To build intuition, recall from Section 5.3.3 that middle-ω banks have the largest gross notional, the smallest net notional, and the largest intermediation volume. These middle-ω banks are predominantly large, because only large banks can recoup their entry cost by conducting a large enough volume of intermediation, in the entry equilibrium. As a result, gross notional increases with size, net notional decreases with size, and intermediation volume increases with size.

Because middle-ω banks are large, have very little net relative to gross notionals, and enter and trade mainly to generate intermediation profits from price dispersion, we consider them to be "dealers." The entry incentives, and resulting trading patterns, for these banks are consistent with the empirical trading patterns seen for dealers in the market for CDS. In practice, banks that are able to set up and maintain successful dealerships are those with broad customer bases that include both counterparties who would like to use CDS to insure credit risk (medium-sized banks with sizable loan portfolios), and counterparties who would like to add credit risk to their portfolios (such as insurance companies and hedge funds). Thus, we do not interpret the model in a strict sense to only include banks, but also other financial institutions which are active in credit markets. Our data

[25] Note that, since we are conditioning on a population of banks who have identical size, this distribution of traders must coincide with the distribution of banks conditional on size.

on CDS holdings at the firm level, however, is constrained to regulated entities that must publicly report their CDS activity.

The last result of the proposition is that price dispersion decreases with size. Because middle-sized banks tend to have extreme ω's, a large number of middle-sized matches involve either two low-ω banks, who trade at low prices, or two high-ω banks, who trade at high prices. The prevalence of either low-price or high-price matches results in a large degree of price dispersion. On the other hand, there are more large-sized banks with middle ω, and therefore more matches at average prices. This effect implies that price dispersion decreases with size.

It is intuitive that large middle-ω banks, which act as dealer banks, should trade with each other at common prices since they all have similar outside options. These dealer banks then trade with typically smaller banks with more extreme ω's, that is, end users or customers, at heterogeneous prices that depend on the customers' post trade positions. In the earlier work of Duffie, Gârleanu, and Pedersen (2005) and Lagos and Rocheteau (2009), it is assumed that market makers or dealers trade in a frictionless market at common prices. In our model, such an inter-dealer market arises endogenously among large banks that are central to the market. One advantage of not assuming a frictionless inter-dealer market is that, empirically, Arora, Gandhi, and Longstaff (2012) show that there is substantial heterogeneity across dealers in CDS spreads offered to the same customer. This seems consistent with the fact that, as in our model, dealer characteristics are also key determinants of CDS spreads.

6.3.2 Concentration

Taking stock of the above results, our model shows that the concentration of gross exposures in large banks is the result of several forces working in the same direction. First, on the extensive margin, smaller banks participate less in the OTC market than large banks: small-sized banks do not participate, middle-sized banks participate only if their per capita endowment, ω, is extreme enough. Second, on the intensive margin, middle-sized banks sign fewer CDS contracts than larger banks. Some of this intensive margin effect is purely mechanical: middle-sized banks have fewer traders, so they sign proportionally fewer CDS contracts. But some of the effect arises because middle-sized banks tend to have extreme ω's, to lie in a flat spot of $g(\omega)$, and to thus find it optimal to use less than their full trading capacity, k. Large-sized banks tend to have average ω, to lie in an increasing spot of $g(\omega)$, and to thus use all their trading capacity. This is another channel driving notional concentration.

Figure 13 shows a heat map of bilateral gross notionals, per capita, across size per-

centiles. Variation along the diagonal of the heat map illustrates that gross notionals are larger amongst larger banks, as we already knew from Proposition 10. The off-diagonal vectors of the heat map offer new information: smaller banks trade more with larger banks than amongst each other. This further illustrates the manner in which, in spite of our assumption that all matching is uniform, large banks endogenously emerge as central counterparties in the OTC market.

6.3.3 Market response to declining frictions

Over the last decade, the volume of trade in OTC markets has grown dramatically, and OTC trading profits have become an ever larger fraction of dealer banks' earnings. In this section we investigate patterns of growth in OTC markets by contrasting two possible scenarios: a decline in the entry cost, c, representing more sophisticated trading technologies, or a relaxation of trading limits, k, representing improvements in operational risk management practices.

Because such comparative statics are difficult to characterize in full generality, we consider a simplified model based on the following assumptions: there are only three possible per-capita endowments, $\omega \in \{0, \frac{1}{2}, 1\}$, the size distribution is Pareto with support $[\underline{S}, \infty)$ for some $\underline{S} > 0$, and the cost of risk bearing is quadratic. Our results, developed formally in Appendix A.10, indicate that in both cases, reducing frictions causes the market to grow, in the sense of increasing gross notional outstanding per capita. But predictions differ markedly in other dimensions. For example, when trading technologies improve, as proxied by a decline in c, the market grows through an increase in intermediation activity, and the net-to-gross notional ratio decreases. In contrast, when risk-management technologies improve, as proxied by an increase in k, the market can grow through an increase in customer-to-customer trades. As a result, the net-to-gross notional ratio can increase. Thus, according to the model, the evolution of the net-to-gross notional ratio can help distinguish a decrease in frictions due to an improvement in trading versus risk-management technologies. These comparative statics can also help to guide policies aimed at reducing the importance of central dealers. Our analysis indicates that improvements to risk management technologies, rather than reductions in entry costs, are more likely to decrease intermediated trade and excess volume.

Decline in entry cost. The comparative static depends on the position of c relative to two thresholds, $\bar{c} > \underline{c}$. When c declines while remaining larger than the upper threshold, $c > \bar{c}$, because of the Pareto size distribution, the market grows proportionally across the ω spectrum: the measure of traders of all types is scaled up by the same constant, so

market composition stays the same. When the entry cost reaches the threshold $c = \bar{c}$, all extreme-ω banks but only part of middle-ω banks have entered the OTC market. That is, when $c = \bar{c}$, the entry threshold satisfies $\Sigma(0) = \Sigma(1) = \underline{S}$, but $\Sigma(\frac{1}{2}) > \underline{S}$. This is because extreme-ω banks always have more incentives to enter than middle-ω banks. Thus, a further decrease in entry cost cannot create more entry of extreme-ω banks, since all have entered, and only causes more entry of middle-ω banks. Thus, when c declines in the intermediate range $c \in [\underline{c}, \bar{c}]$, the OTC market grows through an increase in intermediation. As a result, the ratio of net-to-gross notional decreases. This process continues until c reaches the lower threshold $\underline{c}$, at which point $\Sigma(\frac{1}{2}) = \underline{c}$ and all banks have entered in the OTC market.

Relaxation of trading limits. While a decline in entry costs tends to (weakly) decrease the net-to-gross notional ratio, a relaxation of trading limits has ambiguous effects. When $k \to 1$, the post-trade exposures converge to $g(\omega) = \frac{1}{2}$, and prices are less and less dispersed. As a result, the fraction of middle-ω converges to zero as they have fewer incentives to enter. Near this limiting equilibrium, the volume of customer-to-customer trades increases at the expense of intermediated trades, and the ratio of net-to-gross notional in the OTC market increases. As we discuss in Proposition 19 in the appendix, this pattern of convergence need not be monotonic. This is because an increase in k allows all banks, in particular middle-ω ones, to increase their trading volume and profits, giving them more incentives to enter. In particular, we show that for intermediate values of k, an increase in k can increase the fraction of middle-ω banks in the market, and hence reduce the ratio of net-to-gross notional.

7 Welfare

This section shows that, at the margin of the entry equilibrium, a planner can increase welfare by reducing the entry of middle-ω banks and increasing the entry of extreme-ω banks.

7.1 The welfare impact of marginal changes in entry

Consider any pattern of entry characterized by some continuous threshold $\Sigma(\omega) > \underline{S}$ such that ω-banks enter if and only if $S \geq \Sigma(\omega)$. Let M denote the corresponding measure of traders in the OTC market, and let $n(\omega) > 0$ denote the corresponding distribution of traders. We study the impact of perturbing the measure of traders entering at ω by

$\varepsilon\delta(\omega)$, for some small $\varepsilon > 0$ and some continuous function $\delta(\omega)$. If $\delta(\omega) < 0$, then the measure of traders at ω decreases, and if $\delta(\omega) > 0$, it increases. The changes $\varepsilon\delta(\omega)$ are engineered by changing the entry size threshold from $\Sigma(\omega)$ to $\Sigma_\varepsilon(\omega)$ such that:

$$\Psi\left[\Sigma_\varepsilon(\omega)\right] = Mn(\omega) + \varepsilon\delta(\omega).$$

That is, the total measure of traders in banks of size greater than $\Sigma_\varepsilon(\omega)$ is equal to $Mn(\omega) + \varepsilon\delta(\omega)$. Thus, our perturbation of the distribution $n(\omega)$ is equivalent to a perturbation of the size threshold.

Without loss of generality, we assume that CDS contracts are efficient conditional on entry, i.e., they solve the planning problem of Proposition 1. If the planner can transfer goods (but not assets) across banks at time zero, then $\delta(\omega)$ increases welfare if and only if it increases:

$$\begin{aligned} W(\varepsilon,\delta) \equiv &- \int_0^1 \big(1 - Mn(\omega) - \varepsilon\delta(\omega)\big)\Gamma\left[\omega\right]\, d\omega - \int_0^1 \big(Mn(\omega) + \varepsilon\delta(\omega)\big)\Gamma\left[g_\varepsilon(\omega)\right]\, d\omega \\ &- c\int_0^1 \Phi\left[\Sigma_\varepsilon(\omega)\right]\, d\omega, \end{aligned}$$

where $g_\varepsilon(\omega)$ is the post-trade exposure solving the planning problem conditional on the distribution of traders induced by $Mn(\omega) + \varepsilon\delta(\omega)$. The first term is the cost of risk bearing for traders who stay out of the OTC market, and the second term is the cost of risk bearing for traders who enter. The last term, on the second line, is the sum of all fixed entry costs incurred by bank establishments, with $\Phi\left[S\right] \equiv \int_S^\infty \varphi(x)\, dx$ denoting the measure of banks with sizes greater than S.

To study marginal changes in entry we evaluate the directional derivative $W'(0,\delta)$. Because post-trade exposures conditional on entry are efficient, an envelope theorem of Milgrom and Segal (2002) implies that the marginal impact of entry is found by differentiating $W(\varepsilon,\delta)$ holding the collection of CDS contracts, $\gamma(\omega,\tilde{\omega})$, constant.

Proposition 11 (Directional Derivative)**.** *The derivative of $W(\varepsilon,\delta)$ at $\varepsilon = 0$ is:*

$$W'(0,\delta) = \int_0^1 \delta(\omega)\left(-\frac{c}{\Sigma(\omega)} + \Delta(\omega) + \frac{1}{2}F(\omega) - \frac{1}{2}\int_0^1 F(\tilde{\omega})n(\tilde{\omega})\, d\tilde{\omega}\right) d\omega, \tag{20}$$

where $K(\omega)$ and $F(\omega)$ are the competitive and frictional surpluses defined in Section 6.1.

The intuition for each term is the following. First $\delta(\omega)$ creates additional entry costs: at ω, the marginal bank is of size $\Sigma(\omega)$, so each new ω-trader in this bank must pay a per-capita entry cost $\frac{c}{\Sigma(\omega)}$. The second term, $\Delta(\omega)$, is simply the private value of entry

for this marginal bank at ω. The third and fourth terms capture the gap between the private and social value of entry.

The first part of the gap is $\frac{1}{2}F(\omega)$. Indeed, because of bargaining, the private value of entry incorporates only half of the frictional surplus created by the entrant. Hence $\frac{1}{2}F(\omega)$ represents the portion of the frictional surplus that accrues to incumbents when a marginal ω-bank enters.

The second part of the gap is negative and reflects congestion externalities. Precisely, since the number of matches has to be equal to half the number of traders, when $\delta(\omega)$ traders enter, they create $\delta(\omega)$ new matches with incumbents, and displace $\delta(\omega)/2$ existing matches amongst incumbents.[26] The term $-\frac{1}{2}\int_0^1 F(\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}$ represents the surplus lost from these displaced matches. It is intuitive that this lost surplus is proportional to the average frictional surplus because matches amongst incumbents are, by virtue of uniform matching, effectively displaced at random. Crucially for our welfare result, this loss of surplus is constant with respect to the entrant's type, ω.

7.2 Improving upon the entry equilibrium

Substituting the free entry condition $\Delta(\omega) = \frac{c}{\Sigma(\omega)}$ into (20), we obtain

$$W'(0,\delta) = \frac{1}{2}\int_0^1 \delta(\omega)\left(F(\omega) - \int_0^1 F(\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\right)d\omega.$$

A necessary condition for social optimality is that this integral is equal to zero for all $\delta(\omega)$, and this can only be the case if $\frac{1}{2}F(\omega) = \frac{1}{2}\int_0^1 F(\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}$ for almost all ω. That is, the equilibrium is optimal only if entry compensates incumbents for the congestion externality: the portion of the frictional surplus that accrues to incumbents when the marginal ω-bank enters must be equal to the frictional surplus lost by incumbents from the matches that the marginal bank displaces.

This optimality condition clearly does not hold. As we have discussed above, the frictional surplus $\frac{1}{2}F(\omega)$ that accrues to incumbents is U-shaped in ω. The frictional surplus lost because of displaced matches is constant in ω, and equal to the average frictional surplus accruing to incumbents. Hence, the gap between social and private value is positive for extreme-ω banks and negative for for middle-ω banks, and equal to

[26] Precisely, suppose that there are initially M incumbent traders in the OTC market, and that a small measure δ of new traders enter. Since all traders are matched and since there are two traders per match, the total number of matches is now equal to $(M+\delta)/2$. Hence, the *net* amount of match created is $\delta/2$. At the same time, the number of new matches between entrants and incumbents is, to a first order, equal to $\delta > \delta/2$. This implies that entry displaces $\delta/2$ matches amongst incumbents.

zero on average.

The first implication of this reasoning is that, to a first order, there are no welfare gains from changing the size of the market while keeping its composition, $n(\omega)$, the same. Formally $W'(0,\delta) = 0$ when $\delta(\omega)$ is proportional to $n(\omega)$. In Appendix A.12, we show that this is, in fact, a global result: an entry equilibrium has a socially optimal size given $n(\omega)$. The intuition is that, when the composition of the market does not change, entry is equivalent to creating two separate markets with identical $n(\omega)$, one for the incumbents and one for the entrants. Therefore, proportional entry has no externality on incumbents.

The second implication of this reasoning is that, at the margin, a social planner would like to induce extreme-ω banks (customers) to enter, and middle-ω banks (dealers) to exit. Formally:

Proposition 12. *Assume that D is normally distributed, consider an entry equilibrium in which $n(\omega)$ is U-shaped and symmetric, assume that $\Sigma(\omega) > \underline{S}$ for all ω, and that $F(\omega) \neq 0$ for some positive measure of ω. Then, if $\int_0^1 \delta(\tilde{\omega})\, d\tilde{\omega} = 0$, then $W'(0,\delta) > 0$ when $\delta(\omega)$ is U-shaped and symmetric.*

This result does not mean that middle-ω banks should not enter in the OTC market at all. In our model, one can show that any bank, and in particular a middle-ω bank, always has a positive social value before accounting for the entry cost. This means that, as long as there is partial risk sharing, it is socially optimal to have some large enough $\omega = \frac{1}{2}$ banks entering in the OTC market. In the entry equilibrium, however, the marginal $\omega = \frac{1}{2}$ banks enter too much.

8 Conclusion

The OTC market for CDS's is very large relative to banks' trading assets, and gross notionals are highly concentrated on the balance sheets of just a few large dealer banks. Moreover, the varied bilateral trades executed by banks' many traders create an intricate system of liability linkages. In this paper we have sought to uncover the economic forces which determine this empirical trading structure in the OTC credit derivatives market. To this end, we have developed a model in which banks trade credit default swaps in an OTC market in order to share aggregate credit default risk. The equilibrium distribution of trades in our model has many realistic features. Gross notionals greatly exceed net notionals, and the market is highly concentrated in large banks. Small-sized banks do not participate, middle-sized banks arise as customers that trade at dispersed prices, whereas a small measure of large banks arise as key dealers and trade at near common prices.

Finally, all banks are connected through the bilateral trades of participating banks' many traders. We argue that capturing these positive features gives credence to our model as a laboratory in which to study the normative features of OTC derivatives markets, as well as the policy questions surrounding them.

We study the key normative features of our theoretical market setup, namely the size and composition of the market. We find that, given its composition, the size of the market is optimal: a planner could not improve welfare by adding or subtracting banks while leaving the composition of participating traders' pre-trade risk endowment the same. We show however, that market composition is suboptimal. In particular, the OTC market is too concentrated, in the sense that a planner would want to remove some large dealer banks and replace them with smaller customer banks.

Finally, we argue that it is important to understand the origin of observed trading patterns in order to answer the current regulatory questions surrounding OTC derivatives markets. For example, the proposed "Volcker rule" aims to restrict derivatives trading which is not directly tied to underlying exposures. This may reduce risk taking by banks. However, one potential side effect is a decline in market intermediation and liquidity. Another proposed provision in the Dodd Frank Act aims to restrict exposure to any one counterparty. Are such limits welfare improving? If so, how should each bilateral limit be determined? Current bankruptcy law and capital adequacy regulations seem to favor banks with large offsetting long and short positions. Moreover, policies such as too-big-to-fail and FDIC insurance may provide forces for concentrating derivatives trading in explicitly or implicitly insured institutions. How much of the observed concentration of gross CDS notionals is due to traditional considerations of economies of scale that we study, and how much is driven by regulation favoring large dealer banks? We do not know. In future work, we plan to address questions such as these using the framework we have developed in this paper.

Figure 1: CDS Market Concentration

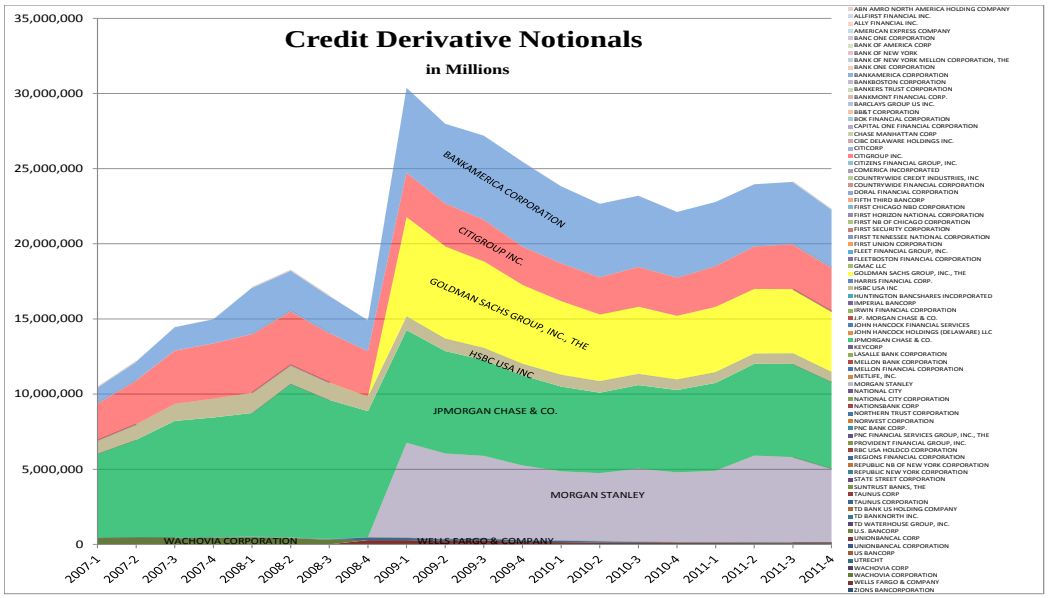

Figure 1 plots gross notionals from 2007-2011 for banks that are among the top 25 bank holding companies in OTC derivatives during that time period. Data are from the OCC quarterly report on bank trading and derivatives activities. Derivatives notionals in millions.

Figure 2: Increasing Returns to Scale in CDS Markets

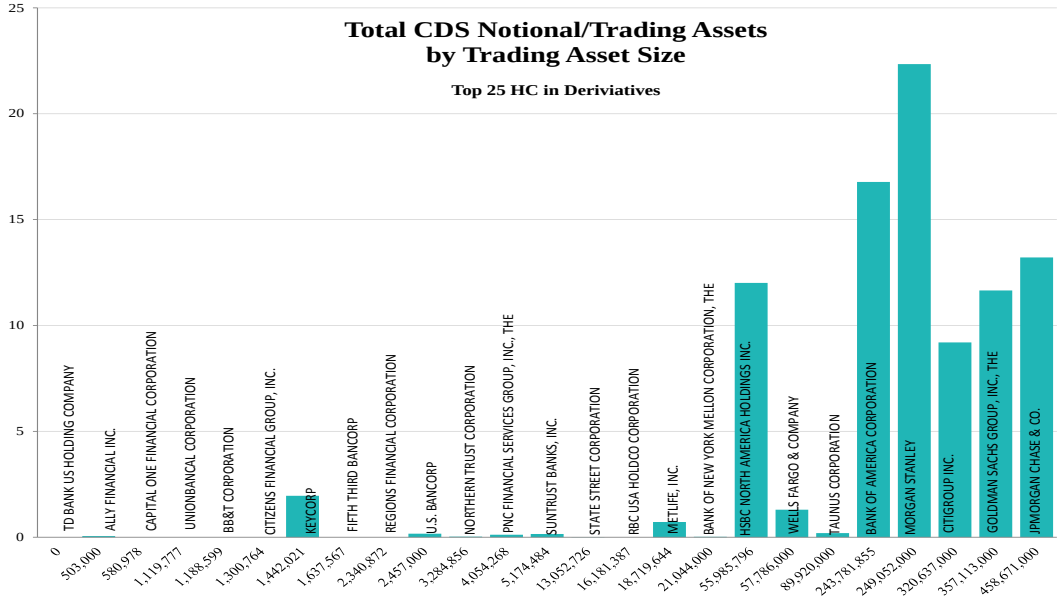

Figure 2 plots gross notional to trading assets by size of trading assets for the top 25 bank holding companies in derivatives according to the OCC quarterly report on bank trading and derivatives activities third quarter 2011. Data are from bank holding companies' FR Y-9C filing from Q3 2011. Trading assets in thousands.

Figure 3: CDS Net to Gross Notional

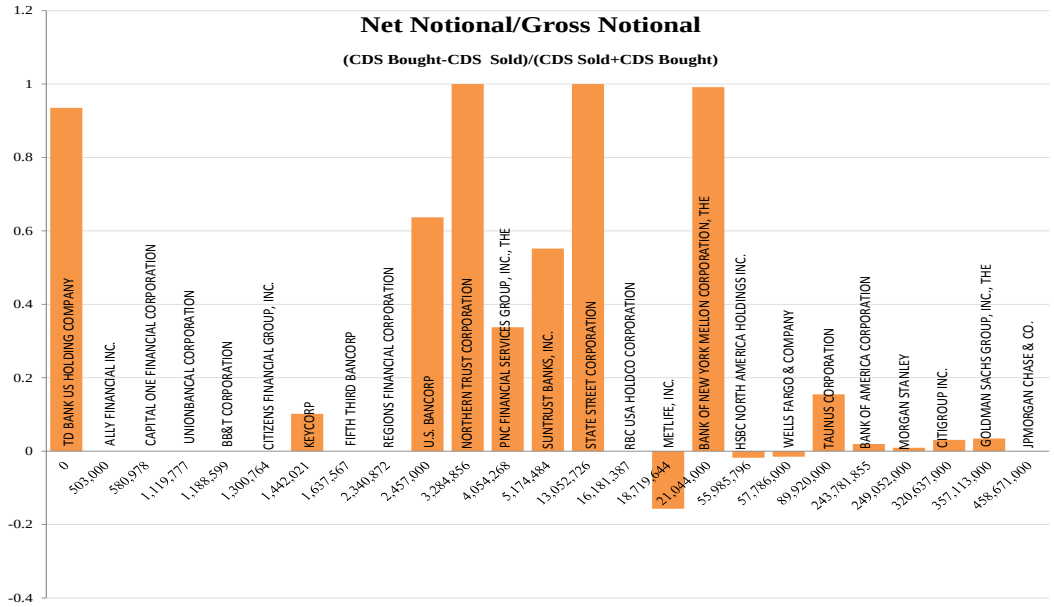

Figure 3 plots net to gross notional for the top 25 bank holding companies in derivatives according to the OCC quarterly report on bank trading and derivatives activities third quarter 2011. Data are from bank holding companies' FR Y-9C filing from Q3 2011. Trading assets in thousands. Empty bars denote zero gross CDS notional.

Figure 4: Fraction of purchased credit derivatives recorded as guarantee

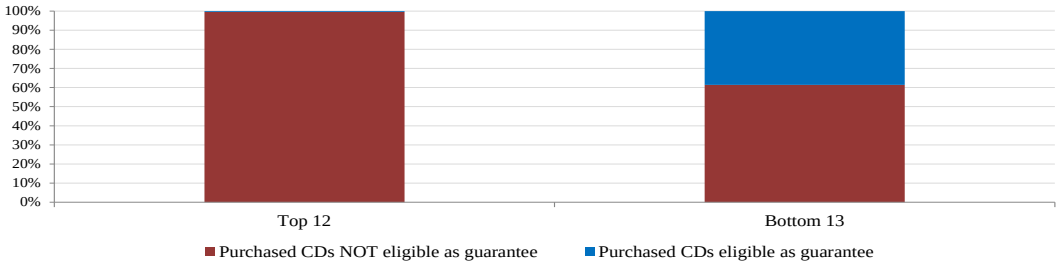

Figure 4 plots the fraction of purchased credit derivatives from Q2 2009 to Q4 2011 that could be counted as a guarantee for regulatory purposes for the larger vs. the smaller top 25 bank holding companies in derivatives. Data are from bank holding companies' FR Y-9C filings. Size is measured by trading assets.

Figure 5: Trade Composition by Counterparty Type

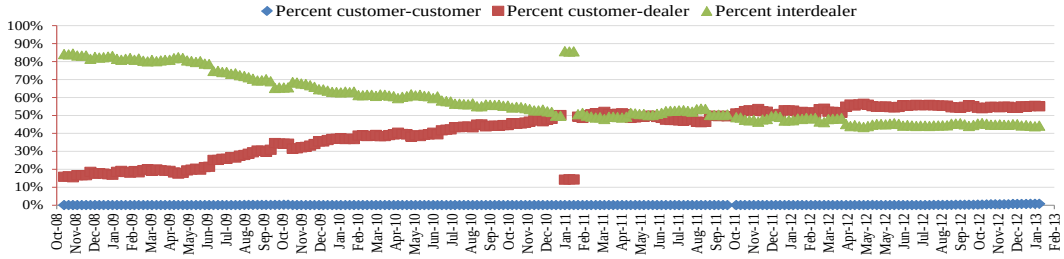

Figure 5 plots trade composition by counterparty type. Data are from the DTCC at http://www.dtcc.com/products/derivserv/data/index.php.

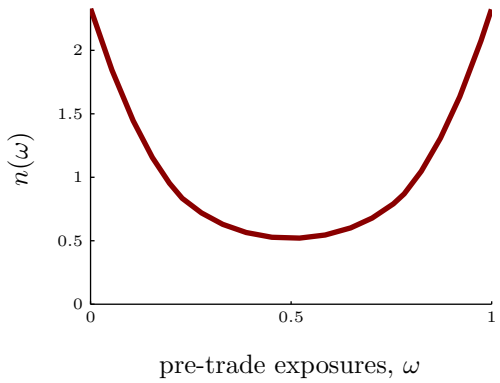

Figure 6: A U-shaped and symmetric distribution.

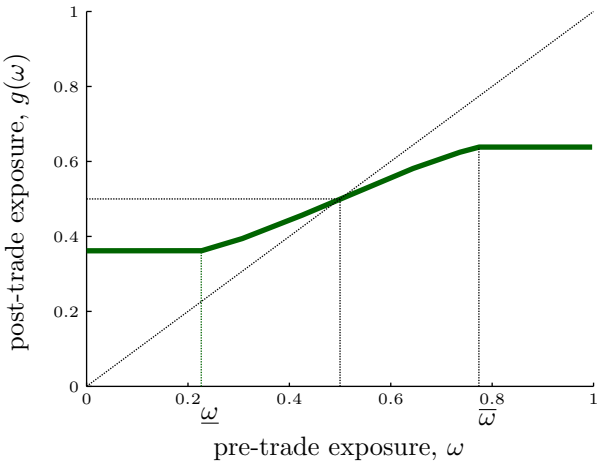

Figure 7: The post-trade exposure function when the traders' distribution is U-shaped and symmetric.

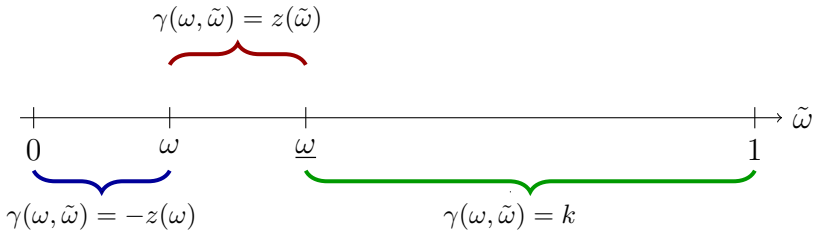

Figure 8: Portfolio of CDS contracts of a bank with pre-trade exposure $\omega \in [0, \frac{1}{2}]$.

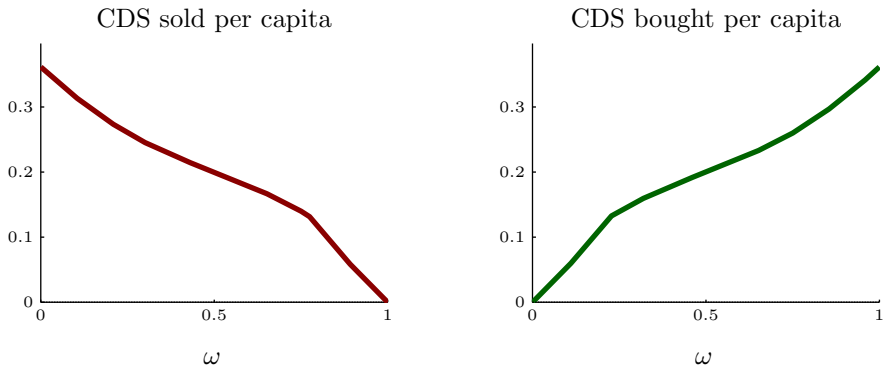

Figure 9: The number of contracts sold, $G^+(\omega)$, and purchased, $G^-(\omega)$.

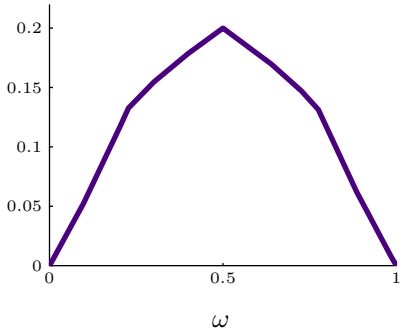

Figure 10: The volume of fully offsetting CDS contracts, $\min\{G^+(\omega), G^-(\omega)\}$.

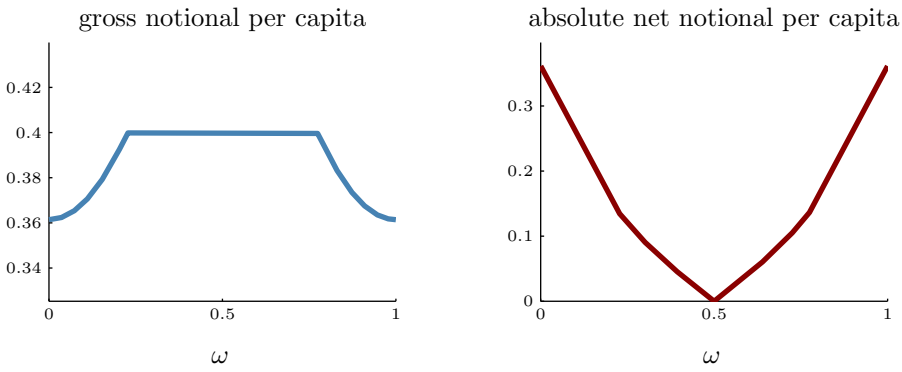

Figure 11: Gross and net notional.

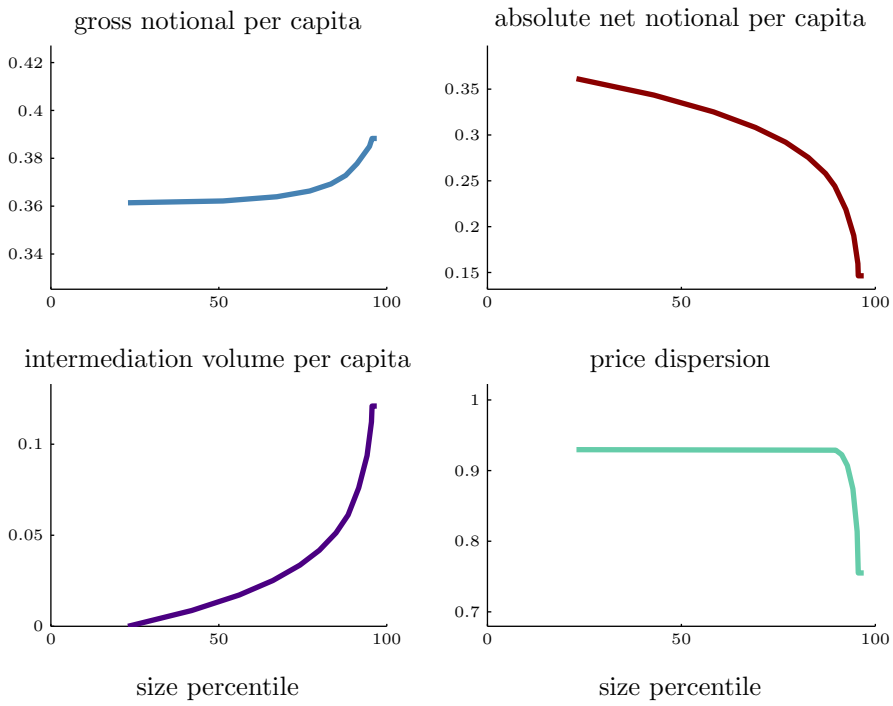

Figure 12: Conditional moments. On all panels, the x-axis is the size percentile of a bank in the overall cross-section. The dotted vertical line indicates the entry threshold, $\Sigma(0)$.

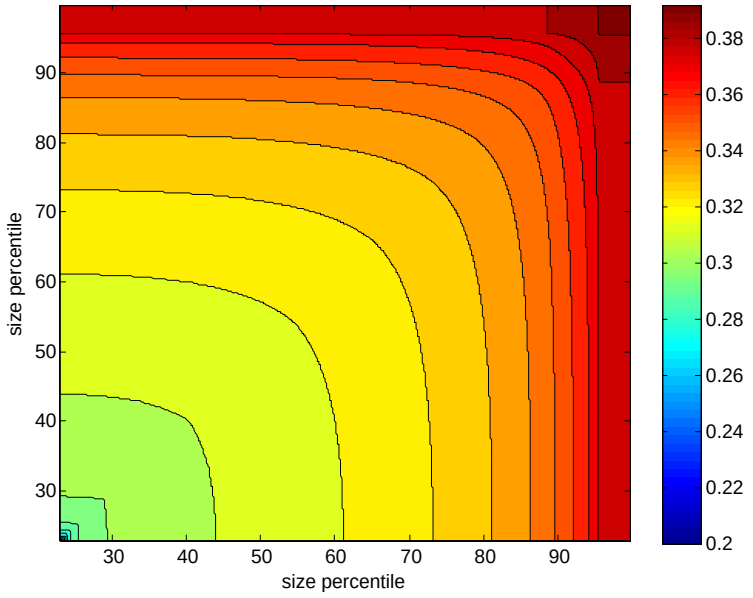

Figure 13: Bilateral gross notionals between size percentiles, per-capita.

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A Proofs

A.1 Proof of Lemma 1

The first derivative of $\Gamma(g)$ is:

$$\Gamma'[g] = \frac{\mathbb{E}\left[De^{\alpha g D}\right]}{\mathbb{E}\left[e^{\alpha g D}\right]},$$

which is positive since D is positive. The second derivative is:

$$\Gamma''[g] = \alpha \frac{\mathbb{E}\left[D^2 e^{\alpha g D}\right]\mathbb{E}\left[e^{\alpha g D}\right] - \mathbb{E}\left[De^{\alpha g D}\right]^2}{\mathbb{E}\left[e^{\alpha g D}\right]^2}.$$

The numerator is positive since, by the Cauchy-Schwarz inequality, we have

$$\mathbb{E}\left[De^{\alpha g D}\right]^2 = \mathbb{E}\left[De^{\alpha g/2 D}e^{\alpha g/2}\right]^2 < \mathbb{E}\left[D^2 e^{\alpha g D}\right]\mathbb{E}\left[e^{\alpha g D}\right].$$

The denominator is positive as well. We thus obtain that $\Gamma''(g) > 0$ as claimed. □

A.2 Proof of Proposition 1

The planner's objective is convex, and its constraint set is convex and bounded. Given that $\gamma(\omega, \tilde{\omega})$ is measurable and bounded by k, $\gamma(\omega, \tilde{\omega})$ belongs to the Hilbert space of square integrable functions. Existence then follows from an application of Proposition 1.2, Chapter II in Eckland and Téman (1987). That all solutions of the planning problem share the same post-trade exposure almost everywhere follows from the strict convexity of the function $\Gamma[g]$.

To show that any solution of the planning problem is the basis of an equilibrium, consider the following variational experiment. Starting from a solution, $\gamma(\omega, \tilde{\omega})$ and $g(\omega)$, consider the alternative feasible allocation $\gamma(\omega, \tilde{\omega}) + \varepsilon\Delta(\omega, \tilde{\omega})$ for some small $\varepsilon > 0$ and some bounded $\Delta(\omega, \tilde{\omega})$ such that

$$\Delta(\omega, \tilde{\omega}) + \Delta(\tilde{\omega}, \omega) = 0 \tag{21}$$
$$\gamma(\omega, \tilde{\omega}) = k \Rightarrow \Delta(\omega, \tilde{\omega}) \leq 0 \tag{22}$$
$$\gamma(\omega, \tilde{\omega}) = -k \Rightarrow \Delta(\omega, \tilde{\omega}) \geq 0. \tag{23}$$

The corresponding post-trade exposure is $g(\omega) + \varepsilon \int_0^1 \Delta(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$. The change in the objective is, up to second-order terms:

$$\begin{aligned}
\delta J =& \varepsilon \int_0^1 n(\omega)\Gamma'[g(\omega)] \int_0^1 \Delta(\omega, \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}\, d\omega \\
=& \frac{\varepsilon}{2} \int_0^1 \int_0^1 \Gamma'[g(\omega)]\, \Delta(\omega, \tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega}\, d\omega + \frac{\varepsilon}{2} \int_0^1 \int_0^1 \Gamma'[g(\omega)]\, \Delta(\omega, \tilde{\omega}) n(\tilde{\omega}) n(\omega) d\tilde{\omega}\, d\omega \\
=& \frac{\varepsilon}{2} \int_0^1 \int_0^1 \Gamma'[g(\omega)]\, \Delta(\omega, \tilde{\omega}) n(\tilde{\omega}) n(\omega)\, d\tilde{\omega}\, d\omega + \frac{\varepsilon}{2} \int_0^1 \int_0^1 \Gamma'[g(\tilde{\omega})]\, \Delta(\tilde{\omega}, \omega) n(\omega) n(\tilde{\omega}) d\omega\, d\tilde{\omega}
\end{aligned}$$

$$
\begin{aligned}
=&\frac{\varepsilon}{2}\int_0^1\int_0^1 \Gamma'\left[g(\omega)\right]\Delta(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}\,d\omega - \frac{\varepsilon}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\Delta(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})d\omega\,d\tilde{\omega}\\
=&\frac{\varepsilon}{2}\int_0^1\int_0^1 \Big(\Gamma'\left[g(\omega)\right]-\Gamma'\left[g(\tilde{\omega})\right]\Big)\Delta(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})\,d\tilde{\omega}\,d\omega.
\end{aligned}
$$

where: the first equality follows trivially; the second equality from relabeling ω by $\tilde{\omega}$ and vice versa; the third equality from the fact that $\Delta(\omega,\tilde{\omega})+\Delta(\tilde{\omega},\omega)=0$; the fourth equality by collecting terms. Since we started from a solution to the planning problem, it must be that $\delta J \geq 0$. For this inequality to hold for any $\Delta(\omega,\tilde{\omega})$ satisfying (21)-(23), it must be the case that $\gamma(\omega,\tilde{\omega})$ and $g(\omega)$ satisfy (11) almost everywhere. Clearly, $\gamma(\omega,\tilde{\omega})$ is basis of an equilibrium, after perhaps modifying it on a measure zero set so that it satisfies (11) everywhere. For brevity we omit the precise construction of such a modification. The details are available from the authors upon request.

Conversely, consider any equilibrium $\gamma(\omega,\tilde{\omega})$ and $g(\omega)$. Given that $\Gamma[g]$ is convex, the difference between the planner's cost for the equilibrium allocation and the planner's cost for any other allocation $\hat{\gamma}(\omega,\tilde{\omega})$ and $\hat{g}(\omega)$ is smaller than:

$$
J-\hat{J}\leq\int_0^1 n(\omega)\Gamma'\left[g(\omega)\right]\int_0^1\Big(\gamma(\omega,\tilde{\omega})-\hat{\gamma}(\omega,\tilde{\omega})\Big)n(\tilde{\omega})d\tilde{\omega}d\omega.
$$

Keeping in mind that $\gamma(\omega,\tilde{\omega})+\gamma(\tilde{\omega},\omega)=0$, we obtain, using the same manipulations as before:

$$
\int_0^1\int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)d\tilde{\omega}\,d\omega=\frac{1}{2}\int_0^1\int_0^1\Big(\Gamma'\left[g(\omega)\right]-\Gamma'\left[g(\tilde{\omega})\right]\Big)\gamma(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})d\omega\,d\tilde{\omega}.
$$

Plugging this back into the expression for $J-\hat{J}$, we obtain:

$$
J-\hat{J}\leq\int_0^1\int_\omega^1\Big(\Gamma'\left[g(\omega)\right]-\Gamma'\left[g(\tilde{\omega})\right]\Big)\Big(\gamma(\omega,\tilde{\omega})-\hat{\gamma}(\omega,\tilde{\omega})\Big)n(\tilde{\omega})n(\omega)d\tilde{\omega}d\omega.
$$

Because of (11) the integrand is negative, implying that $J\leq\hat{J}$ and establishing the claim that $\gamma(\omega,\tilde{\omega})$ solves the planner's problem. □

A.3 Proof of Proposition 2

To prove that $g(\omega)$ is non-decreasing, suppose, constructing a contradiction, that there are $\omega\leq\tilde{\omega}$ such that $g(\tilde{\omega})<g(\omega)$. Then, by (11), it must be the case that for any counterparty x, $\gamma(\omega,x)\leq\gamma(\tilde{\omega},x)$, i.e., bank $\tilde{\omega}$ sells more insurance than bank ω. But since bank $\tilde{\omega}$ starts with weakly larger exposure, $\omega\leq\tilde{\omega}$, the definition of $g(\omega)$ in equation (7) implies that $g(\tilde{\omega})\geq g(\omega)$, which would be a contradiction.

To prove that post-trade exposures are closer together than pre-trade exposures, consider again two banks $\omega\leq\tilde{\omega}$ and bear in mind that we have just shown that $g(\omega)\leq g(\tilde{\omega})$. If $g(\omega)=g(\tilde{\omega})$, then the result is trivially true. Otherwise if $g(\omega)<g(\tilde{\omega})$, then it must be the case that, for any counterparty x, bank $\tilde{\omega}$ sells less insurance than bank ω, i.e. $\gamma(\tilde{\omega},x)\leq\gamma(\omega,x)$. Therefore $\int_0^1\gamma(\tilde{\omega},x)n(x)\,dx\leq\int_0^1\gamma(\omega,x)n(x)\,dx$ which is, by definition of $g(\omega)$, equivalent to $g(\tilde{\omega})-\tilde{\omega}\leq g(\omega)-\omega$. □

A.4 Proof of Proposition 3

All ω-traders in $[\underline{\omega}, \overline{\omega}]$ sell k CDS contracts to any trader $\tilde{\omega} > \overline{\omega}$ and buy k CDS contracts from any trader $\tilde{\omega} < \underline{\omega}$. With traders $\tilde{\omega} \in [\underline{\omega}, \overline{\omega}]$, the number of CDS contracts bought and sold is indeterminate. For any $\omega \in [\underline{\omega}, \overline{\omega}]$ we thus have:

$$g(\underline{\omega}) = g(\overline{\omega}) = g(\omega) = \omega - kN(\underline{\omega}) + \int_{\underline{\omega}}^{\overline{\omega}} \gamma(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + k\left[1 - N(\overline{\omega})\right].$$

Now multiply by $n(\omega)$ and integrate from $\underline{\omega}$ to $\overline{\omega}$ and note that, by (3) it must be the case that:

$$\int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{\overline{\omega}} \gamma(\omega, \tilde{\omega}) n(\omega) n(\tilde{\omega}) d\omega d\tilde{\omega} = 0.$$

In other words, the net trade across all matches $(\omega, \tilde{\omega}) \in [\underline{\omega}, \overline{\omega}]^2$ must be equal to zero. Collecting terms we obtain expression (16). □

A.5 Proof of Proposition 5 and 6

A.5.1 Two preliminary results

First we establish that, when $n(\omega)$ is symmetric around $\frac{1}{2}$, the equilibrium is symmetric as well:

Lemma 2. *Suppose that the distribution of traders satisfies $n(\omega) = n(1-\omega)$. Then equilibrium post-trade exposures are symmetric, i.e. they satisfy $g(1-\omega) = 1-g(\omega)$, and can be implemented by a symmetric collection of CDS contracts, i.e. such that $\gamma(\omega, \tilde{\omega}) = -\gamma(1-\omega, 1-\tilde{\omega})$.*

To see this, consider some equilibrium collection of CDS contracts, $\gamma(\omega, \tilde{\omega})$ and its associated post-trade exposures, $g(\omega)$. Now, the alternative collection of CDS $\hat{\gamma}(\omega, \tilde{\omega}) = -\gamma(1-\omega, 1-\tilde{\omega})$ is feasible and generates post-trade exposures:

$$\begin{aligned}\hat{g}(\omega) &= \omega - \int_0^1 \gamma(1-\omega, 1-\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} = \omega - \int_0^1 \gamma(1-\omega, 1-\tilde{\omega}) n(1-\tilde{\omega})\, d\tilde{\omega} \\ &= 1 - \left(1 - \omega + \int_0^1 \gamma(1-\omega, \hat{\omega}) n(\hat{\omega})\, d\hat{\omega}\right) = 1 - g(1-\omega),\end{aligned}$$

where the first equality holds by definition of $\hat{\gamma}(\omega, \tilde{\omega})$, the second equality because $n(\tilde{\omega}) = n(1-\tilde{\omega})$, and the third equality by change of variables $\hat{\omega} = 1 - \tilde{\omega}$. Now it is easy to see that $\hat{\gamma}(\omega, \tilde{\omega})$ satisfies (11). Indeed, $\hat{g}(\omega) < \hat{g}(\tilde{\omega})$ is equivalent to $g(1-\tilde{\omega}) < g(1-\omega)$, which implies from (11) that $\gamma(1-\tilde{\omega}, 1-\omega) = -\gamma(1-\omega, 1-\tilde{\omega}) = \hat{\gamma}(\omega, \tilde{\omega}) = k$. Since equilibrium post-trade exposures are uniquely determined, we conclude from this that $\hat{g}(\omega) = 1 - g(1-\omega) = g(\omega)$.

To see that $g(\omega)$ can be implemented using a symmetric collection of CDS contracts, consider $\gamma^*(\omega, \tilde{\omega}) = \frac{1}{2}\big(\gamma(\omega, \tilde{\omega}) + \hat{\gamma}(\omega, \tilde{\omega})\big)$, for the same $\hat{\gamma}(\omega, \tilde{\omega})$ defined above. By construction, we have that $\gamma^*(1-\omega, 1-\tilde{\omega}) = -\gamma^*(\omega, \tilde{\omega})$, and $g^*(\omega) = \frac{1}{2}\big(g(\omega) + \hat{g}(\omega)\big) = g(\omega)$, given that we have just shown that $g(\omega)$ is symmetric. □

The second preliminary result concerns the function $H(\omega)$:

Lemma 3. *Let, for $\omega \in [0, \frac{1}{2}]$, $H(\omega) \equiv \int_0^\omega (\omega - \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} - kN(\omega)^2$.*

- *if $k \leq \frac{1}{2}\left[n\left(0\right)\right]^{-1}$, then $H(\omega) \geq 0$.*

- *if* $\frac{1}{2}\left[n\left(0\right)\right]^{-1} < k < 1-2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, *then there is a unique* $\underline{\omega} \in (0, \frac{1}{2})$ *such that* $H(\omega) < 0$ *for* $\omega \in (0, \underline{\omega})$ *and* $H(\omega) > 0$ *for* $\omega \in (\underline{\omega}, \frac{1}{2})$.
- *if* $k \geq 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$, *then* $H(\omega) \leq 0$.

To prove this result, note that $H'(\omega) = N(\omega)\left[1 - 2kn(\omega)\right]$ and keep in mind that $n(\omega)$ is non-increasing over $[0, \frac{1}{2}]$. If $2kn(0) \leq 1$, then $H'(\omega) \geq 0$ over $[0, \frac{1}{2}]$. Given that $H(0) = 0$, this establishes the first point of the Lemma. If $2kn(0) > 1$, then it follows that $H(\omega)$ is initially decreasing and may subsequently become increasing: therefore, the equation $H(\omega) = 0$ has at most one solution in $(0, \frac{1}{2}]$. Such a solution exists if and only if:

$$\begin{aligned} H(\tfrac{1}{2}) \geq 0 &\Longleftrightarrow \tfrac{1}{2}N\left(\tfrac{1}{2}\right) - \int_0^{\frac{1}{2}} \tilde{\omega} n(\tilde{\omega})\, d\tilde{\omega} - kN\left(\tfrac{1}{2}\right)^2 \geq 0 \\ &\Longleftrightarrow k \leq 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \tfrac{1}{2}\right], \end{aligned}$$

since $n(\omega)$ symmetric implies $N(\frac{1}{2}) = \frac{1}{2}$.

Lastly, we verify that $1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right] \geq \frac{1}{2}\left[n(0)\right]^{-1}$, with an equality if and only if $n(\omega)$ is uniform. This follows from two implications of the fact that $n(\omega)$ is non-increasing over $[0, \frac{1}{2}]$. First,

$$N(\tfrac{1}{2}) = \tfrac{1}{2} = \int_0^{\frac{1}{2}} n(\omega)\, d\omega \leq \tfrac{n(0)}{2} \Longrightarrow n(0) \geq 1.$$

Second, a uniform distribution over $[0, \frac{1}{2}]$ first-order stochastically dominates $n(\omega \,|\, \omega \leq \frac{1}{2})$, implying that:

$$\mathbb{E}\left[\omega \,|\, \omega \leq \tfrac{1}{2}\right] \leq \tfrac{1}{2} \Longleftrightarrow 1 - \mathbb{E}\left[\omega \,|\, \omega \leq \tfrac{1}{2}\right] \geq \tfrac{1}{2},$$

with a strict inequality if and only if $n(\omega \,|\, \omega \leq \frac{1}{2})$, and thus $n(\omega)$, is uniform. Combining the two we obtain the desired inequality. □

It follows directly from the Lemma that:

Corollary 13. *For* $\omega \in (0, \frac{1}{2})$, $z(\omega) \leq k$ *if and only if* $H(\omega) \leq 0$.

A.5.2 Proof of the two propositions

Given Lemma 2 it is enough to focus on $\omega \in [0, \frac{1}{2}]$.

When there is no flat spot. Consider first the case when $k \leq \frac{1}{2}\left[n(0)\right]^{-1}$. Then, by Lemma 3, it follows that $z(\omega) \geq k$ for all $\omega \in (0, \frac{1}{2}]$. Therefore, the candidate CDS contracts of Proposition 6 are such that $\gamma(\omega, \tilde{\omega}) = k$ if $\omega < \tilde{\omega}$. The corresponding post trade exposure is thus equal to $g(\omega) = \omega + k\left[1 - 2N(\omega)\right]$. Since $n(\omega)$ is U shaped and symmetric, the maximum of $n(\omega)$ is $n(0)$ and so $2kn(\omega) \leq 1$ for all ω. Therefore the function $g(\omega)$ is non-decreasing over $[0, 1]$. This establishes that the CDS contracts of Proposition 6 are indeed the basis of an equilibrium.

When there are flat and increasing spots. Now assume that $\left[n(0)\right]^{-1} < k < 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$ and consider some $\omega \in [0, \underline{\omega}]$. By Lemma 3, we have that $\gamma(\omega, \tilde{\omega}) = -z(\omega)$ for

$\tilde{\omega} < \omega$, $\gamma(\omega, \tilde{\omega}) = z(\tilde{\omega})$ for $\tilde{\omega} \in (\omega, \underline{\omega})$, and $\gamma(\omega, \tilde{\omega}) = k$ for $\tilde{\omega} \geq \underline{\omega}$. Therefore, the corresponding post-trade exposure is

$$g(\omega) = \omega - \int_0^{\omega} z(\omega)\, n(\tilde{\omega})\, d\tilde{\omega} + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} + k\,[1 - N(\underline{\omega})]\,.$$

Differentiating with respect to ω one directly verifies that $g'(\omega) = 0$ so that $g(\omega)$ is flat over $[0, \frac{1}{2}]$. Next consider $\omega \in (\underline{\omega}, \frac{1}{2}]$. By Lemma 3, we have that $\gamma(\omega, \tilde{\omega}) = -k$ for $\tilde{\omega} < \omega$, and $\gamma(\omega, \tilde{\omega}) = k$ for $\tilde{\omega} > \omega$. Therefore the corresponding post-trade exposure is $g(\omega) = \omega + k\,[1 - 2N(\omega)]$. Differentiating we obtain that $g'(\omega) = 1 - 2kn(\omega)$, which is positive since, by Lemma 3, it must be the case that $H'(\omega) > 0$ for $\omega \in (\underline{\omega}, \frac{1}{2}]$. Taken together, this implies that the CDS contracts of Proposition 6 are indeed the basis of an equilibrium.

When there is full risk sharing. Lastly, assume that $k \geq 1 - 2\mathbb{E}\left[\omega \,|\, \omega \leq \frac{1}{2}\right]$. Then $z(\omega) \leq k$ for all $\omega \in [0, \frac{1}{2}]$. For all $\omega \in [0, \frac{1}{2}]$, $g(\omega)$ becomes

$$g(\omega) = \omega - \int_0^{\omega} z(\omega)\, n(\tilde{\omega})\, d\tilde{\omega} + \int_{\omega}^{\frac{1}{2}} z(\tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega} + z(\tfrac{1}{2})\left[1 - N(\tfrac{1}{2})\right],$$

i.e., the same as above with $\underline{\omega}$ replaced by $\frac{1}{2}$ and k replaced by $z(\frac{1}{2})$. Taking derivatives shows that $g'(\omega) = 0$ and so $g(\omega) = g(\frac{1}{2})$. Now given that $N(\frac{1}{2}) = \frac{1}{2}$, it follows that $g(\frac{1}{2}) = \frac{1}{2}$ and so the CDS contracts of Proposition 6 are the basis of an equilibrium. □

A.5.3 Proof of the results discussed in Section 5.3.3

We first establish:

Proposition 14. *Suppose that $n(\omega)$ is U-shaped, symmetric, positive, and consider the equilibrium CDS contracts of Proposition 6. Then:*

(i) $G^+(\omega) = G^-(1 - \omega)$;

(ii) $G^+(\omega)$ *is decreasing and* $G^-(\omega)$ *is increasing;*

(iii) $G^+(\omega) = G^-(\omega)$ *if and only if* $\omega = \frac{1}{2}$;

(iv) $G^+(\omega) + G^-(\omega)$ *is weakly hump-shaped and symmetric around* $\frac{1}{2}$*: it is increasing over* $[0, \underline{\omega})$*, constant over* $(\underline{\omega}, 1 - \underline{\omega})$ *and decreasing over* $(1 - \underline{\omega}, 1]$;

(v) $\left|G^+(\omega) - G^+(\omega)\right|$ *is U-shaped and symmetric around* $\frac{1}{2}$;

(vi) $\min\{G^+(\omega), G^-(\omega)\}$ *is hump-shaped and symmetric around* $\frac{1}{2}$.

Point (i). The first point of the Proposition follows from the fact that $\gamma(\omega, \tilde{\omega}) = -\gamma(1 - \omega, 1 - \tilde{\omega})$.

Point (ii). Given the first point, to prove the second point we only need to show that $G^+(\omega)$ is decreasing over $[0, \frac{1}{2}]$, and that $G^-(\omega)$ is increasing over $[0, \frac{1}{2}]$. For this we consider various subcases. If $g(\omega)$ is increasing, then $G^+(\omega) = k\,[1 - N(\omega)]$ and so is clearly decreasing. Likewise, $G^-(\omega) = kN(\omega)$ and is clearly increasing. If $g(\omega)$ has flat and possibly increasing spot, i.e. $\underline{\omega} \in$

$(0, \frac{1}{2}]$, then for $\omega \in (\underline{\omega}, \frac{1}{2}]$, $G^+(\omega) = k\,[1 - N(\omega)]$ and $G^-(\omega) = kN(\omega)$, which are respectively decreasing and increasing. For $\omega \in [0, \underline{\omega}]$

$$G^+(\omega) = \int_0^{\underline{\omega}} z(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + k\,[1 - N(\underline{\omega})]\,,$$

which is decreasing. Likewise, when $\omega \in [0, \underline{\omega}]$,

$$G^-(\omega) = N(\omega) z(\omega) = \frac{\int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)}$$

Taking derivatives we obtain

$$\begin{aligned}\frac{dG^-}{d\omega}(\omega) &= \frac{N(\omega)^2 - n(\omega) \int_0^{\omega} \big(\omega - \tilde{\omega}\big) n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2} \\ &\geq \frac{\omega n(\omega) N(\omega) - n(\omega) \int_0^{\omega} \big(\omega - \tilde{\omega}\big) n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2} = \frac{n(\omega) \int_0^{\omega} \tilde{\omega} n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)^2} \geq 0\end{aligned}$$

where the second line follows from the fact that $n(\omega)$ is decreasing over $[0, \frac{1}{2}]$, implying that $N(\omega) \geq \omega n(\omega)$.

Point (iii). The first point implies that $G^+(\frac{1}{2}) = G^-(\frac{1}{2})$. This is the unique solution of $G^+(\omega) = G^-(\omega)$ given that $G^+(\omega)$ is decreasing and $G^-(\omega)$ is increasing.

Point (iv). Given the first point, it is enough to show that $G^+(\omega) + G^-(\omega)$ is non-decreasing over $[0, \frac{1}{2}]$. If $g(\omega)$ is increasing then $G^+(\omega) + G^-(\omega) = k$. Now suppose that $g(\omega)$ has a flat and possibly an increasing spot. For all $\omega \in [\underline{\omega}, \frac{1}{2}]$, $g(\omega)$ is increasing and so $G^+(\omega) + G^-(\omega) = k$. For $\omega \in [0, \underline{\omega}]$, the gross exposure is:

$$G^+(\omega) + G^-(\omega) = z(\omega) N(\omega) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + z(\underline{\omega})\,[1 - N(\underline{\omega})]\,.$$

Now recall:

$$g(\underline{\omega}) = \omega - z(\omega) N(\omega) + \int_{\omega}^{\underline{\omega}} z(\tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega} + z(\underline{\omega})\,[1 - N(\underline{\omega})]\,.$$

This implies:

$$\begin{aligned}G^+(\omega) + G^-(\omega) &= g(\underline{\omega}) - \omega + 2 z(\omega) N(\omega) = g(\underline{\omega}) - \omega + 2 \frac{\int_0^{\omega} (\omega - \tilde{\omega})\, n(\tilde{\omega})\, d\tilde{\omega}}{N(\omega)} \\ &= g(\underline{\omega}) + \frac{\int_0^{\omega} (\omega - 2\tilde{\omega})\, n(\tilde{\omega})}{N(\omega)},\end{aligned}$$

where the first equality follows from plugging in the expression for $z(\omega)$ from Proposition 6. Taking derivatives with respect to ω we find:

$$\begin{aligned}&\frac{N(\omega)\left(-\omega n(\omega)+N(\omega)\right)-n(\omega)\int_0^\omega(\omega-2\tilde{\omega})\,n(\tilde{\omega})\,d\tilde{\omega}}{N(\omega)^2}\\=&\frac{N(\omega)-2n(\omega)\int_0^\omega(\omega-\tilde{\omega})\,n(\tilde{\omega})\,d\tilde{\omega}}{N(\omega)}\geq\frac{n(\omega)}{N(\omega)}\left(\omega-2\int_0^\omega(\omega-\tilde{\omega})\;n(\tilde{\omega})\,d\tilde{\omega}\right),\end{aligned}$$

where the last inequality follows because $n(\omega)$ is non-increasing over $[0,\frac{1}{2}]$, and so $N(\omega)\geq\omega n(\omega)$. The result follows because the term in parentheses is equal to zero when $\omega=0$, and its derivative is $1-2N(\omega)>0$ over $[0,\frac{1}{2})$.

Point (v). Given the first point it is enough to show that $G^+(\omega)-G^-(\omega)$ is decreasing over $[0,\frac{1}{2}]$. For $\omega\in[0,\frac{1}{2}]$, Proposition 2 implies that $g(\frac{1}{2})-g(\omega)\leq\frac{1}{2}-\omega$. Given symmetry, $g(\frac{1}{2})=\frac{1}{2}$, so $g(\omega)\geq 0$ and $|G^+(\omega)-G^-(\omega)|=g(\omega)-\omega$. When $g(\omega)$ is flat, then clearly this is a decreasing function. When $g(\omega)$ is increasing, then $g'(\omega)=1-2kn(\omega)$, and so $g'(\omega)-1=-2kn(\omega)<0$.

Point (vi). From Point (ii) and (iii), it follows that, for $\omega\in[0,\frac{1}{2}]$, $\min\{G^+(\omega),G^-(\omega)\}=G^+(\omega)$ is increasing. □

Lastly we show our results regarding trading volume:

Proposition 15. *Trading volume is non-monotonic in k:*

(i) When k goes to zero, trading volume goes to zero.

(ii) When $k\in[1-2\mathbb{E}\left[\omega\,|\,\omega\leq\frac{1}{2}\right],1]$, trading volume is always greater than the Walrasian volume.

(iii) Assume that $n(\omega)$ is bounded away from zero. Then, when k goes to infinity, there exists an equilibrium collection of CDS contracts generating a trading volume arbitrarily close to the Walrasian volume.

Point (i). Trivially, when $k\to 0$, volume must also go to zero.

Point (ii). Assume that k is greater than $1-2\mathbb{E}\left[\omega\,|\,\omega\leq\frac{1}{2}\right]$ but less than one. Then there is full risk sharing and post trade exposures in the OTC market are the same as in a Walrasian market. As a result, trading volume in the OTC market is at least equal to trading volume in the Walrasian market, where net exposures are equal to gross exposures. Suppose towards a contradiction that volume in the OTC market is exactly equal to its Walrasian counterpart. That is, for any ω, the gross exposure is equal to the absolute net exposure, $|\frac{1}{2}-\omega|$. Since $g(\omega)=\frac{1}{2}=\omega+\int_0^1\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}$, this implies that for $\omega<\frac{1}{2}$, we must have $\gamma(\omega,\tilde{\omega})\geq 0$ and for $\omega>\frac{1}{2}$, $\gamma(\omega,\tilde{\omega})\leq 0$. In other words, the only way there is no excess trading volume is that any given bank ω only sign CDS contracts in one direction. But this also means that there cannot be trades amongst $(\omega,\tilde{\omega})\in[0,\frac{1}{2})^2$, and amongst $(\omega,\tilde{\omega})\in(\frac{1}{2},1]^2$. Therefore, for a $\omega<\frac{1}{2}$ bank:

$$\frac{1}{2}-\omega=\int_{\frac{1}{2}}^1\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\leq\frac{k}{2}.$$

But this inequality cannot hold if $\omega\simeq 0$ and k is less than one, a contradiction.

Point (iii). The construction is based on the following intuition: in the absence of any trading limits, the Walrasian allocation is obtained by trading the quantity $\frac{1}{2} - \omega$ of CDS's with the symmetric bank, $1-\omega$, and nothing with banks $\tilde{\omega} \neq 1-\omega$. When trading capacity is large, then approximate Walrasian volume is obtained by trading with a small measure of banks surrounding the symmetric $1-\omega$ bank. Formally let us assume for this argument that $n(\omega) > 0$ eveywhere so that it is bounded below by $\underline{n} > 0$. Given some integer I, consider the sequence $\omega_i = \frac{1}{2} + \frac{i}{2I}$, for $i \in \{1, \ldots, I\}$ and the symmetric sequence $\omega_{-i} = 1 - \omega_i$. Let $\Omega_i \equiv [\omega_{i-1}, \omega_i]$ and its symmetric $\Omega_{-i} \equiv [\omega_{-i}, \omega_{-i+1}]$. The mean endowment in Ω_i is $\omega_i^\star = \mathbb{E}\big[\omega \,|\, \omega \in \Omega_i\big]$, and by symmetry the mean endowment in Ω_{-i} is $\omega_{-i}^\star = 1 - \omega_i^\star$. Now consider the collection of CDS contracts $\gamma_A(\omega, \tilde{\omega}) + \gamma_B(\omega, \tilde{\omega})$, defined as follows. For $\omega \in \Omega_i$ and $\tilde{\omega} \in \Omega_{-i}$:

$$\gamma_A(\omega, \tilde{\omega}) = \frac{\frac{1}{2} - \omega_i^\star}{\big(N(\omega_i) - N(\omega_{i-1})\big)},$$

and for $\omega \in \Omega_i$ and $\tilde{\omega} \notin \Omega_{-i}$, $\gamma_A(\omega, \tilde{\omega}) = 0$. For $\omega \in \Omega_i$ and $\tilde{\omega} \in \Omega_j$,

$$\gamma_B(\omega, \tilde{\omega}) = \big(\omega_j^\star - \tilde{\omega}\big) - \big(\omega_i^\star - \omega\big).$$

The contracts $\gamma_A(\omega, \tilde{\omega})$ prescribe trades amongst $(\omega, \tilde{\omega})$ belonging to symmetric intervals Ω_i and Ω_{-i}. They are designed to bring a ω-trader's exposure close to $\frac{1}{2}$. Namely after conducting all the trades in $\gamma_A(\omega, \tilde{\omega})$ the exposure of a ω-trader is $\frac{1}{2} - (\omega_i^\star - \omega)$. When I is large enough, $\omega \simeq \omega_i^\star$ and so this exposure is very close to $\frac{1}{2}$. Note that, because the aggregate exposure to default risk is equal to $\frac{1}{2}$, the average residual exposure $\omega_i^\star - \omega$ must be equal to zero.

The contracts $\gamma_B(\omega, \tilde{\omega})$ prescribe a ω-trader to swap his residual endowment, $\omega_i^\star - \omega$, with the residual endowment of everyone else. Therefore, he ends up with the average residual endowment, which is equal to zero as noted above. Taken together, the combined contracts, $\gamma_A(\omega, \tilde{\omega}) + \gamma_B(\omega, \tilde{\omega})$, lead to post-trade exposures $g(\omega) = \frac{1}{2}$.

Clearly, gross notionals created by $\gamma_A(\omega, \tilde{\omega})$ can be made arbitrarily close to their Walrasian counterpart for I large enough. Likewise, the gross notionals created by $\gamma_B(\omega, \tilde{\omega})$ can be made arbitrarily close to zero. Given that $N(\omega_i) - N(\omega_{i-1}) > \underline{n}\,(\omega_i - \omega_{i-1}) = \underline{n}/(2I) > 0$, choosing k large enough makes these contracts feasible, establishing the claim. □

A.6 Proof of Proposition 7

When D is normally distributed, $\Gamma[x]$ is quadratic, and so we have the identity:

$$\Gamma[y] = \Gamma[x] + \Gamma'[x]\,(y - x) + \frac{\Gamma''}{2}\,(y - x)^2,$$

where $\Gamma'' = \alpha \mathbb{V}[D]$ is the constant second derivative of $\Gamma[x]$. It thus follows that competitive surplus must be equal to:

$$K(\omega) = \Gamma[\omega] - \Gamma[g(\omega)] + \Gamma'[g(\omega)]\,\big(g(\omega) - \omega\big) = \frac{\alpha \mathbb{V}[D]}{2}\big(g(\omega) - \omega\big)^2.$$

The property that $K(\omega)$ is U-shaped obtains from the fact that $g'(\omega) \leq 1$ and $g(\frac{1}{2}) = \frac{1}{2}$.

The formula from the frictional surplus is:

$$\begin{aligned} F(\omega) &= \int_0^1 \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\right| n(\tilde{\omega})\, d\tilde{\omega} \\ &= \int_0^{\omega} \big(\Gamma'\left[g(\omega)\right] - \Gamma'\left[g(\tilde{\omega})\right]\big) n(\tilde{\omega})\, d\tilde{\omega} + \int_{\omega}^1 \big(\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega)\right]\big) n(\tilde{\omega})\, d\tilde{\omega}. \end{aligned}$$

Now take derivatives

$$F'(\omega) = \Gamma''\left[g(\omega)\right] g'(\omega)\big(2N(\omega) - 1\big),$$

which is negative for ω below the median, and positive for ω above the median, which in our symmetric case is $\omega = \frac{1}{2}$.

A.7 Proof of Proposition 8

A.7.1 Preliminaries

To apply the fixed point theorem, we first need to establish properties of the operators mapping the distribution of traders, n, to the post-trade exposure function, g, and to the entry payoff, Δ. Consider the set $\mathcal{C}^0([0,1])$ of continuous functions over $[0,1]$, equipped with the sup norm. Let $\mathcal{N}$ be the set of $n \in \mathcal{C}^0([0,1])$ such that $n(\omega) > 0$ almost everywhere and $\int_0^1 n(\omega)\, d\omega = 1$. Let $\mathcal{G}$ be the set of post-trade exposures functions, g, and let $\mathcal{D}$ be the set of entry payoff functions, Δ, generated by distributions in $\mathcal{N}$. Clearly, by Proposition 2, $\mathcal{G}$ is a subset of $\mathcal{C}^0([0,1])$, and so is $\mathcal{D}$. Moreover:

Lemma 4. *The sets $\mathcal{G}$ and $\mathcal{D}$ are equibounded and equicontinuous, and so their closures, $\bar{\mathcal{G}}$ and $\bar{\mathcal{D}}$, are compact.*

For $\mathcal{G}$, boundedness follows because $g(\omega) \in [0,1]$ and equicontinuity because, as shown in Proposition 2, post-trade exposures are Lipchitz with coefficient 1. For $\mathcal{D}$, boundedness follows because g is bounded. For equicontinuity, it is enough to show that all $\Delta \in \mathcal{D}$ are Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. This follows from the observation that $K(\omega) = \Gamma\left[\omega\right] - \Gamma\left[g(\omega)\right] + \Gamma'\left[g(\omega)\right]\big(g(\omega) - \omega\big)$ is Lipchitz with a coefficient independent of n. Indeed, g is Lipchitz with coefficient one, because both $\Gamma\left[x\right]$ and $\Gamma'\left[x\right]$ are continuously differentiable, and because the Lipchitz property over a compact set is preserved by sum, product, and composition. So all we need to show is that the frictional surplus is Lipchitz with a coefficient that is independent from n. To that end, consider $\omega_2 > \omega_1$ and note that:

$$\begin{aligned} \left|F(\omega_1) - F(\omega_2)\right| &= \left| \int_0^1 \left(\left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_2)\right]\right| - \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_1)\right]\right| \right) n(\tilde{\omega})\, d\tilde{\omega} \right| \\ &\leq \int_0^1 \left| \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_2)\right]\right| - \left|\Gamma'\left[g(\tilde{\omega})\right] - \Gamma'\left[g(\omega_1)\right]\right| \right| n(\tilde{\omega})\, d\tilde{\omega} \\ &\leq \int_0^1 \left|\Gamma'\left[g(\omega_2)\right] - \Gamma'\left[g(\omega_1)\right]\right| n(\tilde{\omega})\, d\tilde{\omega} = \left|\Gamma'\left[g(\omega_2)\right] - \Gamma'\left[g(\omega_1)\right]\right|, \end{aligned}$$

where the inequality on the third line follows by an application of the reverse triangle inequality. Since $\Gamma'\left[g\right]$ is Lipchitz, and since g is Lipchitz with coefficient one, the result follows. The compactness of $\bar{\mathcal{G}}$ and $\bar{\mathcal{D}}$ then follows from Arzela-Ascoli Theorem (see Theorem 11.28 in Rudin, 1974). □

Next, we establish a first continuity property:

Lemma 5. *The operator mapping distributions, $n \in \mathcal{N}$, into post-trade exposures, $g \in \mathcal{G}$, is continuous.*

To show this result, consider some $n \in \mathcal{N}$, and let g and γ be the associated equilibrium post-trade exposures and CDS contracts. Let $n^{(p)}$ be a sequence of distributions converging to n, and let $\gamma^{(p)}$ and $g^{(p)}$ be the associated sequences of CDS contracts and post-trade exposures. Keep in mind that we equipped $C^0([0,1])$ with the sup norm, so convergence is uniform. Given that $\bar{\mathcal{G}}$ is compact, in order to show continuity it is sufficient to show that all convergent subsequences of $g^{(p)}$ share the same limit, and that this limit is equal to g. Without loss of generality, we thus assume that $g^{(p)}$ is convergent, and denote its limit by $g^\star$. Since all $g^{(p)}$ are continuous and convergence is uniform, $g^\star$ must be continuous. By construction, $g^{(p)}$, $\gamma^{(p)}$ and $n^{(p)}$ satisfy:

$$g^{(p)}(\omega) = \omega + \int_0^1 \gamma^{(p)}(\omega, \tilde{\omega}) n^{(p)}(\tilde{\omega})\, d\tilde{\omega}.$$

Consider the auxiliary post-trade exposures:

$$\hat{g}^{(p)}(\omega) \equiv \omega + \int_0^1 \gamma^{(p)}(\omega, \tilde{\omega}) n(\tilde{\omega})\, d\tilde{\omega}$$
$$\tilde{g}^{(p)}(\omega) \equiv \omega + \int_0^1 \gamma(\omega, \tilde{\omega}) n^{(p)}(\tilde{\omega})\, d\tilde{\omega}.$$

In words, $\hat{g}^{(p)}$ is the post-trade exposure generated by $\gamma^{(p)}$ if the underlying distribution of traders is n, and $\tilde{g}^{(p)}$ is the post-trade exposure generated by γ if the underlying distribution is $n^{(p)}$. Since $\gamma^{(p)}$ is bounded by k and since $n^{(p)} \to n$, we have $||\hat{g}^{(p)} - g^{(p)}|| \to 0$ and so $\hat{g}^{(p)} \to g^\star$. Likewise, $\tilde{g}^{(p)} \to g$.

Let $J(g, n) \equiv \int_0^1 \Gamma\left[g(\omega)\right] n(\omega)\, d\omega$ be the average cost of risk bearing generated by post-trade exposures g under distribution n. Recall that (γ, g) solves the planning problem of Proposition 1 given n, i.e., it minimizes J amongst feasible risk allocations. Since $(\gamma^{(p)}, \hat{g}^{(p)})$ is feasible for this planning problem, we must have $J(g, n) \leq J(\hat{g}^{(p)}, n)$. Going to the limit $p \to \infty$, we obtain by dominated convergence that $J(g, n) \leq J(g^\star, n)$. Likewise, $g^{(p)}$ solves the planning problem given $n^{(p)}$. Since $(\gamma, \tilde{g}^{(p)})$ is feasible for this planning problem, we must have $J(g^{(p)}, n^{(p)}) \leq J(\tilde{g}^{(p)}, n^{(p)})$. Going to the limit $p \to \infty$, we obtain by dominated convergence that $J(g^\star, n) \leq J(g, n)$. Taken together, we thus have that $J(g^\star, n) = J(g, n)$. Now suppose that $g^\star \neq g$. Then by convexity of $\Gamma[g]$ we have that for all $\lambda \in (0, 1)$:

$$J(\lambda g + (1 - \lambda) g^\star, n) < J(g, n).$$

But $\lambda\gamma + (1 - \lambda)\gamma^{(p)}$ is feasible, generates post trade exposure $\lambda g + (1 - \lambda)\hat{g}^{(p)}$ given n, and converges to $\lambda g + (1 - \lambda) g^\star$ as $p \to \infty$. Therefore, for p large enough, $J(\lambda g + (1 - \lambda)\hat{g}^{(p)}, n) < J(g, n)$, which is impossible given that (γ, g) solves the planning problem. Therefore, $g = g^\star$ almost everywhere, since we restricted attention to n such that $n(\omega) > 0$ almost everywhere. Since g and $g^\star$ are continuous, we obtain that $g = g^*$ everywhere. □

Next we have the corollary:

Corollary 16. *The operator mapping distributions, $n \in \mathcal{N}$, into entry-utility functions, $\Delta \in \mathcal{D}$, is continuous.*

Note that $\Delta(\omega) = A\big(\omega, g(\omega), N(\omega)\big) + B\big(\omega, g, n\big)$, where

$$A(\omega, g, N) = \Gamma[\omega] - \Gamma[g] - \Gamma'[g](\omega - g) + \frac{k}{2}(2N-1)\Gamma'[g]$$
$$B\big(\omega, g(\cdot), n(\cdot)\big) = -\frac{k}{2}\int_0^{\omega} \Gamma'[g(\tilde{\omega})]\, n(\tilde{\omega})\, d\tilde{\omega} + \frac{k}{2}\int_{\omega}^{1} \Gamma'[g(\tilde{\omega})]\, n(\tilde{\omega})\, d\tilde{\omega}.$$

Now given any $n^{(p)} \to n$, we already know that the associated $g^{(p)} \to g$. Note also, that $N^{(p)} \to N$ (uniformly as well). Since $A(\omega, g, N)$ is continuous and $(\omega, g, N) \in [0,1]^3$, it follows that $A(\omega, g^{(p)}(\omega), N^{(p)}(\omega)) \to A(\omega, g(\omega), N(\omega))$ uniformly. Likewise, by dominated convergence, $B\big(\omega, g^{(p)}, n^{(p)}\big) \to B\big(\omega, g, n\big)$ uniformly. □

Lastly an important property for what follows is:

Lemma 6 ($0 \neq \bar{\mathcal{D}}$)**.** *The function $\Delta(\omega) = 0$ does not belong to the closure of $\mathcal{D}$.*

Towards a contradiction, assume it does: there is a sequence $n^{(p)}$ such that the associated $\Delta^{(p)} \to 0$. Note that, by strict convexity of $\Gamma[g(\omega)]$, $\Gamma[\omega] - \Gamma[g(\omega)] + \Gamma'[g(\omega)]\big(g(\omega) - \omega\big) \geq 0$, with an equality if and only if $g(\omega) = \omega$. Given that the frictional surplus is non-negative, this implies that:

$$\Gamma[\omega] - \Gamma\left[g^{(p)}(\omega)\right] - \Gamma'\left[g^{(p)}(\omega)\right]\left(\omega - g^{(p)}(\omega)\right) \to 0,$$

and that all convergent subsequence of $g^{(p)}$ are such that $g^{(p)}(\omega) \to \omega$. Given that $\bar{\mathcal{G}}$ is compact, we thus have that $g^{(p)}(\omega) \to \omega$ uniformly over $\omega \in [0,1]$. Now turning to the last term of $\Delta(\omega)$ evaluated at $\omega = 0$, we have:

$$\int_0^1 \left(\Gamma'\left[g^{(p)}(\tilde{\omega})\right] - \Gamma'\left[g^{(p)}(0)\right]\right) n^{(p)}(\tilde{\omega})\, d\tilde{\omega} \to 0,$$

where we can remove the absolute value since $g^{(p)}(\omega) \geq g^{(p)}(0)$. In particular, for any $\omega > 0$, we have that $\int_{\omega}^1 \left(\Gamma'\left[g^{(p)}(\tilde{\omega})\right] - \Gamma'\left[g^{(p)}(0)\right]\right) n^{(p)}(\tilde{\omega})\, d\tilde{\omega} \to 0$. Given that $g^{(p)}(\tilde{\omega}) \geq g^{(p)}(\omega)$ for $\tilde{\omega} \in [\omega, 1]$, this implies in turn that $\left(\Gamma'\left[g^{(p)}(\omega)\right] - \Gamma'\left[g^{(p)}(0)\right]\right)\left(1 - N^{(p)}(\omega)\right) \to 0$. Since $g^{(p)}(\omega) \to \omega$, we thus have that $N^{(p)}(\omega) \to 1$, i.e., the distribution $n^{(p)}$ converges to a Dirac at $\omega = 0$. The intuition is simple: the only way $\omega = 0$ has no gain from entering, i.e. $\Delta(\omega) = 0$, is if it only meets traders of her kind, i.e., $\tilde{\omega} = 0$ with probability one. But this means that $\omega \neq 0$ must have strictly positive gains from entering. Formally, given that $g^{(p)}(\omega)$ converges towards ω, uniformly over $\omega \in [0,1]$, this implies that, for every $\omega \in (0,1]$, the last term of $\Delta^{(p)}(\omega)$ converges to $\frac{k}{2}\left|\Gamma'(0) - \Gamma'(\omega)\right| > 0$, which is a contradiction.

A.7.2 Properties of the fixed-point equation

For this section it is convenient to rewrite the fixed-point equation as:

$$T[n](\omega) = \frac{\Upsilon[\Delta(\omega)]}{\int_0^1 \Upsilon[\Delta(\tilde{\omega})]\, d\tilde{\omega}}, \text{ where } \Upsilon(x) \equiv \Psi\left(\frac{c}{x}\right). \tag{24}$$

We start by establishing basic properties of the function $\Upsilon(x)$:

Lemma 7. *Let, for $x > 0$, $\Upsilon(x) \equiv \Psi\left(\frac{c}{x}\right)$ and let $\Upsilon(0) = 0$. Then the function $\Upsilon(x)$ is bounded, non-decreasing, continuous, piecewise continuously differentiable with bounded derivative.* □

The function $\Upsilon(x)$ is bounded and non-decreasing since $\Psi(x) \in [0,1]$ and non-increasing. It is obviously continuous for $x > 0$, and it is also continuous at zero since $\lim_{S\to\infty} \Psi(S) = 0$. It is differentiable for all $x > 0$ and $x \neq \frac{c}{\underline{S}}$. For $x \in (0, \frac{c}{\underline{S}})$, $\Upsilon'(x) = -c^2/x^3\varphi(c/x)$. Now recall that $\lim_{x\to 0} \Upsilon'(x) = \frac{1}{c} \times \lim_{S\to\infty} S^3\varphi(S)$, which we assumed exists. Given that $\varphi(S)$ is continuous, this implies in turn that $\Psi'(x)$ is bounded over $\left[0, \frac{c}{\underline{S}}\right)$. For $x > \frac{c}{\underline{S}}$, $\Upsilon'(x) = 0$ and is obviously continuously differentiable and bounded. Moreover,

$$\frac{1}{x}\Psi\left(\frac{c}{x}\right) = \frac{1}{c} \times \frac{c}{x}\int_{\frac{c}{x}}^{\infty} z\varphi(z)\,dz = \frac{1}{c} \times \frac{c}{x}\int_0^{\frac{x}{c}} \frac{1}{y^3}\varphi\left(\frac{1}{y}\right)\,dy \to \frac{1}{c} \times \lim_{S\to\infty} S^3\varphi(S),$$

where the first equality follows from the definition of $\Psi(S)$, the second equality from the change of variable $y = 1/z$, and the third equality follows because of our assumption that $\lim_{S\to\infty} S^3\varphi(S)$ exists. Therefore, $\Upsilon(x)$ is continuously differentiable at zero, implying that its derivative is bounded over $[0, \frac{c}{\underline{S}})$. □

Lemma 8 (Properties of T)**.** *The operator T is continuous and uniformly bounded. The set $T[\mathcal{N}]$ is included in $\mathcal{N}$ and is equicontinuous.*

Continuity follows because $\Upsilon(x)$ is continuous, and because, by Corollary 16, the operator mapping n to Δ is continuous. For boundedness, we first observe that the numerator of (24) is positive and less than one. So all we need to show is that denominator is bounded away from zero. For this it suffices to show that:

$$\inf_{\Delta\in\bar{\mathcal{D}}} \int_0^1 \Upsilon\left[\Delta(\omega)\right] d\omega > 0.$$

Since $\bar{\mathcal{D}}$ is compact by Lemma 4, and since the functional that is minimized is continuous in Δ, it follows that the infimum is achieved for some continuous function $\Delta^* \in \bar{\mathcal{D}}$. By Lemma 6, $\Delta^* \neq 0$. Since $\Psi(x) \geq 0$ with an equality if and only if $x = 0$, it follows that the infimum is strictly positive.

Next we show that $T[\mathcal{N}] \subseteq \mathcal{N}$, i.e., that $T[n](\omega)$ is continuous, satisfies $\int_0^1 n(\omega)\,d\omega = 1$, and $n(\omega) > 0$ almost everywhere. Continuity follows because $\Delta(\omega)$ and $\Upsilon(x)$ are continuous; $\int_0^1 n(\omega)\,d\omega = 1$ follows by construction. To show that $T[n](\omega) > 0$ almost everywhere, we show that there is at most one $\omega^\star$ such that $T[n](\omega^\star) = 0$. Indeed, since $\Upsilon(x) = 0$ if and only if $x = 0$, $T[n](\omega^\star) = 0$ implies $\Upsilon\left[\Delta(\omega^\star)\right] = \Delta(\omega^\star) = K(\omega^\star) = F(\omega^\star) = 0$. Because $F(\omega^\star) = 0$, and keeping in mind that $n(\omega) > 0$ almost everywhere, it follows that $\Gamma'\left[g(\omega)\right] = \Gamma'\left[g(\omega^\star)\right]$ and thus $g(\omega) = g(\omega^\star)$ almost everywhere. Since $g(\tilde{\omega})$ is continuous, it follows that $g(\omega)$ is constant and equal to $\int_0^1 \omega n(\omega)\,d\omega$. But then $K(\omega^*) = 0$ implies that $\omega^\star = \int_0^1 \omega n(\omega)\,d\omega$, and is thus uniquely determined.

Turning to equicontinuity, we first observe that, since $\Psi[x]$ is piecewise differentiable with bounded derivatives, it is Lipchitz. Recall from the proof of Lemma 4 that all $\Delta \in \mathcal{D}$ are Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. Given that the Lipchitz property is preserved by composition, this shows that $\omega \mapsto \Psi\left[\Delta(\omega)\right]$ is Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. Lastly, from argument above, the denominator of $T[n](\omega)$ is bounded away from zero. Taken together, this shows that the function $T[n]$ is Lipchitz with a coefficient that is independent of $n \in \mathcal{N}$. □

A.7.3 Applying the Schauder fixed point theorem

Let M be the uniform upper bound of the operator T we obtained in the proof of Lemma 8. Consider the closed and bounded set $\mathcal{N}_M$ of all $n \in \mathcal{N}$ such that $||n|| \leq M$. Note that $\mathcal{N}$ is convex and that a convex combination of functions bounded by M remains bounded by M. Therefore, $\mathcal{N}_M$ is convex as well. Clearly, the operator T maps $\mathcal{N}_M$ into itself. Moreover, by Lemma 8, the family $T[\mathcal{N}_M]$ is equicontinuous. Taken together all these properties allow us to apply Theorem 17.4 in Stokey and Lucas (1989), establishing the existence of a fixed point. □

A.8 Proof of Corollary 9

The result follows by applying the Schauder fixed point theorem in the set $\mathcal{N}_S$ of $n \in \mathcal{N}_M$ which are U-Shaped and symmetric. Clearly, this set is closed and convex. Moreover, when $\Gamma[x]$ the operator T maps $\mathcal{N}_S$ into itself. The result follows. □

A.9 Proof of Proposition 10

The first step is to derive the distribution of traders conditional on size, $n(\omega \,|\, S)$. The entry threshold at ω, $\Sigma(\omega)$, is a hump-shaped function of ω. This means that, if a bank of size S is active in the market, then its endowment per capita, ω, must be either small enough or large enough. Precisely, for $S \in [\Sigma(0), \Sigma(\frac{1}{2})]$, let $\bar{\omega}(S) \in [0, \frac{1}{2}]$ be the solution of $\Sigma(\omega) = S$. For $S > \Sigma(\frac{1}{2})$, let $\bar{\omega}(S) = \frac{1}{2}$. Then if a bank of size $S \in [\Sigma(0), \infty)$ is active in the OTC market, its endowment per-capita of a bank of size must either belong to $[0, \bar{\omega}(S)]$ or $[1 - \bar{\omega}(S), 1]$. Now recall that the endowment per trader is drawn uniformly conditional on S. This implies that the measure of traders conditional on S must be uniform over its support, that is:

$$n(\omega \,|\, S) = \frac{1}{2\bar{\omega}(S)} \text{ if } \omega \in [0, \bar{\omega}(S)] \cup [1 - \bar{\omega}(S), 1]$$

and $n(\omega \,|\, S) = 0$ otherwise. Clearly, if $S' > S$, then $n(\omega \,|\, S')$ puts more mass towards the middle of the ω spectrum than $n(\omega \,|\, S)$, because middle-ω banks are predominantly large. Mathematically, given symmetry, this property can be expressed as follows:

Lemma 9. *Consider (S, S') in $[\Sigma(0), \Sigma(\frac{1}{2})]^2$. If $S' > S$, then $n(\omega \,|\, S', \omega \leq \frac{1}{2})$ first order stochastically dominates $n(\omega \,|\, S, \omega \leq \frac{1}{2})$.*

Now consider any function $x(\omega)$ that is U-shaped and symmetric and calculate:

$$\mathbb{E}_S[x(\omega)] = \int_0^1 x(\omega) n(\omega \,|\, S)\, d\omega = 2 \int_0^{\frac{1}{2}} x(\omega) n(\omega \,|\, S, \omega \leq \tfrac{1}{2})\, d\omega,$$

where the second equality follows because $x(\omega)$ is symmetric. Now, given that $x(\omega)$ is decreasing over $[0, \frac{1}{2}]$, it follows from the above lemma that $\mathbb{E}_S[x(\omega)]$ is non-increasing. The opposite is true if $x(\omega)$ is hump-shaped and symmetric. Except for price dispersion, the results then follow from Proposition 14. For price dispersion, recall that, given that $\Gamma'[g] = \mathbb{E}[D] + \alpha \frac{\mathbb{V}[D]}{2} g$ and so:

$$R(\omega, \tilde{\omega}) = \mathbb{E}[D] + \frac{\alpha \mathbb{V}[D]}{2} \Big(g(\omega) + g(\tilde{\omega}) \Big)$$

Given that $n(\omega \mid S)$ is symmetric around $\frac{1}{2}$, $\mathbb{E}_S[g(\omega)] = \frac{1}{2}$ and so the conditional mean of CDS price is:

$$\mathbb{E}_S[R(\omega, \tilde{\omega})] = \mathbb{E}[D] + \frac{\alpha \mathbb{V}[D]}{2}.$$

The conditional dispersion is, up to some multiplicative constant:

$$\mathbb{E}_S\left[\left(g(\omega) - \tfrac{1}{2} + g(\tilde{\omega}) - \tfrac{1}{2}\right)^2\right] = 2\mathbb{E}_S\left[\left(g(\omega) - \tfrac{1}{2}\right)^2\right] = 2\int_0^{\frac{1}{2}} \left(g(\omega) - \tfrac{1}{2}\right)^2 n(\omega \mid S, \omega \leq \tfrac{1}{2})\, d\omega,$$

where the first equality follows because ω and $\tilde{\omega}$ are drawn independently, and the second equality follows by symmetry. Now observe that, over $[0, \frac{1}{2}]$, $g(\omega) - \frac{1}{2}$ is non-decreasing and negative, so that $\left(g(\omega) - \frac{1}{2}\right)^2$ is non-increasing. The result then follows from the Lemma. □

A.10 Proof of the results in Section 6.3.3

In this section we formulate and solve the model we use for comparative statics with respect to c and k. The distribution of per-capita endowments is taken to be uniform over $\{0, \frac{1}{2}, 1\}$, and independent from the size distribution. The loss-upon-default, D, is assumed to be normally distributed, so that the cost of risk-bearing, $\Gamma[g]$, is quadratic. Lastly, the distribution of bank size is Pareto: the measure of traders in banks with size greater than some $S \geq \underline{S}$ is $\Psi(S) = (S/\underline{S})^{-\theta}$, for some $\theta > 1$.

A.10.1 Equilibrium in the OTC market, post entry

We first solve for an equilibrium in the OTC market, post entry. We gain tractability by noting that, with three possible values for the endowment ω, the distribution of traders can be parameterized by a single number, the fraction of traders from $\omega = \frac{1}{2}$ banks: $n(\frac{1}{2}) \equiv n$. Given symmetry, it must be the case that $n(0) = n(1) = \frac{1-n}{2}$. We now consider three cases.

Case 1: when $k(1+n) < 1$. Then, we guess and verify that risk sharing is imperfect. Thus, all traders sign k contracts whenever they meet a trader with $\tilde{\omega} \neq \omega$. This implies that:

$$g(0) = kn + k\frac{1-n}{2} = \frac{k(1+n)}{2}.$$

We also have $g(1) = 1 - g(0)$ and $g(\frac{1}{2}) = \frac{1}{2}$. The aggregate gross exposure is:

$$G(k, n) = (1-n)k\frac{1+n}{2} + nk(1-n) = \frac{k(1-n)(1+3n)}{2}. \tag{25}$$

The first term is the exposures of extreme-ω banks. The fraction of $\omega \in \{0, 1\}$ traders is equal to $1-n$. Each such extreme-ω trader transacts with a middle-ω trader with probability n, or with the opposite extreme-ω trader with probability $(1-n)/2$. The total probability of a transaction is $(1+n)/2$, and all transactions are of size k. The second term is the exposure of middle-ω banks. The fraction of middle-ω traders is n. Each such trader transacts with extreme-ω traders with probability $1-n$, and all transactions are of size k. We can calculate, similarly, the absolute net exposure per capita. For extreme-ω banks, net and gross exposures are the same, and for middle-ω banks the net exposure is zero. The same calculation as above thus shows that the

absolute net exposure is:

$$N(k,n) = \frac{k(1-n)(1+n)}{2} \tag{26}$$

Net and gross exposures grow linearly with k. When $k(1+n) = 1$ and full risk sharing is achieved, we have that $N(k) < G(k)$. This is the sense in which the intermediation services of middle-ω banks are essential to achieve full risk sharing. The ratio of net-to-gross exposures:

$$\rho(n) \equiv \frac{N(k,n)}{G(k,n)} = \frac{1+n}{1+3n}, \tag{27}$$

is immediately shown to be decreasing in n.

Case 2.1: if $k(1+n) \geq 1$ but $k(1-n) < 1$. The first inequality means that full risk sharing obtains in equilibrium. The second inequality means that the intermediation services of middle-ω are essential to achieve full risk-sharing. Indeed, if $\omega = 0$ traders only transacted with $\omega = 1$ traders, then $g(0)$ would be equal to $k\frac{1-n}{2} < \frac{1}{2}$. As we will see below this case never arises in an entry equilibrium.

Case 2.2: if $k(1-n) \geq 1$. Then full risk sharing obtains without using the intermediation services of $\omega = \frac{1}{2}$-traders: $\gamma(0,1) = \frac{1}{2}$ and $\gamma(0,\frac{1}{2}) = 0$. In that case net and gross exposures coincide and are both equal to $\frac{1-n}{2}$.

A.10.2 Entry

First recall the proof of Proposition 7, in Section A.6 that:

$$K(\omega) = \frac{\alpha \mathbb{V}[D]}{2}(g(\omega)-\omega)^2 \quad \text{and} \quad F(\omega) = \alpha \mathbb{V}[D] \int_0^1 |g(\omega) - g(\tilde{\omega})|\, n(\tilde{\omega})\, d\tilde{\omega},$$

and $\Delta(\omega) = K(\omega) + \frac{1}{2}F(\omega)$. The entry threshold is $\Sigma(\omega) = \frac{c}{\Delta(\omega)}$. With a Pareto distribution the measure of traders above $\Sigma(\omega)$ is equal to

$$\Psi(\Sigma(\omega)) = \left(\frac{\max\{\Sigma(\omega), \underline{S}\}}{\underline{S}}\right)^{-\theta} = \left(\max\left\{\frac{\Sigma(\omega)}{\underline{S}}, 1\right\}\right)^{-\theta} = \min\left\{\frac{\underline{S}\Delta(\omega)}{c}, 1\right\}^{\theta},$$

where the last equality follows from the definition of the entry threshold. The fixed point equation for the distribution of types becomes:

$$T[n](\omega) = \frac{\min\left\{\frac{\underline{S}\Delta(\omega)}{c}, 1\right\}^{\theta}}{\int_0^1 \min\left\{\frac{\underline{S}\Delta(\tilde{\omega})}{c}, 1\right\}^{\theta} d\tilde{\omega}}$$

Adapting the above formula for a discrete number of types, we obtain after some algebra, gathered in Section A.10.3, that the value of entry for extreme-ω banks is

$$\Delta(0) = \Delta(1) = \begin{cases} \frac{\alpha \mathbb{V}[D] k}{4}\left[1 - \frac{k\left(1-n^2\right)}{2}\right] & \text{if } k(1+n) < 1 \\ \frac{\alpha \mathbb{V}[D]}{8} & \text{if } k(1+n) \geq 1. \end{cases}$$

For middle-ω banks, it is

$$\Delta(\tfrac{1}{2}) = \begin{cases} \frac{\alpha \mathbb{V}[D] k}{4} (1-n) \left[1 - k(1+n)\right] & \text{if } k(1+n) < 1 \\ 0 & \text{if } k(1+n) \geq 1. \end{cases}$$

By symmetry, the value of entry for $\omega = 1$ banks is the same as that of $\omega = 0$ banks. When $k(1+n) \geq 1$, then there is full risk-sharing and so middle-ω banks have no incentives to enter. Lastly, one can verify that $\Delta(0) > \Delta(\frac{1}{2})$, i.e., extreme-$\omega$ banks have stronger incentives to enter than middle-ω banks. The fixed point equation is:

$$n = \frac{F(n)^\theta}{2 + F(n)^\theta}, \qquad \text{where } F(n) \equiv \frac{\min\left\{\frac{\underline{S}\Delta(\frac{1}{2})}{c}, 1\right\}}{\min\left\{\frac{\underline{S}\Delta(0)}{c}, 1\right\}}. \tag{28}$$

Proposition 17. *Equation (28) has a unique solution, n. It is equal to zero if $k \geq 1$, and it is positive and less than $\frac{1}{k} - 1$ if $k < 1$.*

The detailed proof is in Section A.10.4. When $k \geq 1$ then $k(1+n) \geq 1$: thus, there is full risk-sharing in any equilibrium, $\Delta(\frac{1}{2}) = 0$, and so $n = 0$. Conversely, $n = 0$ is indeed an equilibrium. When $k < 1$, the solution of the fixed point equation cannot be such that $k(1+n) \geq 1$. Indeed, when $k < 1$ we need sufficiently positive entry of middle-ω banks to sustain full risk sharing, but if there is full risk sharing middle-ω banks have no incentive to enter. This verifies that Case 2.1 above never arises once we consider entry. The proposition also shows that the symmetric equilibrium is unique. The force that underlies uniqueness is that the entry decisions of middle-ω banks are strategic substitutes. This is because, when more middle-ω banks enter, risk-sharing improves and intermediation profits are eroded.

Our first set of comparative statics is with respect to the fixed cost of entry, c:

Proposition 18 (Changes in c)**.** *Holding all other parameters the same, there are two cost thresholds $\underline{c}$ and $\overline{c}$ such that:*

- *If $c < \underline{c}$, all banks enter and $n = \frac{1}{3}$.*
- *If $c \in (\underline{c}, \overline{c})$, all extreme-$\omega$ banks enter, and only some of the middle-ω banks enter. When c increases, the fraction of middle-ω traders decreases, and the total measure of traders in the market decreases.*
- *If $c \geq \overline{c}$, only some of the extreme-ω and middle-ω banks enter. When c increases, the fraction of middle-ω traders, n, does not not change, but the total measure of traders in the market decreases.*

When c is small enough, $c < \underline{c}$, then all banks enter regardless of their size. When c is in the intermediate range $[\underline{c}, \overline{c}]$, then all extreme-$\omega$ banks and only large enough middle-ω banks enter. Indeed, extreme-ω banks have greater incentives to participate in the market. As c increases, fewer middle-ω banks enter, and the size of the market, as measured by the total measure of active traders, decreases. When c is large enough, then we have partial entry at all ω's. An increase in c reduces the size of the market but does not change its composition. This follows from the Pareto size distribution, which implies that, when c increases, the measures of middle- and extreme-ω traders are scaled down by the same constant.

Next, we turn to comparative statics with respect to k:

Proposition 19 (Changes in k)**.** *Changes in the line limit, k, have non-monotonic effects on intermediation activity:*

- *The measure of middle-ω traders, $\Psi\left[\Sigma(\frac{1}{2})\right]$, is a non-monotonic function of k. It increases with k when k is close to zero, and it goes to zero as $k \to 1$.*
- *The fraction of middle-ω traders, n, is positive when $k = 0$ and equal to zero when $k = 1$. It decreases with k for $k \simeq 0$ and $\simeq 1$, but can increase with k otherwise.*

An increase in k has two opposite effects on middle-ω banks' entry incentives. On the one hand, there is a positive partial equilibrium effect: when k is larger, each trader in a given bank can increase the size of its position and thus earn larger profits. But, on the other hand, there is a general equilibrium effect: risk sharing improves, which reduces intermediation profits. The first effect dominates when $k \simeq 0$, increasing the measure of middle-ω traders. But the second effect dominates when $k \simeq 1$, decreasing the measure of middle-ω traders. To understand the effects on the *fraction* of middle-ω traders, note that, when $k \simeq 0$, an increase in k causes both middle-ω and extreme-ω traders to enter. But extreme-ω traders enter more, resulting in a decrease in n. When $k \simeq 1$, the risk-sharing is almost perfect and so n decreases towards zero.

These comparative statics translate into predictions about the evolution of gross exposures and net exposures as frictions decrease.

Corollary 20 (Exposures)**.** *A reduction in frictions has the following effects on exposures:*

- *When c decreases, gross exposures per capita increase weakly, and the ratio of net to gross exposure decreases weakly.*
- *When k increases, $k \simeq 0$ or $k \simeq 1$, gross exposures increase, and the ratio of net to gross exposures increases.*

One sees that, in both cases, reducing frictions increases trading volume, in the sense of increasing gross notionals outstanding per capita. When c decreases, the increase in trading volume is due to an increase in intermediation. Middle-ω banks enter more, and the ratio of net to gross notional decreases. When k increases, by contrast, the increase in trading volume comes about because of larger customer-to-customer trades, and less intermediation, and the ratio of net to gross notional increases. Therefore, according to the model, the evolution of the net-to-gross notional ratio can help in telling apart a decrease in frictions due to an improvement in entry costs versus risk-management technologies.

A.10.3 Detailed calculations of entry incentives

Let us calculate $\Delta(0)$ when $k(1+n) \leq 1$:

$$\Delta(0) = \frac{\alpha \mathbb{V}[D]}{2}\left[g(0)^2 + k\left\{n\left(\tfrac{1}{2} - g(\omega)\right) + \frac{1-n}{2}\left(g(1) - g(0)\right)\right\}\right].$$

But $g(1) - g(0) = 1 - 2g(0) = 2\left(\frac{1}{2} - g(0)\right)$, and so:

$$\begin{aligned}\Delta(0) &= \frac{\alpha \mathbb{V}[D]}{2}\left[g(0)^2 + k\left(\tfrac{1}{2} - g(\omega)\right)\right] = \frac{\alpha \mathbb{V}[D]}{2}\left[\left(\frac{k(1+n)}{2}\right)^2 + k\left(\tfrac{1}{2} - \frac{k(1+n)}{2}\right)\right] \\ &= \frac{\alpha \mathbb{V}[D]\, k}{4}\left[1 - \frac{k\left(1-n^2\right)}{2}\right]\end{aligned}$$

When $k(1+n)>1$, $\Delta(0)$ is constant equal to $\alpha\mathbb{V}[D]\,k/8$. Similar calculations lead to:

$$\begin{aligned}\Delta(\tfrac{1}{2}) &= \frac{\alpha\mathbb{V}[D]\,k}{2}(1-n)\left[\tfrac{1}{2}-g(0)\right] = \frac{\alpha\mathbb{V}[D]\,k}{2}(1-n)\left(\tfrac{1}{2}-\frac{k(1+n)}{2}\right)\\ &= \frac{\alpha\mathbb{V}[D]\,k}{4}(1-n)\left[1-k(1+n)\right].\end{aligned}$$

When $k(1+n)>1$, $\Delta(\frac{1}{2})$ is constant equal to zero. The fixed point equation for n is then:

$$n = T[n] \quad \text{where} \quad T[n] = \frac{\frac{1}{3}\min\left\{\frac{\underline{S}\Delta(\frac{1}{2})}{c},1\right\}^{\theta}}{\frac{1}{3}\min\left\{\frac{\underline{S}\Delta(\frac{1}{2})}{c},1\right\}^{\theta}+\frac{2}{3}\min\left\{\frac{\underline{S}\Delta(0)}{c},1\right\}^{\theta}} = \frac{F(n)^{\theta}}{2+F(n)^{\theta}},$$

where $F(n)$ is defined in the text.

A.10.4 Proof of Proposition 17

The right-hand size of (28) is continuous, is positive at $n=0$ and equal to 0 at $n=1/k-1$. Thus, by the intermediate value theorem, a solution exists. To establish uniqueness it is enough to show that the right-hand side is non-increasing. We thus distinguish three cases. If both $\Sigma(0)$ and $\Sigma(\frac{1}{2})$ are lower than $\underline{S}$, then $F(n)=1$ and so the property holds. If $\Sigma(0)$ is lower than $\underline{S}$ but $\Sigma(\frac{1}{2})$ is greater than $\underline{S}$, then $F(n)=\underline{S}\Delta(\frac{1}{2})/c$, which is clearly decreasing in $n\in(0,1/k-1)$. If both $\Sigma(0)$ and $\Sigma(\frac{1}{2})$ are greater than $\underline{S}$, then

$$F(n)=\frac{(1-n)(1-k(1+n))}{1-k/2(1-n^2)} \Longrightarrow F'(n)=\frac{-1+k/2+k^2n^2/2+kn}{\left(1-k/2(1-n^2)\right)^2}.$$

Given that $n\in(0,1/k-1)$, we have $k<1/(1+n)$ and so the numerator in the expression of $F'(n)$ is less than:

$$-1+\frac{1}{2(1+n)}+\frac{n^2}{2(1+n)^2}+\frac{n}{1+n}=\frac{1}{2}\left(-\frac{1}{1+n}+\frac{n^2}{(1+n)^2}\right)<0$$

where the last inequality follows because $n^2<1$ and $1/(1+n)>1/(1+n)^2$. This establishes the claim.

A.10.5 Proof of Proposition 18

Given that $\Delta(0)>\Delta(\frac{1}{2})$ we have $\Sigma(0)<\Sigma(\frac{1}{2})$, and thus there are three cases to consider.

When all banks enter: $\Sigma(\frac{1}{2})\leq\underline{S}$. If all banks enter then $n=\frac{1}{3}$ and both $\Sigma(0)$ and $\Sigma(\frac{1}{2})$ are less than $\underline{S}$. In particular, $\Sigma(\frac{1}{2})<\underline{S}$ can be written, given $n=\frac{1}{3}$:

$$c\leq\frac{\alpha\underline{S}\mathbb{V}[D]}{4}\frac{2k}{3}\left(1-\frac{4k}{3}\right)=\underline{c}.$$

Conversely, if this condition is satisfied, $n=\frac{1}{3}$ solves the equilibrium equation (28). Clearly, in that case, the measure of traders in the economy is constant.

When only some extreme- and middle-ω banks enter: $\Sigma(0) \geq \underline{S}$. Then, the fixed point equation, and thus n, does not depend on c. Let $\underline{n}$ denote the solution of the fixed point equation in this case and let

$$\overline{c} \equiv \underline{S}\Delta(0, \underline{n}) = \frac{\alpha \mathbb{V}[D]\, k}{4}\left[1 - \frac{k(1-\underline{n}^2)}{2}\right]\underline{S}.$$

In words, $\overline{c}$ is the cost threshold such that $\Sigma(0) = \underline{S}$, given that $n = \underline{n}$. One can verify that $\overline{c} > \underline{c}$. Clearly, if $\Sigma(0) > \underline{S}$, then $c \geq \overline{c}$. Conversely, if $c \geq \overline{c}$, then $n = \underline{n}$ solves the equilibrium fixed point equation. When $c \geq \overline{c}$, the threshold $\Sigma(0)$ and $\Sigma(\frac{1}{2})$ are increasing in c, and so the total measure of traders in the economy is decreasing in c.

When all extreme-ω banks but only some the middle-ω banks enter: when $\Sigma(0) < \underline{S}$ but $\Sigma(\frac{1}{2}) > \underline{S}$. From the previous two paragraphs, this occurs if and only if $c \in (\underline{c}, \overline{c})$. Because $\Sigma(0) < \underline{S}$, all extreme-$\omega$ traders enter: the measure of extreme-ω traders is thus equal to $2/3$ and does not change with c. Together with the fact that the *fraction* of middle-ω traders, n, is decreasing in c, this implies that the *measure* of middle-ω traders is decreasing in c. Thus, the total measure of traders is decreasing in c.

A.10.6 Proof of Proposition 19

We first establish a number of preliminary results, and then turn to the results stated in the proposition.

Lemma 10. *When $\Sigma(0) > \underline{S}$, the fraction of middle-ω traders decreases with k.*

When $\Sigma(0) > \underline{S}$,

$$F(n,k) = \frac{(1-n)\left[1-k(1+n)\right]}{1-k/2(1-n^2)} \Longrightarrow \frac{\partial F}{\partial k} = -\frac{1-n}{\left[1-k/2(1-n^2)\right]^2}\left\{\frac{1}{2} + n + \frac{n^2}{2}\right\} < 0.$$

But we already know that $\partial F/\partial n < 0$, from the proof of Proposition 17, in Section A.10.4. Thus, the equilibrium fixed-point equation, (28), is increasing in both n and k, establishing the claim.

Lemma 11. *When $\Sigma(\frac{1}{2}) > \underline{S} > \Sigma(0)$, the fraction of middle-ω traders, n, increases in k if $k(1+n) < \frac{1}{2}$, and decreases in k if $k(1+n) > \frac{1}{2}$.*

When $\Sigma(\frac{1}{2}) > \underline{S} > \Sigma(0)$,

$$F(n,k) = \frac{\underline{S}\alpha \mathbb{V}[D]}{4c}\left[1-n\right] \times k \times \left[1-k(1+n)\right].$$

It thus follows that $F(n,k)$ increases with k when $k(1+n) < \frac{1}{2}$ and decreases with k when $k(1+n) > \frac{1}{2}$. But we already know from the proof of Proposition 17, in Section A.10.4, that $\partial F/\partial n < 0$, establishing the claim.[27]

[27] One can easily find parameter values such that, in equilibrium, either $k(1+n) < \frac{1}{2}$ or $k(1+n) > \frac{1}{2}$. To find parameter values such that $k(1+n) < \frac{1}{2}$, one first picks some k such that $k(1+\frac{1}{3}) < \frac{1}{2}$. Then, given this k, one picks $c \in (\underline{c}, \overline{c})$, as constructed in the proof of Proposition 18, so that $\Sigma(\frac{1}{2}) > \underline{S} > \Sigma(0)$. But then we know that the equilibrium n has to be less than $\frac{1}{3}$, and so it follows from $k(1+\frac{1}{3}) < \frac{1}{2}$ that $k(1+n) < \frac{1}{2}$ as well. Likewise, if $k > \frac{1}{2}$ and $c \in (\underline{c}, \overline{c})$, then $k(1+n) > \frac{1}{2}$ and n decreases with k.

Lemma 12. *When $k \simeq 0$, the fraction of middle-ω traders, n, decreases with k, and entry incentives for all banks, $\Delta(0)$ and $\Delta(\frac{1}{2})$, both increase with k.*

Note that $\Delta(0)$ and $\Delta(\frac{1}{2})$ are less than $\alpha\mathbb{V}[D]\,k/4$, which goes to zero when k goes to zero. Therefore, for k sufficiently small, we know that both $\Sigma(0)$ and $\Sigma(\frac{1}{2})$ must be greater than $\underline{S}$, and the first part of the result follows from Lemma 10 that n is a decreasing function of k. Let $n(0)$ denote the limit of n as $k \to 0$. By continuity, $n(0)$ must solve the fixed point equation

$$n - \frac{F(n,0)^\theta}{2 + F(n,0)^\theta} = 0 \Leftrightarrow n - (1-n)^\theta = 0, \text{ where } F(n,k) = \frac{(1-n)(1-k(1+n))}{1 - k/2(1-n^2)}$$

In particular, $n(0) \in (0,1)$. Given that $\partial F(n,k)/\partial n \leq 0$, the partial derivative of the above equation with respect to n must be larger than one. We can thus apply the implicit function theorem and find that n is continuously differentiable with respect to k in a neighborhood of $k = 0$, with bounded derivatives. The second part of the result then follows by differentiating $\Delta(0)$ and $\Delta(\frac{1}{2})$ with respect to k.

Lemma 13. *When $k \simeq 1$, the fraction of middle-ω traders can be written as $n(k) = \max\{n_1(k), n_2(k)\}$, where both $n_1(k)$ and $n_2(k)$ are continuously differentiable and satisfy $n_1(1) = n_2(1) = n_1'(1) = n_2'(1) = 0$.*

For this Lemma we start by noting that since $n \in (0, 1 - 1/k)$, we must have that $n \to 0$ as $k \to 1$. It then follows that $k(1+n) \to 1$, $\underline{S}\Delta(\frac{1}{2})/c \to 0$, and that n must solve either one of the following two equations:

$$n - \frac{F_1(n,k)^\theta}{2 + F_1(n,k)^\theta} = 0, \quad \text{where } F_1(n,k) = \frac{\underline{S}\Delta(\frac{1}{2})}{c} = \frac{\alpha\mathbb{V}[D]}{4}[1-n][1-k(1+n)] \tag{29}$$

$$n - \frac{F_2(n,k)^\theta}{2 + F_2(n,k)^\theta} = 0, \quad \text{where } F_2(n,k) = \frac{\frac{\underline{S}\Delta(\frac{1}{2})}{c}}{\frac{\underline{S}\Delta(0)}{c}} = \frac{[1-n][1-k(1+n)]}{1 - \frac{k}{2}(1-n^2)}. \tag{30}$$

The same calculations as in the proof of Proposition 17, in Section A.10.4, show that each of these equation has a unique solution in $(0, 1-1/k)$, which we denote by $n_1(k)$ and $n_2(k)$ respectively. Clearly, both $n_1(k)$ and $n_2(k)$ go to zero as k goes to one. Moreover the implicit function theorem applies to both equations at $(n,k) = (0,1)$ since both are continuously differentiable in (n,k), with a partial derivative with respect to n that is greater than one. Moreover, since $F_1(0,1) = F_2(0,1) = 0$ and $\theta > 1$, it follows that the partial derivative with respect to k is zero at $(n,k) = (0,1)$. Therefore, $n_1'(1) = n_2'(1) = 0$.

The last thing to show is that $n(k) = \max\{n_1(k), n_2(k)\}$. To see this, note that if $n = n_1(k)$, then it must be the case that $\underline{S}\Delta(0)/c \geq 1$, from which it follows that $F_1(n_1(k),k) \geq F_2(n_1(k),k)$, and so that $n_1(k) \geq n_2(k)$. Similarly, when $n = n_2(k)$, we have that $\underline{S}\Delta(0)/c \leq 1$, that $F_2(n_2(k),k) \geq F_1(n_2(k),k)$, and so $n_2(k) \geq n_1(k)$.

Proof of the proposition. For the first statement of the first bullet point, recall Lemma 12: when k is close to zero, $\Delta(\frac{1}{2})$ is an increasing function of k and $\Sigma(\frac{1}{2}) > \underline{S}$. It thus follows that $\Psi\left[\Sigma(\frac{1}{2})\right]$ is increasing in k. For the second statement of the first bullet point, recall Lemma 13: when $k \to 1$, n goes to zero. At the same time, the total measure of traders in the market cannot exceed one, and so $\Psi\left[\Sigma(\frac{1}{2})\right] \leq n$. The second bullet point follows directly from Lemma 11 and Lemma 10.

A.10.7 Proof of Corollary 20

Gross exposures per capita, $G(n,k)$, are increasing in $n \in [0, \frac{1}{3}]$, and the net-to-gross notional ratio, $\rho(n)$, is decreasing in n. Since n is non-increasing in c, the first bullet point follows.

For the second bullet point, consider first gross exposures. The result when $k \simeq 0$ follows by applying the same argument as in the proof of Lemma 12. When $k \simeq 1$, the argument follows by first noting that, since $G(n,k)$ is increasing in $n \in (0, \frac{1}{3})$, we can write $G(n(k),k) = \max\{G(n_1(k),k), G(n_2(k),k)\}$. Using that the $n_i(k)$'s are continuously differentiable with $n_i(1) = n_i'(1) = 0$, one obtains that $G(n_i(k),k)$ is increasing for $k \simeq 1$, and so is $G(n(k),k)$. The result in the second bullet point about the net-to-gross notional ratio, $\rho(n(k))$, follows because, on the one hand, $\rho(n)$ is a decreasing function of n and, from Proposition 19, $n(k)$ is decreasing in k for $k \simeq 0$ and $k \simeq 1$.

A.11 Proof of Proposition 11

We study each terms of $W(\varepsilon, \delta)$ in turns. To ease exposition, we start with the third term, then move on to the first term, and finally study the second term.

Derivative of the third term. Note that $\Sigma_\varepsilon(\omega) = \Psi^{-1}\left[Mn(\omega) + \varepsilon\delta(\omega)\right]$ and that the derivative of $\Phi \circ \Psi^{-1}[x]$ is $1/\Psi^{-1}[x]$. Therefore, the derivative of the third term is:

$$-c \int_0^1 \frac{\delta(\omega)}{\Sigma(\omega)}\, d\omega.$$

Derivative of the first term. The derivative of the first term is, clearly:

$$\int_0^1 \delta(\omega)\Gamma\left[\omega\right]\, d\omega.$$

Derivative of the second term. For this define $\bar{\delta} \equiv \int_0^1 \delta(\omega)\, d\omega$ and let

$$n_\varepsilon(\omega) \equiv \frac{Mn(\omega) + \varepsilon\delta(\omega)}{M + \varepsilon\bar{\delta}}.$$

Recall that, conditional on entry, the equilibrium allocation of risk is efficient. That is, the second term can be written:

$$-\left(M + \varepsilon\bar{\delta}\right) \times \inf_{\gamma(\omega,\tilde{\omega})} \int_0^1 n_\varepsilon(\omega)\Gamma\left[\omega + \int_0^1 \gamma(\omega,\tilde{\omega})\, n_\varepsilon(\tilde{\omega}) d\tilde{\omega}\right] d\omega,$$

By the envelope theorem (see paragraph below for the detailed formal argument), the derivative of "inf" is equal to the partial derivative of the objective with respect to ε evaluated at the optimal $\gamma(\omega,\tilde{\omega})$. Given that $\frac{\partial n_\varepsilon(\omega)}{\partial \varepsilon}\big|_{\varepsilon=0} = \frac{\delta(\omega) - n(\omega)\bar{\delta}}{M}$, the derivative of the third term is:

$$\begin{aligned}
&-\bar{\delta}\int_0^1 n(\omega)\Gamma\left[g(\omega)\right]\, d\omega - \int_0^1 \left(\delta(\omega) - n(\omega)\bar{\delta}\right)\Gamma\left[g(\omega)\right]\, d\omega \\
&-\int_0^1 n(\omega)\Gamma'\left[g(\omega)\right]\int_0^1 \gamma(\omega,\tilde{\omega})\left(\delta(\tilde{\omega}) - \bar{\delta}n(\tilde{\omega})\right) d\tilde{\omega}d\omega.
\end{aligned}$$

Changing the order of integration and using $\gamma(\omega,\tilde{\omega}) = -\gamma(\tilde{\omega},\omega)$ we obtain that the term on the second line above is:

$$\begin{aligned}&-\int_0^1 n(\omega)\Gamma'\left[g(\omega)\right]\int_0^1 \gamma(\omega,\tilde{\omega})\big(\delta(\tilde{\omega})-\bar{\delta}n(\tilde{\omega})\big)\,d\tilde{\omega}\,d\omega\\ =&\int_0^1 \big(\delta(\omega)-\bar{\delta}n(\tilde{\omega})\big)\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\,d\omega\end{aligned}$$

Some more algebra. Collecting the derivatives of the first, second, and third term we obtain:

$$W'(0,\delta) = \int_0^1 \delta(\omega)\bigg(-\frac{c}{\Sigma(\omega)}+\Gamma\left[\omega\right]-\Gamma\left[g(\omega)\right]+\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\bigg)\,d\omega \tag{31}$$

$$-\bar{\delta}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})\,d\omega d\tilde{\omega}. \tag{32}$$

Just as in the calculation of entry incentives, in Section 6.1, we add and subtract $\int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega} = \Gamma'\left[g(\omega)\right]\big(g(\omega)-\omega\big)$ to the integral in the first line, (31). We obtain that this integral is equal to:

$$\int_0^1 \delta(\omega)\bigg(-\frac{c}{\Sigma(\omega)}+K(\omega)+F(\omega)\bigg)\,d\omega.$$

Now moving on to the second line, (32), we note that:

$$\begin{aligned}&\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega\\ =&\frac{1}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega+\frac{1}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega\\ =&\frac{1}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega-\frac{1}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\tilde{\omega},\omega)n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega\\ =&\frac{1}{2}\int_0^1\int_0^1 \Gamma'\left[g(\tilde{\omega})\right]\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega-\frac{1}{2}\int_0^1\int_0^1 \Gamma'\left[g(\omega)\right]\gamma(\omega,\tilde{\omega})n(\omega)n(\tilde{\omega})\,d\omega d\tilde{\omega}\\ =&\frac{1}{2}\int_0^1\int_0^1 \Big(\Gamma'\left[g(\tilde{\omega})\right]-\Gamma'\left[g(\omega)\right]\Big)\gamma(\omega,\tilde{\omega})n(\tilde{\omega})n(\omega)\,d\tilde{\omega}d\omega=\frac{1}{2}\int_0^1 F(\omega)n(\omega)\,d\omega.\end{aligned}$$

where: the first equality follows trivially; the second equality follows from $\gamma(\tilde{\omega},\omega) = -\gamma(\omega,\tilde{\omega})$; the third equality from relabeling ω by $\tilde{\omega}$ and vice versa; and the last line by collecting terms, using (11), as well as the definition of the frictional surplus. Collecting all terms, and using that $\Delta(\omega) = K(\omega)+\frac{1}{2}F(\omega)$, we arrive at the formula of the proposition.

The formal application of the envelope theorem. Consider the optimization problem

$$\mathcal{K}(\varepsilon) = \min_{\gamma(\omega,\tilde{\omega})}\int_0^1 \phi(\omega,\gamma,\varepsilon)\,d\omega, \tag{33}$$

subject to $\gamma(\omega,\tilde{\omega})+\gamma(\tilde{\omega},\omega)=0$ and $\gamma(\omega,\tilde{\omega})\in[-k,k]$ and where:

$$\begin{aligned}\phi(\omega,\gamma,\varepsilon) &\equiv n_\varepsilon(\omega)\Gamma\left[\omega+\int_0^1\gamma(\omega,\tilde{\omega})\,n_\varepsilon(\tilde{\omega})d\tilde{\omega}\right]d\omega\\ &=\frac{Mn(\omega)+\varepsilon\delta(\omega)}{M+\varepsilon\bar{\delta}}\Gamma\left[\omega+\frac{M}{M+\varepsilon\bar{\delta}}\int_0^1\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}+\frac{\varepsilon}{M+\varepsilon\bar{\delta}}\int_0^1\gamma(\omega,\tilde{\omega})\delta(\tilde{\omega})\,d\tilde{\omega}\right].\end{aligned}$$

Clearly, both $\frac{\partial\phi}{\partial\varepsilon}$ and $\frac{\partial^2\phi}{\partial\varepsilon^2}$ exist. Moreover, since because $\gamma(\omega,\tilde{\omega})$, $n(\tilde{\omega})$ and $\delta(\tilde{\omega})$ are bounded, these derivatives are bounded uniformly in $(\omega,\varepsilon)\in[0,1]^2$. Therefore, using Theorem 9.42 in Rudin (1953), we obtain $\frac{\partial f}{\partial\varepsilon}$ by differentiating $\int_0^1\phi(\omega,\gamma,\varepsilon)\,d\omega$ under the integral sign. Moreover, since $\frac{\partial^2\phi}{\partial\varepsilon^2}$ is bounded uniformly in (ω,ε), it follows that $\frac{\partial\phi}{\partial\varepsilon}$ and thus $\frac{\partial\phi}{\partial\varepsilon}$ is Lipchitz with respect to ε, with a Lipchitz coefficient that is independent from ω. This allows us to apply Theorem 3 in Milgrom and Segal (2002) and assert that:

$$\mathcal{K}'(0)=\lim_{\varepsilon\to0}\int_0^1\frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma_\varepsilon,0)\,d\omega,$$

where γ_ε is, for each ε, a solution of the minimization problem (33). All we need to show is, therefore, that

$$\lim_{\varepsilon\to0}\int_0^1\frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma_\varepsilon,0)\,d\omega=\int_0^1\frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma,0)\,d\omega,$$

where γ is a solution of the minimization problem when $\varepsilon=0$. To that end we first take derivatives with respect to ε:

$$\begin{aligned}M\int_0^1\frac{\partial\phi}{\partial\varepsilon}(\omega,\gamma_\varepsilon,\varepsilon)\,d\omega&=\int_0^1\left[\delta(\omega)-\delta n(\omega)\right]\Gamma\left[\tilde{g}_\varepsilon(\omega)\right]\,d\omega\\ &+\int_0^1 n(\omega)\Gamma'\left[\tilde{g}_\varepsilon(\omega)\right]\left(-\delta\int_0^1\gamma_\varepsilon(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}+\int_0^1\gamma_\varepsilon(\omega,\tilde{\omega})\delta(\tilde{\omega})\,d\tilde{\omega}\right)d\omega,\end{aligned}$$

where $\tilde{g}_\varepsilon(\omega)\equiv\omega+\int_0^1\gamma_\varepsilon(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}$ is the post-trade exposure generated by $\gamma_\varepsilon(\omega,\tilde{\omega})$ if the underlying distribution of traders is $n(\omega)$. Given that $n_\varepsilon(\omega)\to n(\omega)$ uniformly, we know from the proof of Lemma 5 that $\tilde{g}_\varepsilon(\omega)\to g(\omega)$ uniformly. Given the definition of $\tilde{g}_\varepsilon(\omega)$ this also implies that $\int_0^1\gamma_\varepsilon(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}=\tilde{g}_\varepsilon(\omega)-\omega\to g(\omega)-\omega=\int_0^1\gamma(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}$ uniformly. This implies that all terms except perhaps the last one converge to their $\varepsilon=0$ counterparts. To show that the last term converges as well, rewrite it as:

$$\begin{aligned}&\int_0^1\int_0^1 n(\omega)\Gamma'\left[\tilde{g}_\varepsilon(\omega)\right]\gamma_\varepsilon(\omega,\tilde{\omega})\delta(\tilde{\omega})\,d\tilde{\omega}\,d\omega\\ =&\int_0^1\int_0^1 n(\omega)\Big(\Gamma'\left[\tilde{g}_\varepsilon(\omega)\right]-\Gamma'\left[\tilde{g}_\varepsilon(\tilde{\omega})\right]\Big)\gamma_\varepsilon(\omega,\tilde{\omega})\delta(\tilde{\omega})\,d\tilde{\omega}\,d\omega+\int_0^1\int_0^1 n(\omega)\Gamma'\left[\tilde{g}_\varepsilon(\tilde{\omega})\right]\gamma_\varepsilon(\omega,\tilde{\omega})\delta(\tilde{\omega})\,d\tilde{\omega}\,d\omega\\ =&-\int_0^1\int_0^1 n(\omega)\Big|\Gamma'\left[\tilde{g}_\varepsilon(\omega)\right]-\Gamma'\left[\tilde{g}_\varepsilon(\tilde{\omega})\right]\Big|k\delta(\tilde{\omega})\,d\tilde{\omega}\,d\omega-\int_0^1\Gamma'\left[\tilde{g}_\varepsilon(\omega)\right]\delta(\omega)\int_0^1\gamma_\varepsilon(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}\,d\omega,\end{aligned}$$

where the second line follows from subtracting and adding $\Gamma'\left[g_\varepsilon(\tilde{\omega})\right]$, and the third line follows, for the first term, from the optimality condition (11) and, for the second term, from the fact $\gamma(\omega,\tilde{\omega})=-\gamma(\tilde{\omega},\omega)$ and by switching integrating variables. The result follows because, as noted

before, both $\tilde{g}_\varepsilon(\tilde{\omega})$ and $\int_0^1 \gamma_\varepsilon(\omega,\tilde{\omega})n(\tilde{\omega})\,d\tilde{\omega}$ converge uniformly to their $\varepsilon = 0$ counterparts. □

A.12 Equilibrium market size is socially optimal given $n(\omega)$

Go back to the beginning of Section 7.1 and let the planner choose the number M of traders in the OTC market, holding market composition $n(\omega)$ the same. The corresponding entry threshold is $\Sigma_M(\omega)$ and solves $\Sigma_M(\omega) = \Psi^{-1}[Mn(\omega)]$. Moreover the threshold $\Sigma_M(\omega)$ is decreasing in M because $\Psi[S]$ is decreasing in M. Note as well that there is a maximum market size, $\bar{M}$, consistent with $n(\omega)$. The maximum size $\bar{M}$ is attained when the threshold reaches zero for some ω. When M rises above $\bar{M}$, it is no longer possible to increase entry at the extreme and so it is no longer possible to keep market composition the same. Given that the threshold is lowest at $\omega = 0$, $\bar{M}$ solves $\Psi[0] = \bar{M}n(0)$.

For $M \in (0, \bar{M}]$ the derivative of the welfare function with respect to M is given by the directional derivative of Proposition 11, evaluated at $\Sigma_M(\omega)$ and $\delta(\omega) = n(\omega)$:

$$\int_0^1 n(\omega)\left\{\Gamma[g(\omega)] - \Gamma[\omega] - \frac{c}{\Sigma_M(\omega)}\right\} d\omega.$$

Note that $g(\omega)$ only depends on $n(\omega)$ and, therefore, does not depend on M. Therefore, the directional derivative only depends on M through $\Sigma_M(\omega)$. Given that $\Sigma_M(\omega)$ is a decreasing function of M, the derivative is decreasing, the planning problem is strictly concave in M, and the first-order condition for optimality is sufficient.