CRIME AND PUNISHMENT:
AN INTRODUCTORY ANALYSIS IN A NONCOOPERATIVE FRAMEWORK

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Abstract: The purpose of this paper is twofold. First, it seeks to provide the unsophisticated reader with an introduction to modelling issues of crime and punishment; and, second, it seeks to introduce a noncooperative analytical framework as the basic modelling technique to analyze this type of issues. To those purposes, I introduce simple model from which important policy recommendations follow from the noncooperative interaction between criminals and the rest of society.

* The first version of this paper was written over three years ago. Although the presentation has changed substantially over time, the analytical framework and main results are virtually identical to those of the original version. For helpful comments I would like to thank Tom Ulen, participants of the Graduate Student Workshop at the University of Illinois at Urbana-Champaign, and graduate students of the Institute Torcuato Di Tella (Buenos Aires, Argentina). The views expressed below and any errors that may remain are entirely my own.

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I- INTRODUCTION

In the late 60s, Gary Becker (1968) “invaded” for the first time with a formal economic model an area that, until then, had been exclusively dominated by sociologists and psychologists, namely, criminal behavior.[1] Since then, several models have been proposed by economists to explain, within frameworks of utility maximization, that type of behavior. This literature has, in my view, two problems which this paper seeks to overcome.

The first problem with the literature on crime and punishment is that the cooperative framework used to analyze the problem does not seem to be the appropriate one. In most studies, the cooperative nature of the analytical framework follows from the fact that the welfare function to be maximized by (the decent members of) society takes into account the welfare of criminals. That is not, however, the way punishments are decided in the real world; for the decent members of society do not consider the criminals’ welfare when deciding the punishments to be imposed. However, although the *welfare* of criminals does not play a role in the social decision about punishment, the *behavior* of criminals does. In that sense, this paper makes the contribution of introducing a noncooperative analytical framework that recreates such an interaction.

In his Nobel prize lecture, Gary Becker (1993) suggested that, when he originally thought about the issue of crime and punishment, he did it in the way suggested above. He wrote:

> “I was late and had to decide quickly whether to put the car in a parking lot or risk getting a ticket for parking illegally on the street. I calculated the likelihood of getting a ticket, the size of the penalty, and the cost of putting the car in a lot. I decided it paid to take the risk and park the car on the street ... As I walked ... it occurred to me that the city authorities had probably gone through a similar analysis. The frequency of their inspection of parked vehicles and the size of the penalty imposed on violators should depend on their estimates of the type of calculations potential violators like me would make.”[2]

It is precisely in this noncooperative way that I model in this paper the interaction between criminals and the rest of society. In order to do so, I bring the Stackelberg model of oligopoly into the discussion of criminal behavior. My approach is as follows: Criminals behave as Stackelberg followers that, given the probability of apprehension and the severity of the punishment, attempt to maximize their expected utility by choosing an optimal allocation of labor (crime) and leisure. The decent members of society, on the other hand, behave as Stackelberg leaders that, knowing that criminals behave as just explained, take into account that behavior, and attempt to maximize their utility by choosing an optimal allocation of resources between the production of commodities and the production of security, and the optimal severity of punishments.

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[1] Bentham (1931) had informally anticipated many of the results formally derived by Becker (1968).

[2] Becker (1993), pp. 389-390.

A second problem with the literature on crime and punishment is that most of its models are either too simple, almost informal (for example, Cooter and Ulen, 1988, chapters 11 and 12), or so technical (Polinsky and Shavell, 1992) that their important policy recommendations do not reach the mathematically-unsophisticated reader. In that sense, this paper makes the contribution of introducing a simple, although rigorous, model of crime and punishment from which important policy recommendations can be derived.

The rest of the paper is organized as follows. In part II, I introduce the model that criminals and the rest of society use to make their decisions, and address the issue of deterrence. In part III, I address the issue of optimal punishment and derive four propositions, each one being a policy recommendation. And, finally, in part IV, I summarize the most important results of the paper.

II- THE MODEL

Consider a society divided into two types of individuals: criminals and victims; a victim should be thought of as anyone who is not a criminal (hence, the decent members of society are victims). Both criminals and victims make their decisions in isolation, thus being prevented from bargaining; this (certainly realistic) assumption recreates the framework of a noncooperative game. As argued above, this noncooperative analytical framework is a departure from the previous literature.

It is assumed that criminals and victims are rational and that, except in the extreme cases to be considered below, they do not use a utility maximization model to decide whether they will become criminals or victims. Rather, it is assumed that they both have moral constraints that force them to be either criminals or victims. To illustrate, an honest person would not steal an old lady's purse in a dark and lonely street just because he can easily reap a benefit and get away with his crime. This assumption of moral constraints seems to be reasonable for most members of society.

1- The criminal's behavior

Consider a Beckerian framework in which the (representative) criminal produces himself the commodities he consumes.[3] The criminal's utility depends upon two mutually-exclusive states of the world: in one the criminal gets away with his crime; in the other, he is caught and punished. Let α be the probability of apprehension, which is taken as given by the criminal. The criminal's utility in the state of the world in which he is not punished depends on his consumption of a good (g), on his wealth (W), and on some other exogenous variables (Y). The criminal produces the good by combining two inputs, crime (c) and leisure time (l). Crime is (the monetary equivalent of) a harm that the criminal inflicts on his

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[3] Becker's model of consumer behavior assumes that an individual does not derive utility directly from the goods he purchases. Rather, his utility depends on commodities he produces himself by combining the goods he purchases with his own time. Thus, the consumer-producer has to determine the optimal allocation of inputs (goods and time) such that the production of these home-made commodities maximizes his utility. See Becker (1971).

victims (the decent members of society), which he needs as an input in the production of the good he consumes.

In order to produce crime, the criminal needs to allocate some time (t) to this activity; this time may be thought of as the criminal's labor time. Thus, the production function of crime is given by $c=c(t)$, such that $c_t>0$. The good, a commodity whose consumption the criminal enjoys, is produced by combining two inputs, crime and leisure time. Thus, the production function of the good is given by $g=g[c(t),l]$, such that $g_c>0$ and $g_l>0$. Note that g_c is the rate at which the criminal transforms the harm he inflicts on society into a good for himself.

The criminal has to allocate his endowment of time (T) between the production of the good (leisure time) and the production of crime (labor time); hence, $l+t=T$. It thus follows from the set up of the model that the criminal's problem can be viewed as the standard choice between labor and leisure where, in this case, labor is the time the criminal allocates to criminal activities, and leisure is the time he allocates to produce the good he consumes. Further, let the criminal's utility depend on some other exogenous variables not explicitly considered in the model, represented by a vector Y, and on his (exogenously-determined) wealth (W). Thus, by putting together all the arguments considered above, the criminal's utility function when he is not punished (U^{NP}) is given by $U^{NP}=U\{g,W,Y\}=U\{g[c(t),l],W,Y\}$, such that $U_g>0$, $U_{gg}<0$, and $U_W>0$. The sign of U_Y depends on which argument of this vector is being considered; some arguments generate utility and some others disutility.

Consider now the criminal's utility function when he does not get away with his crime; that is, when he is caught and punished. The punishment (p) can take two forms, a fine (F) or (the monetary equivalent of) imprisonment (I), both of which depend on the crime committed by the criminal and on the severity of the punishment (x,y).[4] Thus, the punishment function is given by $p=p(c,x,y)=F(c,x)+I(c,y)$, such that $F_c>0$, $F_x>0$, $I_c>0$, and $I_y>0$; or, alternatively, $p_c>0$, $p_x>0$, and $p_y>0$.[5] This implies that both fines and imprisonment are increasing in the harm caused by the criminal and in the severity of the punishment. Further, assume that $F_{cx}>0$ and $I_{cy}>0$ (hence, $p_{cx}>0$ and $p_{cy}>0$); that is, the marginal punishment is increasing in the severity of the punishment. Finally, let $P=p(c,x,y)-W$ be the net punishment imposed on the criminal, and let the criminal's utility when he does not get away with his crime be a function of the vector Y considered before. Thus, the criminal's utility function when he is punished (U^P) is given by $U^P=U\{P,Y\}=U\{F[c(t),x]+I[c(\mathrm{t}),y]-W,Y\}$, such that $U_P<0$ and $U_{PP}<0$; that is, punishment generates disutility at an increasing rate. Note, however, that U^P is not necessarily negative; even if the criminal is punished, the utility he derives from Y may outweigh the disutility of the

[4] The severity of a fine (x) is a parameter that determines, for each level of harm, the amount of money the criminal must pay. The severity of imprisonment (y), on the other hand, is a parameter that determines, for each level of harm, the length of the prison sentence the criminal must serve.

[5] Note that since the amount of crime the criminal produces depends on the amount of time he allocates to criminal activities, the punishment function he considers is $F[c(t),x]+I[c(t),y]$.

punishment. This, in fact, seems to be the most realistic case. As before, the sign of U_Y depends on which argument of *Y* is considered.

Consider now all the elements at once. The criminal enjoys the consumption of a single good (*g*). In order to produce this good, the criminal needs to commit a crime (*c*), for which he needs to allocate some time (*t*) to criminal activities and some leisure time (*l*) to the enjoyment of this commodity. The production of crime has a negative impact on the rest of society, and therefore criminal activities are punished. However, the punishment (*p*) will be suffered by the criminal only if he is apprehended, which occurs with probability α. This probability, together with the severity of the punishment (x,y), are taken as given by the criminal. Finally, the criminal's utility in both states of the world depends on his wealth (*W*), and on some exogenous variables (*Y*). In this framework, the (representative) criminal acts as a Stackelberg follower whose problem is to allocate his time (*T*) between the production of the good (leisure time) and the production of crime (labor time) in order to maximize his expected utility (*EU*). Formally:

$$\text{Max}_{l,t}\, EU = \alpha U\{F[c(t),x]+I[c(t),y]-W,Y\}+(1-\alpha)U\{g[c(t),l],W,Y\} \tag{1a}$$

$$\text{subject to:} \qquad l+t=T \tag{1b}$$

$$EU \geq \bar{u} \tag{1c}$$

where $\bar{u}$ is the criminal's subsistence level of utility, which is exogenously determined.[6] Replacing the time constraint (l=T-t) into the utility function, maximizing with respect to *t*, and rearranging yields:

$$(1-\alpha)(U_g g_c c_t - U_g g_l) = -\alpha U_P(F_c+I_c)c_t \tag{2}$$

The left-hand side of (2) is the expected net marginal benefit of allocating time to criminal activities, and shows that, when the criminal increases his labor time, he increases his production of crime, which increases the production of the good, which, in turn, increases his utility. Yet, when the criminal allocates more time to criminal activities, he forgoes leisure time, which reduces the production of the good, which, in turn, reduces his utility. The right-hand side of (2) is the expected marginal cost of allocating more time to criminal activities, and shows that, when the criminal increases his labor time, he increases his production of crime, which increases his expected punishment, which, in turn, decreases his utility.

The optimal allocation of time to criminal activities (t^*) solves from (2) and is a function of the parameters of the criminal's model; that is, $t^*=t(\alpha,x,y,T,W,Y)$. In order to focus on the criminal's reaction in response to changes in the probability of apprehension and in the severity of the punishment, this last expression can be simplified to $t^*=t(\alpha,x,y)$. This relationship may be thought of as the criminal's labor supply function, which indicates the criminal's optimal supply of labor for each value of the probability of apprehension and each level of severity. Once the criminal has chosen the optimal amount of time to

[6] The criminal's problem will be solved assuming that (1c) does not bind. Cases in which this constraint does bind are analyzed below.

be allocated to criminal activities, his other choices follow straightforwardly. Of particular importance for further analysis is the criminal's optimal production of crime, which is given by:

$$c^* = c[t^*(\alpha,x,y)] = c(\alpha,x,y) \tag{3}$$

This relationship shows the optimal amount of crime produced by the criminal for each level of the probability of apprehension and each level of severity. Thus, let (3) be the *criminal's reaction function*.

A caveat is in order. The analysis above is valid as long as (1c) does not bind; that is, as long as EU^* (the maximum value of the criminal's expected utility) is larger or equal than (the criminal's subsistence level of utility). When, on the other hand, $EU^* < \bar{u}$, the criminal, after having allocated his time in the most efficient way, cannot attain his subsistence level of utility. This may happen because there exists either a very severe punishment, or a very high probability of apprehension, or a combination of both. Under these circumstances, the criminal may overcome his moral constraints, give up criminal activities, and become a decent member of society. This line of reasoning suggests that a criminal may be induced to quit criminal activities through the imposition of severe punishments or high probabilities of apprehension (or both). This issue is considered in more detail in the next section.

2- The deterrence effect

I address in this section the issue of deterrence; that is, the issue of whether a criminal can somehow be induced to give up his criminal activities. Several models on the topic have been developed in the literature; theoretical and empirical evidence has been found to support both the deterrence and the non-deterrence hypotheses.[7] I derive below formal results that support the existence of a deterrence effect.

Consider the impact of an increase in the probability of apprehension first. Whether or not a criminal can be deterred depends on the sign of the expression $\partial c^*/\partial\alpha=(\partial c^*/\partial t^*)(\partial t^*/\partial\alpha)$. Recall that $(\partial c^*/\partial t^*)$ is positive. Thus, the sign of $\partial c^*/\partial\alpha$, and, therefore, the existence of a deterrence effect depends on the sign of $(\partial t^*/\partial\alpha)$. The sign of this last expression can be found by differentiating (2) with respect to t and α. Thus:

$$\partial t^*/\partial\alpha = [U_g g_c c_t - U_g g_l - U_P(F_c+I_c)c_t]/H < 0 \tag{4}$$

where H is the second-order condition for utility maximization, which is assumed to hold. Equation (4) shows an unambiguous result: a rational criminal can be deterred.[8] That is, if society decides to increase the probability of apprehension, a rational criminal will respond by reducing the time he allocates to criminal activities, and, therefore, his production of crime.

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[7] For an extensive list of models and results on this topic, see Schmidt and Witte (1984), chapter 9, table 9.7.

[8] This can be seen as follows. The second-order condition is assumed to hold; hence, H<0. Further, since the right-hand side of (2) is positive, so has to be the left-hand side, and, given that $(1-\alpha)$ is positive, so has to be the second parenthesis. The numerator of (4) is equal to the second parenthesis in the right-hand side of (2) minus a negative term; hence, this numerator is unambiguously positive. From a positive numerator and a negative denominator, the sign of (4) follows.

Consider now the impact of an increase in the severity of fines. Whether or not a criminal can be deterred depends on the sign of the expression $\partial c^*/\partial x=(\partial c^*/\partial t^*)(\partial t^*/\partial x)$. Since, as before, $(\partial c^*/\partial t^*)$ is positive, the sign of $\partial c^*/\partial x$, and, therefore, the existence of a deterrence effect depends on the sign of $(\partial t^*/\partial x)$. The sign of this last expression can be found by differentiating (2) with respect to t and x. Thus:

$$\partial t^*/\partial x = \{-\alpha c_t[U_{PP}F_x(F_c+I_c)+U_PF_{cx}]\}/H < 0 \tag{5}$$

As in the previous case, (5) shows that a rational criminal can be deterred. That is, an increase in the severity of fines unambiguously induces a rational criminal to reduce the time he allocates to criminal activities, and, therefore, his production of crime.

Finally, consider the impact of an increase in the severity of prison sentences. Whether or not a criminal can be deterred depends on the sign of the expression $\partial c^*/\partial y=(\partial c^*/\partial t^*)(\partial t^*/\partial y)$. Since, as before, $(\partial c^*/\partial t^*)$ is positive, the sign of $\partial c^*/\partial y$ depends on the sign of $(\partial t^*/\partial y)$. The sign of this last expression can be found by differentiating (2) with respect to t and y. Thus:

$$\partial t^*/\partial y = \{-\alpha c_t[U_{PP}I_y(F_c+I_c)+U_PI_{cy}]\}/H < 0 \tag{6}$$

As in the previous two cases, (6) shows that a rational criminal can be deterred. That is, an increase in the severity of prison sentences unambiguously induces a rational criminal to reduce his production of crime. The results that follow from equations (4), (5) and (6) are summarized in the following proposition:

PROPOSITION 1: *A rational criminal can be induced to reduce his criminal activities through an increase in the probability of apprehension or in the severity of the punishment. That is,* $c^*_\alpha<0$, $c^*_x<0$, *and* $c^*_y<0$.

Note that proposition 1 establishes that rational criminals can be deterred through an increase in the probability of apprehension or in the severity of the punishment, but it says nothing about how these three parameters (α,x,y) are set. In fact, they are parameters only from the criminal's point of view; from the rest of society's point of view, the probability of apprehension and the severity of the punishment are variables that have to be optimally determined. I turn now to discuss the framework within which victims consider this problem.

3- The victim's behavior

The (representative) victim's utility depends upon three arguments: the consumption of a good (q), a crime externality (E), and some other exogenous variables (Z) not explicitly considered in the model. The victim's problem is to allocate his endowment of a resource (R) between the production of the good and the production of security in order to deter crime. In other words, the victim faces a trade-off between consumption and security.

The good is produced with a single input (r), which is a portion of the resource R. Thus, the production function of the good is given by $q=q(r)$, such that $q_r>0$. In general, the resource will not be allocated entirely to the production of q; a portion of R (s) will be allocated to the production of security, in order to increase the probability of catching criminals. Hence, $\alpha=\alpha(s)$, such that $\alpha_s>0$, may be thought of as the production function of security. Since R can be allocated only to the production of the good or to the production of security, and its supply is fixed, then, $r+s=R$.

Crime has an obvious negative effect on the victim's utility. This negative effect, which will be referred to as a crime externality (E), has three components: the harm caused by the criminal, the expected cost of imprisonment, and the expected compensation the victim receives from the criminal when the latter is fined. Note that when the criminal is punished with a fine, the victim receives a monetary compensation for the harm he suffered. Yet, this compensation is not certain; it is received by the victim only if the criminal is apprehended, which occurs with probability α. (Hence, the victim's expected compensation is αF.) If the criminal is imprisoned, on the other hand, the victim bears the cost of imprisonment (K), which is increasing in the length of the prison sentence; that is, $K=K(I)$, such that $K_I>0$.[9] Therefore, the crime externality produced by the criminal and suffered by the victim is given by $E=c+\alpha K-\alpha F$.

Consider now all the elements at once. The victim is endowed with a fixed amount of a resource (R). A portion of this resource (r) is allocated to produce a commodity whose consumption the victim enjoys (q), and the rest (s) is allocated to the production of security, to increase the probability of apprehension (α) in order to deter crime. Criminal activities generate a negative impact (E) on the victim's utility. This negative impact increases with the harm caused by criminals (c) and with the expected cost of imprisonment (αK), and decreases with the expected compensation the victim receives from the criminal (αF). Finally, the victim's utility depends on some other variables (Z) not explicitly considered in the model. Thus, the victim's utility function is given by $V=V(q,E,Z)$, such that $V_q>0$, $V_{qq}<0$, $V_E<0$, and $V_{EE}>0$. The sign of V_Z, on the other hand, depends on which argument of the vector Z is considered; some arguments generate utility and some others disutility.

Recall that the victim's choice variables are the probability of apprehension and the severity of the punishment. Note that, when choosing these variables, the victim must take into account the criminal's reaction to his choice. This is due to the fact that the amount of crime the criminal will produce depends on the probability of apprehension and the severity of the punishment chosen by the victim, which depend on the amount of crime the criminal produces, which depend on the probability of apprehension and the severity of the punishment chosen by the victim, and so on. Hence, the victim can be modelled as a Stackelberg leader who maximizes his utility by considering the criminal's optimal response to each choice of the probability of apprehension and the severity of the punishment; that is, by taking into account the criminal's reaction function. Thus, the crime externality considered by the victim is of the form $E=c[\alpha(s),x,y]+\alpha(s)K[I[c(\alpha(s),x,y),y]]-\alpha(s)F[c(\alpha(s),x,y),x]$.

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[9] The cost of imprisonment should be thought of as having two components: a fixed cost (k) and a variable cost; hence, $K=k+K(I)$. Yet, since fixed costs do not affect marginal decisions, for simplicity, only variable costs are considered in the model. Hence, these costs are incurred by the victim with probability α. Note, on the other hand, that fines are assumed to be costless.

Finally, to complete the analysis of the victim's behavior, it is assumed that society's attitude toward crime is proper deterrence; that is, the punishment imposed on the criminal neither overcompensates nor undercompensates the victim.[10] It thus follows that the restriction of proper deterrence amounts to imposing a punishment such that $\alpha F = c + \alpha K$; that is, such that $E=0$. Thus, the victim's problem is to allocate his resource between consumption and security, and to choose the severity of the punishment, in order to maximize his utility, subject to the goal of proper deterrence. Formally:

$$\mathrm{Max}_{r,s,x,y}\ V = V\{q(r),\ c[\alpha(s),x,y]+\alpha(s)K[I[c(\alpha(s),x,y),y]]-\alpha(s)F[c(\alpha(s),x,y),x],\ Z\} \tag{7a}$$

$$\text{subject to:} \quad r+s=R \tag{7b}$$

$$E=0 \tag{7c}$$

$$V \geq \bar{v} \tag{7d}$$

where $\bar{v}$ is the victim's subsistence level of utility, which is exogenously determined.The first-order conditions for this problem, together with the issue of optimal punishment are analyzed below.[11]

III- OPTIMAL PUNISHMENT

Having set up the model within which the criminal and the victim make their decisions, I turn now to analyze the issue of optimal punishment. I consider first a simplified model in order to highlight an important aspect of punishing with fines.

1.- Punishment with fines

When the only form of punishing a criminal is through the imposition of a fine, the analytical framework becomes significantly simpler. In particular, the victim's problem becomes:

$$\mathrm{Max}_{s,x}\ V = V\{q(R-s),\ c[\alpha(s),x]-\alpha(s)F[c(\alpha(s),x),x],\ Z\} \tag{8a}$$

$$\text{subject to:} \quad E=0 \tag{8b}$$

$$V \geq \bar{v} \tag{8c}$$

Thus, the optimal fine (F^*) solves directly from the restriction of proper deterrence; that is:

$$F^* = (1/\alpha)c \tag{9}$$

This result makes intuitive sense. Under the constraint of proper deterrence, the criminal should be punished with a fine equal to (the monetary equivalent of) the harm he inflicts on society. Yet, since the criminal is not punished with certainty, in order to make the expected punishment equal to the harm caused by the criminal, the harm has to be corrected by the inverse of the probability of apprehension. Hence, the optimal fine is larger than the harm caused by the criminal.

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[10] A victim is overcompensated when $\alpha F > c + \alpha K$ (hence, $E<0$), and undercompensated when $\alpha F < c + \alpha K$ (hence, $E>0$).

[11] The victim's problem will be solved assuming that (7d) does not bind. Cases in which this constraint does bind will be analyzed later. To simplify the analysis further, (7b) will be eliminated by replacing r by (R-s) into the utility function. Thus, in what follows, the representative victim will be assumed to maximize such a modified utility function subject to the constraint of proper deterrence.

However, that is not the end of the story. Note that, for any given level of harm, it is always convenient for the victim to increase the fine and to decrease the probability of apprehension so as to achieve the same expected punishment at a lower cost.[12] This leads to the somewhat startling result that, given the level of harm, the victim will choose a probability of apprehension as small as possible ($\alpha \to 0$), and a fine as high as possible ($F \to \infty$).[13] However, the criminal's wealth (W) puts a constraint on the victim's ability to set the fine; for the most the victim can take from the criminal is W. Therefore, an important result is established in the following proposition:

PROPOSITION 2: *The optimal fine is equal to the criminal's wealth. That is,* $F^*=W$.

Note that in the model under consideration all criminals are identical. If this were not the case, a controversial implication of proposition 2 is given by the following proposition:

PROPOSITION 3: *Two criminals with different wealth should be punished differently for the same crime. That is, if* $c_i=c_j$, *and* $W_i \neq W_j$, *then* F_i^* $(=W_i) \neq F_j^*$ $(=W_j)$, *for all* $i \neq j$.

Note that if the criminal is punished with a fine equal to his wealth regardless of the harm he causes, the crime externality becomes $E=c-\alpha W$, which may be positive, negative, or zero, depending on the level harm (c). Put differently, the goal of proper deterrence defeats itself and degenerates into an uniform punishment. Thus, when the criminal is punished with a fine equal to his wealth, the victim's problem becomes:

$$\text{Max}_{s,x}\ V = V\{q(R\text{-}s),\ c[\alpha(s),x]-\alpha(s)W,\ Z\} \tag{10a}$$

$$\text{subject to: } V \geq \bar{v} \tag{10b}$$

and the first-order conditions for this problem are given by:

$$V_E c_\alpha \alpha_s - V_E \alpha_s W = V_q q_r \tag{11}$$

$$V_E c_x = 0 \tag{12}$$

Equation (11) shows the basic trade-off faced by the victim. An increase in the amount of the resource allocated to security increases the victim's utility in two ways. First, it increases the probability of apprehension, which reduces the amount of crime, which decreases the crime externality, which, in turn, increases the victim's utility. Such is the benefit of deterrence. Second, the increase in the probability of apprehension increases the expected compensation received by the victim, which decreases the crime externality, which, in turn, increases the victim's utility. These two benefits are obtained by society at a cost, namely, the consumption the victim forgoes when he reallocates the resource from consumption to security.

Equation (12), on the other hand, reaffirms proposition 2. This equation shows that the only effect of increasing the severity of fines is that of deterring the criminal (which decreases the crime

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[12] This is due to the fact that increasing the fine is costless, but decreasing the probability of apprehension enables the victim to reallocate resources to the production of the good.

[13] This result was originally suggested by Becker (1968), p. 183.

externality, which, in turn, increases the victim's utility), and, therefore, such a punishment should be as severe as possible.[14] In other words, the optimal fine should be as high as possible; that is, equal to the criminal's wealth.

Let an equilibrium be defined as a set $\{c,\alpha,x\}$, such that the utility of both the criminal and the victim are maximized. Further, let s^* and $x^*=\bar{x}$ be the optimal expenditure on security and the severity of the fine, respectively, that solve from the system (11)-(12). Finally, let $\alpha^*=\alpha(s^*)$ be the optimal probability of apprehension, and $c^*=c(\alpha^*,\bar{x})$ the optimal amount of crime that solves (indirectly) from (2). Therefore, an equilibrium in the model is given by the set $\{c^*,\alpha^*,\bar{x}\}$; that is, by the amount of crime, the probability of apprehension and the severity of the fine that maximize the utility of criminals and victims.

2.- Punishment with fines and imprisonment

I relax in this section the assumption that criminals can only be punished with fines. The previous simplification was useful to illustrate the fact that, when fines are used, they should be equal to the criminal's wealth. This result, for the same reasons discussed above, also applies to a more complicated model in which prison sentences are included as a complementary form of punishment. However, note that once the criminal has decided which crimes are profitable to him (given the probability of apprehension and a fine equal to his wealth), a perverse result arises: at the margin, the criminal is given no incentive to reduce the amount of crime he produces; that is, he will commit the most harmful crimes. This is due to the fact that, if the expected punishment is fixed, then the criminal will increase his production of crime as long as causing additional harm increases his utility.[15] In other words, once the criminal has decided which crimes are profitable to him, he cannot be induced to commit the least harmful of these crimes through an increase in fines.[16]

When prison sentences are introduced into the model, the victim's problem becomes:

$$\mathrm{Max}_{s,x,y}\ V = V\{q(R\text{-}s),\ c[\alpha(s),x,y]+\alpha(s)K[I[c(\alpha(s),x,y),y]]\text{-}\alpha(s)W,Z\} \qquad (13a)$$

$$\text{subject to: } V\geq\bar{v} \qquad (13b)$$

and the first-order conditions for this problem are:

$$V_E c_\alpha \alpha_s\text{-}V_E\alpha_s W+V_E\alpha K_I I_c c_\alpha \alpha_s = V_q q_r\text{-}V_E\alpha_s K \qquad (14)$$

$$V_E c_x+V_E\alpha K_I I_c c_x = 0 \qquad (15)$$

$$V_E c_y+V_E\alpha K_I I_c c_y = \text{-}V_E\alpha K_I I_y \qquad (16)$$

Equation (14) shows that prison sentences add an extra benefit and an extra cost to the victim's trade off between consumption and security, compared to the case in which fines are the only form of punishment. The additional benefit is the decrease in the expected cost of prison sentences, which arises

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[14] Let $\bar{x}$ be the maximum level of severity; that is, the x such that $F(c,\bar{x})=W$.

[15] Note that when the criminal is punished with a fine equal to his wealth, the right-hand side of (2) becomes 0. Thus, the only trade off the criminal faces is between labor and leisure.

[16] This result does not contradict the deterrence effect of fines established by (5). Instead, it establishes what happens

in the decrease in crime, which arises in the increase in the probability of apprehension, which, in turn, arises in the increase in the amount of the resource allocated to security. The additional cost, on the other hand, stems from the fact that an increase in the probability of apprehension makes victims more likely to incur in the (variable) cost of imprisonment.

Equation (15) shows that fines should still be set as high as possible; that is, equal to the criminal's wealth. The only difference with respect to the case where fines are the only way to punish criminals is that, in the present case, an increase in the severity of fines generates an extra benefit, namely, a decrease in the expected cost of imprisonment. Finally, equation (16) shows that an increase in the severity of prison sentences generates two benefits to the victim, namely, a reduction in the amount of crime, and a reduction in the expected cost of imprisonment. These are the direct and the indirect benefits of deterrence, respectively. On the other hand, an increase in the severity of prison sentences leads to longer (expected) prison terms, thus increasing the expected cost of imprisonment.

When establishing an optimal punishment structure, it should be noticed that fines are costless but prison sentences are costly. Thus, a criminal should not be imprisoned if he can monetarily compensate society for the harm he caused. Therefore:

PROPOSITION 4: *As long as fines can be imposed, prison sentences should be avoided. That is, if* $\alpha W \geq c$*, then* $F^{*}=W$.

Note, however, that prison sentences can be varied with the extent of the harm caused by the criminal, whereas the optimal fine is fixed. Since, as discussed before, a fixed punishment does not give the criminal any marginal incentive to reduce the harm he causes, then prison sentences are needed in some cases to restore the lost optimality. This follows from the fact that prison sentences force the criminal to face a marginal cost that increases with the harm he causes to society. Thus, an optimal punishment structure (p^{*}) is established in the following proposition:

PROPOSITION 5: *If the harm caused by the criminal cannot be compensated by the highest feasible fine, a prison sentence that increases with the level of harm caused by the criminal should complement a fine equal to the criminal's wealth. That is, if* $\alpha W<c$*, then* $p^{*}=W+I^{*}$*, where* $I^{*}=I(c,y^{*})$.

Finally, an equilibrium in the model is defined as a set $\{c,\alpha,x,y\}$ that maximizes the utility of both the criminal and the victim. Let s^{*}, $x^{*}=\bar{x}$, and y^{*} be the optimal values of the expenditure in security, the severity of the fine, and the severity of prison sentences, respectively, that solve from the system (14)-(16). Further, let $\alpha^{*}=\alpha(s^{*})$ be the optimal probability of apprehension, and $c^{*}=c(\alpha^{*},\bar{x},y^{*})$ the optimal amount of crime that solves (indirectly) from (2). Therefore, an equilibrium in the model is given by a set $\{c^{*},\alpha^{*},\bar{x},y^{*}\}$; that is, by the amount of crime, the probability of apprehension, and the severity of the punishment that maximize the utility of criminals and victims.

A final caveat is in order. The analysis above is valid as long as (13b) does not bind; that is, as long as V^{*} (the maximum value of the victim's utility function) is larger or equal than (the victim's

when a corner solution is obtained. The deterrence effect still holds for fines lower than the criminal's wealth.

subsistence level of utility). When, on the other hand, $V^{*} < \overline{v}$, a decent member of society, after having allocated his resources in the most efficient way, and, after having selected the optimal severity of the punishment, cannot attain his subsistence level of utility. Under these circumstances, he may overcome his moral constraints and become a criminal. This result explains, for example, robberies for necessity.

IV- CONCLUSIONS

I have introduced a model that recreates the noncooperative way in which criminals and the rest of society interact in the real world. I have argued that criminals can be modelled as rational agents that face a labor-supply problem. Thus, given the probability of apprehension and the severity of the punishment, they choose the optimal amount of crime to be committed. I have established that criminals that behave in such a way can be deterred by increases in the probability of apprehension and in the severity of the punishment.

I have further argued that the decent members of society face a trade off between consumption and security that stems from the limited amount of resources to be allocated between the production of commodities and crime deterrence. I have established that these decent members of society should set fines equal to the criminals' wealth, thus implying that criminals with different wealth should be punished differently for the same crime. Further, after having established that as long as fines can be imposed prison sentences should be avoided, I argued that when criminals cannot monetarily compensate society for the harm they caused, an optimal punishment structure calls for the imposition of a prison term in addition to the optimal fine.

In sum, I have introduced a simple noncooperative analytical framework within which issues of crime and punishment can (and should) be analyzed, and from which important policy recommendations can be derived. If modelling economic behavior is about getting closer to recreate the way agents act in the real world, perhaps this paper can be considered a step in the right direction.

REFERENCES

Becker, Gary (1968). “Crime and Punishment: An Economic Approach.” Journal of Political Economy, 76, 169

Becker, Gary (1971). *Economic Theory*. Alfred A. Knopf, Inc.

Becker, Gary (1989). “Make the Punishment Fit the Corporate Crime.” Business Week, March 13, 22.

Becker, Gary (1993). “Nobel Lecture: The Economic Way of Looking at Behavior.” Journal of Political Economy, 101, 385.

Bentham, Jeremy (1931). *Theory of Legislation*. Harcourt Brace Co., New York.

Cooter, Robert and Thomas Ulen (1988). *Law and Economics*. Scott, Foresman and Company.

Ehrlich, Isaac (1975). “The Deterrence Effect of Capital Punishment: A Question of Life and Death.” American Economic Review, 65, 397.

Ehrlich, Isaac (1977). “Capital Punishment and Deterrence: Some Further Thoughts and Additional Evidence.” Journal of Political Economy, 85, 741.

Heineke, J.M. (1978). *Economic Models of Criminal Behavior*. North Holland.

Leung, Siu (1991). “How to Make the Fine Fit the Corporate Crime? An analysis of Static and Dynamic Optimal Punishment Theories.” Journal of Public Economics, 45, 243.

Polinsky, Mitchell and Steven Shavell (1979). “The Optimal Trade-Off between the Probability and Magnitude of Fines.” American Economic Review, 69, 880.

Polinsky, Mitchell and Steven Shavell (1984). “The Optimal Use of Fines and Imprisonment.” Journal of Public Economics, 24, 89.

Polinsky, Mitchell and Steven Shavell (1992). “Enforcement Costs and the Optimal Magnitude and Probability of Fines.” Journal of Law and Economics, 35, 133.

Posner, Richard (1985). “An Economic Theory of the Criminal Law.” Columbia Law Review, 85, 1193.

Schmidt, Peter and Ann Witte (1984). *An Economic Analysis of Crime and Justice. Theory, Methods, and Applications*. Academic Press, Inc.

Wilson, James (1983). *Thinking about Crime*. Basic books, Inc., Publishers.