



The temporal dimension of risk

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Abstract

If returns are stationary, then the risk of an asset in any time frequency can be estimated from the risk of the asset in any other time frequency through a simple linear rescaling. This implies that short-term risk carries reliable information about long-term risk, and both data frequencies and investment horizons are irrelevant when evaluating an asset's risk. However, most series of stock returns are nonstationary, which if ignored may lead investors to make significant mistakes. Using recent data from fourteen European securities markets, I show that investors that mistakenly assume stationarity are bound to underestimate the total and systematic risk (and overestimate the risk-adjusted returns) of European stocks. The underestimation of total risk ranges between .25% and 2.18% a month, and averages almost 1% a month. © 2000 Bureau of Economic and Business Research, University of Illinois. All rights reserved.

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1. Introduction

Two assumptions widely used by both academics and practitioners are that stock prices follow a random walk and that stock returns are normally distributed. Although the balance of the evidence that emerges from the vast literature on these topics is that neither assumption is plausible from an empirical point of view, both assumptions are common in theoretical work. Furthermore, they both hide behind many simple calculations widely performed by practitioners.

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Normality and random walks are useful assumptions, among many other reasons, because the former implies that the standard deviation of returns is an appropriate measure of (total) risk,¹ and the latter implies *iid* (hence, stationary) stock returns.² However, the evidence supporting either assumption is far from satisfactory, which brings us to the main issue considered in this article: How large are the errors an investor can make if he uses some standard assumptions that turn out to be not supported by the data?³

The stationary returns that stem from prices that follow a random walk have a useful property, which is addressed in this article: The risk of a security in any time frequency can be estimated from the risk of the security in any other time frequency through a simple linear rescaling. In the case of total risk as measured by the standard deviation, stationarity implies the sometimes-called $T^{1/2}$ rule; that is, weekly volatility can be estimated by multiplying daily volatility by the square root of 5, annual volatility can be estimated by multiplying monthly volatility by the square root of 12, and so forth. In the case of systematic risk, stationarity implies that betas estimated from daily, weekly, monthly, quarterly or annual data should be the same.

However, if stock prices do not follow a random walk, the linear scaling of risk may be misleading. For if returns are nonstationary, the relationship between the risk of a security in different time frequencies breaks down. And if that is the case, the risk of a security in a given time frequency cannot be reliably estimated from the risk of the security in some other time frequency through a simple linear rescaling.

Both Holton (1992) and Peters (1991, 1994) address the issue of scaling directly and find that, in short horizons, volatility scales at a faster rate than that implied by stationary stock returns. The evidence presented in this article, which also considers a short investment horizon, does support that finding.

The scaling of volatility has also been used as a way to test the random walk hypothesis, mostly through variance ratios. Lo and MacKinlay (1988), using weekly data on NYSE and AMEX stocks, find evidence of positive autocorrelation, particularly in portfolios of small stocks. Fama and French (1988), using monthly data on NYSE stocks, find negative autocorrelations (hence, mean reversion) in long horizons, particularly during the 1926–40 period. Poterba and Summers (1988) find positive autocorrelations in short horizons (less than one year) and negative autocorrelations in long horizons, in both U.S. and non-U.S. data. Richardson and Stock (1989), Richardson (1993), Poon (1996) and Lamoureux and Zhou (1996), on the other hand, argue that the evidence against the random walk hypothesis is weaker than the previously cited (and other) articles suggest.

The main difference between the mean-reversion/variance-ratios literature and this article is that the former focuses on *testing* whether stock prices follow a random walk, whereas this article focuses on the *consequences* of assuming a random walk (hence, stationary stock returns) when stock prices do not follow such process. In other words, the variance-ratios literature looks at the issue of scaling as a way to test the random walk hypothesis, whereas, this article looks at the same issue from the point of view of *quantifying the errors* that stem from assuming that such hypothesis holds when it is not really supported by the data.

Finally, the stationarity of stock returns from which the scaling of risk addressed in this article follows, is at odds with the literature on the ARCH model and its subsequent extensions, such as GARCH, ARCH-M and EGARCH, among many others. This literature

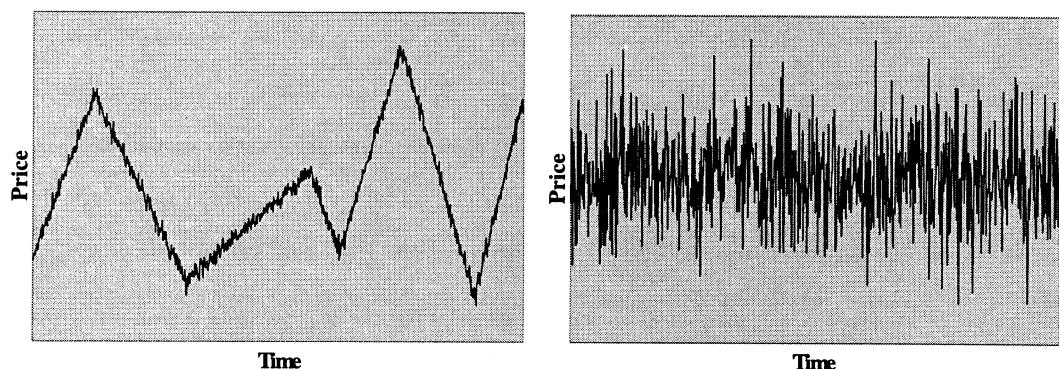


Fig. 1. Short-term risk and long-term risk.

leaves little doubt that stock returns are nonstationary because variances change widely over time. See the pioneering work of Engle (1982), Bollerslev (1986), Engle et al. (1987) and Nelson (1991).

European stock returns, just as returns in most other markets, are nonstationary. Ignoring this fact and using some simple rules of scaling may lead investors to make significant errors in the estimation of risk. The results reported below show that, on average relative terms, estimating monthly volatility from daily data may result in an underestimation of total risk of around 20%; estimating monthly betas from daily data may result in an underestimation of systematic risk of around 16%; and estimating monthly Sharpe ratios from daily data may result in an overestimation of risk-adjusted returns of around 21%.

The rest of the article is organized as follows. I review the nature of the problem in part II. In part III, I present some evidence showing that European stock returns are nonstationary, and, therefore, that European stock prices do not follow a random walk. In part IV, I show that investors that ignore this fact are bound to underestimate the total and systematic risk, and overestimate the risk-adjusted returns of European stocks. In part V, I show that the errors that stem from scaling are much smaller when all returns are measured in a common currency. Finally, in part VI, I summarize the main conclusions of the analysis. An appendix containing tables and a proof concludes the article.

2. The problem

As mentioned above, the random walk theory of stock prices (and the stationarity of stock returns in particular) yields the critical implication that the temporal dimension of risk is irrelevant; that is, the risk of a security in any time frequency can be estimated from the risk of the security in any other time frequency through a simple linear rescaling. However, if returns are nonstationary, the relationship between the risk of a security in different time frequencies breaks down, and both data frequencies and investment horizons become relevant.

To illustrate, consider Figs. 1a and b, which show the prices of two hypothetical securities

over time (neither of which, of course, follows a random walk). A natural question to ask is: Which security is riskier? The answer is not straightforward. The security in Fig. 1a is risky for the long-term investor but not very risky for the short-term investor; the security in Fig. 1b, on the other hand, is risky for the short-term investor but not very risky for the long-term investor. So the next question is: Which security should offer a higher return? Again, the answer is not straightforward; the risk of each security depends on the investor's investment horizon. However, the answer *would* be straightforward if the two securities were random walks. For in that case the risk of both securities in any time frequency could be assessed through its standard deviation or beta, which could then be linearly rescaled to any desired investment horizon. Therefore, investment horizons *are* a relevant issue when analyzing the risk of securities that do not follow a random walk.

Note that the whole issue of time diversification hinges on this point. Those who believe that time diversification applies implicitly believe in mean-reverting returns, whereas, those who believe that it does not apply implicitly believe in prices that follow a random walk. In other words, if stock prices follow a random walk, the argument that in the long run stocks are less risky than bonds does not apply. An excellent review of the issue of time diversification is provided by Kritzman and Rich (1998).

Emerging markets offer another interesting example. As is well known, these markets usually exhibit high short-term volatility; hence, in equilibrium, they should (and do) offer high returns. To illustrate, Erb et al. (1996) report that between September 1979 and March 1995, the U.S. market averaged an annual return of 15.4% (with a standard deviation of 14.8%), whereas, the Philippines and Poland averaged annual dollar returns of 41.7% and 93.3% (with standard deviations of 36.8% and 90.3%), respectively. Thus, if an investor ignores short-term swings and holds on to his shares in these markets long enough (that is, until the risk-return relationship is in equilibrium), he will be rewarded with high returns. Is this long-term investor then being rewarded for risk borne by short-term investors? Are emerging markets very risky for investors whose investment horizon is one day, one month, or even one year, but perhaps not so risky for long-term investors? If stock prices do not follow a random walk, these questions do not have a straightforward answer.⁴

3. Some evidence on European stock returns

I present in this part some evidence showing that European stock returns are nonstationary, and, therefore, that European stock prices do not follow a random walk. To that purpose, I consider a sample of 14 European securities markets, namely, Austria (AUS), Belgium (BEL), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Ireland (IRE), Italy (ITA), Netherlands (NET), Norway (NOR), Spain (SPA), Sweden (SWE), Switzerland (SWI) and the United Kingdom (UK). The behavior of each market is summarized by a Financial Times index measured in local currency. The logarithmic returns computed and used throughout the article include both capital gains and dividends. Sample statistics for the daily and monthly returns of these series, which extend from the beginning of 1990 through the end of 1997, are reported in Tables A1 and A2 in the Appendix.

Table 1 below reports some evidence on the linear and nonlinear dependence of daily and

Table 1
Linear and nonlinear dependence in European stock returns

Market	Daily stock returns						Monthly stock returns							
	ρ_R	ρ_{SR}	SE_ρ	$Q(6)$	p	$Q^2(6)$	p	ρ_R	ρ_{SR}	SE_ρ	$Q(6)$	p	$Q^2(6)$	p
AUS	0.23	0.29	0.02	116.50	0.00	341.52	0.00	0.10	0.33	0.10	9.29	0.16	22.17	0.00
BEL	0.17	0.06	0.02	63.23	0.00	48.11	0.00	0.00	0.14	0.10	3.37	0.76	22.76	0.00
DEN	0.19	0.13	0.02	77.93	0.00	61.03	0.00	0.00	0.02	0.10	2.78	0.84	9.12	0.17
FIN	0.15	0.31	0.02	52.33	0.00	346.51	0.00	0.14	0.05	0.10	3.62	0.73	7.37	0.29
FRA	0.05	0.16	0.02	9.50	0.15	131.77	0.00	0.05	0.15	0.10	2.84	0.83	10.13	0.12
GER	0.01	0.19	0.02	10.25	0.11	185.81	0.00	-0.07	0.42	0.10	4.26	0.64	22.68	0.00
IRE	0.09	0.25	0.02	31.60	0.00	199.88	0.00	0.06	0.06	0.10	9.79	0.13	14.96	0.02
ITA	0.11	0.13	0.02	34.61	0.00	84.05	0.00	-0.02	-0.07	0.10	1.76	0.94	14.97	0.02
NET	0.00	0.19	0.02	6.74	0.35	538.10	0.00	-0.03	0.44	0.10	1.45	0.96	25.94	0.00
NOR	0.11	0.13	0.02	26.41	0.00	213.13	0.00	0.04	-0.08	0.10	1.36	0.97	4.48	0.61
SPA	0.10	0.20	0.02	27.95	0.00	168.60	0.00	0.04	0.30	0.10	1.34	0.97	24.47	0.00
SWE	0.14	0.25	0.02	46.68	0.00	239.90	0.00	0.05	0.11	0.10	6.32	0.39	4.22	0.65
SWI	0.05	0.17	0.02	9.86	0.13	195.28	0.00	0.04	0.21	0.10	6.79	0.34	9.29	0.16
UK	0.07	0.16	0.02	16.91	0.01	106.73	0.00	0.02	0.03	0.10	5.44	0.49	10.26	0.11

ρ_R and ρ_{SR} are the first-order autocorrelations of returns and squared returns, respectively, and SE_ρ is the standard error of both. $Q(6)$ and $Q^2(6)$ are the Ljung-Box statistics for 6 autocorrelations of returns and squared returns, respectively. p represents the p -value of the Ljung-Box test.

monthly European stock returns. The statistics reported include the first-order autocorrelation of returns and squared returns, their standard deviation and the Ljung-Box test for six autocorrelations of returns and squared returns with their respective P values.

As is obvious from the table, and perfectly consistent with findings for other markets, daily stock returns are (at the 5% significance level) not independent. Most markets exhibit at least first-order autocorrelation, and the null hypothesis of uncorrelated daily returns

Table 2
Time-varying volatility

Market	Daily stock returns							Monthly stock returns								
	'90	'91	'92	'93	'94	'95	'96	'97	'90	'91	'92	'93	'94	'95	'96	'97
AUS	1.7	1.2	1.1	0.9	0.8	0.7	0.7	1.2	14.0	8.6	6.7	5.5	4.0	3.9	4.8	6.2
BEL	0.9	0.8	0.7	0.6	0.6	0.5	0.6	0.9	6.4	4.4	3.5	3.8	3.8	3.2	3.1	4.8
DEN	0.8	0.8	1.0	0.8	0.8	0.6	0.6	0.9	5.7	4.2	5.7	4.1	4.4	3.9	2.6	5.2
FIN	0.8	1.3	1.5	1.4	1.1	1.7	1.2	1.7	6.9	8.6	12.6	6.5	6.8	9.2	4.7	8.3
FRA	1.1	1.1	1.0	0.8	1.0	0.9	0.7	1.2	6.6	4.7	4.4	5.1	5.5	3.1	4.1	6.4
GER	1.4	1.2	0.8	0.8	0.9	0.7	0.7	1.4	9.0	4.1	4.4	4.1	4.5	4.2	3.1	7.6
IRE	1.3	1.1	1.0	1.1	0.9	0.6	0.6	1.1	8.5	6.9	4.7	4.7	5.7	2.2	2.8	4.7
ITA	1.1	1.2	1.5	1.3	1.6	1.2	1.2	1.4	7.3	5.8	8.9	8.1	6.7	5.4	6.5	7.9
NET	0.9	0.7	0.6	0.6	0.8	0.6	0.7	1.5	4.4	3.7	3.0	3.6	4.2	2.8	3.2	6.7
NOR	1.2	1.4	1.9	1.1	0.9	0.8	0.7	1.0	7.2	7.7	7.6	5.7	6.2	4.1	3.4	4.0
SPA	1.3	1.1	1.1	0.9	1.1	0.8	0.8	1.3	8.7	5.5	6.3	5.3	5.6	3.5	4.5	6.7
SWE	1.4	1.2	1.5	1.0	1.0	0.9	0.8	1.3	10.3	6.8	8.5	6.1	5.8	4.1	3.3	6.4
SWI	1.2	1.0	0.7	0.7	0.9	0.6	0.7	1.2	7.1	4.1	3.7	3.4	4.2	3.0	3.7	6.1
UK	0.9	0.8	1.0	0.6	0.8	0.6	0.5	0.9	5.6	4.4	5.7	3.0	4.5	2.1	1.9	4.1

Volatility measured by the standard deviation of stock returns and reported in %.

cannot be rejected by the Ljung-Box test in only four markets (FRA, GER, NET and SWI).⁵ In all markets, daily squared returns are clearly autocorrelated, thus implying a pattern of volatility clustering. Monthly returns, on the other hand, are uncorrelated in all markets, and monthly squared returns are uncorrelated in seven markets. Thus, as is typically the case in other markets, the European data shows, first, that daily stock returns are far from independent; and, second, that monthly stock returns are much better behaved than daily stock returns.

Further evidence of non-*iid* stock returns is reported in Table 2, where average daily and monthly volatility (measured by the standard deviation of stock returns) is reported for each market and year in the sample. As is clear from the table, and again fully consistent with findings for other markets, the risk of European markets fluctuates widely over time. In fact, in some markets, the volatility in a given year is over three times higher than the volatility in a different year within the sample period.⁶

The evidence reported thus shows that daily stock returns in the markets considered are far from being *iid*; they are neither independent (nonlinear dependence exists in all markets and linear dependence in most markets) nor identically distributed (variances do change over time). Furthermore, although not reported in Tables 1 and 2, the distributions of daily stock returns in the markets considered also clearly depart from the standard assumption of normality; see Aparicio and Estrada (2000).⁷

4. The danger of implicit assumptions

The finding that European stock prices do not follow a random walk, and that European stock returns are nonstationary, is uninteresting if taken by itself; the same finding has been reported in many other studies and for many other markets. Thus, I go one step further and attempt to quantify the errors an investor could make by ignoring the fact that returns are nonstationary, and *implicitly* assuming stationarity. To that purpose, I consider the total and systematic risk, and the risk-adjusted returns of the fourteen markets in our sample.

4.1. Total risk (standard deviations)

If returns are stationary and $\text{Var}(r_t) = \sigma^2$ for all t , where r_t denotes continuously compounded returns in period t , then it is the case that $\text{Var}(r_1 + \dots + r_T) = T \cdot \text{Var}(r_t) = T \cdot \sigma^2$. Hence, total risk as measured by the standard deviation of returns scales proportionally to $T^{1/2}$. Table 3 below reports a quantification of the errors an investor could make if he estimates monthly volatility on the basis of daily volatility following this simple rule.

The first two columns of the table show the observed volatility, measured by the standard deviation of stock returns, computed on the basis of daily data (ODV) and monthly data (OMV) for the fourteen markets considered. In all markets, there is an average of 22 trading days each month. Thus, under stationarity, the OMV column could be obtained from the ODV column by multiplying the latter by the square root of 22. However, such products, which I will refer to as implied monthly volatility (IMV) are, as shown in the third column, significantly lower than the OMV in all markets.

Table 3
Total risk scaling

Market	ODV	OMV	IMV	OSF	RD	AD
AUS	1.08	7.26	5.08	44.91	1.43	2.18
BEL	0.71	4.36	3.33	37.63	1.31	1.03
DEN	0.79	4.85	3.69	38.00	1.31	1.16
FIN	1.37	8.32	6.42	36.87	1.29	1.89
FRA	0.99	5.17	4.66	27.13	1.11	0.51
GER	1.04	5.52	4.86	28.41	1.14	0.66
IRE	1.00	5.55	4.69	30.76	1.18	0.86
ITA	1.33	7.19	6.24	29.26	1.15	0.96
NET	0.85	4.22	3.97	24.84	1.06	0.25
NOR	1.17	5.91	5.51	25.34	1.07	0.40
SPA	1.08	6.07	5.07	31.45	1.20	0.99
SWE	1.17	6.68	5.48	32.75	1.22	1.21
SWI	0.91	4.77	4.26	27.51	1.12	0.50
UK	0.77	4.13	3.61	28.70	1.14	0.51
Avgs	1.02	5.71	4.78	31.68	1.20	0.94

Volatility measured by the standard deviation of stock returns and reported in %. ODV = Observed daily volatility; OMV = Observed monthly volatility; IMV = Implied monthly volatility; OSF = Observed scaling factor; RD = Relative difference; AD = Absolute difference. $IMV = (22)^{1/2}ODV$; $OSF = (OMV/ODV)^2$; $RD = OMV/IMV$; $AD = OMV - IMV$.

The fourth column shows the observed scaling factor (OSF); that is, the number whose square root, multiplied by the ODV, yields the OMV.⁸ As can be seen from the table, this factor is significantly larger than 22 in all markets, with an average of 31.68. The last two columns report the relative difference (RD) and absolute difference (AD), defined simply as the ratio and the difference between the OMV and the IMV, respectively. The fifth column shows that the OMV is larger than the IMV in all markets, the former being between 6% and 43% larger than the latter in relative terms, and 20% on average. The last column shows that the OMV is between .25% and 2.18% larger than the IMV in absolute terms, and .94% on average.⁹

The scaling of volatility from daily data to quarterly data (not reported in Table 3) yields similar qualitative results. The OSF of 103.41 is far larger than the expected OSF of 66 under stationarity. On average, the observed quarterly volatility is 24% larger in relative terms than that implied by a linear rescaling of daily volatility. In absolute terms, the underestimation of volatility that stems from mistakenly assuming stationarity is on average 2.05% a quarter.

These findings thus support those of several other empirical studies (cited above) that show that, in short horizons, volatility scales at a faster rate than implied by stationary stock returns. They also show that mistakenly assuming stationarity will lead investors to seriously underestimate the total risk of investing in European stocks; such underestimation is on average around 1% *a month*.

4.2. Systematic risk (*betas*)

If returns are stationary, then betas (the typical measure of systematic risk) should be independent from the frequency of the data used to estimate them. In other words, daily,

Table 4
Systematic risk scaling

Market	ODB	OMB	OQB	RD_{DM}	RD_{DO}	RD_{MQ}
AUS	0.86	1.12	1.18	1.31	1.37	1.05
BEL	0.64	0.87	0.87	1.35	1.36	1.00
DEN	0.58	0.85	0.81	1.47	1.40	0.95
FIN	0.80	1.34	1.49	1.67	1.86	1.11
FRA	1.15	1.11	1.10	0.96	0.96	0.99
GER	1.12	1.15	1.17	1.03	1.05	1.02
IRE	0.76	1.05	1.27	1.37	1.67	1.21
ITA	1.04	1.13	1.12	1.08	1.08	0.99
NET	0.95	0.92	0.84	0.96	0.88	0.92
NOR	0.85	0.89	0.66	1.04	0.78	0.75
SPA	1.08	1.14	1.48	1.06	1.37	1.29
SWE	1.09	1.15	1.56	1.05	1.43	1.36
SWI	1.00	0.96	1.12	0.96	1.12	1.17
UK	0.93	0.89	0.82	0.96	0.88	0.92
Avgs	0.92	1.04	1.11	1.16	1.23	1.05

ODB = Observed daily beta; OMB = Observed monthly beta; OQB = Observed quarterly beta; RD_{DM} = Relative difference between daily and monthly betas; RD_{DO} = Relative difference between daily and quarterly betas; RD_{MQ} = Relative difference between monthly and quarterly betas. RD_{DM} = OMB/ODB; RD_{DO} = OQB/ODB; RD_{MQ} = OQB/OMB.

weekly, monthly, quarterly and annual betas should be the same. Table 4 shows that if returns are nonstationary, daily, monthly and quarterly betas may be significantly different.

The table above reports observed daily (ODB), monthly (OMB) and quarterly betas (OQB) estimated using a capitalization-weighted portfolio of European markets. The table also reports the relative differences between daily and monthly betas (RD_{DM}), daily and quarterly betas (RD_{DO}) and monthly and quarterly betas (RD_{MQ}). Although it is not the case in every market that the betas decrease as the frequency of the data increases, that is in fact the case both in most markets and on average. Table 4 shows that, on average, monthly betas are 16% larger than daily betas, quarterly betas are 23% larger than daily betas and quarterly betas are 5% larger than monthly betas. Therefore, use of daily data to compute betas will typically lead investors to underestimate the systematic risk of European stock markets.¹⁰

In order to grasp the magnitude of this difference; consider an “average European company” that needs to estimate its cost of equity in order to discount the annual cash flows of an investment project. If we estimate the cost of equity with the CAPM, we assume an annual risk-free rate of 5% and an annual risk premium of 6%, the annual cost of equity based on the average daily beta reported in Table 4 would be 10.5%. However, using the same risk-free rate and market risk premium, but the average quarterly beta reported in Table 4, the annual cost of equity would be 11.7%; that is, a difference of over 1% a year.

4.3. Risk-adjusted returns (Sharpe ratios)

One way of evaluating the risk-return relationship of a stock or market is by means of the so-called Sharpe ratio, which measures the return obtained per unit of risk borne (or, put

Table 5
Risk-adjusted returns scaling

Market	ODSR	OMSR	IMSR	OSF	RD
AUS	0.0101	0.0328	0.0474	10.53	1.45
BEL	0.0600	0.2128	0.2816	12.56	1.32
DEN	0.0471	0.1662	0.2211	12.44	1.33
FIN	0.0417	0.1493	0.1955	12.82	1.31
FRA	0.0348	0.1452	0.1631	17.43	1.12
GER	0.0388	0.1584	0.1821	16.64	1.15
IRE	0.0554	0.2174	0.2601	15.37	1.20
ITA	0.0255	0.1027	0.1198	16.16	1.17
NET	0.0795	0.3467	0.3728	19.03	1.08
NOR	0.0297	0.1282	0.1392	18.65	1.09
SPA	0.0522	0.2024	0.2448	15.03	1.21
SWE	0.0540	0.2050	0.2531	14.43	1.23
SWI	0.0730	0.3027	0.3425	17.18	1.13
UK	0.0663	0.2689	0.3108	16.47	1.16
Avgs	0.0477	0.1885	0.2238	15.34	1.21

Risk-adjusted returns measured by Sharpe ratios. ODSR = Observed daily Sharpe ratio; OMSR = Observed monthly Sharpe ratio; IMSR = Implied monthly Sharpe ratio; OSF = Observed scaling factor; RD = Relative difference. $IMSR = (22)^{1/2}ODSR$; $OSF = (OMSR/ODSR)^2$; $RD = IMSR/OMSR$.

simply, risk-adjusted returns).¹¹ The first two columns of Table 5 show the observed Sharpe ratio computed on the basis of daily data (ODSR) and monthly data (OMSR). If returns are stationary, Sharpe ratios should scale (just as volatility) proportionally to the square root of time¹²; hence, the OMSR column could be obtained by multiplying the ODSR column by the square root of 22. Such products, which I will refer to as implied monthly Sharpe ratios (IMSR) are reported in the third column.

As can be seen from the table, the OMSRs are lower than the IMSRs in all markets, which is explained as follows. Table A3 in the Appendix shows that stock returns do scale almost exactly proportionally to 22; however, as already noted, volatility scales at a faster rate than the square root of 22. Hence, it must follow that Sharpe ratios scale at a slower rate than the square root of 22, which is confirmed in the fourth column that shows an average OSF of 15.34. Therefore, as is shown in the last column of the table, an investor that mistakenly assumes that stock returns are stationary would overestimate risk-adjusted returns by an average of 21% in relative terms.

The scaling from daily to quarterly Sharpe ratios (not reported in Table 5) yields similar qualitative results. The OSF of 44.07, smaller than that of 66 expected under stationarity, implies an average overestimation of risk-adjusted returns of 26% in relative terms.

5. Dollar returns

The results reported in the previous part show that ignoring that European stock returns are nonstationary, but implicitly using the stationarity assumption, may lead to large errors.

Table 6
Scaling with dollar returns

Market	Total risk		Systematic risk		Risk-adjusted returns	
	OSFV	RDV	RDB _{DM}	RDB _{DQ}	OSFSR	RDSR
AUS	32.52	1.22	1.04	1.09	14.54	1.23
BEL	21.14	0.98	1.01	0.97	22.36	0.99
DEN	23.61	1.04	1.14	0.94	20.02	1.05
FIN	28.18	1.13	1.19	1.24	16.77	1.15
FRA	20.97	0.98	0.91	0.94	22.55	0.99
GER	21.60	0.99	0.96	0.91	21.89	1.00
IRE	22.44	1.01	1.13	1.37	21.06	1.02
ITA	25.05	1.07	0.88	0.80	18.88	1.08
NET	19.52	0.94	0.94	0.82	24.22	0.95
NOR	22.39	1.01	1.00	0.46	21.11	1.02
SPA	28.59	1.14	1.13	1.39	16.54	1.15
SWE	27.17	1.11	1.15	1.18	17.39	1.12
SWI	20.86	0.97	0.88	0.92	22.66	0.99
UK	24.37	1.05	1.04	1.08	19.40	1.06
Avgs	24.17	1.05	1.03	1.01	19.96	1.06

ODB = Observed daily volatility; OMV = Observed monthly volatility; IMV = Implied monthly volatility; OSFV = Observed scaling factor in volatility; RDV = Relative difference in volatility; ODB = Observed daily beta; OMB = Observed monthly beta; OQB = Observed quarterly beta; RDB_{DM} = Relative difference between daily and monthly beta; RDB_{DQ} = Relative difference between daily and quarterly betas; ODSR = Observed daily Sharpe ratio; OMSR = Observed monthly Sharpe ratio; IMSR = Implied monthly Sharpe ratio; OSFSR = Observed scaling factor in Sharpe ratios; RDSR = Relative difference in Sharpe ratios. IMV = $(22)^{1/2}ODV$; OSFV = $(OMV/ODV)^2$; RDV = OMV/IMV; RDB_{DM} = OMB/ODB; RDB_{DQ} = OQB/ODB; IMSR = $(22)^{1/2}ODSR$; OSFSR = $(OMSR/ODSR)^2$; RDSR = IMSR/OMSR.

The results reported below show that such errors are significantly lower if all returns are measured in a common currency (dollars).

In terms of total risk, the first column of Table 6 shows that the average observed scaling factor between daily and monthly volatility is 24.17, fairly close to that of 22 expected under stationarity.¹³ Hence, as the second column shows, the underestimation of monthly volatility that stems from a linear rescaling of daily volatility amounts to just 5% on average relative terms, significantly lower than the 20% obtained for local returns. In absolute terms, the difference between the observed monthly volatility and the monthly volatility implied by daily data (not reported in the table) amounts to just .28% a month on average.

It is shown in the appendix that the slower scaling of the volatility of dollar returns (relative to the scaling of the volatility of local-currency returns) is due to a slower scaling than that suggested by stationarity either in the volatility of exchange-rate returns, or in the covariance between local returns and exchange rate-returns, or both. This issue is not pursued any further here, but the implications for the scaling of exchange-rate risk are obvious.

In terms of systematic risk, the third column of Table 6 shows that monthly betas are, on average, just 3% larger than daily betas in relative terms. Furthermore, the fourth column shows that quarterly betas are just 1% larger than daily betas on average relative terms. Thus, we find again that scaling may not be as dangerous when returns are measured in dollars as when they are measured in local currency.

Finally, Table 6 shows that risk-adjusted returns as measured by the Sharpe ratio scale just a bit slower than they would under stationarity. As shown in the last two columns of Table 6, the observed scaling factor of 19.96 is a bit lower than that of 22 expected under stationarity, thus implying an overestimation of risk-adjusted returns of 6% on average relative terms. This result contrasts sharply with the average overestimation of 21% on average relative terms reported above for local returns.

5. Conclusions

The results reported in this article question some standard practices in the estimation of risk. More precisely, they raise the issue that if stock returns are nonstationary, use of high-frequency data in order to draw inferences about alternative investment horizons may be badly misleading.

The analysis has shown that European daily stock returns are, just as those in most other markets, far from being *iid*. The series of daily returns analyzed exhibit time-varying variances and nonlinear dependence in all cases, and linear dependence in most cases. Under these conditions data frequencies and investment horizons both become relevant issues, and simple linear rules to scale risk may be badly misleading.

If returns are stationary, high-frequency data contains all the relevant information to forecast risk in any other time frequency; in other words, short-term risk can be reliably used to forecast long-term risk. However, if returns are nonstationary, the relationship between risk in different time frequencies breaks down and short-term risk carries no reliable information about long-term risk.

The results reported in this article have shown that ignoring the nonstationarity in the data would typically lead investors to underestimate the total and systematic risk, as well as to overestimate the risk-adjusted returns of European stock returns. More precisely, monthly volatility implied by daily data underestimates the observed monthly volatility by an average of 20% in relative terms, or .94% a month in absolute terms. Furthermore, quarterly volatility implied by daily data underestimates the observed quarterly volatility by an average of 24% in relative terms, and 2.05% a quarter in absolute terms. Monthly betas are on average relative terms 16% larger than daily betas, and quarterly betas are (again on average relative terms) 23% larger than daily betas. Finally, monthly Sharpe ratios implied by daily data are 21% higher than observed monthly Sharpe ratios, and quarterly Sharpe ratios implied by daily data are 26% higher than observed quarterly Sharpe ratios—both on average relative terms.

The results thus reported show that simple rules may lead to big mistakes if the data do not conform to the assumptions implicit in the rules. The nonlinear scaling of volatility, in particular, implies that short-term risk carries no reliable information about long-term risk. Therefore, contrary to predictions from the random-walk theory, time diversification may apply. Furthermore, investors can benefit from buying a stock right after its price has fallen sharply; see De Bondt and Thaler (1985).

The scaling of systematic risk when returns are nonstationary, on the other hand, has important implications for companies. Among them is the fact that, when computing the cost

of equity for the purpose of project evaluation, firms should try to match the frequency of the data used to estimate betas to the timing of the projects' cash flows.

Standard assumptions lead to simple rules, but standard assumptions do not have to be supported by the data; therefore simple rules may lead to misleading results. Practitioners are certainly well advised to carefully balance the trade-off between simplicity and accuracy.

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Notes

1. Under non-normal distributions, more information than just the standard deviation is necessary in order to appropriately estimate risk. To illustrate, if a distribution is not symmetric, then the moment of skewness becomes relevant; if a distribution has fat tails, then the moment of kurtosis becomes relevant.
2. Campbell, Lo and MacKinlay (1997) distinguish three versions of the random walk hypothesis. In the first, stock returns are *iid*; in the second, stock returns are independent but not identically distributed; and in the third, stock returns are uncorrelated but not independent. The linear scaling of risk addressed in this article requires returns to be stationary, and, therefore, they should meet the conditions of the first of these three versions.
3. Aparicio and Estrada (2000) quantify the errors that stem from estimating risk assuming normality when such assumption is not supported by the data; thus, that issue is not addressed in this article. The consensus of the literature is that daily stock returns do not follow a Normal distribution but some alternative specification with fatter tails. Hence, investors that assume normality typically underestimate the risk of their securities. See Mandelbrot (1963), Fama (1965), Peiró (1994) and Aparicio and Estrada (2000), among many others.
4. Perhaps one way of thinking about long-term risk in emerging markets may be in terms of the uncertainty about *when* the risk–return relationship is going to be in equilibrium. That is, at what point in time in the future an investor will be able to liquidate his positions and realize a return consistent with the risk of these markets.
5. Autocorrelation, particularly of the first order, should not surprise due to the well-known problem of nonsynchronous trading that affects indices; see Scholes and Williams (1977), Atchison et al. (1987), and Lo and MacKinlay (1990).
6. It goes without saying that European daily stock returns exhibit significant GARCH(1,1) coefficients (not reported) in all markets.

7. Some evidence on the normality of each distribution of daily stock returns can be gathered from the last four columns of Table A1. Under the assumption of normality, the coefficients of skewness and (excess) kurtosis are asymptotically distributed as $N(0,6/n)$ and $N(0,24/n)$, respectively, where n is the sample size. Hence, values of these standardized coefficients outside the range $[-1.96, 1.96]$ indicate, at the 5% significance level, significant departures from normality. By these standards, Table A1 in the Appendix shows that all but three distributions are significantly skewed (in different directions), and that all fourteen distributions are leptokurtic.
8. More precisely the OSF is the number that solves the *observed* relationship $\sigma_M = (\text{OSF})^{1/2} \sigma_D$, where σ_M and σ_D represent the monthly and daily standard deviation of stock returns, respectively; hence, $\text{OSF} = (\sigma_M/\sigma_D)^2$.
9. It could be argued that the OMV is lower than the IMV simply because of the consideration of 22 trading days, thus omitting the volatility over the weekends. However, note that an OSF of 31.68 indicates a fast scaling even in the extreme case in which volatility over the weekends were assumed to be equal to the volatility of two trading days (which empirical evidence shows it is clearly not the case).
10. Some nonsynchronous-trading adjustments typically implemented when estimating daily betas have been known to academics for many years; see Scholes and Williams (1977). However, it is doubtful that such adjustments have been used by practitioners for that long; see Ibbotson et al. (1997).
11. The Sharpe ratio is computed as $(R_i - R_f)/\sigma_i$, where R_i and σ_i are the return and risk of security i , and R_f is a risk-free rate. The ratios reported in Table 5 ignore the risk-free rate and are computed as R_i/σ_i .
12. Returns and volatility scale proportionally to T and $(T)^{1/2}$, respectively; hence, risk-adjusted returns (the ratio of the two) must scale proportionally to $(T)^{1/2}$.
13. In fact, the difference between the observed scaling factor of 24.17 and that of 22 may very well be accounted for the volatility observed during the weekends.

Appendix

Scaling in the volatility of dollar returns

I show in this section that the slower scaling of the volatility of dollar returns (relative to the scaling of the volatility of local-currency returns) is due to a slower scaling than that suggested by stationarity either in the volatility of exchange-rate returns, or in the covariance between local returns and exchange-rate returns, or both. Let:

$$\sigma_{\$}^2 = \text{Variance of dollar returns}$$

$$\sigma_L^2 = \text{Variance of local-currency returns}$$

$$\sigma_E^2 = \text{Variance of exchange-rate returns}$$

$$\sigma_{LE} = \text{Covariance between local-currency returns and exchange-rate returns}$$

D and M = Subscripts that denote daily and monthly magnitudes, respectively

The relationship between the volatility of dollar returns and local-currency returns is given by

$$\sigma_{\$}^2 = \sigma_L^2 + \sigma_E^2 + 2\sigma_{LE} \Rightarrow \sigma_L^2 = \sigma_{\$}^2 - \sigma_E^2 - 2\sigma_{LE}. \quad (\text{A1})$$

If returns are stationary, the volatility of local-currency returns scales linearly; hence,

$$(22)\sigma_{L,D}^2 = \sigma_{L,M}^2 \Rightarrow (22)(\sigma_{\$,D}^2 - \sigma_{E,D}^2 - 2\sigma_{LE,D}) = (\sigma_{\$,M}^2 - \sigma_{E,M}^2 - 2\sigma_{LE,M}). \quad (\text{A2})$$

However, the evidence reported suggests that the volatility of local-currency returns scales at a faster rate than that implied by stationarity; that is,

$$(22)\sigma_{L,D}^2 < \sigma_{L,M}^2 \Rightarrow (22)(\sigma_{\$,D}^2 - \sigma_{E,D}^2 - 2\sigma_{LE,D}) < (\sigma_{\$,M}^2 - \sigma_{E,M}^2 - 2\sigma_{LE,M}). \quad (\text{A3})$$

The evidence reported also suggests that the volatility of dollar returns scales fairly in line with the rate of scaling implied by stationarity; hence, $(22)\sigma_{\$,D}^2 \approx \sigma_{\$,M}^2$ and Eq. (A3) reduces, after rearranging, to

$$(22)(\sigma_{E,D}^2 + 2\sigma_{LE,D}) > \sigma_{E,M}^2 + 2\sigma_{LE,M}. \quad (\text{A4})$$

Note that (A4) implies that either $(22)\sigma_{E,D}^2 > \sigma_{E,M}^2$, or that $(22)\sigma_{LE,D} > \sigma_{LE,M}$, or both, which is the claim being made.

Table A1
Sample moments of the distributions of European daily stock returns

Market	Mean	SD	Min	Max	Skw	SSkw	Krt	SKrt
AUS	0.01	1.08	-8.44	6.94	-0.08	-1.47	7.87	73.43
BEL	0.04	0.71	-5.55	6.93	-0.01	-0.27	10.71	99.97
DEN	0.04	0.79	-5.89	4.94	-0.22	-4.01	4.94	46.17
FIN	0.06	1.37	-10.79	8.32	-0.33	-6.12	4.68	43.66
FRA	0.03	0.99	-7.25	6.01	-0.21	-3.96	3.51	32.74
GER	0.04	1.04	-10.36	5.81	-0.77	-14.35	9.23	86.20
IRE	0.06	1.00	-9.03	6.09	-0.21	-3.87	9.46	88.30
ITA	0.03	1.33	-8.23	5.51	-0.07	-1.30	2.09	19.50
NET	0.07	0.85	-4.55	5.36	-0.25	-4.66	4.01	37.44
NOR	0.03	1.17	-8.85	10.81	0.24	4.51	10.05	93.86
SPA	0.06	1.08	-8.61	6.81	-0.27	-5.02	5.17	48.28
SWE	0.06	1.17	-6.83	9.32	0.37	6.91	5.40	50.42
SWI	0.07	0.91	-7.20	5.39	-0.60	-11.28	6.27	58.51
UK	0.05	0.77	-3.97	5.55	0.19	3.51	3.20	29.87
EUR	0.05	0.71	-5.70	4.22	-0.50	-9.31	5.34	49.85

Mean returns (Mean), standard deviations (SD), minimum returns (Min), and maximum returns (Max) in local currency and reported in %. Skw = Skewness = m_3/s^3 and Krt = Kurtosis = m_4/s^4-3 , where m_i and s are the i th central sample moment and the sample standard deviation of each distribution, respectively; both coefficients computed with a finite-sample adjustment. SSkw = standardized skewness and SKrt = Standardized kurtosis. EUR is a capitalization-weighted index of the European market. Sample period: Jan/01/90 through Dec/31/97.

Table A2

Sample moments of the distributions of European monthly stock returns

Market	Mean	SD	Min	Max	Skw	SSkw	Krt	SKrt
AUS	0.24	7.26	-25.37	20.35	-0.35	-1.42	2.14	4.38
BEL	0.93	4.36	-11.81	13.63	-0.42	-1.72	1.07	2.20
DEN	0.81	4.85	-10.94	11.80	-0.23	-0.92	-0.24	-0.49
FIN	1.24	8.32	-20.21	21.16	-0.09	-0.38	-0.12	-0.24
FRA	0.75	5.17	-14.47	12.19	-0.33	-1.33	-0.03	-0.05
GER	0.87	5.53	-19.61	15.09	-0.93	-3.77	2.24	4.58
IRE	1.21	5.55	-20.81	20.39	-0.17	-0.69	2.80	5.73
ITA	0.74	7.19	-15.88	21.20	0.40	1.61	-0.13	-0.27
NET	1.46	4.22	-11.93	13.33	-0.31	-1.24	0.64	1.31
NOR	0.76	5.91	-16.27	12.74	-0.44	-1.80	0.15	0.31
SPA	1.23	6.07	-20.37	13.19	-0.58	-2.36	0.67	1.38
SWE	1.37	6.68	-23.74	23.24	-0.54	-2.19	2.17	4.44
SWI	1.44	4.77	-14.34	12.26	-0.70	-2.84	1.42	2.91
UK	1.11	4.13	-8.23	10.93	-0.09	-0.37	0.19	0.40
EUR	1.07	4.08	-11.90	11.11	-0.70	-2.83	1.06	2.18

Mean returns (Mean), standard deviations (SD), minimum returns (Min), and maximum returns (Max) in local currency and reported in %. Skw = Skewness = m_3/s^3 and Krt = Kurtosis = $m_4/s^4 - 3$, where m_i and s are the i th central sample moment and the sample standard deviation of each distribution, respectively; both coefficients computed with a finite-sample adjustment. SSkw = standardized skewness and SKrt = Standardized kurtosis. EUR is a capitalization-weighted index of the European market. Sample period: Jan/90 through Dec/97.

Table A3

Mean returns scaling

Market	ODR	OMR	IMR	OSF	RD	AD
AUS	0.01	0.24	0.24	21.75	0.99	0.00
BEL	0.04	0.93	0.94	21.74	0.99	-0.01
DEN	0.04	0.81	0.82	21.74	0.99	-0.01
FIN	0.06	1.24	1.26	21.74	0.99	-0.01
FRA	0.03	0.75	0.76	21.74	0.99	-0.01
GER	0.04	0.87	0.89	21.74	0.99	-0.01
IRE	0.06	1.21	1.22	21.74	0.99	-0.01
ITA	0.03	0.74	0.75	21.74	0.99	-0.01
NET	0.07	1.46	1.48	21.74	0.99	-0.02
NOR	0.03	0.76	0.77	21.74	0.99	-0.01
SPA	0.06	1.23	1.24	21.74	0.99	-0.01
SWE	0.06	1.37	1.39	21.74	0.99	-0.02
SWI	0.07	1.44	1.46	21.74	0.99	-0.02
UK	0.05	1.11	1.12	21.74	0.99	-0.01
Avgs	0.05	1.01	1.02	21.74	0.99	-0.01

All returns reported in %. ODR = Observed daily returns; OMR = Observed monthly returns; IMR = Implied monthly returns; OSF = Observed scaling factor; RD = Relative difference; AD = Absolute difference. IMR = (22)ODR; OSF = OMR/ODR; RD = OMR/IMR; AD = OMR-IMR.

References

- Aparicio, F., & Estrada, J. (2000). Empirical distributions of stock returns: European securities markets, 1990–95. *European Journal of Finance (forthcoming)*.
- Atchison, M., Butler, K., & Simonds, R. (1987). Nonsynchronous security trading and market index autocorrelation. *Journal of Finance* 42, 111–118.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 31, 307–327.
- Campbell, H., Lo, A., & MacKinlay, C. (1997). *The Econometrics of Financial Markets*. Princeton: Princeton University Press.
- DeBondt, W., & Thaler, R. (1985). Does the stock market overreact. *Journal of Finance* 40, 793–805.
- Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the Variance of U.K. inflation. *Econometrica* 50, 987–1008.
- Engle, R., Lillien, D., & Robbins, R. (1987). Estimating time varying risk premia in the term structure: the ARCH-M model. *Econometrica* 55, 391–407.
- Erb, C., Harvey, C., & Viskanta, T. (1996). Expected returns and volatility in 135 countries. *Journal of Portfolio Management Spring 1996*, 46–58.
- Fama, E. (1965). The behavior of stock prices. *Journal of Business* 37, 34–105.
- Fama, E., & French, K. (1988). Permanent and temporary components of stock prices. *Journal of Political Economy* 96, 246–273.
- Holton, G. (1992). Time: the second dimension of risk. *Financial Analysts Journal November-December 1992*, 38–45.
- Ibbotson, R., Kaplan, P., & Peterson, J. (1997). Estimates of small-stock betas are much too low. *Journal of Portfolio Management Summer*, 104–111.
- Kritzman, M., & Rich, D. (1998). Beware of dogma. *Journal of Portfolio Management Summer*, 66–77.
- Lamoureux, C., & Zhou, G. (1996). Temporary components of stock returns: what do the data tell us? *Review of Financial Studies* 9, 1033–1059.
- Lo, A., & MacKinlay, C. (1988). Stock prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies* 1, 41–66.
- Lo, A., & MacKinlay, C. (1990). An econometric analysis of nonsynchronous trading. *Journal of Econometrics* 45, 181–211.
- Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business* 36, 394–419.
- Nelson, D. (1991). Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* 59, 347–370.
- Peiró, A. (1994). The distribution of stock returns: international evidence. *Applied Financial Economics* 4, 431–439.
- Peters, E. (1991). *Chaos and order in capital markets. a new view of cycles, prices, and market volatility*. New York: John Wiley & Sons.
- Peters, E. (1994). *Fractal market analysis. Applying chaos theory to investment and economics*. New York: John Wiley & Sons.
- Poon, S. (1996). Persistence and mean reversion in UK stock returns. *European Financial Management* 2, 169–196.
- Poterba, J., & Summers, L. (1988). Mean reversion in stock prices. *Journal of Financial Economics* 22, 27–59.
- Richardson, M. (1993). Temporary components of stock prices: a skeptic's view. *Journal of Business and Economic Statistics* 11, 199–207.
- Richardson, M., & Stock, J. (1989). Drawing inferences from statistics based on multiyear asset returns. *Journal of Financial Economics* 25, 323–348.
- Scholes, M., & Williams, J. (1977). Estimating betas from nonsynchronous data. *Journal of Financial Economics* 5, 309–327.