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Mean-Semivariance Behavior: A Note

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Abstract

The most widely-used measure of an asset's risk, beta, stems from an equilibrium in which investors display mean-variance behavior. This criterion assumes that portfolio risk is measured by the variance (or standard deviation) of returns. However, the semivariance is a more plausible measure of risk (as Markowitz himself admits) and is backed by theoretical, empirical, and practical considerations. It can also be used to implement an alternative behavioral criterion, mean-semivariance behavior, that is almost perfectly correlated to expected utility and to the utility of mean compound return.

Key words: Expected utility, Downside risk, Semideviation, Mean-Variance Behavior
JEL classification: G14, G15

1. INTRODUCTION

Risk is a slippery concept and its proper definition, critical for academics and practitioners alike, is under continuous evolution. Though not free from controversy, the most widely-accepted definition of an asset's risk in a diversified portfolio is the asset's beta. This definition of risk stems from an equilibrium in which investors display mean-variance behavior (MVB); that is, a model in which investors choose their optimal portfolio by maximizing a utility function that depends on the mean and variance of the portfolio returns.

Levy and Markowitz (1979) defended MVB as an approximately-correct criterion, in the sense that it yields a level of utility highly correlated to an investor's expected utility. However, when receiving his Nobel prize, Markowitz (1991) stated that "... it can further help evaluate the adequacy of mean and variance, *or alternate practical measures*, as criteria." (Emphasis added.) In addition, he stated that "[p]erhaps... some other measure of portfolio risk will serve in a two parameter analysis ... *Semivariance seems more plausible than variance as a measure of risk, since it is concerned only with adverse deviations*." (Emphasis added.)

In this article, I follow Markowitz's suggestions and evaluate the plausibility of semivariance as a measure of risk, and of mean-semivariance behavior (MSB) as a behavioral model. More precisely, following Levy and Markowitz (1979), I evaluate whether MSB is an approximately-correct criterion, in the sense that it yields a level of utility highly correlated to an investor's expected utility. Furthermore, I analyze the relationship between MSB and an alternative behavioral criterion, namely, the maximization of expected compound return.

2. APPROXIMATING EXPECTED UTILITY

It is a well-established result in modern financial theory that MVB is exactly consistent with expected utility maximization (EUM) under either one of two conditions: 1) That the investor's utility function is quadratic; or 2) that the returns of the investor's portfolio are jointly normally distributed. However, the plausibility of a quadratic utility function is questioned by the fact that it implies than an investor's absolute risk

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aversion is increasing in his wealth, the opposite of what should be reasonably expected. Furthermore, the normality of returns is questioned by loads of data that display skewness, kurtosis, or both.

A Mean-Variance Approximation to Expected Utility

Not ready to give up on what would eventually become the standard behavioral criterion in modern financial theory, Levy and Markowitz (1979) moved to defend MVB from a different perspective: They asked whether an investor choosing a portfolio on the basis of mean and variance would *almost* maximize his expected utility. In other words, they did not question the implausibility of the conditions that make MVB *exactly* consistent with EUM. Rather, they asked whether the simpler choice based on mean and variance would yield a level of expected utility *almost* equal to that obtained by a much more complicated direct maximization of the expected utility function.

Table 1 reproduces Table 1 in Markowitz (1991), which is in turn taken from Levy and Markowitz (1979). The Table shows the correlation coefficients between an investor's expected utility (*EU*) given by

$$EU = (1/T) \cdot \sum_{t=1}^{T} U(R_t) \quad (1)$$

and his approximate expected utility based on MVB (AEU_{MVB}) given by

$$AEU_{MVB} = U(\mu) + (1/2) \cdot \sigma^2 \cdot U''(\mu) \quad (2)$$

where U denotes the investor's utility function, R and T denote returns and the number of returns in the sample, and μ and σ^2 denote the mean and variance of returns.

As Table 1 clearly shows, the correlation between expected utility and approximate expected utility is nearly perfect for three different utility functions and several parameter values. On the basis of this table, Levy and Markowitz (1979) and Markowitz (1991) conclude that MVB is a good approximation to EUM.

Table 1. Levy and Markowitz (1979) and Markowitz (1991) Approximation

Utility Function	Annual Returns 149 Mutual Funds	Annual Returns 97 Stocks	Monthly Returns 97 Stocks	Random Portfolios 5/6 Stocks
$U = \ln(1+R)$	0.997	0.880	0.995	0.998
$U = (1+R)^a$				
$a = 0.1$	0.998	0.895	0.996	0.998
$a = 0.3$	0.999	0.932	0.998	0.999
$a = 0.5$	0.999	0.968	0.999	0.999
$a = 0.7$	0.999	0.991	0.999	0.999
$a = 0.9$	0.999	0.999	0.999	0.999
$U = -e^{-b(1+R)}$				
$b = 0.1$	0.999	0.999	0.999	0.999
$b = 0.5$	0.999	0.961	0.999	0.999
$b = 1$	0.997	0.850	0.997	0.998
$b = 3$	0.949	0.850	0.976	0.958
$b = 5$	0.855	0.863	0.961	0.919
$b = 10$	0.449	0.659	0.899	0.768

Source: Markowitz (1991), Table 1. All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (2).

A Mean-Semivariance Approximation to Expected Utility

An alternative measure of risk that has received recent support from both academics and practitioners, and that Markowitz supported from the start, is the downside standard deviation of returns, or semideviation for short, which for any benchmark return B can be denoted as Σ_B (or simply as Σ if there is no ambiguity about the benchmark) and is given by

$$\Sigma_B \equiv \Sigma = \sqrt{E\{\text{Min}[(R-B), 0]^2\}} \quad (3)$$

As can be noticed by simple inspection of (3), the semideviation gives a positive weight only to the deviations below the benchmark; that is, returns below B increase Σ, but returns above B do not. Essentially, the semideviation defines risk as volatility below the benchmark. Throughout this article the benchmark return used is the arithmetic mean of the relevant distribution of returns.

An approximation to expected utility based on mean and semivariance can be obtained by replacing the variance of returns in (2) by two times the semivariance of returns. Hence, an investor's approximate expected utility based on MSB (AEU_{MSB}) can be calculated with the expression

$$AEU_{MSB} = U(\mu) + (1/2)\cdot(2\Sigma^2)\cdot U''(\mu) = U(\mu) + \Sigma^2\cdot U''(\mu) \tag{4}$$

Note that f the underlying distribution of returns is symmetric, then $2\cdot\Sigma^2 = \sigma^2$. In that case, both (2) and (4) would yield exactly the same level of (approximate) expected utility. However, when the underlying distribution is skewed, then $2\cdot\Sigma^2 \neq \sigma^2$, and (2) and (4) would yield different levels of (approximate) expected utility. More precisely, in the presence of negative skewness, $2\cdot\Sigma^2>\sigma^2$ and $AEU_{MSB}<AEU_{MVB}$; and in the presence of positive skewness, $2\cdot\Sigma^2<\sigma^2$ and $AEU_{MSB}>AEU_{MVB}$.

Table 2. Summary Statistics: Monthly Stock Returns

Developed Markets					Emerging Markets				
Market	μ	σ	Σ	Start	Market	μ	σ	Σ	Start
Australia	0.93	7.21	5.26	Jan-70	Argentina	3.16	17.94	10.02	Jan-88
Austria	0.91	6.08	4.00	Jan-70	Brazil	3.22	17.78	11.85	Jan-88
Belgium	1.29	5.48	3.77	Jan-70	Chile	1.87	7.54	5.20	Jan-88
Canada	1.00	5.55	4.05	Jan-70	China	-0.60	12.93	7.88	Jan-93
Denmark	1.26	5.42	3.71	Jan-70	Colombia	-0.14	9.47	6.46	Jan-93
Finland	1.94	8.60	5.7	Jan-88	Czech Rep.	0.26	9.48	6.81	Jan-95
France	1.25	6.60	4.69	Jan-70	Egypt	1.23	8.79	5.14	Jan-95
Germany	1.14	5.90	4.22	Jan-70	Greece	1.87	11.46	6.65	Jan-88
Hong Kong	2.04	11.31	7.66	Jan-70	Hungary	2.04	12.35	8.5	Jan-95
Ireland	1.07	5.73	3.96	Jan-88	India	0.65	8.95	6.04	Jan-93
Italy	0.90	7.57	5.13	Jan-70	Indonesia	1.34	17.31	9.89	Jan-88
Japan	1.22	6.63	4.56	Jan-70	Israel	1.14	7.51	5.31	Jan-93
Netherlands	1.38	5.14	3.73	Jan-70	Jordan	-0.02	4.49	3.12	Jan-88
New Zealand	0.29	7.00	4.73	Jan-88	Korea	0.67	12.41	7.54	Jan-88
Norway	1.21	7.74	5.47	Jan-70	Malaysia	0.97	10.26	6.97	Jan-88
Portugal	0.61	6.74	4.47	Jan-88	Mexico	2.45	10.56	7.79	Jan-88
Singapore	1.15	8.52	5.96	Jan-88	Morocco	1.00	4.78	3.32	Jan-93
Spain	1.03	6.52	4.58	Jan-70	Pakistan	0.16	11.96	8.13	Jan-93
Sweden	1.50	6.55	4.6	Jan-70	Peru	0.89	9.85	6.79	Jan-93
Switzerland	1.24	5.49	3.91	Jan-70	Philippines	0.87	10.45	7.05	Jan-88
UK	1.24	6.87	4.52	Jan-70	Poland	3.19	18.52	10.48	Jan-93
USA	1.08	4.44	3.22	Jan-70	Russia	3.46	23.61	16.14	Jan-95
Average	1.17	6.69	4.63	N/A	S. Africa	1.03	8.17	5.95	Jan-93
					Sri Lanka	-0.40	9.47	6.65	Jan-93
					Taiwan	1.23	12.34	8.14	Jan-88
					Thailand	0.67	12.64	8.81	Jan-88
					Turkey	2.54	18.39	11.31	Jan-88
					Venezuela	1.52	15.19	10.70	Jan-93
					Average	1.30	11.95	7.81	N/A

μ: Mean return; σ: Standard deviation; Σ: Semideviation. All numbers in %. Data are through December 2000.

The Evidence

The data used to evaluate the relationship between expected utility, the mean-variance approximation, and the mean-semivariance approximation is the entire MSCI database of both developed and emerging markets available at the end of the year 2000, which contains monthly data on 22 developed markets and 28 emerging markets for varied sample periods. Summary statistics for all these markets are reported in Table 2. The utility functions and parameter values used in the evaluation are the same as those used by Levy and Markowitz (1979) and can be seen in Table 1.

Table 3 reports the correlation coefficients between expected utility and the two approximations to expected utility given by (2) and (4). As the Table shows, the mean-variance approximation is in fact very good, with correlation coefficients above 0.9 in all cases (with two exceptions for the highest value of the *b* coefficient of the negative exponential utility function). The Table also shows that the mean-semivariance approximation is just as good. In fact, on average, MSB outperforms MVB in emerging markets and in the combined sample of all markets.

Essentially, three important points follow from Table 3. First, an investor that chose portfolios on the basis of mean and semivariance would in fact almost maximize his expected utility. Second, the approximation to expected utility provided by MSB is on average better than that provided by MVB. And third, MSB can be defended along the same lines used by Levy and Markowitz (1979) to defend MVB.

Table 3. Approximating Expected Utility

Utility Function	All Markets		Developed Markets		Emerging Markets	
	AEUMVB	AEU_{MSB}	AEU_{MVB}	AEU_{MSB}	AEU_{MVB}	AEU_{MSB}
U = ln(1+R)	0.996	0.989	1.000	0.999	0.996	0.991
$U = (1+R)^a$						
a = 0.1	0.997	0.992	1.000	0.999	0.997	0.993
a = 0.3	0.999	0.995	1.000	1.000	0.999	0.996
a = 0.5	1.000	0.998	1.000	1.000	1.000	0.998
a = 0.7	1.000	0.999	1.000	1.000	1.000	0.999
a = 0.9	1.000	1.000	1.000	1.000	1.000	1.000
$U = -e^{-b(1+R)}$						
b = 0.1	1.000	1.000	1.000	1.000	1.000	1.000
b = 0.5	1.000	0.997	1.000	1.000	1.000	0.997
b = 1	0.998	0.988	1.000	0.999	0.998	0.989
b = 3	0.980	0.982	0.999	0.995	0.967	0.972
b = 5	0.951	0.977	0.991	0.988	0.928	0.968
b = 10	0.759	0.825	0.907	0.918	0.731	0.822
Averages	0.973	0.979	0.991	0.991	0.968	0.977

All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (2) and (4).

More on the Semideviation

Although the standard deviation of returns is widely used as a measure of risk, several problems limit its usefulness. First, the standard deviation is an appropriate measure of risk only when the underlying distribution of returns is symmetric. Second, it can be applied straightforwardly as a risk measure only when the underlying distribution of returns is Normal. Third, the two previous conditions, symmetry and normality, are seriously questioned by the empirical evidence. And fourth, although widely used, the equilibrium measure of risk that follows from MVB, beta, is also seriously questioned by the empirical evidence.

In addition, several reasons support the plausibility of the semideviation as a measure of risk. From a practical point of view, investors obviously do not dislike upside volatility; they only dislike downside volatility. Second, the semideviation is more useful than the standard deviation when the underlying distribution of returns is skewed and just as useful when the underlying distribution is symmetric; in other words, the semideviation is at least as useful a measure of risk as the standard deviation. And third, the semideviation combines into one measure the information provided by two statistics, standard deviation and skewness, thus making it possible to use a one-factor model to estimate required returns.

From an empirical point of view, the semideviation has been reported to explain the cross section of returns of emerging markets (Estrada, 2000, and Harvey, 2000), the cross section of industries in emerging markets (Estrada, 2001*a*), and the cross section of Internet stocks returns (Estrada, 2001*b*). Additional support for the semideviation as an appropriate measure of risk can be found in Sortino and van der Meer (1991), Clash (1999), and Sortino, van der Meer, and Plantinga (1999), among others.

3. APPROXIMATING THE UTILITY OF EXPECTED COMPOUND RETURN

All the arguments considered in the previous sections are based on the assumption that maximizing expected utility is the correct behavioral criterion for investors. However, an alternative plausible criterion is for investors to maximize the *expected compound return* of their portfolio. This strategy, discussed at length by Markowitz (1959) and more recently by Hakansson (1971), Booth and Fama (1992) and Wilcox (1997, 1998), consists of maximizing a portfolio's geometric mean return.

Table 4 shows the correlation between the utility of expected compound return, computed as $U = U(1+g)$, where g is the geometric mean return, and both the mean-variance approximation and the mean-semivariance approximation. These correlations, just like those of Table 3, show that both MVB and MSB are very good approximations to the utility of expected compound return, and that on average MSB outperforms MVB. In fact, in this case, MSB outperforms MVB in *both* developed markets and emerging markets.

Table 4. Approximating the Utility of Expected Compound Return

Utility Function	All Markets		Developed Markets		Emerging Markets	
	AEUMVB	AEU_{MSB}	AEU_{MVB}	AEU_{MSB}	AEU_{MVB}	AEU_{MSB}
$U = \ln(1+R)$	0.996	0.989	1.000	0.999	0.996	0.991
$U = (1+R)^a$						
a = 0.1	0.995	0.981	0.999	0.998	0.995	0.985
a = 0.3	0.977	0.956	0.996	0.993	0.980	0.967
a = 0.5	0.940	0.920	0.988	0.985	0.954	0.942
a = 0.7	0.892	0.878	0.977	0.975	0.921	0.913
a = 0.9	0.837	0.832	0.963	0.963	0.884	0.881
$U = -e^{-b(1+R)}$						
b = 0.1	0.837	0.833	0.963	0.963	0.884	0.881
b = 0.5	0.943	0.923	0.988	0.986	0.957	0.944
b = 1	0.996	0.991	1.000	0.999	0.995	0.992
b = 3	0.551	0.694	0.801	0.846	0.462	0.653
b = 5	0.352	0.468	0.529	0.590	0.199	0.361
b = 10	0.263	0.341	0.258	0.299	0.086	0.198
Averages	0.798	0.817	0.872	0.883	0.776	0.809

All numbers in the table show correlation coefficients between the utility of expected compound return, given by $U = U(1+g)$, and approximate expected utility given, by (2) and (4).

4. CONCLUSION

The most-widely used measure of an asset's risk, beta, follows from an equilibrium in which investors display MVB. Levy and Markowitz (1979) defended this criterion as approximately correct, in the sense that it yields a level of utility almost equal to an investor's expected utility. Markowitz (1991) reaffirmed this view and in fact considered the issue important enough to make it the central topic of his Nobel prize lecture.

This article shows that MSB can be defended with the same arguments that Levy and Markowitz (1979) used to defend MVB. The results reported and discussed show that: (i) MSB is consistent with the maximization of expected utility; (ii) MSB is consistent with the maximization of the utility of expected compound return; and (iii) in both cases MSB outperforms MVB. Finally, it provides additional considerations, some practical and some empirical, that support the semideviation as a more plausible measure of risk than the standard deviation. Essentially, this article agrees with Markowitz in that "semivariance seems more plausible than variance as a measure of risk".

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