Rethinking Risk (II): *The Size and Value Effects*

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Return and risk are to finance what benefit and cost are to economics; that is, the two variables at the heart of most decisions. In finance, assessing the return an investor obtained from an asset is trivial, but assessing the risk the investor bore is not. This is simply because risk can be assessed in a wide variety of ways. Uncertainty quantified with volatility is perhaps the most pervasive, but is it the most appropriate for *long*-term investors?

In a recent article, Estrada [2014] argues that volatility may be a misleading measure of risk when investors focus on their long-term terminal wealth. He argues that stocks keep investors more uncertain about their terminal wealth than bonds, and that the ride during the holding period is bumpier with stocks than with bonds. No surprise there. But he also argues that in the long-term stocks offer both more upside potential and, when tail risks strike, better downside protection than bonds.

Put differently, investors that focus on uncertainty, both during and at the end of the holding period, are likely to view stocks as riskier than bonds. Conversely, investors that focus on their long-term terminal wealth are likely to view stocks as less risky than bonds, even if they are concerned with the downside potential of extreme and unlikely adverse scenarios.

The ultimate issue discussed in this article is whether a similar argument can be made when assessing the relative risk of small-cap and large-cap stocks, and that of value and growth stocks. In a nutshell, the answer is yes. The evidence reported here suggests that small-cap and value stocks are respectively riskier than large-cap and growth stocks when the focus is on uncertainty, and less risky when the focus is on long-term terminal wealth, even if investors are concerned with tail risks.

THE ISSUE

Using a comprehensive sample of U.S. stocks, Fama and French [1992] established that small stocks outperform large stocks and value stocks outperform growth stocks. This seminal article spanned a vast literature that ultimately confirmed the existence of the same two patterns in global markets. It also introduced the three-factor model, which implies that returns can be enhanced by increasing a portfolio's exposure to the stock market, the size factor, or the value factor.

The model, however, offers no free lunch; it implicitly argues that small and value stocks are riskier than large and growth stocks, and therefore the higher expected return is simply a compensation for a higher exposure to risk. In fact, part of the vast literature that followed the Fama-French article aimed to

identify more precisely the sources of additional risk, and ultimately to explain why small and value stocks are riskier than large and growth stocks. But are they?

Consider an investor that fully intends to stick to a long holding period, such as a young investor saving for retirement. When assessing the risk of his portfolio, this investor faces at least three important questions. First, how relevant are the short-term fluctuations in the value of his portfolio that will inevitably occur along the way? Second, under what conditions is uncertainty about his terminal wealth detrimental? And third, when focusing on terminal wealth, is uncertainty or downside potential a better way to assess risk?

To be sure, there is more than one way to answer these questions. Preferences do play a critical role; not all investors have the same ability to tolerate risk, and clearly not all of them assess risk in the same way. That said, it seems plausible that investors that intend to stick to their (long) holding period *should* be interested in their terminal wealth, and therefore in the strategies that are likely to provide them with either more upside potential, or better downside protection, particularly when tail risks strike, or both.

Following Estrada [2014], two measures of volatility are considered here, one focusing on the value of a portfolio *at the end* of the holding period (SD_E) and the other focusing on the value of a portfolio *during* the holding period (SD_D). Both SD_E and SD_D are measures of uncertainty, but assess different types of risk. The former assesses the uncertainty an investor bears about how much capital he will have at the end of the holding period; the latter assesses the uncertainty an investor bears during the holding period due to fluctuations in the value of his portfolio. See the appendix for the formal definitions of SD_E and SD_D.

In order to assess terminal wealth in the presence of tail risks, the measure used here is that introduced by Estrada [2014], namely, *lower-tail terminal wealth* (LTW_x). This measure of tail risk is defined as the average terminal wealth in the lower x% of the distribution of terminal wealth (with x being 1%, 5%, and 10%), and aims to assess terminal wealth in extreme and unlikely adverse scenarios. Again, see the appendix for the formal definition of LTW_x.

Finally, for the sake of perspective, the upper-tail terminal wealth (UTW_x), defined as the average terminal wealth in the upper x% of the distribution of terminal wealth (with x again being 1%, 5%, and 10%) is also calculated and discussed in the following section. Again, see the appendix for the formal definition of UTW_x.

EVIDENCE

The sample considered here consists of diversified portfolios of small, large, value, and growth stocks as defined by the widely used Fama-French data available from Ken French's Web page.[1] The sample period starts at the beginning of 1927 and goes through the end of 2013. Returns are annual, nominal, and account for capital gains/losses and dividends. Exhibit 1 summarizes some characteristics of the data.

An initial investment of $100 in four diversified portfolios of small, large, value, and growth stocks is assumed to remain passively invested through the end of each holding period. The holding periods considered are 20 and 30 years, both assumed to be representative of a long investment horizon. Given that there are 87 annual returns in each of the 4 portfolios, then 68 and 58 overlapping holding periods of 20 and 30 years are considered in the analysis.

The discussion largely, but not exclusively, focuses on terminal wealth (the capital accumulated at the end of the holding period) and the distribution of terminal wealth that stems from aggregating all holding periods

EXHIBIT 1
Summary Statistics

This exhibit shows, for the series of annual returns, the arithmetic (AM) and geometric (GM) mean return, standard deviation (SD), semideviation for a 0% benchmark (SSD), lowest (Min) and highest (Max) return, and terminal value at the end of the sample period of $1 invested at the beginning of the sample period (TV) for small, large, value, and growth stocks. The sample period starts at the beginning of 1927 and goes through the end of 2013. Returns are nominal and account for capital gains/losses and dividends. All figures in percent except for TV (in dollars).

	AM	GM	SD	SSD	Min	Max	TV
Small	16.7	12.7	30.3	12.4	–49.6	134.6	31,758
Large	12.9	10.6	21.9	10.0	–51.8	83.9	6,497
Value	17.1	13.2	29.0	12.1	–55.1	117.8	47,465
Growth	12.9	9.9	25.3	11.4	–41.4	102.0	3,693

of the same length. Exhibit 2 reports the results of the analysis; Panel A focuses on the comparison between small and large stocks, and Panel B on the comparison between value and growth stocks.

The Upside

The focus of the analysis is on risk, but a brief analysis of upside potential is useful for perspective. Consistent with previous evidence, Panel A of Exhibit 2 shows that small stocks deliver a higher mean terminal wealth than large stocks; the differences are 38% ($1,575 versus $1,140) and 75% ($5,941 versus $3,403) over 20-year and 30-year holding periods. Similarly, but to an even larger degree, Panel B shows that value stocks deliver a higher mean terminal wealth than growth stocks; the differences are 132% ($2,027 versus $873) and 265% ($8,571 versus $2,351) over 20-year and 30-year holding periods.

All the distributions of terminal wealth considered are positively skewed, as confirmed by the fact that for small, large, value, and growth stocks the mean terminal wealth is higher than the median terminal wealth for both holding periods. That said, as Panels A and B show, focusing on median terminal wealth does not alter the conclusion that a larger terminal wealth is expected from small and value stocks than it is from large and growth stocks.

Furthermore, as Panels A and B show, the other upside potential variables considered point in the same direction, regardless of whether the focus is on the upper 1%, 5%, or 10% of the distribution of terminal wealth (hence on UTW_1, UTW_5, and UTW_{10}). In other words, when focusing on extreme and unlikely favorable scenarios, small and value stocks outperform large and growth stocks by a substantial margin, thus offering a much higher upside potential.

Risk—Focus on Uncertainty

As already mentioned, two measures of volatility are considered in the analysis. One focuses on uncertainty at the end of the holding period (SD_E) and the other on uncertainty during the holding period (SD_D). As Exhibit 2 shows, both point in the same direction and reaffirm one of the central implications of the three-factor model; that is, small and value stocks are riskier than large and growth stocks.

Panel A shows that SD_E is higher for small stocks than for large stocks for both holding periods considered. Panel B further shows that SD_E is higher for value stocks than for growth stocks, also for both holding periods. In words, small and value stocks keep investors more uncertain about how much capital they will have at the end of the holding period than do large and growth stocks. In some cases the difference is substantial, particularly when comparing value and growth stocks.

Furthermore, Panel A shows that SD_D is higher for small stocks than for large stocks, and Panel B that SD_D is higher for value stocks than for growth stocks, in both cases for both holding periods considered. In words,

EXHIBIT 2
The Distribution of Terminal Wealth

This exhibit characterizes the distribution of terminal wealth for an initial investment of $100 in small, large, value, and growth stocks over all possible 20-year and 30-year overlapping holding periods between 1927 and 2013. The statistics reported are the mean, median, and standard deviation (SD_E), as well as the lower-tail terminal wealth (LTW_x) and upper-tail terminal wealth (UTW_x) for x equal to 1%, 5%, and 10%. The volatility of the periodic value of the portfolio during the holding period, averaged across all holding periods of the same length (SD_D), is also reported. The data is described in Exhibit 1. All figures in dollars.

Panel A

	20 Years		30 Years	
	Small	Large	Small	Large
Mean	1,575	1,140	5,941	3,403
Median	1,363	1,032	5,266	3,041
UTW_1	4,036	2,664	17,021	6,218
UTW_5	3,763	2,508	14,623	6,099
UTW_{10}	3,429	2,240	12,977	5,868
SD_E	833	588	3,183	1,267
SD_D	455	322	1,710	1,002
LTW_1	345	199	1,647	1,097
LTW_5	423	239	1,763	1,298
LTW_{10}	541	352	2,125	1,673

Panel B

	20 Years		30 Years	
	Value	Growth	Value	Growth
Mean	2,027	873	8,571	2,351
Median	1,800	730	8,057	1,989
UTW_1	5,227	1,939	18,502	6,218
UTW_5	4,554	1,845	17,488	5,487
UTW_{10}	4,085	1,741	16,423	4,938
SD_E	1,066	435	3,945	1,099
SD_D	586	245	2,449	702
LTW_1	296	225	1,578	1,085
LTW_5	374	283	2,075	1,124
LTW_{10}	499	342	2,694	1,216

small and value stocks take investors along bumpier rides during the holding period than do large and growth stocks. Again, the difference is substantial, particularly when comparing value and growth stocks.

In short, the evidence in Exhibit 2 clearly suggests two things. First, when risk is assessed with uncertainty, small and value stocks are riskier than large and growth stocks; and second, this is the case regardless of whether uncertainty is evaluated during or at the end of the holding period. Both results, however, beg a question: Is uncertainty the best way to assess risk for long-term investors?

Risk—Focus on Terminal Wealth and Tail Risks

Many investors may be perfectly aware than an aggressive strategy is expected to outperform a conservative one, particularly in the long term, and yet may stay away from the former for fear of tail risks. They may fear, for example, having to liquidate a portfolio in a year like 2008, or suffering a period of very low returns such as 2000–2009, however unlikely either event may be. In fact, both Barro [2009] and Bollerslev and Todorov [2011] argue that the observed equity risk premium is consistent with the compensation required by investors to bear tail risks.

Nobody is to tell an investor what lets him sleep at night, but at the very least the evidence may enlighten him about how plausible his fears are. Put differently, having established that small and value stocks are more volatile, both in terms of SD_E and SD_D, than are large and growth stocks, what if the evidence informed this investor that, *even if tail risks strike*, his terminal wealth is very likely to be larger by investing in small and value stocks than by investing in large and growth stocks?

Consider an investor with a 30-year holding period. Suppose this investor is concerned with 1% tail risks; that is, his terminal wealth in the lower 1% of the distribution (LTW_1).[2] Panel A of Exhibit 2 shows that when 1% tail risks stroke small stocks delivered $1,647, which is 50% more than the $1,097 delivered by large stocks; Panel B further shows that value stocks delivered $1,578, which is 45% more than the $1,085 delivered by growth stocks. In words, *in their worst 30-year period, small and value stocks outperformed large and growth stocks by a substantial margin in terms of terminal wealth*.

This conclusion can be generalized if the focus is on 5% tail risks; that is, terminal wealth in the lower 5% of the distribution (LTW_5). Over 30 years, when 5% tail risks stroke, small stocks outperformed large stocks by 36% ($1,763 versus $1,298, as shown in Panel A), and value stocks outperformed growth stocks by 85% ($2,075 versus $1,124, as shown in Panel B). And it can be generalized even further if the focus is on 10% tail risks (LTW_{10}), in which case small and value stocks outperformed large and growth stocks by 27% ($2,125 versus $1,673, as shown in Panel A) and 122% ($2,694 versus $1,216, as shown in Panel B).

Furthermore, these results also generalize for an investor with a 20-year holding period. When 1%, 5%, and 10% risks stroke, Panel A shows that small stocks outperformed large stocks by 73%, 77%, and 54%; and Panel B shows that value stocks outperformed growth stocks by 31%, 32%, and 46%.

Thus, the overall evidence clearly suggests that, *even when 1%, 5%, or 10% tail risks strike*, long-term investors are better off by investing in small and value stocks than by investing in large and growth stocks. And this is the case because, even when tail risks strike, small and value stocks provide better downside protection in the form of a larger terminal wealth than do large and growth stocks.

Further Considerations

The evidence discussed suggests that small and value stocks have more upside potential (mean, median, and UTW_x) than do large and growth stocks. It also suggests that small and value stocks are more volatile than are large and growth stocks, regardless of whether volatility is evaluated during (SD_D) or at the end (SD_E) of the holding period. And it also suggests that small and value stocks deliver a larger terminal wealth than do large and growth stocks when tail risks strike (LTW_x). In these circumstances it makes sense to ask, is volatility really detrimental, or even relevant, for a long-term investor?

Notice that, relative to large and growth stocks, small and value stocks have *both* more upside potential *and* a more limited downside potential. This suggests that the higher SD_E of small and value stocks is mostly *upside* risk; that is, largely one-sided uncertainty about *how much better* (not how much worse) they are expected to perform relative to large and growth stocks. In other

words, the uncertainty a long-term investor faces is largely about how much *more* (not less) capital he will have at the end of his holding period by investing in small and value stocks than by doing so in large and growth stocks.

The evidence discussed also suggests that, for the long-term investor focused on his terminal wealth, short-term volatility (SD_D) is largely a nuisance he has to live with.[3] That said, some investors may not be able to avoid focusing on short-term volatility, even if they realize they have nothing to gain from doing so. Very risk averse investors may be willing to give up a substantial long-term upside potential in exchange for some short-term downside protection. And portfolio managers may have to worry about short-term volatility if investors in their funds do too. For this reason, Kritzman and Rich [2002] advocate to focus on losses throughout, not just at the end, of the holding period.

All that said, it does remain the case that if an investor 1) has a long holding period to which he intends to stick; and 2) focuses on his long-term terminal wealth, then the evidence clearly suggests that he should view small and value stocks as less risky than large and growth stocks, even if he is concerned with tail risks.

ASSESSMENT

Volatility as a measure of risk, in absolute terms measured by the standard deviation of returns, or in relative terms measured by beta, is pervasive. And yet that does not make it the best way to assess risk, or the most appropriate in all circumstances. This is particularly the case when investors have a long-term perspective and focus on their terminal wealth.

The evidence discussed here is unambiguous. It clearly suggests that small and value stocks are more volatile than large and growth stocks, both during and at the end of the holding period. Short-term investors, very risk averse investors, and portfolio managers that have to deal with both are then likely to view small and value stocks as riskier than large and growth stocks.

Yet many investors do take the long view and focus on the endgame. For them, risk is leaving money on the table in the sense of ending with a smaller nest egg than they could have. One reason why this could happen is being struck by an extreme and unlikely adverse scenario, typically referred to as a tail risk.

The evidence discussed here is also unambiguous on this issue. It clearly suggests that small and value stocks are less risky than are large and growth stocks. This is the case because, when tail risks strike, small and value stocks leave investors with a larger terminal wealth than do large and growth stocks.

The evidence also suggests that although small and value stocks keep investors more uncertain about their terminal wealth than do large and growth stocks, this uncertainty is largely upside risk; that is, mostly one-sided uncertainty about how much more (not how much less) capital an investor will have at the end of the holding period by investing in small and value stocks than by investing in large and growth stocks.

The approach proposed here, focusing on terminal wealth and tail risks, aims to distinguish between assets and strategies that are simply volatile from those that are truly risky in the sense of exposing long-term investors to a large decrease in terminal wealth when tail risks strike. From this perspective, small and value stocks are not riskier than are large and growth stocks. In fact, small and value stocks offer both more upside potential and better downside protection when tail risks strike than do large and value stocks.

In short, then, this article questions a straightforward conclusion regarding the relative risk of small and large stocks, and that of value and growth stocks. Different measures of risk point in different directions, and, therefore, which asset is riskier depends on an investor's holding period and the way he assesses risk. In fact, long-term investors that focus on their terminal wealth should defy the conventional wisdom, even if they are concerned with tail risks; they should think of small and value stocks as being less risky than large and growth stocks.

APPENDIX

This appendix borrows heavily from Estrada [2014]. Consider an initial investment of $100 in four diversified portfolios of small, large, value, and growth stocks, and assume it remains passively invested through the end of each holding period. The lengths of the holding period considered are 20 and 30 years. Given that there are 87 annual returns in each portfolio, then 68 and 58 overlapping holding periods of 20 years (1927–1946, 1928–1947, …) and 30 years (1927–1956, 1928–1957, …) are considered here.

Formally, the volatility of terminal wealth across all holding periods of the same length (SD_E) is given by

$$SD_E = \left\{(1/S)\cdot\sum_{s=1}^{S}(W_{Ts} - EW_T)^2\right\}^{1/2} \quad (1)$$

where S is the number of holding periods; T is the length of the holding period; W_T is the capital accumulated at the end of each holding period of length T; EW_T is the mean terminal wealth across all holding periods of length T; and s indexes holding periods. When $T = 20$, $S = 68$; and when $T = 30$, $S = 58$.

The volatility of the periodic value of the portfolio during a given holding period, averaged across all holding periods of the same length (SD_D), is given by

$$SD_D = (1/S)\cdot\sum_{s=1}^{S}\sigma_s \quad (2)$$

where σ_s is in turn given by

$$\sigma_s = \left\{(1/T)\cdot\sum_{t=1}^{T}(W_t - EW_s)\right\}^{1/2} \quad (3)$$

where W is the periodic (in our case, annual) value of the portfolio during each holding period; EW_s is the mean wealth over each holding period; and t indexes years.

Lower-tail terminal wealth (LTW_x) is defined as the average terminal wealth in the lower x% of the distribution of terminal wealth; that is,

$$LTW_x = (1/N_L)\cdot\sum_{n=1}^{N_L}W_{Tn} \quad (4)$$

where N_L is the number of observations in the lower x% of the distribution; n indexes observations (terminal wealth levels); and x takes a value of 1, 5, and 10.

For the sake of perspective, the upper-tail terminal wealth (UTW_x), defined as the average terminal wealth in the upper x% of the distribution of terminal wealth, is also calculated and is given by

$$UTW_x = (1/N_U)\cdot\sum_{n=1}^{N_U}W_{Tn} \quad (5)$$

where N_U is the number of observations in the upper x% of the distribution.

ENDNOTES

[1]See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[2]In the sample considered here, a 1% tail risk amounts to the worst-case scenario. This is because there are 58 overlapping 30-year holding periods, and 1% of 58 rounded to the nearest integer is 1.

[3]Estrada [2013a] discusses the relationship between short-term risk and long-term risk, assessing the latter as suggested by Warren Buffett. Furthermore, Estrada [2013b] discusses in a comprehensive way the general issue of time diversification.

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