EMPIRICAL DISTRIBUTIONS OF STOCK RETURNS:
SCANDINAVIAN SECURITIES MARKETS, 1990-95

Felipe Aparicio
and
Javier Estrada [*] [**]

Carlos III University (Madrid, Spain)

Department of Statistics and Econometrics
and
Department of Business

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Abstract: The assumption that daily stock returns are normally distributed has long been disputed by the data. In this article we test (and clearly reject) the normality assumption using time series of daily stock returns for the four Scandinavian securities markets. More importantly, we fit to the data four alternative specifications, find empirical support for the scaled-*t* distribution, and quantify the magnitude of the error that stems from predicting the probability of obtaining returns in specified intervals by using the Normal distribution.

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[**] Corresponding author: Javier Estrada, Departamento de Empresa, Universidad Carlos III, 28903 Getafe, Madrid, Spain. TEL: (34-1) 624-9578, FAX: (34-1) 624-9875, EMAIL: <estrada@eco.uc3m.es>.

I- INTRODUCTION

The assumption that stock returns are normally distributed is widely used, implicitly or explicitly, in theoretical finance. The popularity of this assumption may perhaps stem from the fact that normally-distributed stock returns are an implication of the random walk theory of stock prices.[1]

However, from a theoretical point of view, the normality of stock returns is questionable if information does not arrive linearly to the market, or, even if it does, if investors do not react linearly to its arrival. In both cases, a leptokurtic distribution of stock returns should be observed. If information arrives to the market in infrequent clumps instead of in a linear fashion, investors would be forced to react similarly; in other words, if the distribution of information is leptokurtic, so should be the distribution of stock returns. Alternatively, if information arrives to the market linearly but investors ignore it until trends are well in place, and then react in a cumulative fashion to all the information ignored up to that point, a leptokurtic distribution of stock returns would also be obtained;[2] see Peters (1991). Thus, both arguments suggest that the distribution of stock returns should have fatter tails than expected under the Normal distribution.[3]

Empirical evidence against the normality assumption, on the other hand, has been mounting since the pioneering articles by Mandelbrot (1963), Fama (1965), and Clark (1973). Mandelbrot (1963) argued that price changes can be characterized by a stable Paretian distribution with a characteristic exponent less than 2, thus exhibiting fat tails and an infinite variance.[4] He directly tested the infinite-variance hypothesis by computing the sample variance of a large number of samples containing the returns of cotton prices, and found that the variances did not converge to any limiting value. Rather, they evolved in an erratic fashion, just as would be expected under the infinite-variance hypothesis.

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[1] This follows from the fact that, if stock prices follow a random walk, then stock returns should be *i.i.d*. And, if enough *i.i.d*. returns are collected, the central limit theorem implies that the limiting distribution of these returns should be Normal.

[2] Note that under this second theory, unlike under the first, changes in stock prices depend on past information, thus contradicting the efficient market hypothesis.

[3] The leptokurtosis in stock returns motivated the proliferation of ARCH-type models, which seek to incorporate the information contained in the tails of a distribution of stock returns into time series models. For a literature review, see Bollerslev, Chou, and Kroner (1992).

[4] When the characteristic exponent of a stable Paretian distribution is exactly equal to 2, then the Normal distribution is obtained. Hence, the latter is a special case of the former.

Fama (1965), using the thirty stocks of the Dow Jones Industrial Average, confirmed Mandelbrot's (1963) hypothesis that a stable Paretian distribution with a characteristic exponent less than 2 describes stock returns better than a Normal distribution. Thus, since stable Paretian markets tend to evolve in jumps (rather than continuously and smoothly like Gaussian markets), he concluded that stocks are riskier than indicated by the standard deviation of a Normal distribution.[5]

The infinite variance of stable Paretian distributions, and the fact that if stock returns follow this distribution then the usual statistical tools may be badly misleading, led many researchers to look for alternatives. Clark (1973) argued in favor of a finite-variance subordinated stochastic process and found that a member of this class (the lognormal distribution) fitted data on cotton futures prices better than a stable Paretian distribution.

More recently, using weekly data for the period 1928-89, Peters (1991) found that the distribution of the S&P500 stock returns exhibits negative skewness, fat tails, and a high peak. He also found that the probability of a three-sigma event under the empirical distribution of stock returns is roughly twice as large as the probability that would be expected under a Normal distribution.

We consider in this article the normality of the distribution of stock returns of the four Scandinavian securities markets, which have received less attention than other markets; see, however, Booth et al. (1992), Frennberg and Hansson (1993), and Booth et al. (1994). We start by describing the data and testing the normality assumption for daily stock returns (which we clearly reject in all markets) in part II. In part III, we introduce the statistical distributions to be fitted to the data. In part IV, we fit those specifications to the data and find strong support for the scaled-*t*, which cannot be rejected at any reasonable significance level in any market. In part V, we quantify the magnitude of the error that stems from predicting the probability of obtaining returns in specified intervals by using the Normal (rather than the scaled-*t*) distribution, and find that such specification significantly underestimates the risk of investing in Scandinavian stocks. Finally, in part VI, we summarize the main findings of our study. An appendix with figures concludes the article.

II- DATA AND TESTS OF NORMALITY

[5] In fact, in a stable Paretian market, the sample standard deviation cannot be used as a meaningful measure of risk. This is due to the fact that this statistic exhibits a very erratic behavior in the sense that, as the sample size increases, it does not converge to any given value.

The sample under consideration consists of the four Scandinavian securities markets, namely, Denmark (DEN), Finland (FIN), Norway (NOR), and Sweden (SWE). The behavior of each of these markets is summarized by the Financial Times Actuaries Indices, published daily in the *Financial Times*. All four indices are expressed in local currencies. We also analyze the distribution a European index (EUR) and a World index (WOR).[6] The sample period extends from January 1, 1990, through December 31, 1994; the behavior of the six indices considered during this period is shown in part A1 of the appendix.

The series analyzed for each market is the series of returns, where returns are defined as $R_t = 100[\ln(I_t) - \ln(I_{t-1})]$, where R_t and I_t are the return and the index in day t, respectively. Table 1 below summarizes some relevant information about the empirical distributions of stock returns under consideration. The statistics reported are the mean, standard deviation, minimum and maximum return during the sample period, coefficients of skewness and kurtosis, and standardized coefficients of skewness and kurtosis.

TABLE 1: Sample Moments of the Distributions

Market	Mean	SD	Min	Max	Skw	SSkw	Krt	SKrt
DEN	-0.0030	0.8232	-5.8997	4.9312	-0.0936	-1.3803	5.9536	43.8846
FIN	0.0377	1.2440	-5.4757	5.2919	0.2328	3.4316	2.1259	15.6700
NOR	0.0069	1.3307	-8.8584	10.8018	0.3662	5.3988	8.8477	65.2171
SWE	0.0282	1.2504	-6.8453	9.3145	0.5003	7.3755	5.7016	42.0272
EUR	0.0105	0.6908	-6.6946	4.4285	-0.7935	-11.6979	10.8207	79.7604
WOR	-0.0023	0.6535	-4.2796	3.9281	-0.0142	-0.2088	5.5388	40.8272

Mean returns, standard deviations (SD), minimum returns (Min), and maximum returns (Max) are all expressed in percentages. Skw = Skewness = m_3/s^3 and Krt = Kurtosis = m_4/s^4-3, where m_i and s are the ith central sample moment and the sample standard deviation of each distribution, respectively; both coefficients are computed with a finite-sample adjustment. SSkw = Standardized skewness and SKrt = Standardized kurtosis. Sample size = 1,304 for all markets.

Preliminary evidence on the normality of each distribution of stock returns under consideration can be gathered from the last four columns of Table 1; that is, by considering the third and fourth central moments of each distribution. Under the assumption of normality, the coefficients of skewness and excess kurtosis are asymptotically distributed as $N(0,6/T)$ and $N(0,24/T)$, respectively, where T is the sample size. Hence, values of these standardized coefficients (SSkw and SKrt, respectively) outside the range [-1.96,1.96] indicate, at the 5% significance level, significant departures from normality.

Table 1 shows that not all the distributions are negatively skewed, as daily data from the U.S. typically shows. Three distributions (DEN, EUR, WOR) display negative skewness and the

[6] EUR is an equally-weighted index of thirteen European securities markets (Austria, Belgium, Denmark, England, Finland, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, and Switzerland), and WOR is an index computed on the basis of 2,249 stocks worldwide.

other three (FIN, NOR, SWE) display positive skewness, although the observed asymmetry is not significant in two (DEN, WOR) of the six markets considered. In addition, the last column of Table 1 shows that *all* six distributions are clearly leptokurtic, thus exhibiting fat tails (and high peaks). The departures from normality detected by the coefficients of standardized skewness and kurtosis can also be seen in the histograms displayed in part A2 of the appendix, where Normal distributions generated by the sample mean and standard deviation of each market are shown together with the observed histograms.

The coefficients of standardized skewness and kurtosis provide strong evidence about departures from normality, but more formal conclusions can be reached through the tests of normality reported below in Table 2. Although the three tests use different information,[7] the results of all three point in the same direction, namely, to the outright rejection of the normality assumption in Scandinavian securities markets.

TABLE 2: Tests of Normality

	Goodness of Fit			Kolmogorov-Smirnov		Jarque-Bera	
Market	Statistic	df	*p*-value	Statistic	*p*-value	Statistic	*p*-value
DEN	118.647	7	0.0000	0.0716	3.2e-06	1,927.768	0.0000
FIN	123.383	13	0.0000	0.0612	1.2e-04	257.335	0.0000
NOR	79.617	8	5.8e-14	0.0594	2.0e-04	4,282.456	0.0000
SWE	128.317	9	0.0000	0.0626	7.3e-05	1,820.680	0.0000
EUR	111.082	6	0.0000	0.0764	4.8e-07	6,498.599	0.0000
WOR	89.716	9	1.9e-15	0.0629	6.6e-05	1,666.898	0.0000

The goodness-of-fit test follows a Chi-square distribution with the degrees of freedom (df) indicated above. The asymptotic critical value for the Kolmogorov-Smirnov test at the 5% significance level is 0.038. The Jarque-Bera test is asymptotically distributed as a Chi-square with 2 degrees of freedom; its critical value at the 5% significance level is 5.99.

The results in Table 2 should come as no surprise; virtually all studies that use daily data also reject the normality of stock returns. In order to test what specification describes the data better than the Normal distribution, we consider in the next part four alternative distributions that allow for the characteristics of the data discussed above; we then fit such distributions to the data in the following part.

III- ALTERNATIVE DISTRIBUTIONS FOR STOCK RETURNS

The results reported and discussed above indicate that the six markets we consider are characterized by somewhat skewed distributions with fat tails and high peaks. As a result, we

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[7] The Jarque-Bera test uses information on the third and fourth moments of a distribution. The goodness-of-fit test divides a distribution in intervals and compares, across intervals, the observed returns with those that would be expected if the underlying distribution were Normal. Finally, the Kolmogorov-Smirnov test computes the maximum distance between an observed cumulative distribution and the Normal cumulative distribution.

consider in this part three specifications that allow for leptokurtosis and one that also allows for skewness.

The Logistic Distribution. This distribution, which is very similar to the Normal but has thicker tails, was first suggested as appropriate to model stock returns by Smith (1981), and subsequently tested by Gray and French (1990) and Peiró (1994). The density function of the logistic distribution can be written as

$$f(x) = \frac{\exp\left(\dfrac{x-\mu}{\alpha}\right)}{\alpha\left[1+\exp\left(\dfrac{x-\mu}{\alpha}\right)\right]^2} \quad , \tag{1}$$

where μ ($-\infty<\mu<\infty$) is a location parameter and α ($\alpha>0$) is a dispersion (or scale) parameter. If R_t follows a logistic distribution, then $E(R_t)=\mu$ and $Var(R_t)=\sigma^2=(\pi^2/3)\alpha^2$.

The Scaled-*t* Distribution. Praetz (1972), Blattberg and Gonedes (1974), Gray and French (1990), and Peiró (1994) have reported that this specification fits stock returns better than many competing alternatives. The density function of the scaled-*t* distribution is given by

$$f(x) = \frac{\Gamma\left(\dfrac{v+1}{2}\right)}{\Gamma(v/2)\sqrt{\pi(v-2)\sigma^2}} \cdot \left[1+\frac{(x-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \quad , \tag{2}$$

where $\Gamma(\bullet)$ represents the gamma function, μ ($-\infty<\mu<\infty$) and σ^2 ($\sigma^2>0$) represent a location and a dispersion parameter, respectively, and v ($v>0$) is a degrees of freedom parameter. If R_t follows a scaled-*t* distribution and $v>2$, then $E(R_t)=\mu$ and $Var(R_t)=\sigma^2$.

The Exponential Power Distribution. Hsu (1982) and Gray and French (1990) have argued that this specification, which displays fat tails that shrink at an exponential rate and a high peak, provides a reasonably-good fit to stock return data. The density function of the exponential power distribution is given by

$$f(x) = \frac{\exp\left[-\dfrac{1}{2}\left|\dfrac{x-\mu}{\alpha}\right|^{\left(\frac{2}{1+\beta}\right)}\right]}{2^{\left(\frac{3+\beta}{2}\right)}\,\alpha\,\Gamma\left(\dfrac{3+\beta}{2}\right)} \quad , \tag{3}$$

where μ ($-\infty<\mu<\infty$), α ($\alpha>0$), and β ($-1<\beta\leq1$) are a location, a dispersion, and a shape parameter, respectively. This last parameter, in particular, measures the kurtosis of the distribution. More precisely, $\beta<0$ implies a platykurtic distribution, the Normal distribution is obtained when $\beta=0$, and fat tails and a high peak are obtained when $0<\beta\leq1$, with the thickness of the tails increasing in β.[8] If R_t follows an exponential power distribution, then $E(R_t)=\mu$ and $Var(R_t) = \sigma^2 = 2^{(1+\beta)} \cdot \frac{\Gamma[3(1+\beta)/2]}{\Gamma[(1+\beta)/2]} \alpha^2$.

Mixtures of Two Normal Distributions. An alternative to assuming that stock returns are generated from a single distribution is to assume that they are generated by a mixture of distributions. Press (1967) argued that stock returns may be generated by the interaction of a continuous diffusion (Brownian motion) process and a discontinuous jump (Poison) process, where the former captures continuous changes in stock prices and the second models large informational shocks. Kon (1984) also argues in favor of (and finds empirical support for) this specification. The density function of a mixture of two Normal distributions is given by

$$\begin{aligned} f(x) &= \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{-\left[\frac{(x-\mu_1)^2}{2\sigma_1^2}\right]} \quad , \quad \textit{with probability } \lambda \\ &= \frac{1}{\sqrt{2\pi\sigma_2^2}} \cdot e^{-\left[\frac{(x-\mu_2)^2}{2\sigma_2^2}\right]} \quad , \quad \textit{with probability } (1-\lambda) \,, \end{aligned} \tag{4}$$

where μ_i ($-\infty<\mu_i<\infty$) and σ_i^2 ($\sigma_i^2>0$) are location and dispersion parameters, respectively. This mixture implies that stock returns are drawn from a Normal distribution with mean μ_1 and standard deviation σ_1 with probability λ, and from a Normal distribution with mean μ_2 and standard deviation σ_2 with probability (1-λ). If R_t follows such mixture of distributions, then $E(R_t)=\mu=\lambda\mu_1+(1-\lambda)\mu_2$ and $Var(R_t)=\sigma^2=\lambda\{(\mu_1-\mu)^2+\sigma_1^2\}+(1-\lambda)\{(\mu_2-\mu)]^2+\sigma_2^2\}$. Of all the specifications we consider, this mixture of two Normal distributions is the only one that allows for skewness in the data.[9]

IV- ESTIMATION OF PARAMETERS AND GOODNESS-OF-FIT TESTS

We report in Table 3 below the (maximum-likelihood) estimations that result from fitting the theoretical distributions described in the previous part to the series of daily stock returns of the six markets under consideration.

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[8] For β=1, the double-exponential distribution is obtained.

[9] The coefficient of skewness (k_3) that follows from the mixture of two Normal distributions is given by

$$k_3 = \frac{\lambda\left[(\mu_1-\mu)^2+3(\mu_1-\mu)\sigma_1^2\right]+(1-\lambda)\left[(\mu_2-\mu)^2+3(\mu_2-\mu)\sigma_2^2\right]}{\left\{\lambda\left[(\mu_1-\mu)^2+\sigma_1^2\right]+(1-\lambda)\left[(\mu_2-\mu)^2+\sigma_2^2\right]\right\}^{3/2}} .$$

TABLE 3: Parameter Estimates

		DEN	FIN	NOR	SWE	EUR	WOR
N:	μ:	-0.00301	0.03770	0.00689	0.02819	0.01052	-0.00233
	σ:	0.82253	1.24302	1.32966	1.24940	0.69032	0.65305
L:	μ:	-0.00153	0.01425	-0.00854	0.01305	0.02457	-0.00399
	α:	0.41885	0.66172	0.67926	0.64014	0.34677	0.33609
S-*t*:	μ:	-0.00072	0.00629	-0.01467	0.00929	0.03002	-0.00481
	σ:	0.86401	1.28016	1.30499	1.27511	0.69273	0.66291
	ν:	3.34660	4.26650	4.25530	3.69760	3.63030	3.83590
EP:	μ:	-0.00000	-0.00000	-0.00000	0.00000	0.02780	-0.00319
	α:	0.28459	0.51754	0.52770	0.43700	0.26122	0.27211
	β:	0.99999	0.86572	0.88132	1.00000	0.91345	0.84280
MN:	μ_1:	-0.01645	-0.05409	0.00135	-0.00012	-0.21775	0.01230
	σ_1:	1.45330	0.82717	1.06820	0.91374	1.51840	1.20920
	μ_2:	0.00578	0.23524	0.10965	0.22347	0.03788	-0.00536
	σ_2:	0.54173	1.82780	3.66050	2.55450	0.50032	0.46024
	λ:	0.21064	0.68276	0.94890	0.87338	0.10703	0.17163
	k_3:	-0.00157	0.16450	0.03141	0.10426	-0.14151	0.01247

N=Normal. L=Logistic. S-*t*=Scaled-*t*. EP=Exponential Power. MN=Mixture of two Normal distributions. The coefficient of skewness (k_3) follows from the expression in footnote 9.

At least two things are worth noting from Table 3. First, recall that the Normal distribution and the *t*-distribution tend to converge as the degrees of freedom of the latter increase. However, the table shows that the estimated degrees of freedom of the scaled-*t* distributions are very small in all markets (between 3 and 4.5), thus indicating that these empirical distributions diverge significantly from the Normal, particularly in the tails. Second, recall that the parameter β of the exponential power distribution is a measure of its kurtosis, that for β=0 the Normal distribution is obtained, and that β is increasing in the thickness of the tails (with an upper bound at β=1). Table 3 shows that β is larger than .8 in all markets and larger than .9 in three markets (DEN, SWE, EUR). This provides additional evidence of departures from normality, and, in particular, of the thickness of the tails of the empirical distributions under consideration.

In order to compare the relative fit of the theoretical distributions considered, we performed goodness-of-fit tests. To that purpose, we divided the range of returns into 20 equal, non-overlapping intervals contained in the range [-10%,10%]. The results of these tests are shown below in Table 4.

TABLE 4: Goodness-of-fit Tests

	N	*p*-value	L	*p*-value	S-*t*	*p*-value	EP	*p*-value	MN	*p*-value
DEN	1.3e06	0.0000	233.2	0.0000	14.7	0.5467	29.4	0.0214	30.7	0.0061
FIN	577.2000	0.0000	43.6	0.0004	16.8	0.3987	19.7	0.2340	16.6	0.2781
NOR	2.2e10	0.0000	749.5	0.0000	21.5	0.1601	111.1	2.2e-16	19.1	0.1612
SWE	2.2e09	0.0000	1,351.9	0.0000	20.7	0.1903	94.8	3.2e-13	40.7	0.0002
EUR	4.9e14	0.0000	28,777.7	0.0000	11.8	0.7576	1,195.6	0.0000	121.7	0.0000
WOR	1.7e06	0.0000	189.2	0.0000	6.4	0.9832	40.5	0.0007	18.2	0.1978

N = Normal; L = Logistic; S-*t* = Scaled-*t*; EP = Exponential Power; MN = Mixture of two Normal distributions. The goodness of fit test follows a Chi-square distribution with *p-k-1* degrees of freedom, where *p* is the number of intervals and *k* is the number of parameters estimated for each distribution. Degrees of freedom are 17 for N and L, 16 for S-*t* and EP, and 14 for MN.

This table shows that, as expected, the Normal distribution provides the worst fit among all the specifications considered, being clearly rejected in all markets. The logistic distribution does not fit much better than the Normal, and is also rejected at any reasonable significance level in all markets. The exponential power distribution also provides a very poor fit, not being rejected (at the 5% significance level) in only one market (FIN).

Table 4 shows partial support for a mixture of two Normal distributions, which cannot be rejected (at the 5% significance level) in three markets (FIN, NOR, WOR).[10] Finally, the scaled-*t* is the distribution that provides the best fit, not being rejected in any market at any reasonable significance level. This overall support for the scaled-*t* distribution confirms results reported by Peiró (1994) for different markets and sample periods.

V- ERRORS IMPLIED BY THE NORMALITY ASSUMPTION

The tests of normality reported in part II establish that the distributions of stock returns of the six markets analyzed exhibit significant departures from normality. In addition, the goodness-of-fit tests reported in part IV establish that a scaled-*t* distribution cannot be rejected at any reasonable significance level in any market. In this part, we quantify the error that can be made by predicting the probability of obtaining returns in specified intervals by assuming an underlying Normal distribution, thus ignoring the additional information provided by the scaled-*t* distribution.

In order to assess this error, we first estimate the (unconditional) probability of obtaining returns in a given interval using the parameters previously estimated (and reported in Table 3) for the Normal distribution; we subsequently repeat this process for the twelve intervals we consider. Then we estimate the same probability using the parameters previously estimated (and reported in Table 3) for the scaled-*t* distribution for the same twelve intervals. We finally compare, one by one, the probability of obtaining returns in each interval. The results of our estimations are reported below in Table 5.

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[10] Kon (1984) fits mixtures of up to five Normal distributions to the thirty stock of the Dow Jones Industrial Average. He finds that a mixture of four Normal distributions best fits seven stocks, a mixture of three Normal distributions best fits eleven stocks, and a mixture of two Normal distributions best fits the remaining twelve stocks.

TABLE 5: Probabilities of Obtaining Returns in Specified Intervals

		$[\bar{x}, \bar{x}+s]$	$[\bar{x}+s, \bar{x}+2s]$	$[\bar{x}+2s, \bar{x}+3s]$	$[\bar{x}+3s, \bar{x}+4s]$	$[\bar{x}+4s, \bar{x}+5s]$	$[\bar{x}+5s, \bar{x}+6s]$
DEN:	N:	0.34144	0.13585	0.02136	0.00131	0.00003	2.83e-7
	S-*t*:	0.39052	0.08591	0.01703	0.00472	0.00170	0.00074
FIN:	N:	0.34144	0.13585	0.02136	0.00131	0.00003	2.83e-7
	S-*t*:	0.36763	0.09442	0.01837	0.00448	0.00140	0.00053
NOR	N:	0.34144	0.13585	0.02136	0.00131	0.00003	2.83e-7
	S-*t*:	0.37961	0.08966	0.01630	0.00385	0.00118	0.00044
SWE:	N:	0.34144	0.13580	0.02136	0.00131	0.00003	2.83e-7
	S-*t*:	0.38109	0.08683	0.01676	0.00438	0.00149	0.00061
EUR:	N:	0.34144	0.13586	0.02136	0.00131	0.00003	2.83e-7
	S-*t*:	0.40060	0.08958	0.01670	0.00430	0.00146	0.00060
WOR:	N:	0.34144	0.13585	0.02136	0.00131	0.00003	2.83e-7
	S-*t*:	0.38354	0.09017	0.01723	0.00438	0.00145	0.00058

N = Normal; S-*t* = Scaled-*t*. Each number shows the probability of obtaining a return in the specified interval under the specified distribution. Each distribution is centered around its sample mean ($\bar{x}$), and the length of each interval is equal to each distribution's sample standard deviation (s), both taken from Table 1. Both N and S-*t* are symmetric distributions; hence, predictions are reported only for one half of each distribution.

Table 5 shows that the probability of obtaining returns in any given interval is very different depending on whether the Normal or the scaled-*t* are assumed as the underlying distribution. Note that the probability of obtaining returns in the interval $[\bar{x}, \bar{x}+s]$ is higher under the scaled-*t* distribution in all markets, that the opposite is the case in the intervals $[\bar{x}+s, \bar{x}+2s]$ and $[\bar{x}+2s, \bar{x}+3s]$, and that the situation reverses again for the interval $[\bar{x}+3s, \bar{x}+4s]$ and all intervals beyond; that is, the probability of obtaining returns three or more standard deviations from the mean is higher under the scaled-*t* distribution in all markets. Furthermore, note that the difference between the probability predicted by each distribution increases dramatically as we move away from the mean. The probability of obtaining a return between three and four, four and five, and five and six standard deviations away from the mean is, on average, 3.3, 48.2, and 2,061.2 times respectively higher under the scaled-*t* distribution.

The previous results show that investors that predict the probability of obtaining returns by assuming an underlying Normal distribution may significantly underestimate the risk of investing in Scandinavian securities markets. This underestimation, as the numbers above show, is particularly severe in the tails of the distribution; that is, when predicting the probability of large (positive or negative) returns. Returns three and more standard deviations away from the mean, which occur with a negligible probability under the Normal distribution, occur *much* more frequently under a scaled-*t* distribution.

VI- CONCLUSIONS

The evidence against the assumption that daily stock returns are normally distributed has been mounting for over thirty years. Most of the empirical evidence analyzes U.S. data, although some recent studies have considered European markets. In this article, we used data from the first half of the decade to test the hypothesis that stock returns in the Scandinavian markets are normally distributed.

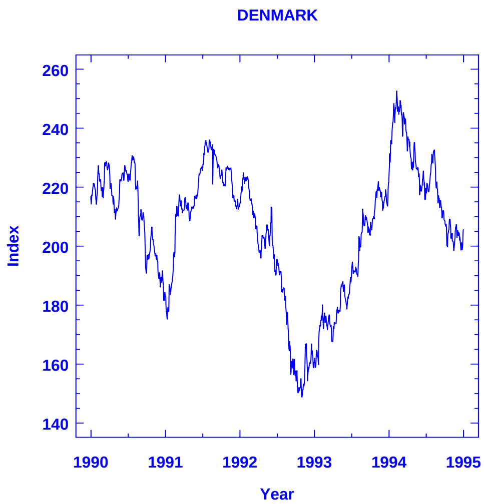
We started by describing the data and testing the hypothesis of normality. Not surprisingly, the distributions of daily stock returns analyzed show fat tails and high peaks, as well as skewness in different directions. These results are fully consistent with those found for many other markets and reported in many other studies.

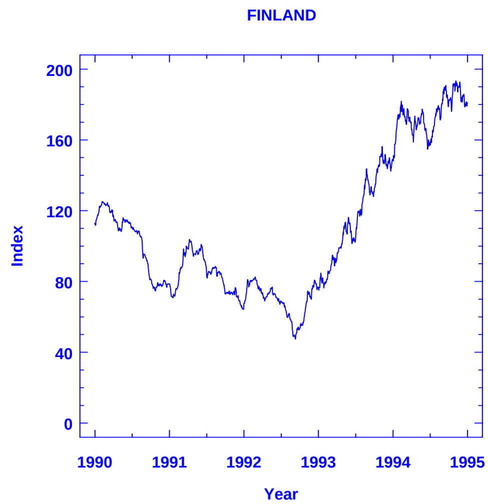
We then fitted the Normal distribution to the data, as well as four alternative specifications, all of which exhibit fat tails and one that also allows for skewness. Predictably, we found that the Normal distribution exhibited the worst fit in all markets. We also found that neither the logistic nor the exponential power distributions provide a good fit to the empirical distributions of Scandinavian stock returns. However, we found partial support for a mixture of two normal distributions, which cannot be rejected in three markets, and strong support for the scaled-*t* distribution, which cannot be rejected in any market at any reasonable significance level.

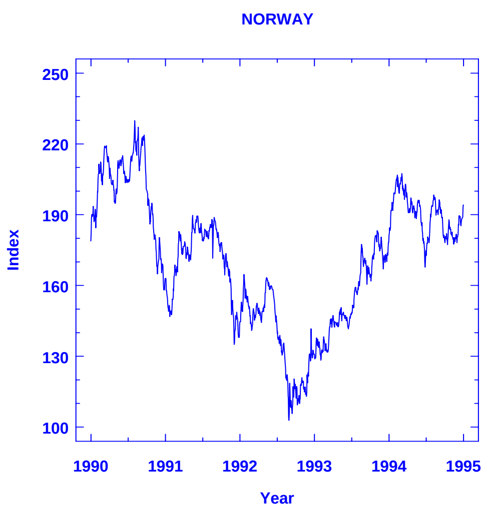
Having established that the scaled-*t* (rather than the Normal) distribution properly describes daily returns in Scandinavian securities markets, we attempted to quantify the error that can be made by predicting the probability of obtaining returns in specified intervals by using the Normal (instead of the scaled-*t*) distribution. We showed that such errors can be very large, particularly in the tails, and therefore conclude that booms and crashes in Scandinavian securities markets are much more likely to occur than what a Normal distribution would predict.

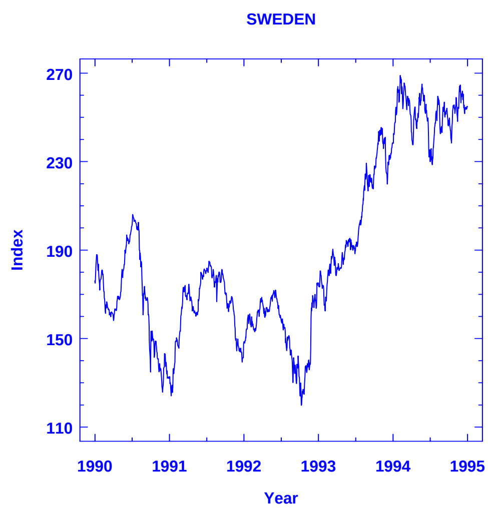
APPENDIX

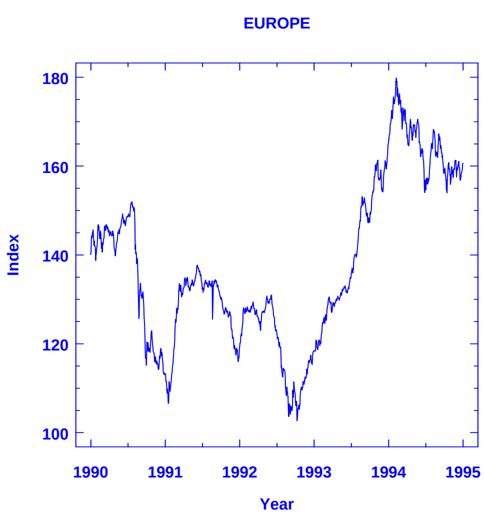
A1- MARKET BEHAVIOR

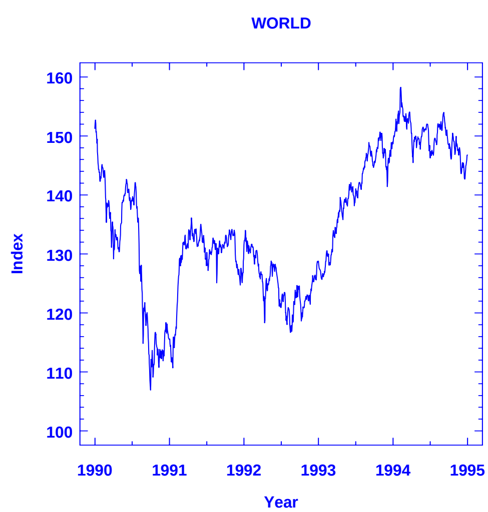




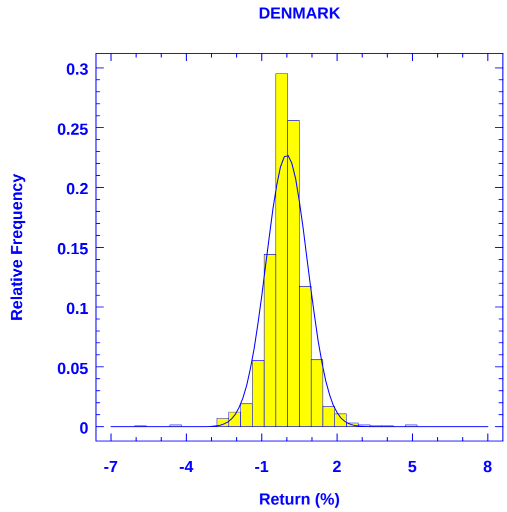


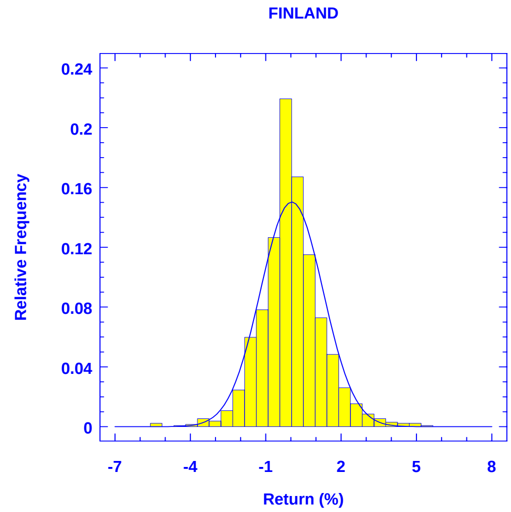


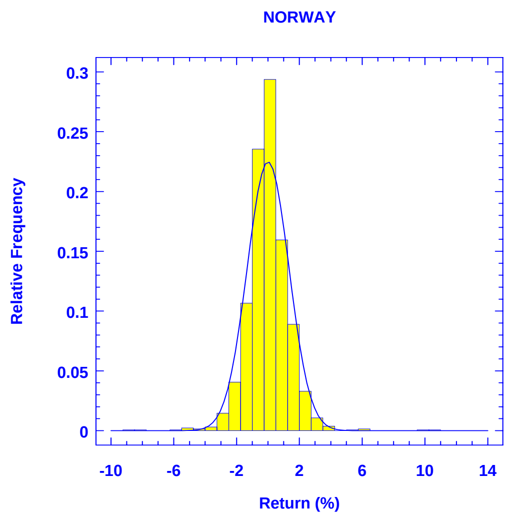


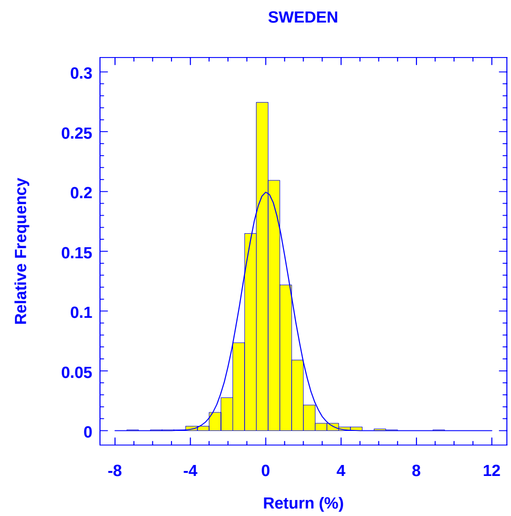


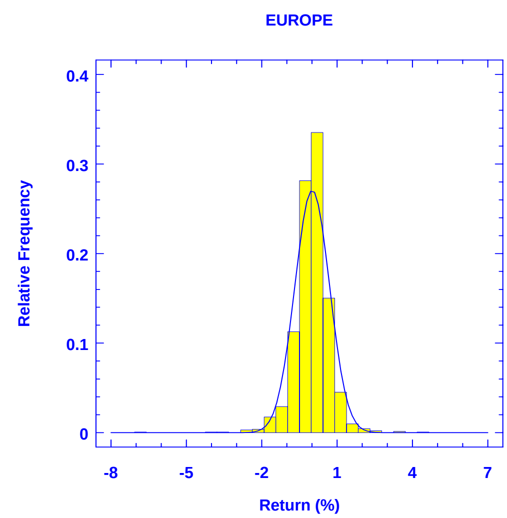
A2- HISTOGRAMS

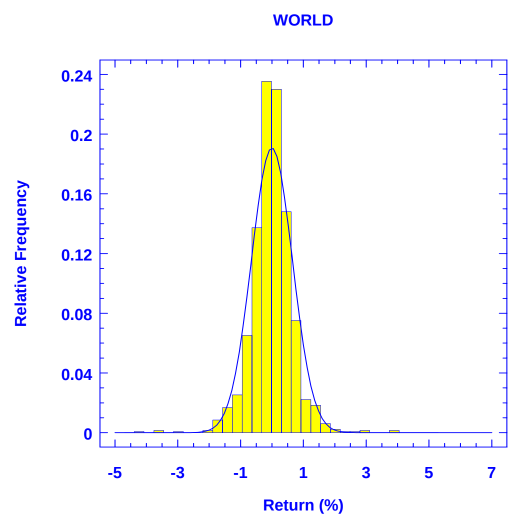












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