Do Firms Mitigate or Magnify Capital Misallocation? Evidence from Plant-Level Data*,**

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July 17, 2016

Abstract

More than half of the cross-plant dispersion in average capital returns occurs across plants within the same firm. Even though firms allocate investment very differently across their plants, they do not equalize average returns of capital across their plants; in recessions, the share of within-firm dispersion even increases to 60%. We reconcile these findings in a model of multi-plant firms, physical adjustment costs and credit constraints. When multi-plant firms are credit constrained, they can utilize internal capital markets by concentrating internal funds on investment in only a few of their plants in a given period and rotating funds to another set of plants in the future. The resulting increase in within-firm dispersion of capital returns is hence not a symptom of misallocation within the firm, but rather indicates the mitigating of credit constraints in external financial markets. Economies with multi-plant firms produce more aggregate output despite *more* dispersion in capital returns compared to economies with single-unit firms. The efficiency gains from bringing down misallocation in emerging single-plant-firm economies to the level of developed multi-plant-firm economies are even larger than previously thought.

KEYWORDS: Misallocation, Productivity Dispersion, Investment, Plants vs. Firms, Internal vs. External Capital Markets.
JEL CODES: E2, G3

*This draft is partly based on our earlier paper "Financial Frictions and Investment Dynamics in Multi-Plant Firms." We thank seminar participants at Duke, Oxford, Penn State, Iowa, UW Madison, Arizona State, Ente Einaudi, Boston College, UT Austin, Census-CES, the Federal Reserve, the Dutch Central Bank, the Barcelona Summer Forum, the NBER Summer Institute, the Ifo Conference on Macroeconomics and Survey Data, the Midwest Macro Meetings, the Empirical Macro Workshop and the Annual Meeting of German Economists Abroad as well as Klaus Adam, Gadi Barlevy, Marco Bassetto, Christian, Bayer, David Berger, Jeff Campbell, John Cochrane, Russ Cooper, François Gourio, John Haltiwanger, Hugo Hopenhayn, Ben Moll, Michèle Tertilt and Mark Wright for helpful discussions.

**Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed. The latest version of this paper can be downloaded at http://papers.ssrn.com/abstract=2731594

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1 Introduction

A considerable body of recent research has documented a large, persistent and ubiquitous degree of productivity dispersion across production units, leading in turn to a revival of interest in the causes and consequences of resource misallocation. For instance, Hsieh and Klenow (2009) have found substantial aggregate output increases if one managed to lower this dispersion by improving the allocation of resources across production units. Alternatively interpreting dispersion as a consequence of exogenous increases in uncertainty, Bloom (2009) has shown the recessionary effects of higher productivity dispersion. The implicit assumption in the literatures growing out of these seminal papers is that a high level of productivity dispersion is a sign of resource misallocation in an economy and therefore reduces welfare. But does higher productivity dispersion always depress output and welfare? In this paper, we explore the opposite hypothesis: we show theoretically that optimal behavior of agents may lead to increased productivity dispersion. Novel empirical evidence from data on multi-plant firms supports our hypothesis. We then build a model of investment in multi-plant firms which, calibrated to U.S. manufacturing, attributes up to half of the empirically observed productivity dispersion to optimal firm behavior rather than misallocation. As a consequence, our theoretical and empirical work suggests that the gains from reducing inefficient distortions in developing economies to the level of the U.S. are considerably larger than previously thought.

To illustrate our theoretical argument, consider an economy with many production units which are perfectly identical in terms of technology, productivity and capital. Given their state, each unit would find it optimal to undertake an investment project of a fixed size, but cannot do so for a lack of access to external finance. With this extreme financing constraint, no investment activity takes place and productivity dispersion across units – conceptualized as the dispersion in capital returns – remains zero. Suppose instead that there is a benevolent planner with a limited amount of funds. These funds suffice to finance the investment project in some but not all plants. So the planner optimally finances the investment project in as many plants as the funds permit. Productivity dispersion would then rise, reflecting investment in some plants and no investment in others. A naive interpretation of this higher productivity dispersion would assume that the latter allocation is less efficient than the former where no investment takes place at all. Yet, in reality, the higher dispersion in the latter case is the outcome of a benevolent planner doing his best to overcome a financing constraint and generating more investment, capital and output in the aggregate.

This example demonstrates that productivity dispersion – commonly taken as an indicator for insufficient factor reallocation – may rise or fall with efficiency and thus aggregate output, while reallocation intensity and aggregate output are monotonically linked. It is therefore unclear how much of the empirically observed productivity dispersion reflects inefficient distortions and how much reflects (constrained) efficient behavior. This ambiguity warrants an empirical investigation

in the origins of productivity dispersion. While research in the above mentioned literatures are concerned with the impact of misallocation and uncertainty on the aggregate economy, there is little work on why and where in the economy such significant dispersion originates in the first place. Yet, identifying the agents and frictions that are mainly responsible for the substantial productivity dispersion is crucial to assess how much of that dispersion reflects constrained optimal behavior versus true misallocation. This knowledge in turn is relevant for policymakers interested in fostering growth and mitigating recessions. In the present paper, we fill this gap in the literature by jointly studying the dispersion of capital returns between firms *and* between plants within firms. As a starting point, we document two novel empirical findings based on the plant-level data from the U.S. Annual Survey of Manufactures: First, more than half of the overall dispersion originates across plants within the same firm rather than between firms. Second, when overall dispersion rises in recessions, we show that it rises primarily within firms rather than between firms. Our findings are quantitatively significant: a typical firm may increase its output by about 28% if it shifted an optimal amount of resources towards high-return plants. In strong downturns such as the 1980/82 recession, this potential output gain increases to 35%.

This focus on both the plant and the firm is relevant for a number of reasons: First, it allows us to highlight the fact that the firm and the market are two fundamentally different allocation mechanisms. In the micro-founded macroeconomic literature, plants are typically treated as independent decision makers. Yet, the allocation of resources across plants could conceivably be affected if they are part of an integrated firm or are required to share scarce firm inputs such as logistics and external credit. Consequently, understanding how firms work differently than markets may shed new light on the causes of resource (mis)allocation or how the economy copes with aggregate disturbances such as credit or uncertainty shocks.

Second, our findings potentially inform us on whether firms or markets are the most appropriate determinants of allocative efficiency and economic growth: if the dispersion of capital returns across plants reflects resource misallocation, then our empirical findings may indicate that firms are in fact inferior to markets at allocating inputs. In other words, distorsions such as principal-agent problems, transaction costs and asymmetric information may not be mitigated significantly by firms.

Third, the economy-wide effects of aggregate shocks are often largely shaped by frictions that hamper resource reallocation at the micro level. While a large variety of micro-level frictions has been suggested and studied in the literature, little effort has been made to understand at what granular level these frictions matter quantitatively. Since we are studying the dispersion of capital returns, we give some examples of frictions that interfere with capital reallocation and investment in general. For instance, some studies have highlighted the lumpy nature of investment at the *plant* level (see Doms and Dunne (1998) and Gourio and Kashyap (2007)), while others have found that capital expenditures at the level of the *firm* is fairly smooth (Eberly et al. (2012)). In the literature

on uncertainty shocks, Bloom et al. (2012) combine such shocks with non-convex adjustment costs of capital that are most pertinent at the plant level. Christiano et al. (2014), in contrast, focus instead on financial constraints, a class of frictions that are of relevance at the level of the firm. These two separate ways of modeling the effects of uncertainty shocks require empirical productivity dispersion to fluctuate at different aggregation levels. Hence, documenting at which level of aggregation the dispersion of capital returns is most prevalent and volatile could be informative about the type of reallocation frictions that are most relevant.

All of the above reasons are strong arguments to investigate further the extent and dynamics of productivity dispersion. In this paper, our focus is on questions related to the link between within-firm dispersion in capital returns and investment activity. For instance, why do firms tolerate such large dispersion of capital returns across their plants? As an allocation mechanism of scarce resources, are firms less efficient than the market? Or is this dispersion simply optimal given the constraints, both internal and external, faced by the firm? How much do firms discriminate in allocating capital across their plants? At first sight, they appear to do so to a large extent: we find that dispersion in investment rates across plants is also large and more important within than between firms. In line with other studies highlighting the lumpy nature of investment (see Doms and Dunne (1998) and Gourio and Kashyap (2007)), we find that this dispersion is largely driven by investment spikes in a small number of plants. Yet, the correlation between a plant's capital return and a plant's investment within a firm is not very strong even when the firm invests as a whole. Moreover, we find that this correlation is even lower in periods of tight credit, exactly when the within-firm dispersion of capital returns is largest. Eisfeldt and Rampini (2006) document a similar mis-timing of capital returns dispersion and capital reallocation between firms. Though we confirm their evidence, we find the larger portion of dispersion and mis-timed reallocation to occur between plants within firms.

This last finding hints at a potential role for *external* financial frictions in shaping the degree of capital misallocation *within* the firm. To investigate this channel, we build a novel dynamic investment model of a multi-plant firm that features technological, organizational and financial interdependencies and constraints. The purpose of the model is threefold. First, we wish to establish how much of the empirically observed dispersion in capital returns and investment rates can be explained by our model. Second, we investigate the relative importance of the various frictions and their interactions as determinants of this apparent misallocation. Third, we study how much of the countercyclical dispersion in capital returns could result from optimal firm behavior.

Because of the focus in the literature on single-plant models, we are required to build a new model of a firm operating multiple distinct plants. Yet, in shaping our framework, we are mindful of the many frictions and imperfections that have been suggested in models of capital accumulation and reallocation (see Caballero (1999) for an overview). The investment literature, for example, has argued that "technological" frictions such as a fixed cost of investing are crucial to replicate the

lumpy aspect of investment documented empirically (see among others Abel and Eberly (1994), Caballero et al. (1995), Cooper and Haltiwanger (2006), Gourio and Kashyap (2007)). Others have instead focused on the central role played by financing frictions, arguing that they offer a natural explanation for the documented role of cash flow in investment regressions (see among others Fazzari et al. (1988), Gilchrist and Himmelberg (1995)). Most of these papers have considered frictions in isolation.[1] We are the first to nest real and financial frictions in a model of a multi-plant firm. While some, mostly technological, frictions matter at the level of the plant, others such as financial or organizational imperfections affect the firm as a whole. We argue that modeling a multi-plant structure is crucial to understanding the effects of various frictions and their interactions. For instance, focusing solely on firm-level frictions would counterfactually predict the absence of any dispersion in capital returns across plants within firms. On the other hand, with only plant-level frictions, one would expect to see no difference between plants in single-unit versus multi-unit firms. our understanding about which micro-level frictions matter for aggregate outcomes may be affected by the multi-plant structure of firms in the economy.[2] Though its first aim is to characterize the investment policy of multi-plant firms, our model will also enable us to address questions about the aggregate effects of plant- and firm-level frictions.

In our model, firms operate plants that face both fixed and convex adjustment costs while the firm organizes internal and costly external financing of investment. When firms are credit constrained, they leverage internal capital markets and focus investment on only a few plants *even if most of their plants are equally productive*. This credit constrained investment policy leads to a rise in the dispersion of capital returns within the firm. In other words, had the firm unlimited access to external funds, it would invest in a way to minimize further this dispersion, to the extent allowed by the technological constraints it faces. This is consistent with Asker et al. (2014) and Bartelsmann et al. (2013) who have shown that non-convexities such as fixed investment adjustment costs or overhead labor can lead to endogenous dispersion of capital returns in equilibrium. Our model contains both features, but we show that their impacts can be magnified by the presence of costly external finance. In the face of such financing constraints, we document that the firm's optimal policy is to allocate internal funds to only a handful of its plants in a given period, postponing investment projects in its other plants. This staggering in the allocation of scarce resources implies that an externally credit constrained firm will display heightened capital returns and investment dispersion across its plants for several periods. The serial correlation of large investment projects, however, will increase in credit-constrained firms compared to their unconstrained counterparts. Using micro data from the U.S. Annual Survey of Manufactures, we find empirical support for these predictions at the micro and macro levels.

[1] Gomes (2001) and Eisfeldt and Muir (2013) who have combined real and financial frictions in a unified model of a firm operating one plant are notable exceptions.

[2] We study this question in a companion paper Kehrig and Vincent (2015): "Financial Frictions and Investment Dynamics in Multi-Plant Firms."

As described above, our model can explain dispersion of capital returns and investment rates within firms qualitatively. But dispersion may result from various other reasons beyond the scope of our model mechanism. We therefore analyze the quantitative predictions of our model. Calibrating it to micro-level moments in our data, we find that introducing financial firm-level constraints increases capital returns dispersion within the firm by 43% compared to dispersion already caused by technological non-convexities at the plant level. This suggests that our mechanism is quite powerful in generating within-firm heterogeneity with is substantial and countercyclical. This also means that dispersion of capital returns within financially-constrained firms is not necessarily an indicator of misallocation or inefficient investment policies, but rather the by-product of the firm's effort to mitigate external financial frictions. On the contrary, if the firm did not leverage its internal capital market, dispersion might remain at a lower level, but overall investment, growth and output would be lower as well. Internal capital markets thus help an economy cope more easily with periods of financial tightness.

In a counterfactual exercise, we find that the existence of internal capital markets in multi-plant firms can increase within-firm dispersion by up to 20%, yet also raise aggregate investment and output by up to 8% and 5%, respectively, relative to an economy in which the pooling of resources within the firm is shut down. We view this finding as a cautionary tale to interpreting higher productivity dispersion as a sign of resource misallocation.

We see our project as a first step into modeling how the organizational structure of a firm may impact the micro-level adjustment of capital, as well as understanding the role of firms for efficiency. Some theoretical research has been done on the efficiency of internal versus external capital markets: Stein (1997), Malenko (2012) study mostly principal-agent problems between a firm's owner and manager in a single-plant setup. Whether internal capital markets are more or less efficient is theoretically ambiguous: Gertner et al. (1994) show that division managers may exploit imperfect monitoring by firm headquarters to build up "inefficient empires." On the other hand, Scharfstein and Stein (2000) say that firms are likely to monitor investment spending better than outside financiers thus improving capital allocation. study information problems in a hierarchy with several managers and show that. Eisfeldt and Papanikolaou (2013) stress the importance of organizational or intangible capital at the firm level in order to understand a firm's productivity albeit without the multi-plant dimension we are interested in. With the exception of Lamont (1997), Schoar (2002), Giroud (2013), empirical research on within-firm dynamics is scarce. That being said, our paper currently falls short of thoroughly studying the general equilibrium effects on multi-plant firms in the spirit of Thomas (2002) and Khan and Thomas (2008). On the empirical front, we believe that taking seriously this firm-plant dichotomy and the joint investment dynamics of plants within a firm offers a new dimension for the researcher to identify various types of frictions.

Our paper is organized as follows. In Section 2, we describe the data and show evidence on the importance of the within-firm dimension for the dispersion of capital returns and investment.

Section 3 describes our multi-plant model of the firm and analyses its predictions when an external financing constraint is introduced. Section 4 conducts various quantitative exercises geared to understand the nature of productivity dispersion and carry out counterfactual exercises. In Section 5, we investigate whether the model predictions are borne out in the micro-level data; Section 6 concludes.

2 Empirical Motivation

2.1 Data sources and variables of interest

We use Census data described in Appendix A.1. Our main object of interest are the dispersions of real returns of capital and the real investment rates across plants in the economy. They reflect the differences in capital returns and capital reallocation, respectively. In a standard frictionless economy, agents should choose investment rates in a way to equalize expected returns on capital. Hsieh and Klenow (2009), Asker et al. (2014) among others assess misallocation by studying the dispersion of revenue total factor productivity, denoted TFPR. While this is appropriate in their context, it is not in ours where we want to proceed to study investment. Investment should not necessarily flow towards the units with the highest TFPR if they already operate a large capital stock. The investment flow is shaped by the expected capital return. This is defined as the proceeds of one unit of capital at the end of next period – the value of undepreciated capital plus its marginal product – divided by the cost of next period's capital in this period. We define industry output as the numeraire, and the price of next period's capital k' in terms of this period's numeraire denoted by $P_t^{k'}$. Let δ denote the industry-wide depreciation rate, MPK_{nt} the marginal product of capital in plant n in year t, y_{nt} real value added and k_{nt} the capital stock, then the expected return, $\mathbb{E}\mathcal{R}_{nt+1}$, and its variance across plants in an industry are

$$\begin{aligned}
\mathbb{E}\mathcal{R}_{nt+1} &= \mathbb{E}\frac{P_{t+1}^{k}(1-\delta)+MPK_{nt+1}}{P_t^{k'}}-1 \\
Var_t(\mathbb{E}\mathcal{R}_{nt+1}) &= Var_t\left(\mathbb{E}MPK_{nt+1}\right)/(P_t^{k'})^2 \\
&\propto Var_t\left(\frac{\mathbb{E}y_{nt+1}}{k_{nt+1}}\right)/(P_t^{k'})^2 \qquad (1)
\end{aligned}$$

last proportionality statement is true when the production function is Cobb-Douglas. Note that neither the depreciation rate, δ, nor the price of next period's capital in terms of next period's output, P_{t+1}^{k}, affect the cross-sectional dispersion within an industry.

In order to study the dispersion of capital returns in equation (1) as well as that of investment rates requires from us to compute real value added, the real capital stock and real investment, denoted by i_{nt}. To obtain real value added, we first compute nominal value added as sales less

intermediate and energy inputs, corrected for inventory changes and resales, and deflate the resulting measure by the 6-digit NAICS shipment price deflator from the NBER-CES manufacturing database. The real capital stock, k_{nt}, is the sum of structure and equipment capital each of which are expressed as real replacement values at current market conditions. These replacement values for both structure and equipment capital are individually computed by first using the perpetual inventory method on book values, second by transforming these nominal book values into market values and finally deflating this measure using 3-digit NAICS industry price deflators. Like capital, we compute real investment as the sum of real structure and equipment investment by deflating the respective nominal investment expenditures by the 3-digit NAICS industry investment price deflators from the BEA. Our capital measure denotes the beginning-of-year stock values while our investment and value added measures refer to flow values during the year.

2.2 Returns dispersion and reallocation within firms

The goal of our empirical work is to document and study the dispersion of capital returns in a given industry where different agents compete with each other over resources to allocate. In particular, we are interested in assessing the relative importance of dispersion across firms and plants, as well as its evolution over the business cycle.

To get an idea of the complexity of the typical firm in the U.S. manufacturing sector, Table 3 shows moments about the distribution of the number of plants per manufacturing firm. We present moments for both the ASM panel, the dataset used in our study, as well as for the manufacturing portion of the LBD for comparison purposes. Our preferred specification is the capital-weighted average, in order to take into account the relevant magnitude of firms. According to this measure, the typical manufacturing firm in the U.S. operates on average 36 plants, relative to 2 plants if we use an unweighted average. This obviously reflects the fact that most capital is concentrated in firms that operate many plants. Using employment weights gives results that are only slightly smaller in magnitude than those using capital weights, suggesting that our results do not hinge on an extremely skewed capital distribution. The observations labeled "LBD" come from the Longitudinal Business Database which reports employment and firm affiliation of all active plants (though no investment, output or capital data). This measure contains all active plants while the ASM panel is biased towards larger plants. As far as the number of plants per firm is concerned, the ASM panel is a fairly representative sample of the underlying universe which is captured in the LBD. All this evidence shows that multi-unit firms are very prevalent and that they account for the majority of aggregate employment, capital and production. Moreover, the large number of plants per manufacturing firm suggests that firms have to consider a complex hierarchy of plants when allocating resources inside their firm.

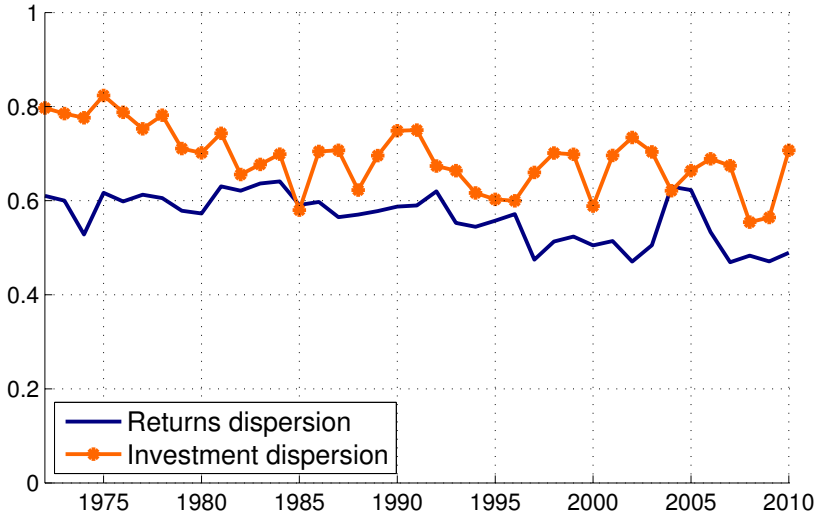
We denote the weighted cross-sectional variance of capital returns displayed in equation (1) by σ_t. Since we do not measure expected returns, we approximate them by realized capital returns.

This is a good approximation if capital is chosen one period in advance, all other inputs are chosen statically and total factor productivity is sufficiently persistent. All these assumptions are plausible and widely used in the macroeconomic and investment literature. In a given industry, capital returns dispersion can be decomposed into the dispersion between firms, denoted by σ_t^B, and the average dispersion between plants within firms, denoted by σ_t^W:

$$\begin{aligned}\sigma_t &= \sum_n \omega_{njt} \left(\mathcal{R}_{njt} - \overline{\mathcal{R}}_t\right)^2 \\ &= \underbrace{\sum_j \omega_{jt} \left(\mathcal{R}_{jt} - \overline{\mathcal{R}}_t\right)^2}_{\sigma_t^B \text{ between}} + \underbrace{\sum_j \omega_{jt} \underbrace{\sum_{n \in j}^{N_j} \tilde{\omega}_{njt} \left(\mathcal{R}_{njt} - \mathcal{R}_{jt}\right)^2}_{\sigma_{jt}^W \text{ within firm } j}}_{\sigma_t^W \text{ average within}}\end{aligned} \tag{2}$$

where n indicates the plant, j the firm, $\mathcal{R}_{njt}$ the capital return of plant n belonging to firm j in year t, $\mathcal{R}_{jt}$ the average return in firm j and $\overline{\mathcal{R}}_t$ the average level of returns in the economy. We focus on weighted dispersion to capture the economic relevance and avoid small economically unimportant outliers driving our results. These weights are embedded in the ω's: the share of plant n's capital stock in the economy is denoted by $\omega_{njt} = k_{njt}/k_t$, the share of the plant's capital stock in its own firm by $\tilde{\omega}_{njt} = k_{njt}/k_{jt} = \omega_{njt}/\omega_{jt}$ and the share of firm j's capital stock in the economy by $\omega_{jt} = k_{jt}/k_t$.

Figure 1: Within-firm share of overall dispersion



The three terms from the decomposition derived in equation (2) are plotted in Figure 8. Three

results stand out: First, overall dispersion of capital returns in the economy is considerable: the variance of capital returns across plants is about 1.042. We can appreciate this number more when we look at the coefficient of variation which is about 1.401. This means that a plant one standard deviation above the mean produces $\sqrt{1.042}/0.728 \approx 1.401$ times the value added with the same capital stock as the average plant in the economy. Reallocating capital to high-return plants should hence result in a considerable boost in aggregate output. If one interprets capital returns dispersion as misallocation, this means that the economy foregoes a lot of income. We refrain from computing a hypothetical number at this point because we are aware that any such number would depend on the distribution of returns, capital stocks and various frictions which would need to be assessed for any serious income gain from reallocation.

The second conclusion of our accounting exercise is that the majority of dispersion occurs inside firms rather than between firms: about 62% of the overall variance, about 0.641, is explained by variation of capital returns within firms. The share of within-firm variation was even larger in the beginning of the sample until the mid-1990s. The aforementioned benefits from reallocation would thus firstly stem from reallocation across the production plants within a given firm. Again, we can better interpret the between-firm and within-firm dispersion using the coefficient of variation. Comparing a plant one standard deviation above the average firm capital return to the average plant, the former produces 82.1% more value added than the latter with the same capital stock. This is significant. If we assume all plants operate the same Cobb-Douglas production function without any further frictions or complementarities between plants, the typical firm could increase output by 28.4% if they managed to equalize capital returns.

3 A model of the multi-plant firm

In this section, we describe, solve, simulate and analyze a simple model of a firm comprised of several plants. We study how various plant- and firm-level frictions interact with the optimal allocation of capital by the firm across its plants. The interdependency of a plant's capital allocation varies. At one extreme, the firm is a collection of disconnected plants: decisions are made on a plant-by-plant basis, without any interactions between them. We show that in the presence of frictions at the firm level, the firm alters the size and timing of plant-level investment plans which may result in dispersed capital returns across its plants. At the other extreme, firm-level frictions may induce the firm to treat several perfectly identical plants differently. After describing and solving the problem of the multi-plant firm, we will examine how much dispersion in capital returns a calibrated version of our model can generate and how much capital returns dispersion remains unexplained.

3.1 The problem of the firm

We focus on the basic problem of a firm that operates two plants n, $n = A, B$. We limit our model to only two plants in an effort to keep the numerical analysis of our model, which features non-convex policies, computationally feasible. A larger number of plants would increase the state vector of a firm which contains the capital stock and the technology level of each plant; in addition to that, any interactions between plants within the same firm would increase exponentially in the number of plants thus complicating the analysis without adding insight in the underlying fundamental economic mechanisms. Our choice to limit attention to two-plant firms allows us to consider the interactions of several plants within the organizational unit of the same firm while minimizing the computational burden. We first consider the technology and constraints at the level of the individual plant before extending the analysis to the level of the firm. In the following, lower case letters refer to plant variables, upper case letters to firm variables and bold upper case letters refer to vectors of a firm's plant variables. For better readability, we omit time subscripts and denote variables in the next period by a prime.

3.1.1 Technology and frictions at the plant level

Each plant n is characterized by total factor productivity which may include a firm-specific component that is common to both plants. The plant operates a Cobb-Douglas production function which combines the beginning-of-period capital stock k_{nt} and other variable inputs in order to produce output y_{nt}. While capital is fixed throughout the period, we assume that plants can freely choose any other variable inputs in perfectly competitive markets. This means we can substitute out any static first-order condition for variable inputs and write plant revenues net of variable factor costs as

$$y_{nt} = z_{nt} k_{nt}^{\alpha}. \tag{3}$$

z_{nt} contains plant total factor productivity and prices of other statically chosen production factors and α is the scaled production elasticity of capital. The productivity level of plant n which belongs to firm j consists of a firm component common to both plants in the firm and an idiosyncratic plant component; both evolve as follows:

$$z_{njt} = \rho^{p} z_{njt-1} + \eta_{njt}$$
$$z_{jt} = \rho^{f} z_{jt-1} + \eta_{jt}.$$

with $\mathbb{E}\eta_{njt} = \mathbb{E}\eta_{jt} = 0$, $V(\eta_{njt}) = (\sigma^{f})^2$ and $V(\eta_{jt}) = (\sigma^{f})^2$.

The capital stock of plant n depreciates every period at rate δ and grows with investment i_{nt},

so it evolves over time according to the conventional expression

$$k_{nt+1} = (1-\delta)k_{nt} + i_{nt}.$$

As documented in a number of studies (see Cooper and Haltiwanger (1993), Cooper et al. (1999), Doms and Dunne (1998), Caballero and Engel (1999) among others), investment dynamics at the plant level are characterized by lumpiness: multiple periods of inactivity (no or only small amounts of maintenance investment) are followed by "investment spikes."[3] The traditional modeling feature used to reproduce this stylized fact is to introduce a fixed cost of investing: the firm must pay a certain cost, ψk_{nt}, if investment is greater than zero. Such costs can arise because investment activity – no matter how small or large – has a disruptive effect on production activities in the short run, for example. The parameter ψ regulates how much revenue is foregone when the plants needs to shut down production in order to install new capital. As a result of aggregation, firm-level investment activity will be less lumpy, as documented in Eberly, Rebelo and Vincent (2012).

In addition to this non-convex adjustment cost, we include a traditional quadratic adjustment cost. This convex adjustment cost captures the notion that larger investment projects become increasingly disruptive with size.[4] The parameter γ below captures the importance of this margin.

To summarize, frictions at the plant level will be expressed as:

$$\theta\left(i_{nt}, k_{nt}\right) = \left[\psi \mathbb{I}\left\{\frac{i_{nt}}{k_{nt}} > \vartheta\right\} + \gamma\left(\frac{i_{nt}}{k_{nt}}\right)^2\right] k_{nt} \tag{4}$$

where $\mathbb{I}$ is an indicator function equal to 1 if the plant investment rate is above ϑ; ψ is a parameter regulating the foregone sales if the plant undergoes an investment, and γ regulates the impact of the quadratic adjustment cost. Everything is scaled by the plant's capital stock k_{nt} in order to eliminate size differences.

Combining equations (3) and (4) above, plant cash flow is

$$\pi_{nt} = z_{nt} k_{nt}^{\alpha} - \theta(i_{nt}, k_{nt}). \tag{5}$$

3.1.2 Technology and frictions at the firm level

What sets plants in a multi-unit firm apart from their identical counterparts in single-unit firms? What are the economic benefits the firm provides to its own plants? This section discusses the allocation of funds within the firm.

A firm collects the cash flow from all plants and decides on how to allocate funds across its

[3] Investment spikes are usually defined as investment rates exceeding 15% or 20%.

[4] This formulation is similar to assuming lower profitability during large capital adjustments which has been documented by Power (1998), Sakellaris (2004).

plants for investment projects. This means firm cash flow is

$$\Pi_t = \pi_{At} + \pi_{Bt} - \Phi. \tag{6}$$

Managing firm-wide operations requires overhead inputs such as firm management which every firm needs to pay in order to be functional. This overhead fixed cost, denoted by Φ, reflects items such as management costs, overall firm infrastructure, expenditures for R&D, marketing etc. Hence, compared to single-unit firms, multi-unit firms benefit from operating two plants with the same corporate overhead.

Second, while all production and investment activities take place at the level of the individual plant, only the firm is capable of organizing external finance. This assumption is realistic and sensible: while large and complex firms like General Electric operate hundreds of plants, only the firm issues bonds, borrows from banks or raises equity. Typically, the firms then allocate these funds in an internal capital market to individual plants. Consistent with this empirical pattern, we assume that it is the firm that co-ordinates investment plans across all its plants, organizes financing of investment through either internal cash flow or external borrowing and allocates funds to plants where investment is put in place. Only if desired firm-wide investment exceeds firm cash flow, the firm attempts to borrow amount B_{t+1} at net interest rate R_t so that all investment gets financed:

$$i_{At} + i_{Bt} = I_t \leq \Pi_t + B_{t+1}. \tag{7}$$

Organizing external financing, however, is an imperfect process. Following the literature on financial frictions, we assume there are two types of frictions: First, there is a financial participation cost ζK_t if the firm wants to borrow at all. This cost reflects the effort to establish a relationship with a lender, expenditures for various information and disclosure requirements and other administrative expenses independent of the loan amount. Like the real investment adjustment cost above, this cost is scaled by the capital stock. Second, we assume that the firm can divert a fraction $1/\lambda$ of the loan amount B_{t+1} to its private benefit. This diversion of resources cannot be observed or prevented by the lender. As a consequence, the lender will require collateral which it can seize in case it discovers ex post that the firm did divert funds. We assume that firms pledge a fraction ξ of the value of their capital stock, $K_t = (k_{At} + k_{Bt})$, as collateral. Given the posted collateral, lenders will limit the loan such that the diverted loan amount never exceeds the collateral in order to avoid ex post moral hazard unprofitable for the borrower:

$$\frac{B_{t+1}}{\lambda} \leq \xi K_t. \tag{8}$$

Note that one can think of $\lambda\xi$ as the maximum leverage the lender is willing to tolerate. We

summarize the cost of external finance in the following function

$$\Theta(B_{t+1}, K_t) = \begin{cases} 0 & \text{if } B_{t+1} = 0 \\ \zeta K_t & \text{if } 0 < B_{t+1} \leq \xi\lambda K_t \\ \infty & \text{if } B_{t+1} > \xi\lambda K_t. \end{cases} \tag{9}$$

Note that $\Theta(\cdot)$ captures the net present value of the borrowing cost associated with B_{t+1}. The interest rate payments next periods have a net present value of $\beta R_t B_{t+1}$. Since there is no risk in our model, the firm's borrowing rate R_t is equal to the risk-free interest rate which in turn equals the inverse of the discount rate, so the net present value of the linear part of borrowing costs exactly equals B_{t+1}. Of course, this would change once we introduce any risk of bankruptcy along the lines of Townsend (1979) which would drive up R_t above the risk-free rate and make the middle portion of the borrowing cost in equation (9) monotonically increasing in the borrowing amount B_{t+1}.

Total cost of investment in a given plant depends on the investment amount in that plant, the combined investment in the rest of the firm, whether or not the firm needs to borrow and if it possibly runs into the collateral constraint. The total cost of investment in plant A then consists of fixed and quadratic adjustment costs (real costs $\theta(i_{At}/k_{At})$ in equation (4)) as well as fixed borrowing costs (financial costs $\Theta(B_{t+1}, K_t)$ in equation (9)). The latter part depends on how much the other plant in the firm, plant B, invests as it dictates how fast and how much the firm needs to borrow. Thus, investment in one plant imposes an externality on investment in the rest of the firm because it depletes internal funds and imposes a borrowing cost that is shared by the entire firm.

We plot the total cost of investment in Figure 2 to illustrate the multiple non-convexities and how the interaction of investment across plants shape the cost of investment for the firm. Note that in these plots we assume that ψ, the parameter that regulates the fixed costs of investing in a plant, is "small" in the sense that the minimum investment in one plant can be financed using internal funds of the firm. If they were excessive, even the minimum investment to justify the fixed investment adjustment costs would require borrowing. In that case, the effective fixed cost of investing in any plant would be $(\psi + \zeta)k_{nt}$.

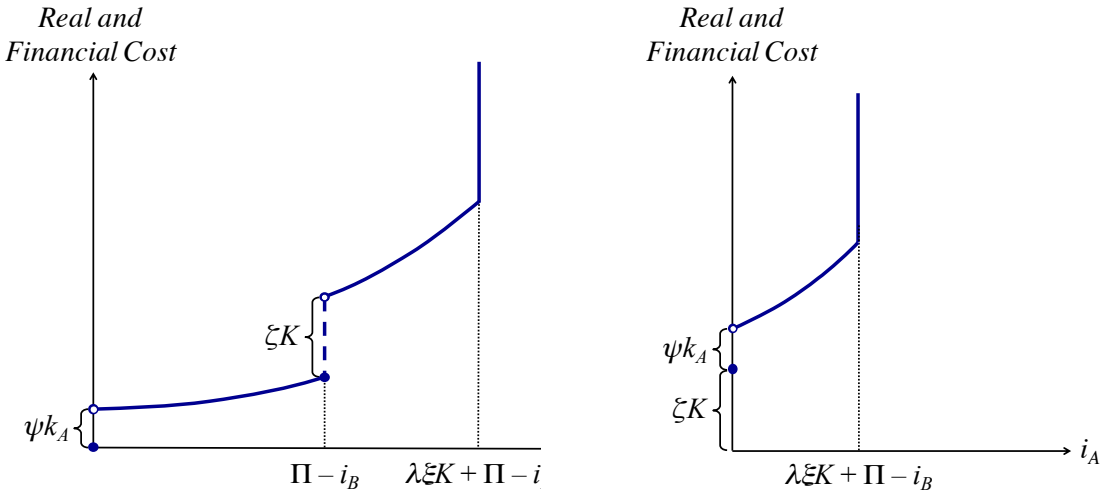
3.1.3 Firm value and firm policy

We define the vectors of technology levels and capital stocks within the firm as $\mathbf{Z_t} = \{z_{At}, z_{Bt}\}$ and $\mathbf{K_t} = \{k_{At}, k_{Bt}\}$, respectively. Given the plant-level fixed adjustment cost, the firm's state will consist of the distribution of capital stocks $\mathbf{K_t}$ and technology levels $\mathbf{Z_t}$ across plants within the firm. The firm chooses investment in either plant A and B in order to maximize firm value which consists of the net present value of discounted future gross profits net of investment and borrowing costs. When choosing the investment levels in either plant, the firm takes into account

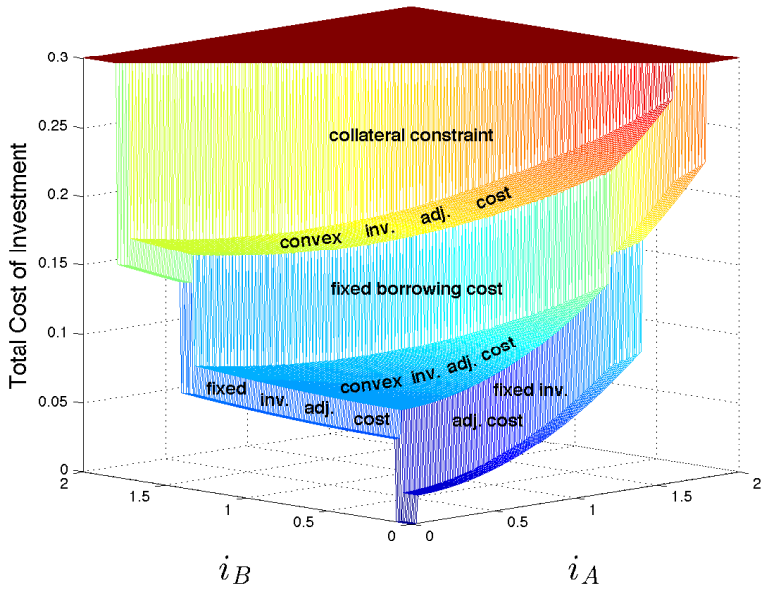
Figure 2: Total cost of investment

(a) Total cost of investing in plant A when i_{Bt} can be fully financed internally

(b) Total cost of investing in plant A when i_{Bt} cannot be financed internally



(c) Total investment costs from the firm's perspective



Note: Panel (a) on the left displays total cost of investing in plant A when investment in the rest of the firm does not exceed internal funds $0 \leq i_{Bt} \leq \Pi_t$. Then, small amounts of i_{At} can be financed with left over internal funds $(\Pi_t - i_{Bt})$ without incurring borrowing costs. Any investment exceeding that amount makes the cost level jump due to the fixed borrowing cost ζK_t. Panel (b) on the right displays the case when the firm already needs to borrow to finance investment in the rest of the firm. Even zero investment in plant A means fixed and linear borrowing costs $\zeta K_t + R(i_{Bt} - \Pi_t)$. Investment in either case is always limited by the collateral constraint: $B_{t+1} \leq \xi\lambda K \Leftrightarrow i_{At} \leq \xi\lambda K + \Pi_t - i_{Bt}$. Panel (c) shows the the total cost jointly for $k_{At} = 1$, $k_{Bt} = 3$, $\zeta = \psi = 0.02$, $\gamma = 0.04$, $\vartheta = 0.03$, $\xi = 0.05$, $\lambda = 2$ and $\Pi_t = 1.4$.

the various adjustment costs and whether or not borrowing is required to finance the desired level of investment. The firm's problem can be written in recursive form as:

$$V(\mathbf{Z_t}, \mathbf{K_t}) = \max_{i_{At}, i_{Bt}, B_{t+1}} \left\{ \Pi_t - I_t - \Phi - \Theta(B_{t+1}, K_t) + \beta \mathbb{E} V(\mathbf{Z_{t+1}}, \mathbf{K_{t+1}}) \right\}$$
$$s.t. \quad k'_{nt} = (1-\delta) k_{nt} + i_{nt} \quad \forall n = A, B$$
$$B_{t+1} \leq \xi \lambda K_t.$$
$$I_t \leq \Pi_t + B_{t+1}.$$

This value function embeds three non-convexities: the fixed investment adjustment cost, ψ, the fixed borrowing cost, ζ, and the collateral constraint $\xi\lambda$.[5] All three non-convexities give rise to "inaction regions" in the firm's state space where the firm may choose to not change investment and/or borrowing even though the underlying productivity shocks z do. In those regions, the realized capital return will also differ from that of a frictionless investment model. The three non-convex investment/borrowing costs make the solution of the firm problem fairly complex and there are consequentially at most seven different[6] cases of investment-borrowing decisions each of which reflect investment and borrowing (in)activity in the various parts of the firm.

We illustrate the investment policy for plant A and the firm's borrowing policy in Figure 3 for a given amount of investment in plant B assuming that internal funds Π_t are sufficient to finance small amounts of investment. In principle, investment in either plant i_{At} is monotone in its productivity z_{At}. The fixed investment adjustment costs prevent small amounts of investment that would not justify paying the fixed cost ψk_{At}. So investment will not change unless z exceeds a threshold $z^0(k_{At})$. At $z^0(k_{At})$ the expected benefit from investing, $\mathbb{E}\frac{\partial V'}{\partial k'_{At}} i_{At}$, equals the fixed costs, ψk_{At}. Because the latter are modeled proportional to k_{At} and because of decreasing returns to scale, the first threshold will depend on k_{At}. Note that z^0 does not depend on firm variables. Investment jumps and then increases monotonically in productivity. The quadratic investment adjustment cost regulated by γ increase the marginal cost of investing but do not break the monotonicity. Plant investment increases until the firm's internal funds are exhausted at $z^1_{At}(k_{At}; z_{Bt}, k_{Bt})$. Notice that this second threshold of productivity for plant A depends on investment in the rest of the firm, i.e. i_{Bt}, and vice versa. This is because investment expenditures in the other plants entail a negative externality for plant A as it depletes internal funds of the firm.

As with the fixed adjustment cost at the plant level, the fixed borrowing costs, ζK_t, prevent

[5] We refer to these frictions as non-convexities even though the last one is strictly speaking only a non-differentiability; but it has similar results for the dispersion of capital returns within the firm as the non-convexities.

[6] With three binary decisions, there are eight cases, but when neither plant invests the firm would never borrow thus making one case redundant. We restrict the investment fixed adjustment cost ψ to be small. If it were extremely high, even the minimum investment which justifies paying the fixed cost would require firm borrowing and there would only be four cases. Then, there is no role for an internal capital market to finance investment because it never suffices even for the minimum investment in one of the two plants.

the firm from borrowing small external funds. This means there is another region of productivities $[z^1_{At}(\cdot), z^2_{At}(\cdot)]$ where investment does not respond to productivity. Only at productivities above $z^2_{At}(z_{At}, k_{At}; z_{Bt}, k_{Bt})$ is it worthwhile for the firm to pay the fixed borrowing cost. This third threshold depends on investment needs in the entire firm and hence depends on the levels and the distribution of capital stocks and the productivity shocks. Investment again increases monotonically in productivity above $z^2_{At}(\cdot)$ until productivity threshold $z^3_{At}(z_{At}, k_{At}; z_{Bt}, k_{Bt})$ at which point the firm's borrowing demands are limited by the collateral constraint.

3.2 The effects of financial constraints on capital allocation and capital returns

In the previous section, we described how investment at the individual plant is affected by firm-level frictions. Understanding these micro-level dynamics is helpful to gain intuition about the *joint dynamics* of capital returns and capital reallocation within the firm, which is central to the model. We will now describe the effect of each firm-level financial constraint on the dispersion of returns and investment rates and further moments that can be empirically checked in the data. We will explain how interactions between the plants within the firm shape the investment and borrowing policy qualitatively and then assess these patterns quantitatively.

For the quantitative analysis, we solve the model using a value function iteration procedure which is described in detail in Appendix B. Once we obtain the optimal investment and borrowing policy, we simulate a panel of 1,000 two-plant firms for $1,000$ periods and study the resulting within-firm and between-firm dynamics of investment and capital returns. We repeat this exercise several times changing the calibration of the various investment and financing frictions to assess their quantitative effects.

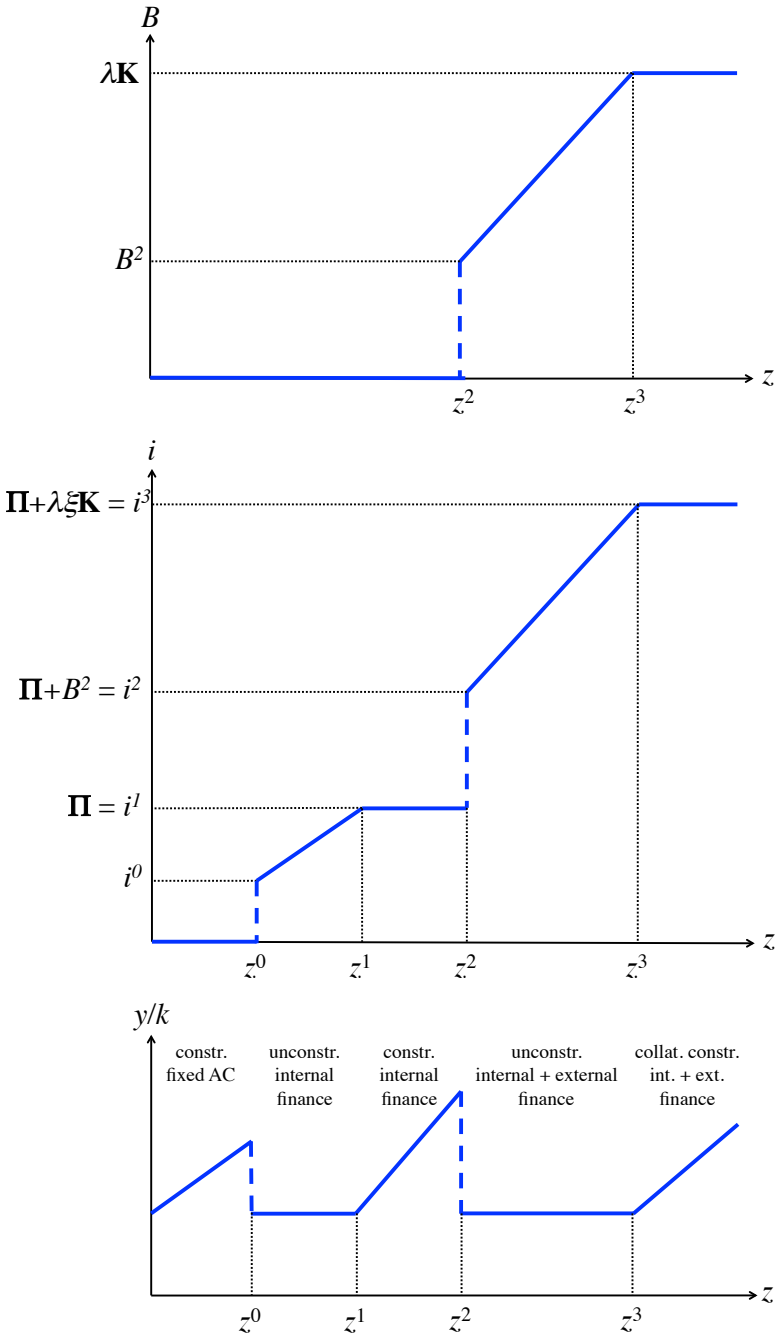
3.2.1 Benchmark model

The model with only quadratic investment adjustment costs is our benchmark. We choose this because comparing the effect of frictions to a completely frictionless model would make many moments such as capital returns dispersion zero thus rendering quantitative comparisons useless. We exclude fixed investment costs from the benchmark because this non-convexity will itself be a significant and known source of returns dispersion which we want to compare to the new firm-level frictions.

3.2.2 Effects of fixed borrowing costs: Less frequent borrowing, lumpier investment

The fixed borrowing costs imply that the firm will only borrow amounts large enough to justify paying the fixed costs. The firm tries to smooth these costs over time by "saving", which in our model happens through capital accumulation. More capital makes it less likely future borrowing will be needed as often, which in turn reduces the incidence of the fixed borrowing costs. This

Figure 3: Firm borrowing and plant investment



Note: Schematic illustration of firm borrowing, plant investment policy functions and realized capital returns. This illustration assumes that the other plant does not invest at all and that physical adjustment costs ψ are smaller than fixed borrowing costs ζ, so that firms start investing without having to borrow; otherwise z^0 and z^2 would coincide.

results in prolonged periods during which the firm will not borrow and finance all – if any – investment internally. Realized capital returns then fluctuate with productivity shocks and will start to diverge as the firm refrains from borrowing and investing in order to align capital returns. As a consequence, within-firm dispersion of capital returns may reflect the fixed borrowing costs faced by the firm rather than misallocation. Below, we will study the quantitative impact of this firm friction on within-firm dispersion.

Our model makes further predictions that we can utilize to study the effects of fixed borrowing costs. First, fixed borrowing costs lead to investment spikes that are synchronized across plants within the firm. Since the firm only borrows rarely, it will then find it optimal to borrow large amounts and pay the fixed investment adjustment costs in both plants. In those periods of firm-wide investment, the correlation between investment and capital returns is high, while the opposite is true during the long periods of internally-financed small investment projects. The latter tend to dominate, leading to lower overall correlation between investment and capital returns. Finally, because fixed borrowing costs make investment at the plant level even lumpier, the autocorrelation becomes even more negative and so does investment at the firm level.

3.2.3 Effects of collateral constraints: Staggered investment across plants

Collateral constraints also limit firm access to external finance. Unlike borrowing costs, however, they prevent borrowing large sums and do not discourage borrowing small amounts: in that sense, they have an effect opposite to that of fixed borrowing costs. The firm will only run into that constraint if it wants to borrow large sums, which happens when both plants receive large positive technology shocks. The firm will try to smooth out this type of borrowing constraint by staying away from it which means to borrow more often and smaller amounts. This constraint is particularly binding when both plants receive a technology shock. Because it faces a borrowing limit, the firm utilizes its internal capital market to finance capital projects in one plant at a time. In other words, it rotates investment plans across its plants. Seen over time, investment across the individual plants within a firm looks staggered.

Because the collateral constraint makes the firm borrow more often and smaller amounts, the correlation between investment and capital returns increases. Perhaps most interestingly, the correlation of investment within the firm drops sharply to negative reflecting the staggering pattern of investment across a firm's plants. Consistent with that, the firm avoids large investment spikes in its plants at the same time.

4 Quantitative analysis

4.1 Calibration

Table 1 summarizes the parameter values used for the quantitative analysis. Most values are based on moments from the ASM dataset and are in line with calibrated parameters generally used in the investment literature. Of note, the fixed investment adjustment costs are lower than those estimated in Cooper and Haltiwanger (2006), Asker et al. (2014). This is because the fixed borrowing costs ζ reinforce the effects of fixed investment adjustment costs ψ. As a result, estimating a model that does not take into account firm structure will confound the two and bias the inferred value for ψ upwards. In addition, when we use the value from Cooper and Haltiwanger (2006), we find that the minimum investment projects needed to justify the fixed adjustment costs are so large that they usually require borrowing. While this case may be theoretically possible, it seems not very plausible that even minimal investment cannot be financed out of the combined firm's internal funds.

Given the lack of hard evidence to support the calibration of the borrowing fixed cost ζ, we remain conservative and pick a small value equal to only 0.01% of the firm capital stock. Also, we start with no firm fixed production cost, which mostly acts as a scale factor anyway.

Table 1: Model Calibration

Parameter	Meaning	Value	Target/Source
β	Discount rate	0.98	Long-run real interest rate
α	Production elasticity	0.636	ASM data
δ	Depreciation rate	0.089	Mean from BLS Mfg capital tables
ψ	Fixed inv. adj. cost	0.039	Cooper and Haltiwanger (2006)
		0.080	Asker et al. (2014)
γ	Quadratic inv. adj. cost	0.049	Cooper and Haltiwanger (2006)
ζ	Fixed borrowing cost	10^{-4}	See text
ξ	Size of collateral	0.1	10% of capital can be posted as collateral
λ	Collateral constraint	2.5	Borrowing firm can divert 40% of loan
Φ	Firm fixed cost	0	See text
σ^f	TFP shock plant	0.25	ASM & Cooper and Haltiwanger (2006)
σ^p	TFP shock firm	0.173	ASM & Cooper and Haltiwanger (2006)

4.2 Financial constraints, investment activity and dispersion

In order to better understand the behavior of the firm, we now turn our attention to the model simulations. We focus on the impact of external financial constraints on various moments to get a better sense of the behavior of the multi-plant firm.

First, we plot in the the top panel of Figure 4 the relationship between the tightness of the collateral constraint, determined by the parameter λ, and the degree of dispersion of logged ex-

pected capital returns ($\log(y_{nt}/k_{nt})$) and investment (i_{nt}/k_{nt}) within the firm. Both moments are normalized to 1 in the scenario where the collateral constraint is not binding ($\lambda = \infty$). As the constraint is tightened (λ gets smaller), dispersion within the typical firm rises sharply, increasing by close to 40% for investment and 20% for capital returns once $\lambda = 2.5$ (firm cannot borrow more than 25% its capital stock). The increase in dispersion eventually slows down, and dispersion can even fall as the constraint becomes so binding that the firm rarely invests at all.

From the second panel of Figure 4, we can see where the increase in dispersion comes from: as the collateral constraint limits the ability of the firm to borrow, the correlation in investment activity across plants within the firm drops dramatically from its benchmark value of 0.4 in the unconstrained case. In fact, for low-enough values of λ, the cross-plant investment correlation turns *negative*, reaching a trough of -0.17 with our parameterization.

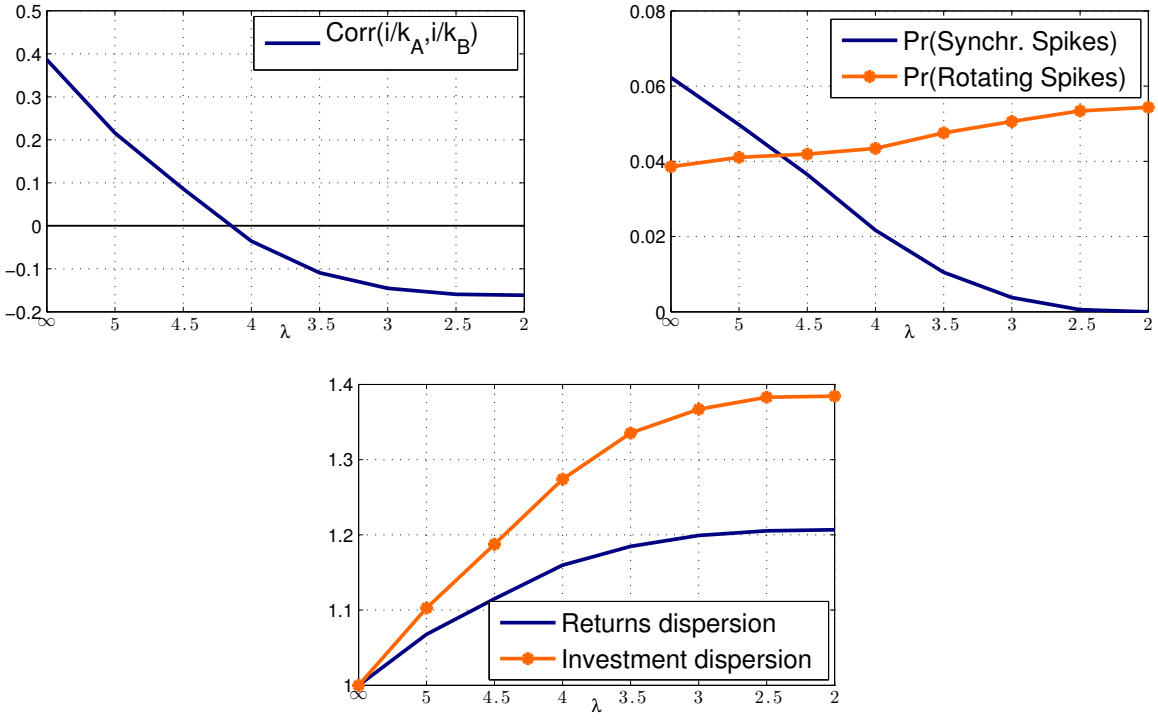
The third panel illustrates the mechanics behind the drop in correlation. Here, we plot two additional moments. The first one represents the probability of observing synchronized investment spikes, i.e. investment rates above 15% in both plants at the same time. As the collateral constraint is tightened and access to internal finance is limited, the firm cannot allocate large amounts of capital to both plants at the same time, even following a positive firm-level shock. Conceivably, an option for the firm would be to invest smaller amounts in both firms to restrain how much it needs to borrow on capital markets. But because of the presence of investment fixed costs, the firm would rather instead opt for desynchronization of investment activity across its plants. That is, in the event of large positive firm-level shock, the firm allocates capital first to the plant with the highest (expected) return on capital, then does the same for the second plant in the following period. This is why the frequency of rotating spikes, i.e. an investment spike in one plant followed by a spike in the other plant, rises as external financing becomes more limited.

4.3 Aggregate implications: the social value of multi-plant firms

In the previous section, we studied how a multi-unit firm optimally allocates capital across its plants when it is faced with limited access to external financing in addition to the traditional frictions found in the literature. We found that the firm tends to stagger investment activity across its plants to limit borrowing activity. As a result, dispersion of investment but also capital returns within the firm rises.

In what follows, we ask ourselves a different but related question: could the rise in dispersion observed within the firm be socially optimal, i.e. associated with higher aggregate output? To investigate this issue, we simulate two almost identical economies, but for the presence of internal capital markets. The first economy is similar to the exercise we have done so far: we simulate a panel of 1,000 two-plant firms for 1,000 periods, where each firm can pool capital across its plants to post as collateral. The second economy also consists of 1,000 two-plant firms subjected to the same shocks. This time, however, each plant within the firm is considered to be a standalone unit:

Figure 4: Effect of external financial constraints on the multi-plant firm economy



it must borrow on its own by posting its specific capital as collateral. Besides that, it faces exactly the same constraints and frictions as the plants in multi-unit firms. In other words, we create a "wall" between the plants that bars them from pooling resources when comes the time to access external financial markets. This exercise allows us to specifically isolate the role of the firm in creating internal capital markets and allocating new investment across its plants.[7]

Table 2 compares a number of moments across the two economies under the presence of collateral constraints ($\lambda = 2.5$), as well as the case in which such constraints are absent ($\lambda = \infty$). In this latter scenario, the single-plant and multi-plant firm economies are by definition identical. In line with our previous findings, we find that the presence of financial frictions raises dispersion in a world of multi-unit firms. With standalone plants, however, dispersion tends to fall relative to the unconstrained benchmark. Similarly, the correlation of investment across plants turns negative within multi-unit firms, yet increases somewhat once the plants cannot pool resources and need to borrow on their own.

The bottom panel of the table pertains to the aggregates across our economies: we sum up output, investment and capital across all firms for each period, and take the time-series average. One can notice a striking result: the multi-plant firm is capable of undoing most if not all the effets of financial frictions, with aggregate variables remaining almost unchanged relative to the unconstrained benchmark. This is true even if the presence of collateral constraints *raises* dispersion of capital returns. On the other hand, the same frictions imposed on an economy populated by firms comprised of standalone plants exhibits lower aggregate output *and* lower dispersion.

Table 2: Comparing multi-plant (MUF) and single-plant (SUF) firms

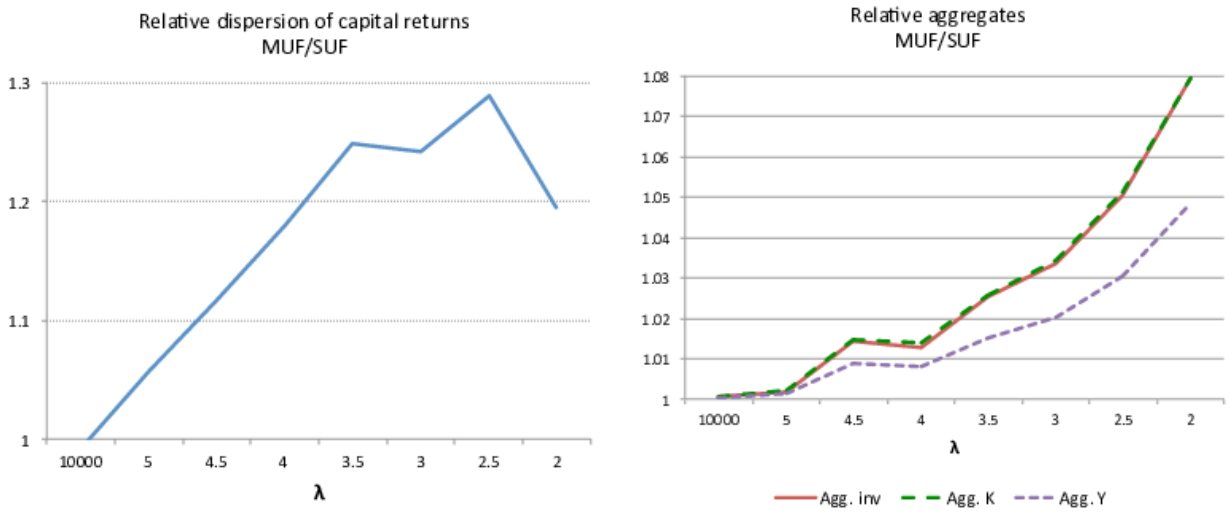
Moment	No frictions	$\lambda = 2.5$	
	(MUF = SUF)	MUF	SUF
1. Cross-sectional moments			
$CV(E[\log(y/k)])$	1	1.241	0.929
Share of within-firm dispersion of $E[\log(y/k)]$	0.274	0.373	0.257
Share of within-firm dispersion of i/k	0.335	0.657	0.276
$Corr(i/k_{At}, i/k_{Bt})$	0.374	-0.159	0.465
$Pr(i/k > 0.15)$	0.163	0.281	0.191
2. Aggregate Moments			
Aggr I	0.447	0.447	0.425
Aggr K	0.502	0.502	0.477
Aggr Y	0.122	0.122	0.118
Aggr V	295.9	295.8	288.1

We investigate this finding further by simulating the impact of a tightening of the collateral

[7] We leave to future work the simulation of a full general equilibrium model with an endogenous interest rate.

constraint in our two economies. The left panel of 5 shows the relative dispersion of capital returns as λ decreases, i.e. borrowing becomes more limited. In line with our previous findings, we can see that as the constraint is tightened, the multi-unit firm optimally decides to generate *more* dispersion, even if it could always decide to standalone-plants case by shutting down internal capital markets. This optimal strategy at the firm level translates into higher levels of aggregate investment, capital and output relative to an identical economy but for the fact that plants cannot pool resources to access external finance.

Figure 5: Multi-plant vs. single-plant firms, dispersion and aggregates



Note: Each value corresponds to the ratio of a specific moment in the multi-plant-firms economy (MUF) relative to the same moment for the economy composed of firms that cannot pool capital across plants (SUF). Ratios of average within-firm capital returns dispersion (left panel) and aggregate investment, capital and output (right panel) are plotted for various values of the collateral constraint parameter λ.

To summarize, the insight from our model is that higher capital returns dispersion can be an outcome of constrained efficient behavior by multi-plant firms. This higher dispersion is not a sign of resource misallocation in the economy, but instead of an optimal decision by the firm to stagger investment activity in order to circumvent financial constraints.

In other words, our findings cast doubt on the notion that dispersion in capital returns necessarily indicates inefficiencies.

5 Collateral constraints in multi-plant firms: empirical evidence

The previous sections documented a set of new facts and proposed a new theoretical model which challenged the notion that high dispersion in capital returns necessarily reflects misallocation. While

our model is consistent with many data facts, we now want to demonstrate its empirical relevance more directly. To do that, we verify the crucial features of our model mechanism in firms which operate exactly two plants as in our model. The crux of the model mechanism was a collateral constraints which limited external finance. The tighter this financing constraint, the more relevant a firm's internal capital market becomes which the firm uses to finance investment in at least a subset of its plants. This meant that the more financially constrained, the more dispersed capital returns, capital reallocation and the more prevalent "rotating investment spikes" as opposed to "synchronized investment spikes." We check for these patterns in two-plant firms in the data. As an empirical proxy for the tightness of the collateral constraint, we choose the amount of the pledgeable collateral which we identify as proportional to the plant's capital stock.

5.1 Evidence for collateral constraints I: Staggering investment

We first check if unconstrained two-plant firms exhibit more synchronized investment spikes in the wake of a firm-level technology shock. As the pledgeable collateral of a firm shrinks, our model would predict that synchronized investment spikes become less common and firms transition towards "rotating investment spikes."

We first estimate the likelihood of synchronized investment spikes after a firm-wide technology shock using the following probit model:

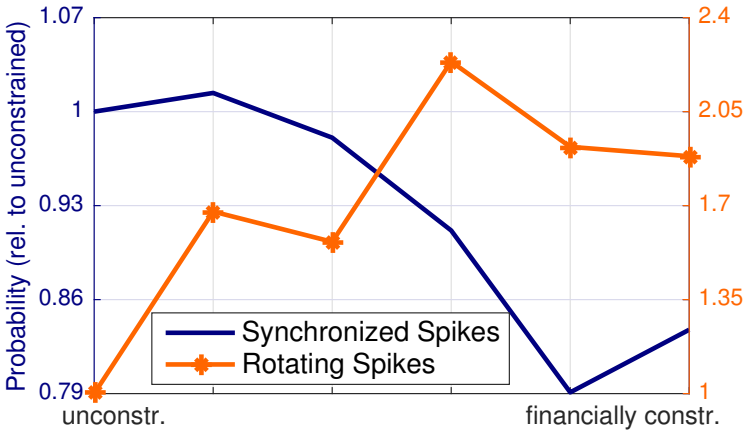
$$Pr(X_{jt}^{sync} = 1|X_{jt}) = \Phi\left(X_{jt}\beta\right) \tag{10}$$

where X_{jt} is a vector of controls which includes is the firm's level of the productivity shock, η_{jt}, and the dummy variable X_{jt}^{sync} is defined as follows

$$X_{jt}^{sync} = \begin{cases} 1 & \text{if } \frac{i_{At}}{k_{At}} > 0.15 \text{ and } \frac{i_{Bt}}{k_{Bt}} > 0.15 \\ 0 & \text{otherwise.} \end{cases}$$

We focus on the firm's investment response to a firm shock because this provides the cleanest example of the changing investment patterns as the financial constraint tightens. We estimate equation (10) separately for ten deciles of two-plant firms along the value of the pledgeable collateral and evaluate the marginal likelihood of a synchronized investment spike in the wake of a firm-wide technology shock, i.e. a shock experienced by both plants. For disclosure requirements we have to smooth the results of these deciles using a cross-sectional rolling-window average of five adjacent deciles. Normalizing the probability of a "synchronized investment spike" to unity for the least financially constrained firms, we plot the normalized probabilities in Figure 6 (solid dark blue line). It shows that the financially most constrained firms are only 80% as likely as the least constrained firms to respond to a firm-wide technology shock with an investment spike in both plants.

Figure 6: Synchronized and rotating investment spikes after firm TFP shocks



In a similar vein, we estimate probit models where we regress a dummy variable if the firm experiences a "rotating investment spike," that is, both plants experience an investment spike in the wake of a firm productivity shock, but just in subsequent periods. We estimate

$$Pr(X_{jt}^{rotate} = 1|X_{jt}) = \Phi\left(X_{jt}\beta\right) \tag{11}$$

where X_{jt} is a vector of controls which includes is the firm's level of the productivity shock, η_{jt}, and the dummy variable X_{jt}^{rotate} is defined as follows

$$X_{jt}^{rotate} = \begin{cases} 1 & \text{if } \frac{i_{At}}{k_{At}} > 0.15 \text{ and } \frac{i_{Bt}}{k_{Bt}} < 0.15 \text{ and } \frac{i_{At+1}}{k_{At+1}} < 0.15 \text{ and } \frac{i_{Bt+1}}{k_{Bt+1}} > 0.15 \\ 1 & \text{if } \frac{i_{At}}{k_{At}} < 0.15 \text{ and } \frac{i_{Bt}}{k_{Bt}} > 0.15 \text{ and } \frac{i_{At+1}}{k_{At+1}} > 0.15 \text{ and } \frac{i_{Bt+1}}{k_{Bt+1}} < 0.15 \\ 0 & \text{otherwise.} \end{cases}$$

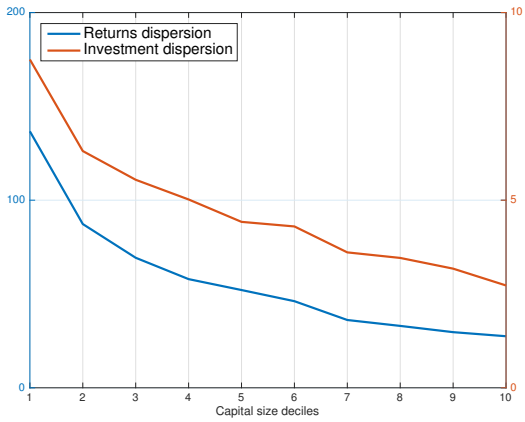
Again, we evaluate the marginal probabilities, smooth them across deciles and normalize the probability in the least constrained group to unity. The results are plotted as the starred orange line in Figure 6. In line with the model, the financially most constrained firms are more likely to respond to firm shocks with a rotating investment spike. That is, the firms contemporaneously invests in only one plant *and* invests in the other plant in the subsequent period. The empirical difference of the likelihood of a rotating investment spike increases a lot with our proxy measure of financial constraint: The most constrained firms are twice as likely to revert to rotating investment spikes than financially unconstrained ones. All in all, we view this evidence as strong support for the presence of our model mechanism.

5.2 Evidence for collateral constraints II: Within-firm dispersion

Lastly, we consider the within-firm dispersion in capital returns and capital reallocation. According to our model, firms which are more credit constrained should experience more dispersion while firms with a lot of pledgeable collateral should have an easier time managing to equate returns. We compute the two dispersion measures by decile of pledgeable collateral and divide by the mean level in each decile to account for level differences. Unlike the standard deviation, the resulting coefficient of variation is dimensionless and can be easily compared across deciles. Again, we smooth out the results across deciles as in the previous subsection and plot the results in Figure 7.

Indeed, computing the within-firm dispersion measures shows that investment dispersion monotonically declines in our proxy for credit constraints: the most constrained firms have investment dispersion within firms that is about three times as large as that for the least constrained firms. The difference in capital returns is even stronger: the most constrained firms are about six times as dispersed as the least constrained firms.

Figure 7: Productivity and investment dispersion (CV) within 2-plant firms



6 Conclusion

This paper showed that dispersion in capital returns need not indicate distortions. Motivated by the evidence that dispersion mostly occurred within firms rather than across firms, we built a model of a firm operating several plants. Such firms dispose over an internal capital market that helps easing external financial constraints and supports aggregate investment, capital and output. Most importantly, economies with multi-plant firms may well exhibit more dispersion in capital returns than economies with single-plant firms, but still produce more aggregate output with the same technologies.

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A Additional empirical evidence

A.1 Data

We mainly use confidential data on manufacturing establishments collected by the U.S. Census Bureau which comprise the Annual Survey of Manufactures (ASM), the Census of Manufactures (CMF) and the Longitudinal Business Database (LBD). These data inform us about age, output, capital stocks, investment expenditures and other inputs at the level of the individual establishment. In the manufacturing sector, Census defines an "establishment" as a business location where the principal activity is production; we hence think of an "establishment" as a production plant. The Census data also contain information about the ownership of each plant (denoted by the variable `FIRMID`[8]) which allows us to construct the hierarchical plant structure of "firms" necessary for our main object of interest, the within-firm and between-firm component of heterogeneity in returns, productivity and reallocation.

From the Census of Manufactures (CMF) and the Annual Survey of Manufactures (ASM) we construct a large dataset of plants in the U.S. manufacturing sector. In order to obtain a consistent longitudinal panel, we limit attention to the ASM and the ASM portion of the CMF data (identified by establishment type `ET`=0). By focusing on the ASM portion in all years, we automatically eliminate all administrative observations (identified by `AR`=1) which are imputed mainly off industry means and would thus corrupt moments of the distribution we are interested in. Our resulting panel spans the years 1972-2010, which allows us to study both long-run features of the dispersion of capital returns and reallocation as well as business cycle properties over six recessions including the "Great Recession" of 2008/09. Every year, we observe about 55k plants which total up to 2.1 million observations.

We combine the Census data with industry-level data from several publicly available sources: input and output price deflators from the NBER-CES Manufacturing Industry Database (NBER-CES), various asset data from the the Capital Tables published by the Bureau of Labor Statistics (BLS) and the Fixed Asset Tables published by the Bureau of Economic Analysis (BEA). Unless otherwise noted, all datasets are at annual frequency. Most of the information contained in the non-Census datasets (BEA, BLS, NBER-CES) other than the manufacturing data are merely needed to estimate productivity.[9] To avoid outliers driving our results about dispersion and the investment-productivity link, we winsorize the 1% tails of the productivity and investment rate distributions in a given year.

A.2 Returns dispersion

A.3 Investment dispersion

A.3.1 Investment dispersion is highest within firms

The previous section documented a significant and countercyclical dispersion of capital returns across plants within firms. Next, we investigate to which extent economic agents respond to these

[8] Song et al. (2015) identify firms off the `EIN`, the employer identification number, which comes from tax records. Since we are interested in organizational control rather than tax liability and because the same `FIRMID` may operate hundreds of `EIN`s for tax purposes, we prefer `FIRMID` to indicate firms.

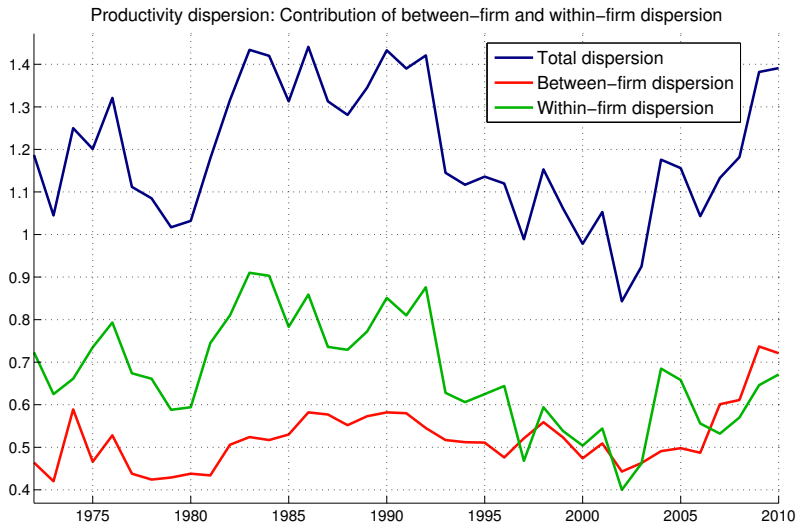
[9] For more details about the primary data and their transformation needed to estimate productivity, see the description in the appendix to Kehrig (2015).

Table 3: Number of plants per firm in U.S. manufacturing

Dataset	Weights	Mean	Bottom quartile	Median	Top quartile
ASM	unweighted	1.9	1	1	1
ASM	L-weighted	30.5	2	12	41
ASM	K-weighted	35.9	5	20	50
LBD	unweighted	1.2	1	1	1
LBD	L-weighted	31.0	1	7	39

Note: ASM sample are the $\approx 50k$ annual plant-level observations in the ASM 1972-2010. The LBD sample contains all active manufacturing plants 1976-2009. K- and L-weighted refer to means and quartiles that are weighted by capital, employment and production, respectively.

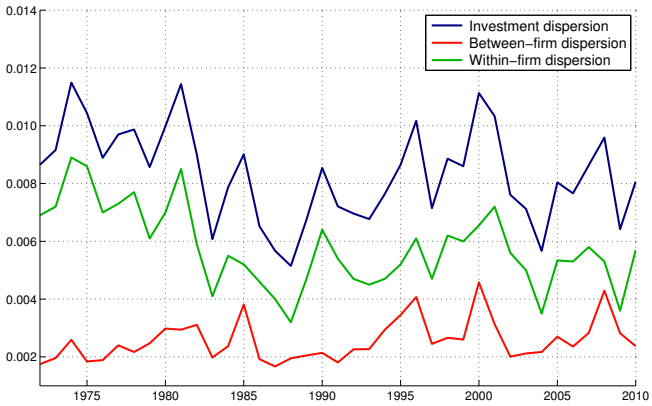
Figure 8: Productivity dispersion between and within firms



differences by shifting resources to these high-return plants. Standard neoclassical investment theory (see Hayashi (1982)) predicts that capital should be reallocated until its returns are equalized. In the context of our finding of a prominent and countercyclical within-firm dispersion, we are particularly interested in determining whether firms reallocate capital optimally, as proxied by investment rates.[10]

Analogous to capital returns dispersion, we use the decomposition in equation (2) to study the dispersion of investment rates. If investment rates differ across plants, this means that the economy discriminates when allocating capital across plants. We want to know what portion of overall dispersion caused by different agents – firms or markets – is quantitatively more important, more volatile and more cyclical. After studying the properties of the investment rate distribution, we will look at any links between capital returns and investment rates at the micro level.

Figure 9: Investment dispersion between and within firms



Our analysis based on investment rates shares many of the conclusions based on capital returns dispersion. First, investment dispersion is sizable. The cross-sectional standard deviation amounts to 0.091. This means that a plant with one positive standard deviation has an investment rate of about 9.1% above the average plant. Given that the average investment rate in the average U.S. manufacturing plant is about 5.8% and given that our investment rate is non-negative by construction, this is a considerable spread.

Second, the within-firm dispersion is once again the main factor in driving overall heterogeneity: as the first row in Table 4 indicates, it accounts for about 70% of the long-run overall dispersion, especially in the first half of the sample. It thus appears that firms do differentiate a lot when they allocate investment across their plants though it is unclear at this point if they build capital in high-return places or not. We will examine that below.

Third, as with capital returns dispersion, it is the within-firm investment dispersion which is cyclically more sensitive. When we choose aggregate manufacturing investment as the cyclical

[10]While we are mindful that capital reallocation may also take the form of resales of old capital previously installed elsewhere, we focus on investment as our sole measure of capital change because capital resales and retirements are not well measured in all years of our data. According to published aggregates of the "Annual Survey of Capital Expenditures," another Census survey, expenditures for used capital amount to less than 5% of aggregate investment in manufacturing, so omitting capital resales seems negligible.

indicator (see third row in Table 4), we find that within-firm dispersion is much more cyclically sensitive than the between-firm component of dispersion. The within-firm component is moreover responsible for the ups and downs in overall dispersion as the fourth row in Table 4 shows.

There is a fourth feature about investment dispersion which stands out: Unlike cross-sectional dispersion measures of other variables which are *countercyclical*, investment dispersion is *procyclcial*. This specific fact was first documented by Bachmann and Bayer (2014) using German firm-level data. We confirm their finding for U.S. manufacturing firms, but refine this fact at the same time in a way that dispersion across plants within firms is not only stronger but also more procyclical than between-firm dispersion. Like Eisfeldt and Rampini (2006) we note that the benefits of capital reallocation – the dispersion of capital returns – are countercyclical while the actions of capital reallocation – the dispersion of investment rates – are procyclical. These authors documented this misaligned cyclicality using U.S. firm data. Again, we refine their result in that we show how most of these unaligned cyclicalities occur within firms rather than between firms. This suggests that one can interpret the results of both Bachmann and Bayer (2014) and Eisfeldt and Rampini (2006) as a lower bound as they both miss an even larger and cyclically sensitive portion of capital returns and investment dispersion hidden in their respective datasets. The procyclicality of investment dispersion and its components is illustrated in the correlogram in Figure 14.

A.3.2 The importance of large investment projects

The empirical evidence so far paints a picture that may seem puzzling in several ways: Firms tolerate a large degree of capital returns dispersion across their plants. Within a given period, they allocate capital very differently across their plants. Yet the timing of investment and capital returns dispersion does not appear to conform with the predictions of basic economic theory, raising concerns that firms may not allocate capital optimally across their plants. Or firms may be prevented from equalizing returns within the firm by various frictions and adjustment costs which throw a spoke in the wheel of optimal capital reallocation. Fixed investment adjustment costs are one prominent such friction. Many empirical investment studies have emphasized their importance and how they result in "investment spikes," that is, short bursts of strong investment activity are followed by periods of relative calmness. Such lumpy investment may result in both investment dispersion and prevent an equalization of capital returns. Asker et al. (2014) have shown that this non-convexity can explain a larger share of dispersion in revenue productivity across plants.

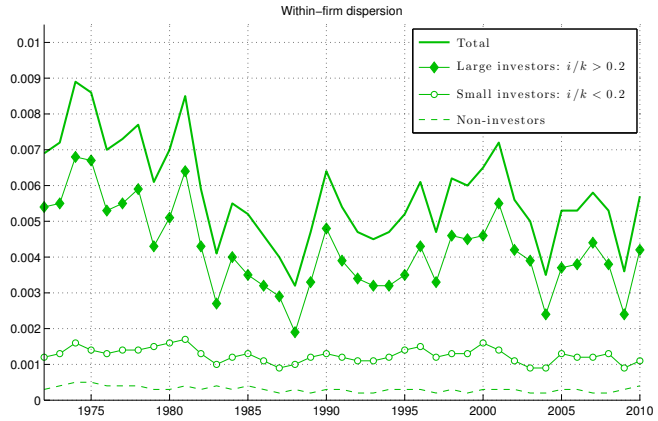
We therefore decompose the within-firm dispersion component into three parts: the portion driven by large investment spikes ($big \equiv \{n : (i/k)_{nt} \geq 0.2\}$), small investment projects ($sm \equiv \{n : 0.01 \leq (i/k)_{nt} < 0.2\}$) and investment inactivity ($no \equiv \{n : (i/k)_{nt} < 0.01\}$):

$$\begin{aligned}\sigma_t^W &= \sigma_t^{W\,big} + \sigma_t^{W\,small} + \sigma_t^{W\,no} \\ &= \sum_j \omega_{jt} \left\{ \sum_{n\in j;\left(\frac{i}{k}\right)_{nt}>0.2}^{N_{jt}^{big}} \omega_{njt} \left[\left(\frac{i}{k}\right)_{njt}^{big} - \overline{\left(\frac{i}{k}\right)}_{jt} \right]^2 + \sum_{n\in j}^{N_{jt}^{sm}} \omega_{njt} \left[\left(\frac{i}{k}\right)_{njt}^{sm} - \overline{\left(\frac{i}{k}\right)}_{jt} \right]^2 + \sum_{n\in j}^{N_{jt}^{0}} \omega_{njt} \left[\left(\frac{i}{k}\right)_{njt}^{no} - \overline{\left(\frac{i}{k}\right)} \right.\right.\end{aligned}$$

We plot the results of this decomposition in Figure 10.

As one can easily see from the figure, investment heterogeneity almost exclusively stems from large investment projects, that is, plants receiving large additions to their capital stock. This result is consistent with the previous empirical literature that emphasized the discrete nature of investment

Figure 10: Investment dispersion within firms and lumpy investment



(see Doms and Dunne (1998), Gourio and Kashyap (2007), Cooper and Haltiwanger (2006) among others) and implies that any multi-plant model of a firm should feature some non-convexity at the level of individual plants that gives rise to "lumpy investment."

A.3.3 Accounting for investment inactivity

In this section, we examine if factors outside our model could drive the empirically documented dispersion of investment rates. We consider lumpy investment (in a more narrow, conventional sense) as well as entry and exit. Both factors might have the potential to cause the volatile and procyclical investment dispersion we see in the data.

A natural question is to find out if this results is driven by the changes in the extensive margin, i.e. the measure of plants investing at all. The extensive margin can vary due to birth and death of plants (assuming that new-born young plants invest more to grow and dying plants are shrunk and invest less) and lumpy investment. The lumpiness is a more serious concern because previous research has documented that investment is in fact lumpy (see Doms and Dunne (1998), Cooper and Haltiwanger (2006), Gourio and Kashyap (2007)). To test if lumpy investment drives the observed investment heterogeneity, we limit the analysis to the plants that have strictly positive investment.[11] These results are displayed in Figure 11.

Although investment dispersion changes a bit on average, the magnitude of this change is rather small and the overall cyclicality patterns are preserved as well. So we conclude that investment dispersion is not driven by the extensive margin. Although we focus on weighted investment dispersion here, this robustness check still gives a significantly positive correlation of the unweighted dispersion (albeit it is weaker). As we show in the main body of the paper, lumpy investment in a broader sense – where lumpiness is defined as large investment rates exceeding 20% rather than non-negative investment activity – does in fact cause the majority of within-firm investment heterogeneity.

Figure 11: Investment Dispersion

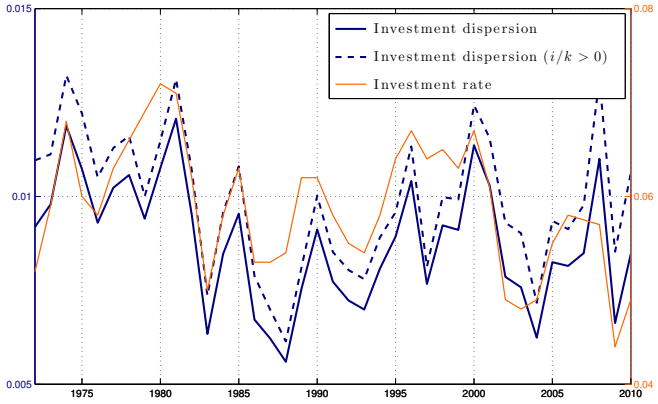
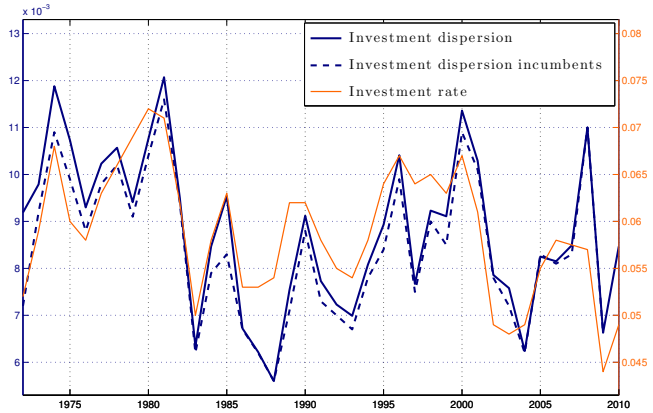


Figure 12: Investment dispersion of mid-age firms



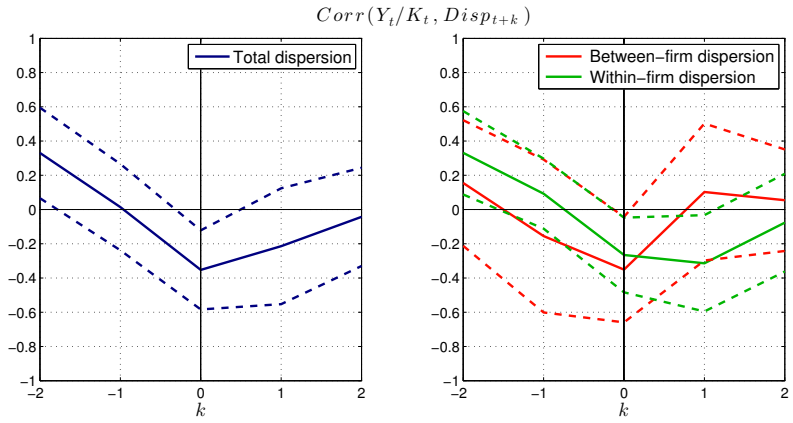
A.3.4 Accounting for entry and exit of plants

A high investment dispersion could come about because firms open new plants and investment in them heavily in booms. Entry of new plants is procyclical, so a procyclical investment dispersion could result from a procyclical and entry and heavy start-up investment. Countercyclical exit has a similar effect. To control for that issue, we drop all plants that are active for two years or less and will not exit within the next two years. The between-firm within-firm decomposition on the remaining "mid-age" plants looks similar to the one of the overall sample. The results are displayed in Figure 12.

A.4 Cyclicality of returns and investment dispersion

A.4.1 Dispersion of capital returns within firms is countercyclical

Figure 13: Cyclicality of between and within dispersion of capital returns



Third, we focus on the cyclical nature of dispersion in capital returns. It is well known that it is larger in recessions than in booms. To cite some of this literature, results in this spirit have been shown empirically for capital returns in Compustat data by Eisfeldt and Rampini (2006), for TFP levels by Kehrig (2015) and for TFP innovations by Bloom et al. (2012). This means that the benefits from reallocation are highest in recessions. As displayed in Figure 13, the dispersion in our sample, too, is negatively correlated with the cycle. This negative correlation between overall dispersion and aggregate output per capital is displayed in the left panel of Figure 13. More interestingly, though, we find the within-firm component of dispersion to be more cyclically sensitive than the between-firm component. Though both components are countercyclical, the within-firm component is more strongly countercyclical and also for several periods compared to the between-firm dispersion.

This third result is remarkable for a number of reasons: If one interprets dispersion as a cause of cycles as Bloom et al. (2012), Christiano et al. (2014), then our within-firm result suggests that cycles manifest themselves within firms rather than between firms. This means that looking at the granular firm level like Gabaix (2011) underestimates the potential that dispersion fluctuations

[11] Following the literature, we define positive investment as plants with an investment of less than 1%.

have for aggregate fluctuations. Since fluctuations in heterogeneity at the sub-granular level of plants within firms are larger and more cyclical, this suggests one should place an emphasis on the multi-unit firm when studying micro-founded business cycles. In addition to that, the empirical uncertainty literature has almost always interpreted plants as individually independent decision makers facing various reallocation frictions. These reallocation frictions often affect interactions among firms or interactions between firms and their lenders. The smaller and somewhat less cyclical between-firm dispersion of capital returns thus casts doubt on some particular model mechanisms based on frictions between firms rather than between plants. Lastly, the countercyclical dispersion of capital returns means that any output benefits from reallocations emerge within the firm. It is hence the firm that needs to shift resources across its various plants rather than markets or lenders to shift resources between firms to capture higher gains from reallocation in recessions.

A.4.2 Investment dispersion within and between firms is procyclical

The year-by-year decomposition of the investment rate dispersion is plotted in Figure 9; its moments are displayed in Table 4.

Table 4: Dispersion of investment rates

	Dispersion of i/k...		
	overall	between firms	between plants within firms
Variance	σ	σ^B	σ^W
Average	$8.35 * 10^{-3}$	$2.56 * 10^{-3}$	$5.79 * 10^{-3}$
Volatility	$1.61 * 10^{-3}$	$0.71 * 10^{-3}$	$1.38 * 10^{-3}$
$Corr(I/K, ...)$	0.719^{***}	0.25^{**}	0.72^{***}
	(0.065)	(0.121)	(0.064)
$Corr(\sigma_t, ...)$	1	0.524^{***}	0.900^{***}
		(0.101)	(0.033)
Coefficient of Variation			
Average	0.144	0.044	0.109
Volatility	0.020	0.011	0.047

Note: See notes to Table **??**.

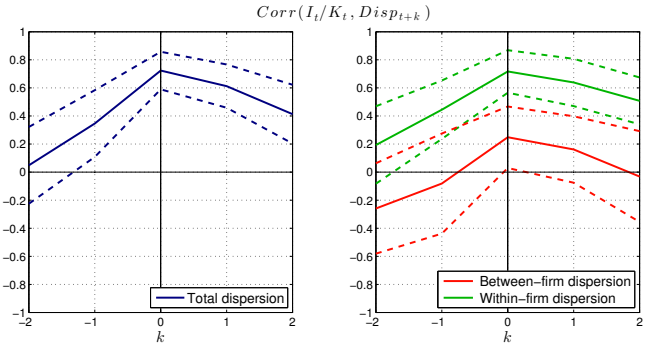
A.5 Statistics on capital intensity

In the main body of this paper, we considered moments of output per capital thus approximating capital returns. The inverse of this variable, capital per output is also of interest for researchers. We present some simple moments about this variable in Table 5.

A.6 Robustness: Are Firms Random Collection of Plants?

That most of the investment and productivity dispersion originates within firms proved to be a robust to explanations such as lumpy investment and entry and exit of production units. But before one jumps to conclusion about the (ir-)relevance of firms one should consider the possibility

Figure 14: Cyclicality of investment dispersion



Note: Correlation of aggregate manufacturing investment (I/K) with overall investment dispersion (σ_t, left panel), between-firm (σ_t^B) and within-firm (σ_t^W) investment dispersion (right panel).

Table 5: Moments about Capital intensity

	Capital intensity k/y	
	Total Capital	Equipment & Software
Mean	1.260	0.697
Median	0.954	0.520
XS StD	1.240	0.691
XS IQR	1.007	0.612
XS Coeff. of Skewness	2.799	2.610
Autocorrelation	0.844	0.851

Note: Capital k is the replacement value in 2005-$, output y is production in 2005-$ in the ASM panel 1972-2011. Cross-sectional moments are computed year by year in an unweighted fashion and then averaged across all years. 1% tails are winsorized in every year.

that firms are just random collection of plants. This means that plant-level investment could be purely driven by plant-specific characteristics. If that was true, firm affiliation would not matter because any firm manager would decide on the same investment. Within-firm dispersion would then be a product of pure chance without any economic meaning.

We want to find out if the between-/within-firm differences in capital returns and investment dispersion is such a statistical coincidence. We do that by randomly assigning plants to firms and redoing the decomposition outlined in equation (2). If dispersion within firms was just a statistical coincidence, then the between-firm, within-firm decomposition *might* differ. If on the other hand, we do find that "random firms" do produce a statistically different decomposition of dispersion between and within firms, then we could reject the hypothesis that firms are mere random collection in a statistical sense. Of course, this does not imply that firms are coherent units in an economically beneficial way.

Table 6: Between-within Decomposition of K-weighted Dispersion

Statistic	Actual Firms	Random Firms I	Random Firms II
σ	0.0084	0.0084	0.0084
σ^B	0.0026	0.0036	0.0031
		(0.0003)	(0.0002)
σ^W	0.0058	0.0048	0.0053
		(0.0003)	(0.0002)

Note: Table displays the average (over time) investment dispersion overall (σ_t) and its decomposition into between-firm (σ_t^B) and within-firm dispersion (σ_t^W). Random Firms I refers to a perturbation of plants and their investment rate within a given year and 3-digit NAICS industry, Random Firms II refers to a random perturbation of plants and their investment rate within given years, 4-digit NAICS industries and the four Census regions. The standard errors below the between and within firm component denote the time series average of the standard deviation across 100 bootstraps.

In order to test the hypothesis that firm affiliation is irrelevant, we construct 100 randomised samples where in each sample we replace the investment rate of a plant with a randomly drawn investment rate from another plant in the same year and the same 3-digit NAICS industry. We call this perturbation Random Sample I. We keep the year and industry fixed to control for the fact that investment patterns and the nature of firm affiliation differ across industries. Of course, this is rather a coarse grid and we carry out a second, finer randomisation where we replace plants within 4-digit NAICS industries and the four Census Regions (Northeast, Midwest, West, South) to additionally control for regional investment patterns; we call this finer perturbation Random Sample II. Across these two random samples, we perform the same between- and within-firm decomposition of investment rates. Naturally, every random sample will yield a slightly different decomposition, so we use the standard deviation across the different random draws as error bands. The results are displayed in Table 6 and show that the value of the between- and within-firm variances in the actual data are outside the 95% error bands of the random draws. So we conclude that firms are not random collection of plants and that considering firm-level frictions matter for plant-level investment outcomes.

B Model solution

— to be done —

C Estimating technology for multi-plant firms

C.1 Setup

Our quantitative analysis rests on the calibrated values for the production function and technology shocks. Identifying technology shocks is hence a crucial determinant of our quantitative analysis. Firm-level and plant-level shocks regulate how much capital returns dispersion emerge from the volatility of shock processes rather than constraints the firm faces when choosing its internal resource allocation. In this section we describe how we identify parameters α, ρ^f, ρ^p, σ^f and σ^p in the model. We estimate these consistent with the assumptions of the economic model in Section 3. In doing that, we extend the estimation of technology shocks in Cooper and Haltiwanger (2006) to a multi-plant setting. In addition to identifying technology shocks in a setting for multi-plant firms, we also refine their previous empirical work by defining a 3-digit NAICS industry as the common environment for firms. This allows production functions, long-run technology growth and TFP shocks to differ across those industries.

Plants face an inverse demand curve $p = y^{-\nu}$, so sales are $y^{1-\nu}$. Output y is produced using a Cobb-Douglas technology with capital k and other variable inputs which are chosen free of adjustment costs in markets where the firm is price taker. Assuming that the production elasticities of all variable inputs sum to ξ, gross profits of plant n which belongs to firm j at time t can be written (in logs) as

$$\pi_{njt} = \tag{12}$$

$$log(1-\xi) + (1-\nu) y_{njt}$$

$$= \tag{13}$$

$$log(1-\xi) + (1-\nu)\left[\frac{\tilde{\alpha}}{1-\xi} k_{njt} + tfp_{njt}\right]. \tag{14}$$

In contrast to the body of the paper, all variables are logged for ease of reading. We assume that log technology follows a stochastic process with trend growth

$$tfp_{njt} = \tilde{A} + \tilde{g}t + \tilde{A}_t + \tilde{\alpha}_{nj} + \tilde{z}_{njt} + \tilde{z}_{jt}. \tag{15}$$

Trend growth $\tilde{g}$ is common across all units as is the scaling parameter $\tilde{A}$ and the stochastic component $\tilde{A}_t$ which is stationary and fluctuates around zero. Every plant has a fixed technology level $\tilde{\alpha}_{nj}$ which embeds long-run plant profitability. In addition to the aggregate effect $\tilde{A}_t$, the stochastic portion of technology contains a firm component, $\tilde{z}_{jt}$, and an idiosyncratic plant component, $\tilde{z}_{njt}$. The firm component is defined as the firm average, so that z_{njt} denotes a plant's deviation from the firm average technology, that is: $\sum_{n \in j} z_{njt} = 0 \ \forall t$. Substituting equation (15) into equation

(14) yields

$$\begin{aligned}\pi_{njt} &= \log(1-\xi) + (1-\nu)\left[\frac{\tilde{\alpha}}{1-\xi}k_{njt} + \tilde{A} + \tilde{g}t + \tilde{A}_t + \tilde{\alpha}_{nj} + \tilde{z}_{njt} + \tilde{z}_{jt}\right] \\ &= \alpha k_{njt} + \overline{A} + gt + A_t + \alpha_{nj} + z_{njt} + z_{jt} \end{aligned} \tag{16}$$

where $\alpha = \frac{(1-\nu)\tilde{\alpha}}{1-\xi}$, $\overline{A} = \log(1-\xi) + (1-\nu)\tilde{A}$, $g = (1-\nu)\tilde{g}$ and analogous definitions for A_t, α_{nj}, z_{njt} and z_{jt}. If one abstracts from common and long-run productivity factors, equation (16) corresponds to the logged version of equation (3) in the model section. We can also map this equation into the data: we measure π_{njt} as the log value of sales less all variable input costs (salaries, wages, fringe benefits, materials and energy) deflated by the 5-digit shipment price deflator of the NBER-CES manufacturing database and k_{njt} as the log value of the real stock of structure and equipment capital.

We assume that both plant- and firm-specific components follow AR(1) stochastic processes:

$$\begin{aligned} z_{njt} &= \rho^p z_{njt-1} + \eta_{njt} \\ z_{jt} &= \rho^f z_{jt-1} + \eta_{jt}. \end{aligned}$$

with $\mathbb{E}\eta_{njt} = \mathbb{E}\eta_{jt} = 0$, $V(\eta_{njt}) = (\sigma^f)^2$ and $V(\eta_{jt}) = (\sigma^f)^2$. This persistence in technology processes is quite realistic, but prevents a direct empirical implementation of equation (16). There are several ways to overcome the problem which we detail below.

C.2 Identifying technology: Preferred benchmark

C.2.1 Estimating the production function and firm-level technology shocks

We can achieve identification of all parameters in equation (16) if we assume that the plant fixed productivity effects sum to zero inside firms. This is not a strong a assumption for two reasons: First, all α_{nj} have to sum up to zero – any common long-run component is contained in $\overline{A}$ – which is not very restrictive because "common" means 3-digit NAICS industry here. Second, the median firm in U.S. manufacturing operates 36 plants which samples a considerable number of plant fixed effects to sum to zero. We allow the stochastic processes to be different for firms and plants as highlighted above. To get overcome the problem of autocorrelated error terms in equation (16), we aggregate this equation to the firm level and then take quasi differences:

$$\begin{aligned} \pi_{jt} &= \alpha k_{jt} + \overline{A} + gt + A_t + z_{jt} \qquad (17) \\ &= \alpha k_{jt} + \overline{A} + gt + A_t + \rho^f z_{jt-1} + \eta_{jt} \\ \pi_{jt} - \rho^f \pi_{jt-1} &= \alpha k_{jt} - \rho^f \alpha k_{jt-1} + (1-\rho^f)\overline{A} + \rho^f g + (1-\rho^f)gt + A_t - \rho^f A_{t-1} + \eta_{nj}. \qquad (18) \end{aligned}$$

where $\pi_{jt} = \frac{1}{N_j}\sum_{n\in j}\pi_{jt}$ is the average log profit across the N_j plants of firm j (analogously for capital). We also used the fact that $\frac{1}{N_j}\sum_{n\in j} z_{njt} = \frac{1}{N_j}\sum_{n\in j}\alpha_{nj} = 0$. Since η_{jt} are iid over time and across firms, we can estimate equation (18). Since it's over identified, we use GMM with the following instruments: lagged and twice lagged values of π_{jt}, current, lagged and twice lagged values of k_{jt} and a complete set of time dummies to capture $(1-\rho^f)gt + A_t - \rho^f A_t$. This way, we do not have to specify a stochastic process for the aggregate technology shock.

C.2.2 Estimating plant-level components of TFP

We identify plant-level technology shocks from equation (16) as follows. Define the following two objects at the plant and firm level respectively:

$$X_{njt} \equiv \pi_{njt} - \alpha k_{njt} = \overline{A} + gt + A_t + \alpha_{nj} + z_{njt} + z_{jt}$$
$$X_{jt} \equiv \pi_{jt} - \alpha k_{jt} = \overline{A} + gt + A_t + z_{jt}.$$

We can compute the difference between those two objects and use it to estimate plant-level technology shocks in a fixed effects panel regression:

$$\begin{aligned} Y_{njt} \equiv X_{njt} - X_{jt} &= \alpha_{nj} + z_{njt} \\ &= \alpha_{nj} + \rho^p z_{njt-1} + \eta_{njt} \\ &= \alpha_{nj} + \rho^p \left[Y_{njt-1} - \alpha_{nj} \right] + \eta_{njt} \\ &= (1-\rho^p)\alpha_{nj} + \rho^p Y_{njt-1} + \eta_{njt} \end{aligned} \tag{19}$$

This gives usï¿½ estimates for the outstanding parameters ρ^p and σ^p which we report alongside the other estimates for the median industry in Table 7.

Table 7: Estimates

	Aggregate	Firm level		Plant level	
	α	ρ^f	σ^f	ρ^p	σ^p
(Ia)	0.6268	0.7360	0.8533	0.2806	0.4985
	(0.0028)	(0.0016)	(???)	(0.0121)	(???)
(Ib)	0.6268	0.7136	0.8628	0.3414	0.5939
	(0.0118)	(0.0073)	(???)	(0.0037)	(???)
(Ic)	0.5924	0.6980	0.8407	0.3423	0.5955
	(0.0206)	(0.0131)	(???)	(0.0071)	(???)
(II)	0.6268	0.1627	0.6153	0.2806	0.4985
	(0.0028)	(0.0502)	(???)	(0.0121)	(???)

Note: Results from the GMM estimation of equation (18) and the fixed effects panel regression in equation (19), standard errors in parentheses. Row Ia reports GMM estimates for α and the firm process (ρ_f and σ^f) and FE panel estimates for the plant process (ρ_f and σ^f); row (II) reports GMM estimate of α and FE panel estimates for both the firm and the plant processes.
(Ib) is like (Ia) except that all parameters are estimated at the NAICS-3 industry level and then averaged using value added industry weights. (Ic) does the whole thing at the NAICS-4 level.
Estimating everything at the 3-digit NAICS industry level and then reporting median industry values is fairly similar.

C.3 Identifying technology: General case

We also present a more general identification than above which proceeds along the lines of Blundell and Bond (1998). In particular, we relax the assumption $\sum_{n\in j}\alpha_{nj}=0$. This means firms within an industry are allowed to have long-run productivity differences. This added flexibility comes at the cost of potentially weaker identification. For filtering out firm-level fixed effects requires differencing equation (17) and then taking again the same quasi differences as above to overcome the persistence problem.

$$\begin{aligned}
\Delta\pi_{jt} &= \alpha\Delta k_{jt} + g + \Delta A_t + \Delta z_{jt} \\
&= \alpha\Delta k_{jt} + g + \Delta A_t + \rho^f \Delta z_{jt-1} + \Delta\eta_{jt} \\
&= \alpha\Delta k_{jt} + g + \Delta A_t + \rho^f \left[\Delta\pi_{jt-1} - \alpha\Delta k_{jt-1} - g - \Delta A_{t-1}\right] + \Delta\eta_{jt} \\
\Delta\pi_{jt} - \rho^f\Delta\pi_{jt-1} &= \alpha\Delta k_{jt} - \rho^f\alpha\Delta k_{jt-1} + (1-\rho^f)g + \Delta A_t - \rho^f\Delta A_{t-1} + \Delta\eta_{jt}. \qquad (20)
\end{aligned}$$

As in our benchmark case, we estimate equation (20) with GMM. As instruments, we choose lagged values of gross profits and current and lagged values of capital as above. But in this setup, where we need to instrument for growth rates rather than levels, these instruments are notoriously weak. This weakness manifests itself in estimates which are quite different from the accepted standard in the literature (see Cooper and Haltiwanger (2006) or Kehrig (2015)), so we are not confident they make a solid calibration target for your quantitative work. We get $\hat{\alpha}=0.17$, $\hat{\rho}^f=-0.15$. In principle, one could follow the subsequent procedures outlined above to obtain estimates for the aggregate and plant-level components of technology.

C.4 Identifying technology: Previous standard

Previous work that estimated technology shocks in the context of investment models ignored the firm dimension. This implicitly assumed the same persistence of plant and firm technology and also assumed no plant-level fixed effects. If we were to follow that procedure, the same persistence allows us to write $\zeta_{njt}=z_{njt}+z_{jt}=\rho\zeta_{njt-1}+\eta^{\zeta}_{njt}$. Then, we taking quasi differences as above:

$$\begin{aligned}
\pi_{njt} &= \alpha k_{njt} + \overline{A} + gt + A_t + \zeta_{njt} \\
&= \alpha k_{njt} + \overline{A} + gt + A_t + \rho^{\zeta}\zeta_{njt-1} + \eta^{\zeta}_{nj} \\
\pi_{njt} - \rho^{\zeta}\pi_{njt-1} &= \alpha k_{njt} - \rho^{\zeta}\alpha k_{njt-1} + (1-\rho^{\zeta})\overline{A} + \rho^{\zeta}g + (1-\rho^{\zeta})gt + A_t - \rho^{\zeta}A_{t-1} + \eta^{\zeta}_{nj}.
\end{aligned}$$

We estimate this equation by GMM using as current and up to twice lagged values of capital and lagged values of gross profits. In principle, one could follow the subsequent procedures outlined above to obtain estimates for the aggregate and plant-level components of technology.

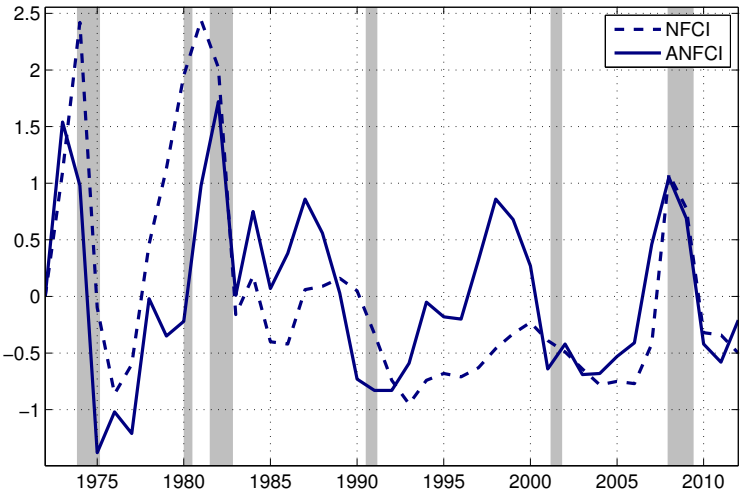
D How financial constraints shape investment empirically

D.1 Credit Constraints

D.1.1 Introducing time-varying financial frictions

In order to determine whether the predictions of the model with time-varying financial frictions are in line with what we observe in the data, we need some measure of financial conditions that covers a long enough time period. The National Financial Conditions Index (NFCI) from the Federal Reserve Bank of Chicago seems well suited to our purposes. The NFCI is a weighted average of a large number variables of financial activity, relative to their means. The index is therefore centered around zero by construction. We will be using the Adjusted National Financial Conditions Index (ANFCI), a version that isolates the component of financial conditions that is orthogonal to current economic conditions, allowing us to focus solely on the impact of fluctuations in credit tightness without worrying about endogeneity. Figure 15 shows the evolution of both the raw and adjusted NFCI.

Figure 15: Credit Tightness in the U.S. Economy



Note: Annualised time series of the National Financial Conditions Index in the raw version (dashed line) and the version that is adjusted for endogenous responses of financial indicators to non-financial shocks (solid line). Shaded areas are NBER recessions.

As one can see, the adjusted indicator lines nicely with NBER recessions in the 1970's, 1980's and in 2008/09 while it does not increase before or during the 1991 and the 2001 recession – that financial indicators such as credit spreads did not catch those recessions is a well-known fact and we follow the literature to interpret these as non-financial recessions. In the subsequent analysis, we will use the ANFCI time series as an exogenous variable that constrains a firm's ability to borrow funds for investment purposes.

D.1.2 Empirical joint dynamics in 2-plant firms

In this section we move away from comparative statics by incorporating a two-state process for the financial friction: the firm will alternate between states with low or high degrees of financial frictions. This exercise allows us to run on the simulated data regressions that are similar to the empirical specifications discussed in the next section where we use a time-series index of financial conditions to determine how investment dynamics are affected by changes in financial frictions.

For this exercise, we continue to set the fixed cost of borrowing, ζ, to 1% of the capital stock. This parameter will be time-invariant and can be interpretred for example as the management costs related to preparing a loan application and interacting with the financial intermediary The other borrowing friction parameter, η, is time-varying: it can take the values 0 (low borrowing cost) or 5 (high borrowing cost) depending on the degree of credit tightness, and the probability of switching between the two states is equal to 0.1. All the other parameters of the model are unchanged.

We simulate the model and run regressions on the simulated data. The move to regression analysis is important if we want to link the predictions of the model with the data: while our model allows us to perfectly isolate the role of borrowing constraints, in the real world investment dynamics may be affected by multiple sources of heterogeneity unrelated to financial frictions. The use of controls is therefore crucial.

It should be noted that we are in no way trying to calibrate the size and relative importance of the firm-level financial frictions at this point. Therefore, what we are interested in determining whether the model predictions are in line with what we observe in the data from a qualitative, not quantitative, standpoint.

Table 8 shows results for plant-level investment regressions. Each regression uses the investment-to-capital ratio for plant A at time t as the dependent variable (the two plants are perfectly symmetric in the model), $(i/k)_{At}$. In order to determine the impact of firm-level financial frictions on investment dynamics, we define a dummy variable, ς_t, equal to one if credit is tight in period t (i.e. $\eta = 5$) and zero otherwise.

The first two regressions of Table 8 focus on the role of output/cash flow variables in explaining movements in investment. Not surprisingly, plant-level investment is on average lower in periods of high borrowing costs ($\varsigma_t = 1$). In the top panel we can see that a 10% increase in the output-to-capital ratio of the plant raises its i/k ratio by about 1.3%. Interestingly, plant A's investment is also affected by plant B shocks, to a lesser degree. There are two possible reasons for this result. First, simultaneous increases in output at both plants A and B are potentially indicative of a firm-wide shock. Since firm shocks are more persistent than plant-level shocks, the optimal decision is to invest more.

Second, the internal finance channel is also at play: in a context where funds are scarce and borrowing costly (recall that $\zeta = 0.01K$ in all states), a good shock in plant B generates precious cash flow that can be used to finance investment in plant A. This is also evident in the results for the second regression of the same table. There, plant B's output is replaced by cash flow at the level of the firm, net of adjustment and fixed operating costs (notice that we do not use logs as cash flow is sometimes negative) Not only is plant A investment higher when firm cash flows are higher (conditional on the plant-specific shock), but we can see that this dependence on the firm's financial resources is particularly strong when credit is tight: the coefficient on cf_t/k_t more than doubles when $\varsigma_t = 1$. In other words, the existence of internal capital markets is particularly relevant when external financial constraints are more binding.

The last regression of Table 8 looks at the correlation in investment activity across plants within the firm. In periods where credit is cheap, $(i/k)_{At}$ is a positive function of investment activity in plant B, though the relationship is somewhat weak: a 1 percentage point increase in $(i/k)_{Bt}$ raises $(i/k)_{At}$ by less than 0.1 percentage point. The relationship, however, changes dramatically when credit is tight: in periods where $\eta = 5$, the same 1 p.p. change in investment at plant B leads instead to a *fall* of almost 0.5 percentage point in $(i/k)_{At}$ as investment activity in one plant crowds out investment elsewhere. With scarce funds, the firm selects the most profitable projects, postponing others as their marginal benefit is outweighed by the marginal cost of external funds.

The firm-level regression in Table 8 highlights another impact of financial frictions in multi-unit firms: the serial correlation of firm-level investment increases. As shown earlier in Table 2, our baseline calibration implies that the autocorrelation of i_t/k_t is slightly negative, which should not be too surprising as these firms are very small. However, a financial constraint shock makes firm-level investment significantly smoother. On potential explanation for this result is related to our earlier findings: when credit is tight, it makes the firm less likely to invest in both plants at the same time in order to avoid costly borrowing. To understand why this may lead to higher autocorrelation of investment for the firm, consider the example of a positive firm-level shock. Since both z_{At} and z_{Bt} are now higher, both plants would now like to invest to reach their new optimal level of capital. But given limited cash flows, this implies that the firm would need to obtain costly funds on capital markets. Instead, it will sometimes find it optimal to stagger investment: plant A invests today, while plant B waits until tomorrow. By construction, this makes firm-level investment smoother. We show additional evidence for this kind of behaviour below.

In Table 10 we instead focus on investment spikes, having showed earlier that they were the main contributor to aggregate investment empirically. All regressions are linear probability models. In the first panel, the dependent variable is a dummy equal to 1 if the investment-to-capital ratio in plant A is greater than 15%, our threshold for a spike. Both output levels for plants A and B have a positive impact on the probability of a spike, in line with what we found earlier. Our focus, however, is on the spike indicator for plant B: very clearly, the spike activity in one plant does matter for the probability of a spike in the other. In periods of tight credit ($\varsigma_t = 1$), the probability of observing an investment spike in plant A is 0.5 percentage point lower if plant B is already spiking.

Another way to confirm this finding is to see whether occurrences of double spikes (i.e. spikes in both plants) is more or less likely when external funds are more costly. Conditional on firm cash flow and capital stock (coefficients not reported), the second panel of the same table shows that the probability of observing both plants spiking in the same period basically falls to zero when credit is tight.

Finally, the last regression in Table 10 revisits the question of staggered investment across plants. We regress the spike indicator for plant A on its own lag, the spike indicator for plant B at time t as well as its lagged value, both by itself and multiplied by the credit dummy, ς_t. Our focus is on this last variable: if the firm is more likely to stagger investment activity when borrowing is costly, the coefficient on the interaction term should be positive. This is what we obtain, with a value of 0.036, though the effect seems to be relatively small.

All these results seem to indicate that the firm-level financial frictions, whether in the form of fixed or quadratic costs, alter significantly plant-level investment dynamics. In summary, the firm optimises the timing of investment spikes, making sure that joint spikes are avoided in order to minimise the need to borrow in a given period. Instead, following a positive firm-level shock

Table 8: Plant-level investment regressions

Constant		0.419^{***}
Credit tightness	ς_t	-0.075^{***}
Output plant A	$\log(y/k)_{At}$	0.129^{***}
	$\varsigma_t \cdot \log(y/k)_{At}$	-0.014^{***}
Output plant B	$\log(y/k)_{Bt}$	0.063^{***}
	$\varsigma_t \cdot \log(y/k)_{Bt}$	-0.011^{**}
R^2		0.33
Constant		1.647^{***}
Credit tightness	ς_t	-0.074^{***}
Output plant A	$(y/k)_{At}$	0.584^{***}
Firm cash flow	$(cf/k)_t$	0.246^{***}
	$\varsigma_t \cdot (cf/k)_t$	0.356^{***}
R^2		0.34
Constant		1.882^{***}
Credit tightness	ς_t	0.017^{***}
i/k plant B	$(i/k)_{Bt}$	0.085^{***}
	$\varsigma_t \cdot (i/k)_{Bt}$	-0.545^{***}
Cash flow firm	$(cf/k)_t$	-0.007
	$\varsigma_t \cdot (cf/k)_t$	0.882^{***}
R^2		0.40

Note: Dependent variable is i/k of plant A. $\varsigma_t = 1$ in periods of high borrowing cost, 0 otherwise. Controls such as capital stock or cash flow are included but not always reported. *, ** and *** indicate significance at the 10, 5 and 1 percent level respectively.

Table 9: Firm-level investment regressions

$$(i/k)_t = \beta_0 + \beta_1 \varsigma_t + \beta_2 \log(i/k)_{t-1} + \beta_3 \varsigma_t \cdot \log(i/k)_{t-1} + \varepsilon_t$$

Variable		Estimate
Constant	β_0	0.419^{***}
Credit tightness	β_1	-0.038^{***}
Lagged firm investment	β_2	-0.034^{***}
... interacted with credit tightness	β_3	0.125^{***}
R^2		0.44

Note: Dependent variable is i/k of the firm. $\varsigma_t = 1$ in periods of high borrowing cost, 0 otherwise. Controls such as capital stock and firm cash flow are included but not reported. *, ** and *** indicate significance at the 10, 5 and 1 percent level respectively.

affecting both plants, one plant spikes immediately while the other waits one period to invest. This, in turn, makes investment aggregated at the firm level more serially correlated in periods of tight credit. In the next section, we investigate whether these predictions are borne out empirically using a proxy for financial conditions.

D.2 Investment Dynamics of Plants, of Firms and Within Firms

We first focus on the time series properties of investment rates at the plant level, the firm level and the joint investment dynamics of plants within firms. Table 2 is our benchmark.

D.2.1 Autocorrelation of plant-level investment

We start by examine the autocorrelation of investment at the plant and the firm level. Table 2 shows how plant-level investment becomes less and firm-level investment becomes more autocorrelated in the model. The latter is a response of firms where costly external credit induces the firm to smooth borrowing and thus investment. Predictions about the autocorrelation of plant-level investment are not as sharp and that is reflected in the data: Table 11 displays the estimates of regressing plant-level investment on the credit constraints indicator (denoted by ς_t). The estimates from the panel regression (our preferred specification) are not significant. A simple pooled OLS regression indicates that in tight credit times investment becomes more autocorrelated.

The predictions about firm-level investment are much sharper. Table 11 reveals that investment at the firm level significantly becomes more autocorrelated: A standard deviation to credit tightness almost doubles the autocorrelation. The effect at the firm level is much stronger than any change at the plant level where the autocorrelation increases only by 10%. Interestingly, tight credit itself does not significantly lower investment levels as the estimate of β_3 is not significantly negative as one may expect – and neither it is with the plant regression in Table 11.

D.2.2 Investment Correlation Within Firms

A particularly sharp implication of credit constraints is that investment across the two plants within the firm falls; Table 2 shows in fact that it may even become negative. This obviously reflects the

Table 10: Spike ($i/k > 0.15$) regressions

	Dependent variable	$\mathbf{I}\{(i/k)_{At} > 0.15\}$
Constant		6.261^{***}
Credit tightness	ς_t	-0.039^{***}
Output plant A	$\log(y/k)_{At}$	0.504^{***}
Output plant B	$\log(y/k)_{Bt}$	0.330^{***}
Spike plant B	$\mathbf{I}\{(i/k)_{Bt} > 0.15\}$	-0.254^{***}
	$\varsigma_t \cdot \mathbf{I}\{(i/k)_{Bt} > 0.15\}$	-0.261^{***}
R^2		0.44
	Dependent variable	$\mathbf{I}\{(i/k)_{At}, (i/k)_{Bt} > 0.15\}$
Constant		0.069^{***}
Credit tightness	ς_t	-0.075^{***}
R^2		0.13
	Dependent variable	$\mathbf{I}\{(i/k)_{At} > 0.15\}$
Constant		6.651^{***}
Credit tightness	ς_t	-0.108^{***}
Spike plant B	$\mathbf{I}\{(i/k)_{Bt} > 0.15\}$	-0.478^{***}
Lag spike plant A	$\mathbf{I}\{(i/k)_{At-1} > 0.15\}$	-0.048^{***}
Lag spike plant B	$\mathbf{I}\{(i/k)_{Bt} > 0.15\}$	0.061^{**}
	$\varsigma_t \cdot \mathbf{I}\{(i/k)_{Bt} > 0.15\}$	0.036^{**}
R^2		0.48

Note: Dependent variable is spike dummy. $\varsigma_t = 1$ in periods of high borrowing cost, 0 otherwise. *, ** and *** indicate significance at the 10, 5 and 1 percent level respectively.

Table 11: Credit Constraints and Plant- and Firm-Level Investment

left panel (plant): $(i/k)_{nt} = \beta_0 + \beta_1(i/k)_{nt-1} + \beta_2\varsigma_t \cdot (i/k)_{nt-1} + \beta_3\varsigma_t + \varepsilon_{nt}$
right panel (firm): $(i/k)_{jt} = \beta_0 + \beta_1(i/k)_{jt-1} + \beta_2\varsigma_t \cdot (i/k)_{jt-1} + \beta_3\varsigma_t + \varepsilon_{jt}$

Coefficient	OLS	Panel	Coefficient	OLS	Panel
	(0.0471)	(0.0638)		(0.0565)	(0.0614)
β_2	0.0145^{**}	0.0109	β_2	0.0264^{***}	0.0288^{***}
	(0.0069)	(0.0095)		(0.0074)	(0.0075)
β_3	0.0004	0.0017	β_3	0.0003	0.0003
	(0.0008)	(0.0011)		(0.0007)	(0.0007)
Controls	Yes	Yes	Controls	Yes	Yes
N	49k	17k	N	24k	9k

fact that in times of tight credit the firm needs to scale back investment in general. So if one plants invests a lot, the the other one probably suffers when credit is tight.

Table 12 confirms this prediction of the model. The estimates of β_2 are significantly negative across both panel and OLS regressions. The estimate imply that a doubling of investment in the other plant lowers the investment rate in the other plant by two percentage points.

Table 12: Credit Constraints and Within-Firm Investment

$$(i/k)_{At} = \beta_0 + \beta_1 (i/k)_{Bt} + \beta_2 \varsigma_t \cdot (i/k)_{Bt} + \varepsilon_{nt}$$

Coefficient	OLS	Panel
	(0.0017)	(0.0016)
β_2	–0.0035**	–0.0031**
	(0.0014)	(0.0013)
Controls	Yes	Yes
N	66k	66k

D.2.3 Probability of investment spikes and spike size

We look at the likelihood of investment spikes in episodes of tight credit. It's not obvious whether there will be more or less investment spikes when credit is tight. On the one hand, spikes will happen less often because tight credit limits overall investment resources. Then, tight credit results in a "lumpiness" effect because investment looks more lumpy. On the other hand, tight credit makes the firm smooth its borrowing so that investment spikes become more frequent. If that latter outcome prevails, one would expect investment spikes to become smaller; we label this latter effect the "smoothing effect." We test both of these possible predictions and display the results of this regression in Table 13. The results are overall weak and borderline significant. But if at all, one sees that the probability of investment spikes increases and the level of investment spikes decreases significantly.

D.2.4 Joint distribution of investment spikes within firms

While predictions about single investment spikes are not conclusive, the model has fairly strong predictions about the *joint* distribution of investment spikes within firms. When credit is tight, a firm cannot allow both of its plants to undergo an investment spike in the same period – at least not if it's financially constrained. As a consequence, the likelihood to see both plants undergoing an investment spike should drop significantly when credit is dear. This is clearly borne out in the data as Table 2 shows. An it does show up in the data as well: When credit gets tight, the likelihood to undergo an investment spike when the other plant undergoes one is 2% lower than when credit is loose. In that latter scenario, we can expect that one plant spiking raises the likelihood of the other one spiking by 18% – probably reflecting a positive firm-specific productivity shock.

Note that this logic of reduced simultaneous spiker plants only applies to financially constrained firms. If a firm wasn't financially constrained, we would expect the coefficient β_2 to be zero. To test

Table 13: Credit Constraints and Investment Spikes

$$\mathbf{I}\{(i/k)_{nt} > 0.15\} = \beta_0 + \beta_1 \varsigma_t + \beta_2 \varsigma_t \cdot \mathbf{I}\{\text{Public}\} + \varepsilon_{nt}$$
$$(i/k)_{nt} = \tilde{\beta}_0 + \tilde{\beta}_1 \varsigma_t + \tilde{\beta}_2 \varsigma_t \cdot \mathbf{I}\{\text{Public}\} + \varepsilon_{nt} \quad \forall n, \text{ s.t. } (i/k)_{nt} > 0.15$$

Coefficient	OLS	Panel
β_0	0.0411***	0.0519***
	(0.0.131)	(0.0092)
β_1	0.0002	0.0003
	(0.0012)	(0.0011)
β_2	–0.0055	–0.0004
	(0.0067)	(0.0066)
Controls	Yes	Yes
N	66k	66k

Coefficient	OLS	Panel
$\tilde{\beta}_0$	.	
$\tilde{\beta}_1$	negative (significant)	
$\tilde{\beta}_2$	positive (significant) ⇒ publicly traded firms unaffected	
Controls	Yes	Yes
N	66k	66k

for that hypothesis, we include an interaction term of credit tightness and the other plant spiking with a dummy variable that indicates whether or not the firm is publicly traded or not. The idea is that publicly traded firm probably are not affected by ς_t. As we can see, this is borne out in the data, albeit it's borderline significant.

D.2.5 Serial correlation of single investment spikes

So what can firms that are credit constrained do if both of its plants are so productive that ideally they should btw undergo investment projects? If credit is tight and thus external finance particularly costly, then it may see no other possibility than to focus its funds on investing in one plant and postponing investment in the other plant. We call this spacing out of investment spikes "adjacent investment spikes" and test for them by regressing a dummy variable that indicates such "adjacent spike"-firms on credit conditions. The unconditional probability (without any especially tight credit) is about 10% (see Table **??**) and the regression results in Table 15 tell us that this probability drops by 0.7%.[12]

[12] In this analysis, we consider firm that undergo exactly one spike today and one or two spikes last year. Instead, we should have restricted the sample to only those firms that have *at least* 2 spikes within a two year window and then see if they are more likely to stretch the at least 2 spikes out or if they choose to do it simultaneously and also consider interaction terms with the publicly traded dummy. Since this sample is smaller than the one we consider here, we see our estimates as a lower bound.

Table 14: Credit Constraints and Firm-level Investment

$$\mathbf{I}\{(i/k)_{At} > 0.15\} = \beta_0 + \beta_1 \mathbf{I}\{(i/k)_{Bt} > 0.15\} + \beta_2 \varsigma_t \mathbf{I}\{(i/k)_{Bt} > 0.15\} + \beta_3 \varsigma_t \mathbf{I}\{(i/k)_{Bt} > 0.15\} \mathbf{I}\{\text{Public}\} + \varepsilon_{nt}$$

Coefficient	OLS	Panel
β_0	0.0378***	0.0463***
	(0.0130)	(0.0089)
β_1	0.1846***	0.1812***
	(0.0056)	(0.0056)
β_2	–0.0211***	–0.0205***
	(0.0051)	(0.0051)
β_3	0.0237*	0.0201
	(0.0132)	(0.0135)
Controls	Yes	Yes
N	66k	40k

Table 15: Credit Constraints and Serial Correlation of Investment Spikes

$$\mathbf{I}\left\{(i/k)_{At}, (i/k)_{At-1} > 0.15\right\} = \beta_0 + \beta_1 \varsigma_t + \beta_2 \varsigma_t \cdot (i/k)_{Bt} + \varepsilon_{nt}$$

Coefficient	OLS	Panel
β_0	0.2148***	0.1836***
	(0.0354)	(0.0274)
β_1	0.0069**	0.0069**
	(0.0030)	(0.0032)
β_2	–0.0141	–0.0141
	(0.0130)	(0.0133)
Controls	Yes	Yes
N	17k	9k