Financial Frictions in Production Networks*

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April 26, 2016

Abstract

We study how an economy's production structure determines the response of aggregate output and employment to sectoral financial shocks. In our framework, economic production is organized in an input-output network in which firms face financial constraints on their working capital. We show how sectoral financial shocks propagate through the network and manifest at the aggregate level through two channels: a fall in total factor productivity and an aggregate labor wedge distortion. The strength of each channel depends on the overall network architecture and the location of shocks. Finally, we calibrate our model to the U.S. input-output tables and use it to quantitatively assess the role of the network multiplier within the context of the recent Financial Crisis and the Great Recession.

Keywords: Production Networks, Financial Frictions, Business Cycles

JEL classification: C67, G10, E42

*We thank Daron Acemoglu, Marios Angeletos, Enghin Atalay, Vasco Carvalho, Gabriel Chodorow-Reich, Andrew Foerster, Mark Gertler, Michael Gofman, Bob Hall, James Hamilton, Pete Klenow, Ricardo Lagos, Ezra Oberfield, Valerie Ramey, Ricardo Reis, Alireza Tahbaz-Salehi, Pierre-Daniel Sarte, and Randy Wright for their valuable feedback and suggestions. We owe special thanks to V.V. Chari, Ian Dew-Becker, Chad Jones, Marco Ortiz, and Alberto Trejos for their insightful conference discussions of our paper. Finally, this paper benefitted greatly from the constructive comments of colleagues and seminar participants at Columbia, the Federal Reserve Bank of Richmond, UCSD, USC, the Paris School of Economics, Ente Einaudi, and Cambridge University, as well as the helpful feedback of organizers and participants of the following conferences: the Chicago Booth Junior Finance Symposium, the Central Bank of Peru 30th Research Meeting of Economists, the 7th Annual NBER Conference on Macroeconomics Across Time and Space hosted by the Federal Reserve Bank of Philadelphia, the 2013 SED meetings, the 2013 NBER Summer Institute EFG Meeting, the 2014 AEA meetings, the Wisconsin School of Business 5th Annual Conference on Money, Banking and Asset Markets, the 2015 Tepper-LAEF Macro Finance Conference, and the 2016 LAEF Pecuniary Externalities Conference.

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1 Introduction

Since the onset of the Great Recession, financial frictions have been reinstated at the forefront of business cycle research. A predominant view suggests that the tightening of credit to firms following the 2007-2008 Financial Crisis was a main contributor to the recession, the most significant and prolonged contraction since the Great Depression. Yet, the precise mechanics of how financial shocks ultimately affect aggregate output and employment are not fully understood. This paper is a qualitative and quantitative study of how financial shocks to firms are transmitted through input-output linkages when the economy's production is organized in a complex network.

Glancing downward from an airplane window, anyone can appreciate the outstanding complexity involved in economic production. Warehouses, silos, loading levers, etc., adjoin manufacturing and energy plants. A grid of highways and railroads connects producers, allowing materials to flow from one production site to another. Through this process, labor, capital, and intermediate goods are eventually transformed into final products, sold by retailers to their ultimate consumers. All in all, the economy's production of gross domestic product constitutes an amazingly sophisticated yet harmonious system.

Most of modern macroeconomics abstracts away from this network complexity. With perfectly competitive markets and the absence of frictions, standard models wrap all of the intermediate steps and intricate details of the production process into one representative firm. We are thus accustomed to neglect the richness of the economy's production structure. However, given the vast amount of interfirm and intersectoral trade observed in the world, it is difficult to imagine that the organization of the economy's production does not matter—particularly when firms in the economy are subject to financial constraints.

Within the representative firm framework, macro models that associate output losses to events in financial markets face quantitative challenges. First, when constraints are placed only on the funding of investment, these models often struggle to deliver significant output fluctuations.[1] Furthermore, there is evidence that in the aggregate, non-financial corporations can finance their capital expenditures entirely from retained earnings and dividends alone. This suggests that perhaps only a fraction of firms may be constrained in accessing credit.[2] Thus, viewing these events through the lens of a representative firm model may lead to the potential conclusion that financial constraints can play only a minor role in explaining business cycle fluctuations.[3]

Our paper investigates whether introducing working capital constraints on firms' purchases of intermediate goods and input-output linkages across these firms (or sectors) can help resolve these challenges. There are reasons to believe that a model with input-output linkages would be desirable. When firms are interconnected in a production network, they may require funding to purchase not

[1] See e.g. Kocherlakota (2000) and Cordoba and Ripoll (2004).

[2] See Chari et al. (2008), Kahle and Stulz (2013), and Shourideh and Zetlin-Jones (2012).

[3] Typically, quantitative models that have been successful in delivering large output fluctuations from financial shocks do so only by adding more frictions such as nominal rigidities, see e.g., Bernanke et al. (1999).

only primary inputs, like capital and labor, but also intermediate good inputs. This implies that firms may be constrained even if they appear to have enough internal funds to cover their investment and labor expenditures. More importantly, though, financial constraints that affect only a small fraction of firms in the economy may have disproportionately large effects on the aggregate economy.

To understand why, consider a series of firms connected via a supply chain: each firm produces an intermediate good which is then sold to the subsequent firm along the chain. If one firm along that chain becomes financially constrained, it is forced to invest less, hire less labor, and purchase less intermediate goods relative to its unconstrained optimum. As the constraint on this firm tightens, it reduces its output and as a result the price of its good rises. This price hike leads to a reduction in the intermediate good demand of the subsequent firm along the chain. As a result, that firm's output also falls and the price of its good rises. But this in turn leads to a reduction in the intermediate good demand of the following firm, and so on. As a result, the aggregate effect of the financial shock is amplified as it propagates along the supply chain.[4]

More generally, in an economy with interconnected production units, a negative financial shock that hits only a subset of firms may adversely affect not only the constrained firms' output but also the output of their trading partners, the output of their trading partners' partners, and so on. The production network can thereby generate a multiplier which magnifies the aggregate effects of financial shocks.

This paper analyzes this mechanism both qualitatively and quantitatively. In so doing, we seek to answer the following questions: can financial shocks that disrupt a small number of sectors lead to an aggregate economic recession? By how much are financial shocks amplified by input-output linkages—in particular, how does the multiplier described above depend on the particular network structure of the economy? How does the propagation of financial shocks differ from the propagation of productivity shocks? Given the U.S. production structure, which sectors are the most vulnerable within the network and which create the most vulnerabilities? In sum, we hope to provide a novel explanation for why firms at the micro level may face seemingly harmless financial distortions, yet at the macro level the economy experiences a large recession.

Our approach. Our approach to answering these questions consists of building on the static multi-sector network model of Acemoglu et al. (2012) to incorporate financial constraints.[5] In this framework, firms within a productive sector operate a Cobb-Douglass technology which uses as inputs labor supplied by the household and intermediate goods produced by other sectors. The matrix of cross-sector input requirements defines the input-output production network.

Within this multi-sector input-output economy we introduce financial frictions that constrain the

[4] At the micro level, note that the subsequent firms along the chain experience only an increase in the price of their intermediate goods which induce them to demand less inputs. Thus, these firms perceive only supply-chain problems, not financial problems. Though there is some evidence of a large credit-crunch during the Great Recession, see e.g., Campello et al. (2010) or Chodorow-Reich (2014), conflicting survey data reveals that only a small portion of firms found funding costs to be among their major concerns. Our model could thus be consistent with the latter.

[5] Acemoglu et al. (2012) itself builds on the seminal multi-sector model of Long and Plosser (1983).

working capital of firms. That is, we impose a simple pledgeability constraint in which firms must pledge a fraction of their revenue in order to finance their inputs–both labor and intermediate goods. This constraint creates a distortionary wedge between the firm's marginal revenue and marginal cost. In more microfounded versions, we show that this wedge may be derived alternatively from interest rates on working capital or enforcement constraints. Thus, although our baseline approach is a bit reduced-form, it serves as a valuable benchmark for distilling the effects of financial shocks in a production network. We then consider the general equilibrium of this economy in which a household consumes a basket of final goods and makes an endogenous labor supply decision.

Given our model, we develop a solution method for characterizing the equilibrium allocations and prices within this general multi-sector network economy with distortionary wedges. We first show that this economy aggregates to yield a Cobb-Douglas aggregate production function which maps aggregate labor supply into aggregate value added, i.e. GDP. We then show how the aggregate effects of sectoral financial shocks can be decomposed into two channels whose strengths depend on the overall network architecture and the location of shocks.

The first channel through which financial frictions affect aggregate output is through a multiplicative factor on the aggregate production function. More specifically, financial frictions manifest themselves at the aggregate level as a negative shock to Total Factor Productivity (TFP), or what some refer to as the *efficiency wedge*.[6] The second channel through which financial frictions affect aggregate output is by inducing a distortionary wedge between the household's marginal rate of substitution between consumption and labor and the economy's marginal rate of transformation. That is, in addition to the efficiency wedge, financial frictions manifest themselves at the macro level as an aggregate *labor wedge*.

Our decomposition of the aggregate effects of financial frictions into these two channels provides insight along a few important dimensions. First, it allows us to showcase the disparity between financial frictions and sectoral productivity shocks in terms of their aggregate implications. Second, it permits us to analyze how the overall network structure amplifies financial shocks along these two channels. In particular, moving away from our general network framework, we apply our equilibrium characterization and decomposition to several extremely simple and specific network economies. Through these examples we illustrate how the efficiency wedge and the labor wedge respond differentially to financial shocks and how the strength of these responses are governed by the economy's particular input-output architecture.

We first study two stark and polar opposite cases: the vertical economy and the horizontal economy. In the *vertical economy*, production is organized in a vertical supply chain as described above. The first sector in the chain hires labor to produce a good which is sold to the second sector. That sector uses this intermediate good as an input to produce a good which is sold to the following sector, and so on. The final sector produces a good which is sold to the household. This pure vertical structure is similar to the supply chain economies seen in Kiyotaki and Moore

[6] See e.g. Chari et al. (2007).

(1997a) and Kim and Shin (2012).[7] In this example, we find that financial frictions *do not* induce an aggregate efficiency wedge. That is, no matter the severity of the financial constraints, labor cannot be misallocated—due to the pure vertical structure, there is only one route through which labor is transformed into the final good. However, we show that financial frictions do generate an aggregate labor wedge in this economy: as financial constraints tighten, the real wage must fall in order to clear the labor market, thereby inducing a distortionary wedge between the marginal product of labor and the household's marginal rate of substitution.

In our second example, we examine the pure *horizontal economy.* In this economy, firms produce in isolation: they each hire labor to produce final goods which are consolidated into the household's consumption basket. This horizontal structure is typical of standard macroeconomic models, e.g. Dixit Stiglitz economies. We find that in this economy asymmetric financial shocks lead to misallocation of labor across sectors thereby reducing aggregate TFP. However, a common financial shock does not distort the relative use of hours across sectors, and hence does not generate an efficiency wedge. Finally, in either case, sectoral shocks contribute additively to the labor wedge.

Following these two examples, we explore two more economies which are particular hybrids of the vertical and horizontal economies, both of which yield further insights as to how the network structure dictates the response of aggregate variables to underlying financial shocks.

Quantitative Analysis. Finally, after developing our theoretical framework and results, we apply our model to a particular network structure: that of the U.S. economy. We calibrate the production network to the U.S. input-output tables provided by the Bureau of Economic Analysis (BEA). We then quantitatively assess our mechanism within the context of the 2007-2008 Financial Crisis and Great Recession.

First, given our calibration of the movement in financial constraints during this period, our model predicts a fall in aggregate output from 2007 to 2009 of around 4% to 28%, with 4% representing our lower bound estimate and 28% representing our upper bound. When we moreover decompose the fall in output across the two aggregate wedges, we find that around 20% of the fall is due to movement in the efficiency wedge while 80% results from the endogenous response in labor supply.

Our model's predicted fall in output of 4-28% is in stark contrast to an alternative economy in which sectors are configured in a purely horizontal structure with zero interaction with one another. In this alternative horizontal economy, our model predicts a smaller fall in output of around 2-4.5%. We call the ratio of the response of output in our benchmark U.S. input-output network to the response of output in the horizontal economy the *network liquidity multiplier*. We thus obtain a network liquidity multiplier of 1.8 to 6.3. This is our quantitative measure of the amplification resulting from the network structure.

Our quantitative analysis furthermore allows us to take a closer look at the U.S. input-output structure itself to understand the role played by individual sectors. In particular, we find that the

[7] See also the work of Gofman (2015) and Kalemli-Ozcan et al. (2014) which build on the Kim and Shin (2012) model.

most vulnerable sectors are those that serve as the primary suppliers to manufacturing industries: metal products, chemical products, fabricated metal, hydrocarbons, and the like. On the other hand, the most influential sectors are those closest to final goods production, and in particular, the automobile industry.

Finally, while the simplicity of our environment is certainly advantageous, we acknowledge that it poses certain limitations to our quantitative analysis. First, our model restricts production to be Cobb-Douglas and hence imposes an elasticity of substitution of one across all intermediate good inputs. A second limiting feature is that of a completely mobile labor supply: labor can be immediately reallocated across sectors without any search or adjustment costs, e.g. training or education. Together these yield a high degree of substitution across sectors thereby dampening the aggregate response to sectoral shocks. An extension of our model that relaxes either assumption would most likely amplify financial shocks beyond what we find in this paper.[8] Despite these limitations, we believe our paper makes a first step towards understanding both the qualitative and quantitative implications of financial frictions in production network economies.

Related Literature

Our paper is related to three broad strands of macroeconomic research: the literature on the aggregate implications of financial frictions, the literature on intersectoral production networks, and the literature on the macro effects of sectoral (or firm-level) distortions. In what follows we detail how our work complements and contributes to each of these literatures.

Financial Frictions in Macroeconomics. The pioneering models of financial frictions in macroeconomics are those of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997b). This work, and much of the literature that followed, consider representative firm environments in which financial frictions distort the firm's investment decision only.[9] However, a common finding is that these models are unable to generate strong responses in aggregate output from financial shocks alone. One reason is fairly simple: new investment is only a small proportion of the total capital stock, see e.g. Kocherlakota (2000), Cordoba and Ripoll (2004). Chari et al. (2007) moreover argue that these models are not consistent with the little evidence they find in the aggregate data for counter-cyclical investment wedges.[10]

A second generation of models introduces financial frictions in ways that generate stronger effects on output. One approach is to instead constrain the firm's labor demand margin, see e.g. Jermann

[8]Recent work by Atalay (2015) shows that departing from Cobb-Douglas technologies to allow for more general CES structures can lead to much greater amplification of TFP shocks (since the estimated elasticity of substitution appears to be lower than one).

[9]Presumably, the first papers were influenced by corporate finance models in which the firm's primary financing challenge is the funding of large, infrequent investment projects.

[10]Furthermore, within the context of the Great Recession, Chari et al. (2008), Kahle and Stulz (2013) and Shourideh and Zetlin-Jones (2012) show that at least in the aggregate, non-financial corporations could fund their capital expenditures entirely through retained earnings, therefore challenging the assertion that firms were constrained during the crisis.

and Quadrini (2011) and Bigio (2015). Although this avenue may deliver stronger responses, it also has its drawbacks: Bigio (2015) finds that the same reasons that lead to strong output responses also leads to counterfactual interest rates and profits. Another approach is to depart from the representative firm environment by introducing heterogeneous producers as in Buera and Moll (2012) or Midrigan and Xu (2014). In these models, financial shocks constrain the scale of the most productive firms so that inputs are reallocated to less productive firms, thereby lowering total output. The challenge for these models is that in order to create strong output responses, one needs both (i) a substantial amount of heterogeneity in firm productivity, and (ii) large flows of capital and labor reallocation.

Our contribution to this literature is to explore a new mechanism that may deliver strong responses in output and employment at the macro level, yet still allow for small financial disturbances at the micro level. In particular, we consider financial frictions that constrain the working capital of firms: both hiring as well as intermediate good purchases. As such, our paper is one of the first papers to highlight the role that interfirm trade plays in amplifying financial shocks.

An early paper by Kiyotaki and Moore (1997a) looks at how shocks are propagated along a supply chain composed of firms that borrow and lend from one another. Following this work, Kim and Shin (2012) consider a moral hazard incentive problem of producers linked through a production chain. While these models study carefully the trade credit problem of suppliers and customers, they focus on this problem within a very narrow and specific network: the purely vertical supply chain.[11] Thus, relative to this work we are the first to characterize the amplification and propagation of shocks within a general network structure and to decompose the aggregate effects into an efficiency and labor wedge. Finally, we believe our paper is the first to quantitatively test this mechanism within the context of the 2007-2008 Financial Crisis and the Great Recession.

Intersectoral Production Networks. Dating back to Long and Plosser (1983), there is an extensive macroeconomic literature focused on how input-output linkages affect the transmission of sectoral productivity shocks; see e.g. Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu et al. (2012). This literature has generally focused on the question of whether idiosyncratic sectoral productivity shocks transform into macroeconomic fluctuations as they propagate through the intermediate good network. This issue was the focal point of the debate between Horvath (1998, 2000) and Dupor (1999). More recently, Acemoglu et al. (2012) provide a clear answer, showing that the rate of decay of idiosyncratic shocks depends critically on certain characteristics of the network structure.[12]

[11] See also the work of Gofman (2015) and Kalemli-Ozcan et al. (2014) which build on the Kim and Shin (2012) model. We also study the pure vertical structure as a special case of our general network economy. However, our model completely abstracts from any time dimension or agency problem of trade credit.

[12] In particular, one of their central results is that a key determinant of the contribution of sector-specific shocks to aggregate volatility is the coefficient of variation of the intersectoral network's degree distribution; that is, if there are large asymmetries in the degree distribution of the intersectoral network, then micro shocks may indeed have aggregate effects. This insight is closely related to the pioneering work of Gabaix (2011) who demonstrated that idiosyncratic firm-level shocks can become macro-level fluctuations when the firm size distribution is heavy-tailed.

Our model builds on the multi-sector network models of Acemoglu et al. (2012) and Long and Plosser (1983) but incorporates financial constraints on firm production. Thus, in contrast to this literature, our paper redirects the focus away from productivity shocks towards the role of financial frictions. The frictions in our model operate as would any other type of distortionary wedge at the sectoral level; as such, one may think of our model as a generalization of Acemoglu et al. (2012) to include sectoral wedges (as well as to allow for more flexible preferences and technology).[13]

Given our focus on financial shocks or more generally, sectoral distortions, we believe our contribution vis-à-vis this literature is both methodological and substantive. Methodologically, we provide a complete characterization of equilibrium allocations and prices in a multi-sector network economy with sectoral distortions and an endogenous labor supply. Given this characterization, we find that the aggregate effects of financial frictions and productivity shocks differ substantially. In particular, we show how financial frictions manifest themselves at the aggregate level as an efficiency wedge as well as an aggregate labor wedge and we demonstrate how particular network structures amplify these effects.

A secondary contribution of our paper to this literature is our finding that distortionary sectoral wedges are sufficient to break Hulten's theorem; see Hulten (1978). In a multi-sector input-output model, Hulten's theorem states that a particular sector's influence, i.e. the total effect of its own productivity on aggregate output, is equal to its equilibrium share of sales; see also the results in Gabaix (2011) and Acemoglu et al. (2012). As such, data on network structures become irrelevant for aggregate fluctuations when revenues serve as sufficient statistics for such effects. We show that this direct equivalence is no more the case when distortionary sectoral wedges are introduced.

Finally, a third contribution of our paper relative to this literature is to quantify these effects. In Section 5 we quantitatively assess the role the input-output network played in amplifying financial frictions during the 2007-2008 financial crisis and the ensuing recession. This portion of our paper is thus similar in spirit to that of Foerster et al. (2011) as well as Atalay (2015); both of these papers seek to quantify the contribution of sectoral productivity shocks to business cycle fluctuations during the post-war period. In contrast to their work, we study the network effects of tighter financial constraints during the financial crisis and hence focus our quantitative analysis on the period 2006-2012.[14]

The Macroeconomic Effects of Micro-level Distortions. Finally, a broad literature examines how micro-level distortions may lead to a loss of total factor productivity at the aggregate level through misallocation effects. See for example, the seminal work of Banerjee and Duflo (2005); Chari et al. (2007); Hsieh and Klenow (2009); Restuccia and Rogerson (2008), and more recently, Midrigan and

[13]To be precise, if we shut down the financial frictions in our model and allow only for productivity shocks, our baseline model is equivalent to Acemoglu et al. (2012) with greater generality in preferences, technology, as well as an endogenous labor supply choice.

[14]Furthermore, Foerster et al. (2011) conduct their analysis using the Federal Reserve Board's dataset on industrial production, a dataset that is richer than ours but at the same time spans only the goods-producing sectors of the U.S. economy while our dataset covers all sectors.

Xu (2014) and Hopenhayn (2014). This work generally focuses on economies with simple Dixit-Stiglitz production structures in which intermediate good trade is limited and all firms essentially operate in isolation. With respect to this literature, our paper generalizes these misallocation effects to the class of network economies with intermediate good trade.

Thus, along these lines our paper is most closely related to a smaller set of papers that have studied the effects of firm or sector-level distortions in economies with intermediate good trade: Jones (2011), Jones (2013), Ciccone (2002), Basu (1995), and Yi (2003). With the exception of Jones (2013), these papers have in general focused on the aggregate effects of distortions within *specific* input-output structures.[15] In contrast, our paper studies a fully general network of input-output linkages, thereby allowing us to compare aggregate outcomes across different networks.

Jones (2013) likewise analyses a general input-output structure growth model with sectoral distortions and shows that input misallocation at the micro level affects total factor productivity at the aggregate level. In particular, Jones (2013) considers how different input-output structures may help explain the large cross-country income differentials observed in the data.

While there are clear similarities between our paper and Jones (2013)—both examine how input-output linkages amplify sectoral distortions and result in disparate aggregate outcomes—we believe our paper makes a few distinct contributions relative to his work. First, in terms of our model and analysis, we incorporate an endogenous labor supply decision which allows us to examine how sectoral distortions may manifest themselves as an aggregate labor wedge. Our analysis therefore decomposes the aggregate effects of financial frictions into two channels: those that work through the efficiency wedge and those that generate a labor wedge. We furthermore explore how the responses of these aggregate wedges to financial shocks differ across various network structures. Our decomposition thereby provides further insight as to how the network structure amplifies sectoral distortions at the aggregate level.

The second main difference is again in regards to our quantitative analysis. While Jones (2013) studies the implications of input-output networks within the context of explaining long-run cross-country growth differentials, we explore this mechanism in the context of business cycle fluctuations, in particular the recent financial crisis and recession. We believe that this particular application in fact gives us an advantage in calibrating these distortions, as technological parameters governing returns to scale are arguably less likely to move at business cycle frequency; this calibration method will be made clear in the quantitative section of our paper.

Layout

[15] Jones (2011) examines a single-intermediate input economy and focuses on different elasticities of substitution. Ciccone (2002) similarly develops a multiplier formula on the growth of an economy with intermediate good production. In the economic fluctuations literature, Basu (1995) shows that in a demand-driven business cycle model, the combination of counter-cyclical markups (distortions) and input-output production can result in pro-cyclical TFP. Finally, Yi (2003) finds that tariffs can be amplified when intermediate goods are traded through multiple stages of production.

The remainder of this paper is organized as follows. Section 2 introduces our model of a general, multi-sector, input-output economy with financial frictions. In Section 3 we characterize the equilibrium of this economy and present our main results on the aggregate implications of financial constraints in a network economy. In Section 4 we then investigate a few special cases of simple networks in order to provide intuition. This concludes the purely theoretical section of our paper. Following this, Section 5 provides a quantitative assessment of the model applied to the Financial Crisis and the Great Recession. Section 6 concludes.

2 The General Production Network Model

In this section we introduce our general multi-sector input-output economy. The economy is populated by a representative household and N production sectors indexed by $i = 1, \ldots, N$. Each sector consists of a continuum of identical and competitive firms each producing an identical good. However, while goods are identical across all firms within a sector, goods are differentiated across sectors. We index goods by $j = 1, \ldots N$, with the understanding that there is a one-to-one mapping between sectors and goods.[16] We thus use the indices i and j interchangeably, to denote either the sector i or the good j produced by sector $j = i$.

Production. Firms are perfectly competitive. We assume that the technology and the financial constraint is the same across all firms within a sector. This implies the existence of a representative firm per sector. We thus simply refer to the sector as the individual production unit, with the understanding that underlying each sector is a representative firm.

Sector i produces output according to the following Cobb-Douglas technology

$$y_i = (z_i x_i)^{\eta_i} . \tag{1}$$

where y_i is the output of sector i, z_i is an input-augmenting sector-specific productivity, and x_i is a composite of inputs used by sector i. The parameter $\eta_i \in (0, 1)$ implies that there is decreasing returns to scale in sector i.[17]

The composite x_i of sectoral inputs is given by the Cobb-Douglas composite

$$x_i \equiv \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{w_{ij}} \right)^{1-\alpha_i}$$

[16] There does not necessarily have to be a one-to-one mapping between sectors and goods. In fact the U.S. input-output tables provided by the BEA do not assume this (see discussion in Data Appendix C). For the purposes of this paper, however, we abstract from this issue and assume that all firms within a sector produce only one good, the good that corresponds to that sector.

[17] To motivate the decreasing returns to scale assumption, we assume that in the short run there is a fixed factor in production, say, capital or land.

where ℓ_i is sector i's labor input and x_{ij} is the amount of commodity j used by sector i.[18] The exponent $w_{ij} \in [0,1]$ denotes the share of good j in the total intermediate input use of sector i. Note that if sector i does not use a good j as an input to production, then $w_{ij} = 0$.[19] Finally, without loss of generality, we assume that $\sum_{j=1}^{N} w_{ij} = 1, \forall i \in \{1, ..., N\}$.[20]

Next, we impose financial constraints on the sector's purchase of intermediate inputs. Firm i takes prices as given and maximizes profits subject to the financial constraint:

$$\ell_i + \sum_{j=1}^{N} p_j x_{ij} \leq \chi_i p_i y_i \tag{2}$$

where the wage is normalized to 1 and p_j denotes the price of good j. The expenditure of firm i on all inputs is constrained to be less than χ_i of its earnings. Therefore χ_i parameterizes the financial constraint for sector i. This representation of a financial constraint can be adapted to reflect interest rate costs, limited enforcement, moral-hazard, and funding costs for trade-credit; we present a possible microfoundation for this constraint stemming from a limited enforcement problem in Appendix D.

The objective of the representative firm of each sector is to choose inputs and output so as to maximize profits subject to its financial constraint (2), taking the wage and prices as given. We assume all firms are owned by the representative household.

The Household. The preferences of the representative household are given by

$$U(c, \ell) \equiv \frac{c^{1-\gamma}}{1-\gamma} - \frac{\ell^{1+\epsilon}}{1+\epsilon}.$$

where c is the household's final consumption basket and ℓ is the household's labor supply. Here, $\gamma \geq 0$ parametrizes the income elasticity of labor supply,[21] and $\epsilon > 0$ corresponds to the inverse Frish elasticity of labor supply. The final consumption basket of the household is a composite of the differentiated goods in the economy.

$$c = \prod_{j=1}^{N} c_j^{v_j}$$

where c_j is the household's consumption of good j. The parameter $v_j \in [0,1]$ is the household's expenditure share on good j. If the household does not consume good j, then $v_j = 0$. Again without

[18] In terms of notation in the firm's production function, without loss of generality we treat labor differently from intermediate goods. This notation will prove useful later in an auxiliary step in which we treat labor supply as exogenous.

[19] In general, w_{ii} need not equal zero as sectors may use their own output as an input to production.

[20] This is without loss of generality because we may always rescale the value of η_i.

[21] Note that risk aversion and intertemporal substitution play no role in our setting because all idiosyncratic risk is insurable and the model is static. Therefore, γ controls only the sensitivity of labor supply to income for a given wage. Note that when $\gamma = 0$, there is no income effect (as in GHH preferences).

loss of generality we set $\sum_{j=1}^{N} v_j = 1$.[22]

The budget constraint of the household is given by

$$\sum_{j=1}^{N} p_j c_j \leq \ell + \sum_{i=1}^{N} \pi_i$$

where π_i are the profits of sector i. Thus, total expenditure must be weakly less than the household's labor income plus dividends. The household's objective is to choose consumption and labor so as to maximize utility subject to its budget constraint, taking the wage and prices as given.

Market Clearing. The output of any given sector may either be consumed by the household or used by other sectors as an input to production. Commodity market clearing for each good j is thus given by

$$y_j = c_j + \sum_{i=1}^{N} x_{ij}, \forall j \in \{1, \dots N\}. \tag{3}$$

Similarly, labor market clearing satisfies

$$\ell = \sum_{i=1}^{N} \ell_i.$$

Finally, we set household labor to be the numeraire, thereby normalizing the wage to 1.

The Input-Output Matrix. We let $\mathbf{W}$ denote the $N \times N$ input-output matrix of the economy with entries w_{ij} as follows

$$\mathbf{W} \equiv \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1N} \\ w_{21} & w_{22} & & \\ \vdots & & \ddots & \\ w_{N1} & & & w_{NN} \end{bmatrix}. \tag{4}$$

We let $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_N)'$ and $\boldsymbol{\eta} = (\eta_1, \dots, \eta_N)'$, denote the vector of labor shares and the vector of decreasing returns to scale parameters for each firm, respectively. Furthermore, the vector of household expenditure shares is given by $\mathbf{v} = (v_1, v_2, \dots, v_N)'$. Note that the rows of the matrix $\mathbf{W}$ sum to one due to our normalization that $\sum_{j=1}^{N} w_{ij} = 1, \forall i$. However, the columns of the matrix $\mathbf{W}$ need not sum to one. The column sum is what is known as the weighted outdegree: this corresponds to the share of sector i's output in the input supply of the entire economy.[23]

For future reference we let $\mathbf{e}_n$ denote a column vector of 1s of length n and we let $\mathbb{I}_n$ denote the identity matrix of size $n \times n$. Finally, for shorthand we occasionally use $\mathbf{1} \equiv \mathbf{e}_N$.

Equilibrium Definition. We define an equilibrium for this economy as follows.

[22] We may always rescale the value of γ.

[23] Finally, input-output relationships between different sectors can equivalently be represented by a directed weighted graph on n vertices, each corresponding to a particular sector of the economy.

Definition 1 *A competitive equilibrium consists of a vector of commodity prices* $\mathbf{p} = (p_1, p_2, \ldots, p_N)$, *a consumption bundle* $\{c_j\}_{j=1}^N$, *and sectoral output, input, and labor allocations* $\left\{y_i, \{x_{ij}\}_{j=1}^N, \ell_i\right\}_{i=1}^N$ *such that*
(i) the household and firms are at their respective optima,
(ii) prices clear commodity markets, and
(iii) the wage, normalized to 1, clears the labor market.

Remarks on the Model. Without financial frictions, the model presented here is similar to the model of Acemoglu et al. (2012), which itself builds on the original multi-sector model of Long and Plosser (1983). To be precise, if we shut down the financial frictions in our model so that all firms operate unconstrained and we allow only for productivity shocks, our baseline model is equivalent to Acemoglu et al. (2012) but with greater generality in preferences and technology. In particular, in terms of the technology we allow for decreasing returns to scale in firm production, heterogeneity across firms in their returns to scale, as well as heterogeneity in their labor shares. In terms of preferences we allow for heterogeneity in final consumption shares across different goods and for the household to have an endogenous labor-leisure choice (as opposed to a fixed endowment of labor). This direct connection to Acemoglu et al. (2012) is made even clearer in Section 3 in which we show that our equilibrium allocation nests that found in Acemoglu et al. (2012) as a special case.

Aside from these cosmetic differences, however, the main difference between our model and that of Acemoglu et al. (2012), as well as with the entire Long and Plosser (1983) literature, is that within this general input-output network framework, we embed financial constraints on firms' intermediate good purchases.

We believe there remain a few more important aspects of production economies that our simple model is not capturing. First, much like the previous literature, our paper maintains the assumption of Cobb-Douglas technology and thereby imposes unitary elasticity of substitution across intermediate goods. This is done purely for tractability. However, while this assumption makes the analysis simpler, it does mean that we lose the flexibility that, say, a constant elasticity of substitution (CES) production function would provide.

Along these lines, a recent paper by Atalay (2015) examines quantitatively the contribution of sectoral shocks to business cycle fluctuations. A key contribution of Atalay (2015) is to relax the standard assumption of a unitary elasticity of substitution and to instead impose CES production functions. In his quantitative analysis, Atalay estimates the elasticity of substitution across goods to be lower than one—that is, more inelastic than Cobb-Douglas. Similarly, using very disaggregated micro-level evidence from the U.S. Census, Boehm et al. (2014) empirically estimate the elasticity of substition across intermediate goods and find it to be incredibly low, near zero, suggesting that production functions are close to Leontief in the short run.

Hence, were we to impose instead a CES production function and use the aforementioned empirical estimates of either Atalay (2015) or Boehm et al. (2014) for the elasticity of substitution, this would only help to amplify the aggregate effects of financial frictions we find in our paper. That is, were substitution more inelastic—if intermediate goods could not move so easily from a constrained firm to an unconstrained firm—then it would be nearly impossible for reallocation to mitigate any financial disruptions in the production network. We thus conjecture that financial constraints would have a more pronounced effect on aggregate output than what we find in our paper.[24]

Related to this last point, our model furthermore allows for completely frictionless labor mobility across sectors. Clearly if there were some search or adjustment costs for labor to move across firms or sectors due to, say, re-training or re-education, these frictions would also contribute to less reallocation of inputs from financially constrained sectors to unconstrained sectors. This would again exacerbate the effects of sectoral financial shocks on aggregate fluctuations. For this reason and the reason stated above, we believe our estimates for the aggregate effects of financial frictions may be thought of as a lower bound on their true effects.

Finally, our model takes the input-output network structure to be completely exogenous. Of course, the introduction of financial constraints on working capital purchases implies that there would be some rationale for vertical integration to overcome these constraints. While the idea of endogenizing the network structure is clearly intriguing, we believe this is beyond the scope of this paper. In particular, given the recessionary focus of our quantitative section, the assumption of a fixed network in the short run seems fairly reasonable.[25]

3 Equilibrium Characterization of the General Model

We now turn to characterizing the general equilibrium of this economy. In order to solve for equilibrium allocations, we proceed in three steps outlined here.

First, in subsection 3.1 we characterize the optimality conditions of the sector-level representative firms and the optimality conditions of the representative household. We show that the introduction of financial constraints leads to distortionary wedges in the firms' optimality conditions.

The second step in our solution method is to then use these optimality conditions along with market clearing conditions to solve for equilibrium allocations given a fixed quantity of labor input. That is, in subsection 3.2 we for a moment ignore the household's optimal labor supply decision and instead characterize aggregate consumption in partial equilibrium for an exogenous aggregate

[24] Of course the elasticity of substitution depends on the time horizon. In the long run, firms would have the ability to write new contracts with new firms, thus the elasticity of substitution would be much higher than in the short run. However, because our paper is concerned with business cycle fluctuations, we believe that an elasticity of intermediate good substitution below 1 is the more appropriate estimate.

[25] A recent paper by Oberfield (2011) develops a rich theory in which the network structure of production is endogenous. Firms in his model however do not face financial constraints.

labor supply. The solution to this intermediate step thereby traces out the *aggregate production function* of this economy, allowing us to highlight the effect of financial frictions on aggregate TFP. In this way we identify the *efficiency wedge* channel.

The final step in our solution method is to reincorporate the household's endogenous labor supply decision and solve for the general equilibrium of the economy. In subsection 3.3 we thus combine the household's labor optimality condition with the aggregate production function derived in the previous step. Together these conditions allow us to fully characterize equilibrium aggregate output, consumption, and labor. We furthermore demonstrate how the sectoral financial frictions in this network economy manifest themselves at the macro level as an aggregate *labor wedge* distortion, in addition to the efficiency wedge highlighted previously.

To conclude, in subsection 3.4 we graphically illustrate the aggregate effects of financial frictions on GDP via both channels: the efficiency wedge and the labor wedge. All proofs for this section are provided in Appendix A.

3.1 Firm and Household Optimality

Firms. We first consider the representative (sector-level) firm's problem, which may be written explicitly as follows.

Problem 1 *Given the real wage and the vector of prices, the representative firm of sector i chooses inputs $\{x_{ij}\}_{j=0}^{N}$ and output y_i in order to maximize profits*

$$\max p_i y_i - \ell_i - \sum_{j=1}^{N} p_j x_{ij}$$

subject to the firm's production function

$$y_i = \left[z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{w_{ij}} \right)^{1-\alpha_i} \right]^{\eta_i}, \tag{5}$$

and financial constraint

$$\ell_i + \sum_{j=1}^{N} p_j x_{ij} \leq \chi_i p_i y_i. \tag{6}$$

We solve for the firm's optimal choice of inputs, labor, and output in two steps. The first step is an expenditure minimization problem: given an arbitrary amount of total output, we solve for the firm's cost-minimizing choice of inputs and labor that would achieve that output. The next step is to then solve the firm's optimal level of output. The solution to the firm's dual problem is presented in the following lemma.

Lemma 1 *Let $u_i = \ell_i + \sum_{j=1}^{N} p_j x_{ij}$ denote the total expenditure of sector i on inputs (including its expenditure on labor) and let $g_i = p_i y_i$ denote the total revenue of sector i. Given a vector of prices and the real wage, the firm's optimal choice of inputs and output satisfies the following conditions:*

(i) expenditure on any particular good j is proportional to the firm's total expenditure

$$p_j x_{ij} = (1 - \alpha_i)\, w_{ij} u_i \tag{7}$$

$$\ell_i = \alpha_i u_i \tag{8}$$

(ii) total expenditure and total revenue satisfy

$$u_i = \phi_i \eta_i g_i \quad \text{where} \quad \phi_i = \min\left\{\frac{\chi_i}{\eta_i}, 1\right\} \tag{9}$$

The first part of this lemma states that expenditure on any individual input is proportional to the firm's total expenditure on inputs, where α_{ij} is the constant of proportionality. This constant proportionality holds no matter the firm's total expenditure of inputs–a classic result of the Cobb-Douglas technology structure.

The second part of this lemma states that total firm expenditure is proportional to its revenue. However, this proportionality now depends on the financial friction, parameterized by ϕ. In particular, if the firm is unconstrained, then expenditure is equal to its total revenue times its returns to scale parameter; that is, $u_i = \eta_i g_i$. This is the profit maximizing choice of scale for the firm in the absence of financial constraints. However, if the firm is financially constrained, it cannot reach its profit-maximizing scale. Instead, equation (6) holds with equality. The firm's financial constraint thus binds when $\chi_i < \eta_i$.

The Household. We next consider the representative household's problem, written as follows.

Problem 2 *Given the real wage, the vector of prices, and the vector of firm profits, the household chooses consumption and labor in order to maximize utility*

$$\max \frac{c^{1-\gamma}}{1-\gamma} - \frac{\ell^{1+\epsilon}}{1+\epsilon}$$

subject to

$$c = \prod_{j=1}^{N} c_j^{v_j}$$

and the household's budget constraint

$$\sum_{j=1}^{N} p_j c_j \leq \ell + \sum_{i=1}^{N} \pi_i.$$

In a fashion similar to the firm's problem, we solve for the household's optimal choice of consumption over goods and labor supply in two steps. We first solve the household's expenditure minimization problem: given a total amount of final consumption, we find the household's expenditure-minimizing choice over commodities. The expenditure minimizing cost of these goods results in the ideal price of the household's consumption basket. Given this price (and therefore the real wage), we then solve for the household's optimal choice of total consumption and labor supply. The solution to the household's dual problem is presented in the following lemma.

Lemma 2 *Let $u_0 = \sum_{j=1}^{N} p_j x_{0j}$ denote the household's total expenditure on the differentiated goods. Given a vector of prices and the real wage, the household's optimal choices of consumption and leisure are characterized as follows*

(i) expenditure on any particular good j is proportional to the household's total expenditure

$$p_j c_j = v_j \bar{p} c \tag{10}$$

where $\bar{p}$ is the ideal price index for the household's consumption basket, given by

$$\bar{p} = \prod_{j=1}^{N} \left(\frac{p_j}{v_j} \right)^{v_j}. \tag{11}$$

(ii) labor and total consumption must jointly satisfy

$$\frac{\ell^{\epsilon}}{c^{-\gamma}} = \frac{1}{\bar{p}} \quad \text{and} \quad u_0 = \ell + \sum_{i=1}^{N} \pi_i, \tag{12}$$

where $u_0 = \bar{p} c$.

Again due to the Cobb-Douglas structure of preferences, we obtain the classic result that household expenditure on any individual good is proportional to the household's total expenditure, where v_j is the constant of proportionality. Total expenditure on consumption is denoted by $\bar{p}c$, where $\bar{p}$ is the ideal price of the final consumption basket, given by equation (11).

Given the price of consumption, and hence the real wage, the second part of this lemma provides the household's optimality condition between consumption and labor. Equation (12) equates the marginal rate of substitution between consumption and labor with the the real wage $1/\bar{p}$. Finally, the household's optimal consumption allocation must satisfy its budget constraint.

Tax Representation. There is a simple and direct isomorphism between our economy with financial frictions and the exact same input-output economy with sectoral taxes. Consider an economy in which firms do not face any financing constraints, but instead face sector-specific taxes

on their revenues. In this economy, the firm's problem would thus be written as

$$\max_{x_i} (1-\tau_i)\, p_i \left(z_i x_i\right)^{\eta_i} - \ell_i - \sum_{j=1}^{N} p_j x_{ij}.$$

where τ_i is the sales tax on sector i. The optimality conditions for the firm would then be identical to those presented in Lemma 1, with the condition relating total expenditure to total revenue in (9) replaced by $(1-\tau_i)\,\eta_i g_i = u_i$. Hence, the firm's problem and solution are equivalent across the two economies with a tax that satisfies $(1-\tau_i) = \phi_i$.

More generally, this representation demonstrates how our environment is isomorphic to an input-output economy with sectoral distortions. In the following two subsections, we show how these sectoral distortions propagate through the network and manifest themselves at the macro level in the form of an aggregate efficiency wedge and an aggregate labor wedge. All of our theoretical insights will thus apply equally well to input-output economies with sectoral wedges—irrespective of whether these wedges originate in proportional taxes, mark-ups, market imperfections, or any number of frictions. The main point is to show how the input-output structure magnifies these distortions and through which channels.

3.2 The Aggregate Production Function and the Efficiency Wedge

The second part of our solution method is to combine market clearing conditions with the firms' and the household's optimality conditions (from Lemmas 1 and 2) in order to characterize aggregate output and consumption given an exogenously fixed amount of labor. For a moment we thus discard the household's optimal labor supply decision. After a series of intermediate steps (we solve for firm sales, then prices, and finally allocations), we characterize aggregate consumption given a fixed labor supply, thereby tracing out the aggregate production function of this economy.

Equilibrium Sales. Our first step is to solve for equilibrium sales and expenditure of each sector given a fixed labor supply. We let $\mathbf{g} \equiv (g_1, \ldots, g_N)'$ denote the vector of sectoral sales and $\mathbf{u} \equiv (u_1, \ldots, u_N)'$ the vector of sectoral expenditures. Furthermore, we let $\boldsymbol{\phi} \equiv (\phi_1, \ldots, \phi_N)'$ denote the vector of financial frictions across firms with ϕ_i defined in (9).

As seen in Lemmas 1 and 2, the Cobb-Douglas structure of preferences and technology implies constant expenditure shares of firms and households over the commodities. Substituting these constant expenditure shares into the economy's market clearing conditions and combining this with the household's budget constraint admits a fixed point in sectoral sales and expenditures. The solution to this fixed point is given in the following proposition.

Proposition 1 *Take the household's supply of labor ℓ as given. For given ℓ, an equilibrium will be characterized by a vector of sectoral sales* $\mathbf{g}$ *and a vector of firm expenditures* $\mathbf{u}$ *such that*

$$\mathbf{g}(\boldsymbol{\phi}) = \mathbf{a}(\boldsymbol{\phi})\,\ell \qquad \textit{and} \qquad \mathbf{u}(\boldsymbol{\phi}) = (\boldsymbol{\phi} \circ \boldsymbol{\eta} \circ \mathbf{a}(\boldsymbol{\phi}))\,\ell \tag{13}$$

where

$$\mathbf{a}(\boldsymbol{\phi}) \equiv \left[\mathbb{I}_N - \left((\mathbf{1}-\boldsymbol{\alpha})\,\mathbf{e}_N^{'} \circ \mathbf{W}\right)' \circ \left(\mathbf{e}_N\left(\boldsymbol{\phi}\circ\boldsymbol{\eta}\right)'\right) - \mathbf{v}\left(\mathbf{e}_N - (\boldsymbol{\phi}\circ\boldsymbol{\eta})\right)'\right]^{-1}\mathbf{v} \tag{14}$$

and $\circ$ *denotes the Hadamard (entrywise) product.*

Proposition 1 characterizes the vector of equilibrium sales and the vector of equilibrium expenditures given a fixed labor supply. This proposition shows equilibrium revenue and expenditure of each sector is linear in aggregate labor. Furthermore, equilibrium sales and expenditures clearly depend on $\boldsymbol{\phi}$, the vector of financial frictions, as well as $\mathbf{W}$, the production network.

Equilibrium Prices. The next step is to calculate equilibrium prices. Using the equilibrium sales vector from Proposition 1 along with the sectoral production functions, we may easily back out the vector of equilibrium sectoral prices.

Proposition 2 *Given the vector of equilibrium sales,* $\mathbf{g}$*, the vector of sectoral prices satisfies*

$$\log\mathbf{p}(\boldsymbol{\phi}) = -\mathbf{B}\left[\boldsymbol{\eta}\circ(\log\mathbf{z}+\log\boldsymbol{\phi}+\log\boldsymbol{\eta}+\boldsymbol{\kappa}) - (\mathbf{e}_N-\boldsymbol{\eta})\circ\log\mathbf{g}(\boldsymbol{\phi})\right] \tag{15}$$

where $\boldsymbol{\kappa} = (\kappa_1,\ldots,\kappa_N)'$ *is an* $N\times 1$ *vector of constants and* $\mathbf{B}$ *is an* $N\times N$ *matrix defined by*

$$\mathbf{B} \equiv \left[\mathbb{I}_N - \left((\boldsymbol{\eta}\circ(\mathbf{1}-\boldsymbol{\alpha}))\,\mathbf{e}_N^{'}\right)\circ\mathbf{W}\right]^{-1}. \tag{16}$$

The aggregate price level (the ideal price for the household's consumption basket) is thus given by

$$\log\bar{p}(\boldsymbol{\phi}) = \mathbf{v}'\log\mathbf{p}(\boldsymbol{\phi}) - \mathbf{v}'\log\mathbf{v}. \tag{17}$$

From Proposition 2 we see immediately that equilibrium prices also depend on $\boldsymbol{\phi}$, the vector of financial frictions, as well as $\mathbf{W}$, the production network matrix. We postpone our discussion of the intuition for these effects until after we characterize equilibrium allocations. **Equilibrium Allocations.** Finally, given equilibrium sales, expenditures, and prices, we may then back out equilibrium allocations. Note that the vector of sectoral gross output can be computed as follows: $\log\mathbf{y}(\boldsymbol{\phi}) = \log\mathbf{g}(\boldsymbol{\phi}) - \log\mathbf{p}(\boldsymbol{\phi})$. We are however interested in sectoral value added. We thus denote sector i's value added by μ_i and further denote the vector of sectoral value added by $\boldsymbol{\mu} = (\mu_1,\ldots,\mu_N)'$. Given sales, expenditures, and prices, sector i's real value added is given by

$$\mu_i = \bar{p}(\boldsymbol{\phi})^{-1}\left(g_i - (u_i - \ell_i)\right), \tag{18}$$

that is, firm sales minus firm expenditure on inputs (not including labor costs).

Next, real GDP is simply equal to the consumption of the household,

$$GDP = c = \bar{p}(\boldsymbol{\phi})^{-1}u_0 \tag{19}$$

where $u_0 = \ell + \mathbf{e}_N' (\mathbf{g}(\boldsymbol{\phi}) - \mathbf{u}(\boldsymbol{\phi}))$ is the household's wealth. It is easy to check that sectoral value added (18) summed across all sectors is equal to aggregate GDP (19). Substituting firm sales and expenditure $\mathbf{g}(\boldsymbol{\phi})$ and $\mathbf{u}(\boldsymbol{\phi})$ from (13) into equation (19), we may express real GDP as follows

$$GDP = \bar{p}(\boldsymbol{\phi})^{-1} (1 + \psi(\boldsymbol{\phi})) \ell \tag{20}$$

where the scalar $\psi(\boldsymbol{\phi})$ satisfies

$$\psi(\boldsymbol{\phi}) \equiv \mathbf{e}_N' ((\mathbf{e}_N - (\boldsymbol{\phi} \circ \boldsymbol{\eta})) \circ \mathbf{a}(\boldsymbol{\phi})). \tag{21}$$

This $\psi(\boldsymbol{\phi})$ term arises due to the fact that in our economy firms earn profits. Profits, which are simply a fraction $(1 - \phi_i \eta_i)$ of each sector's sales, are paid to the household in the form of dividends thereby augmenting household income. In the special case with constant returns to scale and no financial frictions, i.e. $\phi_i = 1$ and $\eta_i = 1$ for all i, firms make zero profit and the $\psi(\boldsymbol{\phi})$ term disappears. In this case, $GDP = \bar{p}(\mathbf{1})^{-1} \ell$.

Equation (20) thereby provides a simple expression for aggregate GDP as household income (including profits) deflated by the aggregate price level. The endogeneity of the price level, however, means that we must combine equation (20) with our characterization of equilibrium prices in Proposition 2 in order to obtain equilibrium GDP in terms of primitives alone. The result of this is presented in the following theorem.

Theorem 1 *The log of equilibrium GDP may be expressed as the following function of productivity shocks, financial frictions, and aggregate labor:*

$$\begin{aligned} \log GDP &= \mathbf{q} \log \mathbf{z} + \mathbf{q} \log \boldsymbol{\phi} + \log \ell \\ &\quad + \log(1 + \psi(\boldsymbol{\phi})) - \mathbf{d} \log \mathbf{g}(\boldsymbol{\phi}) + K, \end{aligned} \tag{22}$$

where $\log \mathbf{g}(\boldsymbol{\phi}) = \log \mathbf{a}(\boldsymbol{\phi}) + \log \ell$ *is the sectoral sales vector,* $\mathbf{q}$ *is a* $1 \times N$ *row vector given by*

$$\mathbf{q} \equiv (\mathbf{v}'\mathbf{B}) \circ \boldsymbol{\eta}' = \left(\mathbf{v}' \left[\mathbb{I}_N - \left((\boldsymbol{\eta} \circ (\mathbf{1} - \boldsymbol{\alpha})) \mathbf{e}_N' \right) \circ \mathbf{W} \right]^{-1} \right) \circ \boldsymbol{\eta}', \tag{23}$$

$\mathbf{d}$ *is a* $1 \times N$ *row vector given by*

$$\mathbf{d} \equiv \mathbf{v}' \left(\left(\mathbf{e}_N (\mathbf{1} - \boldsymbol{\eta})' \right) \circ \mathbf{B} \right), \tag{24}$$

and K *is a scalar constant.*

Theorem 1 thereby provides a partial equilibrium characterization of GDP. We consider this a partial equilibrium characterization because the labor supply is fixed. In order to decompose the aggregate effects of financial frictions and understand how they are altered by the input-output network, let us first consider the special case of our model without frictions.

Special Case: no Financial Frictions and CRS Technology. Suppose there are no financial frictions: $\phi_i = 1$ for all i. Further suppose that all sectors operate constant returns to scale technologies: $\eta_i = 1$ for all i. The CRS assumption implies that $\mathbf{d} = (0, \ldots, 0)$ while the combination of both CRS and no financial frictions implies that firms make zero profits, $\psi(\boldsymbol{\phi}) = 0$. As a result, GDP in this special case may be expressed simply as follows[26]

$$\log GDP = \mathbf{q} \log \mathbf{z} + \log \ell + const \tag{25}$$

where

$$\mathbf{q} = \mathbf{v}'\mathbf{B} = \mathbf{v}' \left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N' \circ \mathbf{W} \right) \right]^{-1}. \tag{26}$$

Therefore, in this special case real GDP is log-linear in labor. Real GDP is furthermore a log-linear combination of the sectoral productivity shocks, represented by the vector $\log \mathbf{z}$. The coefficients on these productivity shocks are given by the elements of the vector $\mathbf{q}$, often referred to as the "influence vector" (see e.g. Acemoglu et al. (2012)). Each component of the influence vector provides the total effect on real GDP of the corresponding sector's productivity.[27]

To understand what determines the influence vector, in particular the role played by the input-output network, we examine more closely its main component, the matrix $\mathbf{B}$. This matrix is known as the Leontief Inverse and may be expanded as follows

$$\left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N' \circ \mathbf{W} \right) \right]^{-1} = \mathbb{I}_N + \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N' \circ \mathbf{W} \right) + \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N' \circ \mathbf{W} \right)^2 + \cdots.$$

This expansion provides a useful way of understanding how individual productivity shocks propagate through the input-output network and affect aggregate output. The first term in this expansion (the identity matrix) gives the direct effect of a sectoral productivity shock on its own production: a one-percent increase in the productivity of sector i leads to a one-percent increase in sector i's output and a fall in its price.[28] This increases the demand for intermediate good i by sector i's customers, thereby leading to an increase in their output as well as a fall in their prices; these

[26] To simplify, we abstract from the constant K as it is simply a function of exogenous parameters.

[27] To make expression (26) even simpler, suppose that the labor share is constant across all sectors, that is, assume $\alpha_i = \bar{\alpha}$ for all i. Furthermore assume that also the household's consumption shares are constant across all commodities, i.e. $v_i = 1/N$. In this special case, the influence vector reduces to

$$\mathbf{q} = \frac{1}{N} \mathbf{e}_N' \left[\mathbb{I}_N - (1 - \bar{\alpha}) \, \mathbf{W} \right]^{-1}.$$

This is the influence vector in Acemoglu et al. (2012); their model is nested as a special case of our model. Thus, one can think of equation (26) as a generalization of the influence vector in Acemoglu et al. (2012) to allow for heterogeneity in labor shares across industries and heterogeneous consumption shares across goods.

[28] Note that in Proposition 2, the Leontief inverse matrix also appears in the equation determining equilibrium prices, thereby coinciding with the price effects explained here. In fact, in this frictionless CRS special case, the price vector reduces to

$$\log \mathbf{p} = -\mathbf{B} \log \mathbf{z}.$$

secondary effects are reflected in the second term of this expansion. This in turn leads to a rise in demand for their respective goods and a subsequent increase in the output of sector i's customers' customers; these effects are reflected in the third term of this expansion, and so on.

Therefore, the Leontief Inverse matrix $\mathbf{B}$ captures the entire infinite hierarchy of network effects as sectoral productivity shocks propagate downstream through input-output linkages.[29] Clearly, the particular network structure plays an important role in determining the magnitude and distribution of these effects. Finally, in order to obtain the influence vector, one must transform these output effects into value added by pre-multiplying by the household's consumption share vector, $\mathbf{v}$.

Lastly, another way to interpret the influence vector is to consider the share of total labor that eventually travels through sector i on its path towards becoming a final good. Consider the following: each sector hires labor and purchases intermediate inputs, yet the only primary input in the economy is labor. As labor flows through the production network it is at every stage transformed into new intermediate goods, and it eventually becomes a final good. Through this transformation process it is augmented by each sector's productivity shocks. Thus, one may think of the influence vector as a measure of the labor supply that eventually travels through that firm. If very few resources flow through a certain firm, this firm has very little effect on aggregate output. This interpretation can be related to recent findings in Hopenhayn (2014).

General Case with both Financial Frictions and DRS Technology. We now move away from the frictionless benchmark and allow for binding financial constraints: $\phi_i \in (0, 1)$. With financial constraints, we must also impose decreasing returns to scale in firm technology to ensure that an optima exists; that is, we assume $\eta_i \in (0, 1)$. In this case, we obtain equation (22) in Theorem 1 for the aggregate production function.

Comparing equation (22) to the corresponding equation (25) in the frictionless benchmark, we see that there are a few substantive changes. Consider the first line in equation (22). Aggregate GDP is log-linear in the sectoral financial constraints, represented by the vector $\log \boldsymbol{\phi}$, as with the sectoral productivities. In particular, the coefficients on the sectoral financial frictions are determined by the same influence vector $\mathbf{q}$. The influence vector in (23), compared to that in the frictionless benchmark (26), now includes the vector $\boldsymbol{\eta}$; that is, the effects of financial frictions and productivity shocks are now attenuated by the sectors' decreasing returns to scale.

Despite this change, the intuition for how the influence vector captures the entire hierarchy of network effects remains the same. In particular, a tightening of the financial constraint of sector i leads to a fall in sector i's output and an increase in its price.[30] This results in a fall in the output of sector i's customers and an increase in their prices, which in turn leads to a fall in the output of sector i's customers' customers, and so on. The cumulative effect is once again captured in the Leontief inverse matrix.

Note that, unlike productivity shocks, sectoral financial frictions generate two additional terms

[29] The Leontief inverse matrix is also related to the notion of Bonacich Centrality. See Bonacich (1987).

[30] Again see Proposition 2 for the network effects on sectoral prices.

that appear in the second line of equation (22). The first of these terms is given by $1 + \psi(\boldsymbol{\phi})$. Because of binding financial constraints and decreasing returns, firms make profits. These profits are paid to the households in the form of dividends, which are then spent on final good consumption. Financial frictions thereby alter the equilibrium allocation through profit "leakage": the allocation with frictions differs from one in which these resources were to remain within the firms and were spent on intermediate inputs within the production network.[31]

The second of these terms is given by $-\mathbf{d} \log \mathbf{g}(\boldsymbol{\phi})$ where the vector $\mathbf{d}$ is defined in (24) and $\log \mathbf{g}(\boldsymbol{\phi})$ is the vector of sectoral sales. This term originates in an indirect effect stemming from the firms' decreasing returns to scale of production. As one tightens financial constraints, firms must reduce their production and operate at a lower scale (given their productivity). Due to decreasing returns to scale, operating at a lower scale means that the constrained firm is in fact more efficient: it implies a greater marginal return to production from any additional input. This leads to a fall in the firm's price, and hence an increase in the demand for its good. This is a fairly subtle but attenuating indirect effect which hinges completely on the decreasing returns to scale assumption. In the limit in which technology approaches constant returns ($\eta_i \to 1$), all entries of the vector $\mathbf{d}$ approach zero: $\mathbf{d} \to (0, \ldots, 0)$, and this term disappears.

The Efficiency Wedge. Theorem 1 provides an expression for GDP, or aggregate consumption, as a function of a fixed labor supply. It therefore traces out the aggregate production function of the economy. By exponentiating this function, we may rewrite the aggregate production function in the following way which highlights the effect of financial frictions on aggregate TFP in this economy.

Corollary 1 *The aggregate production function in this economy is given by*

$$GDP = \bar{z}(\mathbf{z})\, \zeta(\boldsymbol{\phi})\, \ell^{\bar{\eta}} \tag{27}$$

where $\bar{z}(\mathbf{z}) \equiv \exp\{\mathbf{q} \log \mathbf{z}\}$, $\bar{\eta} \equiv 1 - \mathbf{d}\mathbf{e}_N$, *and*

$$\zeta(\boldsymbol{\phi}) \equiv \exp\{\mathbf{q} \log \boldsymbol{\phi} - \mathbf{d} \log \mathbf{a}(\boldsymbol{\phi})\} (1 + \psi(\boldsymbol{\phi})). \tag{28}$$

Corollary 1 provides a simple representation of the aggregate production function. Not surprisingly, our economy aggregates to a Cobb-Douglas aggregate production function. In this expression, $\bar{z}(\mathbf{z})$ represents the total effect of sectoral productivity shocks on the aggregate production function, while $\zeta(\boldsymbol{\phi})$ represents the total effect of sectoral financial frictions. Following the terminology used in Chari et al. (2007), we may call the product $\bar{z}(\mathbf{z})\, \zeta(\boldsymbol{\phi})$ the *efficiency wedge*. Therefore, movements in either productivity or financial frictions manifest themselves as shocks to TFP in the aggregate.

[31] Note that if instead the profits of the firm were not given to the household and instead simply thrown out, or if they were taxed completely by the government (and not used for consumption), the $\psi(\boldsymbol{\phi})$ term would disappear.

In particular, financial constraints generate misallocation of resources across sectors. That is, relative to the frictionless economy, financial frictions may cause factors of production to be used inefficiently across sectors and may thereby lead to a fall in total factor productivity. However, the extent to which these frictions generate movements in the efficiency wedge depends crucially on the network structure. As articulated previously, the aggregate effect of any particular any sectoral distortion is determined by the Leontief inverse matrix, which itself depends on the input-output matrix. Thus, the effects of financial distortions on aggregate TFP depend on where these distortions are located and how they are amplified by the network architecture.

To put this result in context, an extensive literature has studied how distortions at the firm or sector level can lead to input misallocation, thereby manifesting itself as a loss in aggregate TFP. See for example, the work of Chari et al. (2007); Hsieh and Klenow (2009); Restuccia and Rogerson (2008), and more recently, Midrigan and Xu (2014) and Hopenhayn (2014).[32] All of the aforementioned work, however, has focused on economies with simple horizontal production structures, e.g. Dixit-Stiglitz, in which intermediate good trade is limited and all firms essentially operate in isolation.

With respect to this literature, Corollary 1 generalizes these misallocation effects to the class of network economies of intermediate good trade; in this sense our work is most closely related to that of Jones (2011, 2013); Basu (1995), Ciccone (2002) and Yi (2003). We postpone our discussion of the relation between our paper and this work until the following subsection.

A Comment on Hulten's Theorem. Finally, we conclude this subsection by demonstrating that our model breaks what is known as Hulten's theorem; see Hulten (1978). This theorem states that a sector's influence, i.e. the total effect of that sector's productivity on aggregate output, is equal to its equilibrium share of sales.[33] We first give a formal statement of this result and show under which conditions it holds in our model; following this we discuss its relevance within the Long and Plosser (1983) production network literature.

Consider the share of sales vector $\mathbf{a}(\boldsymbol{\phi})$ in our economy, characterized in equation (14). To make the analysis simpler, consider the CRS limit in which $\eta_i \to 1$ for all i. In this case, the equilibrium sales vector reduces to

$$\mathbf{a}(\boldsymbol{\phi}) = \left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha})\,\mathbf{e}_N' \circ \mathbf{W}\right)' \circ (\mathbf{e}_N \boldsymbol{\phi}') - \mathbf{v}\,(\mathbf{e}_N - \boldsymbol{\phi})'\right]^{-1} \mathbf{v}. \tag{29}$$

Second, consider the influence vector $\mathbf{q}$ as defined in (23).[34] In the limit case of CRS technology,

[32] One may liken our result to one shown in Chari et al. (2007) in which they show that in an input-financing friction model, frictions may manifest themselves primarily as efficiency wedges. We clearly generalize their result to a network economy and we will later show that in our economy a labor wedge will also emerge.

[33] The equilibrium sales shares are sometimes referred to as the Domar weights.

[34] If one were to consider the effect of financial frictions on aggregate GDP, it would be clearly non-log-linear, due to the fact that equation (22) is not log-linear in the ϕ's. Thus, it is almost immediate that the effect of financial frictions is the not the same as the aggregate sales vector.

the transposed influence vector[35] reduces to

$$\mathbf{q}' = \left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha})\,\mathbf{e}_N' \circ \mathbf{W}\right)'\right]^{-1} \mathbf{v}. \tag{30}$$

It is clear that the vectors in (29) and (30) are not identical due the presence of financial frictions, or more generally, distortionary sectoral wedges. Financial frictions constrain the production of sectors and hence divert a portion of revenue to the households in the form of profits thereby altering the sales vector. If instead there were no sectoral wedges, the vectors would then be equivalent. In particular, the vector of sales would be given by

$$\mathbf{a}(\mathbf{1}) = \left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha})\,\mathbf{e}_N' \circ \mathbf{W}\right)'\right]^{-1} \mathbf{v},$$

and hence equal to the influence vector in (30). This confirms the Hulten (1978) result, as well as versions seen in Gabaix (2011) and Acemoglu et al. (2012). That is, in the absence of frictions, there is an exact equivalence between the influence vector and the equilibrium sales vector.

Proposition 3 *In the special case without financial frictions and with constant returns to scale, the influence vector is identical to the vector of sectoral sales:* $\mathbf{q}' = \mathbf{a}(\mathbf{1})$. *However, with sectoral wedges, even in the limit case of constant returns to scale, the influence vector is not equivalent to the vector of sectoral sales:* $\mathbf{q}' \neq \mathbf{a}(\boldsymbol{\phi})$.

Why is this particular result relevant? The Long and Plosser (1983) input-output literature has generally focused on the question of whether idiosyncratic sectoral productivity shocks transform into macroeconomic fluctuations via propagation through the intersectoral network. As mentioned in our introduction, more recently Acemoglu et al. (2012) provide a clear answer, showing that the rate of decay of idiosyncratic shocks crucially depends on certain characteristics of the network structure. In particular, they show that a key determinant of the contribution of sector-specific shocks to aggregate volatility is the coefficient of variation of the intersectoral network's degree distribution; that is, if there are large asymmetries in the degree distribution across sectors, then the rate of decay of idiosyncratic sectoral shocks is significantly diminished.

Acemoglu et al's insight is closely related to the important work of Gabaix (2011) who proves that the rate of decay of idiosyncratic firm-level shocks falls decidedly when the firm size distribution is heavy-tailed, e.g. when firm sizes are power-law distributed. While the firm size distribution is exogenous in Gabaix's baseline model, in Acemoglu et al. (2012) the sectoral size distribution is an endogenous equilibrium object that depends on the structure of the intersectoral network. Despite this endogeneity, however, Acemoglu et al find that the equilibrium vector of sectoral sizes (sales) is exactly equal to the influence vector, as in Hulten's theorem and as replicated above. In other

[35] We take the transpose of this vector in order to make it a column vector and thereby comparable to $\mathbf{a}$.

words, the share of sales of a particular sector is a *sufficient statistic* for the extent to which a shock to that sector affects aggregate output.

One may then infer that the role that the asymmetric degree distribution plays in Acemoglu et al. (2012) in determining the transmission of micro shocks into macro fluctuations is akin to the role played by the fat-tailed size distribution in Gabaix (2011). One way to thereby interpret the results in Acemoglu et al. (2012) vis-à-vis those of Gabaix (2011) is that they highlight the importance of network interactions as a possible microfoundation for the sectoral size distribution and they show that highly central sectors will in equilibrium be larger and more influential. However, taking a more critical view, Hulten's theorem informs us that the intersectoral network does not matter insofar as it determines the sectoral size distribution; given this sufficient statistic, the network plays no further role in and of itself. This result, coupled with the fact that we can easily observe the equilibrium sizes of firms and sectors in the data, implies that there is little need to examine the rich data on network interactions as nothing further can be learned from network structures about aggregate fluctuations.[36]

We show that this line of criticism ceases to hold water when distortionary sectoral wedges are introduced. Proposition 3 states that generically in an intermediate good network model with sectoral wedges, the equilibrium vector of sectoral sizes is not equal to the vector of sectoral influence. The equivalence between the two vectors holds only in the special case of a perfectly frictionless economy, thereby nesting the result found in Acemoglu et al. (2012). Instead, the introduction of distortionary sectoral wedges is enough to break Hulten's theorem and, by direct implication, the intersectoral network plays an important and independent role in the transmission of micro shocks into macro fluctuations.

3.3 General Equilibrium and the Aggregate Labor Wedge

The third and final step in our solution method is to reincorporate the household's endogenous labor supply decision and solve for the general equilibrium allocations.[37] From Lemma 2, we have that the household's optimal choice between consumption and labor satisfies

$$\frac{c^{-\gamma}}{\ell^{\epsilon}} = \bar{p}. \tag{31}$$

We thus combine this condition with the aggregate production function derived in the previous step and solve for equilibrium consumption and labor. This yields the following result.

Theorem 2 *Let* $\beta \equiv \frac{1-\gamma}{\epsilon+\gamma}$. *Equilibrium aggregate output and equilibrium labor are given by the*

[36] Note, however, that the intersectoral network imposes certain restrictions on sectoral comovement. Empirical evidence for these patterns has been found by Foerster et al. (2011).

[37] That is, we locate the point on the aggregate production function that simultaneously satisfies the household's optimal choice between consumption and labor.

following functions of productivity and financial friction shocks:

$$\log GDP(\boldsymbol{\phi}) = \Gamma_q \left[\mathbf{q}\log \mathbf{z} + \mathbf{q}\log \boldsymbol{\phi} - \mathbf{d}\log \mathbf{a}(\boldsymbol{\phi}) + K\right] + \Gamma_\psi \log\left(1+\psi(\boldsymbol{\phi})\right), \tag{32}$$

$$\log \ell(\boldsymbol{\phi}) = \Lambda_q \left[\mathbf{q}\log \mathbf{z} + \mathbf{q}\log \boldsymbol{\phi} - \mathbf{d}\log \mathbf{a}(\boldsymbol{\phi}) + K\right] - \Lambda_\psi \log\left(1+\psi(\boldsymbol{\phi})\right), \tag{33}$$

in which the scalars Γ_q, Γ_ψ, Λ_q, *and* Λ_ψ *are given by*

$$\Gamma_q = \tfrac{1+\beta}{1+\beta\mathbf{d}\mathbf{e}_N}, \quad \Gamma_\psi = 1 - \tfrac{\gamma}{1-\gamma}\tfrac{\beta(1-\mathbf{d}\mathbf{e}_N)}{1+\beta\mathbf{d}\mathbf{e}_N}, \quad \Lambda_q = \tfrac{\beta}{1+\beta\mathbf{d}\mathbf{e}_N}, \quad \textit{and} \quad \Lambda_\psi = \tfrac{\gamma}{1-\gamma}\tfrac{\beta}{1+\beta\mathbf{d}\mathbf{e}_N}. \tag{34}$$

Theorem 2 provides closed-form solutions for both equilibrium GDP and labor in this economy. In comparison to Theorem 1, we see that when labor supply is endogenous, the effect of individual productivity and financial shocks on aggregate output are essentially the same as when labor supply is exogenous, modulo a scalar multiple on the influence vector given by Γ_q. As a result, the interpretation of equation (32) is similar to that discussed previously. For intuition regarding the sign and magnitude of the scalar Γ_q, however, we again consider the following special limit case of constant returns to scale.

Limit case of CRS Technology. Consider the limit case as the sectoral production functions approach constant returns to scale.[38] In this limit all entries of $\mathbf{d}$ approach zero: $\mathbf{d} \to (0, \ldots, 0)$, which implies that $\Gamma_q \to \frac{\epsilon+1}{\epsilon+\gamma} > 0$ and $\Gamma_\psi \to \frac{\epsilon}{\epsilon+\gamma} > 0$. Therefore, we may rewrite the equations (32) and (33) as follows:

$$\log GDP(\boldsymbol{\phi}) = \frac{\epsilon+1}{\epsilon+\gamma}\left(\mathbf{q}\log \mathbf{z} + \mathbf{q}\log \boldsymbol{\phi}\right) + \frac{\epsilon}{\epsilon+\gamma}\log\left(1+\psi(\boldsymbol{\phi})\right) + const, \text{ and} \tag{35}$$

$$\log \ell(\boldsymbol{\phi}) = \frac{1-\gamma}{\epsilon+\gamma}\left(\mathbf{q}\log \mathbf{z} + \mathbf{q}\log \boldsymbol{\phi}\right) - \frac{\gamma}{\epsilon+\gamma}\log\left(1+\psi(\boldsymbol{\phi})\right) + const, \tag{36}$$

where we have abstracted from the constant terms.

Recall that in the aggregate production function, the aggregate effects of sectoral productivity shocks and sectoral financial constraints are dictated by the components of the influence vector, $\mathbf{q}$ (abstracting from the profits effect and that of decreasing returns to scale.) When labor supply is endogenous these log-linear effects remain intact, except that the influence vector is now multiplied by the strictly positive coefficient $\frac{\epsilon+1}{\epsilon+\gamma}$, as seen in equation (35).

Similarly, in equation (36) we observe that the effects of productivity shocks and financial frictions on equilibrium labor also take the familiar log-linear form. However, whether labor is increasing or decreasing in these shocks depends on the sign of the coefficient $\frac{1-\gamma}{\epsilon+\gamma}$, in particular whether γ is greater than or less than 1.

To understand this, note that an increase in the real wage always has both a positive substitution effect and a negative income effect on labor supply. Here, one may think of the term $\frac{1}{\epsilon+\gamma}$ as controlling the positive substitution effect while the term $-\frac{\gamma}{\epsilon+\gamma}$ controls the negative income effect.

[38] We take the limit in which $\eta_i \to 1$ for all i.

The income effect is evident from how labor supply responds to movements in $1+\psi(\boldsymbol{\phi})$. Dividends increase the household's capital income; this leads to a negative income effect on labor supply without any corresponding substitution effect. The term $-\frac{\gamma}{\epsilon+\gamma}$ thereby parameterizes this effect.

Therefore, the higher the γ, the greater the income effect. Furthermore, the extent to which labor is either increasing or decreasing in productivity and financial shocks hinges entirely on whether γ is greater than or less than 1. If $\gamma < 1$, then the income effect is outweighed by the substitution effect, and as a result labor responds positively to productivity or the loosening of financial frictions. On the other hand, if $\gamma > 1$, then the income effect outweighs the substitution effect and in this case labor falls in response to greater productivity or the loosening of financial constraints.[39]

Considering again aggregate value added, GDP always responds positively to greater productivity or the loosening of financial frictions. However, whether the scalar multiple $\frac{\epsilon+1}{\epsilon+\gamma}$ on the influence vector in equation (35) is greater or less than one clearly depends on whether γ is greater than or less than one.[40] Thus, as long as the income effect is sufficiently weak, the introduction of endogenous household labor supply amplifies the log-linear effects of productivity and financial shocks.

General Case. Next, moving away from the CRS limit, the overall picture is not as clear. For this we provide a sufficient condition for the sign of these multipliers to be positive.

Lemma 3 *$\gamma < 1$ is a sufficient condition for $\beta > 0$. Given this condition, $\Gamma_q > 0$ and $\Lambda_q > 0$.*

The restriction that the household's preference parameter γ be less than one may not be considered standard. However, note that γ in our environment solely parameterizes the income elasticity of labor, and as such, this seems a reasonable assumption.[41] Similar to using Greenwood-Hercowitz-Huffman (GHH) preferences, by assuming γ be less than one, we are essentially imposing a very weak income effect. In this case, the substitution effect always outweighs the income effect, and labor rises in response to an increase in the real wage. For the rest of this paper, we thus continue under the assumption that $\gamma < 1$ and move forward with the understanding that Γ_q and Λ_q are both strictly positive multipliers.

The Aggregate Labor Wedge. Given equilibrium aggregate consumption and labor, we may now characterize and examine the aggregate labor wedge in this economy. Again following the terminology of Chari et al. (2007), we define the aggregate labor wedge, $1-\tau_\ell(\boldsymbol{\phi})$, implicitly by

$$(1-\tau_\ell(\boldsymbol{\phi}))\left(\bar{\eta}\frac{c(\boldsymbol{\phi})}{\ell(\boldsymbol{\phi})}\right) = \frac{\ell(\boldsymbol{\phi})^\epsilon}{c(\boldsymbol{\phi})^{-\gamma}}. \tag{37}$$

[39] Finally, in the knife-edge case in which $\gamma = 1$ (i.e. log utility), the income effect exactly cancels out with the substitution effect, and labor does not respond at all to these shocks through the real wage. It will however respond through the profits term.

[40] The scalar is greater than 1 when $\gamma < 1$, and less than 1 if $\gamma > 1$.

[41] Note that this model is static, so γ does not control the typical elasticity of intertemporal substitution. Similarly, there is no uncertainty, so γ neither parameterizes risk aversion.

as the wedge that arises between the aggregate marginal product of labor, computed from the aggregate production function given in (27), and the marginal rate of substitution between consumption and labor. A large literature has empirically documented significant movement of this wedge at the business cycle frequency; see e.g., Hall (1997), Rotemberg and Woodford (1999), Chari et al. (2007), and Shimer (2009).[42] Using the results of Theorem 2, we find that the aggregate labor wedge that arises in the equilibrium of this economy may be characterized simply as follows.

Proposition 4 *In equilibrium an aggregate labor wedge emerges which satisfies*

$$\log\left(1-\tau_{\ell}\left(\boldsymbol{\phi}\right)\right)=-\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right)-\log\bar{\eta}. \tag{38}$$

We therefore see that the underlying financial frictions in this economy manifest themselves at the macro level not only as an efficiency wedge but also as an aggregate labor wedge. Furthermore, we find that the labor wedge is driven solely by movements in the profits function: $1+\psi\left(\boldsymbol{\phi}\right)$.

To understand this result, consider a tightening of the financial constraint of a particular sector i. This tightening will have two effects. First there is the misallocation effect of diverting inputs from this constrained sector to another sector, thereby leading to a loss in aggregate TFP as discussed previously.

However, there is a second effect that results from this tightening. As sector i is forced to reduce its production below its optimal level, it charges a price for its good above its marginal cost of producing. This price markup at the sectoral level contributes to a price markup at the aggregate level, thereby translating into an aggregate labor wedge. That is, the household faces a distorted real wage–a real wage lower than would have occurred in a frictionless economy. This distorts the household's optimal consumption-labor choice, thereby leading to lower total GDP.

In fact, to see this clearly suppose sector i is the only sector in the economy, as in a representative firm model. Then a tightening of this sector's financial constraint would in fact not lead to any misallocation as there would be no way for labor to be diverted away from this firm. There would thus be no loss in TFP. Instead, this tightening would lead to a labor wedge distortion: in order to induce the sector to hire enough labor to clear the market, the real wage must fall. This is equivalent to the firm charging a mark-up of price over marginal cost. This labor wedge distortion would thus lead to less aggregate value added in the economy.

Therefore, financial frictions, unlike productivity shocks, lead to a labor wedge at the aggregate level. The clean characterization of this wedge in Proposition 4 is due to the fact that the profits function $1+\psi\left(\boldsymbol{\phi}\right)$ coincides with the aggregate markup. As financial constraints restrict output, each sector charges a mark-up over its marginal costs. This mark-up translates into profits paid to the household in the form of dividends. Therefore, the function $\psi\left(\boldsymbol{\phi}\right)$ characterizes not only the profits accrued to the household's income, but also represents how these sectoral mark-up distortions

[42] Karabarbounis (2014) shows that most of the time variation in the labor wedge is on the household rather than on the firm side. However, this is not true during the recent recession.

manifest themselves as an aggregate labor wedge. Finally, note that the shape and magnitude of the aggregate labor wedge crucially depend on the network structure: the network matrix $\mathbf{W}$ is a key determinant of the $\psi(\boldsymbol{\phi})$ function.

Relation to the literature. Finally, as mentioned previously, there are a few papers that have examined the effects of sectoral wedges in macroeconomic models with intermediate good trade across sectors. These papers include Jones (2011, 2013); Basu (1995), Ciccone (2002), and Yi (2003). With the exception of Jones (2013), the papers cited here have analyzed the aggregate effects of distortions within specific input-output structures. In contrast, our model allows for a completely general network of input-output linkages and provides a solution method for characterizing the economy's equilibrium allocations and prices given any network configuration.

Jones (2013) builds a growth model with a general input-output structure and sectoral distortions and shows that input misallocation at the micro level affects total factor productivity at the aggregate level. In this sense, Theorem 1 in our paper relates to Jones's work. The main argument made in Jones (2013) is that input-output linkages generate a powerful multiplier on misallocation which can help explain the large cross-country income differentials observed in the data.

Hence, there are clear similarities between our paper and Jones (2013): both examine how different network structures amplify sectoral distortions and result in disparate aggregate outcomes.[43] There are, however, two substantial differences. First, in terms of our model and analysis, we incorporate an endogenous labor supply decision. This allows us to examine how sectoral distortions generate an aggregate labor wedge, as in Proposition 4. Our analysis therefore decomposes the aggregate effects of frictions in production networks into two channels: their effects on the efficiency wedge and their effects on the labor wedge.

In particular, in the following section, Section 4, we explore a few simple network economies and show how different network structures can generate very different responses in these two wedges. Our decomposition thereby provides a deeper understanding of how the network structure amplifies the aggregate effects of sectoral distortions.

The second main difference is in regards to our quantitative analysis. While Jones looks at the implications of input-output networks within the context of long-run cross-country growth differentials, we explore its implications in the context of business cycles. In particular our quantitative analysis focuses primarily on the distortionary effects of financial frictions during the recent financial crisis and recession (see Section 5).

[43] The models are essentially similar. Jones (2013) allows for international trade, while our model allows for decrease returns to scale.

3.4 An Economics 101 Description

To conclude this section, we briefly illustrate the aggregate effects of financial shocks using two simple graphs.[44] Consider first Figure 1a which illustrates the aggregate equilibrium outcomes when financial frictions are not present. The x-axis corresponds to labor (hours) and the y-axis to GDP (or aggregate consumption). The blue-ticked curve plots the equilibrium production function: aggregate output as a function of aggregate labor as presented in Theorem 1. This curve summarizes the technological frontier of the economy. The red-solid line, on the other hand, represents the household's outermost indifference curve between consumption and labor that is feasible given the technological frontier. As such, without frictions the equilibrium allocation of labor and consumption occurs where the indifference curve lies tangent to the production function, as is taught in any undergraduate economics textbook. We mark this allocation with a red dot. Finally, the thin dashed black line simply indicates the hyperplane that runs through this tangency point, its slope equal to the equilibrium price of consumption.

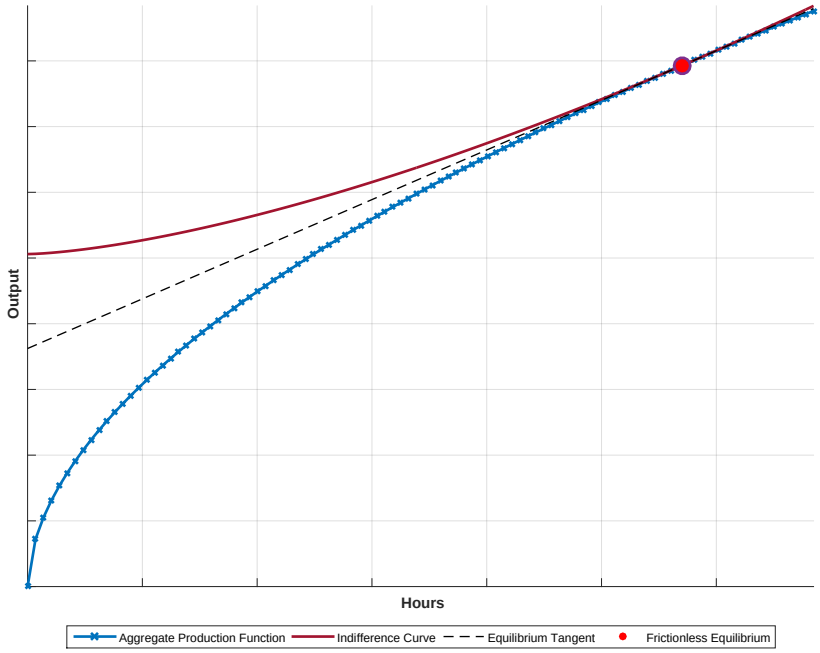
Figure 1b then illustrates the aggregate effects of financial shocks. Depending on the network architecture and which sectors are affected, two changes occur in equilibrium when frictions are introduced. First, financial frictions generically lead to sectoral misallocation of inputs which reduces the aggregate productivity of the economy. As a result, relative to Figure 1a, the blue-ticked production function in Figure 1b shifts inward; the extent of this shift depends on the network structure.

Second, as firms become financially constrained, an aggregate labor wedge emerges. Graphically, this implies that the equilibrium no more occurs at the tangency point of the household's indifference curve and the aggregate production function; instead there exists a wedge between the marginal rate of substitution and the marginal product of labor. We mark this new equilibrium allocation with a red diamond in Figure 1b. It is evident that at this new allocation, the household's indifference curve and the aggregate production function meet but do not lie tangent to one another. As a result, financial frictions overall lead to a fall in both aggregate output and labor (as characterized in Theorem 2).

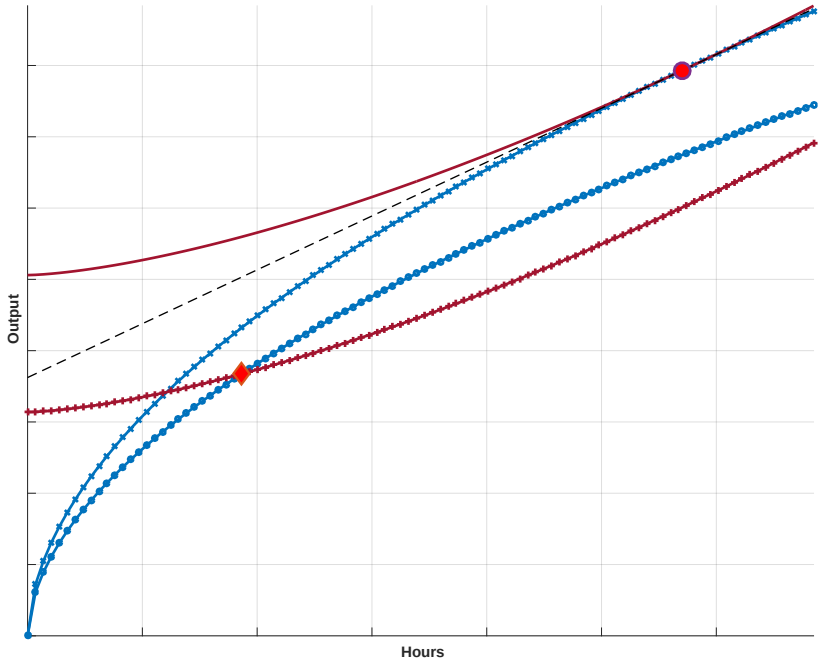
4 Explorations of Simple Network Economies

In this section we explore the effects of financial frictions in a few simple economies, each of which is nested as a special case of the general framework presented in the previous section. Each economy consists of three sectors, indexed by $i = 1, 2, 3$, yet in each case they are arranged in different

[44] The figures constructed here are generated using the same parameters values as in our benchmark calibration described in Section 5.



(a) Aggregate Production Function and Consumption Labor Isocurve



(b) Effects of Financial Shocks

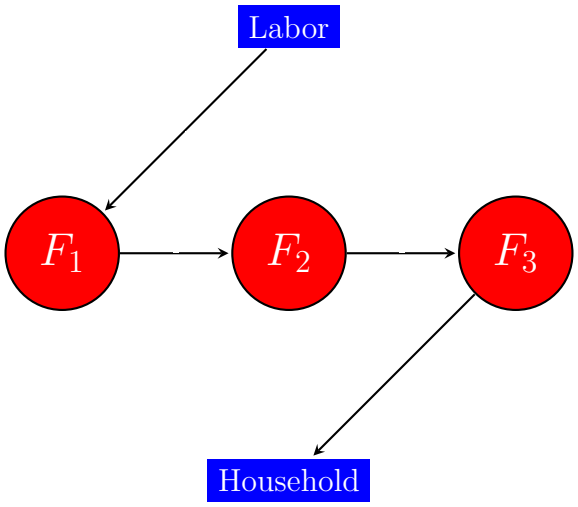


Figure 2: Vertical-Chain Network

production structures. Given their simple nature, these economies help convey some basic intuition as to how different network structures alter the aggregate effects of financial shocks.

We first study two distinct and polar opposite economies: the vertical and the horizontal economy. In the vertical economy production is arranged in a vertical supply chain connecting all three sectors. In contrast, in the horizontal economy each sector operates completely in isolation from one another. Finally, we close this section by considering two more economies that are natural hybrids of the vertical and horizontal economies. All proofs for this section are provided in Appendix B.

The Vertical Economy. In the first of these economies, production is organized in a vertical supply chain. Firm 1 purchases labor and produces an intermediate good purchased by firm 2. Firm 2 uses this input to produce another intermediate good purchased by firm 3. Finally firm 3 uses this input to produce the final good consumed by the household. See the diagram in Figure 2. This structure is similar to the vertical economies seen in Gofman (2015), Kiyotaki and Moore (1997a), Kim and Shin (2012) and Kalemli-Ozcan et al. (2014).

Accordingly, we write the firms' production functions as follows

$$y_1 = (z_1 \ell_1)^{\eta_1}, \quad y_2 = (z_2 x_{21})^{\eta_2}, \quad y_3 = (z_3 x_{32})^{\eta_3}.$$

Market clearing in commodities 1 and 2 are given by $x_{21} = y_1$ and $x_{32} = y_2$. The household consumes only the good produced by the third sector: $c = y_3$. Finally, labor is supplied only to firm 1; labor market clearing thereby dictates $\ell = \ell_1$.

For simplicity we consider the limit economy in which technology approaches constant returns to scale: $(\eta_1, \eta_2, \eta_3)' \to \mathbf{e}_3$. In this limit economy, applying the results of Theorem 2, we obtain an influence vector given by

$$\mathbf{q} = (1, 1, 1).$$

Thus, all financial shocks are equally important. As mentioned previously, one way to interpret this is to consider the share of labor that flows through each firm. In this vertical production chain, there is only one route through which labor can be transformed into the final consumption good. Thus, all labor flows through each and every firm. Hence, any friction affecting any part of the supply chain will have a 1-for-1 effect on output; moreover, no link in the supply chain is more important than any other.

In the following proposition we characterize GDP as well as the efficiency and labor wedges in this economy. In order to focus on the effects of financial frictions, we simply assume that productivities are constant.[45]

Proposition 5 *In the simple vertical production network with constant productivities and in the limit as technology approaches constant returns to scale, the log of equilibrium GDP is given by*

$$\log GDP\left(\boldsymbol{\phi}\right) = \frac{1}{\epsilon+\gamma}\left(\log\phi_1 + \log\phi_2 + \log\phi_3\right) + const.$$

The equilibrium efficiency and labor wedges are given by

$$\zeta\left(\boldsymbol{\phi}\right) = 1 \quad \text{and} \quad 1 - \tau_\ell\left(\boldsymbol{\phi}\right) = \phi_1\phi_2\phi_3.$$

As expected, aggregate GDP is affected symmetrically by each financial shock. Perhaps more surprisingly though, in this economy financial frictions *do not* induce an efficiency wedge. Ex post, the reason is quite intuitive. In this economy there is no misallocation of labor: due to the vertical production structure, there is only one route through which labor may travel in order to be transformed from primary input to final consumption. Even if an individual firm is constrained, labor cannot be misallocated away from that firm along its path. Thus, the aggregate production function is not altered by sectoral distortions: given an aggregate amount of labor, the same quantity of total output is produced.[46]

However, while financial frictions do not create an efficiency wedge, they do induce an aggregate labor wedge. In particular, the individual frictions compound to generate the labor wedge. If financial constraints tighten, the final good price level must increase (i.e. the real wage must fall) in order for firm 1 to demand the same number of workers so that labor market clears (and consequently for firm 2 to demand the goods produced by firm 1, and for firm 3 to demand the goods produced by firm 2). Therefore, the real wage must fall relative to the aggregate marginal product of labor and as a result a labor wedge emerges. Of course, the financial frictions among these firms affect the aggregate labor wedge symmetrically, as each sector is equally important.

[45] For example, let $\log \mathbf{z} = (0, 0, 0)'$.

[46] Other related models of vertical production chains include Bak et al. (1993), Kremer (1993), Levine (2012). Effects on TFP in those models occurs either because output can be disrupted or delayed at some point in the chain.

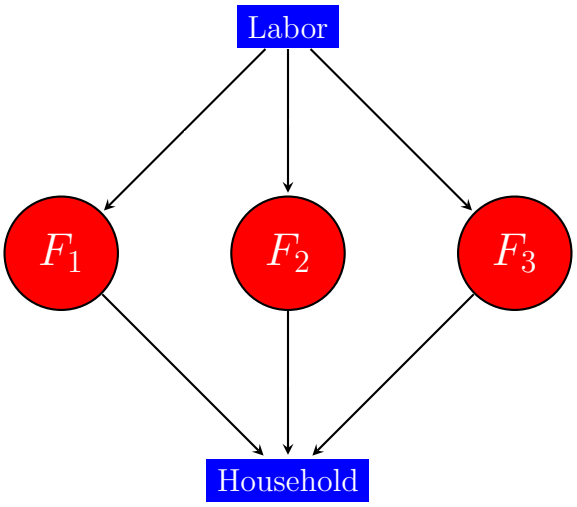


Figure 3: Horizontal Network

The Horizontal Economy. The next economy we consider is the polar opposite of the pure vertical economy: the completely horizontal economy. In this economy, each firm uses labor as its sole input to produce a differentiated final good consumed by the household. All sectors thus operate completely in isolation from one another: there is no intermediate good trade among these firms. See Figure 3. We believe this horizontal network is particularly relevant as it is the typical structure of most standard macroeconomic models, for example those with Dixit-Stiglitz preferences.

More specifically, we write the production functions of these firms as follows:

$$y_1 = (z_1\ell_1)^{\eta_1}\,, \quad y_2 = (z_2\ell_2)^{\eta_2}\,, \quad y_3 = (z_3\ell_3)^{\eta_3}\,.$$

The goods of each sector are consolidated into a final consumption basket consumed by the household:

$$c = \prod_{j=1}^{3} c_j^{1/3_j}, \tag{39}$$

where for simplicity we have assumed that the household consumption shares are equal across the three goods. Market clearing conditions for these commodities are given by by $c_j = y_j$ for each j, while market clearing in the labor market satisfies $\ell_1 + \ell_2 + \ell_3 = \ell$.

We again consider the limit case as technology approaches constant returns to scale.[47] In this limit economy, we obtain an influence vector given by

$$\mathbf{q} = (1/3, 1/3, 1/3)\,.$$

Again all shocks are equally important. This symmetry is expected as all firms produce in isolation and are identical in terms of their technology and share of household consumption.

[47] That is, we let $(\eta_1, \eta_2, \eta_3)' \to \mathbf{e}_3$.

However, unlike the vertical economy, the influence of each sector on aggregate output is only 1/3 rather than 1. To understand this, consider the share of labor which flows through each firm in the frictionless economy. As each of the three firms hire the same amount of labor and have equal shares of the household's consumption, it must be the case that they share the labor supply equally. Thus 1/3 of the labor supply flows through each firm in the frictionless economy, coinciding with the firm's influence.

The following proposition characterizes GDP as well the efficiency and labor wedges in this economy (again abstracting away from productivity shocks).

Proposition 6 *In the simple horizontal production network with constant productivities and in the limit as technology approaches constant returns to scale, the log of equilibrium GDP is given by*

$$\log GDP(\boldsymbol{\phi}) = \frac{\epsilon+1}{\epsilon+\gamma}\frac{1}{3}\left(\log\phi_1 + \log\phi_2 + \log\phi_3\right) - \frac{\epsilon}{\epsilon+\gamma}\log\left(\phi_1+\phi_2+\phi_3\right) + const.$$

The equilibrium efficiency and labor wedges are given by

$$\zeta(\boldsymbol{\phi}) = \frac{\left(\phi_1\phi_2\phi_3\right)^{1/3}}{\frac{1}{3}\left(\phi_3+\phi_2+\phi_1\right)} \quad \text{and} \quad 1-\tau_\ell(\boldsymbol{\phi}) = \frac{1}{3}\left(\phi_1+\phi_2+\phi_3\right).$$

As expected, aggregate GDP is affected symmetrically by each of the financial shocks. The same is true for the impact of each of these shocks on either the efficiency or the labor wedge. Note that in contrast to the vertical economy, each constraint now contributes additively to the aggregate labor wedge.

An interesting case in this economy to consider is when all financial shocks are symmetric. That is, suppose that each financial constraint is equally tight: $\phi_i = \bar{\phi}$. We then find that

$$\zeta(\boldsymbol{\phi}) = 1 \quad \text{and} \quad 1-\tau_\ell(\boldsymbol{\phi}) = \bar{\phi}.$$

Therefore in the case of symmetric financial shocks there is no efficiency wedge and only a labor wedge.

Why is there no efficiency wedge? If all firms are equally constrained, then labor cannot be inefficiently misallocated from one firm to another. A common financial shock does not affect the *relative* use of hours across sectors: it results in the exact same allocation of labor across firms as in a completely frictionless economy. Therefore, the lack of misallocation implies no movement in the efficiency wedge, or aggregate TFP.

On the other hand, in the case of symmetric financial shocks a labor wedge emerges. The reason for this is the same as before: if financial constraints tighten, it acts like a tax–the real wage must fall relative to the marginal product of labor in order for firms to demand enough labor for the market to clear. [48]

[48] This is isomorphic to the effect of a constant proportional tax levied across all sectors and the tax proceeds

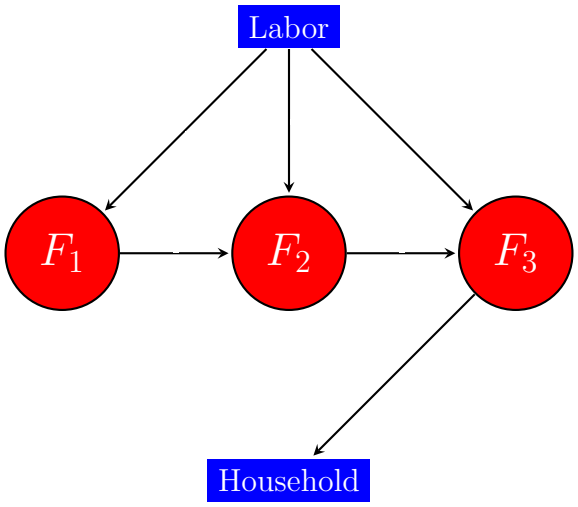


Figure 4: Hybrid-Chain Network

Finally, in the more general case in which financial constraints are asymmetric, an efficiency wedge does emerge. This is because labor inefficiently moves from a more constrained firm to a less constrained firm thereby resulting in misallocation relative to the frictionless economy. How input misallocation in horizontal economies manifests itself as an efficiency wedge at the aggregate level is a topic that has been studied quite extensively, see e.g. and Hsieh and Klenow (2009), Restuccia and Rogerson (2008), and Hopenhayn (2014).

The Hybrid Chain economy. We now consider an economy that is a particular hybrid between both the vertical and the horizontal economies examined above. In what we call the hybrid chain economy, firms are arranged as before in a vertical supply chain in which firm 1 sells an intermediate good to firm 2, firm 2 sells an intermediate good to firm 3, and firm 3 sells a final good to the household. However, unlike the previous pure vertical supply chain, we now allow labor to be an input for all firms in the economy, not just the most upstream firm. See Figure 4.

The production functions of these firms are thus written accordingly:

$$y_1 = (z_1 \ell_1)^{\eta_1}, \quad y_2 = \left(z_2 \ell_2^{\alpha_2} x_{21}^{1-\alpha_2}\right)^{\eta_2}, \quad y_3 = \left(z_3 \ell_3^{\alpha_3} x_{32}^{1-\alpha_3}\right)^{\eta_3}.$$

The household consumes the final good produced by firm 3.[49] Clearly, this hybrid economy nests the pure vertical economy when the labor shares of firms 2 and 3 are equal to zero: $\alpha_3 = \alpha_2 = 0$.

In the CRS limit of the hybrid chain economy, we obtain an influence vector given by

$$\mathbf{q} = \left((1-\alpha_2)(1-\alpha_3), 1-\alpha_3, 1\right). \tag{40}$$

rebated lump-sum to the households.

[49] Market clearing in commodities 1 and 2 are given by $x_{21} = y_1$ and $x_{32} = y_2$, while the household consumes the final good: $c = y_3$. Labor market clearing implies $\ell_1 + \ell_2 + \ell_3 = \ell$.

First, note that this nests the vertical economy influence vector when $\alpha_3 = \alpha_2 = 0$, in which case $\mathbf{q} = (1, 1, 1)$. Relative to the vertical economy in which the influence of all firms is equal to one, the effect of firm 1 and firm 2's financial constraints are attenuated as labor can now be used as a substitutable input into firm 3's production. Thus, if firms 1 or 2 are constrained, firm 3 can substitute away from its intermediate good by hiring more workers. Therefore, it is no more the case that there is only route through which labor may flow en route to become the final good.

To understand the entries of the influence vector, consider how much labor flows through each firm in the undistorted economy. In the frictionless economy, all labor flows through firm 3. Thus, its influence is 1. For firm 2, all labor flows through firm 2 except for the α_3 share that only goes to the most downstream firm, firm 3. Therefore, the amount of labor which flows throw firm 2 is $1 - \alpha_3$. Finally for firm 1, its labor share of the aggregate production function is $(1 - \alpha_2)(1 - \alpha_3)$, as the rest flows directly through firm 2 and firm 3. As a result, firm 3 has the greatest influence, followed by firm 2, then firm 1. Sectoral financial shocks thus have greater aggregate effects in this economy if they hit more downstream firms.

The Hybrid Circle Economy. Finally, we consider one last economy which is similar to the previous hybrid chain economy, but instead we tie the two ends of the chain together so that production becomes a closed circle. Suppose that the production functions are given by,

$$y_1 = \left(z_1 \ell_1^{\alpha} x_{13}^{1-\alpha}\right)^{\eta_1}, \quad y_2 = \left(z_2 \ell_2^{\alpha} x_{21}^{1-\alpha}\right)^{\eta_2}, \quad y_3 = \left(z_3 \ell_3^{\alpha} x_{32}^{1-\alpha}\right)^{\eta_3}.$$

Thus, each firm uses both labor and an intermediate good, where for simplicity we set the labor share to be constant across all sectors. As before, firm 2 uses as input the good produced by firm 1, and firm 3 uses as input the good produced by firm 2. In addition, however, to close the circle firm 1 now uses as input the good produced by firm 3.

As in the horizontal economy, we assume the household consumes a basket composed equally of all three goods, as in (39).[50] Clearly, this hybrid economy nests the pure horizontal economy when the labor share of all firms, α, is equal to 1.

In the CRS limit of the hybrid circle economy, we obtain an influence vector given by

$$\mathbf{q} = \frac{1}{3}\left(\frac{1 + (1-\alpha) + (1-\alpha)^2}{1 - (1-\alpha)^3}\right)(1, 1, 1). \tag{41}$$

Note that this nests the horizontal economy influence vector when $\alpha = 1$. Moving away from the horizontal economy, a smaller labor share increases the influence of each sector symmetrically.

In fact, consider the limit as the labor share approaches zero. In this limit, the entries of the

[50] Market clearing in each of the commodities are given by $c_1 + x_{21} = y_1$, $c_2 + x_{32} = y_2$, and $c_3 + x_{13} = y_3$. Labor market clearing implies $\ell_1 + \ell_2 + \ell_3 = \ell$.

influence vector diverge to infinity:

$$\lim_{\alpha \to 0} \mathbf{q} = (\infty, \infty, \infty) .$$

This intuition for this divergence is the following: in this limit the circle structure becomes essentially an infinite loop of the primary labor input. As labor travels through the input-output system to eventually become household consumption, it goes through all three firms over and over and over again. As a result, a financial shock to any sector has a tremendous effect on aggregate consumption.[51]

5 Quantitative Assessment

This section provides a quantitative assessment of the model. We calibrate the production network in our model to match the United States input-output tables provided by the Bureau of Economic Analysis. We then test the model's predictions for aggregate output for the years following the 2007-2008 Financial Crisis. The goal of this analysis is to provide a quantitative sense of how much the input-output network may amplify the response of GDP to movements in financial constraints. We also take a close look at cross-sectional effects—that is, we examine which sectors may generate the largest aggregate effects and which sectors are the most vulnerable to aggregate shocks.

5.1 A Visual Overview of the US Input-Output Architecture

Input-Output data is provided by the Bureau of Economic Analysis; at the five digit industry level there are 384 industries, and the input-output matrix contains nearly 138,000 entries. This is a lot of information. Thus, before diving into the details of this data, we first try to provide a visual overview of how the production network of the US economy is structured.[52]

Production Network Flow Diagram. Figure 5 presents a Sankey (flow) diagram that illustrates intermediate input flows and added value flows across 2-digit sectors. This figure contains a number of curves that connect vertical bars. Each vertical bar on the left-hand side corresponds to a particular 2-digit sector. The size (length) of each bar corresponds to the dollar-value of intermediate goods produced by each of these sectors. The vertical bars in the middle also correspond to 2-digit sectors. Finally, the shaded curves emanating from the left-hand side bars represent the

[51] This example is similar to one found in Jones (2013)–see the simple example in Section 3.1 of that paper. In Jones's example, intermediate goods produced today are used in tomorrow's production, thereby creating an intermediate-good chain across periods of production akin to the intermediate-good chain across firms presented here. As in that example, the share of intermediate good usage in the production function leads to a macro-level multiplier effect on micro-level distortions.

[52] The visual patterns we present here are based on our construction of the U.S. Input-Output matrix that we discuss in detail in the following section.

scaled flows of intermediate good commodities produced by these sectors and purchased by the sectors represented by the middle vertical bars.

Another set of curves emerge from the middle vertical bar sectors and flow towards the set of vertical bars on the right-hand side. The vertical bars on the right-hand side represent final uses of commodities: consumption, investment, and government expenditures. These curves thus represent the flows of commodities produced by the sectors to their final uses.

The length of the vertical bars in the middle is the max of their intermediate good inflows and final goods outflows. Thus a sector that receives a large inflow from the left but generates only a small outflow to the right is mostly a buyer and producer of intermediate inputs. On the other hand, a sector that receives a small inflow from the left but generates a large outflow to the right is more like a final goods sector.

Several generalizations about the U.S. production network can be directly observed from this Sankey plot. First, there appear to be three broad categories of sectors. A first group—the primary sectors—supply a large outflow of primary production inputs (materials) to other sectors but do not receive a large inflow of inputs nor do they account for a significant share of final good uses. This group of sectors is composed of agriculture, mining, and utilities.

A second group is made up of sectors that either purchase a large amount of intermediate goods from other sectors and/or supply a large amount of intermediate inputs in relation to their added value. We can think of these sectors as the bulk of the U.S. supply chain. These sectors include manufacturing, construction, and service sectors that support production. We see from the Sankey diagram that manufacturing is both the largest intermediate producer and the largest user of intermediate inputs. At the same time, manufacturing also produces a large proportion of aggregate consumption and investment goods. Construction plays a small role as an intermediate goods producer since it produces investment goods that enter production as capital investment but not as intermediate inputs. Furthermore, this group also includes service sectors that support these production industries including professional services (lawyers, accountants, management firms), supply chain services (transportation, warehousing), information services (IT and advertisement), and wholesale.

Finally, a third set of sectors are those that do not use a large amount of intermediate goods nor do they supply a large amount of intermediate goods to other sectors. Instead, these sectors primarily employ labor and capital in their production and supply their final product to consumers. Among these we find retail and other services, education, health, government, and retail.[53]

Input-Output Matrix Heatmap. Figure 6 presents a heatmap of the input-output matrix for the United States in 2006 at the five-digit industry level. As mentioned above, there are 384 five-digit sectors. In this figure, each row represents a sector supplying inputs to the sectors represented by each column. The entries of the matrix correspond to the share of intermediate good usage of

[53]Note that when a commodity is sold to retail, it is not counted as an input because the product is not transformed.

each sector in a given column. Lighter colors reflect more intensive input usage while darker colors reflect less intensive input usage. In this figure we have also added labels to groups of sectors to indicate their two-digit sector family.

Several additional patterns emerge from this figure. First, we observe that the diagonal features light colors. This reflects the fact that many intermediate good transactions occur within the same sector, even at the five-digit level. We would expect this diagonal term to vanish as we reach higher and higher levels of disaggregation. Thus, for our quantitative results we test their robustness with an alternative specification in which we set all diagonal elements to zero.

Next, we find a high concentration of light colors along certain rows. This indicates particular sectors that supply inputs to many other sectors. For example, the rows corresponding to utilities are light for almost all column entries reflecting the need for energy by all sectors. A similar pattern is found for service sectors that support production—a reflection that there is a substantial amount of outsourcing of professional tasks outside the firm.

Finally, we again find that the manufacturing sectors are special: we see light colors both directed towards and emerging from manufacturing sectors. The manufacturing cluster of sectors are therefore both buyers and sellers of a significant amount of intermediate goods amongst themselves. We further observe that sectors that belong to construction demand many inputs from other sectors, but supply few inputs to other sectors. Again, this is because construction often produces investment goods that are treated as final uses not as intermediate goods.

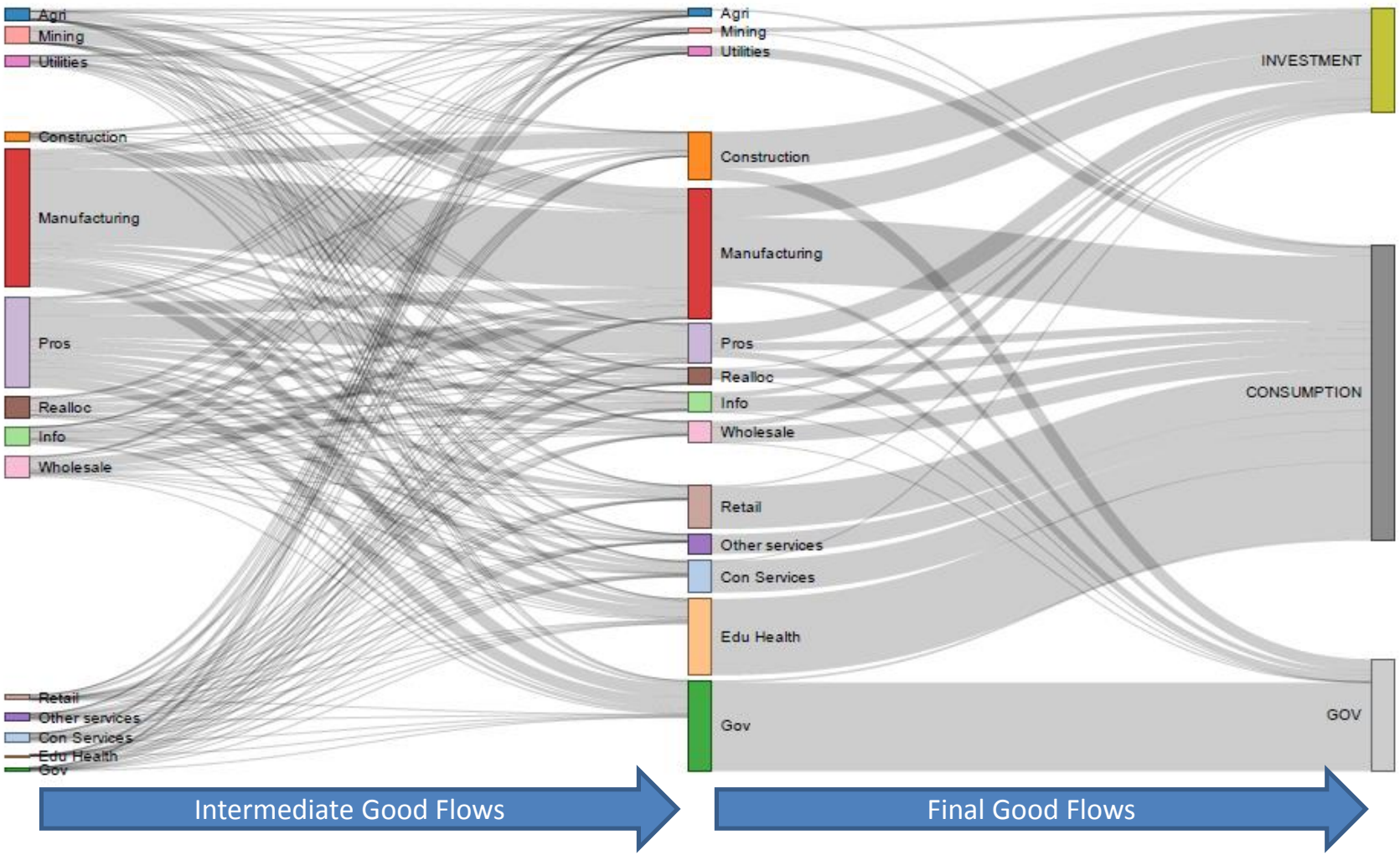


Figure 5: **Flows of Intermediate Inputs and Added Value.**

Note: The width of gray curves are proportional to cross-sectoral flows. Total sectoral flows.

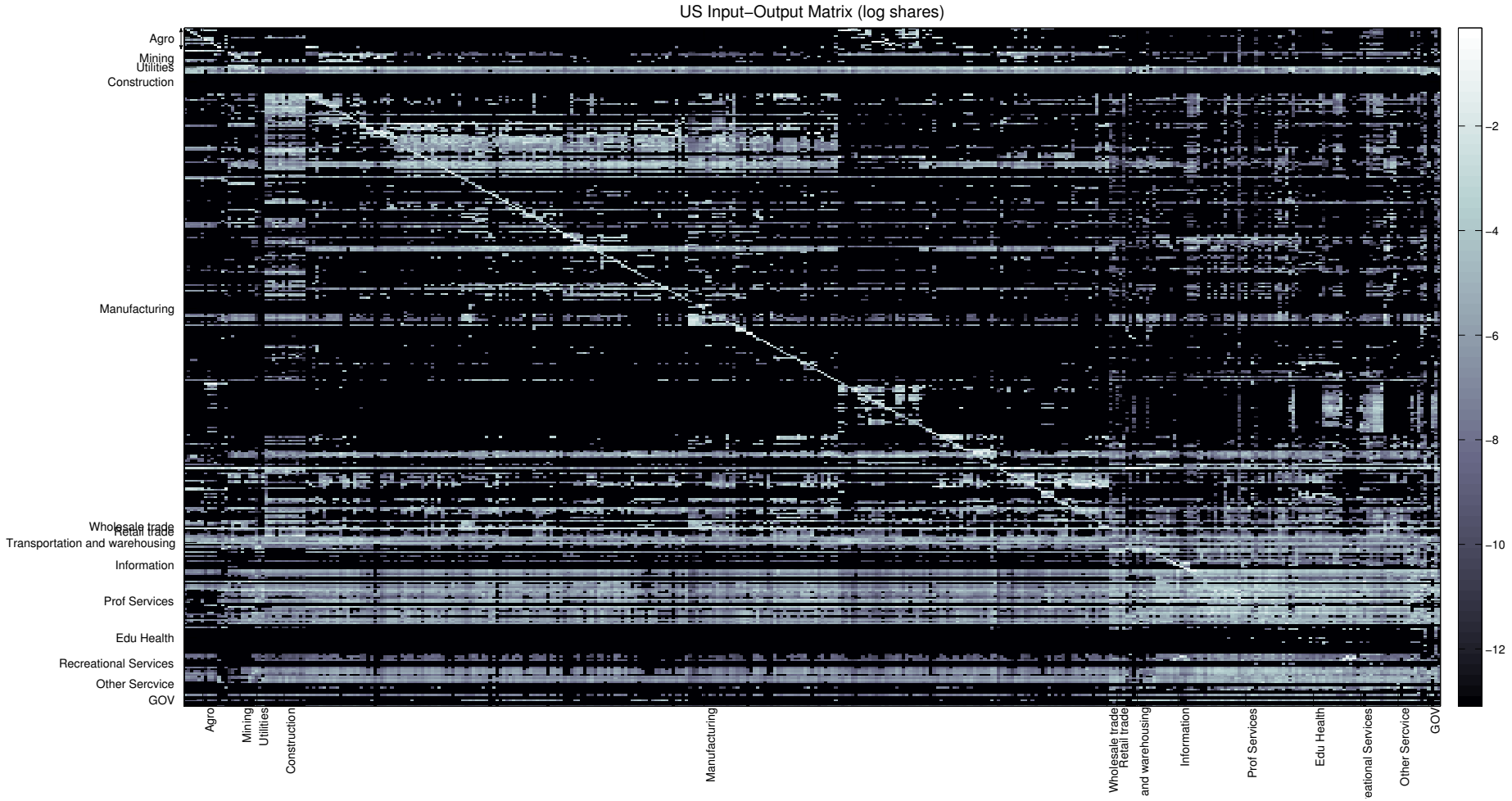


Figure 6: US 5-Digit Input-Output Matrix

Note: Rows in the matrix correspond to sectors supplying intermediate goods to the sectors located in the column. The colors indicate the logarithm of sectoral flows.

5.2 Data and Calibration

5.2.1 The Data

The Bureau of Economic Analysis (BEA) provides data on gross sales for all sectors. Expenditures by each sector on sectoral commodities are recorded in the U.S. Input-Output tables (US IO) where sectors are classified according to the North American Industry Classification System (NAICS). The BEA reports data for two-digit and three-digit industry levels annually from 1997 to 2012 and data for the five-digit industry level every five years. There are 15, 65, and 384 sectors at the two, three, and five-digit industry classification levels, respectively.

We work with the BEA's Use of Commodities by Industries (USES) table. The USES table is organized in the following way: each row corresponds to a sectoral commodity and each column corresponds to either an industry or component of aggregate demand (consumption, investment, etc). An entry in this table reports the expenditures by the column industry on the commodity produced by the corresponding row in billions of US dollars. We use this table to construct the Input-Output production matrix of the US that corresponds to our closed-economy static model. Appendix C descibes this construction in great detail.[54]

5.2.2 Calibration of Preferences, Technology, and the Input-Output Network

Preference Parameters. The household's preference parameters γ and ϵ control the household's labor supply. As discussed earlier, γ governs the wealth effect while ϵ governs the substitution effect. We set $\gamma = 0$ as our baseline value to shut down any wealth effect; for robustness we report results for greater values of gamma. We follow Hall (2009) and set $\epsilon = 1/2$ in our baseline which implies a Frisch elasticity of labor supply of 2. For robustness we report our results also for lower labor supply elasticities.[55]

Expenditure Shares. Under Cobb-Douglas utility, the household's expenditure shares over commodities is constant and parameterized by the vector $\mathbf{v} = (v_1, v_2, \ldots, v_N)'$. As described above, the BEA reports the sales of each sector as final uses. We treat final uses as the analogue of the household's expenditure shares in the model.[56] That is, let $\hat{u}^c_{i,t}$ be the final uses in the data for the commodity produced by industry i in year t. We calibrate $v_{i,t}$ as follows

$$v_{i,t} = \frac{\hat{u}^c_{i,t}}{\sum_{j=1:N} \hat{u}^c_{j,t}}.$$

[54] That is, in Appendix C we descibe how we treat imports, exports, inventories, and investment, as well as the Finance, Insurance, and Real-Estate (FIRE) industries.

[55] An implied Frisch elasticity of labor supply of 2 is higher than the value found in micro studies but is standard in the macroeconomics literature. Macroeconomic models with frictionless labor markets often need a highly elastic labor supply to generate large output responses. These high elasticities can be obtained through indivisible labor. A useful discussion is found in Chetty et al. (2011) and Ljungqvist and Sargent (2011).

[56] Following the corrections we discuss in Appendix C, we construct final uses as domestic absorption minus net imports.

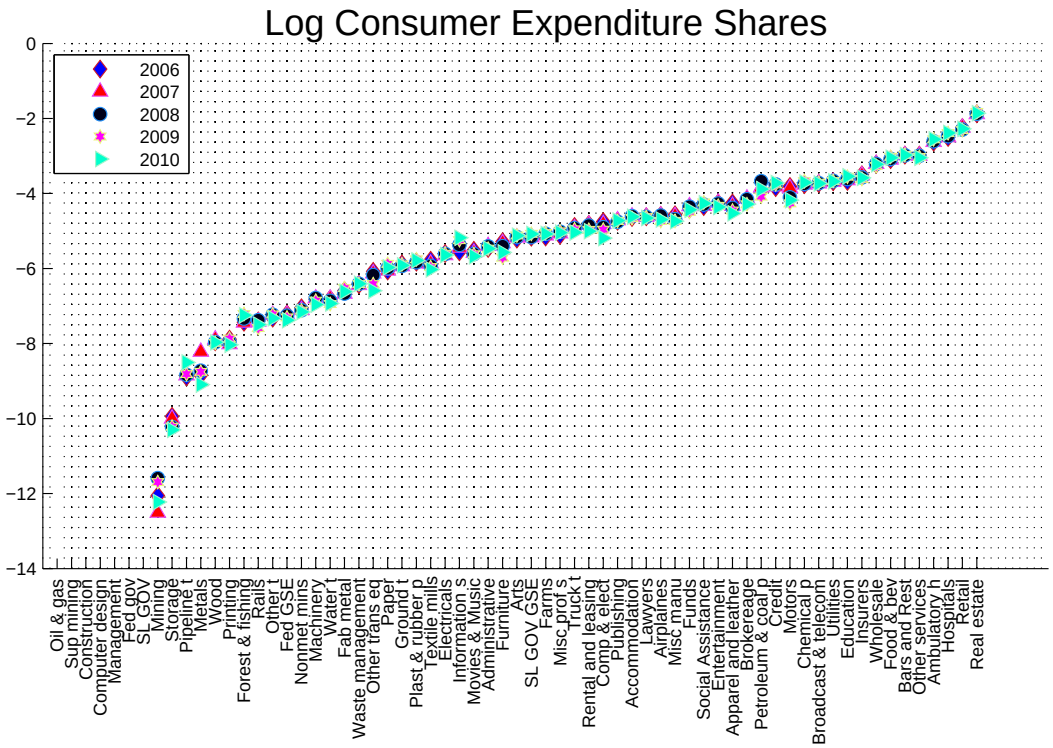


Figure 7: Measured Consumer Shares

Figure 7 reports the calibrated values of $\log v_{i,t}$, for each sector at the 3-digit industry classification level annually from 2006-2010. From this figure we see that expenditure shares at this level are fairly stable from year to year.

Technology and the Input-Output Matrix. For each industry i we calibrate the parameters that govern technology: its labor share α_i, its decreasing returns to scale parameter η_i, and its expenditure shares $\{w_{ij}\}_{j=1:N}$. The latter comprise the entries of the input-output matrix $\mathbf{W}$.

Calibration of the parameters α_i and $\{w_{ij}\}_{j=1:N}$ is straight-forward. Each entry of a USES table reports the expenditure by the sector corresponding to a given column on the commodity supplied by the sector corresponding to a given row. The USES table includes an extra row for labor expenses. Since our firms have a Cobb-Douglas production technology, we proceed in the same way as we did for households: Let $\hat{u}_{ij,t}$ be the expenses of sector i in commodities produced by sector j in year t. We calibrate the entries of the input-output matrix in our model according to:

$$w_{ij,t} = \frac{\hat{u}_{ij,t}}{\sum_{j=1:\hat{N}} \hat{u}_{ij,t}}.$$

The values of this calibration are used in the construction of the input-output matrix heatmap in Figure 6 as discussed earlier. Next, to calibrate $\alpha_{i,t}$, let $\hat{w\ell}_{i,t}$ denote the labor expenses of sector i in year t. Let $\hat{u}_{i,t}$ denote the sector i's total expenditure in year t including the industry's labor

expenses. That is, $\hat{u}_{i,t} = \hat{w}\hat{\ell}_{i,t} + \sum_{j=1:\hat{N}} \hat{u}_{ij,t}$. We thus calibrate the sectoral labor shares as follows:

$$\alpha_{i,t} = \frac{\hat{w}\hat{\ell}_{i,t}}{\hat{u}_{i,t}} = \frac{\hat{w}\hat{\ell}_{i,t}}{\hat{w}\hat{\ell}_{i,t} + \sum_{j=1:\hat{N}} \hat{u}_{ij,t}}.$$

There is one additional technology parameter: the η_i governing the sector's returns to scale. This parameter cannot be calibrated directly from observables, as we explain next.

5.2.3 Calibration of Financial Frictions and Returns to Scale

The main challenge we face in the calibration of our model is that of separately identifying the decreasing returns to scale parameter η_i from the financial wedge ϕ_i of each sector. In particular, note that in the equilibrium of our model the ratio of total firm costs (expenditure) to total firm revenue satisfies:

$$\frac{u_{i,t}}{p_{i,t}y_{i,t}} = \phi_{i,t}\eta_i, \tag{42}$$

for each sector i. Therefore, data on sectoral expenditure and revenue do not allow us to separately identify the parameters η_i and $\phi_{i,t}$—they only help us to identify their product. This is in fact a generic problem. Concerning this particular issue, Jones (2013) writes, "there is a fundamental identification problem: we see data on observed intermediate good shares and we do not know how to decompose that data into distortions and differences in technologies."

To overcome this obstacle, we take the following baseline calibration strategy. Our model assumes that technology, in this case the decreasing returns to scale parameter η_i, is fixed over the length of the business cycle. Given this assumption, one may then attribute all movement in the costs-to-sales ratio over the financial crisis to movement in the wedges. Specifically, as long as η_i remains fixed, all variation in the the left hand side of equation (42) must necessarily be driven by variation in $\phi_{i,t}$.

We believe this is a reasonable strategy as it is rather unlikely that technology shares vary substantially at business cycle frequency. We describe the details of this baseline strategy next. However, in addition we employ two alternative strategies for calibrating the wedges that rely more heavily on their interpretation as financial frictions. We describe those strategies following our description of the baseline.

Baseline calibration of η_i and $\phi_{i,t}$. We first construct a benchmark measure for the level of η_i. From our model we know that η_i must lie in the interval $(0, 1)$ and similarly $\phi_{i,t}$ must lie in $(0, 1]$. We use this information to obtain a lower bound on η_i. In particular, let $\hat{s}_{i,t}$ denote the (observable) total sales of industry i in year t, i.e. the empirical analogue of $p_{i,t}y_{i,t}$. We may then construct a yearly measure of the costs-to-sales ratio for each sector: $\hat{u}_{i,t}/\hat{s}_{i,t}$. According to equation (42), this ratio is equal to the product $\phi_{i,t}\eta_i$.

Consider the time series of this ratio for one industry over the multiple years in our sample. Under the model constraints that η_i is constant over all t and the upper bound on $\phi_{i,t}$ of 1, we infer that the lowest possible value for η_i that would be consistent with the data is the *highest* costs-to-sales ratio in the series.[57] We label this lower bound $\underline{\eta}_i$.

Thus, given the time series data for industry i, the only admissible values for η_i are those that lie in $[\underline{\eta}_i, 1)$. For most exercises, we choose to set our benchmark calibration for η_i at the lower bound: $\eta_i = \underline{\eta}_i$. By setting this benchmark, we are implicitly assuming that in the year with the highest cost-to-sales ratio, there are no financial frictions in this industry, i.e. $\phi_{i,t} = 1$. Therefore, by setting $\eta_i = \underline{\eta}_i$ we minimize the level of financial frictions, and hence obtain the most conservative measures from our model for the effects of these frictions.[58]

Once we set our benchmark of $\eta_i = \underline{\eta}_i$, we back out a time series for $\phi_{i,t}$ simply by applying equation (42) directly to the data. We denote this benchmark measurement by $\hat{\phi}_{i,t}$. Figure 8 shows the values of $\hat{\phi}_{i,t}$ from this calibration for the sub-sample of years around the Great Recession, 2006-2010. It is clear that the year 2009, considered the trough of the Great Recession, features low imputed values for $\phi_{i,t}$ on average across sectors.[59]

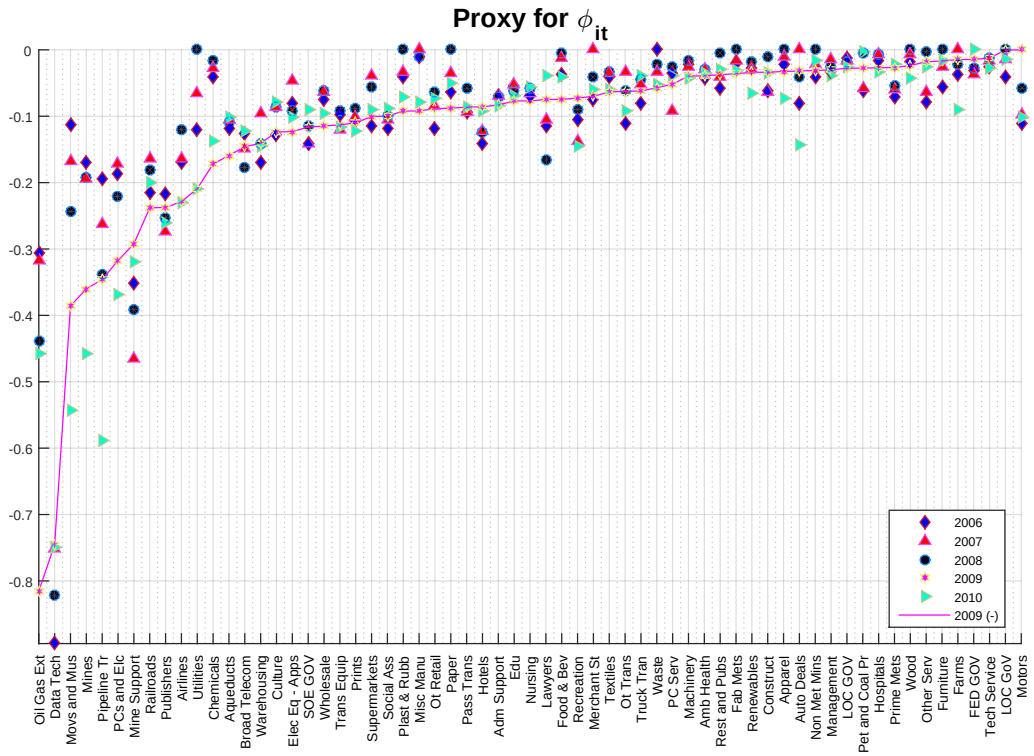


Figure 8: Measured $\hat{\phi}_i$

[57] That is, in years in which the cost-to-sales ratio is lower, it can only be attributed to falls in $\phi_{i,t}$ and not a lower value of η_i, otherwise $\phi_{i,t}$ would have to greater than 1 in the year with the highest cost-to-sales ratio.

[58] In our experiments, we obtain greater aggregate responses from financial frictions when we set η_i to be higher than $\underline{\eta}_i$. We will also report results for η_i near 1.

[59] NBER determined that a trough in business activity occurred in the U.S. economy in June 2009.

This baseline calibration clearly interprets $\phi_{i,t}$ as a catch-all for all sectoral distortions: it makes no distinction as to whether this wedge results from financial frictions, market power, taxes, etc. While we believe that movement in these distortions were likely due to financial constraints during the financial crisis, we cannot rule out the possibility that this variation may be due to other distortionary factors. We thus construct two alternative measurements for $\phi_{i,t}$ using data direclty related to financial frictions and expenses. We present these alternative measures next.

Alternative Measure of $\phi_{i,t}$ using FIRE sectors. In the construction of our input-output matrix, we omit all Finance, Insurance, and Real-Estate (FIRE) industries. We exclude payments to these industries as they do not appear to represent expenditure on intermediate goods in production (see Appendix C for details). However, payments to the FIRE industries may be indicative of the costs firms face in financing their operations. Following this logic, our second method uses information in these expenses as an independent source for calibrating ϕ_i.

Under this method, we maintain the same benchmark value of $\eta_i = \underline{\eta}_i$, but we compute a new measurement for the movement in financial frictions. For each sector i, we sum up its total expenses paid to all FIRE sectors in year t and denote this sum as $\hat{u}_{iFIRE,t}$. We choose as a benchmark the year 2007. For all years following 2007 we construct a new sequence of financial wedges as follows:

$$\phi_{i,t}^{FIRE} = \hat{\phi}_{i,2007} + \left[\left(\frac{\hat{u}_{iFIRE,t}}{\sum_{j=1:N} \hat{u}_{ij,t}} \right)^{-1} - \left(\frac{\hat{u}_{iFIRE,2007}}{\sum_{j=1:N} \hat{u}_{ij,2007}} \right)^{-1} \right].$$

To understand this formulation, suppose that firms finance their intermediate good purchases through short-term loans made by firms in the financial industry. Payments to the FIRE sector could thus be interpreted as the net interest rate payments on these working capital loans, and the ratio $\hat{u}_{iFIRE,t} / \sum_{j=1:N} \hat{u}_{ij,t}$ may be interpreted as the interest rate. The inverse of this ratio would then be analogous to the financial wedge $\phi_{i,t}$.[60] We then take the difference between the wedge induced by these interest rate payments in year t and the wedge induced by these payments in our benchmark year 2007 in order to obtain the net change in $\phi_{i,t}$.[61]

One important observation is that payments to FIRE industries do not fully capture a sector's financial expenses; it simply captures the expenditure on financial industry services as accounted for by the BEA. Yet, part of firms' financial expenses will be captured by their operating revenues which are not counted as financial expenses directly. Thus, while $\phi_{i,t}^{FIRE}$ is not a perfect measure of financial frictions, movements in this variable may provide some sense of the changing financing needs of non-financial firms during the crisis and recession.

[60] In Appendix D we show explicitly how the interest on working capital would enter the firm's first-order condition in the same way as the $\phi_{i,t}$ wedge.

[61] That is, we normalize this measure so that $\phi_{i,t}^{FIRE}$ is equal to $\hat{\phi}_{i,t}$ in the benchmark year 2007.

Alternative Measure of $\phi_{i,t}$ using Excess-Bond Premia. Our third and final method for calibrating ϕ_i is a direct measure of financing costs based on the excess-bond premia measurements of Gilchrist and Zakrajšek (2012).[62]

Gilchrist and Zakrajšek (2012) construct excess-bond premia in several steps. First, the authors compare the price of a synthetic bond reconstructed from coupon payments on actual bonds versus the yields on Treasury Bills. From that price difference, the authors factor a component that predicts corporate defaults and a component that is orthogonal to credit risk. The latter component yields a time series for excess-bond premia which the authors show is correlated with multiple measures of economic activity and financial distress. The authors also build sectoral indices for excess-bond premia based on the 12 industry classification of Fama and French widely used in the finance literature. We map their classification to the NAICS industry classification and obtain a time series, $EBP_{i,t}^{GZ}$, for the excess-bond premia for each of our industries.

Again using 2007 as our benchmark year, for all following years we construct a new sequence for financial shocks as follows:

$$\phi_{i,t}^{GZ} = \left(\frac{1 + EBP_{i,t}^{GZ}}{1 + EBP_{i,2007}^{GZ}} \right)^{-1} \hat{\phi}_{i,2007}.$$

To understand this formulation we interpret an increase in a sector's excess-bond premia as relating to greater financing costs for this sector. Thus, as with our previous measurement, movements in $EBP_{i,t}^{GZ}$ may be interpreted as movements in the short-term interest rate faced by this sector, hence the inverse of $1 + EBP_{i,t}^{GZ}$ is analogous to the financial wedge $\phi_{i,t}$.[63]

One caveat to this strategy is that the measurement $\phi_{i,t}^{GZ}$ does not fully capture movements in financial costs because it factors out solvency risk, it only takes into account the orthogonal component. According to Gilchrist and Zakrajšek (2012) the excess-bond premium captures only about one half of a firm's financing cost. Another potential issue with this measurement is that the industry classifications in the BEA data are finer—multiple NAICS sectors belong to the same Fama and French sector.

Comparison of Measurements. We thus obtain three empirical and independently-constructed proxies for the financial wedge $\phi_{i,t}$. By construction all three measures coincide with one another in 2007 (our benchmark year). Here we report some summary statistics for these measures, $\left\{ \hat{\phi}_{i,t}, \phi_{i,t}^{FIRE}, \phi_{i,t}^{GZ} \right\}$, and take a brief look at how they relate to one another.

First, under all measures we find that the year in which financial constraints are the tightest on average is 2009, the year largely considered the trough of the Great Recession. The average drop in this year for our benchmark measure $\hat{\phi}_{i,t}$ is -0.0230; this is equivalent to a 2.3% increase in working capital interest rates. This drop is the largest for all three of our measures. The average drop in

[62] We thank Simon Gilchrist and Egon Zakrajšek for kindly sharing their data with us.

[63] Again we normalize this measure so that $\phi_{i,t}^{GZ}$ is equal to $\hat{\phi}_{i,t}$ in the benchmark year 2007.

$\phi_{i,t}^{FIRE}$ in 2009 is -0.0015, but again, we believe this measure does not capture the full extent of external financing costs. Finally, the average drop using the excess bond-premia method is only -0.0082, about one third of the drop of the benchmark method. If the excess-bond premium captures only about one half of a firm's financing cost, then our benchmark method and this method yield drops of the same order of magnitude.

The cross-sectional standard deviations of $\left\{\hat{\phi}_{i,t}, \phi_{i,t}^{FIRE}, \phi_{i,t}^{GZ}\right\}$ also differ substantially. These are 0.060, 0.014, and 0.003, respectively. Thus, in terms of cross-sector variation our benchmark method yields the greatest dispersion.

Finally, we compute the correlation between the changes in each of these measures between 2007 to 2009. The measured series for $\left(\hat{\phi}_{i,2009} - \hat{\phi}_{i,2007}\right)$ and $\left(\phi_{i,2009}^{FIRE} - \phi_{i,2007}^{FIRE}\right)$ feature a statistical correlation of 0.29 which is significant at the 5% level. The correlation between $\left(\hat{\phi}_{i,2009} - \hat{\phi}_{i,2007}\right)$ and $\left(\phi_{i,2009}^{GZ} - \phi_{i,2007}^{GZ}\right)$ is close to zero and non-significant.

5.3 Experiments

We now conduct various experiments in the calibrated model with our measures.

5.3.1 Correlations with sectoral output.

First we examine how our measurements correlate with sectoral output. Our model has two predictions related to sectoral output. First, sectoral output should be positively correlated with the financial wedge $\phi_{i,t}$, that is, tighter financial constraints should lower sectoral output. Second, our model also predicts network spillovers: sector i's output will be affected not only by its own financial shock, but also by the financial shock to sector j if the two sectors are connected. Which sector affects which should depend on the input-output structure.

We test these predictions with several regressions, the results of which are summarized in Table 1. The right-hand side dependent variable in each regression is the change in gross sales of a given sector i in year t relative to its gross sales in the benchmark year 2007.[64] The first regressor is the change in our measurement in the financial wedge relative to the benchmark year $\left(\phi_{i,t} - \phi_{i,2007}\right)$. The first column reports the results of this regression for our benchmark measure $\hat{\phi}_{i,t}$, while the following two columns correspond to each of our alternative measures, $\phi_{i,t}^{FIRE}$ and $\phi_{i,t}^{GZ}$.

Our results suggests that at least for our benchmark measure, there exists a positive and significant relationship between sectoral output growth and $\hat{\phi}_{i,t}$. The second regressor in each of these columns is the household expenditure shares, $\nu_{i,t}$, to capture any effects arising from relative demand for that sector. We find that the effect of expenditure shares is also highly significant.

Next, to study the presence of network effects without yet using our model, for each sector i we calculate a ranking of external sectors j that generate the largest drop in the sales of sector i after a 1% drop in ϕ_j—we denote by n_m^i the identity of the sector that has the m-th ranked effect on

[64] Observations correspond to the pairs $\{i, t\}$ for all years from 2006 to 2010.

	Benchmark	FIRE	GZ
	$\Delta p_i y_i$	$\Delta p_i y_i$	$\Delta p_i y_i$
$\Delta\phi_{i,t}$	30.7*	311***	-16.5
$\Delta\nu_{i,t}$	2177 ***	2362.4***	2434***
$\Delta\phi_{n_1^i,t}$	93.6 ***	-185.1	43.5
$\Delta\phi_{n_2^i,t}$	16.1	-61.1	-11.0
$\Delta\phi_{n_3^i,t}$	1.2	55.9	-107.6
$\Delta\phi_{n_4^i,t}$	2.7	33.9	-59.3
$\Delta\phi_{n_5^i,t}$	-10.6	29.6	31.8
Observations	315	315	315
R^2	0.2348	0.184	0.11

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Table 1: Measurement of ϕ and Sectoral Growth

sector i in year t. Thus, $\hat{\phi}_{n_m^i,t}$ is the value of ϕ_j for the external sector that has the m-th largest effect on i. We include the $\phi's$ of the first five ranked external sectors as regressors in the three columns of Table 1.

From column one, we see that our regression also detects the presence of network effects up to the first ranked external sector, but the coefficients are not significantly different from zero for higher-ranking sectors. From the second column we see that in our second method, our proxy $\phi_{i,t}^{FIRE}$ is also significantly correlated with sectoral growth, but the network effects are no longer significant under this measure. Finally, the third method, the EBP method, shows no significance. This could be due to the fact that we are looking at all sectors in this regression.

5.3.2 Application to the Great Recession

We now examine how much our model can help explain the fall in aggregate output during the Great Recession. In a first experiment we impute our measured values of ϕ_i for the years 2007 and 2009 into our calibrated model. We then compute the model's predicted change in several macroeconomic aggregates. The results of this exercise are reported in Table 2.

Table 2 reports the fall in aggregate output, hours worked, and efficiency (TFP), for two different versions of our calibration: one in which the decreasing returns to scale parameter η_i is set to $\underline{\eta}_i$ and another in which it is set near 1. Recall that the admissible interval for this parameter is $[\underline{\eta}_i, 1)$; we are thus considering the two extremes. In either case, we report the results for our baseline calibration of the sectoral financial frictions.

We consider the case of $\eta_i = \underline{\eta}_i$ as providing the lower bound for our estimated responses. To understand this, recall that when $\eta_i = \underline{\eta}_i$, the value of $\hat{\phi}_i$ is calibrated to its highest possible value (given the assumption that all variation in the costs-to-sales ratio is a result of financial shocks). In this case we are thereby minimizing the extent of financial frictions. In contrast, when $\eta_i \to 1$,

i.e. the constant returns to scale limit, we consider this case as providing the model's upper bound for estimated responses. Under this value, $\hat{\phi}_{i,t}$ takes on its lowest possible value; we are thereby maximizing the extent of financial frictions.

Finally, for comparison, for each of these two cases we also report the response of macro aggregates when we alter the input-output network to be completely horizontal (as in our simple example). That is, we consider an economy in which we set all entries of the input-output matrix to zero: $w_{ij} = 0$ for all i,j. This economy is consistent with a horizontal economy, similar to that used in most macroeconomic models. All other parameter values are kept the same. The idea behind comparing responses across the two networks is to identify the extent to which the U.S. input-output structure plays a role in amplifying financial frictions.

From Table 2 we first see that when $\eta_i = \underline{\eta}_i$, the model predicts an output drop of almost 4%. This response corresponds roughly to a 5% decline in hours aggravated by a 1% drop TFP. This latter effect is entirely coming from the misallocation of inputs. As mentioned above, these results may be interpreted as the lower bound of our effects.

We next compare column one to column two: the U.S. network economy to the horizontal economy. When $\eta_i = \underline{\eta}_i$, we find that for the alternative horizontal network, the response in output and hours is almost halved: they fall to around 2.1% and 2.7%, respectively. Furthermore, the fall in efficiency in the horizontal economy is almost zero: even though sectors may feature different decreasing returns to scale, there is little misallocation across sectors. Therefore all of the response in output and hours results from movements in the labor wedge in this economy.

We next examine the third column. When the economy is closer to constant returns to scale, and hence the calibrated $\hat{\phi}_{i,t}$ is lower, we find a much greater response in aggregate output, hours, and TFP. In particular, we see a large output drop of 28%. This response corresponds to a decline in hours of approximately 22% and an 8% drop TFP. The disparity between this economy and the economy with lower returns to scale is due to the fact that the curvature of the production function partially offsets the misallocation effects (as shown explicitly in our theoretical analysis).

Furthermore, the closer we are to constant returns to scale, the greater the difference between the calibrated U.S. input-output economy and the horizontal economy (see column four). When $\eta_i = 1$, the fall in output in the horizontal economy is only 4.5%: a much smaller response compared to the 28% drop when our model is calibrated to the U.S. input-output table.

Finally, let us call the ratio of the response of output in the benchmark calibration of the U.S. input-output network to the response of output in the horizontal economy the *network liquidity multiplier*. This ratio represents the amplification that results from taking into account the complex network economy. In our calibration with $\eta_i = \underline{\eta}_i$ we obtain a multiplier of approximately 1.8. On the other hand, when $\eta_i \to 1$ the response of output in either network is much larger, but the network liquidity multiplier in particular rises to 6.3. Therefore, in this experiment, one can think of 1.8 and 6.3 as the lower and upper bounds of the network multiplier effect.

	Δ % 2007 - 2009		Δ % 2007 - 2009	
Returns-to-Scale:	$\eta_i = \underline{\eta}_i$		$\eta_i \to 1$	
Variable	US IO	Horizontal	US IO	Horizontal
Output	-3.8%	-2.1%	-28.3%	-4.5%
Hours Worked	-4.9%	-2.7%	-21.79%	-4.3%
Efficiency	-0.9%	0.0%	-8.4%	0.13%

Table 2: Model Fit - Great Recession

Cross-Sectional Responses. We further compare the response of individual sectoral output in our model to that found in the data. That is, we compare the output responses across all sectors in the entire cross section for the year 2009 vis-*à*-vis the output responses for 2008.

Figure 9 plots gross sales growth for every sector from 2008 to 2009, both in the model and in the data. An immediate observation is that sectoral growth rates vary much more in the data than in the model. This simply means that our model is not capturing all determinants of cross-sectional variation, which is not surprising. However, despite these differences, the correlation between growth rates in our model and in the data is remarkably high at 0.44.

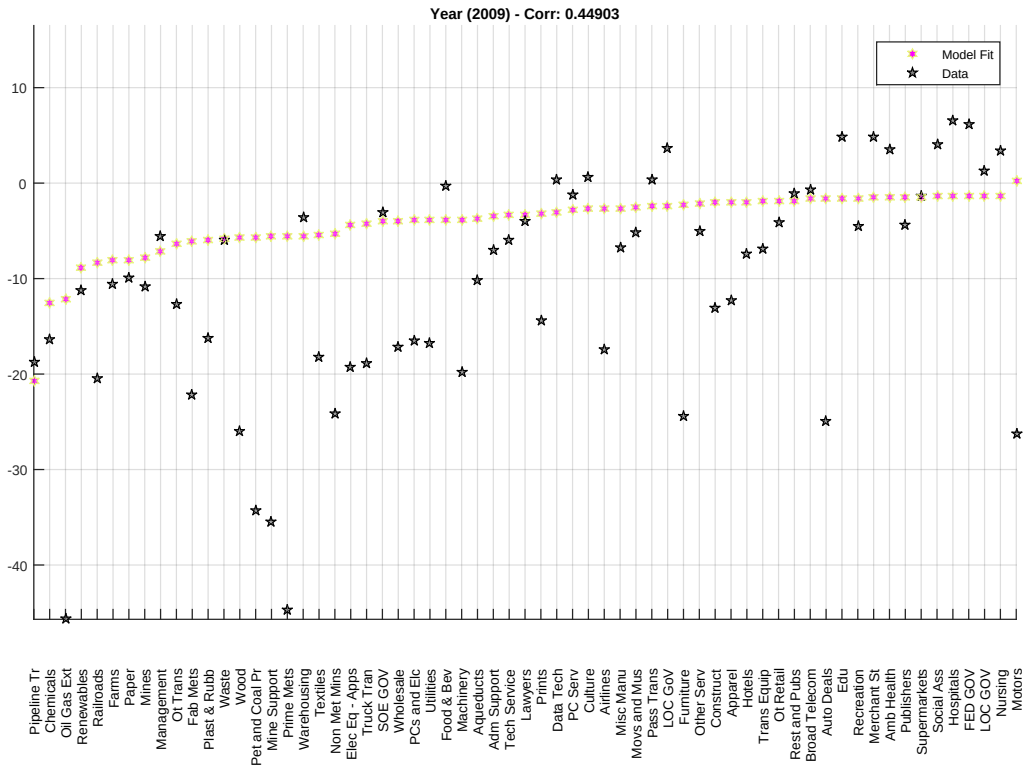


Figure 9: Measured ϕ_i

5.3.3 The Aggregate and Cross-Sectional Impact of Symmetric Financial Shocks

When we feed our model with the changes in $\phi_{i,t}$ calibrated from the data, we are shocking sectors in different magnitudes. We now instead report the effects of a perfectly symmetric 1% fall in $\phi_{i,t}$ in every sector. We ask the following: what does our model predict for the fall in aggregate output in response to this symmetric shock and how does this response compare to that of a completely horizontal economy?

The results of this exercise are summarized in Table 3. Each column corresponds to a different input-output network leaving all other parameters fixed. Again we set $\eta_i = \underline{\eta}_i$ and compare the completely horizontal economy, i.e. $w_{ij} = 0$ for all i,j, in column one, to the U.S. input-output structure in column two. The last two columns repeat the same exercises for the constant returns to scale limit, $\eta_i \to 1$.

The first row of Table 3 reports the fall in TFP in response to a perfectly symmetric 1% decrease in ϕ across all sectors. This also corresponds to the response of GDP under completely inelastic labor supply. The following rows then report the response of aggregate output under different values for the household's (labor supply) preference parameters γ and ϵ.

The first thing to note is that in the horizontal economy, productivity does not react at all when all sectors face a symmetric shock. As we explain in the theoretical characterization, all of the effects on efficiency stem from the misallocation of labor across sectors, hence a common shock to all sectors has no effect. Here, even though there are some differences in the degree of decreasing returns to scale and labor shares across sectors, the overall effect of the common financial shock is near zero.

Next, for the second column, we see that given the U.S. input-output network we see only a small drop in TFP, around .1%. While this is still larger than in the horizontal economy, it suggests that a symmetric financial shock appears to have little effect on aggregate efficiency even in the network economy. However, in the constant returns to scale limit, the effect on TFP is larger.

We now examine the response of aggregate output to the symmetric financial shock. The response of GDP takes into account not only the effect of the financial constraints on aggregate TFP, but also the endogenous labor supply response in hours. First, we see that the stronger the labor supply elasticity, the stronger the response of hours and hence the larger the drop in output. Comparing the results between our baseline parameter choice of $\epsilon = 0.5$ and that when we set $\epsilon = 2$, i.e. a lower labor supply elasticity, we find that the fall in output is more than twice as large in our baseline calibration. Second, as one would expect, an increase in γ, i.e. an increase in the income effect on labor supply, tends to dampen the overall response of hours.

Finally, in the constant returns to scale limit (the last two columns of Table 3), all GDP responses are amplified. In particular, we see that a 1% financial shock across all sectors results in a 4.6% drop in aggregate output in the U.S. input-output economy. Furthermore, the network liquidity multiplier—the ratio between the fall in the IO economy and that in the horizontal economy—increases to 2.5 for this exercise.

Returns-to-Scale:	$\eta_i = \underline{\eta}_i$		$\eta_i \to 1$	
	Horizontal	US IO	Horizontal	US IO
Δ Efficiency	0%	-0.1%	0%	-0.8%
Δ GDP for $\gamma = 0$, $\epsilon = 0.5$	-0.8%	-1.2%	-2.0%	-4.6%
Δ GDP for $\gamma = 0$, $\epsilon = 1$	-0.6%	-0.9%	-1.0%	-2.3%
Δ GDP for $\gamma = 0$, $\epsilon = 2$	-0.3%	-0.5%	-0.5%	-1.1%
Δ GDP for $\gamma = 0.5$, $\epsilon = 0.5$	-0.3%	-0.5%	-1.0%	-2.7%

Table 3: Model Responses to 1% drop in all ϕ_i

Vulnerable Sectors. We use our model to explore which sectors are the most affected after a symmetric fall in $\phi_{i,t}$ of 1% across all sectors. We call these the most vulnerable sectors. Figures 10a and 10b present the responses of gross commodity output for 2- and 5-digit level sectors, respectively.

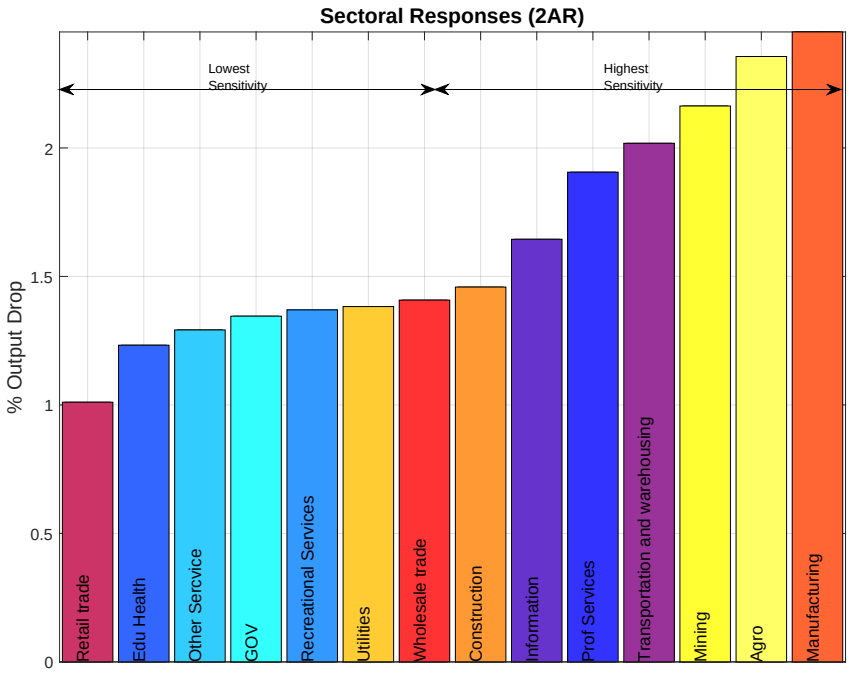
Figure 10a reports the percentage fall in output for all 2-digit sectors. Figure 10b, on the other hand, reports the percentage fall in output for the most and the least sensitive 5-digit sectors to the aggregate shock. Figure 10a can also be used as a two-digit color code: that is, the colors of the bars in Figure 10b correspond to the same color of the two-digit level sector of which each sector belongs to.

We find that the sectors most affected by the aggregate liquidity shock belong to a cluster of sectors that supply primary inputs (materials) to the manufacturing industry. These sectors include the processing of metal products, chemical products, textile products, etc. Recall the location of these sectors in Figure 5: these sectors are located upstream in the production chain and are the most primary suppliers.

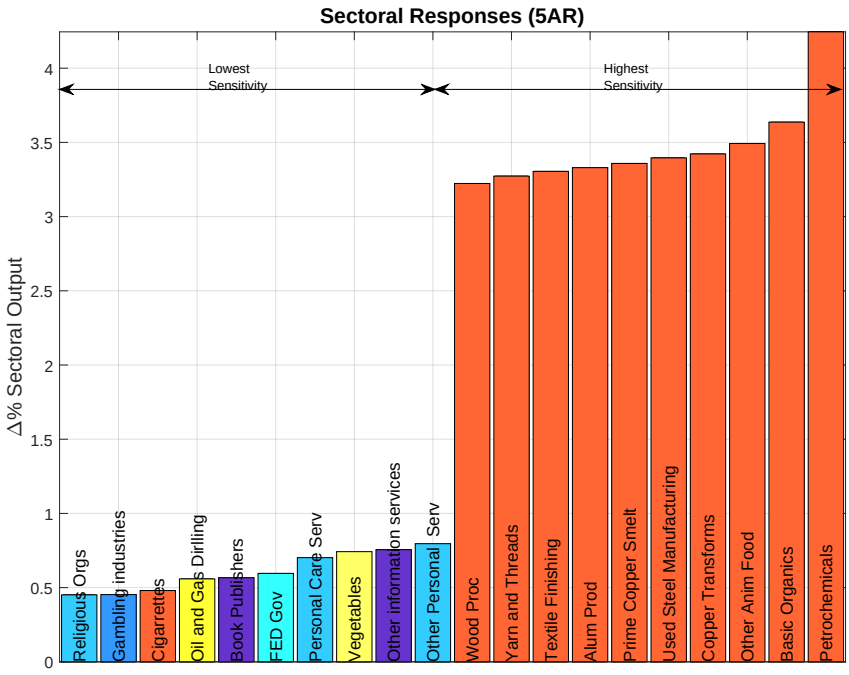
On the other hand, sectors that are the least sensitive to the aggregate financial shock belong to miscellaneous professional services and government. These sectors interact very little with the rest of the input-output economy.

Influential Sectors. Finally, in our last experiment, we study which sectors have the greatest impact on aggregate output. That is, we consider an individual financial shock to each sector and ask which that induce the largest fall in aggregate GDP.

We present the answer to this question in two ways. The first way of answering ranks sectors according to which sectors generate the largest drop in GDP after a 1% drop in the sector's individual $\phi_{i,t}$. The second way of answering instead scales the response of output to each sector's shock by the sector's consumption share. Recall that the influence vector is determined not only by the Leontief inverse, but also by the vector of sectoral consumption shares; see equation (23). Thus, the consumption share of a sector plays an oversized role in determining the response of GDP to



(a) Sectoral Sensitivity to Aggregate Shock - 2 Digits



(b) Sectoral Sensitivity to Aggregate Shock - 5 Digits

that sector's financial shock. Our second ranking thus normalizes this effect allowing us to identify the true network amplification of sectoral frictions.

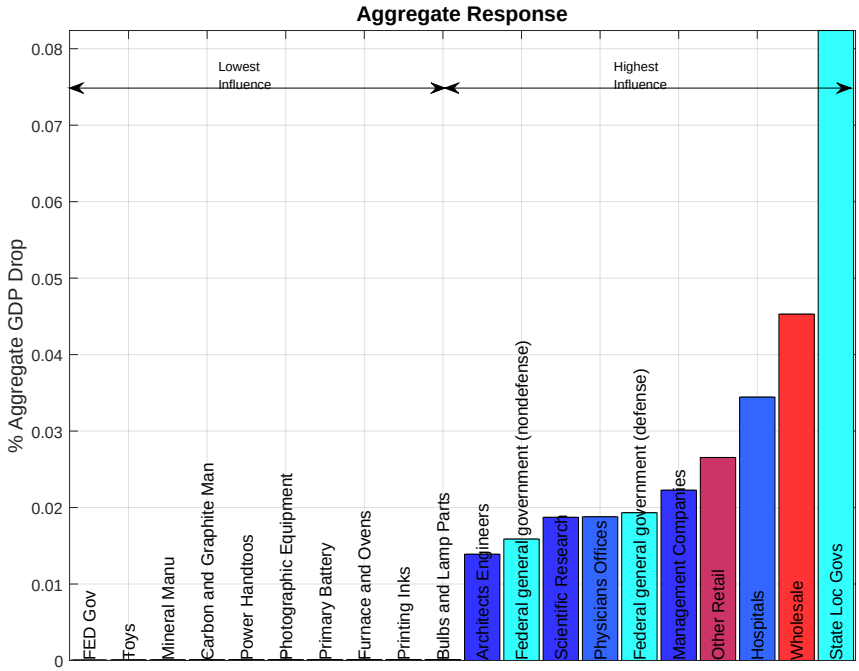
Corresponding to the first ranking, Figure 11a reports the sectors that lead to the largest fall in aggregate output. As anticipated, we find that the sectors with the greatest influence on aggregate output are those with the largest consumption shares: *government (federal, state, and local) including defense spending, wholesale, retail, and hospitals*. We also find that the set of sectors that provide services to support these industries are highly influential as well.

We now turn to the second ranking, reported in Figure 11b, which normalizes these effects by sectoral consumption shares.[65] We find that the sectors in the manufacturing cluster of the automobile industry in fact yield the greatest influence: *Cars, Vehicle Bodies, Motor Parts, Car Seating and Interior, Heavy Trucks, Motor Homes, and Trailers*. Thus, while it is often noted that the volatility of the automobile manufacturing industry is large in comparison to other industries, its pronounced effect on aggregate output is typically attributed to the durability of cars, see e.g. (Ramey and Vine, 2006). Here we instead highlight that the complexity of the production process and the location of the auto industry within the U.S. input-output network also plays a significant role in determining its influence on GDP.

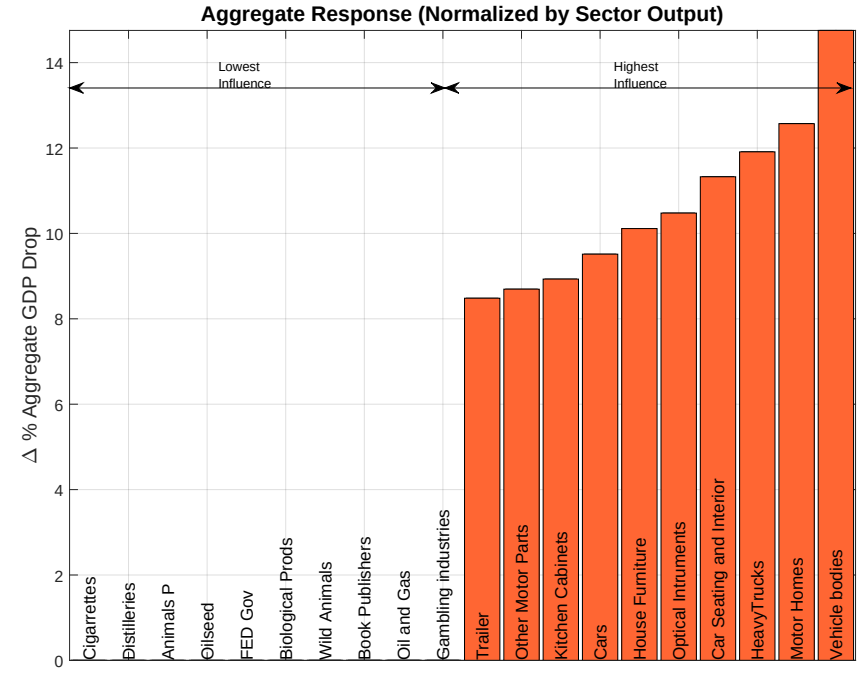
5.3.4 Further Discussion of Robustness and Sensitivity

Finally, for all of our experiments we perform several sensitivity checks. First, we check our results for all levels of disaggregation when possible. In addition, we check our results using the IO tables before and after redefinitions—see data appendix for more details. Finally, we also test if the exclusion of the diagonal term of the input-output matrix alters our results. For none of these variations do we find that our results change significantly.

[65] The units on the y-axis in Figure 11b is not informative as we have divided the aggregate response by the consumption share of each sector.



(a) Aggregate Response to Sector Specific Shock - Levels



(b) Aggregate Response to Sector Specific Shock - Weighted by Sales

6 Conclusion

This paper studies whether the network structure of production in an economy matters both qualitatively and quantitatively for the transmission of financial shocks. We formulate a static general equilibrium economy in which firms buy and sell intermediate goods from one another and use these goods as inputs to production. Within this general production network, we introduce financial frictions as a constraint on firms' working capital purchases. We then consider how an exogenous tightening of these constraints may be amplified as its effect travels throughout the production network.

Our first theoretical result is an aggregation result which shows how financial frictions at the sector level manifest themselves as an efficiency wedge and a labor wedge at the aggregate level. The exact mapping from sectoral wedges to aggregate output depends on the network's particular architecture and the location of shocks. We show that with financial frictions, the vector of equilibrium sales of sectors is not a sufficient statistic for sectoral influence. We furthermore provide several simple analytic examples of different network structures, and show how the propagation of liquidity shocks differs across networks.

Finally, we use data from the U.S. Input-Output tables to calibrate the production network in our model. We ask what is the multiplier generated by these network effects. Our model predicts a range for the network liquidity multiplier of 1.8 to 6.3 during the Great Recession.

Our theory is, of course, incomplete. As we mention throughout, there are limitations to our analysis, in particular, the model's restriction to Cobb-Douglas technologies and zero adjustment cost of labor reallocation. In tandem, an elasticity of substitution of one across intermediate goods and a perfectly mobile labor supply renders a much more resilient production structure for withstanding financial shocks. If it were instead more difficult to reallocate inputs across sectors in the short run, then we conjecture that movements in financial constraints would have an even more pronounced effect on aggregate output. We thus suspect that departing from perfectly mobile labor supply or relaxing Cobb-Douglas technologies for a CES structure, as in Atalay (2015), would be fruitful in generating more amplification. We furthermore think that including dynamic aspects, such as capital, inventory, and durable goods, should also be explored.

Moreover, in this paper we remain agnostic about the sources of financial shocks. We believe our simplified model serves as a useful starting point for studying the effects of financial frictions in production networks. That said, clearly considering more deeply the origins of credit constraints would yield further insight. Along these lines, very recent work by Altinoglu (2016) and Luo (2016) embed a credit network into a production network structure. In their models, intermediate good trade is financed by supplier trade credit. Thus, by adding direct credit linkages, Altinoglu (2016) and Luo (2016) generate further predictions regarding the correlation structure of sectoral output and the propagation of shocks.

Finally, in our model, the network structure is completely exogenous and the firms within

the network remain fixed over time. We do this to keep our model tractable, but we clearly believe that the endogeneity of supply-chain links and the formation of the production network are important considerations; for an interesting treatment of this issue see Oberfield (2011). Moreover, the extensive margin of firm entry and exit within a production network is examined carefully in recent work by Baqaee (2015).

In conclusion, we hope that this paper has brought to light the important role the production network can play in propagating and amplifying financial shocks. We hope that our work will prompt new research avenues studying the transmission of financial shocks across firms and sectors within the economy.

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A Proofs of Results in Section 3

Proof of Lemma 1. We solve the firm's dual problem as described in Problem 1. First let $q_i x_i$ denote the firm i's total expenditure on inputs, where q_i is a composite of input costs:

$$q_i x_i \equiv \min \ell_i + \sum_{j=0}^{N} p_j x_{ij}, \text{ subject to } x_i = \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{w_{ij}} \right)^{1-\alpha_i}.$$

Firm i maximizes its profits,

$$\max p_i y_i - q_i x_i,$$

subject to

$$\begin{aligned} y_i &= (z_i x_i)^{\eta_i}, \text{ and} \\ q_i x_i &\leq \chi_i p_i y_i. \end{aligned}$$

If the financial constraint isn't binding, then firm optimality implies $q_i x_i = \eta_i p_i y_i$. On the other hand, if the financial constraint is binding, we have that $q_i x_i = \chi_i p_i y_i$. Therefore,

$$q_i x_i = \min \{\chi_i, \eta_i\} p_i y_i = \min \left\{ \frac{\chi_i}{\eta_i}, 1 \right\} \eta_i p_i y_i.$$

We may thus write

$$q_i x_i = \phi_i \eta_i p_i y_i, \quad \text{where} \quad \phi_i = \min \left\{ \frac{\chi_i}{\eta_i}, 1 \right\}.$$

The variable $\phi_i \in [0, 1]$ represents the firm-specific wedge that arises between the firm's marginal cost and marginal revenue. This proves part (ii) of Lemma 1.

Next, given some amount of output, the firm solves its cost minimization problem over the individual inputs. This is given by

$$q_i x_i \equiv \min \ell_i + \sum_{j=0}^{N} p_j x_{ij},$$

subject to

$$x_i = \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{w_{ij}} \right)^{1-\alpha_i}.$$

The first order conditions with respect to x_{ij} and ℓ_i are given by

$$\begin{aligned} p_j - \lambda_i (1 - \alpha_i) w_{ij} \frac{x_i}{x_{ij}} &= 0, \text{ for all } j, \text{ and} \\ 1 - \lambda_i \alpha_i \frac{x_i}{\ell_i} &= 0, \end{aligned}$$

where λ_i is the multiplier on the constraint. Thus $p_j x_{ij} = \lambda_i (1-\alpha_i) w_{ij} x_i$ and $\ell_i = \lambda_i \alpha_i x_i$. Summing over the expenditure on all inputs, we get

$$q_i x_i = \ell_i + \sum_{j=1}^{N} p_j x_{ij} = \lambda_i \alpha_i x_i + \sum_{j=1}^{N} \lambda_i (1-\alpha_i) w_{ij} x_i = \lambda_i x_i.$$

We thus have that $\lambda_i = q_i$, which implies that $p_j x_{ij} = (1-\alpha_i) w_{ij} u_i$ and $\ell_i = \alpha_i u_i$ as in part (i) of Lemma 1. QED.

Proof of Lemma 2. We solve the household's dual problem as described in Problem 2. Let $u_0 = \bar{p}c$ denote the household's total expenditure on goods, where $\bar{p}$ is the cost-minimizing composite of final good prices. The household maximizes

$$\max \frac{c^{1-\gamma}}{1-\gamma} - \frac{\ell^{1+\epsilon}}{1+\epsilon}$$

subject to its budget constraint

$$\bar{p}c \leq \ell + \sum_{i=1}^{N} \pi_i$$

The first order conditions for this problem yields the following optimality condition

$$\frac{c^{-\gamma}}{\ell^{\epsilon}} = \bar{p}$$

where $\bar{p}$ is the real price of consumption relative to labor. This proves part (ii) of Lemma 2.

Next we solve the household's expenditure minimization problem over individual goods. This is given by

$$\bar{p}c = \min \sum_{j=1}^{N} p_j c_j \quad \text{subject to} \quad c = \prod_{j=1}^{N} c_j^{v_j}.$$

The household's first order conditions for this problem are given by

$$p_j - \lambda_0 v_j \frac{c}{c_j} = 0, \text{ for all } j,$$

where λ_0 is the multiplier on the constraint. Thus $p_j c_j = \lambda_0 v_j c$. Summing up the expenditure over all goods, we get

$$\bar{p}c = \sum_{j=1}^{N} p_j c_j = \sum_{j=1}^{N} \lambda_0 v_j c = \lambda_0 c.$$

We thus have that $\lambda_0 = \bar{p}$, which implies $p_j c_j = v_j \bar{p} c = v_j u_0$, as in part (i) of Lemma 2.

Finally, we solve for the ideal price index as follows:

$$c = \prod_{j=1}^{N} \left(\frac{v_j}{p_j} \bar{p} c \right)^{v_j} = \prod_{j=1}^{N} \left(\frac{v_j}{p_j} \right)^{v_j} \bar{p} c.$$

Therefore, we obtain

$$\bar{p} = \prod_{j=1}^{N} \left(\frac{p_j}{v_j} \right)^{v_j}$$

as the ideal price index for the household. QED.

Proof of Proposition 1. From market clearing, total revenue of a sector $g_i = p_i y_i$ must satisfy $g_i = p_i c_i + \sum_{j=1}^{N} p_i x_{ji}$. From firm and household optimality conditions for expenditure on each good, (7) and (10), we obtain the following equation

$$g_i = v_i u_0 + \sum_{j=1}^{N} (1 - \alpha_j) \, w_{ji} u_j, \tag{43}$$

where u_0 denotes total household expenditures. Stacking equation (43) for each sector atop one another, we have

$$\begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} u_0 + \left(\begin{bmatrix} 1-\alpha_1 & 1-\alpha_2 & \cdots & 1-\alpha_N \\ 1-\alpha_1 & 1-\alpha_2 & & \\ \vdots & & \ddots & \\ 1-\alpha_1 & 1-\alpha_2 & & 1-\alpha_N \end{bmatrix} \circ \begin{bmatrix} w_{11} & w_{21} & \cdots & w_{N1} \\ w_{12} & w_{22} & & \\ \vdots & & \ddots & \\ w_{1N} & & & w_{NN} \end{bmatrix} \right) \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \tag{44}$$

where $\circ$ denotes the Hadamard product (entrywise product). Next, let $\mathbf{1} - \boldsymbol{\alpha} \equiv \mathbf{e}_N - \boldsymbol{\alpha}$. We thus write the system of equations in (44) as follows

$$\mathbf{g} = \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N' \circ \mathbf{W} \right)' \mathbf{u} + \mathbf{v} u_0, \tag{45}$$

where $\circ$ denotes the Hadamard (entrywise) product. Thus, using market clearing and the optimality of expenditure on each good, we can relate total revenue of each firm to the total expenditures of each sector and to the expenditure of the household.

Next, noting that $\pi_i = g_i - u_i$, household expenditures must satisfy the household budget constraint given by

$$u_0 = \ell + \sum_{i=1}^{N} (g_i - u_i) = \ell + \mathbf{e}_N' (\mathbf{g} - \mathbf{u}).$$

Substituting this expression for u_0 into our market clearing equation (45) yields

$$\mathbf{g} = \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N^{'} \circ \mathbf{W} \right)^{'} \mathbf{u} + \mathbf{v} \left(\ell + \mathbf{e}_N^{\prime} \left(\mathbf{g} - \mathbf{u} \right) \right). \tag{46}$$

Next, from firm optimality condition (9) we have that firm expenditures satisfy $u_i = \phi_i \eta_i g_i$. Stacking this equation atop one another sector-by-sector, we have

$$\mathbf{u} = \begin{bmatrix} \phi_1 \eta_1 g_1 \\ \phi_2 \eta_2 g_2 \\ \vdots \\ \phi_N \eta_N g_N \end{bmatrix} = \boldsymbol{\phi} \circ \boldsymbol{\eta} \circ \mathbf{g}. \tag{47}$$

Substituting (47) for $\mathbf{u}$ into (46), yields

$$\mathbf{g} = \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N^{'} \circ \mathbf{W} \right)^{'} (\boldsymbol{\phi} \circ \boldsymbol{\eta} \circ \mathbf{g}) + \mathbf{v} \mathbf{e}_N^{\prime} \left(\mathbf{g} - (\boldsymbol{\phi} \circ \boldsymbol{\eta} \circ \mathbf{g}) \right) + \mathbf{v} \ell,$$

which with a bit of algebraic manipulation we may re-write as

$$\mathbf{g} = \left[\left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N^{'} \circ \mathbf{W} \right)^{'} \circ \left(\mathbf{e}_N \left(\boldsymbol{\phi} \circ \boldsymbol{\eta} \right)^{\prime} \right) + \mathbf{v} \left(\mathbf{e}_N - (\boldsymbol{\phi} \circ \boldsymbol{\eta}) \right)^{\prime} \right] \mathbf{g} + \mathbf{v} \ell.$$

Total revenue thereby satisfies

$$\mathbf{g} = \left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N^{'} \circ \mathbf{W} \right)^{'} \circ \left(\mathbf{e}_N \left(\boldsymbol{\phi} \circ \boldsymbol{\eta} \right)^{\prime} \right) - \mathbf{v} \left(\mathbf{e}_N - (\boldsymbol{\phi} \circ \boldsymbol{\eta}) \right)^{\prime} \right]^{-1} \mathbf{v} \ell,$$

where $\mathbb{I}_N$ is the the identity matrix of size $N \times N$. This equation, along with equation (47), gives us (13), with the vector $\mathbf{a}(\boldsymbol{\phi})$ as defined in (14). QED.

Proof of Proposition 2. Substituting (7) into (5), we have that total revenue must satisfy

$$g_i = p_i \left[u_i z_i \left(\alpha_i \right)^{\alpha_i} \left((1 - \alpha_i) \prod_{j=1}^{N} \left(\frac{w_{ij}}{p_j} \right)^{w_{ij}} \right)^{1 - \alpha_i} \right]^{\eta_i}.$$

Taking logs of both sides of this equation results in the following expression

$$\begin{aligned} \log g_i \;\; = \;\; & \log p_i + \eta_i \log z_i + \eta_i \log u_i \\ & + \eta_i \left[\alpha_i \log \alpha_i + (1 - \alpha_i) \log (1 - \alpha_i) + (1 - \alpha_i) \sum_{j=1}^{N} w_{ij} \left(\log w_{ij} - \log p_j \right) \right] \end{aligned}$$

Therefore

$$\log g_i = \log p_i + \eta_i \log z_i + \eta_i \log u_i + \eta_i \left[\kappa_i - (1-\alpha_i) \sum_{j=1}^{N} w_{ij} \log p_j \right] \tag{48}$$

where κ_i is a constant for each sector given by,

$$\kappa_i \equiv \alpha_i \log \alpha_i + (1-\alpha_i) \log (1-\alpha_i) + (1-\alpha_i) \sum_{j=1}^{N} w_{ij} \log w_{ij}.$$

Stacking equation (48) atop one another sector-by-sector, we get

$$\begin{bmatrix} \log g_1 \\ \log g_2 \\ \vdots \\ \log g_N \end{bmatrix} = \begin{bmatrix} \log p_1 \\ \log p_2 \\ \vdots \\ \log p_N \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{bmatrix} \circ \left(\begin{bmatrix} \log z_1 \\ \log z_2 \\ \vdots \\ \log z_N \end{bmatrix} + \begin{bmatrix} \log u_1 \\ \log u_2 \\ \vdots \\ \log u_N \end{bmatrix} + \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \vdots \\ \kappa_N \end{bmatrix} \right)$$
$$- \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{bmatrix} \circ \begin{bmatrix} 1-\alpha_1 \\ 1-\alpha_2 \\ \vdots \\ 1-\alpha_N \end{bmatrix} \circ \left(\begin{bmatrix} (w_{11}, w_{12}, ...w_{1N}) \\ (w_{21}, w_{22}, ...w_{2N}) \\ \vdots \\ (w_{N1}, w_{N2}, ...w_{NN}) \end{bmatrix} \begin{bmatrix} \log p_1 \\ \log p_2 \\ \vdots \\ \log p_N \end{bmatrix} \right),$$

which we may re-write in vector form as

$$\log \mathbf{g} = \log \mathbf{p} + \boldsymbol{\eta} \circ (\log \mathbf{z} + \log \mathbf{u} + \boldsymbol{\kappa}) - \boldsymbol{\eta} \circ (\mathbf{1} - \boldsymbol{\alpha}) \circ (\mathbf{W} \log \mathbf{p}),$$

where $\boldsymbol{\kappa} \equiv (\kappa_1, \ldots, \kappa_N)'$. We use this expression to solve for the log vector of prices $\mathbf{p}$ as follows:

$$\log \mathbf{p} - \boldsymbol{\eta} \circ (\mathbf{1} - \boldsymbol{\alpha}) \circ (\mathbf{W} \log \mathbf{p}) = \log \mathbf{g} - \boldsymbol{\eta} \circ (\log \mathbf{z} + \log \mathbf{u} + \boldsymbol{\kappa}).$$

Next, taking logs of equation (47), we have that $\log \mathbf{u} = \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \log \mathbf{g}$. Substituting this for $\log \mathbf{u}$ into the above equation yields

$$\log \mathbf{p} - \boldsymbol{\eta} \circ (\mathbf{1} - \boldsymbol{\alpha}) \circ (\mathbf{W} \log \mathbf{p}) = \log \mathbf{g} - \boldsymbol{\eta} \circ (\log \mathbf{z} + \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \log \mathbf{g} + \boldsymbol{\kappa}),$$

which with some algebraic manipulation becomes

$$\left[\mathbb{I}_N - \left((\boldsymbol{\eta} \circ (\mathbf{1} - \boldsymbol{\alpha})) \, \mathbf{e}_N' \right) \circ \mathbf{W} \right] \log \mathbf{p} = (\mathbf{e}_N - \boldsymbol{\eta}) \circ \log \mathbf{g} - \boldsymbol{\eta} \circ (\log \mathbf{z} + \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \boldsymbol{\kappa}).$$

Therefore,

$$\log \mathbf{p} = \left[\mathbb{I}_N - \left((\boldsymbol{\eta} \circ (\mathbf{1} - \boldsymbol{\alpha})) \, \mathbf{e}_N' \right) \circ \mathbf{W} \right]^{-1} \left[(\mathbf{e}_N - \boldsymbol{\eta}) \circ \log \mathbf{g} - \boldsymbol{\eta} \circ (\log \mathbf{z} + \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \boldsymbol{\kappa}) \right].$$

We thus obtain equation (15) in Proposition 2 with the properly defined matrix $\mathbf{B}$.

We obtain the aggregate price index as follows. Taking the log of equation (11) we have

$$\log \bar{p} = \sum_{j=1}^{N} v_j \left(\log p_j - \log v_j \right).$$

Re-writing this in vector form gives us (17). QED.

Proof of Equation (20). Real GDP in this economy is simply household consumption. Household consumption satisfies the budget constraint, $\bar{p}c = \ell + \sum_{i=1}^{N} \pi_i$. Therefore, real GDP must satisfy

$$c = \bar{p}^{-1} \left(\ell + \mathbf{e}_N' \left(\mathbf{g} - \mathbf{u} \right) \right). \tag{49}$$

Value added for each sector is sectoral sales minus input costs aside from labor, divided by the aggregate price level:

$$\mu_i = \bar{p}^{-1} \left(g_i - \left(u_i - \ell_i \right) \right).$$

It's clear that aggregating over value added for all sectors results in aggregate consumption:

$$\sum_{i=1}^{N} \mu_i = \frac{1}{\bar{p}} \left[\sum_{i=1}^{N} \left(g_i - u_i \right) + \sum_{i=1}^{N} \ell_i \right] = c.$$

Next, substituting in from (13) for $\mathbf{g}$ and $\mathbf{u}$ into equation (49) for real GDP, we have that

$$c = \bar{p}^{-1} \left(\ell + \mathbf{e}_N' \left(\mathbf{a} \left(\boldsymbol{\phi} \right) \ell - \left(\boldsymbol{\phi} \circ \boldsymbol{\eta} \circ \mathbf{a} \left(\boldsymbol{\phi} \right) \right) \ell \right) \right).$$

With a little bit of algebraic manipulation we may re-write this as

$$c = \bar{p}^{-1} \left(1 + \mathbf{e}_N' \left(\left(\mathbf{e}_N - \left(\boldsymbol{\phi} \circ \boldsymbol{\eta} \right) \right) \circ \mathbf{a} \left(\boldsymbol{\phi} \right) \right) \right) \ell.$$

Therefore, real GDP satisfies (20) where the scalar $\psi \left(\boldsymbol{\phi} \right)$ is as defined in (21). QED.

Proof of Theorem 1. We first obtain an expression for the ideal price index in terms of sectoral sales. From (11) we have that the aggregate price index satisfies

$$\log \bar{p} = \mathbf{v}' \log \mathbf{p} - \mathbf{v}' \log \mathbf{v}, \tag{50}$$

where the vector of sectoral prices is given by

$$\log \mathbf{p} = \mathbf{B} \left[\left(\mathbf{e}_N - \boldsymbol{\eta} \right) \circ \log \mathbf{g} - \boldsymbol{\eta} \circ \left(\log \mathbf{z} + \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \boldsymbol{\kappa} \right) \right]. \tag{51}$$

With a little bit of algebraic manipulation, the vector of sectoral prices may be re-written as

$$\log \mathbf{p} = \left(\left(\mathbf{e}_N \left(\mathbf{e}_N - \boldsymbol{\eta}\right)'\right) \circ \mathbf{B}\right) \log \mathbf{g} - \mathbf{B}\left(\boldsymbol{\eta} \circ \left(\log \mathbf{z} + \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \boldsymbol{\kappa}\right)\right). \tag{52}$$

Next, taking logs of equation (20) and substituting in our expression for the log price index from (50), we have that GDP must satisfy

$$\log GDP = -\mathbf{v}' \log \mathbf{p} + \mathbf{v}' \log \mathbf{v} + \log\left(1 + \psi\left(\boldsymbol{\phi}\right)\right) + \log \ell.$$

Next, using (52) to replace the vector of sectoral prices in the above equation gives us

$$\begin{aligned} \log GDP \;\; = \;\; & -\mathbf{v}' \left(\left(\mathbf{e}_N \left(\mathbf{e}_N - \boldsymbol{\eta}\right)'\right) \circ \mathbf{B}\right) \log \mathbf{g}\left(\boldsymbol{\phi}\right) + \mathbf{v}'\mathbf{B}\left(\boldsymbol{\eta} \circ \left(\log \mathbf{z} + \log \boldsymbol{\phi} + \log \boldsymbol{\eta} + \boldsymbol{\kappa}\right)\right) \\ & + \log \ell + \log\left(1 + \psi\left(\boldsymbol{\phi}\right)\right) + \mathbf{v}' \log \mathbf{v}. \end{aligned}$$

Finally, from Proposition 1 we have that $\log \mathbf{g}\left(\boldsymbol{\phi}\right) = \log \mathbf{a}\left(\boldsymbol{\phi}\right) + \mathbf{e}_N \log \ell$. Substituting this into the above equation and collecting terms, we obtain the following expression:

$$\begin{aligned} \log GDP \;\; = \;\; & -\mathbf{d} \log \mathbf{a}\left(\boldsymbol{\phi}\right) + \left(1 - \mathbf{d}\mathbf{e}_N\right) \log \ell + \left(\left(\mathbf{v}'\mathbf{B}\right) \circ \boldsymbol{\eta}'\right)\left(\log \mathbf{z} + \log \boldsymbol{\phi}\right) \\ & + \log\left(1 + \psi\left(\boldsymbol{\phi}\right)\right) + K, \end{aligned}$$

where

$$\mathbf{d} \equiv \mathbf{v}' \left(\left(\mathbf{e}_N \left(\mathbf{e}_N - \boldsymbol{\eta}\right)'\right) \circ \mathbf{B}\right) \quad \text{and} \quad K \equiv \mathbf{v}'\mathbf{B}\left(\boldsymbol{\eta} \circ \left(\log \boldsymbol{\eta} + \boldsymbol{\kappa}\right)\right) + \mathbf{v}' \log \mathbf{v}$$

This verifies equation (22) in Theorem 1. QED.

Proof of Corollary 1. Follows from main text.

Proof of Proposition 3. Follows from main text.

Proof of Theorem 2. First, in the household's optimality condition (31) between consumption and labor, we substitute in our expression for the household's consumption from equation (20). This gives us

$$\frac{\left(\bar{p}^{-1}\left(1 + \psi\left(\boldsymbol{\phi}\right)\right)\ell\right)^{-\gamma}}{\ell^{\epsilon}} = \bar{p}.$$

Rearranging, we have that equilibrium labor must satisfy

$$\ell = \bar{p}^{\frac{\gamma - 1}{\epsilon + \gamma}} \left(1 + \psi\left(\boldsymbol{\phi}\right)\right)^{-\frac{\gamma}{\epsilon + \gamma}}.$$

Taking logs of both sides of this equation and letting $\beta \equiv \frac{1-\gamma}{\epsilon+\gamma}$, we may re-write this as

$$\log \ell = -\beta \log \bar{p} - \frac{\gamma}{\epsilon+1}(1+\beta)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right). \tag{53}$$

Using our expression (50) for the price index, we may re-express (53) as

$$\log \ell = -\beta\left(\mathbf{v}'\log \mathbf{p} - \mathbf{v}'\log \mathbf{v}\right) - \frac{\gamma}{\epsilon+1}(1+\beta)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right).$$

Furthermore combining this with equation (52) for the sectoral price vector, we get

$$\begin{aligned}\log \ell &= -\beta\mathbf{v}'\left(\left(\mathbf{e}_N\left(\mathbf{e}_N-\boldsymbol{\eta}\right)'\right)\circ\mathbf{B}\right)\log\mathbf{g}\left(\boldsymbol{\phi}\right) - \mathbf{B}\log\mathbf{z} - \mathbf{B}\left(\boldsymbol{\eta}\circ\left(\log\boldsymbol{\phi}+\log\boldsymbol{\eta}+\boldsymbol{\kappa}\right)\right)\right) \\ &\quad +\beta\mathbf{v}'\log\mathbf{v} - \frac{\gamma}{\epsilon+1}(1+\beta)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right).\end{aligned}$$

Finally, substituting $\log\mathbf{g}\left(\boldsymbol{\phi}\right) = \log\mathbf{a}\left(\boldsymbol{\phi}\right) + \mathbf{e}_N\log\ell$ into the above expression gives us

$$\begin{aligned}\log \ell &= -\beta\mathbf{v}'\left(\left(\mathbf{e}_N\left(\mathbf{e}_N-\boldsymbol{\eta}\right)'\right)\circ\mathbf{B}\right)\left(\log\mathbf{a}\left(\boldsymbol{\phi}\right)+\mathbf{e}_N\log\ell\right) + \beta\mathbf{v}'\mathbf{B}\log\mathbf{z} \\ &\quad +\beta\mathbf{v}'\mathbf{B}\left(\boldsymbol{\eta}\circ\left(\log\boldsymbol{\phi}+\log\boldsymbol{\eta}+\boldsymbol{\kappa}\right)\right) + \beta\mathbf{v}'\log\mathbf{v} - \frac{\gamma}{\epsilon+1}(1+\beta)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right).\end{aligned}$$

Collecting terms,

$$\begin{aligned}\left(1+\beta\mathbf{v}'\left(\left(\mathbf{e}_N\left(\mathbf{e}_N-\boldsymbol{\eta}\right)'\right)\circ\mathbf{B}\right)\mathbf{e}_N\right)\log\ell &= -\beta\mathbf{v}'\left(\left(\mathbf{e}_N\left(\mathbf{e}_N-\boldsymbol{\eta}\right)'\right)\circ\mathbf{B}\right)\log\mathbf{a}\left(\boldsymbol{\phi}\right) + \beta\mathbf{v}'\mathbf{B}\log\mathbf{z} \\ &\quad +\beta\mathbf{v}'\mathbf{B}\left(\boldsymbol{\eta}\circ\left(\log\boldsymbol{\phi}+\log\boldsymbol{\eta}+\boldsymbol{\kappa}\right)\right) + \beta\mathbf{v}'\log\mathbf{v} \\ &\quad -\frac{\gamma}{\epsilon+1}(1+\beta)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right).\end{aligned}$$

Next, using our definitions of $\mathbf{q}_z$, $\mathbf{q}_\phi$, $\mathbf{d}$ and K from Theorem 1, we rewrite the above expression as follows

$$\begin{aligned}\left(1+\beta\mathbf{d}\mathbf{e}_N\right)\log\ell &= -\beta\mathbf{d}\log\mathbf{a}\left(\boldsymbol{\phi}\right) + \beta\mathbf{q}_z\log\mathbf{z} + \beta\mathbf{q}_{\boldsymbol{\phi}}\log\boldsymbol{\phi} \\ &\quad +\beta K - \frac{\gamma}{\epsilon+1}(1+\beta)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right).\end{aligned}$$

Dividing through by the scalar $\left(1+\beta\mathbf{d}\mathbf{e}_N\right)$ we obtain the following expression for equilibrium labor

$$\begin{aligned}\log\ell &= \frac{\beta}{1+\beta\mathbf{d}\mathbf{e}_N}\left[\mathbf{q}_{\mathbf{z}}\log\mathbf{z} + \mathbf{q}_{\boldsymbol{\phi}}\log\boldsymbol{\phi} - \mathbf{d}\log\mathbf{a}\left(\boldsymbol{\phi}\right) + K\right] \\ &\quad -\frac{\gamma}{\epsilon+1}\frac{(1+\beta)}{1+\beta\mathbf{d}\mathbf{e}_N}\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right)\end{aligned} \tag{54}$$

This coincides with equation (33) in the theorem with the scalars Λ_q and Λ_ψ defined in (34).

Finally, we may then substitute (54) into the aggregate production function (22). This results

in the following expression for aggregate output

$$\begin{aligned}\log GDP &= \left(1+\beta\frac{1-\mathbf{de}_N}{1+\beta\mathbf{de}_N}\right)\left[\mathbf{q_z}\log\mathbf{z}+\mathbf{q}_{\boldsymbol{\phi}}\log\boldsymbol{\phi}-\mathbf{d}\log\mathbf{a}\left(\boldsymbol{\phi}\right)+K\right]\\ &\quad+\left(1-\frac{\gamma}{\epsilon+1}\left(1+\beta\right)\frac{\left(1-\mathbf{de}_N\right)}{1+\beta\mathbf{de}_N}\right)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right)\end{aligned}$$

This coincides with equation (32) in the theorem with scalars Γ_q and Γ_ψ defined in (34). QED.

Proof of Lemma 3. All entries of the vector $\mathbf{d}$ are necessarily positive. The parameter restriction $\gamma\in(0,1)$ implies that $\beta=\frac{1-\gamma}{\epsilon+\gamma}$ is strictly positive. Together, these imply

$$\Gamma_q=\frac{1+\beta}{1+\beta\mathbf{de}_N}>0\quad\text{ and }\quad\Lambda_q=\frac{\beta}{1+\beta\mathbf{de}_N}>0$$

QED.

Proof of Proposition 4. From our definition (37) we have that the labor wedge satisfies

$$\left(1-\tau\left(\boldsymbol{\phi}\right)\right)=\bar{\eta}^{-1}\frac{\ell^{1+\epsilon}}{c^{1-\gamma}}.$$

Taking logs of both sides of this expression yields

$$\log\left(1-\tau\left(\boldsymbol{\phi}\right)\right)=\left(1+\epsilon\right)\log\ell-\left(1-\gamma\right)\log c+const.$$

Next we substitute in our expressions of equilibrium GDP and labor from (32) and (33) and collect terms. This yields

$$\begin{aligned}\log\left(1-\tau\left(\boldsymbol{\phi}\right)\right) &= \left(\left(1+\epsilon\right)\Lambda_q-\left(1-\gamma\right)\Gamma_q\right)\left[\mathbf{q}\log\mathbf{z}+\mathbf{q}\log\boldsymbol{\phi}-\mathbf{d}\log\mathbf{a}\left(\boldsymbol{\phi}\right)+K\right]\\ &\quad-\left(\left(1+\epsilon\right)\Lambda_\psi+\left(1-\gamma\right)\Gamma_\psi\right)\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right)-\log\bar{\eta}.\end{aligned}$$

Note that

$$\left(1+\epsilon\right)\Lambda_q-\left(1-\gamma\right)\Gamma_q=0,\quad\text{ and }\quad\left(1+\epsilon\right)\Lambda_\psi+\left(1-\gamma\right)\Gamma_\psi=1.$$

As a result, we have

$$\log\left(1-\tau\left(\boldsymbol{\phi}\right)\right)=-\log\left(1+\psi\left(\boldsymbol{\phi}\right)\right)-\log\bar{\eta}$$

as in (38). QED.

B Proofs of Results in Section 4

Proof of Proposition 5. In the vertical economy, the vector of labor shares is $\boldsymbol{\alpha} = (1, 0, 0)'$, the vector of household consumption shares is $\mathbf{v} = (0, 0, 1)'$, and the input-output matrix is given by

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{55}$$

Note that while the entries in the first row do not add up to one, this is a non-issue because firm 1's labor share is 1 (and hence its input share is zero).

We consider the limit in which firm technologies approach constant returns to scale (CRS). In this case, we may compute the influence vector from equation (26),

$$\mathbf{q} = \mathbf{v}' \left[\mathbb{I}_N - \left((\mathbf{1} - \boldsymbol{\alpha}) \, \mathbf{e}_N' \right) \circ \mathbf{W} \right]^{-1}.$$

Substituting in this economy's values for $\mathbf{v}$, $\boldsymbol{\alpha}$, and $\mathbf{W}$, this expression becomes:

$$\mathbf{q} = (0, 0, 1) \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right)^{-1}.$$

The influence vector in this economy is hence given by

$$\mathbf{q} = (1, 1, 1).$$

As for the profits distortion function, $\psi(\boldsymbol{\phi})$, we have that

$$\psi(\boldsymbol{\phi}) \equiv \mathbf{e}_N' \left((\mathbf{e}_N - (\boldsymbol{\phi} \circ \boldsymbol{\eta})) \circ \mathbf{a}(\boldsymbol{\phi}) \right) = (1, 1, 1) \left(\begin{bmatrix} 1 - \phi_1 \\ 1 - \phi_2 \\ 1 - \phi_3 \end{bmatrix} \circ \mathbf{a}(\boldsymbol{\phi}) \right), \tag{56}$$

where the vector of sales, $\mathbf{a}(\boldsymbol{\phi})$, is given by (29). Again, plugging in our values for for $\mathbf{v}$, $\boldsymbol{\alpha}$, and $\mathbf{W}$, we obtain the following expression for sales:

$$\mathbf{a}(\boldsymbol{\phi}) = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 - \phi_1 \\ 1 - \phi_2 \\ 1 - \phi_3 \end{bmatrix}' \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Therefore, $\mathbf{a}(\boldsymbol{\phi})$ must satisfy

$$\mathbf{a}(\boldsymbol{\phi}) = \begin{bmatrix} 1 & -\phi_2 & 0 \\ 0 & 1 & -\phi_3 \\ \phi_1 - 1 & \phi_2 - 1 & \phi_3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Taking the inverse of this matrix, we get that the vector of sales is given by

$$\mathbf{a}(\boldsymbol{\phi}) = \begin{bmatrix} 1/\phi_1 \\ 1/\phi_1\phi_2 \\ 1/\phi_1\phi_2\phi_3 \end{bmatrix}. \tag{57}$$

Note that this sales vector does not coincide with the influence vector $\mathbf{q}$ unless $\phi_i = 1$ for all i.

Substituting (57) into (56), we have that

$$\psi(\boldsymbol{\phi}) = (1,1,1)\left(\begin{bmatrix} 1-\phi_1 \\ 1-\phi_2 \\ 1-\phi_3 \end{bmatrix} \circ \begin{bmatrix} 1/\phi_1 \\ 1/\phi_1\phi_2 \\ 1/\phi_1\phi_2\phi_3 \end{bmatrix}\right) = \frac{1-\phi_1}{\phi_1} + \frac{1-\phi_2}{\phi_1\phi_2} + \frac{1-\phi_3}{\phi_1\phi_2\phi_3}.$$

The profits distortion hence satisfies

$$1 + \psi(\boldsymbol{\phi}) = \frac{1}{\phi_1\phi_2\phi_3},$$

which implies $1 - \tau_\ell(\boldsymbol{\phi}) = \phi_1\phi_2\phi_3$. Furthermore, the efficiency wedge is given by

$$\zeta(\boldsymbol{\phi}) = \exp\{\mathbf{q}\log\boldsymbol{\phi}\}(1 + \psi(\boldsymbol{\phi})) = \frac{\phi_1\phi_2\phi_3}{\phi_1\phi_2\phi_3} = 1.$$

Finally, aggregate value added must satisfy (35). Substituting in our expressions for $\mathbf{q}$ and $\psi(\boldsymbol{\phi})$, we get

$$\log GDP(\boldsymbol{\phi}) = \frac{\epsilon+1}{\epsilon+\gamma}(\log\phi_1 + \log\phi_2 + \log\phi_3) - \frac{\epsilon}{\epsilon+\gamma}(\log\phi_1 + \log\phi_2 + \log\phi_3) + const.$$

As a result, log GDP is given by

$$\log GDP(\boldsymbol{\phi}) = \frac{1}{\epsilon+\gamma}(\log\phi_1 + \log\phi_2 + \log\phi_3) + const,$$

as stated in the proposition. QED.

Proof of Proposition 6. In the horizontal economy, the vector of labor shares is $\boldsymbol{\alpha} = (1,1,1)'$, the vector of household consumption shares is $\mathbf{v} = (1/3, 1/3, 1/3)'$, and the input-output matrix is

given by

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{58}$$

All entries in the input-output matrix are zero as no commodity is bought by any firm as an input.[66]

We again consider the limit in which firm technologies approach constant returns to scale (CRS) and compute the influence vector from equation (26). Substituting in this economy's values for $\mathbf{v}$, $\boldsymbol{\alpha}$, and $\mathbf{W}$, we obtain the following expression for the influence vector:

$$\mathbf{q} = (1/3, 1/3, 1/3) \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)^{-1}.$$

The influence vector in this economy is hence given by

$$\mathbf{q} = (1/3, 1/3, 1/3).$$

As for the profits distortion function, $\psi(\boldsymbol{\phi})$, we have that

$$\psi(\boldsymbol{\phi}) \equiv \mathbf{e}_N' \left((\mathbf{e}_N - (\boldsymbol{\phi} \circ \boldsymbol{\eta})) \circ \mathbf{a}(\boldsymbol{\phi}) \right) = (1,1,1) \left(\begin{bmatrix} 1-\phi_1 \\ 1-\phi_2 \\ 1-\phi_3 \end{bmatrix} \circ \mathbf{a}(\boldsymbol{\phi}) \right), \tag{59}$$

where the vector of sales, $\mathbf{a}(\boldsymbol{\phi})$, is given by (29). Again, plugging in our values for for $\mathbf{v}$, $\boldsymbol{\alpha}$, and $\mathbf{W}$, we obtain the following expression for sales:

$$\mathbf{a}(\boldsymbol{\phi}) = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \begin{bmatrix} 1-\phi_1 \\ 1-\phi_2 \\ 1-\phi_3 \end{bmatrix}' \right)^{-1} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}.$$

Therefore, $\mathbf{a}(\boldsymbol{\phi})$ must satisfy

$$\mathbf{a}(\boldsymbol{\phi}) = \begin{bmatrix} 1 - \frac{1}{3}(1-\phi_1) & -\frac{1}{3}(1-\phi_2) & -\frac{1}{3}(1-\phi_3) \\ -\frac{1}{3}(1-\phi_1) & 1 - \frac{1}{3}(1-\phi_2) & -\frac{1}{3}(1-\phi_3) \\ -\frac{1}{3}(1-\phi_1) & -\frac{1}{3}(1-\phi_2) & 1 - \frac{1}{3}(1-\phi_3) \end{bmatrix}^{-1} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}.$$

[66] Again, note that for all rows, the entries do not add up to one. This again does not matter because all firms have a labor share of 1.

Taking the inverse of this matrix, we get that the vector of sales is given by

$$\mathbf{a}(\boldsymbol{\phi}) = \frac{1}{(\phi_3 + \phi_2 + \phi_1)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{60}$$

Note that this sales vector does not coincide with the influence vector $\mathbf{q}$ unless $\phi_i = 1$ for all i.

Substituting (60) into (59), we find that the profits distortion satisfies

$$1 + \psi(\boldsymbol{\phi}) = \frac{1}{\frac{1}{3}(\phi_3 + \phi_2 + \phi_1)}.$$

which implies $1 - \tau_\ell(\boldsymbol{\phi}) = \frac{1}{3}(\phi_3 + \phi_2 + \phi_1)$. Furthermore, the efficiency wedge is given by

$$\zeta(\boldsymbol{\phi}) \equiv \exp\{\mathbf{q} \log \boldsymbol{\phi}\}(1 + \psi(\boldsymbol{\phi})) = \frac{(\phi_1 \phi_2 \phi_3)^{\frac{1}{3}}}{\frac{1}{3}(\phi_3 + \phi_2 + \phi_1)}.$$

Finally, aggregate value added must satisfy (35). Substituting in our expressions for $\mathbf{q}$ and $\psi(\boldsymbol{\phi})$, we find that log GDP is given by

$$\log GDP(\boldsymbol{\phi}) = \frac{\epsilon + 1}{\epsilon + \gamma} \frac{1}{3} (\log \phi_1 + \log \phi_2 + \log \phi_3) - \frac{\epsilon}{\epsilon + \gamma} \log(\phi_1 + \phi_2 + \phi_3) + const,$$

as stated in the proposition. QED.

Proof of Equation (40). In the hybrid chain economy, the vector of labor shares is $\boldsymbol{\alpha} = (1, \alpha_2, \alpha_3)$, the vector of household consumption shares is $\mathbf{v} = (0, 0, 1)$, and the input-output matrix is the same as in the vertical economy and thereby given by (55). Again in the CRS limit we may compute the influence vector from equation (26). Substituting in this economy's values for $\mathbf{v}$, $\boldsymbol{\alpha}$, and $\mathbf{W}$, we obtain the following expression for the influence vector:

$$\mathbf{q} = (0, 0, 1) \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 - \alpha_2 & 1 - \alpha_2 & 1 - \alpha_2 \\ 1 - \alpha_3 & 1 - \alpha_3 & 1 - \alpha_3 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right)^{-1}.$$

Thus,

$$\mathbf{q} = (0, 0, 1) \begin{pmatrix} 1 & 0 & 0 \\ -(1 - \alpha_2) & 1 & 0 \\ 0 & -(1 - \alpha_3) & 1 \end{pmatrix}^{-1}.$$

Computing this inverse, we find that the influence vector in this economy is given by

$$\mathbf{q} = ((1 - \alpha_2)(1 - \alpha_3), 1 - \alpha_3, 1)$$

as stated in (40). QED.

Proof of Equation (41). In the hybrid circle economy, the vector of labor shares is $\boldsymbol{\alpha} = (\alpha, \alpha, \alpha)'$, the vector of household consumption shares is $\mathbf{v} = (1/3, 1/3, 1/3)'$, and the input-output matrix is given by

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{61}$$

Again in the CRS limit we may compute the influence vector from equation (26). Substituting in this economy's values for $\mathbf{v}$, $\boldsymbol{\alpha}$, and $\mathbf{W}$, we obtain the following expression for the influence vector:

$$\mathbf{q} = \frac{1}{3}(1,1,1)\left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - (1-\alpha)\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right]^{-1}.$$

Thus,

$$\mathbf{q} = \frac{1}{3}(1,1,1)\begin{pmatrix} 1 & 0 & -(1-\alpha) \\ -(1-\alpha) & 1 & 0 \\ 0 & -(1-\alpha) & 1 \end{pmatrix}^{-1}.$$

Computing this inverse, we find that the influence vector in this economy is given by

$$\mathbf{q} = \frac{1}{3}\left(\frac{1+(1-\alpha)+(1-\alpha)^2}{1-(1-\alpha)^3}\right)(1,1,1)$$

as stated in (41). QED.

C Construction of the Input-Output Tables

We work with the Use of Commodities by Industries After Redefinitions (USES AR).[67] The USES AR tables reported by the BEA are organized in the following way: every row corresponds to a sectoral commodity and every column to an industry or component of aggregate demand (consumption, investment, etc). An entry of that table reports the expenditures on the commodity of the corresponding row by the industry of the corresponding column (in billions of US dollars). We use this table to reconstruct the Input-Output table of the US that corresponds to our model, but its construction of this tables merits some discussion.

The NIPA categories are based on attributes of the goods and services (commodities) produced by an industry. The Uses AR table is constructed from another table called the the Uses of Commodities by Industries Before Redefinitions (Uses BR). The data in the Uses BR table is constructed from firm level data. However, several firms produce goods that fall into different product categories. The Uses AR thus reorganizes the data in the Uses BR by reassigning a portion the commodity uses of a given industry to another industry when the firms in the former also produce the product of latter.

First, we make adjustments so that there are the same number of input sectors as output sectors. This ensures that we have a symmetric IO table. Not all commodities correspond to industries at the five digit level. To construct a symmetric IO table, we assign the uses of Secondary smelting and alloying of aluminum (331314) sector to merge Alumina refining and primary aluminum production (33131a). We do the same for the Federal electric utilities (S00101) sector which is allocated to Federal general government (nondefense) (S00600) sector and for State and local government passenger transit (S00201) and State and local government electric utilities (S00202) which are both allocated to State and local general government (S00203). Finally, the input-output tables also report entries for used goods and scrap. We reassign these rows across all other rows according to their proportion.

Treatment of Finance, Insurance, and Real-Estate Financing (FIRE). Second, we treat the Finance, insurance, real estate, rental, and leasing (FIRE) sectors as a special sector. The Uses table reports transfers from each sector to each of the the FIRE industries. However, we do not want to treat financial, insurance and real estate payments as intermediate production inputs. The reason for this that we interpret production function as related to the physical of production: if a given sector purchases a given volume of production inputs, production would be the same. In contrast, higher interest rates, insurance premia, or rental rates will only affect the the distribution of the value added of each sector —the portion going to the capital share and the rest to their financing.

Thus, we exclude the rows and columns corresponding to the Finance, Insurance, and Real-

[67] For robustness we also check that our results hold if we use the USES Before Redefinitions table (USES BR).

NIPA	Sector Description
52A000	Monetary authorities and depository credit intermediation
522A00	Nondepository credit intermediation and related activities
523A00	Securities and commodity contracts intermediation and brokerage
523900	Other financial investment activities
524100	Insurance carriers
524200	Insurance agencies, brokerages, and related activities
525000	Funds, trusts, and other financial vehicles
531000	Real estate
532100	Automotive equipment rental and leasing
532A00	Consumer goods and general rental centers
532400	Commercial and industrial machinery and equipment rental and leasing
533000	Lessors of nonfinancial intangible assets

Table 4: FIRE Sectors

Estate (FIRE) sectors from our I-O matrix. That is, we do not treat the income of FIRE sectors as a production input, but rather, as part of capital gains. We assign the commodity share of the FIRE sectors to part of the capital share of production. In turn, the purchases of the financial sector are considered as part of final uses. To avoid double counting, we don't include this share in GDP accounting. We will use the flows from industries to the financial sector when we investigate sectoral wedges.

At the five digit level, these industries correspond to:

Treatment of Exports and Imports. Our model is a closed economy while the actual IO-tables include a foreign sector, i.e. imports and exports. In the data, several industries show a negative trade balance, which in principle means that certain intermediate inputs could be partially supplied by an external sector.

However, turns out that for all industries, except for oil, domestic absorption exceeds any negative trade balance. Even at the five digit level, most inputs used in the US are produced within the US. In fact, most commodities in the US are both imported and produced domestically, and used as inputs and final goods. There are some commodities that are imported but for all commodities except oil, that are produced in the US, final uses exceed imports. Thus, we treat all imports as if they are consumed by households —in the case of oil, we assume ignore oil imports. In the calibration, we leave aside issued of foreign trade. We will think of households as purchasing US produced goods, exporting some of these and importing the rest for final consumption only. Thus, we treat all US production as final uses. When we consider the household's expenditure shares on domestic production, we exclude imports but include exports as part of the shares.

Treatment of Inventories. Finally, we correct for inventories. Part of the final uses of a given sector is inventory that will be used in final periods. Our model is static so there is no inventory

accumulation. In practice, inventories can be stored as final goods, or as materials.

To account for inventories, we subtract inventories from total investment, and redistribute the dollar value of inventories supplied by a given sector by adding a share of total inventories to the uses by other sectors. The share of inventories we add to uses is proportional to the uses of each sector over total uses. For example, we raise the sales of cotton mills to the textile industry by adding the stock of cotton inventories times the proportion of cotton sold to the textile industry relative to all industries.

D Microfoundations for the Financial Constraint

Finally we discuss a possible microfounded interpretation of the financial constraint imposed on these firms. We consider a few interpretations of the model that capture the essence of the financial frictions in the model. This will allow us to decompose the constraint of the firm into limited enforcement constraints and higher financial costs.

Suppose a firm faces a limited enforcement constraint. Firm i maximizes profits,

$$\Pi_i = \max_{\sigma_i, x_i} p_i y_i - \bar{p}_i x_i$$

subject to

$$\begin{aligned} y_i &= z_i x_i^{\alpha_i} \\ (1-\sigma_i)\,\bar{p}_i x_i &\leq w_i \qquad (62) \\ (1-\theta_i)\,p_i y_i &\leq p_i y_i - \sigma_i \bar{p}_i x_i. \qquad (63) \end{aligned}$$

In the expression above $\bar{p}_i$ is the marginal cost of x_i which by the assumption that $\sum_{j\in N_i} \alpha_{ij} = 1$, is constant and independent of x_i. The first constraint is the technological constraint of the firm. The second constraint states that a fraction $(1-\sigma_i)$, chosen by the firm, must be paid up-front with an liquid funds w_i, where w_i is given exogenously. Thus, $\sigma_i \bar{p}_i x_i$ is the amount of trade credit obtained from suppliers. Changes in the availability of these liquid funds are the focus of interest in this paper.

In addition we assume, following the contracting literature, that a firm may pledge at most θ_i of their revenue to pay their suppliers. If the firm chooses to default on its suppliers, they lose a fraction θ_i of the firm's income. Hence, upon default, the firm keeps the fraction $(1-\theta_i)\,p_i y_i$. Thus, the third constraint is an incentive constraint that states that the fraction of output they get to keep should they choose to default on suppliers must exceed the revenue firm's expect to make minus the amount it owes after it pays for part of its inputs with liquid funds.

By rearranging this constraint, we obtain an equivalent constraint, $\sigma_i \bar{p}_i x_i \leq \theta_i p_i y_i$. This one reads that the amount that the firm can owe to its suppliers after it paid from a certain fraction in advance must not exceed the pledgeable amount of output, $\theta_i p_i y_i$.

In the specific examples that follow, we let the liquid funds to be some proportion of the firms output $w_i = \omega_i p_i y_i$. Then,

$$\begin{aligned} (1-\sigma_i)\,c_i x_i &\leq \omega_i p_i y_i \qquad (64) \\ c_i x_i &\leq \theta_i p_i y_i \qquad (65) \end{aligned}$$

so combining both constraints,

$$\omega_i + \theta_i = \chi_i$$

so θ_i is the fraction of trade credit and ω_i is the working capital of the firm.

E Additional Graphs

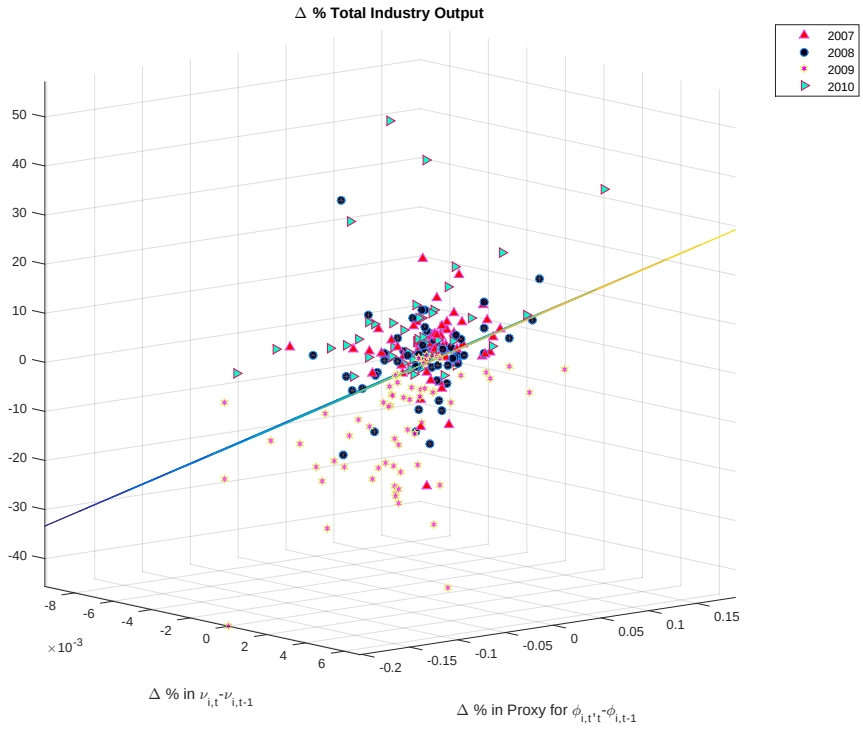

Figure 12: Regression Fit - Expenditure Shares and ϕ vs. Sectoral Output