

The Retirement Glidepath: An International Perspective

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Abstract

All individuals need to decide how much to save during their working years, how much to spend during retirement, and the asset allocation of their portfolio in both periods. A portfolio's exposure to stocks and bonds affects key variables, such as the probability of portfolio failure, the degree of downside protection, and the expected bequest; how should this exposure evolve during retirement is the ultimate issue explored in this article. After considering declining-equity, rising-equity, and static glidepaths, the comprehensive international evidence from 19 countries and the world market over 110 years considered here ultimately suggests that both an all-equity portfolio and a 60-40 stock-bond allocation are simple and very effective strategies for retirees to implement.

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1. Introduction

Saving for retirement and spending during retirement requires individuals to choose their savings rate, withdrawal rate, and asset allocation. The savings rate concerns the accumulation period, the withdrawal rate concerns the retirement or distribution period, and the asset allocation concerns both periods. This last variable is the main focus of this article.

Lifecycle strategies, on which target-date funds are based, gradually decrease a portfolio's exposure to stocks, and increase its exposure to bonds, thus making the portfolio increasingly conservative over time. A well-known example of this approach is the 'age in bonds' rule, which suggests that the proportion of bonds in an individual's portfolio should be equal to his age, and that of stocks equal to 100 minus his age. Thus, an individual should have a 70-30 stock-bond allocation when he is 30 and a 35-65 allocation when he is 65. Some variations of this rule have been proposed, such as exposure to stocks equal to 110-Age or 120-Age, but their common characteristic is a declining-equity glidepath.¹

This conventional wisdom, however, is being challenged. Focusing on the accumulation period and using a comprehensive sample of 19 countries and two regions over a 110-year period, Estrada (2014a) finds that strategies that *increase* the exposure to stocks over time, thus

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¹ Dolvin et al (2010) find that most target-date funds implement similar strategies, most of them closely resembling the 120-Age rule.

making the portfolio increasingly *aggressive*, provide both a higher upside potential and better downside protection than do lifecycle strategies. Shiller (2005), Basu and Drew (2009), Arnott (2012), and Arnott et al (2013) also argue that a rising-equity glidepath may be a better choice for individuals saving for retirement than a declining-equity glidepath.

Furthermore, focusing on the retirement period, Pfau and Kitces (2014) argue that a rising-equity glidepath lowers the probability of failure relative to a declining-equity glidepath. Put differently, if retirees make their portfolios more aggressive over time, they would face a lower probability of running out of money than if they followed a lifecycle strategy. Interestingly, however, Pfau and Kitces (2014) side with the conventional wisdom (a declining-equity glidepath) during the accumulation period, and ultimately recommend a U-shaped glidepath. In other words, they suggest that individuals should gradually decrease their exposure to stocks (and increase their exposure to bonds) while saving for retirement, have a minimum exposure to stocks at the time of retirement, and then gradually increase their exposure to stocks (and decrease their exposure to bonds) during retirement.

As far as the retirement period is concerned, the comprehensive international evidence discussed in this article points in the opposite direction; that is, individuals should *decrease* their exposure to stocks (and increase their exposure to bonds) during retirement, thus making their portfolios increasingly conservative over time. Interestingly, then, the combined results of Estrada (2014a) and those discussed in this article suggest that individuals should implement an *inverted* U-shaped glidepath; that is, they should gradually increase their exposure to stocks while saving for retirement, have a maximum exposure to stocks at the time of retirement, and then gradually decrease their exposure to stocks during retirement. This recommendation, that follows from a thorough empirical analysis, is the opposite of the U-shaped glidepath that Pfau and Kitces (2014) suggest.

That said, the recommendation of an inverted U-shaped glidepath applies when considering only rising-equity and declining-equity strategies. Estrada (2014a) focuses on the accumulation period and finds support for both an all-equity strategy and a 60-40 stock-bond allocation. This article focuses on the retirement period but the evidence points in the same direction; that is, it supports a portfolio fully invested in stocks as the best overall strategy (although many retirees would be reluctant to implement it), and a 60-40 stock-bond allocation as a very effective strategy (which may be psychologically easier for retirees to implement). Given that many individuals may fail to periodically adjust their portfolios in the methodical way required by rising-equity or declining-equity glidepaths, the all-equity and the 60-40 strategies during *both* the accumulation and the retirement periods seem to have simplicity and evidence on their side.

The literature on sustainable retirement portfolios, and more specifically on asset allocation during retirement, to which this article contributes is almost exclusively based on US data. One of the main contributions of this article is to broaden the scope of the analysis by considering a comprehensive international sample consisting of 19 countries and the world market over the 110-year period between 1900 and 2009.

The rest of the article is organized as follows. Section 2 discusses in more detail the issue at stake and some of the relevant contributions on this topic; section 3 discusses the evidence; and section 4 provides an assessment. An appendix with tables concludes the article.

2. The Issue

A retiree's proper management of his nest egg requires a careful balancing of two financial risks. On the one hand, the retiree may spend too much and outlive his savings; on the other hand, the retiree may unnecessarily lower his lifestyle and end up with an unintended bequest. A massive literature on sustainable retirement portfolios ultimately seeks to guide retirees on how to properly balance these risks. It is widely acknowledged that Bengen (1994) is the seminal article that inspired the vast amount of research produced on this topic.

Bengen (1994) pioneered the idea of considering withdrawal rates over all possible historical rolling (overlapping) periods. He aimed to find how many years a portfolio would have lasted given an initial withdrawal rate and subsequent inflation-adjusted withdrawals, performing the evaluation at the beginning of every year starting in 1926.² Given a 50-50 stock-bond allocation he found that a 3% withdrawal rate would have never exhausted a portfolio in less than 50 years, and a 4% withdrawal rate would have never exhausted a portfolio in less than 33 years. He called a 5% withdrawal rate 'risky' and withdrawal rates 6% or higher 'gambling' because they would have exhausted a portfolio much sooner over many historical periods. He also called the 4% withdrawal rate 'safe' because it never exhausted a portfolio in less than 30 years, which he thought of as the minimum requirement of portfolio longevity. This was the origin of the well-known and widely-used '4% rule.'

2.1. Some Relevant Differences

The vast literature spanned by Bengen (1994) does not offer a consensus regarding a sustainable withdrawal rate for retirees. This is the case because different articles consider different methodologies, time periods, assets, asset allocations, acceptable failure rates, and

² The initial withdrawal rate is defined as the initial withdrawal relative to the value of the portfolio at the beginning of retirement. Unless otherwise stated, in this literature a 'withdrawal rate' typically refers to the *initial* withdrawal rate, implicitly assuming subsequent inflation-adjusted withdrawals. Note that this implies that the *current* withdrawal rate (the withdrawal relative to the value of the portfolio at any point in time) can fluctuate widely over time.

retirement periods, to name but some differences, and therefore reach very different conclusions both on the sustainability of the 4% withdrawal rate and on the specific withdrawal rate recommended to retirees.

Most of the articles in the literature rely on one of two methodologies, historical rolling (overlapping) periods and Monte Carlo (or bootstrapping) simulations. Bengen (1994, 1996, 1997) and Cooley et al (1998) are early applications of the first methodology; Pye (2000) and Ameriks et al (2001) are early applications of the second. Cooley et al (2003b) compare both approaches and find that their results and recommendations sometimes are similar and sometimes differ. They do not take sides on which methodology is better and ultimately argue that whichever approach happens to more accurately reflect the (unknown) distribution of future returns will produce the more plausible results and recommendations.

The articles in the literature also differ in the assets they consider. Although most articles focus on stocks and bonds, different types of stocks and bonds and different asset classes were introduced over time. Bengen (1997) considers small-cap stocks; Pye (2000) considers TIPS; Cooley et al (2003a) consider international (EAFE) stocks; Guyton (2004) considers value stocks; and Cassaday (2006) considers real estate and commodities.

An important aspect, which differs widely across the articles in the literature, is the failure rate considered to be acceptable to a retiree. In other words, different withdrawal rates imply different probabilities of portfolio depletion before the end of the retirement period, some of which a retiree may find acceptable and some of which he may not. On one extreme, Cooley et al (2003b, 2011) argue that a 25% failure rate is reasonable; on the other, Terry (2003) argues that failure rates 5% or higher are unacceptable. Spitzer et al (2007) plot a relationship between withdrawal rates and failure rates and highlight that a 4% withdrawal rate can be thought of as safe as long as a 6% probability of failure is acceptable.

The retirement periods considered in the literature also vary widely. Although 30 years seems to be by far the most widely-used alternative (and the one used in this article), on one extreme Cooley et al (2005) focus on a five-year period, and on the other Blanchett and Frank (2009) consider up to 50 years. Some articles take a different approach and base the *expected* retirement period on mortality tables, such as Milevsky and Robinson (2005), Stout and Mitchell (2006), and Sheikh et al (2014).

Finally, many articles in the literature consider an initial withdrawal rate and subsequent inflation-adjusted withdrawals, such as Bengen (1994, 1996), who pioneered the approach. Many other articles, however, consider a wide variety of dynamic withdrawal rules, most of them depending on portfolio performance. Some add simple floors and ceilings to the withdrawals, such as Bengen (2001) and Jaconetti et al (2013); some add more complex floors and ceilings,

such as Guyton and Klinger (2006) and Stout (2008);³ some make periodic re-evaluations of life expectancy (Dus et al, 2005), the probability of failure (Blanchett and Frank, 2009), or several variables (Sheikh et al, 2014); and some link the withdrawal rate to fundamental variables such as the cyclically-adjusted P/E ratio (Kitces, 2008; Pfau, 2011; and Kitces and Pfau, 2014).

2.2. The Evolution of Asset Allocation During Retirement

Most of the articles in the literature consider different asset allocations. In his pioneering article, for example, Bengen (1994) bases most of his discussion on a 50-50 stock-bond allocation but also considers portfolios with 0%, 25%, 75%, and 100% in stocks (and the rest in bonds). Considering different asset allocations, however, is different from considering how the asset allocation should *evolve* during retirement, which is the focus of this article.⁴ Three possibilities are considered here, namely, declining-equity (DE) strategies, rising-equity (RE) strategies, and static strategies.

Bengen (1994) does not explicitly consider the evolution of the asset allocation during retirement, but he does recommend a 50-75% exposure to stocks and argues that it “can be maintained throughout retirement.” Bengen (1996), in turn, explicitly considers whether the asset allocation should be adjusted during the retirement period. More precisely, he considers annual reductions in the allocation to stocks between 0.5% and 3%; finds a negative relationship between the rate of decrease of the allocation to stocks and sustainable withdrawal rates; and ultimately recommends to phase down the exposure to stocks at the annual rate of 1% (as the ‘age in bonds’ rule would). Sheikh et al (2014) also recommend a DE strategy, and therefore an increasingly-conservative portfolio, during retirement.

Unsurprisingly, not everybody agrees with this recommendation. In fact, some argue just the opposite and recommend an RE strategy. Spitzer and Singh (2006, 2007) suggest that retirees should first make withdrawals from the bond portion of their portfolios, and start withdrawing from stocks only after bonds are depleted. This recommendation would gradually reduce the exposure to bonds in the portfolio, thus implying an RE glidepath and an increasingly aggressive portfolio. Pfau and Kitces (2014) explicitly compare DE and RE strategies during retirement and find that the latter, which they recommend, expose retirees to a lower probability of failure.

³ It is far from clear that more complex rules improve upon simpler ones. In fact, some of the complex rules in the literature seem to be meticulously designed to work well (or better than simpler alternatives) in sample. This overfitting of the data often leads to poor behavior out of sample.

⁴ The articles that consider different asset allocations, but not its evolution during the retirement period, tend to agree that a higher exposure to stocks is more likely to support a higher withdrawal rate. Early recommendations, such as Cooley et al (1998), suggest an exposure to stocks of at least 50%; Bengen (1994) recommends a 50-75% exposure, and Milevsky et al (1997) argue that many retirees would benefit from a 70-100% exposure.

An intermediate possibility is a static or constant-equity strategy. Blanchett (2007) considers several types of rising/declining/static-equity strategies; finds that despite their simplicity static allocations are “remarkably efficient” distribution strategies; and concludes that a 60-40 stock-bond allocation is likely to be optimal for most retirees. Cohen et al (2010) argue that for any given DE strategy, a static strategy with a higher risk-adjusted return can be created and ultimately recommend a 32-68 stock-bond static allocation for retirees. Kitces and Pfau (2014) also consider several types of rising/declining/static-equity strategies and find that a 60-40 stock-bond allocation is nearly optimal in most situations. The results discussed in the next section also yield support both to static strategies in general and (the all-equity strategy notwithstanding) to a 60-40 stock-bond allocation in particular.

A final possibility is a strategy in which the exposure to stocks neither declines or rises at a predetermined rate nor does it remains constant; rather, the asset allocation is dynamically adjusted depending on the value of some observable (technical or fundamental) variable. Garrison et al (2010), for example, use a 12-month moving average of large-cap stocks to determine whether a portfolio should be fully invested in bonds or stocks. Pfau (2012), in turn, uses the cyclically-adjusted P/E ratio to determine whether the exposure to stocks should be 25%, 50%, or 75%, with the rest invested in bonds. Both articles find support for a dynamic, valuation-based asset allocation approach.

3. Evidence

The vast majority of the evidence on sustainable retirement portfolios is based on US data. Exceptions are Ho et al (1994) and Milevski et al (1997) that use Canadian data; Dus et al (2005) that use German data; and Pfau (2010) that uses a sample similar to that used in this article but focusing on the narrow question of the sustainability of the 4% rule. Hence, one of the main contributions of this article is to provide a broad international perspective on the impact of asset allocation on the sustainability of retirement portfolios.

3.1. Data and Methodology

The sample considered here is the Dimson-Marsh-Staunton (DMS) dataset, described in detail in Dimson, Marsh, and Staunton (2002). The sample contains annual returns for stocks and government bonds over the 1900-2009 period. Returns for individual countries are real (adjusted by local inflation) and in local currency; returns for the world market are real (adjusted by US inflation) and in dollars. In all cases returns account for both capital gains/losses and cash flows (dividends or coupons). Exhibit A1 in the appendix summarizes some characteristics of all the series of stock and bond returns in the sample.

The analysis is based on a \$1,000 nest egg at the beginning of retirement, an initial withdrawal rate of 4%, and a 30-year retirement period. At the beginning of each year, \$40 are withdrawn, the portfolio is rebalanced right after to the target allocation for the year, and then it compounds at the observed return of stocks and bonds for that year. This process is repeated at the beginning of each year during the 30-year retirement period, at the end of which the portfolio has a terminal wealth or bequest that may be positive or 0.⁵ The first 30-year retirement period is 1900-1929, the next is 1901-1930, and the last two are 1979-2008 and 1980-2009, for a total of 81 rolling (overlapping) periods.

The analysis focuses on the failure rate, defined as the proportion of the 81 retirement periods in which the portfolio is depleted before 30 years; if history is any guide, this failure rate should be a good proxy for the expected probability of portfolio failure. The analysis also focuses on the distribution of terminal wealth or bequest, which results from aggregating the 81 wealth levels at the end of each of the 81 retirement periods considered.

3.2. Dynamic Strategies

As already discussed, DE strategies feature allocations that become more conservative over time. Four such strategies are considered here. The first starts fully invested in stocks and ends up fully invested in bonds; that is, it starts with a 100-0, and ends with a 0-100, stock-bond allocation. The other three DE strategies considered are similar. They begin with 90-10, 80-20, and 70-30, and respectively end with 10-90, 20-80, and 30-70 stock-bond allocations. In all cases, the asset allocation between the beginning and the end of each 30-year retirement period changes annually and linearly over time.

These four DE strategies are evaluated against their mirrors; that is, RE strategies that start and end with opposite allocations to stocks and bonds. To illustrate, the DE strategy that starts fully invested in stocks and ends fully invested in bonds is evaluated against a mirror strategy that starts fully invested in bonds and ends fully invested in stocks; the DE strategy that starts with a 90-10 (and ends with a 10-90) stock-bond allocation is evaluated against a mirror strategy that starts with a 10-90 (and ends with a 90-10) stock-bond allocation; and so forth. Importantly, note that DE strategies and their respective RE mirrors spend the same amount of time invested in stocks and bonds only differing on when they do so.

Results for these eight dynamic strategies are presented in Exhibit 1 for the US, the world market, and the average country (that is, a cross-sectional average of all 19 countries in the sample). The exhibit reports failure rates and some characteristics of the distribution of terminal

⁵ Given that the analysis is performed in real terms, an annual withdrawal of \$40 is equivalent to an initial withdrawal of \$40 and subsequent inflation-adjusted annual withdrawals.

wealth or bequest across the 81 retirement periods considered. Exhibit A2 in the appendix presents the same analysis on a country-by-country basis.

Exhibit 1: Dynamic Strategies

This exhibit shows summary statistics for eight declining-equity and rising-equity strategies evaluated over 81 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1980-2009. All strategies consider a starting capital of \$1,000 and annual withdrawals of \$40 in real terms; start invested in stocks in the proportion indicated by the left end of the ranges shown in the first row; finish invested in stocks in the proportion indicated by the right end of those ranges; and linearly shift between stocks and bonds, rebalancing at the beginning of each year, from the left to the right end of the ranges. The failure rate (Failure) is the proportion of the 81 retirement periods in which the portfolio was depleted before 30 years. The statistics that describe the distribution of terminal wealth or bequest across the 81 retirement periods include the mean; median; standard deviation (SD); average wealth in the lower 1% (P1), 5% (P5), and 10% (P10) tail; and average wealth in the upper 1% (P99), 5% (P95), and 10% (P90) tail. The third panel (Average) shows averages across the 19 countries in the sample. The data is described in Exhibit A1 in the appendix. All figures in dollars except for failure rates (in %).

	100-0	0-100	90-10	10-90	80-20	20-80	70-30	30-70
<i>USA</i>								
Failure	8.6	21.0	6.2	17.3	4.9	11.1	4.9	8.6
Mean	1,388	851	1,336	901	1,283	954	1,230	1,009
Median	947	171	873	293	908	424	951	527
P99	7,017	4,102	6,304	3,958	5,613	4,006	4,950	4,029
P95	5,913	3,824	5,493	3,553	5,063	3,468	4,627	3,476
P90	4,781	3,579	4,490	3,323	4,233	3,234	3,967	3,185
SD	1,412	1,178	1,314	1,108	1,224	1,056	1,146	1,027
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	9	0	14	0	45	0	67	2
<i>World</i>								
Failure	14.8	38.3	14.8	38.3	17.3	38.3	18.5	35.8
Mean	1,229	593	1,146	638	1,065	685	987	734
Median	648	258	672	382	615	439	594	520
P99	6,341	2,232	5,817	2,447	5,305	2,669	4,807	2,990
P95	5,499	2,175	4,971	2,330	4,555	2,618	4,215	2,913
P90	5,039	2,078	4,619	2,213	4,212	2,357	3,819	2,546
SD	1,525	739	1,392	740	1,269	765	1,157	814
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	0	0	0	0	0	0	0	0
<i>Average</i>								
Failure	31.4	49.4	31.1	45.7	31.5	42.8	32.2	39.4
Mean	1,330	710	1,267	761	1,204	819	1,139	880
Median	579	102	551	169	520	244	511	320
P99	12,333	6,239	11,296	6,374	10,397	6,658	9,631	7,100
P95	8,991	4,749	8,416	4,923	7,866	5,162	7,315	5,471
P90	6,659	3,782	6,278	3,906	5,920	4,101	5,567	4,338
SD	2,214	1,254	2,069	1,286	1,933	1,336	1,805	1,402
P1	0	0	0	0	0	0	0	0
P5	0	0	1	0	1	0	0	0
P10	7	0	8	0	8	0	7	0

Consider the results for the US first. The failure rates for the DE strategies (8.6%, 6.2%, 4.9%, and 4.9%) are in all cases substantially lower than those for their respective RE mirrors (21.0%, 17.3%, 11.1%, and 8.6%). In fact, failure rates for the former are less than half those for

the latter, with the exception of the 70-30 strategy and its mirror. Thus, DE strategies during retirement lead to much lower expected probabilities of portfolio failure.

DE strategies also accumulate more wealth by the end of the retirement period than do RE strategies. This is the case regardless of whether the expected bequest is evaluated with the mean terminal wealth (\$1,388, \$1,336, \$1,283, and \$1,230 versus \$851, \$901, \$954, and \$1,009) or the median terminal wealth (\$947, \$873, \$908, and \$951 versus \$171, \$293, \$424, and \$527). Furthermore, in particularly good retirement periods (those occurring less than 1%, 5%, or 10% of the time and quantified by P99, P95, and P90 in Exhibit 1), DE strategies expose retirees to a much higher upside potential than do RE strategies.

DE strategies keep retirees more uncertain about their terminal wealth or bequest than do RE strategies, as indicated by their larger standard deviations (\$1,412, \$1,314, \$1,224, and \$1,146 versus \$1,178, \$1,108, \$1,056, and \$1,027).⁶ But consider the P1, P5, and P10 figures, which represent the average terminal wealth during particularly bad retirement periods, defined as those that occur less than 1%, 5%, or 10% of the time. These figures are what Estrada (2014*b*, 2014*c*) defines as lower-tail terminal wealth, a measure of long-term risk that focuses on extreme and unlikely adverse scenarios; and in this case they show that DE strategies provide the same or better downside protection (in the form of a higher terminal wealth or bequest) when tail risks strike than do RE strategies.

For this reason, it is problematic to view DE strategies as riskier than RE strategies. As argued by Estrada (2014*b*, 2014*c*), standard deviation is a questionable measure of risk if, relative to a less volatile strategy, a more volatile strategy provides *both* a higher upside potential and better downside protection when tail risks strike, as is the case here when comparing DE and RE strategies. In such cases, a larger standard deviation only indicates higher uncertainty about how much *better off* (not worse off) a retiree will be after 30 years, which may be thought of as *upside* risk.

Results for the world market are similar. As shown in the 'World' panel of Exhibit 1, relative to RE strategies, DE strategies have substantially lower failure rates; accumulate larger mean and median bequests; have higher upside potential in particularly good periods; provide the same downside protection in particularly bad periods;⁷ and keep retirees more uncertain about their bequest, but only in terms of how much higher (not lower) it will be.

⁶ Note that the standard deviations reported do not assess how much a retirement portfolio fluctuates *over time*; rather, they assess the dispersion of the distribution of *terminal* wealth or bequest. In other words, these standard deviations measure uncertainty about the bequest, not the variability of a portfolio during the retirement period.

⁷ Note that the 0s for all strategies in the worst 1%, 5%, and 10% scenarios indicate that all strategies fail at least 10% of the time, which is obviously confirmed by the failure rates in the first row of the panel.

Finally, as the 'Average' panel of Exhibit 1 indicates, results are once again similar for the average country in the sample. Relative to RE strategies, DE strategies have substantially lower failure rates, though the differences are not as large as they are for the US and the world market; accumulate larger mean and median bequests; have higher upside potential in particularly good periods; provide overall better downside protection in particularly bad periods; and keep retirees more uncertain about how much higher (not lower) their bequest will be. As already mentioned, Exhibit A2 in the appendix reports results of a similar analysis on a country-by-country basis.

3.3. Static Strategies

Contrary to the findings and recommendation of Pfau and Kitces (2014) for the US, based on Monte Carlo simulations, the evidence from the comprehensive international sample considered here, based on historical rolling periods, suggests that individuals should gradually *reduce* their exposure to stocks (and increase their exposure to bonds) during retirement. Doing so instead of the opposite would expose retirees to a lower probability of failure, a higher expected bequest, a higher upside potential in particularly good periods, and better downside protection in particularly bad periods. Should retirees make their portfolios increasingly conservative over time then? Not necessarily.

The first two columns of Exhibit 2 report averages for the four DE (100-0, 90-10, 80-20, and 70-30) and RE (0-100, 10-90, 20-80, and 30-70) strategies considered in Exhibit 1. The next three columns, labeled 60×30, 50×30, and 40×30, report results for three static strategies that respectively invest 60%, 50%, and 40% in stocks (and the rest in bonds) during the 30-year retirement period, rebalancing to those proportions at the beginning of each year. The reason for focusing on these three strategies at this time is simply because they are neither too aggressive nor too conservative; more static strategies are considered in the next section. Given that the strategy that invests 60% in stocks displays the best overall behavior among the three static strategies considered in this section, the discussion that follows is focused on its comparison to DE and RE strategies.

As before, consider the US first. The 60×30 strategy has a lower failure rate than both the average DE and the average RE strategies considered here. In fact, it ties for the lowest failure rate of all those reported in Exhibit 1 (those for the 80-20 and 70-30 strategies) at 4.9%. At the same time, the 60×30 strategy outperforms all other strategies in terms of the expected bequest, with mean and median values of \$1,437 and \$1,155. In terms of upside potential in particularly good periods (P99, P95, and P90), the 60×30 strategy outperforms all the RE strategies, and underperforms all but one of the DE strategies, considered in Exhibit 1.

Exhibit 2: Static Strategies

This exhibit shows summary statistics for three static strategies evaluated over 81 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1980-2009, as well as averages for the four declining-equity (DE) and the four rising-equity (RE) strategies in Exhibit 1. The static strategies remain invested 60%, 50%, and 40% in stocks during 30 years (respectively labeled 60×30, 50×30, and 40×30), with the rest invested in bonds. All strategies consider a starting capital of \$1,000, annual withdrawals of \$40 in real terms, and rebalancing at the beginning of each year. The failure rate (Failure) is the proportion of the 81 retirement periods in which the portfolio was depleted before 30 years. The statistics that describe the distribution of terminal wealth or bequest across the 81 retirement periods include the mean; median; standard deviation (SD); average wealth in the lower 1% (P1), 5% (P5), and 10% (P10) tail; and average wealth in the upper 1% (P99), 5% (P95), and 10% (P90) tail. The third panel (Average) shows averages across the 19 countries in the sample. The data is described in Exhibit A1 in the appendix. All figures in dollars except for failure rates (in %).

	DE	RE	60×30	50×30	40×30
<i>USA</i>					
Failure	6.2	14.5	4.9	7.4	9.9
Mean	1,309	929	1,437	1,119	841
Median	920	354	1,155	804	543
P99	5,971	4,024	4,615	4,227	3,924
P95	5,274	3,580	4,428	3,931	3,436
P90	4,368	3,330	4,079	3,497	3,012
SD	1,274	1,092	1,202	1,042	913
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	34	1	86	22	0
<i>World</i>					
Failure	16.4	37.7	16.0	28.4	37.0
Mean	1,107	663	1,062	850	682
Median	632	400	721	578	419
P99	5,568	2,584	4,307	3,859	3,400
P95	4,810	2,509	3,869	3,549	3,210
P90	4,422	2,298	3,493	3,085	2,739
SD	1,336	764	1,083	959	846
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i>Average</i>					
Failure	31.6	44.3	31.2	34.6	40.4
Mean	1,235	792	1,235	1,008	808
Median	540	209	604	435	275
P99	10,914	6,593	9,936	8,257	6,756
P95	8,147	5,076	7,424	6,292	5,276
P90	6,106	4,032	5,752	4,897	4,139
SD	2,005	1,320	1,849	1,580	1,339
P1	0	0	0	0	0
P5	1	0	1	0	0
P10	8	0	11	1	0

The 60×30 strategy has a lower standard deviation than the average DE strategy, and a higher standard deviation than the average RE strategy. However, as already mentioned, downside protection measured by terminal wealth when tail risks strike may be a more relevant measure of long-term risk. And in this regard, as the P1, P5, and P10 figures show, the 60×30 strategy provides the same or better downside protection than all the DE and RE strategies

considered here. Thus, the 60×30 strategy has the lowest failure rate, the largest expected bequest, and provides the best overall downside protection in particularly bad periods.

As the 'World' panel of Exhibit 2 shows, the overall results for the world market are rather similar to those for the US. The 60×30 strategy has a lower failure rate (16.0%) than both the average DE and RE strategies, although it is marginally higher than the failure rate of the 100-0 and 90-10 strategies (14.8%). It is, however, less than half of the failure rate of all the RE strategies reported in Exhibit 1. The 60×30 strategy underperforms some DE strategies in terms of the mean bequest, but outperforms all DE and RE strategies in terms of the median bequest. When (1%, 5%, and 10%) tail risks strike, all the strategies considered leave retirees with their portfolios depleted, thus providing no downside protection in particularly bad periods.

Finally, as the 'Average' panel of Exhibit 2 shows, the results are again similar for the average country in the sample. The failure rate of the 60×30 strategy (31.2%) is lower than that of the average DE strategy (but very marginally higher than the 31.1% of the 90-10 strategy), and substantially lower than that of all the RE strategies. The 60×30 strategy underperforms two DE strategies in terms of the mean bequest, but outperforms all DE and RE strategies in terms of the median bequest. And when (1%, 5%, and 10%) tail risks strike, the 60×30 strategy provides the best overall downside protection.

In short, then, a simple static strategy that invests 60% in stocks and 40% in bonds, rebalanced once a year, produces the best overall performance. It does not outperform all DE and RE strategies in every characteristic considered, but it does provide retirees with the lowest or near-lowest failure rates, the highest or near-highest expected bequest, good upside potential, and overall best downside protection.

3.4. Why Not 100% Stocks?

Many retirees would view having their portfolio fully, or even heavily, invested in stocks as a very risky strategy, and few advisors would dare to recommend it. Although risk is in the eyes of the beholder, the interesting question is *why* an all-equity portfolio is considered such a risky alternative. The answer, in fact, is far from clear. To shed light on this issue, 11 static strategies were considered, with allocations to stocks of 100%, 90%, 80%, ..., 20%, 10%, and 0%, and the rest being invested in bonds. Only the results for the best of these strategies (as defined below) for each country are reported here.

Exhibit 3 shows in the first column results for a strategy that remains fully invested in stocks during the 30 years of retirement (100×30) for the US, the world market, and the average country. The second column (Best Dynamic) shows results for the best of the eight dynamic strategies reported in Exhibit 1, defined as the one with the lowest failure rate. (If two or more strategies have the same failure rate, the one with the highest mean bequest is selected as the

best strategy.) The third column (Best Static) shows results for the best of the static strategies mentioned above, *not including* the strategy that fully invests in stocks.⁸ Finally, the last column shows results for a strategy that remains invested 60% in stocks over 30 years (60×30); this column is the same as the third column of Exhibit 2. Exhibit A4 in the appendix reports results of a similar analysis on a country-by-country basis.

Consider again the US first and ask why (in fact, *whether*) the 100×30 strategy is riskier than the other strategies shown in the exhibit. Relative to the best dynamic and static strategies considered here, the strategy that fully invests in stocks has the lowest failure rate (tied with the best static strategy, which in this case is 90% invested in stocks) at 3.7%; provides the same or better downside protection when tail risks strike (measured by P1, P5, and P10); and provides much higher upside potential (measured by the mean, median, P90, P95, and P99). Therefore, as discussed before, the higher standard deviation of this strategy only indicates uncertainty about how much better off (not worse off) a retiree will be after 30 years.

Results for the world market and the average country in the sample are similar. Relative to the best dynamic and static strategies considered here, the strategy that fully invests in stocks has the lowest failure rate (6.2% for the world market and 26.4% for the average country), provides the same or better downside protection when tail risks strike, and provides much higher upside potential. The only exception to this statement is in terms of downside protection as measured by P1 for the average country in the sample; in this case, the best static strategy provides a slightly higher terminal wealth (\$12 versus \$3 for the all-equity strategy). Importantly, note that this panel shows *average* figures for the best static strategy for each country, and which one is the best changes across countries.⁹ In other words, no *single* static strategy can be said to provide better downside protection when 1% tail risks strike than the all-equity strategy.

In short, although a strategy that fully invests a retirement portfolio in stocks can be *perceived* as riskier than most alternatives, is that really the case? Is a strategy that has the lowest probability of failure, provides the same or better downside protection, and higher upside potential really riskier than other strategies simply because a retiree is more uncertain about (how much *higher* will be) his bequest? If not, then having a retirement portfolio fully invested in stocks is a strategy that should be seriously considered by retirees.

Exhibit 3: Why Not 100% Stocks?

⁸ In other words, this column shows the best of the 10 static strategies that invest 90%, 80%, ..., 10%, or 0% in stocks. The strategy that invests 100% in stocks is shown separately in the first column (100×30).

⁹ To illustrate, although for most countries the best static strategy is 90% in stocks, for Switzerland and Japan it is 50% in stocks. As already mentioned, the best static strategy excludes the all-equity strategy.

This exhibit shows summary statistics for strategies evaluated over 81 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1980-2009. All strategies consider a starting capital of \$1,000, annual withdrawals of \$40 in real terms, and rebalancing at the beginning of each year. The failure rate (Failure) is the proportion of the 81 retirement periods in which the portfolio was depleted before 30 years. The statistics that describe the distribution of terminal wealth or bequest across the 81 retirement periods include the mean; median; standard deviation (SD); average wealth in the lower 1% (P1), 5% (P5), and 10% (P10) tail; and average wealth in the upper 1% (P99), 5% (P95), and 10% (P90) tail. Two strategies remain 100% and 60% invested in stocks during 30 years (respectively labeled 100×30 and 60×30), with the rest invested in bonds. The 'Best Dynamic' strategy is the one with the lowest failure rate of all those reported in Exhibit 1; in case of equal failure rates, the strategy selected is the one with the highest mean. The 'Best Static' strategy is the one with the lowest failure rate among those invested 90%, 80%, ..., 10%, or 0% in stocks; in case of equal failure rates, the strategy selected is the one with the highest mean. The third panel (Average) shows averages across the 19 countries in the sample. The data is described in Exhibit A1 in the appendix. All figures in dollars except for failure rates (in %).

	100×30	Best Dynamic	Best Static	60×30
<u>USA</u>				
Failure	3.7	4.9	3.7	4.9
Mean	3,077	1,283	2,607	1,437
Median	2,505	908	2,085	1,155
P99	11,858	5,613	9,360	4,615
P95	10,851	5,063	8,250	4,428
P90	9,263	4,233	7,260	4,079
SD	2,764	1,224	2,171	1,202
P1	0	0	0	0
P5	20	0	7	0
P10	149	45	149	86
<u>World</u>				
Failure	6.2	14.8	7.4	16.0
Mean	2,367	1,229	1,968	1,062
Median	1,945	648	1,696	721
P99	8,641	6,341	6,687	4,307
P95	7,708	5,499	6,110	3,869
P90	6,944	5,039	5,614	3,493
SD	2,062	1,525	1,692	1,083
P1	0	0	0	0
P5	0	0	0	0
P10	46	0	7	0
<u>Averages</u>				
Failure	26.4	29.3	26.8	31.2
Mean	2,360	1,239	1,894	1,235
Median	1,304	564	1,084	604
P99	18,318	9,999	14,446	9,936
P95	13,063	7,800	10,247	7,424
P90	10,094	5,948	7,940	5,752
SD	3,293	1,922	2,578	1,849
P1	3	0	12	0
P5	36	1	33	1
P10	84	10	71	11

3.5. Some Further Considerations

This section briefly considers five issues that complement the discussion of the previous three sections. First, it is important to highlight once again that the focus of the analysis here is on the evolution of the asset allocation during retirement, not on the initial withdrawal rate, which is considered to be 4%. Because when 1% and 5% tail risks strike most of the strategies

considered leave retirees with their portfolios depleted before 30 years, there seems to be little reason to explore higher withdrawal rates. That said, all the implications of the analysis above hold when the initial withdrawal rate considered is 3%. The implications of the analysis also hold for retirement periods of 25 and 35 years.

Second, how safe has the '4% rule' been *globally* over a 30-year retirement period? Results obviously vary by country, but as Exhibit 4 shows, the overall conclusion is not very optimistic. Based on the best strategy of those considered here (that is, the one that yields the lowest failure rate), the '4% rule' has never failed in Canada and New Zealand; had a low failure rate in Denmark (1.2%), South Africa (2.5%), Australia (3.7%), and the US (3.7%); and a high failure rate in Belgium (50.6%), France (56.8%), and Italy (64.2%). The failure rates for the world market and for the average country in the sample have been 6.2% and 26.4%. Hence, consistent with the results reported by Pfau (2010), the '4% rule' has not been nearly as safe globally as it has been in the US.

Exhibit 4: How Safe Is the 4% Rule Globally?

This exhibit shows failure rates (Failure) for 19 countries, the world market, and the average country (the average across the 19 countries in the sample). Failure rates are defined as the proportion of the 81 retirement periods considered in which the portfolio was depleted before 30 years, and are shown for the strategy that yields the minimum value of all those considered here. The data is described in Exhibit A1 in the appendix. All figures in %.

Country	Failure	Country	Failure	Country	Failure
Australia	3.7	Ireland	28.4	Spain	40.7
Belgium	50.6	Italy	64.2	Sweden	12.3
Canada	0.0	Japan	37.0	Switzerland	23.5
Denmark	1.2	Netherlands	19.8	UK	6.2
Finland	34.6	New Zealand	0.0	USA	3.7
France	56.8	Norway	37.0	World	6.2
Germany	49.4	South Africa	2.5	Averages	26.4

Third, Pfau and Kitces (2014) find support for RE strategies during retirement and justify their findings with the notion of sequence of returns risk. This concept highlights the fact that, when withdrawals are made from a portfolio, the same set of returns has a very different impact depending on *when* the returns occur. More precisely, if large negative returns occur at the beginning of the retirement period, the portfolio is far more likely to be depleted than if the same returns occurred by the end of such period; see, for example, Basu et al (2012). This is a plausible argument and perhaps applies to the simulations discussed in Pfau and Kitces (2014). However, the support for DE strategies found here (at least when compared to RE strategies) calls into question how relevant sequence of returns risk has been *empirically*, when evaluated on the basis of a comprehensive international sample over a long period of time. In other words, however plausible in theory, sequence of returns risk does not seem to have been a key determinant of portfolio failure in this broad sample.

Fourth, it may be argued that the mean and median terminal wealth are critical when evaluating strategies during the accumulation period but are far less relevant when focusing on the retirement period. This may be so because in the first case the mean and median terminal wealth measure the size of the nest egg, whereas in the second case they measure the size of an unintended bequest. Although this may be a plausible argument, it is important to keep in mind that all the strategies evaluated here were subject to the same 4% spending rule. In other words, everything else equal, *including the same level of withdrawals*, a higher bequest does make a strategy more desirable.

Finally, note that the asset allocations considered in this article either increase or decrease the equity exposure at a predetermined rate, or they keep it constant. There is no attempt here to tie the evolution of the asset allocation to the time-varying valuation of stocks, as Garrison et al (2010) and Kitces and Pfau (2014) do. Although neither approach is excessively complicated, most retirees are unlikely to be able to properly implement valuation-based asset allocation unless they are helped by a financial planner.

4. Assessment

What proportion of the portfolio to spend and the portfolio's asset allocation are two of the most important decisions a retiree needs to make periodically. This article focuses on the latter and at the same time provides some insight on the sustainability of the former. The recommendations here are based on results from a comprehensive international sample of 19 countries and the world market over a 110-year period.

Pfau and Kitces (2014) compare declining-equity and rising-equity strategies during retirement and recommend retirees to implement the latter. However, the results discussed here point in the opposite direction; that is, when deciding exclusively between these two types of strategies, retirees should choose a declining-equity glidepath. In fact, making their portfolios increasingly conservative (rather than aggressive) during retirement would help retirees to lower the probability of portfolio failure, increase their expected bequest, and obtain better downside protection when tail risks strike.

Kitces and Pfau (2014) temper their previous enthusiasm for rising-equity strategies, argue that a static 60-40 stock-bond allocation is nearly optimal in most situations, and conclude that it is "remarkably effective as a retirement asset allocation." Similarly, Blanchett (2007) finds that despite their simplicity static allocations are "remarkably efficient" distribution strategies and concludes that "the optimal allocation for most retirees is likely a balanced portfolio, such as a 60 percent equity and 40 percent fixed income/cash allocation." These conclusions, based on evidence from the US, point in the same direction as those from this article, based on evidence from a comprehensive international sample.

More precisely, the international evidence discussed here suggests that a static 60-40 stock-bond allocation would, relative to dynamic and more complicated asset allocation strategies, lead retirees to face lower probabilities of portfolio failure, expect higher bequests, and obtain better downside protection when tail risks strike. In fact, both evidence and simplicity support this static strategy, which is particularly relevant for those retirees that may fail to periodically adjust their portfolios in the methodical way required by rising-equity or declining-equity glidepaths. Furthermore, the combined results in Estrada (2014a) and those discussed here support the implementation of this simple strategy during *both* the accumulation and the retirement periods.

That said, although many individuals would be reluctant to hold portfolios fully invested in stocks, the evidence does support this strategy. In fact, during both the accumulation period (Estrada, 2014a) and the retirement period considered here, an all-equity strategy generally outperforms all others, including the 60-40 allocation, in terms of the probability of failure, upside potential, and downside protection when tail risks strike. This last variable is particularly important because, as a measure of risk, it is (or should be) far more relevant to retirees than uncertainty about their bequest or time variability in the value of their portfolio.

The financial world is becoming increasingly complex, often for all the wrong reasons; and yet simple strategies, however underrated, are sometimes hard to beat. This certainly applies to the many and varied recommendations that retirees have received from financial planners over the years. And yet a simple, static all-equity portfolio or a 60-40 stock-bond allocation are not only easy for retirees to implement but also supported by the comprehensive evidence discussed here. Their exposure to stocks and bonds is of course not the only important financial decision retirees need to make, but implementing a simple and effective asset allocation certainly is a good starting point.

Appendix

Exhibit A1: Summary Statistics

This exhibit shows, for the series of annual returns, the arithmetic (AM) and geometric (GM) mean return, standard deviation (SD), semideviation for a 0% benchmark (SSD), and lowest (Min) and highest (Max) return for all the stock and government bond markets in the Dimson-Marsh-Staunton (DMS) dataset over the 1900-2009 period. Returns for individual countries are real (adjusted by local inflation) and in local currency; returns for the world market are real (adjusted by US inflation) and in dollars. In all cases returns account for both capital gains/losses and cash flows (dividends or coupons). All figures in %.

	AM	GM	SD	SSD	Min	Max
<i>Stocks</i>						
Australia	9.1	7.5	18.2	9.3	-42.5	51.5
Belgium	5.2	2.5	23.6	12.6	-57.1	109.5
Canada	7.2	5.8	17.2	8.5	-33.8	55.2
Denmark	6.7	4.9	20.7	8.9	-49.2	107.8
Finland	9.1	5.1	30.3	14.1	-60.8	161.7
France	5.7	3.1	23.5	12.6	-42.7	66.1
Germany	8.1	3.0	32.2	15.1	-90.8	154.6
Ireland	6.5	3.8	23.1	12.2	-65.4	68.4
Italy	6.2	2.1	29.0	15.8	-72.9	120.7
Japan	8.6	3.8	29.8	15.5	-85.5	121.1
Netherlands	7.1	4.9	21.8	10.4	-50.4	101.6
New Zealand	7.6	5.9	19.7	9.2	-54.7	105.3
Norway	7.2	4.1	27.4	11.9	-53.6	166.9
South Africa	9.5	7.2	22.5	9.2	-52.2	102.9
Spain	6.0	3.8	22.1	11.1	-43.3	99.4
Sweden	8.6	6.2	22.8	10.9	-43.6	89.8
Switzerland	6.1	4.3	19.8	10.3	-37.8	59.4
UK	7.2	5.3	20.0	9.9	-57.1	96.7
USA	8.2	6.2	20.3	10.6	-38.0	56.5
World	6.9	5.4	17.7	9.4	-40.4	70.1
<i>Bonds</i>						
Australia	2.3	1.4	13.2	7.7	-26.6	62.2
Belgium	0.6	-0.1	12.0	8.3	-30.6	40.5
Canada	2.5	2.0	10.4	5.5	-25.9	41.7
Denmark	3.6	3.0	11.6	5.1	-18.2	50.1
Finland	1.0	-0.3	13.7	11.1	-69.5	30.2
France	0.7	-0.2	13.0	9.7	-43.5	35.9
Germany	0.7	-2.0	15.6	12.7	-95.0	62.5
Ireland	2.1	1.1	14.6	7.9	-34.1	61.2
Italy	-0.4	-1.6	14.1	11.9	-64.3	28.7
Japan	1.5	-1.2	20.1	15.0	-77.5	69.8
Netherlands	1.8	1.4	9.4	5.2	-18.1	32.8
New Zealand	2.4	2.0	9.0	4.9	-23.7	34.1
Norway	2.4	1.7	12.2	7.0	-48.0	62.1
South Africa	2.2	1.7	10.4	5.9	-32.6	37.1
Spain	2.0	1.4	11.7	7.0	-30.2	53.2
Sweden	3.2	2.5	12.4	6.1	-36.7	68.2
Switzerland	2.5	2.1	9.3	4.3	-21.4	56.1
UK	2.2	1.3	13.6	7.2	-30.7	59.0
USA	2.4	1.9	10.1	5.3	-19.4	35.1
World	2.2	1.7	10.3	5.6	-27.1	31.7

Exhibit A2: Dynamic Strategies

This exhibit shows summary statistics for eight declining-equity and rising-equity strategies evaluated over 81 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1980-2009. The strategies and notation are those described in Exhibit 1. The data is described in Exhibit A1. All figures in dollars except for failure rates (in %).

	100-0	0-100	90-10	10-90	80-20	20-80	70-30	30-70
<i><u>Australia</u></i>								
Failure	7.4	42.0	8.6	33.3	11.1	30.9	12.3	29.6
Mean	2,233	993	2,113	1,100	1,989	1,224	1,861	1,349
Median	1,080	42	1,041	195	1,089	480	1,229	705
P99	7,855	8,863	7,619	8,733	7,415	8,586	7,640	8,424
P95	7,643	6,049	7,416	6,141	7,225	6,268	7,014	6,378
P90	7,161	4,739	6,825	4,818	6,530	5,015	6,257	5,314
SD	2,362	1,643	2,236	1,645	2,118	1,661	2,010	1,698
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	13	0	2	0	0	0	0	0
<i><u>Belgium</u></i>								
Failure	51.9	63.0	53.1	60.5	54.3	60.5	56.8	61.7
Mean	632	453	618	472	604	492	587	513
Median	0	0	0	0	0	0	0	0
P99	6,586	5,768	6,371	5,806	6,116	5,818	5,981	5,805
P95	6,003	4,671	5,889	4,761	5,750	4,894	5,587	5,028
P90	4,440	3,404	4,362	3,538	4,268	3,666	4,160	3,783
SD	1,424	1,117	1,399	1,152	1,372	1,186	1,344	1,219
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	0	0	0	0	0	0	0	0
<i><u>Canada</u></i>								
Failure	3.7	40.7	2.5	29.6	2.5	24.7	4.9	21.0
Mean	1,366	947	1,314	971	1,261	1,001	1,210	1,035
Median	977	161	909	286	808	397	739	517
P99	5,064	7,399	4,826	7,039	4,559	6,667	4,745	6,287
P95	4,583	5,069	4,344	4,867	4,208	4,660	4,085	4,437
P90	3,981	4,372	3,863	4,151	3,786	3,977	3,699	3,793
SD	1,169	1,477	1,144	1,402	1,132	1,333	1,135	1,272
P1	0	0	0	0	0	0	0	0
P5	5	0	15	0	11	0	0	0
P10	51	0	66	0	66	0	44	0
<i><u>Denmark</u></i>								
Failure	8.6	39.5	7.4	35.8	8.6	30.9	8.6	23.5
Mean	1,345	1,169	1,342	1,194	1,336	1,217	1,325	1,241
Median	592	159	564	223	521	351	477	380
P99	10,035	12,145	10,208	11,533	10,298	10,900	10,299	10,252
P95	7,757	9,331	7,812	9,123	8,005	9,036	8,172	8,937
P90	6,374	7,215	6,580	7,230	6,765	7,207	6,921	7,220
SD	1,914	2,299	1,968	2,274	2,019	2,248	2,067	2,219
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	3	0	7	0	6	0	5	0

Exhibit A2: Dynamic Strategies (Cont.)

	100-0	0-100	90-10	10-90	80-20	20-80	70-30	30-70
<i><u>Japan</u></i>								
Failure	38.3	43.2	38.3	42.0	38.3	40.7	38.3	38.3
Mean	2,878	657	2,612	816	2,350	994	2,094	1,187
Median	1,987	152	1,902	352	1,768	597	1,636	775
P99	23,167	5,645	18,805	5,990	16,159	6,268	13,667	6,479
P95	18,083	5,115	15,576	5,172	13,173	5,184	10,913	5,429
P90	12,922	3,635	11,365	3,845	9,992	4,183	8,666	4,670
SD	4,224	1,184	3,657	1,236	3,139	1,325	2,676	1,466
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	0	0	0	0	0	0	0	0
<i><u>Netherlands</u></i>								
Failure	22.2	56.8	19.8	55.6	22.2	50.6	21.0	43.2
Mean	1,156	865	1,107	870	1,060	879	1,015	895
Median	421	0	410	0	340	0	341	106
P99	10,115	4,632	9,491	5,013	8,822	5,382	8,118	5,739
P95	9,273	4,143	8,654	4,470	8,022	4,914	7,384	5,358
P90	7,203	3,720	6,864	3,923	6,514	4,203	6,146	4,570
SD	2,222	1,280	2,096	1,302	1,971	1,351	1,848	1,422
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	0	0	0	0	0	0	0	0
<i><u>New Zealand</u></i>								
Failure	3.7	34.6	3.7	29.6	3.7	23.5	4.9	18.5
Mean	1,568	575	1,468	660	1,365	751	1,260	845
Median	928	153	945	238	951	393	925	597
P99	9,310	4,432	8,373	4,321	7,422	4,209	6,837	4,193
P95	7,985	3,209	7,416	3,309	6,804	3,507	6,157	3,840
P90	6,062	2,585	5,606	2,745	5,152	2,957	4,711	3,267
SD	1,847	857	1,710	881	1,574	927	1,440	998
P1	0	0	0	0	0	0	0	0
P5	4	0	3	0	3	0	0	0
P10	36	0	30	0	19	0	8	0
<i><u>Norway</u></i>								
Failure	43.2	66.7	43.2	63.0	46.9	59.3	53.1	58.0
Mean	818	644	836	693	845	743	852	784
Median	94	0	86	0	14	0	0	0
P99	12,130	7,250	11,649	6,741	10,973	6,863	10,113	7,942
P95	7,373	4,815	7,487	5,010	7,489	5,367	7,367	5,974
P90	4,920	4,002	5,032	4,134	5,114	4,430	5,154	4,723
SD	1,816	1,328	1,826	1,374	1,818	1,443	1,790	1,525
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	0	0	0	0	0	0	0	0
<i><u>South Africa</u></i>								
Failure	6.2	40.7	6.2	32.1	6.2	22.2	7.4	14.8
Mean	1,855	709	1,753	810	1,646	927	1,535	1,050
Median	1,307	366	1,252	562	1,064	682	1,079	833
P99	9,778	6,175	8,721	6,440	7,727	6,688	7,612	6,916
P95	8,395	3,992	7,691	4,174	7,119	4,343	6,537	4,497
P90	6,881	3,163	6,310	3,345	5,785	3,511	5,261	3,685
SD	2,047	1,033	1,871	1,072	1,709	1,116	1,563	1,169
P1	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0
P10	25	0	23	0	20	0	17	0

Exhibit A3: Static Strategies

This exhibit shows summary statistics for three static strategies evaluated over 81 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1980-2009, as well as averages for the four declining-equity (DE) and the four rising-equity (RE) strategies in Exhibit A2. The strategies and notation are those described in Exhibit 2. The data is described in Exhibit A1. All figures in dollars except for failure rates (in %).

	DE	RE	60×30	50×30	40×30
<i><u>Australia</u></i>					
Failure	9.9	34.0	13.6	18.5	32.1
Mean	2,049	1,167	2,093	1,601	1,197
Median	1,110	355	1,474	1,144	638
P99	7,632	8,651	9,094	8,057	7,081
P95	7,325	6,209	7,739	6,548	5,523
P90	6,693	4,972	6,982	5,838	4,802
SD	2,181	1,662	2,159	1,828	1,527
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	4	0	0	0	0
<i><u>Belgium</u></i>					
Failure	54.0	61.4	56.8	59.3	60.5
Mean	610	483	621	552	481
Median	0	0	0	0	0
P99	6,264	5,799	6,733	5,809	4,941
P95	5,807	4,838	5,851	5,233	4,615
P90	4,307	3,598	4,410	3,982	3,540
SD	1,385	1,169	1,429	1,284	1,140
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Canada</u></i>					
Failure	3.4	29.0	1.2	14.8	18.5
Mean	1,288	989	1,389	1,114	879
Median	858	340	938	663	421
P99	4,798	6,848	6,129	5,514	4,885
P95	4,305	4,758	4,579	4,103	3,618
P90	3,832	4,073	4,053	3,572	3,262
SD	1,145	1,371	1,302	1,180	1,071
P1	0	0	0	0	0
P5	8	0	12	0	0
P10	57	0	36	0	0
<i><u>Denmark</u></i>					
Failure	8.3	32.4	4.9	11.1	21.0
Mean	1,337	1,205	1,373	1,288	1,198
Median	538	278	530	437	342
P99	10,210	11,208	10,424	10,020	9,451
P95	7,936	9,107	8,784	8,609	8,307
P90	6,660	7,218	7,298	7,140	6,880
SD	1,992	2,260	2,172	2,151	2,097
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	5	0	12	0	0

Exhibit A3: Static Strategies (Cont.)

	DE	RE	60×30	50×30	40×30
<i><u>Finland</u></i>					
Failure	41.4	44.4	40.7	43.2	45.7
Mean	1,249	840	1,462	1,126	827
Median	287	334	529	342	184
P99	12,579	8,568	14,529	11,167	8,216
P95	10,803	6,292	10,850	8,757	6,752
P90	7,549	4,521	8,196	6,371	4,747
SD	2,490	1,485	2,606	2,045	1,543
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>France</u></i>					
Failure	64.5	64.8	61.7	63.0	63.0
Mean	935	939	976	959	926
Median	0	0	0	0	0
P99	10,841	10,050	11,911	10,838	9,681
P95	8,796	8,370	9,257	8,749	8,201
P90	6,777	6,795	7,195	7,002	6,687
SD	2,180	2,185	2,327	2,235	2,113
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Germany</u></i>					
Failure	62.7	52.2	56.8	56.8	58.0
Mean	1,510	691	1,201	1,014	841
Median	0	17	0	0	0
P99	30,065	4,711	17,985	13,231	9,367
P95	15,077	3,190	9,226	7,104	5,314
P90	10,007	2,807	6,607	5,231	4,077
SD	3,956	984	2,504	1,945	1,482
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Ireland</u></i>					
Failure	39.2	59.0	38.3	43.2	53.1
Mean	1,069	680	1,086	889	721
Median	190	0	258	98	0
P99	12,187	9,190	13,765	11,374	9,186
P95	8,903	6,744	9,538	8,275	7,097
P90	6,398	4,790	6,546	5,739	5,032
SD	2,138	1,612	2,262	1,954	1,670
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Italy</u></i>					
Failure	75.6	83.0	75.3	76.5	80.2
Mean	399	114	287	249	209
Median	0	0	0	0	0
P99	7,138	2,325	5,549	4,757	3,915
P95	5,271	1,788	3,924	3,567	3,116
P90	3,396	1,074	2,434	2,174	1,872
SD	1,218	402	890	801	693
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0

Exhibit A3: Static Strategies (Cont.)

	DE	RE	60×30	50×30	40×30
<i><u>Japan</u></i>					
Failure	38.3	41.0	38.3	37.0	38.3
Mean	2,483	914	2,053	1,610	1,239
Median	1,823	469	1,407	1,140	1,014
P99	17,950	6,096	13,678	9,133	5,704
P95	14,436	5,225	10,619	7,133	4,930
P90	10,736	4,083	8,850	6,224	4,256
SD	3,424	1,303	2,720	1,942	1,383
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Netherlands</u></i>					
Failure	21.3	51.5	24.7	34.6	44.4
Mean	1,085	877	1,164	943	758
Median	378	26	446	271	135
P99	9,136	5,192	7,851	6,649	5,599
P95	8,333	4,721	7,648	6,373	5,191
P90	6,682	4,104	6,383	5,373	4,421
SD	2,034	1,339	1,905	1,615	1,346
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>New Zealand</u></i>					
Failure	4.0	26.5	3.7	12.3	23.5
Mean	1,415	708	1,320	1,049	811
Median	937	345	914	657	296
P99	7,986	4,289	6,176	5,576	5,081
P95	7,090	3,467	5,368	4,847	4,251
P90	5,383	2,888	4,397	3,903	3,414
SD	1,643	916	1,312	1,193	1,056
P1	0	0	0	0	0
P5	3	0	9	0	0
P10	23	0	34	0	0
<i><u>Norway</u></i>					
Failure	46.6	61.7	53.1	55.6	59.3
Mean	838	716	949	838	718
Median	48	0	0	0	0
P99	11,216	7,199	11,668	9,335	7,133
P95	7,429	5,291	8,057	6,839	5,691
P90	5,055	4,322	5,753	5,068	4,382
SD	1,812	1,417	1,955	1,682	1,420
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>S. Africa</u></i>					
Failure	6.5	27.5	6.2	7.4	17.3
Mean	1,697	874	1,777	1,298	907
Median	1,176	611	1,496	1,083	613
P99	8,460	6,555	8,758	7,310	6,029
P95	7,435	4,251	6,880	5,364	4,065
P90	6,060	3,426	5,460	4,300	3,326
SD	1,798	1,098	1,632	1,327	1,070
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	21	0	48	4	0

Exhibit A3: Static Strategies (Cont.)

	DE	RE	60×30	50×30	40×30
<i><u>Spain</u></i>					
Failure	47.8	61.4	45.7	51.9	59.3
Mean	355	323	386	331	278
Median	32	0	34	0	0
P99	8,588	6,369	9,379	7,884	6,481
P95	4,274	3,844	4,928	4,185	3,476
P90	2,547	2,519	2,828	2,449	2,079
SD	1,111	941	1,233	1,044	866
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Sweden</u></i>					
Failure	14.5	29.3	16.0	16.0	18.5
Mean	1,796	1,103	1,792	1,413	1,081
Median	821	299	870	580	398
P99	19,352	8,275	16,044	13,086	10,290
P95	14,760	6,607	12,615	10,185	7,939
P90	10,612	5,441	9,325	7,522	5,977
SD	3,405	1,705	2,898	2,357	1,881
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>Switzerland</u></i>					
Failure	32.1	24.4	27.2	24.7	27.2
Mean	782	717	869	745	626
Median	522	413	676	586	466
P99	3,908	4,636	4,467	4,340	4,178
P95	3,251	3,492	3,619	3,398	3,163
P90	2,620	2,763	2,924	2,630	2,377
SD	873	881	913	827	756
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0
<i><u>UK</u></i>					
Failure	23.5	43.8	23.5	24.7	37.0
Mean	1,256	787	1,240	1,004	805
Median	622	126	749	463	177
P99	13,091	5,279	10,027	8,585	7,227
P95	8,288	4,670	7,143	6,337	5,560
P90	6,333	3,878	5,563	5,018	4,489
SD	2,042	1,254	1,717	1,558	1,410
P1	0	0	0	0	0
P5	0	0	0	0	0
P10	0	0	0	0	0

Exhibit A4: Why Not 100% Stocks?

This exhibit shows summary statistics for strategies evaluated over 81 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1980-2009. The strategies and notation are those described in Exhibit 3. The data is described in Exhibit A1. All figures in dollars except for failure rates (in %).

	100×30	Best Dynamic	Best Static	60×30
<i><u>Australia</u></i>				
Failure	3.7	7.4	4.9	13.6
Mean	4,898	2,233	4,068	2,093
Median	3,637	1,080	3,010	1,474
P99	14,395	7,855	12,492	9,094
P95	13,791	7,643	12,057	7,739
P90	13,045	7,161	11,232	6,982
SD	4,092	2,362	3,480	2,159
P1	0	0	0	0
P5	11	0	0	0
P10	235	13	152	0
<i><u>Belgium</u></i>				
Failure	50.6	51.9	51.9	56.8
Mean	856	632	807	621
Median	0	0	0	0
P99	10,698	6,586	9,705	6,733
P95	8,147	6,003	7,555	5,851
P90	5,853	4,440	5,548	4,410
SD	1,963	1,424	1,841	1,429
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Canada</u></i>				
Failure	1.2	2.5	0.0	1.2
Mean	2,815	1,314	2,417	1,389
Median	2,121	909	1,954	938
P99	9,662	4,826	7,668	6,129
P95	8,886	4,344	7,306	4,579
P90	8,053	3,863	6,609	4,053
SD	2,355	1,144	1,958	1,302
P1	0	0	190	0
P5	173	15	240	12
P10	286	66	284	36
<i><u>Denmark</u></i>				
Failure	3.7	7.4	1.2	4.9
Mean	1,582	1,342	1,550	1,373
Median	824	564	794	530
P99	9,902	10,208	10,383	10,424
P95	8,108	7,812	8,447	8,784
P90	6,788	6,580	7,088	7,298
SD	1,947	1,968	2,047	2,172
P1	0	0	0	0
P5	15	0	24	0
P10	123	7	99	12

Exhibit A4: Why Not 100% Stocks? (Cont.)

	100×30	Best Dynamic	Best Static	60×30
<i><u>Finland</u></i>				
Failure	34.6	40.7	39.5	40.7
Mean	2,903	1,248	2,556	1,462
Median	837	288	812	529
P99	29,086	12,367	25,697	14,529
P95	20,459	10,612	17,827	10,850
P90	16,321	7,436	14,379	8,196
SD	5,155	2,443	4,511	2,606
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>France</u></i>				
Failure	56.8	63.0	59.3	61.7
Mean	862	935	913	976
Median	0	0	0	0
P99	14,521	9,895	14,195	11,911
P95	9,661	8,299	9,826	9,257
P90	6,777	6,741	7,035	7,195
SD	2,363	2,175	2,405	2,327
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Germany</u></i>				
Failure	56.8	49.4	56.8	56.8
Mean	2,085	580	1,837	1,201
Median	0	69	0	0
P99	46,868	2,390	38,156	17,985
P95	21,440	2,301	17,904	9,226
P90	13,908	2,175	11,836	6,607
SD	5,829	746	4,832	2,504
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Ireland</u></i>				
Failure	28.4	38.3	29.6	38.3
Mean	1,964	1,125	1,747	1,086
Median	771	219	616	258
P99	24,107	12,240	21,577	13,765
P95	14,642	8,943	13,503	9,538
P90	10,796	6,607	9,580	6,546
SD	3,624	2,179	3,281	2,262
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Italy</u></i>				
Failure	64.2	75.3	69.1	75.3
Mean	470	420	402	287
Median	0	0	0	0
P99	7,529	7,477	7,269	5,549
P95	4,173	5,513	4,293	3,924
P90	3,135	3,572	2,836	2,434
SD	1,100	1,277	1,043	890
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0

Exhibit A4: Why Not 100% Stocks? (Cont.)

	100×30	Best Dynamic	Best Static	60×30
<i><u>Japan</u></i>				
Failure	39.5	38.3	37.0	38.3
Mean	4,472	2,878	1,610	2,053
Median	1,478	1,987	1,140	1,407
P99	45,352	23,167	9,133	13,678
P95	36,301	18,083	7,133	10,619
P90	26,674	12,922	6,224	8,850
SD	8,608	4,224	1,942	2,720
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Netherlands</u></i>				
Failure	21.0	19.8	22.2	24.7
Mean	2,259	1,107	1,681	1,164
Median	987	410	836	446
P99	14,948	9,491	11,205	7,851
P95	13,105	8,654	10,373	7,648
P90	10,524	6,864	8,479	6,383
SD	3,284	2,096	2,554	1,905
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>New Zealand</u></i>				
Failure	0.0	3.7	0.0	3.7
Mean	2,579	1,568	2,248	1,320
Median	2,378	928	2,063	914
P99	7,681	9,310	7,743	6,176
P95	6,880	7,985	6,191	5,368
P90	6,264	6,062	5,702	4,397
SD	1,782	1,847	1,628	1,312
P1	51	0	37	0
P5	285	4	233	9
P10	408	36	331	34
<i><u>Norway</u></i>				
Failure	37.0	43.2	37.0	53.1
Mean	1,189	836	1,168	949
Median	135	86	96	0
P99	20,231	11,649	18,435	11,668
P95	11,520	7,487	11,065	8,057
P90	7,380	5,032	7,216	5,753
SD	2,797	1,826	2,653	1,955
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>S. Africa</u></i>				
Failure	2.5	6.2	2.5	6.2
Mean	4,682	1,855	3,789	1,777
Median	4,072	1,307	3,333	1,496
P99	16,331	9,778	14,167	8,758
P95	15,198	8,395	12,784	6,880
P90	12,796	6,881	10,375	5,460
SD	3,632	2,047	2,971	1,632
P1	0	0	0	0
P5	181	0	119	0
P10	383	25	326	48

Exhibit A4: Why Not 100% Stocks? (Cont.)

	100×30	Best Dynamic	Best Static	60×30
<i><u>Spain</u></i>				
Failure	40.7	46.9	42.0	45.7
Mean	590	368	547	386
Median	154	62	121	34
P99	15,292	8,692	13,947	9,379
P95	7,738	4,277	7,119	4,928
P90	4,328	2,554	3,968	2,828
SD	1,971	1,122	1,804	1,233
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Sweden</u></i>				
Failure	17.3	12.3	16.0	16.0
Mean	3,664	1,740	3,162	1,792
Median	1,932	807	1,530	870
P99	27,962	18,585	24,280	16,044
P95	22,118	14,154	20,029	12,615
P90	17,376	10,196	15,457	9,325
SD	5,279	3,255	4,697	2,898
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>Switzerland</u></i>				
Failure	33.3	23.5	24.7	27.2
Mean	1,373	712	745	869
Median	1,022	383	586	676
P99	5,274	4,676	4,340	4,467
P95	4,721	3,487	3,398	3,619
P90	4,225	2,784	2,630	2,924
SD	1,418	892	827	913
P1	0	0	0	0
P5	0	0	0	0
P10	0	0	0	0
<i><u>UK</u></i>				
Failure	6.2	22.2	12.3	23.5
Mean	2,514	1,359	2,139	1,240
Median	1,925	718	1,628	749
P99	16,350	15,181	14,719	10,027
P95	10,462	9,142	9,639	7,143
P90	8,287	6,916	7,410	5,563
SD	2,612	2,274	2,337	1,717
P1	0	0	0	0
P5	0	0	0	0
P10	14	0	0	0

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