Maximum Withdrawal Rates: *An Empirical and Global Perspective*

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A critical problem faced by an individual during retirement is how to withdraw as much as possible from his portfolio subject to two constraints, namely, an expected probability of failure and a desired bequest. The latter is rather trivial given that it is a parameter the retiree can set at any desired level, including 0; the former can be ascertained, at least approximately, through historical evidence or simulations.

When dealing with this problem, the retiree faces two sources of uncertainty: how long he is going to live and the returns of his portfolio during those years. Of these, this article focuses on the latter. To deal with this uncertainty, the retiree can adjust two variables, the withdrawal rate and the portfolio's asset allocation. Of these, this article focuses on the former.

Ultimately, this article asks and aims to respond to the following question: Given a retirement portfolio, an expected retirement period, and a desired bequest, what is the *maximum* initial withdrawal a retiree could make if he aims to keep his purchasing power constant during his retirement? The answer to this question is explored both analytically and empirically.

Analytically, this problem has a closed-form solution, referred to here as the *maximum withdrawal rate*. This variable, which is at the center of this article, has at least two critical applications: It provides a way to assess how likely a retiree is to sustain a target level of inflation-adjusted withdrawals during his retirement, and it provides a comprehensive way to evaluate retirement strategies. The focus here is largely, though not exclusively, on the first application.

Empirically, the evidence shows that maximum withdrawal rates have varied widely across countries and over time; therefore, characteristics of the distributions of this variable are reported and discussed. The analysis considers 21 countries and the world market over a 115-year period, as well as 11 asset allocations, making it the first study to provide such a comprehensive historical assessment of maximum withdrawal rates.

The rest of the article is organized as follows. The following section explores in more detail the issue at stake by discussing some withdrawal rules, introducing some important issues with an example, and developing the analytical framework. Next we discuss the evidence with a comprehensive global database that includes 21 countries and the world market over the 115 years between 1900 and 2014. The final section provides an assessment. An appendix with tables concludes the article.

THE ISSUE

The first part of this section briefly discusses fixed and variable withdrawals, as well as some of the relevant literature on both. The following part discusses an example with five scenarios considering different returns, withdrawals, and bequests, which provides an intuitive way to introduce some of the issues addressed in this article. The third part discusses the concept of maximum withdrawal rate, its relevance, and related literature.

Fixed and Variable Withdrawals

For the purpose of the discussion here, withdrawals can be thought of as divided into fixed and variable. The former can in turn be divided into nominal and real, although fixed nominal withdrawals have plausibly received little attention; no retiree would be happy with an ever-decreasing purchasing power. Fixed real withdrawals, on the other hand, have been the subject of a massive literature.

Bengen [1994] pioneered this line of research by arguing that an initial withdrawal rate (IWR) of 4%, with annual withdrawals subsequently adjusted by inflation (hence fixed real withdrawals) was "safe" in the sense that, historically, this strategy never depleted a portfolio in less than 30 years. This was the origin of the widely used and abused "4% rule" that still is the subject of heated debate.

Those that support the rule tend to highlight historical evidence from the United States. In fact, over the 115 years between 1900 and 2014, a 60-40 portfolio of U.S. stocks and bonds had a failure rate of 4.7%, and portfolios with at least 70% in U.S. stocks had an even lower (3.5%) failure rate.[1] Those that find the rule problematic point to either its performance in other markets, where it led to much higher failure rates (Pfau [2010]), or to current market conditions that suggest lower-than-historical expected returns for stocks and bonds (Crook [2013]). The debate on the pros and cons of the 4% rule, and on fixed real withdrawals more generally, is sure to continue.

Variable withdrawals encompass a broad set of strategies in which withdrawals are adjusted over time, typically based on changing life expectancy (Dus et al. [2005]), changing market conditions (Estrada [2016b]), or both (Stout and Mitchell [2006]). Withdrawals depending on market conditions, in particular, are the subject of a vast literature too long to review here; Suarez et al. [2015] and Clare et al. [2016] provide partial reviews.

Unsurprisingly, both fixed and variable withdrawal rules have pros and cons. Fixed withdrawals are generally easy to understand and implement, and in the case of fixed real withdrawals, they have the desirable characteristic of preserving purchasing power. However, they have the drawback of not adjusting to changing market conditions (or life expectancy), which may lead to depletion of a retirement portfolio earlier than desired.

Variable withdrawals, in turn, do adjust to changing conditions and therefore reduce or even eliminate the risk of early depletion. However, they typically are more difficult to understand and implement (see, for example, Stout [2008]) and may require a retiree to reduce his real or even nominal consumption, which he naturally may be reluctant to do.[2]

By definition, the maximum withdrawal rate as defined below belongs to the category of fixed *real* withdrawals. As such, it easy to understand and implement (though in this case not necessarily trivial to calculate) and has the desirable characteristic of keeping purchasing power constant throughout the retirement period. However, if the maximum withdrawal rate is updated periodically during the retirement period, then it is likely to lead to variable withdrawals in both nominal and real terms. (More on this later.)

Returns, Withdrawals, and Bequest

Consider Exhibit 1, which contemplates five scenarios (S1 through S5) based on a $1,000 retirement portfolio, a 30-year retirement period, annual withdrawals (*W*), and real returns. All scenarios but S5 are based on a 4% IWR; all scenarios but S4 are based on maintaining the purchasing power of the first withdrawal constant over time.[3] The figures in the exhibit show the value of a retiree's portfolio (*P*) at the end of each year, before making the annual withdrawal for the following year, as well as the annual withdrawals in S4 and S5.

The first column shows a scenario (S1) in which the portfolio returns just keep up with inflation; in this case, the retiree would be able to sustain withdrawals for only 25 years.[4] In other words, if a portfolio earns a 0% real return, a strategy of withdrawing 4% of the portfolio in the first year, and subsequently adjusting the annual withdrawals by inflation, would lead to a depleted portfolio five years earlier than desired.

The second column shows a scenario (S2) in which a retiree seeks to 1) withdraw 4% of his retirement

EXHIBIT 1

Five Scenarios

	S1	S2	S3	S4		S5	
Year	***P***	***P***	***P***	***P***	***W***	***P***	***W***
0	1,000	1,000	1,000	1,000	40	1,000	63
1	960	973	1,010	1,010	42	986	63
2	920	945	1,021	1,019	43	970	63
3	880	917	1,032	1,027	45	954	63
4	840	888	1,044	1,034	47	938	63
5	800	859	1,057	1,039	48	920	63
6	760	830	1,070	1,043	50	901	63
7	720	800	1,084	1,044	52	882	63
8	680	770	1,099	1,044	54	861	63
9	640	740	1,114	1,042	56	839	63
10	600	709	1,131	1,037	59	816	63
11	560	678	1,148	1,030	61	792	63
12	520	646	1,166	1,020	63	767	63
13	480	614	1,185	1,007	66	740	63
14	440	582	1,205	991	68	712	63
15	400	549	1,226	971	71	683	63
16	360	515	1,248	947	74	652	63
17	320	482	1,271	919	76	619	63
18	280	447	1,296	887	79	584	63
19	240	413	1,321	850	82	548	63
20	200	378	1,349	808	86	510	63
21	160	342	1,377	760	89	470	63
22	120	306	1,407	706	92	428	63
23	80	269	1,439	646	96	383	63
24	40	232	1,472	579	100	337	63
25	0	195	1,507	504	104	287	63
26	0	157	1,544	422	108	236	63
27	0	118	1,583	331	112	181	63
28	0	79	1,623	231	116	124	63
29	0	40	1,666	121	121	63	63
30	0	0	1,711	0	0	0	0

Notes: This exhibit shows five scenarios based on a starting portfolio of $1,000, a 30-year retirement period, annual withdrawals (W), and real returns. The first four scenarios (S1, S2, S3, and S4) are based on a 4% IWR; the fifth scenario (S5) is based on a 6.3% IWR; all scenarios but S4 are based on constant withdrawals in real terms. The first scenario is based on a 0% return; the second on a 1.31% return; the third on a 5.24% return; the fourth on a 5.24% return and withdrawals increasing at a 3.88% rate; and the fifth on a 5.24% return. In all cases, P indicates the value of a portfolio at the end of the year, before making the annual withdrawal for the following year. All figures are in dollars.

portfolio in the first year; 2) annually adjust all subsequent withdrawals by inflation; and 3) leave no bequest. For these conditions to be met, the retiree's portfolio would need to generate a 1.31% annual return above inflation. If the annual real return were less than that, he would run out of money earlier than 30 years; if it were higher than that, he would leave a positive bequest.

The third column shows a scenario (S3) in which the portfolio generates an annual real return of 5.24%, which is the historical return of a 60-40 portfolio of U.S. stocks and (long-term) bonds.[5] In this scenario, the retiree withdraws 4% of his retirement portfolio in the first year and adjusts all subsequent annual withdrawals by inflation; as a result, he leaves a bequest of $1,711. Importantly, note that this is more than 70% larger than his portfolio at the beginning of retirement, in *real* terms, and equivalent to almost 43 years of annual (inflation-adjusted) withdrawals.

This last scenario suggests that, if the retiree had intended to leave no bequest, he could have increased his withdrawals substantially, thus being able to afford a better retirement. He could have done this in at least two ways, which are contemplated in the last four columns of Exhibit 1. The fourth and fifth columns show a scenario (S4) in which a retiree withdraws 4% of his portfolio at the beginning of retirement, increases his withdrawals at the annual *real* rate of 3.88%, and exhausts his portfolio at the end of 30 years.[6] The last two columns show a scenario (S5) in which a retiree withdraws 6.3% of his portfolio at the beginning of retirement, adjusts his annual withdrawals by inflation, and exhausts his portfolio at the end of 30 years.[7]

In scenarios S4 and S5 the retiree enjoys a higher standard of living at the expense of his heirs; put differently, relative to S3, in these two scenarios the retiree enjoys higher withdrawals during retirement but leaves no bequest.[8] Note than in S4 the retiree starts with the same withdrawal as he would in S3, but then enjoys higher and increasing withdrawals (in real terms) throughout retirement; his last withdrawal ($121) is over three times larger than in S3. In S5, on the other hand, the retiree enjoys annual withdrawals ($63) almost 59% higher in real terms than in S3 every year of his retirement.

The underlying message from Exhibit 1 is that given a 30-year retirement period and the historical returns of a 60-40 portfolio of U.S. stocks and bonds, a 4% IWR, and subsequent inflation-adjusted withdrawals may leave a very substantial bequest.[9] Exhibit 2 drives this point home with broader evidence from the U.S. market. It shows the mean and median of the distribution of terminal wealth based on a starting portfolio of $1,000, a 4% IWR, subsequent withdrawals adjusted by inflation, and 86 rolling 30-year retirement

EXHIBIT 2

The Distribution of Terminal Wealth—USA

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
Mean ($)	3,232	2,789	2,383	2,009	1,667	1,356	1,082	848	665	536	433
Median ($)	2,881	2,457	1,925	1,460	1,171	793	520	249	40	0	0
Mean (YW)	80.8	69.7	59.6	50.2	41.7	33.9	27.1	21.2	16.6	13.4	10.8
Median (YW)	72.0	61.4	48.1	36.5	29.3	19.8	13.0	6.2	1.0	0.0	0.0
Failure (%)	3.5	3.5	3.5	3.5	4.7	8.1	15.1	25.6	40.7	64.0	65.1

Notes: This exhibit shows the mean and median for the distribution of terminal wealth (bequest) for the U.S. market, for 11 static asset allocations with stock-bond proportions between 100-0 (all stocks) and 0-100 (all bonds), over 86 rolling 30-year retirement periods, beginning with 1900–1929 and ending with 1985–2014. All strategies are based on a starting portfolio of $1,000, a 4% IWR, subsequent annual withdrawals adjusted by inflation, and annual rebalancing to the stock-bond allocations in the first row. The first two rows show the mean and median in real dollars; the next two rows show the mean and median in terms of years of real withdrawals (YW); and the last row shows failure rates in %. The data are described in Exhibit A1 in the appendix.

periods between 1900 and 2014, for 11 static stock-bond allocations, annually rebalanced.

The first two rows show the mean and median terminal wealth in real dollars; for perspective, recall that the retirement portfolio starts with $1,000. The third and fourth rows show the same mean and median terminal wealth but expressed in years of real ($40) withdrawals. As these two rows make clear, retirees implementing moderate to aggressive asset allocations would have left, on average, two or more *decades* of real withdrawals as bequest. This may have been their intention, but if it was not, they could have substantially increased their withdrawals (hence their standard of living) during retirement, and left no (or a smaller) bequest.

The last row of Exhibit 2 shows the failure rate of each strategy; that is, the proportion of the 86 retirement periods considered in which the strategy failed (ran out of money before the end of the retirement period). For strategies with allocations to stocks 50% or higher, all failure rates are under 10%; for more conservative allocations, they all are above 15%.

Consider, for example, the 60-40 strategy, which had a failure rate of 4.7%. Although this may be plausibly viewed as a "tolerable" failure rate, it led to a mean (median) bequest equivalent to 42 (29) years of real withdrawals. Retirees that followed this strategy, and had no intention of leaving a bequest, could have made higher withdrawals and enjoyed a higher standard of living. This leads to the central issue considered in this article: Given a desired bequest, what are the maximum inflation-adjusted withdrawals an individual could make during his retirement?

The Maximum Withdrawal Rate

Given a retirement portfolio, an expected retirement period, a desired bequest, and the goal of keeping purchasing power constant during retirement, only the uncertain portfolio returns prevent the calculation of the maximum annual withdrawals a retiree could make. To formalize the discussion, let P_t be the value of a retirement portfolio at the end of period t, and R_t be the real return of the portfolio during period t. Furthermore, let W be the initial withdrawal, which will remain constant, in real terms, every year during the T-year retirement period. Then, the value of the portfolio at time 0 (the beginning of retirement), after taking the withdrawal for the following year, is

$$P_0 - W. \tag{1}$$

At the end of the first year, after the portfolio's return for the year and after taking the withdrawal for the following year, the value of the portfolio is

$$(P_0 - W)(1 + R_1) - W, \tag{2}$$

and similarly, at the end of the second year, after the portfolio's return for the year and after taking the withdrawal for the following year, the value of the portfolio is

$$[(P_0 - W)(1 + R_1) - W](1 + R_2) - W. \tag{3}$$

Continuing in the same fashion through year T (the end of retirement) and solving for W yields

$$W^* = \frac{P_0(1+R_1)(1+R_2)\cdots(1+R_T) - P_T}{(1+R_1)\cdots(1+R_T) + (1+R_2)\cdots(1+R_T) + (1+R_3)\cdots(1+R_T) + \cdots + (1+R_T)}, \qquad (4)$$

where W^* denotes the maximum withdrawal given the goal of leaving a bequest of P_T. Note that W^* is a constant annual amount, measured in real dollars. Hence, the *maximum withdrawal rate* (MWR) is given by

$$MWR = W^*/P_0. \qquad (5)$$

Finally, inserting (4) into (5) and assuming that the retiree's goal is to leave no bequest ($P_T = 0$) yields

$$MWR = \frac{(1+R_1)(1+R_2)\cdots(1+R_T)}{(1+R_1)\cdots(1+R_T) + (1+R_2)\cdots(1+R_T) + (1+R_3)\cdots(1+R_T) + \cdots + (1+R_T)}. \qquad (6)$$

Expression (6) is at the heart of this article and therefore requires some interpretation. Note, first, that MWR is the proportion of the portfolio withdrawn *at the beginning* of retirement; hence, it is an *initial* withdrawal rate. Second, the MWR (measured in percent), together with the value of the portfolio at the beginning of retirement, determine the initial withdrawal W^* (measured in dollars); that is, $W^* = \text{MWR}{\cdot}P_0$. Third, by definition, W^* remains constant, in real terms, throughout the retirement period. Finally, the MWR as written in (6) assumes that the retiree has the goal of leaving no bequest.[10]

Importantly, note that the MWR can only be calculated ex post, once the returns of the portfolio during the retirement period are known. In other words, it is a measure of the best a retiree could have done *had he known* the future returns of his portfolio. Obviously, in practice, no retiree knows what the returns of his portfolio will be; still, the MWR is an essential tool for at least two reasons.

First, it provides a way to assess how likely a retiree is to sustain a target level of constant real withdrawals during his retirement; this likelihood can be estimated from the (historical or simulated) distribution of MWRs. This is the application emphasized by Suarez et al. [2015]. Second, it provides a comprehensive way to evaluate retirement strategies, simply because the higher a strategy's MWR, the higher the standard of living it enables. From this perspective, the MWR can be used to evaluate the (historical or simulated) performance of different strategies. This is the application emphasized by Clare et al. [2016]. The focus of this article is largely, though not exclusively, on the first application.

Previous articles in the literature consider a similar analytical framework. Blanchett et al. [2012] derive an expression similar to (4) and call it the *sustainable spending rate*; Suarez et al. [2015] and Clare et al. [2016] do the same and call it the *perfect withdrawal amount*. Miller [2016] does not provide a formal analytical framework but discusses the idea behind Expression (6) and calls it, as is done here, the *maximum withdrawal rate*.

EVIDENCE

This section discusses the global evidence on MWRs based on 21 countries and the world market over the 115 years between 1900 and 2014. The first part discusses the data and methodology, the second part discusses the results from the analysis, and the third part digresses on a few issues.

Data and Methodology

The sample considered is the Dimson-Marsh-Staunton (DMS) database, described in detail in Dimson, Marsh, and Staunton [2002, 2016]. It contains annual returns for stocks and long-term government bonds over the 1900–2014 period for 21 countries and the world market. Returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for both capital gains/losses and cash flows (dividends or coupons). Exhibit A1 in the appendix summarizes some characteristics of all the series of stock and bond returns in the sample.

The analysis is based on a $1,000 portfolio at the beginning of retirement, annual withdrawals, and a 30-year retirement period. At the beginning of each year the annual withdrawal is made, the portfolio is then rebalanced to the target asset allocation for the year, and then it compounds at the observed return of stocks and bonds for that year. This process is repeated at the beginning of each year during the 30-year retirement period, at the end of which the terminal wealth or bequest is, by design, 0.

The observed MWR from Expression (6) is calculated for each 30-year retirement period, asset allocation, and country considered. Given the 1900–2014 sample period, 86 rolling (overlapping) retirement periods are considered, beginning with 1900–1929 and ending with 1985–2014. Furthermore, 11 stock-bond allocations are evaluated, ranging between 100-0 (all stocks) and 0-100 (all bonds), with nine allocations (90-10, 80-20, ..., 20-80, and 10-90) in between. Finally, as already mentioned, 21 countries and the world market are included in the sample. This set up yields a total of 20,812 observed MWRs (946 per country, and 86 per country and asset allocation), which are discussed immediately below.[11]

Maximum Withdrawal Rates—A Comprehensive History

The ultimate goal of this article is to provide a comprehensive historical perspective on MWRs. Unlike Suarez et al. [2015] and Clare et al. [2016], who focus on one country, one asset allocation, and Monte Carlo simulations, the analysis here broadens the perspective by considering 21 countries, 11 asset allocations, and 86 historical retirement periods (rather than simulations).

For each country and asset allocation, the distribution of MWRs results from aggregating the MWR for each of the 86 retirement periods considered. Exhibit 3 reports some parameters that characterize the distribution of MWRs for the United States, the world market, and the average country (a cross-sectional average of the 21 countries in the sample) for 11 static allocations; these parameters are the mean, median, and standard deviation (SD), as well as the 1% (P1), 5% (P5), and 10% (P10) cutoff MWRs in the lower tail, and the 10% (P90), 5% (P95), and 1% (P99) cutoff MWRs in the upper tail. Exhibit A2 in the appendix reports the same information for the other 20 countries in the sample.

To illustrate the usefulness of the figures reported in Exhibit 3 (and A2) consider the U.S. market (Panel A) and the 60-40 stock-bond allocation, as well as a retiree who expects to live 30 years in retirement, aims to keep his purchasing power constant during this period, and plans to leave no bequest. If this retiree were considering withdrawing $55,000 from a $1 million portfolio (hence a 5.5% MWR), then he would face a 50% probability of failure. Similarly, if this retiree were considering withdrawing $38,000, $39,000, or $42,000, then he would face a 1%, 5%, or 10% probability of failure, as the P1, P5, and P10 figures indicate.

Consider now a diversified portfolio of global stocks and bonds, as shown in Panel B of Exhibit 3 (World), and again for the sake of concreteness, a 60-40 allocation. Although the median MWR for the global 60-40 allocation (5.2%) is not much lower than that for the U.S. 60-40 allocation (5.5%), a retiree who does not wish to face more than a 1%, 5%, or 10% probability of failure should consider substantially lower MWRs (2.9%, 3.0%, and 3.3%, as opposed to 3.8%, 3.9%, and 4.2%). Or, put differently, the 3.8%, 3.9%, and 4.2% MWRs, which have 1%, 5%, and 10% probability of failure when investing in a U.S. 60-40 portfolio, have higher probabilities of failure when investing in the global 60-40 portfolio.

Finally, Panel C describes the historical distribution of MWRs for the average country in the sample. This panel shows, for each variable, an equally weighted average of the figures across the 21 countries in the sample. Going back once again to the 60-40 allocation, a retiree in the average country that considers withdrawing $48,000 from a $1 million portfolio (hence a 4.8% MWR) faces a 50% probability of failure. Furthermore, if this retiree wanted to limit the probability of failure to 5% (10%), then he should start by withdrawing a much lower $26,000 ($28,000) of his portfolio.

A similar analysis can be made for any individual country or asset allocation; and if the distribution of MWRs has not changed substantially, then the figures in Exhibits 3 and A2 could be used to reliably assess the expected probability of failure of any target MWR. Furthermore, although these two exhibits show some selected summary statistics and cutoff points, probabilities of failure for any target MWR can be estimated directly from the empirical distributions of MWRs.

Some Further Considerations

Two related issues are discussed in this section: First, the variability of MWRs across countries and over time, and its implications for the welfare of retirees in different countries and historical periods; and second, whether the MWR needs to be selected just once at the beginning of retirement, or periodically during the retirement period.

It often is the case that averages hide substantial variability, and the figures in Panel C of Exhibit 3 are no exception. Exhibit A2 reveals the extent of the variability in MWRs across countries. For the most aggressive asset allocation (100-0), the median MWR was over 7% in some countries (Australia, South Africa) and just over 3.5% in some others (Germany, Italy). In the bottom 5% of the distribution, the MWR was over 4.5% in some countries (New Zealand, South Africa), and well under 1% in some others (Austria, Japan). In other

EXHIBIT 3

The Distribution of Maximum Withdrawal Rates

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
Panel A: USA											
Mean	7.1	6.9	6.6	6.4	6.1	5.8	5.5	5.1	4.8	4.5	4.1
Median	6.6	6.5	6.2	5.8	5.5	5.2	4.9	4.5	4.1	3.7	3.4
SD	2.3	2.1	1.9	1.8	1.7	1.6	1.6	1.6	1.6	1.7	1.7
P1	3.5	3.8	3.9	3.9	3.8	3.7	3.5	3.4	3.2	2.9	2.4
P5	4.1	4.0	4.2	4.1	3.9	3.9	3.8	3.6	3.3	3.0	2.6
P10	4.4	4.4	4.4	4.4	4.2	4.0	3.9	3.7	3.5	3.1	2.7
P90	10.4	9.9	9.4	8.8	8.5	8.3	8.0	7.7	7.4	7.1	6.7
P95	11.1	10.4	10.2	9.7	9.5	9.4	9.4	9.2	8.8	8.7	8.6
P99	12.5	11.4	10.9	10.6	10.5	10.5	10.4	10.3	10.2	10.1	9.9
Panel B: World											
Mean	6.5	6.3	6.1	5.8	5.6	5.3	5.1	4.8	4.6	4.3	4.0
Median	5.9	5.7	5.5	5.3	5.2	5.0	4.9	4.6	4.4	4.1	3.8
SD	2.3	2.2	2.1	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.9
P1	3.3	3.2	3.1	3.0	2.9	2.8	2.7	2.5	2.4	2.2	2.0
P5	3.5	3.4	3.3	3.1	3.0	2.9	2.8	2.7	2.5	2.3	2.0
P10	3.8	3.7	3.6	3.4	3.3	3.1	3.0	2.8	2.6	2.4	2.2
P90	10.2	9.9	9.2	8.4	8.1	7.5	7.4	7.2	7.0	6.7	6.4
P95	10.9	10.4	10.1	10	9.9	9.9	9.7	9.5	9.2	9.0	8.7
P99	11.7	10.8	10.7	10.7	10.7	10.7	10.7	10.6	10.5	10.4	10.2
Panel C: Averages											
Mean	5.9	5.8	5.7	5.5	5.3	5.1	4.8	4.6	4.3	4.0	3.7
Median	5.4	5.3	5.1	5.0	4.8	4.6	4.3	4.0	3.8	3.5	3.1
SD	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.2	2.1	2.1	2.0
P1	2.2	2.4	2.4	2.4	2.3	2.3	2.2	2.1	1.9	1.7	1.5
P5	2.7	2.7	2.7	2.7	2.6	2.5	2.4	2.2	2.0	1.8	1.6
P10	3.0	3.0	3.0	2.9	2.8	2.7	2.5	2.3	2.2	2.0	1.7
P90	9.7	9.3	9.1	8.8	8.5	8.3	8.0	7.7	7.5	7.2	6.7
P95	12.3	11.8	11.4	11.0	10.5	10.1	9.7	9.2	8.9	8.6	8.2
P99	15.8	15.1	14.5	13.9	13.3	12.6	11.9	11.3	10.8	10.2	9.8

Notes: This exhibit shows summary statistics for the distribution of maximum withdrawal rates (MWRs), each as defined in Expression (6) in the text, for the United States, the world market, and the average country (a cross-sectional average of the 21 countries in the sample), for 11 static asset allocations with stock-bond proportions between 100-0 (all stocks) and 0-100 (all bonds), over 86 rolling (overlapping) 30-year retirement periods, beginning with 1900–1929 and ending with 1985–2014. All strategies are based on a starting portfolio of $1,000, annual withdrawals adjusted by inflation, and annual rebalancing to the stock-bond allocations in the first row. The summary statistics that describe the distribution MWRs across the 86 retirement periods considered are the mean, median, standard deviation (SD); 1% (P1), 5% (P5), and 10% (P10) cutoff MWRs in the lower tail; and 10% (P90), 5% (P95), and 1% (P99) cutoff MWRs in the upper tail. The data are described in Exhibit A1 in the appendix. All figures are in %.

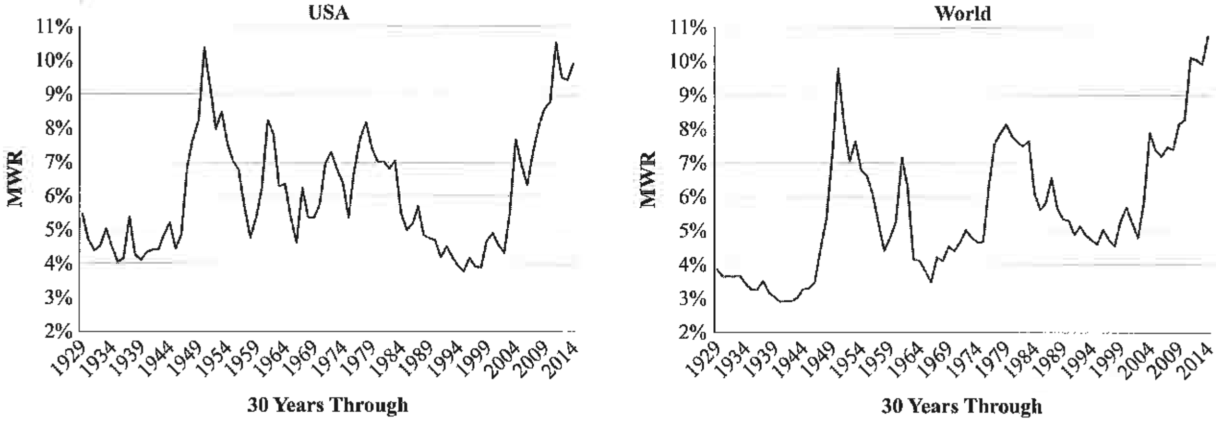
words, individuals from different countries had vastly different standards of living in retirement.

In order to get a glimpse of the variability of MWRs over time, beyond that provided by the cutoff points in Exhibit 3, Exhibit 4 shows the MWRs for the United States and the world market, for the 60-40 allocation, for each of the 30-year retirement periods considered, beginning with the 30 years through 1929 and ending with the 30 years through 2014. As the exhibit clearly shows, the variability was indeed substantial.

Consider the figure on the left (USA) first. Had an individual following a 60-40 allocation in the U.S. retired in 1982 (1969), he would have been able to make an initial withdrawal of 10.5% (3.9%), adjust all subsequent withdrawals by inflation, and end his retirement in 2011 (1998) with no bequest left behind. The difference in the standard of living between these retirees is thus huge; interestingly, they retired less than a decade and a half apart from each other.

EXHIBIT 4

The Variability of Maximum Withdrawal Rates



Notes: This exhibit shows, for the United States and the world market, and the 60-40 allocation, the evolution of MWRs between the first (1900–1929) and the last (1985–2014) 30-year retirement periods considered. All MWRs are based on a starting portfolio of $1,000, annual withdrawals adjusted by inflation, and annual rebalancing. The data are described in Exhibit A1 in the appendix.

A similar situation was faced by retirees following a 60-40 allocation but diversified globally in both stocks and bonds, as the figure on the right (World) shows. Had an individual been lucky to retire in 1985 (unlucky to retiree in 1913), he would have been able to initially withdraw 10.7% (2.9%) of his portfolio, adjust all subsequent withdrawals by inflation, and end his retirement in 2014 (1942) with no bequest. Again, the difference in the standard of living between these retirees is huge.

Needless to say, although the very different experience of all these retirees can only be observed ex post, a tool to determine ex ante whether an individual is retiring at a "good" or a "bad" time would be extremely valuable, among other things to determine an appropriate asset allocation. Unfortunately, although some tools have been proposed for this purpose, research in this area is still incipient and rather tentative. Pfau [2011], for example, proposes to estimate forward-looking IWRs on the basis of the earnings yield, the dividend yield (with both earnings and dividends smoothed over 10 years), and the yield on 10-year Treasury notes prevailing at the beginning of retirement.

The second issue to discuss in this section is whether a retiree should estimate an MWR at the beginning of retirement, and then simply adjust his annual withdrawals by inflation; or, alternatively, should re-estimate the MWR periodically. In both cases, at the beginning of retirement an individual would consider some MWRs and their probability of failure, and then choose an MWR with a failure rate that he finds acceptable.[12] The difference between these two approaches stems from what the retiree does from the second year on.

A periodic re-estimation of MWRs implies that, year after year, a new distribution of MWRs needs to be estimated, each time shrinking the retirement period by one year, and each time the trade-off between different MWRs and their probability of failure needs to be reconsidered. For a 30-year retirement period, this implies estimating 30 distributions of MWRs and assessing the trade-off mentioned 30 times. This is computationally demanding as well as time-consuming.

Furthermore, although the *process* by which the MWR is periodically selected remains unchanged period after period, the annual *withdrawal* is likely to change. Put differently, the periodic re-estimation of MWRs is likely to lead to variable withdrawals, which at least in principle are less desirable than constant (real) withdrawals. Thus, on the negative side, a periodic re-estimation of MWRs implies burdensome calculations and most likely variable withdrawals. That said, on the positive side, variable withdrawals adapt to changing market and life expectancy conditions, thus reducing the probability of failure.

Alternatively, selecting just one MWR at the beginning of retirement, and then adjusting subsequent withdrawals by inflation, has the obvious advantage of simplicity; it also has the desirable characteristic of maintaining purchasing power constant during retirement.

On the other hand, this alternative leads to an MWR and subsequent withdrawals that do not adjust to changing market and life expectancy conditions, which may increase the probability of failure. The jury is still out on which of these two alternative approaches to estimate MWRs makes retirees better off.

ASSESSMENT

Failure rates as a proxy for the expected probability of failure have become a standard way of evaluating retirement strategies. However, this useful tool does have some shortcomings, one of which is that a strategy with a low historical failure rate may have also left large unintended bequests.[13] Enter then the MWR, which by definition leaves no bequest, or, alternatively, a chosen level of bequest.

The analysis in this article considers 11 asset allocations, 21 countries and the world market, 86 retirement periods, and 115 years of history, making it the first study to provide such a comprehensive historical assessment of MWRs. The evidence shows, perhaps unsurprisingly, that MWRs varied very substantially across countries and over time. Importantly, this implies that individuals who retired in some countries or at certain points in time had vastly different standards of living than those who retired in other countries or at other points in time.

The MWR is a useful tool that can be used both to assess how likely a retiree is to sustain a target level of inflation-adjusted withdrawals, and to provide a comprehensive evaluation of retirement strategies; the emphasis here was largely on the first application. Research on MWRs is still incipient, but the attractive characteristics of this novel tool make it a serious candidate to be the subject of much more research in the foreseeable future.

APPENDIX

EXHIBIT A1

Summary Statistics

Panel A: Stocks

	AM	GM	SD	SSD	Min	Max
Australia	8.9	7.3	17.9	9.2	–42.5	51.5
Austria	4.6	0.6	30	15.6	–60.1	127.1
Belgium	5.4	2.7	23.7	13	–48.9	105.1
Canada	7.2	5.8	16.9	8.4	–33.8	55.2
Denmark	7.2	5.3	20.7	8.9	–49.2	107.8
Finland	9.3	5.3	30	13.9	–60.8	161.7
France	5.7	3.2	23.1	12.3	–41.5	66.1
Germany	8.2	3.2	31.7	14.7	–90.8	154.6
Ireland	6.8	4.2	22.9	11.9	–65.4	68.4
Italy	5.9	1.9	28.5	15.6	–72.9	120.7
Japan	8.8	4.1	29.6	15.2	–85.5	121.1
Netherlands	7.1	5	21.4	10.3	–50.4	101.6
New Zealand	7.8	6.1	19.4	9	–54.7	105.3
Norway	7.2	4.2	26.9	11.7	–53.6	166.9
Portugal	8.4	3.4	34.4	15.3	–76.6	151.8
South Africa	9.5	7.4	22.1	9	–52.2	102.9
Spain	5.9	3.7	21.9	11	–43.3	99.4
Sweden	8.0	5.8	21.2	10.8	–42.5	67.5
Switzerland	6.3	4.5	19.5	10.1	–37.8	59.4
UK	7.1	5.3	19.6	9.7	–57.1	96.7
USA	8.5	6.5	20	10.4	–37.6	56.3
World	6.6	5.2	17.4	9.4	–41.0	68.2

Panel B: Bonds

	AM	GM	SD	SSD	Min	Max
Australia	2.5	1.7	13.2	7.6	–26.6	62.2
Austria	4.9	–3.8	51.2	20.1	–94.4	441.6
Belgium	1.6	0.4	15	9.9	–45.6	62.3
Canada	2.8	2.2	10.4	5.4	–25.9	41.7
Denmark	3.9	3.3	11.9	5.1	–18.2	50.1
Finland	1.5	0.2	13.7	10.9	–69.5	30.2
France	1.1	0.2	13	9.5	–43.5	35.9
Germany	1.3	–1.4	15.8	12.4	–95.0	62.5
Ireland	2.7	1.6	15.1	8	–34.1	61.2
Italy	0.2	–1.2	14.7	11.8	–64.3	35.5
Japan	1.7	–0.9	19.7	14.7	–77.5	69.8
Netherlands	2.2	1.7	9.8	5.2	–18.1	32.8
New Zealand	2.5	2.1	9	4.8	–23.7	34.1
Norway	2.6	1.9	12	6.8	–48.0	62.1
Portugal	2.5	0.8	18.7	11.2	–49.7	82.4
South Africa	2.4	1.9	10.4	5.9	–32.6	37.1
Spain	2.5	1.8	12.6	7.1	–30.2	53.2
Sweden	3.5	2.8	12.7	5.9	–37.0	68.2
Switzerland	2.7	2.3	9.4	4.3	–21.4	56.1
UK	2.4	1.6	13.7	7.1	–30.7	59.4
USA	2.5	2	10.4	5.3	–18.4	35.1
World	2.5	1.9	11.3	6	–32.0	46.7

Notes: This exhibit shows, for the series of annual returns over the 1900–2014 period, the arithmetic (AM) and geometric (GM) mean return, standard deviation (SD), semideviation for a 0% benchmark (SSD), lowest return (Min), and highest return (Max). All returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for capital gains/losses and cash flows (dividends or coupons). All figures are in %.

EXHIBIT A2
The Distribution of Maximum Withdrawal Rates

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
Australia											
Mean	7.8	7.4	7.1	6.7	6.3	6.0	5.6	5.2	4.8	4.4	4.0
Median	7.8	7.5	7.0	6.5	5.9	5.4	4.9	4.4	3.9	3.4	3.0
SD	2.3	2.2	2.2	2.2	2.2	2.1	2.1	2.1	2.1	2.1	2.1
P1	2.7	2.8	2.9	2.9	2.9	2.9	2.9	2.6	2.3	1.9	1.7
P5	4.0	4.0	3.9	3.8	3.7	3.4	3.0	2.7	2.5	2.1	1.7
P10	5.1	4.9	4.5	4.2	3.9	3.6	3.3	3.0	2.7	2.3	2.0
P90	11.1	10.2	10.0	9.7	9.5	9.1	8.9	8.6	8.5	8.3	7.7
P95	12.1	11.5	11.3	11.0	10.7	10.4	10.0	9.4	9.0	8.7	8.4
P99	13.7	13.5	13.3	13.1	12.8	12.5	12.2	11.9	11.5	11.2	10.8
Austria											
Mean	4.1	4.4	4.6	4.6	4.6	4.5	4.3	4.0	3.6	3.2	2.7
Median	3.9	4.1	4.3	4.4	4.5	4.6	4.3	4.0	4.1	3.6	1.9
SD	3.1	3.0	3.1	3.1	3.2	3.3	3.3	3.2	3.0	2.8	2.5
P1	0.3	0.4	0.4	0.4	0.3	0.3	0.2	0.1	0.1	0.0	0.0
P5	0.3	0.4	0.4	0.4	0.3	0.3	0.2	0.1	0.1	0.1	0.0
P10	0.4	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.1	0.1	0.0
P90	9.1	9.1	9.4	9.5	9.3	9.0	8.2	7.9	7.7	7.4	6.1
P95	11.0	11.1	10.8	10.9	10.6	10.2	10.3	9.6	8.9	8.0	7.1
P99	12.6	11.9	11.9	12.6	14.0	14.7	15.0	14.7	13.2	10.1	7.5
Belgium											
Mean	4.7	4.7	4.6	4.5	4.4	4.3	4.1	4.0	3.8	3.6	3.4
Median	3.9	4.0	3.9	3.9	3.8	3.8	3.7	3.6	3.6	3.5	3.3
SD	2.8	2.7	2.6	2.5	2.4	2.3	2.3	2.2	2.1	2.1	2.0
P1	1.5	1.7	1.6	1.5	1.5	1.4	1.3	1.2	1.1	1.0	0.9
P5	1.8	1.8	1.7	1.6	1.6	1.5	1.4	1.3	1.2	1.1	0.9
P10	2.1	2.0	1.9	1.9	1.8	1.6	1.5	1.4	1.3	1.1	1.0
P90	8.6	8.2	8.0	7.9	7.8	7.6	7.4	7.3	7.0	6.8	6.6
P95	12.0	11.8	11.5	11.2	10.9	10.5	10.1	9.7	9.3	8.8	8.3
P99	14.3	13.8	13.3	12.8	12.2	11.7	11.1	10.5	9.9	9.3	8.7
Canada											
Mean	6.8	6.6	6.4	6.2	6.0	5.7	5.4	5.1	4.9	4.6	4.3
Median	6.5	6.3	6.1	5.9	5.6	5.3	4.8	4.5	4.1	3.7	3.3
SD	2.0	1.8	1.7	1.6	1.6	1.6	1.6	1.7	1.8	1.8	1.9
P1	3.9	4.2	4.2	4.1	3.9	3.7	3.4	3.2	3.0	2.7	2.5
P5	4.5	4.4	4.3	4.3	4.1	3.8	3.5	3.3	3.0	2.8	2.5
P10	4.7	4.6	4.5	4.3	4.1	3.9	3.7	3.5	3.3	3.1	2.6
P90	10.0	9.2	8.8	8.4	8.2	8.0	8.1	7.7	7.6	7.2	7.1
P95	10.8	10.1	9.6	8.8	8.6	8.8	8.7	8.6	8.7	8.8	8.8
P99	11.7	11.0	11.0	11.1	11.0	11.0	10.8	10.7	10.4	10.2	10.3
Denmark											
Mean	6.0	5.9	5.9	5.8	5.7	5.6	5.5	5.3	5.2	5.0	4.9
Median	5.5	5.3	5.2	5.1	4.9	4.7	4.6	4.5	4.3	4.1	3.9
SD	1.7	1.8	1.8	1.9	1.9	2.0	2.0	2.1	2.1	2.2	2.2
P1	3.5	3.6	3.8	3.9	3.8	3.7	3.5	3.4	3.2	3.0	2.7
P5	4.1	4.1	4.2	4.1	4.0	3.9	3.7	3.5	3.3	3.1	2.8
P10	4.6	4.4	4.3	4.2	4.1	4.0	3.8	3.6	3.4	3.2	3.1
P90	8.0	8.1	8.1	7.9	8.0	8.2	8.2	8.3	8.4	8.4	8.5
P95	11.0	11.2	11.4	11.6	11.7	11.6	11.4	11.1	10.7	10.4	10.0
P99	12.7	12.7	12.7	12.6	12.5	12.3	12.0	11.9	11.8	11.8	11.6

(continued)

EXHIBIT A2 *(continued)*

The Distribution of Maximum Withdrawal Rates

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
Finland											
Mean	6.1	5.9	5.6	5.4	5.1	4.8	4.5	4.2	3.8	3.4	3.1
Median	5.7	5.6	5.5	5.3	5.2	4.9	4.6	4.4	4.1	3.8	3.4
SD	3.2	3.1	3.0	2.9	2.7	2.6	2.5	2.3	2.2	2.0	1.8
P1	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.1	1.0	0.9	0.7
P5	2.0	1.9	1.8	1.7	1.6	1.4	1.3	1.2	1.1	1.0	0.8
P10	2.3	2.2	2.1	1.9	1.8	1.7	1.5	1.4	1.2	1.1	0.9
P90	11.1	11.0	10.5	9.8	9.4	8.6	8.2	7.5	6.8	6.1	5.5
P95	12.7	12.1	11.2	10.9	10.6	10.3	9.8	8.9	8.3	7.5	6.7
P99	14.3	13.8	13.2	12.6	11.9	11.1	10.2	9.4	8.7	8.0	7.2
France											
Mean	4.9	4.8	4.7	4.5	4.4	4.2	4.1	3.9	3.7	3.5	3.3
Median	3.9	3.8	3.6	3.5	3.5	3.4	3.1	2.8	2.4	2.2	1.9
SD	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.7	2.7	2.7	2.7
P1	1.2	1.2	1.1	1.1	1.0	0.9	0.9	0.8	0.7	0.6	0.4
P5	2.0	2.0	1.9	1.8	1.6	1.3	1.1	1.0	0.8	0.6	0.5
P10	2.5	2.5	2.4	2.2	1.9	1.6	1.3	1.1	0.9	0.7	0.5
P90	9.0	9.2	9.3	9.2	9.2	9.2	9.2	9.1	8.9	8.6	8.2
P95	12.0	12.0	11.9	11.7	11.5	11.2	10.8	10.4	10.0	9.5	8.9
P99	15.7	15.3	14.8	14.3	13.7	13.1	12.4	11.7	10.9	10.1	9.4
Germany											
Mean	5.4	5.2	5.0	4.8	4.5	4.3	4.0	3.7	3.4	3.2	2.9
Median	3.7	3.8	3.6	3.5	3.6	3.6	3.4	3.3	3.1	2.8	2.4
SD	5.2	4.8	4.5	4.1	3.8	3.4	3.1	2.9	2.6	2.5	2.3
P1	1.1	1.1	1.1	1.0	0.9	0.8	0.8	0.7	0.6	0.4	0.3
P5	1.2	1.2	1.1	1.0	1.0	0.9	0.8	0.7	0.6	0.4	0.3
P10	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.7	0.6	0.5	0.4
P90	10.3	10.1	9.7	9.3	8.8	8.5	8.2	7.8	7.3	6.4	5.7
P95	16.7	15.5	14.3	13.1	12.0	10.8	9.6	8.5	7.9	7.1	6.5
P99	33.2	30.6	27.8	24.9	22.0	19.0	16.0	13.1	10.4	7.9	7.0
Ireland											
Mean	5.9	5.8	5.6	5.5	5.3	5.1	4.9	4.6	4.4	4.1	3.9
Median	5.3	5.1	4.9	4.8	4.5	4.3	4.0	3.7	3.3	2.9	2.6
SD	2.7	2.7	2.6	2.5	2.5	2.5	2.4	2.4	2.4	2.3	2.3
P1	2.9	2.8	2.8	2.7	2.7	2.6	2.5	2.4	2.3	2.1	1.8
P5	3.0	2.9	2.9	2.8	2.7	2.7	2.6	2.5	2.4	2.2	2.1
P10	3.1	3.0	2.9	2.9	2.8	2.8	2.7	2.6	2.5	2.4	2.2
P90	10.6	9.7	9.3	8.9	8.9	8.9	8.8	8.4	8.1	7.6	7.5
P95	11.9	12.0	11.5	11.4	11.4	11.2	11.2	11.2	11.2	10.4	9.8
P99	15.9	15.6	15.2	14.7	14.1	13.4	12.7	12.1	11.8	11.4	11.1
Italy											
Mean	4.4	4.2	4.1	3.9	3.7	3.5	3.3	3.1	2.8	2.6	2.4
Median	3.6	3.4	3.2	3.0	2.9	2.8	2.7	2.7	2.5	2.4	2.2
SD	2.5	2.4	2.3	2.2	2.2	2.1	2.1	2.0	2.0	1.9	1.8
P1	1.3	1.5	1.4	1.3	1.1	0.9	0.7	0.5	0.4	0.3	0.2
P5	1.6	1.7	1.6	1.4	1.1	0.9	0.7	0.6	0.4	0.3	0.2
P10	1.9	1.9	1.9	1.8	1.6	1.2	0.9	0.7	0.5	0.3	0.2
P90	8.5	8.6	8.3	7.7	7.2	6.7	6.6	6.0	5.6	5.2	4.7
P95	10.1	9.3	9.4	9.3	9.0	8.6	7.9	7.8	7.8	7.7	7.6
P99	11.3	10.9	10.4	9.8	9.6	9.5	9.3	9.1	8.8	8.4	8.0

(continued)

EXHIBIT A2 *(continued)*

The Distribution of Maximum Withdrawal Rates

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
Japan											
Mean	6.3	6.1	5.8	5.6	5.3	5.0	4.6	4.3	4.0	3.6	3.3
Median	6.4	6.5	6.3	6.2	6.0	5.8	5.5	5.2	4.7	4.2	3.7
SD	5.0	4.7	4.3	3.9	3.6	3.3	3.0	2.8	2.6	2.5	2.4
P1	0.5	0.4	0.4	0.3	0.3	0.2	0.2	0.1	0.1	0.1	0.0
P5	0.5	0.4	0.4	0.4	0.3	0.3	0.2	0.2	0.1	0.1	0.0
P10	0.5	0.5	0.5	0.4	0.4	0.3	0.2	0.2	0.1	0.1	0.1
P90	11.6	10.9	10.2	9.4	8.6	8.2	7.9	7.8	7.0	7.1	6.7
P95	19.0	17.7	16.2	14.6	12.6	10.9	9.5	8.3	8.1	7.8	7.6
P99	22.8	20.8	18.9	16.8	14.7	12.6	10.5	9.4	8.6	8.1	7.9
Netherlands											
Mean	6.3	6.1	6.0	5.8	5.6	5.3	5.1	4.8	4.6	4.3	4.0
Median	5.3	5.3	5.1	5.0	5.0	4.6	4.4	3.9	3.8	3.5	3.3
SD	2.9	2.6	2.4	2.3	2.1	2.0	1.9	1.8	1.8	1.8	1.8
P1	2.9	3.2	3.4	3.4	3.3	3.2	3.1	3.0	2.7	2.4	2.1
P5	3.1	3.3	3.4	3.5	3.6	3.5	3.3	3.1	2.8	2.4	2.1
P10	3.4	3.5	3.7	3.7	3.6	3.6	3.5	3.2	2.9	2.5	2.1
P90	10.8	10.1	9.9	9.6	9.1	8.6	8.1	7.7	7.6	7.1	6.7
P95	12.5	12.1	11.6	11.1	10.5	10.0	9.4	8.8	8.4	8.0	8.0
P99	15.6	15.1	14.4	13.8	13.0	12.3	11.5	10.7	9.8	9.3	9.5
New Zealand											
Mean	6.8	6.6	6.4	6.2	5.9	5.6	5.3	5.0	4.7	4.4	4.1
Median	6.8	6.5	6.3	6	5.6	5.2	4.9	4.6	4.2	3.9	3.4
SD	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
P1	4.1	4.0	4.0	3.9	3.8	3.6	3.4	3.2	3.0	2.7	2.4
P5	4.7	4.6	4.4	4.2	4.0	3.8	3.6	3.3	3.0	2.7	2.4
P10	4.9	4.8	4.6	4.4	4.2	4.0	3.7	3.5	3.2	2.9	2.5
P90	8.8	8.4	8.4	8.2	7.9	8.0	7.8	7.4	7.3	7.0	6.5
P95	10.5	10.5	10.3	9.9	9.6	9.5	9.2	8.8	8.7	8.3	7.5
P99	11.2	10.9	11.1	11.2	11.1	10.8	10.3	9.8	9.1	9.3	9.3
Norway											
Mean	5.2	5.2	5.2	5.1	5.0	4.9	4.8	4.7	4.5	4.3	4.0
Median	4.4	4.3	4.2	4.2	4.0	3.8	3.8	3.6	3.5	3.3	3.0
SD	2.4	2.4	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.1	2.1
P1	2.2	2.3	2.5	2.6	2.7	2.7	2.7	2.6	2.5	2.4	2.3
P5	2.6	2.7	2.7	2.8	2.8	2.8	2.8	2.7	2.6	2.5	2.4
P10	2.8	2.9	3.0	3.0	3.1	3.0	2.9	2.8	2.8	2.6	2.4
P90	8.2	8.3	8.4	8.5	8.7	8.7	8.5	8.1	8.1	8.0	7.9
P95	10.2	10.3	10.3	10.2	10.0	9.8	9.4	9.2	9.2	9.1	8.9
P99	14.4	14.1	13.7	13.2	12.6	12.0	11.4	11.8	12.2	12.5	12.8
Portugal											
Mean	5.7	5.7	5.6	5.4	5.3	5.1	4.8	4.6	4.3	4.0	3.7
Median	4.4	4.4	4.3	4.4	4.3	4.4	4.2	4.0	3.5	2.9	2.4
SD	3.8	3.5	3.3	3.1	3.0	2.9	2.8	2.8	2.8	2.8	2.9
P1	0.9	1.1	1.2	1.3	1.4	1.5	1.5	1.4	1.2	0.9	0.7
P5	2.0	2.2	2.4	2.5	2.4	2.1	1.9	1.5	1.2	1.0	0.7
P10	2.5	3.0	2.9	2.7	2.6	2.3	2.0	1.6	1.3	1.0	0.7
P90	10.2	9.5	9.2	9.3	9.4	9.2	9.5	8.5	8.9	8.3	7.2
P95	14.3	13.8	13.2	12.5	11.7	11.8	11.8	10.5	10.7	11.0	11.1
P99	22.0	21.5	20.7	19.6	18.3	16.7	14.9	13.0	11.6	11.4	11.5

(continued)

EXHIBIT A2 *(continued)*

The Distribution of Maximum Withdrawal Rates

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
South Africa											
Mean	7.7	7.4	7.0	6.6	6.2	5.8	5.4	5.0	4.6	4.2	3.8
Median	7.3	7.0	6.6	6.1	5.7	5.3	4.9	4.6	4.3	3.7	3.3
SD	2.7	2.5	2.3	2.1	1.9	1.8	1.7	1.6	1.5	1.4	1.4
P1	3.8	3.8	3.7	3.6	3.6	3.5	3.4	3.1	2.8	2.4	2.1
P5	4.6	4.5	4.3	4.1	3.9	3.7	3.5	3.3	3.0	2.5	2.1
P10	4.8	4.8	4.7	4.5	4.3	4.1	3.8	3.6	3.2	2.8	2.4
P90	11.4	11.0	10.3	9.5	8.6	8.0	7.5	7.0	6.6	6.1	5.6
P95	12.8	11.7	10.7	10.1	9.6	9.0	8.5	7.9	7.4	6.8	6.3
P99	16.4	15.5	14.8	14.3	13.7	13.2	12.5	11.9	11.2	10.5	9.7
Spain											
Mean	4.9	4.8	4.8	4.7	4.6	4.5	4.3	4.2	4.0	3.8	3.6
Median	4.4	4.4	4.3	4.3	4.1	4.0	3.9	3.6	3.5	3.3	3.1
SD	2.9	2.8	2.6	2.5	2.3	2.2	2.1	1.9	1.8	1.7	1.6
P1	1.5	1.7	1.8	1.9	2.1	2.2	2.2	2.2	2.1	2.0	1.9
P5	1.9	2.0	2.2	2.3	2.3	2.3	2.5	2.4	2.3	2.1	2.0
P10	2.2	2.3	2.5	2.7	2.7	2.6	2.6	2.6	2.5	2.4	2.1
P90	7.4	7.1	6.7	6.7	6.6	6.4	6.3	6.1	5.9	5.7	5.5
P95	12.3	12.1	11.8	11.4	11.0	10.6	10.1	9.7	9.2	8.5	7.9
P99	17.0	16.4	15.7	15.0	14.3	13.5	12.7	11.9	11.1	10.2	9.3
Sweden											
Mean	6.4	6.3	6.1	6.0	5.8	5.6	5.3	5.1	4.9	4.6	4.4
Median	6.6	6.3	5.9	5.6	5.3	4.9	4.6	4.3	4.1	4.1	4.0
SD	3.1	2.8	2.6	2.4	2.2	2.1	2.0	1.9	1.9	2.0	2.0
P1	1.9	2.1	2.4	2.7	3.0	3.2	3.1	2.7	2.4	2.1	1.8
P5	2.1	2.4	2.7	3.1	3.3	3.4	3.5	3.6	3.1	2.6	2.3
P10	2.6	2.9	3.2	3.4	3.6	3.8	3.8	3.7	3.3	2.9	2.5
P90	9.3	9.1	8.8	9.0	9.4	9.3	8.6	8.7	8.4	8.4	7.4
P95	13.5	12.8	12.0	11.7	11.3	10.9	10.4	9.9	9.4	8.9	8.4
P99	17.3	16.4	15.5	14.5	13.5	12.5	11.4	11.3	11.8	12.3	12.7
Switzerland											
Mean	5.7	5.6	5.5	5.4	5.3	5.2	5.1	4.9	4.7	4.5	4.3
Median	5.5	5.5	5.5	5.5	5.3	5.2	5.0	4.8	4.5	4.3	4.0
SD	2.2	2.1	1.9	1.8	1.7	1.6	1.5	1.4	1.4	1.3	1.3
P1	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.0	3.0	3.1	3.1
P5	2.7	2.7	2.8	3.0	3.0	3.1	3.2	3.2	3.2	3.2	3.1
P10	2.8	2.9	3.0	3.1	3.2	3.2	3.3	3.3	3.3	3.3	3.2
P90	8.6	8.1	8.1	7.6	7.2	6.9	6.8	6.5	6.2	5.9	5.6
P95	9.8	9.4	9.0	8.5	8.1	7.6	7.1	7.1	7.4	7.6	7.7
P99	11.3	11.1	11.2	11.3	11.3	11.3	11.3	11.2	11.1	10.9	10.8

(continued)

EXHIBIT A2 *(continued)*

The Distribution of Maximum Withdrawal Rates

Stocks-Bonds	100-0	90-10	80-20	70-30	60-40	50-50	40-60	30-70	20-80	10-90	0-100
UK											
Mean	6.8	6.5	6.3	6.0	5.8	5.5	5.2	4.9	4.6	4.3	4.1
Median	6.1	6.0	5.8	5.6	5.2	4.8	4.4	4.0	3.6	3.3	2.9
SD	2.5	2.3	2.2	2.2	2.1	2.1	2.1	2.1	2.1	2.1	2.2
P1	3.6	3.6	3.5	3.3	3.2	3.1	2.9	2.8	2.5	2.1	1.7
P5	4.0	3.8	3.7	3.5	3.4	3.2	3.1	2.8	2.7	2.5	2.1
P10	4.2	4.0	3.8	3.7	3.5	3.4	3.2	3.0	2.8	2.6	2.4
P90	10.2	9.9	9.4	9.3	9.0	8.7	8.4	8.3	8.1	7.7	7.2
P95	11.5	11.2	10.9	10.6	10.3	9.9	9.6	9.2	8.8	8.5	8.3
P99	15.6	14.7	13.8	12.9	12.0	12.0	12.0	11.9	11.9	11.8	11.7

Notes: This exhibit shows summary statistics for the distribution of maximum withdrawal rates (MWRs), each as defined in Expression (6) in the text, for 20 of the 21 countries in the sample, for 11 static asset allocations with stock-bond proportions between 100-0 (all stocks) and 0-100 (all bonds), over 86 rolling (overlapping) 30-year retirement periods, beginning with 1900–1929 and ending with 1985–2014. All strategies are based on a starting portfolio of $1,000, annual withdrawals adjusted by inflation, and annual rebalancing to the stock-bond allocations in the first row. The summary statistics that describe the distribution MWRs across the 86 retirement periods considered are the mean, median, standard deviation (SD), 1% (P1), 5% (P5), and 10% (P10) cutoff MWRs in the lower tail; and the 10% (P90), 5% (P95), and 1% (P99) cutoff MWRs in the upper tail. The data are described in Exhibit A1. All figures are in %.

ENDNOTES

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[1] See Estrada [2017], who reports historical failure rates for 11 asset allocations and 21 countries over the 1900–2014 period. He also introduces the concepts of shortfall years and sustained percentage, which aim to refine and complement the failure rate, and reports their values for the same allocations, countries, and sample period.

[2] Directly or indirectly, annuities are often featured in discussions of withdrawal strategies; see, for example, Sexauer et al. [2012] on a strategy combining a laddered TIPS portfolio and a deferred nominal annuity, and Waring and Siegel [2015] on an annually recalculated virtual annuity.

[3] Given the use of real returns, this implies a constant withdrawal of $40 in scenarios S1, S2, and S3, and a constant withdrawal of $63 in scenario S5.

[4] Note that by the end of year 24 the portfolio still has $40, which is for the final withdrawal that will enable the retiree to live through the following year.

[5] This is based on the DMS database (described below) over the 1900–2014 period.

[6] The 3.88% annual growth rate in real withdrawals is the solution of the problem. Put differently, if a retiree starts with $1,000, expects to live 30 years, earns a 5.24% annual return, desires to leave no bequest, and makes a first withdrawal of $40, then increasing his withdrawals by 3.88% a year in real terms would exhaust his portfolio by the end of this period.

[7] The 6.3% IWR is the solution of the problem. Put differently, if a retiree starts with $1,000, expects to live 30 years, earns a 5.24% annual return, desires to leave no bequest, and aims to keep his purchasing power constant in real terms, then taking 6.3% of his portfolio at the beginning of retirement would exhaust his portfolio by the end of this period.

[8] Obviously, a desired bequest of $0 can be replaced by any other level of desired bequest; the numbers would change but not the intuition of the results just discussed.

[9] To be sure, this is the case for the U.S. market. The 4% rule has a mixed record in other markets; see Estrada [2016a, 2017]. Also, and importantly, note that all the scenarios in Exhibit 1 consider a constant annual return, which assumes sequence risk away. On this important issue, related to the sequence of returns, the cash flows taken from a portfolio, and the failure rate, see Frank and Blanchett [2010].

[10] This assumption will be maintained throughout this article. Assuming that the retiree desires to leave a bequest of P_T instead is trivial and does not change the intuition of any of the results discussed.

[11] As in most of the related literature, transaction costs and taxes are not accounted for. Cook et al. [2015] do consider taxes, and in particular the difference between implementing several strategies in a taxable account, a tax-deferred account, and a tax-exempt account.

[12] The probability of failure can be estimated from the distribution of MWRs. This distribution could be empirical,

such as those summarized in Exhibits 3 and A2, or one generated by way of simulations, as those considered by Suarez et al. [2015] and Clare et al. [2016].

[13] On some of the shortcomings of the failure rate, see Milevsky [2016] and Estrada [2017].

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