Mean-Semivariance Behaviour: An Alternative Behavioural Model

Javier Estrada

The most widely used measure of an asset's risk, beta, stems from an equilibrium in which investors display mean-variance behaviour. This behavioural criterion assumes that portfolio risk is measured by the variance (or standard deviation) of returns, which is a questionable measure of risk. The semivariance of returns is a more plausible measure of risk (as Markowitz himself admits) and is backed by theoretical, empirical and practical considerations. It can also be used to implement an alternative behavioural criterion, mean-semivariance behaviour, that is almost perfectly correlated to both expected utility and the utility of mean compound return. Although the analytical framework and results are general, they are particularly relevant for emerging markets.

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1. Introduction

Risk is a slippery concept and its proper definition, critical for academics and practitioners alike, is under continuous evolution. Though not free from controversy, the most widely accepted definition of an asset's risk in a

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diversified portfolio is the asset's beta. This definition of risk, in turn, stems from an equilibrium in which investors display mean-variance behaviour (MVB), that is, a model in which investors choose their optimal portfolio by maximising a utility function that depends solely on the mean and variance of the portfolio returns.

Levy and Markowitz (1979) defended MVB as an approximately correct criterion in the sense that it yields a level of utility highly correlated to an investor's expected utility.[1] However, when receiving the Nobel prize for Economics in 1990, Markowitz (1991) stated that '...it can further help evaluate the adequacy of mean and variance, *or alternate practical measures*, as criteria' (emphasis added). In addition, he stated that '[p]erhaps...some other measure of portfolio risk will serve in a two parameter analysis.... *Semivariance seems more plausible than variance as a measure of risk, since it is concerned only with adverse deviations*' (emphasis added).

In this article, I follow Markowitz' suggestions and evaluate the plausibility of semivariance as a measure of risk, and of mean-semivariance behaviour (MSB) as a behavioural criterion. More precisely, following Levy and Markowitz (1979), I evaluate whether MSB is an approximately correct criterion in the sense that it yields a level of utility highly correlated to an investor's expected utility. I also outline several additional reasons for which semivariance is a better measure of risk than variance and analyse the relationship between MSB and an alternative behavioural criterion, namely, the maximisation of expected compound return.

The rest of this article is organised as follows. Section 2 tackles MVB and some extensions, as well as the Levy-Markowitz (1979) defence of this criterion. Section 3 provides a similar defence of MSB, suggests additional reasons that support both the MSB criterion and the semideviation as a measure of risk, and discusses the relationship between MSB and the maximisation of expected compound return. Finally, Section 4 contains some concluding remarks.

2. Mean-Variance Behaviour (and Extensions)

It is a well established result in modern financial theory that MVB is consistent with expected utility maximisation (EUM) under either one of two conditions: (*a*) that the investor's utility function is quadratic; or (*b*) that the returns of the investor's portfolio are jointly normally distributed.

[1] See also, Kroll et al. (1984), Pulley (1981, 1985) and Reid and Tew (1986).

In either case, the optimal portfolio chosen by an investor who maximises a utility function that depends on only two parameters, the mean and the variance of the portfolio returns, would be the same portfolio chosen by the investor if he maximised his expected utility function directly.

However, the plausibility of a quadratic utility function is questioned by the fact that it implies that an investor's absolute risk aversion is increasing in his wealth, although the opposite would be reasonably expected. Furthermore, the normality of returns is questioned by loads of data that display either skewness or kurtosis (or both).[2]

2.1 A mean-variance approximation to expected utility

Not ready to give up on what would eventually become the standard behavioural criterion in modern financial theory, Levy and Markowitz (1979) moved to defend MVB from a different perspective: they asked whether an investor choosing a portfolio on the basis of mean and variance would *almost* maximise his expected utility. In other words, they did not question the implausibility of the conditions that make MVB *exactly* consistent with EUM; rather, they asked whether the simpler choice based on mean and variance would yield a level of expected utility *almost* equal to that obtained by a much more complicated direct maximisation of the expected utility function. They also proposed to measure the goodness of the approximation by the correlation between expected utility and its approximation.

Table A.1 in the Appendix reproduces Table 1 in Markowitz (1991), which, in turn, has been taken from Levy and Markowitz (1979). The table shows the correlation coefficients between an investor's expected utility (EU) given by

$$EU = (T/1) \cdot \sum_{t=1}^{T} U(R_t) \qquad (1)$$

and his approximate expected utility based on MVB (AEU_{MVB}) given by

$$AEU_{MVB} = U(\mu) + (1/2) \cdot \sigma^2 \cdot U''(\mu) \qquad (2)$$

[2] Although monthly stock returns in developed markets do not seem to depart significantly from normality, high frequency returns in these markets and returns in emerging markets do depart significantly from this assumption; see Aparicio and Estrada (2001), Estrada (2000 and 2001).

where U denotes the investor's utility function, R and T denote returns and the number of returns in the sample and μ and σ^2 denote the mean and variance of returns.

As Table A.1 clearly shows, the correlation between expected utility and approximate expected utility is nearly perfect for three different utility functions and several parameter values. On the basis of this table, Levy and Markowitz (1979) and Markowitz (1991) conclude that MVB is a good approximation to EUM. Academics and practitioners seemed to agree, as they widely use beta as a measure of risk, which follows from an equilibrium in which investors display MVB.

2.2 Two further approximations to expected utility

The behavioural criterion proposed by Markowitz uses the first two terms of a Taylor approximation (around mean return) to expected utility. If an additional term is added to (2), we obtain an approximate expected utility, based not just on mean and variance but also on skewness (AEU_{Skw}). Such an approximation is given by

$$AEU_{Skw} = U(\mu) + (1/2) \cdot \sigma^2 \cdot U''(\mu) + (1/6) \cdot Skw \cdot U'''(\mu) \qquad (3)$$

where Skw denotes the skewness in the returns of the investor's portfolio. The importance of skewness in the assessment of risk has been stressed by Chen et al. (2001), Harvey and Siddique (2000) and Leland (1999) among others.

Furthermore, if an additional term is added to (3), we then obtain an approximate expected utility, based not just on mean, variance and skewness but also on kurtosis (AEU_{Krt}). Such an approximation is given by

$$AEU_{Krt} = U(\mu) + (1/2) \cdot \sigma^2 \cdot U''(\mu) + (1/6) \cdot Skw \cdot U'''(\mu) + (1/24) \cdot Krt \cdot U''''(\mu) \qquad (4)$$

where Krt denotes the kurtosis in the returns of the investor's portfolio. The importance of both skewness and kurtosis in the assessment of risk has been stressed by Aparicio and Estrada (2001) and Bekaert et al. (1998) among others.

2.3 Is MVB a good approximation to expected utility?

The data used to evaluate the relationship between expected utility and different approximations to expected utility are the entire Morgan Stanley Capital International (MSCI) database of both developed and emerging markets for the year 2000. This database contains monthly data on 22 developed markets and 28 emerging markets for varied sample periods, some starting as far back as January 1970. Summary statistics for all these markets, together with the earliest month for which data are available for each market, are reported in Table A.2 in the Appendix.

The utility functions and parameter values used in the evaluation are the same as those used by Levy and Markowitz (1979). I thus use a logarithmic utility function, $U = \ln(1 + R)$; a power utility function, $U = (1 + R)^a$, for several values of a; and a negative exponential utility function, $U = -e^{-b(1+R)}$, for several values of b. The details of the computations, as well as plausible values for a and b, are discussed in the Appendix; the results are shown in Table 1, which reports correlation coefficients between expected utility and three different approximations to expected utility based on mean-variance, mean-variance-skewness, and mean-variance-skewness-kurtosis.

Table 1
EUM, MVB and Extensions

Utility	*All Markets*			*Developed Markets*			*Emerging Markets*		
Function	AEU_{MVB}	AEU_{Skw}	AEU_{Krt}	AEU_{MVB}	AEU_{Skw}	AEU_{Krt}	AEU_{MVB}	AEU_{Skw}	AEU_{Krt}
$U = \ln(1+R)$	0.996	0.997	0.999	1.000	1.000	1.000	0.996	0.997	0.999
$U = (1+R)^a$									
$a = 0.1$	0.997	0.998	0.999	1.000	1.000	1.000	0.997	0.998	0.999
$a = 0.3$	0.999	0.999	1.000	1.000	1.000	1.000	0.999	0.999	1.000
$a = 0.5$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$a = 0.7$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$a = 0.9$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$U = -e^{-b(1+R)}$									
$b = 0.1$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$b = 0.5$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$b = 1$	0.998	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000
$b = 3$	0.980	0.979	0.994	0.999	0.995	1.000	0.967	0.967	0.991
$b = 5$	0.951	0.910	0.954	0.991	0.966	0.996	0.928	0.869	0.930
$b = 10$	0.759	0.577	0.689	0.907	0.759	0.979	0.731	0.526	0.626
Average	0.973	0.955	0.970	0.991	0.977	0.998	0.968	0.946	0.962

Note: All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (2), (3) and (4).

Focus on the AEU_{MVB} columns first, which indicate the correlation between expected utility and its mean-variance approximation. Consistent with the results reported by Levy and Markowitz (1979), MVB does seem to be a very good approximation to expected utility; all correlation coefficients are well above 0.900, with two exceptions for the highest value of the b coefficient of the negative exponential utility function ($b = 10$). These columns also show that AEU_{MVB} seems to approximate expected utility in developed markets slightly better than it does in emerging markets.

The mean-variance approximation is so good that, as the AEU_{Skw} and AEU_{Krt} columns show, there is very little room for improvement. In fact, considering skewness or both skewness and kurtosis *worsens* the approximation to expected utility in a few cases. We can therefore conclude, unsurprisingly, that Markowitz' insight was correct: MVB does provide a good approximation to EUM.

3. Mean-Semivariance Behaviour

In this section I show that MSB is an approximately correct criterion in the same sense that Levy and Markowitz (1979) showed MVB to be approximately correct. I also provide some reasons why standard deviation is an implausible measure of risk and some other reasons why semideviation is a plausible measure of risk. Finally, I argue that MSB is also an approximately correct criterion with respect to an alternative behavioural model, namely, the maximisation of expected compound return.

3.1 The semideviation

Although the standard deviation of returns is widely used as a measure of risk, several problems limit its usefulness. First, the standard deviation is an appropriate measure of risk only when the underlying distribution of returns is symmetric. Second, it can be applied directly as a risk measure only when the underlying distribution of returns is normal. Third, the two previous conditions, symmetry and normality, are seriously questioned by the empirical evidence. Finally, although widely used, beta, the equilibrium measure of risk that follows from MVB, is also seriously questioned by the empirical evidence.

An alternative measure of risk that has received support from both academics and practitioners and that Markowitz supported from the start (Markowitz 1959, Chapter IX), is the downside standard deviation of returns,

or semideviation for short, which for any benchmark return B can be denoted as Σ_B and is given by

$$\Sigma_B = \sqrt{E\{Min[(R-B),0]^2\}} \qquad (5)$$

where R denotes returns. As can be noticed by a simple review of (5), the semideviation gives a positive weight only to the deviations below the benchmark; that is, returns below B increase Σ_B, but returns above B do not. Essentially, the semideviation defines risk as volatility below the benchmark.[3]

Some reasons that support the plausibility of the semideviation as an appropriate measure of risk are discussed in Section 3.3. But before turning to them, let us follow the pathbreaking insight of Levy and Markowitz (1979) and ask whether an investor choosing portfolios on the basis of mean and semivariance would almost maximise his expected utility.

3.2 A mean-semivariance approximation to expected utility

An approximation to expected utility based on mean and semivariance can be obtained by replacing the variance of returns in (2) by twice the semivariance of returns. Hence, an investor's approximate expected utility based on MSB (AEU_{MSB}) can be calculated with the expression

$$AEU_{MSB} = U(\mu) + (1/2)\cdot(2\Sigma^2)\cdot U''(\mu) = U(\mu) + \Sigma^2 \cdot U''(\mu) \qquad (6)$$

The rationale for this approximation is the following. If the underlying distribution of returns is symmetric, then $2\cdot\Sigma^2 = \sigma^2$. In that case, both (2) and (6) would yield exactly the same level of (approximate) expected utility. However, when the underlying distribution is skewed, then $2\cdot\Sigma^2 \neq \sigma^2$ and (2) and (6) would yield different levels of (approximate) expected utility. More precisely, in the presence of negative skewness, $2\cdot\Sigma^2 > \sigma^2$ and $AEU_{MSB} < AEU_{MVB}$ and in the presence of positive skewness, $2\cdot\Sigma^2 < \sigma^2$ and $AEU_{MSB} > AEU_{MVB}$.

The details of all the computations are provided in the Appendix; the results of the analysis are reported in Table 2.

[3] Throughout this article, I have used the arithmetic mean return (μ) of each distribution as the benchmark return B and for ease of notation, from now on I will write Σ_μ simply as Σ.

Table 2
EUM and MSB

Utility Function	*All Markets*	*Developed Markets*	*Emerging Markets*
$U = \ln(1+R)$	0.989	0.999	0.991
$U = (1+R)^a$			
$a = 0.1$	0.992	0.999	0.993
$a = 0.3$	0.995	1.000	0.996
$a = 0.5$	0.998	1.000	0.998
$a = 0.7$	0.999	1.000	0.999
$a = 0.9$	1.000	1.000	1.000
$U = -e^{-b(1+R)}$			
$b = 0.1$	1.000	1.000	1.000
$b = 0.5$	0.997	1.000	0.997
$b = 1$	0.988	0.999	0.989
$b = 3$	0.982	0.995	0.972
$b = 5$	0.977	0.988	0.968
$b = 10$	0.825	0.918	0.822
Average	**0.979**	**0.991**	**0.977**

Note: All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (6).

Table 2, similar in aim and scope to Exhibit A1 used by Levy and Markowitz (1979) to argue that MVB is an approximately correct criterion with respect to expected utility, shows that MSB is also an approximately correct criterion in the same sense. The three columns show correlation coefficients well above 0.900 in all cases, with two exceptions for the highest value of the b coefficient of the negative exponential utility function $b = 10$. These columns also show, as was the case for AEU_{MVB}, that AEU_{MSB} performs slightly better in developed markets than in emerging markets.

A comparison between Tables 1 and 2 suggests that MSB provides an approximation to expected utility virtually identical to that provided by MVB for all utility functions and parameters in both developed markets and emerging markets. For high values of the b coefficient of the negative exponential utility function ($b \geq 3$), however, MSB tends to outperform the MVB. In fact, on average, the mean-semivariance approximation outperforms the mean-variance approximation in emerging markets and in the full sample of all markets. (Both approximations have the same average performance in developed markets.)

Although the correlations between expected utility and AEU_{MVB} are virtually identical to those between expected utility and AEU_{MSB}, an interesting question to ask is whether these two sets of correlations are statistically indistinguishable from each other. The answer to such a question, however, is not straightforward for we run into the problem of comparing non-nested hypotheses; that is, when comparing the power of AEU_{MVB} and AEU_{MSB} to explain the variability of expected utility, AEU_{MSB} cannot be considered a special case of AEU_{MVB}.

Although there is no widely accepted test to determine whether two competing non-nested models have a significantly different explanatory power, the J-test proposed by Davidson and MacKinnon (1981) provides some evidence in the right direction. Consider the two competing models

$$H_0: EU_i = \alpha_0 + \alpha_1 \cdot AEU_{MVB,i} + u_i \quad (7)$$

$$H_1: EU_i = \beta_0 + \beta_1 \cdot AEU_{MSB,i} + v_i \quad (8)$$

where EU denotes expected utility, the αs and βs are coefficients to be estimated and u and v are error terms.

The J-test consists of first estimating the αs and the βs, generating the predicted values from equation (7), $\hat{\alpha}_0 + \hat{\alpha}_1 \cdot AEU_{MVB,i}$ and equation (8), $\hat{\beta}_0 + \hat{\beta}_1 \cdot AEU_{MSB,i}$, then running the regressions

$$EU_i = \alpha_0 + \alpha_1 \cdot AEU_{MVB,i} + \alpha_2 \cdot (\hat{\beta}_0 + \hat{\beta}_1 \cdot AEU_{MSB}) + u_i' \quad (9)$$

$$EU_i = \beta_0 + \beta_1 \cdot AEU_{MSB,i} + \beta_2 \cdot (\hat{\alpha}_0 + \hat{\alpha}_1 \cdot AEU_{MVB}) + v_i' \quad (10)$$

and finally testing for the significance of α_2 and β_2. The idea is that if model H_0 is correct, then the fitted values of model H_1 should have no explanatory power in (9) and α_2 should not be significant when evaluated with the standard t-test. Similarly, if model H_1 is correct, then the fitted values of model H_0 should have no explanatory power in (10) and β_2 should not be significant when evaluated with the standard t-test.

Table 3 reports the p-values for the t-tests on the significance of α_2 in (9) and of β_2 in (10) using all the markets in the sample. As can be seen from the table, α_2 is significant in all cases and β_2 is significant in all but one case. Thus, the J-tests indicate that neither approximation significantly outperforms the other when explaining the variability of expected utility, and that AEU_{MSB} significantly outperforms AEU_{MVB} in the case of the negative exponential utility function for $b = 10$.

Table 3
J-Tests

	Log	Power U					Exponential U					
	U	$a = 0.1$	$a = 0.3$	$a = 0.5$	$a = 0.7$	$a = 0.9$	$b = 0.1$	$b = 0.5$	$b = 1$	$b = 3$	$b = 5$	$b = 10$
α_2	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
β_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.14

Note: All numbers in the table show the *p*-values of the α_2 and β_2 coefficients from equations (9) and (10).

Essentially, three important points follow from Tables 1–3. First, that an investor who chooses portfolios on the basis of mean and semivariance would in fact almost maximise his expected utility. Second, that the approximation to expected utility provided by MSB is, on average, at least as good as that provided by MVB. Finally, that MSB can be defended along the same lines used by Levy and Markowitz (1979) to defend MVB.

3.3 More on the semideviation

Having established that MSB is an approximately correct criterion in the sense that it yields a level of utility highly correlated to an investor's expected utility, let us now consider some additional reasons that support the plausibility of the semideviation as a measure of risk. Some of these reasons are practical, some others are empirical.

From a practical point of view, first, investors obviously do not dislike upside volatility; they only dislike downside volatility. Second, the semideviation is more useful than the standard deviation when the underlying distribution of returns is asymmetric and just as useful when the underlying distribution is symmetric; in other words, the semideviation is, at least, as useful a measure of risk as the standard deviation. Finally, the semideviation combines into one measure the information provided by two statistics, variance and skewness, thus making it possible to use a one-factor model to estimate required returns.

From an empirical point of view, the semideviation has been reported to explain the cross section of returns of emerging markets (Estrada 2000; Harvey 2000), the cross section of industries in emerging markets (Estrada 2001) and the cross section of Internet stocks returns (Estrada 2004). Additional support for the semideviation as an appropriate measure of risk can be found in Clash (1999), Sortino and van der Meer (1991) and Sortino et al.(1999) among others.

3.4 Maximising expected compound return

All the arguments considered in the previous sections are based on the assumption that maximising expected utility is the correct behavioural criterion for investors. However, an alternative plausible criterion for investors is to maximise the *expected compound return* of their portfolio, a strategy sometimes associated with long-term investing and discussed at length by Markowitz (1959).[4]

Hakansson (1971) and more recently Booth and Fama (1992) and Wilcox (1997, 1998) also support the maximisation of expected compound return, an approach that essentially consists of maximising the geometric mean return of a portfolio (or a portfolio's 'rate of growth' in Markowitz' words).

Let R and r denote simple (arithmetic) and logarithmic (continuously compounded) returns respectively and let μ and σ^2 be the mean and variance of R; then, by definition, $r = \ln(1+R)$. Furthermore, approximating by Taylor the expected value of r around μ we obtain

$$E(r) = E\{\ln(1+R)\} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} + \frac{(1/3)\cdot Skw}{(1+\mu)^3} - \frac{(1/4)\cdot Krt}{(1+\mu)^4} + \cdots \quad (11)$$

Equation (11) shows why investors like mean return and positive skewness and dislike variance and kurtosis; the last two produce a drag on expected compound return. This equation also shows that the maximisation of expected compound return implies a logarithmic utility function for terminal wealth (compare [11] with [A11] in the Appendix), which adds to the plausibility of this criterion.

Table 4 shows the correlation between the utility of mean compound return, computed as $U = U(1 + g)$, where g is geometric mean return of R, and both the mean-variance approximation (AEU_{MVB}) and the mean-semivariance approximation (AEU_{MSB}).[5] These correlations show that both MVB and MSB are very good approximations to the utility of mean compound return.

[4] Markowitz (1959) in fact devotes an entire chapter of his book (Chapter VI, 'Return in the Long Run') to this issue, plus an additional chapter ('Note on Chapter VI') in his 1991 revision of the same book.

[5] Although Table 4 does not show it, the correlation between expected utility and the utility of mean compound return is very high (above 0.9) for the three utility functions and all parameters (with the exception of the negative exponential utility function for values of $b \geq 3$). In other words, the criteria of maximising expected utility and maximising mean compound return are very similar to each other.

These correlations also show, just like Tables 1 and 2, that the average performance of MSB is better than the average performance of MVB and that this average performance is better in *both* developed markets and emerging markets.

Table 4
Maximising Expected Compound Return, MVB and MSB

Utility Function	*All Markets* AEU_{MVB}	*All Markets* AEU_{MSB}	*Developed Markets* AEU_{MVB}	*Developed Markets* AEU_{MSB}	*Emerging Markets* AEU_{MVB}	*Emerging Markets* AEU_{MSB}
$U = \ln(1+R)$	0.996	0.989	1.000	0.999	0.996	0.991
$U = (1+R)^a$						
$a = 0.1$	0.995	0.981	0.999	0.998	0.995	0.985
$a = 0.3$	0.977	0.956	0.996	0.993	0.980	0.967
$a = 0.5$	0.940	0.920	0.988	0.985	0.954	0.942
$a = 0.7$	0.892	0.878	0.977	0.975	0.921	0.913
$a = 0.9$	0.837	0.832	0.963	0.963	0.884	0.881
$U = -e^{-b(1+R)}$						
$b = 0.1$	0.837	0.833	0.963	0.963	0.884	0.881
$b = 0.5$	0.943	0.923	0.988	0.986	0.957	0.944
$b = 1$	0.996	0.991	1.000	0.999	0.995	0.992
$b = 3$	0.551	0.694	0.801	0.846	0.462	0.653
$b = 5$	0.352	0.468	0.529	0.590	0.199	0.361
$b = 10$	0.263	0.341	0.258	0.299	0.086	0.198
Average	**0.798**	**0.817**	**0.872**	**0.883**	**0.776**	**0.809**

Note: All numbers in the table show correlation coefficients between the utility of mean compound return, given by $U = U(1+g)$, and approximate expected utility, given by (2) and (6).

In sum, the whole analysis supports the idea that an investor who maximises a utility function that depends on mean and semivariance would also maximise both expected utility and the utility of expected compound return. In other words, although both MVB and MSB provide a good approximation to expected utility and the utility of expected compound return, MSB appears to be at least as good an approximation as MVB. This finding, plus the practical and empirical considerations discussed above, make the semideviation an ideal variable for a two-parameter utility function and a behavioural model from which further results and implications could be derived.

4. Conclusions

The most widely used measure of an asset's risk, beta, follows from an equilibrium in which investors display MVB. Levy and Markowitz (1979) defended this criterion as approximately correct in the sense that it yields a level of utility almost equal to an investor's expected utility. Markowitz (1991) reaffirmed this view and in fact considered the issue important enough to make it the central topic of his lecture while accepting the Nobel prize.

This article shows that MSB can be defended with the same arguments that Levy and Markowitz (1979) used to defend MVB. It also provides additional considerations, some practical and some empirical, that support the semideviation as a more plausible measure of risk than the standard deviation. It reports results showing that MSB is not only consistent with the maximisation of expected utility but also with the maximisation of the utility of expected compound return. Those considerations and results should round up the reasons for which MSB is a more plausible criterion than MVB. Essentially, this article agrees with Markowitz that 'semivariance seems more plausible than variance as a measure of risk.'

A fair question to ask is, if MSB is the correct behavioural model, then in this framework what is the appropriate measure of risk of an asset in a diversified portfolio? In other words, what is the counterpart of beta in a downside risk framework? It turns out that a 'downside beta' can be defined and articulated into a one-factor model (similar to the Capital Asset Pricing Model [CAPM]) that can be used to generate required returns. As shown in Estrada (2002a,b), it turns out that this downside beta explains the cross section of stock returns better than beta, particularly in emerging markets.

APPENDIX

Table A.1
Expected Utility and Approximate Expected Utility

Utility Function	*Annual Returns 149 Mutual Funds*	*Annual Returns 97 Stocks*	*Monthly Returns 97 Stocks*	*Random Portfolios 5/6 Stocks*
$U = \ln(1+R)$	0.997	0.880	0.995	0.998
$U = (1+R)^a$				
$a = 0.1$	0.998	0.895	0.996	0.998
$a = 0.3$	0.999	0.932	0.998	0.999
$a = 0.5$	0.999	0.968	0.999	0.999
$a = 0.7$	0.999	0.991	0.999	0.999
$a = 0.9$	0.999	0.999	0.999	0.999
$U = -e^{-b(1+R)}$				
$b = 0.1$	0.999	0.999	0.999	0.999
$b = 0.5$	0.999	0.961	0.999	0.999

(*Table A.1 contd.*)

(*Table A.1 contd.*)

Utility Function	*Annual Returns 149 Mutual Funds*	*Annual Returns 97 Stocks*	*Monthly Returns 97 Stocks*	*Random Portfolios 5/6 Stocks*
$b = 1$	0.997	0.850	0.997	0.998
$b = 3$	0.949	0.850	0.976	0.958
$b = 5$	0.855	0.863	0.961	0.919
$b = 10$	0.449	0.659	0.899	0.768

Source: Markowitz (1991), Table 1.
Note: All numbers in the table show correlation coefficients between expected utility, given by (1) and approximate expected utility, given by (2).

Table A.2
Summary Statistics (Monthly Stock Returns)

Developed Markets					*Emerging Markets*				
Market	*μ*	*σ*	*Σ*	*Start*	*Market*	*μ*	*σ*	*Σ*	*Start*
Australia	0.93	7.21	5.26	Jan'70	Argentina	3.16	17.94	10.02	Jan'88
Austria	0.91	6.08	4.00	Jan'70	Brazil	3.22	17.78	11.85	Jan'88
Belgium	1.29	5.48	3.77	Jan'70	Chile	1.87	7.54	5.20	Jan'88
Canada	1.00	5.55	4.05	Jan'70	China	−0.60	12.93	7.88	Jan'93
Denmark	1.26	5.42	3.71	Jan'70	Colombia	−0.14	9.47	6.46	Jan'93
Finland	1.94	8.60	5.70	Jan'88	Czech Rep.	0.26	9.48	6.81	Jan'95
France	1.25	6.60	4.69	Jan'70	Egypt	1.23	8.79	5.14	Jan'95
Germany	1.14	5.90	4.22	Jan'70	Greece	1.87	11.46	6.65	Jan'88
Hong Kong	2.04	11.31	7.66	Jan'70	Hungary	2.04	12.35	8.50	Jan'95
Ireland	1.07	5.73	3.96	Jan'88	India	0.65	8.95	6.04	Jan'93
Italy	0.90	7.57	5.13	Jan'70	Indonesia	1.34	17.31	9.89	Jan'88
Japan	1.22	6.63	4.56	Jan'70	Israel	1.14	7.51	5.31	Jan'93
Netherlands	1.38	5.14	3.73	Jan'70	Jordan	−0.02	4.49	3.12	Jan'88
New Zealand	0.29	7.00	4.73	Jan'88	Korea	0.67	12.41	7.54	Jan'88
Norway	1.21	7.74	5.47	Jan'70	Malaysia	0.97	10.26	6.97	Jan'88
Portugal	0.61	6.74	4.47	Jan'88	Mexico	2.45	10.56	7.79	Jan'88
Singapore	1.15	8.52	5.96	Jan'88	Morocco	1.00	4.78	3.32	Jan'93
Spain	1.03	6.52	4.58	Jan'70	Pakistan	0.16	11.96	8.13	Jan'93
Sweden	1.50	6.55	4.60	Jan'70	Peru	0.89	9.85	6.79	Jan'93
Switzerland	1.24	5.49	3.91	Jan'70	Philippines	0.87	10.45	7.05	Jan'88
UK	1.24	6.87	4.52	Jan'70	Poland	3.19	18.52	10.48	Jan'93
USA	1.08	4.44	3.22	Jan'70	Russia	3.46	23.61	16.14	Jan'95
Average	**1.17**	**6.69**	**4.63**	**n.a.**	South Africa	1.03	8.17	5.95	Jan'93
					Sri Lanka	−0.40	9.47	6.65	Jan'93
					Taiwan	1.23	12.34	8.14	Jan'88
					Thailand	0.67	12.64	8.81	Jan'88
					Turkey	2.54	18.39	11.31	Jan'88
					Venezuela	1.52	15.19	10.70	Jan'93
					Average	**1.30**	**11.95**	**7.81**	**n.a.**

Notes: μ: Mean return; σ: Standard deviation; Σ: Semideviation. All numbers in percentage. Data through December 2000.

Expected Utility and Approximate Expected Utility Calculations

I briefly discuss here the equations used to evaluate the relationship between EUM, MVB and MSB. For all three utility functions, an investor's expected utility (*EU*) is defined as in (1); that is,

$$EU = (T/1) \cdot \sum_{t=1}^{T} U(R_t) \quad \text{(A1)}$$

where *U* denotes the investor's utility function and *R* and *T* denote returns and the number of returns in the sample respectively. It thus follows that the expected utility of an investor who has a logarithmic, power or negative exponential utility function is given by the following respectively.

$$EU = (T/1) \cdot \sum_{t=1}^{T} \ln(1+R_t) \quad \text{(A2)}$$

$$EU = (T/1) \cdot \sum_{t=1}^{T} \ln(1+R_t)^a \quad \text{(A3)}$$

$$EU = (T/1) \cdot \sum_{t=1}^{T} -e^{-b(1+R_t)} \quad \text{(A4)}$$

Let μ, σ^2, and Σ^2 be the mean, variance and semivariance of any given series of returns respectively. The approximate expected utility of an investor who displays MVB (AEU_{MVB}) and has a logarithmic, power or negative exponential utility function is respectively given by

$$AEU_{MVB} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} \quad \text{(A5)}$$

$$AEU_{MVB} = (1+\mu)^a + (1/2)\cdot\sigma^2\cdot a(a-1)(1+\mu)^{a-2} \quad \text{(A6)}$$

$$AEU_{MVB} = -e^{-b(1+\mu)} - (1/2)\cdot\sigma^2\cdot b^2\cdot e^{-b(1+\mu)} \quad \text{(A7)}$$

Furthermore, let *Skw* and *Krt* be the moments of skewness and kurtosis respectively of any given series of returns. Then an approximate expected utility based on mean, variance and skewness (AEU_{Skw}) of an investor that displays a logarithmic, power or negative exponential utility function is respectively given by

$$AEU_{Skw} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} + \frac{(1/3)\cdot Skw}{(1+\mu)^3} \quad \text{(A8)}$$

$$AEU'_{Skw} = (1+\mu)^a + (1/2)\cdot\sigma^2\cdot a(a-1)(1+\mu)^{a-2} + (1/6)\cdot Skw\cdot a(a-1)(a-2)(1+\mu)^{a-3} \quad (A9)$$

$$AEU_{Skw} = -e^{-b(1+\mu)} - (1/2)\cdot\sigma^2\cdot b^2\cdot e^{-b(1+\mu)} + (1/6)\cdot Skw\cdot b^3\cdot e^{-b(1+\mu)} \quad (A10)$$

An approximate expected utility based on mean, variance, skewness and kurtosis (AEU_{Krt}) of an investor that displays a logarithmic, power or negative exponential utility function, on the other hand, is respectively given by

$$AEU_{Krt} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} + \frac{(1/3)\cdot Skw}{(1+\mu)^3} - \frac{(1/4)\cdot Krt}{(1+\mu)^4} \quad (A11)$$

$$AEU_{Krt} = (1+\mu)^a + (1/2)\cdot\sigma^2\cdot a(a-1)(1+\mu)^{a-2} + (1/6)\cdot Skw\cdot a(a-1)(a-2)(1+\mu)^{a-3} + (1/24)\cdot Krt\cdot a(a-1)(a-2)(a-3)(1+\mu)^{a-4} \quad (A12)$$

$$AEU_{Krt} = -e^{-b(1+\mu)} - (1/2)\cdot\sigma^2\cdot b^2\cdot e^{-b(1+\mu)} + (1/6)\cdot Skw\cdot b^3\cdot e^{-b(1+\mu)} - (1/24)\cdot Krt\cdot b^4\cdot e^{-b(1+\mu)} \quad (A13)$$

Finally, the approximate expected utility of an investor who displays MSB (AEU_{MSB}) and has a logarithmic, power or negative exponential utility function is respectively given by

$$AEU_{MSB} = \ln(1+\mu) - \frac{\Sigma^2}{(1+\mu)^2} \quad (A14)$$

$$AEU_{MSB} = (1+\mu)^a + \Sigma^2\cdot a(a-1)(1+\mu)^{a-2} \quad (A15)$$

$$AEU_{MSB} = -e^{-b(1+\mu)} - \Sigma^2\cdot b^2\cdot e^{-b(1+\mu)} \quad (A16)$$

In order to evaluate the relationship between MVB, MSB and the maximisation of expected compound return, the expressions for AEU_{MVB} and AEU_{MSB} are the same as above. The equations for the utility of mean compound return for an investor that displays a logarithmic, power or negative exponential utility function, on the other hand, are respectively given by

$$U = \ln(1+g) \quad (A17)$$

$$U = (1+g)^a \quad (A18)$$

$$U = -e^{-b(1+g)} \quad (A19)$$

where g is the geometric mean of R.

Empirical Estimates of Risk Aversion

Both the logarithmic and the power utility functions exhibit constant relative risk aversion (RRA), whereas the negative exponential utility function exhibits an RRA that increases with wealth. The empirical evidence on the topic is mixed: Cohn et al. (1975) find that RRA decreases with wealth; Friend and Blume (1975) that RRA is constant in wealth; and Siegel and Hoban (1982) that RRA increases with wealth.

Regarding plausible values for the coefficient of the power utility function, Plantinga and de Groot (2002) argue that given the utility function $U = W^{1-\lambda}/(1-\lambda)$, where W denotes wealth, plausible values for λ lie between 1 and 2, with 2 indicating a moderate degree of risk aversion. On the other hand, regarding the negative exponential utility function, Markowitz (1991) argues that someone with $b = 10$ would have 'very strange' preferences, to the point that this individual would prefer a certain 10 per cent return over a 50–50 chance of breaking even or obtaining a 100,000 per cent return.

Javier Estrada is at the IESE Business School, Av. Pearson 21, Barcelona.
E-mail: jestrada@iese.edu

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