From Failure to Success: *Replacing the Failure Rate*

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Retirement strategies can be evaluated in many different ways. The failure rate, a proxy for the probability of failure, is arguably the variable most widely used for this purpose. It measures how often a strategy failed to sustain a withdrawal plan, thus depleting a portfolio before a retiree's death, over the historical or simulated retirement periods evaluated.

The failure rate is both very widely used and very useful, although not free from shortcomings. Estrada [2017] argues that this variable accounts for *how often* a strategy failed but ignores *by how much* it failed. Two strategies may have failed in 10% of the retirement periods evaluated, but if during those failing periods one sustained a given level of withdrawals, on average, twice as long as the other, then it can hardly be said that these two strategies had the same performance.[1]

In order to overcome this shortcoming, Estrada [2017] proposes a new variable, shortfall years, which seeks to complement, rather than replace, the failure rate. Shortfall years aims to measure the average number of years a strategy failed to support withdrawals, over all the retirement periods in which it failed. The combination of the failure rate and shortfall years, referred to here as the F–S_Y framework, enables a broader evaluation of retirement strategies.

Shortfall years, however, is not free from shortcomings. For now, it suffices to highlight that because it aims to complement rather than replace the failure rate, then it takes two variables to evaluate retirement strategies. Needless to say, it would be more convenient to have just one variable providing an equally comprehensive evaluation; that is, precisely, the goal of the framework introduced in this article.

In order to overcome the somewhat burdensome joint use of the failure rate and shortfall years, the new framework proposed in this article moves the focus from failure to success. More precisely, instead of focusing on the frequency and magnitude of a strategy's failure, the approach proposed focuses on its success. A new variable, years sustained, is introduced; and the ratio between its expected value and standard deviation, risk-adjusted success, or RAS, is the single variable proposed here to provide a comprehensive evaluation of retirement strategies.

The framework suggested is applied to the selection of a single strategy, of the 11 asset allocations evaluated, for each of the 21 countries and the world market in the sample. Given the 115-year sample period considered for each country, the results discussed here provide a comprehensive evaluation of the relative performance of aggressive and conservative asset allocation strategies during retirement.

Needless to say, the retirement problem is analogous to a wealth management problem when the latter is focused on financing an individual's lifestyle after retiring from work. Furthermore, in the same way that a retiree may implement a strategy that fails (that is, it depletes a portfolio too early), more generally an individual may implement a strategy that fails to achieve whatever goal was set for the portfolio.

The rest of the article is organized as follows. The first section discusses in more detail the issue at stake and particularly the two variables introduced in this article, years sustained and risk-adjusted success; the next session discusses the global evidence and the selection of the best strategy in each country; and the following section provides an assessment. An appendix with the formal background and tables concludes the article.

THE ISSUE

This section starts with a brief literature review; then discusses the main variables featured in this article by way of an example; and finally discusses the intuition behind the framework proposed. The formal background is relegated to the appendix.

Literature Review

The evaluation of retirement strategies has a long history that is typically traced back to the groundbreaking article by Bengen [1994], who focuses on determining a "safe" initial withdrawal rate (IWR); his analysis does not focus on the failure rate directly but on portfolio longevity instead. However, his analysis does put failure at the center of the discussion by considering a 4% IWR safe precisely because historically it never depleted a portfolio (failed) before 30 years.

Ho, Milevsky, and Robinson [1994] develop a theoretical model that aims to minimize the probability that a retiree fails to achieve the return needed to satisfy his target consumption during his expected retirement period. Hence, failing in this framework amounts to achieving a return lower than that needed to satisfy the target consumption.

Bengen [1996] considers historical 30-year retirement periods and calculates, for different IWRs, the probability (better thought of as a historical frequency) that a portfolio will last for 30 years. Hence, he actually focuses on the *success* rate, rather than on the failure rate, although the latter is obviously 1 minus the former.

Milevsky, Ho, and Robinson [1997] consider a model in which the goal is to minimize the probability of depleting a portfolio before the uncertain end of the retirement period (that is, the probability of failing). Their model incorporates mortality tables and gender, among other variables, and enables a retiree to change her consumption level during retirement to affect the probability of failure.

Cooley, Hubbard, and Walz [1998] focus on the success rate (1 minus the failure rate), which they define as the percentage of all past periods supported by a portfolio despite withdrawals. This success rate is similar to Bengen's [1996] probability that a portfolio will last 30 years, except for the fact that Cooley, Hubbard, and Walz consider retirement periods from 15 to 30 years.

In research focusing on asset allocation or withdrawal rates during the 2000s and beyond, the failure rate became ubiquitous. It has also been articulated into models in very different ways. Das et al. [2010], for example, define risk as the probability of failing to reach the threshold level in different mental accounts, and allow for different attitudes toward risk to vary by account. They consider behavioral individuals that view their portfolios as a collection of mental account subportfolios, with each account having a goal and threshold level.

More recently, Chhabra [2015] and Brunel [2015] both favor a goals-based wealth management framework based on the achievement of personal goals.[2] This approach starts with an individual's personal goals and then optimizes financial assets and human capital around those goals. Importantly, in this framework risk is viewed as the probability of failing to achieve a goal, which is different from the traditional definition of failure (that is, depleting a retirement portfolio before death). In fact, this approach links each goal with a minimum required probability of achieving it, over the time horizon that applies to it.

Even more recently, Parker [2016], who subscribes to a goals-based approach, offers the following example. Consider a client that in order to satisfy his goals needs to convert \$1 into \$1.28 in five years. The probability of ending with less than \$1.28 is risk; reward, in turn, is any wealth obtained beyond \$1.28. Thus, an individual that ended with \$1.35 has obtained, on a goal-adjusted basis, an additional 5% in wealth that can be used to fund another goal.

The Failure Rate, Shortfall Years, and Shortcomings

Consider Exhibit 1, which contemplates four hypothetical retirement strategies, S1 through S4. All strategies are evaluated over 100 retirement periods, each of them 30 years long. Just to simplify this introductory discussion, and for reasons that will be clearer later, it is assumed that these four strategies leave no bequest in any of the retirement periods evaluated.

As Panel A of Exhibit 1 shows, S1 and S3 failed only once, thus having a 1% failure rate; S2 and S4 failed much more often, 99 and 25 times, thus having 99% and 25% failure rates. Importantly, not all four strategies sustained withdrawals during the same number of years when they failed. S1 and S2 both fell short by 15 years, thus supporting 15 years of withdrawals. However, because S1 failed only once and S2 failed 99 times, over the 100 retirement periods considered, S1 fell short by 15 years and S2 did so by 1,485 years.

S3 and S4, on the other hand, failed by the same 25 years over the 100 retirement periods considered, but they did so in a very different way. S3 failed only once, by 25 years, thus supporting just five years of withdrawals; S4 failed 25 times, by just one year each time, thus supporting 29 years of withdrawals in each failing period.

If these four strategies were evaluated solely on the basis of the failure rate, S1 and S3 would be clearly preferred over S4, which in turn would be clearly preferred over S2. Are then S1 and S3 equally attractive from a retiree's perspective? Clearly not, which is where the *shortfall years* variable introduced by Estrada [2017] comes in. When S1 failed, it fell short by 15 years; when S3 failed, it fell short by 25 years. Therefore, S1 is clearly preferred over S3.

The joint use of the failure rate (F) and shortfall years (S_Y), referred to here as the F–S_Y framework, thus enables a clear choice between S1 and S3, although at the cost of having to deal with two variables rather than with just one. That said, shortfall years used by itself has a shortcoming of its own. Consider S1 and S2, both of which fell short by 15 years (on average) when they failed. However, S1 failed just one time and S2 failed 99 times; thus, S1 and S2 are hardly identical from a retiree's perspective, as the shortfall years variable seems to suggest. This is why, again, the failure rate and shortfall years need to be used jointly rather than independently.

And yet even that may not be enough; to see why, consider S3 and S4. The failure rate would clearly suggest choosing S3 over S4; shortfall years would clearly suggest the opposite. Thus, the F–S_Y framework would provide a retiree with a trade-off to evaluate rather than with a clear choice between these two strategies. This is, precisely, where the framework introduced in this article comes in.

EXHIBIT 1
Four Strategies

	S1	S2	S3	S4
Panel A				
Number of failures	1	99	1	25
Failure rate (F)	1.0%	99.0%	1.0%	25.0%
Years short when the strategy failed, per failure	15	15	25	1
Total years short when the strategy failed	15	1485	25	25
Shortfall years (S_Y)	15	15	25	1
Panel B				
Years sustained when the strategy fails (Y_{S-F})	15	15	5	29
$E(Y_S)$	29.85	15.15	29.75	29.75
$SD(Y_S)$	1.49	1.49	2.49	0.43
RAS	20.00	10.15	11.96	68.70

Notes: This exhibit shows four hypothetical retirement strategies, S1 through S4, evaluated over 100 retirement periods, each of them 30 years long. The failure rate (F) is the number of retirement periods a strategy failed, relative to the total number of retirement periods evaluated; shortfall years (S_Y) is the average number of years a strategy fell short, over all the retirement periods in which it failed; years sustained when a strategy fails (Y_{S-F}) is the average number of years a strategy sustained withdrawals over all the retirement periods in which it failed; $E(Y_S)$ and $SD(Y_S)$ are the expected value and standard deviation of the years sustained variable (Y_S); risk-adjusted success (RAS) is the ratio between $E(Y_S)$ and $SD(Y_S)$. All variables formally are defined in expressions A-1–A-6 in the appendix.

Years Sustained and Risk-Adjusted Success

This section discusses the intuition behind the framework proposed in this article; the formal background is relegated to the appendix. Beginning from the F–S_Y framework as informally defined in the previous section (and formally in the appendix,) and considering that in any given retirement period a strategy may succeed or fail, the variable introduced here, *years sustained* (Y_S), takes different values in success and in failure.

Over all the periods in which a strategy failed, the years sustained variable measures the average number of years the strategy sustained withdrawals. Because the average is taken over failing periods, it should be clear that the resulting figure will be lower than the length of the retirement period considered (L).

However, over all the years in which a strategy succeeded, years sustained still measures the average number of years the strategy sustained withdrawals. However, because the average in this case is taken over successful periods, it should be clear that the resulting figure will be higher than the length of the retirement period considered. To be sure, this is the case because unless the portfolio lasted exactly L years, it left behind a bequest, which can be expressed in years of inflation-adjusted withdrawals, thus pushing the average taken above L.

Given that the failure rate is a proxy for the probability of failure, which implies that a strategy fails with probability F and succeeds with probability $1-F$, years sustained is a random variable for which its expected value and standard deviation can be calculated. The other variable introduced here, *risk-adjusted success* (RAS), is precisely the ratio between the expected value and the standard deviation of years sustained. Note that, when evaluating two strategies, the one with a higher RAS had superior performance; this is the case because a higher RAS indicates more years of withdrawals sustained or less uncertainty about the years of withdrawals sustained.

To drive home the usefulness of Y_S and RAS, consider now panel B of Exhibit 1. The failure rate, shortfall years, or their joint use provide a framework to unambiguously choose between S1 and S2 (S1, due to a lower F and the same S_Y), between S1 and S3 (S1, due to the same F and a lower S_Y), and between S2 and S4 (S4, due to both a lower F and a lower S_Y). However, these two variables do not provide an unambiguous choice between S1 and S4 (S1 has a lower F but a higher S_Y), between S2 and S3 (S2 has a higher F but a lower S_Y), or between S3 and S4 (S3 has a lower F but a higher S_Y).

Enter then Y_S, its expected value and standard deviation, and ultimately RAS, all calculated as in expressions A-3–A-6 in the appendix. RAS now reveals the better performing strategy in the three previous ambiguous comparisons: S4 performed better than S1, S3 performed better than S2, and S4 performed better than S3. RAS also reveals that S4 delivered the best overall performance; this is the case because S4 sustained the largest number of annual withdrawals relative to the uncertainty about those withdrawals.

Unlike the F-S_Y framework, which is based on two variables, both focusing on failure, the framework built around Y_S, focuses on success and ultimately on just one variable, the RAS ratio introduced here. This approach is applied in the next section to select the best strategy, of the 11 asset allocations considered, for each of the 21 countries and world market in the sample.

EVIDENCE

This section discusses the global evidence based on 21 countries and the world market over the 115 years from 1900 to 2014. The first part discusses the data and methodology; the second part discusses the evidence on the failure rate, years sustained, and risk-adjusted success for 11 asset allocation strategies; and the third part elaborates on some related issues.

Data and Methodology

The sample considered is the Dimson–Marsh–Staunton database, described in detail in Dimson, Marsh, and Staunton [2002, 2016]. It contains annual returns for stocks and long-term government bonds over the 1900–2014 period for 21 countries and the world market. Returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for both capital gains/losses and cash flows (dividends or coupons). Exhibit A1 in the appendix summarizes some characteristics of all the series of stock and bond returns in the sample.

The analysis is based on a \$1,000 portfolio at the beginning of retirement, a 4% initial withdrawal rate, annual withdrawals, and a 30-year retirement period.[3] At the beginning of each year the annual withdrawal is made, the portfolio is rebalanced to the target asset allocation for the year, and then it compounds at the observed return of stocks and bonds for that year. This process is repeated at the beginning of each year during the 30-year retirement period, at the end of which the portfolio has a terminal wealth or bequest that may be positive or 0. The first 30-year retirement period considered is 1900–1929 and the last one is 1985–2014, for a total of 86 rolling (overlapping) periods.

The analysis considers 11 stock–bond allocations ranging 100/0 (all stocks) and 0/100 (all bonds), with nine allocations in between (90/10, 80/20, ..., 20/80, and 10/90). The goal of the analysis that follows is to apply the framework proposed, and particularly the RAS ratio, to the selection of the best asset allocation for each country in the sample.

Evaluation of Strategies

Exhibit 2 shows the failure rate (F), shortfall years (S_Y), and risk-adjusted success (RAS) for the U.S. and world markets, for the 11 asset allocations considered. Exhibit A2 in the appendix shows the same variables, as well as years sustained in failure (Y_{S-F}) and years sustained in success (Y_{S-S}), for all the countries in the sample.[4] In both exhibits, the highest RAS across all the strategies evaluated for each country is highlighted in bold.

First, consider the U.S. market in the top half of Exhibit 2. The failure rate would favor an aggressive strategy, with at least 70% invested in stocks, but it would not provide a clear choice among the four most aggressive strategies. Furthermore, the F–S_Y framework would lead to the selection of the 80/20 strategy, for which the exhibit shows the lowest S_Y among the four allocations with the lowest F. However, when the focus shifts to years sustained, and its expected value and standard deviation are combined into the RAS ratio proposed here, the 70/30 stock–bond allocation is revealed as the best performing strategy. Thus, in this case, the F–S_Y framework and RAS would both select rather similar (and relatively aggressive) strategies, although not exactly the same one.

Consider now the world market, in the bottom half of Exhibit 2. The failure rate would select the most aggressive strategy, which happens to be the one that would also be selected by shortfall years; in fact, the lowest F and S_Y are those for the 100/0 allocation. Furthermore, this allocation is also the one that has the highest RAS. In this case, then, both the F–S_Y framework and RAS would select the same strategy, and in particular the most aggressive of those considered.

Perhaps unsurprisingly, the results for the other countries in the sample reported in Exhibit A2 show that the best performing allocation varies markedly from country to country. In five countries and in the world market, RAS selects the most aggressive (100/0) strategy; in three countries RAS selects the most conservative (0/100) strategy; and in most of the other countries, the strategy selected by RAS has at least 60% of equity exposure. Interestingly, the results in Exhibit A2 show that in a few cases the F–S_Y framework and RAS lead to the selection of a rather different strategy; in three countries (Belgium, Germany, and Spain) RAS selects a fairly more conservative strategy than that suggested by the F–S_Y framework.

EXHIBIT 2
Risk-Adjusted Success

Stocks/Bonds	100/0	90/10	80/20	70/30	60/40	50/50	40/60	30/70	20/80	10/90	0/100
USA											
F	3.5	3.5	3.5	3.5	4.7	8.1	15.1	25.6	40.7	64.0	65.1
S_Y	5.7	3.3	2.0	2.7	3.0	2.9	3.2	3.9	4.9	5.6	7.9
RAS	6.7	7.2	7.7	**8.0**	7.3	5.9	4.5	3.6	2.8	1.9	1.9
World											
F	14.0	15.1	16.3	20.9	23.3	29.1	36.0	36.0	38.4	45.3	59.3
S_Y	5.8	6.2	7.1	7.4	8.4	8.5	8.6	10.5	11.3	10.6	9.7
RAS	**3.5**	3.4	3.3	3.0	2.8	2.6	2.3	2.3	2.2	2.1	1.8

Notes: This exhibit shows, for 11 asset allocations with stock-bond proportions ranging 100/0 (all stocks) to 0/100 (all bonds), and for the U.S. and world markets, the failure rate (F), shortfall years (S_Y), and risk-adjusted success (RAS), as defined in expressions A-1, A-2, and A-6 in the appendix, over 86 rolling 30-year retirement periods, beginning with 1900–1929 and ending with 1985/2014. All strategies are based on a starting capital of $1,000, a 4% initial withdrawal rate, subsequent annual withdrawals adjusted by inflation, and annual rebalancing to the stock–bond allocations in the first row. F is expressed in % and S_Y in years. The data is described in Exhibit A1 in the appendix.

Why does RAS favor the 70/30 strategy over the 80/20 strategy in the United States? Why do RAS and the F–S_Y framework lead to the same choice in the world market? Why does RAS suggest more conservative strategies in some countries than those suggested by the F–S_Y framework? These are some of the issues discussed in the next section.

Some Further Thoughts

The Sharpe ratio, which is widely used to evaluate the performance of funds and strategies, uses volatility (the standard deviation of returns) as a measure of risk. Although it arguably is the variable most widely used to assess risk, volatility has several shortcomings, two of which deserve special attention for our purposes. First, equal deviations above and below the mean are treated symmetrically; that is, the standard deviation of returns does not distinguish between upside and downside volatility, although investors obviously do. Second, an asset with very low volatility is bound to have a very high Sharpe ratio, largely regardless of how low the asset's return may be.[5]

The reason this is relevant is because RAS is also a ratio of expected return to risk, and the latter is assessed with the standard deviation of Y_S. This implies that the same caveats that apply to the standard deviation of returns and the Sharpe ratio also apply to the standard deviation of Y_S and RAS. In particular, although a large dispersion is not *necessarily* bad, as it may be the case when an asset has a very substantial upside, the standard deviation always treats it as such.

Consider why RAS selects the 70/30 strategy over the 80/20 strategy selected by the F–S_Y framework in the United States. As Exhibit A2 shows, the latter allocation not only has a higher $E(Y_S)$ but also has both a higher Y_{S-F} (28.0 versus 27.3) and a higher Y_{S-S} (91.7 versus 82.0); that is, it supports more years of withdrawals both when it fails and when it succeeds. However, the Y_{S-S}–Y_{S-F} spread for the 80/20 strategy (63.7) is higher than that for the 70/30 strategy (54.7), which is reflected in a higher $SD(Y_S)$ and ultimately in a lower RAS.[6]

Interestingly, note that the 80/20 strategy has a slightly higher Y_{S-F} than the 70/30 strategy, but a substantially higher Y_{S-S}. This implies that most of the larger Y_{S-S}–Y_{S-F} spread of the former strategy, which results in a higher $SD(Y_S)$, is due to its higher upside. Thus, just like the volatility of returns does not distinguish between a large negative and a large positive fluctuation, neither does the volatility of Y_S; a departure from the mean is "bad" regardless of whether it is above or below it. This weakness of the RAS ratio is similar to that of the Sharpe ratio, and therefore those who frequently use the latter should have no problem using the former.

A similar reasoning explains most of the strategies selected by RAS, including those in markets in which the F–S_Y framework selects a fairly different strategy. Of the three markets in which RAS selects a more conservative strategy than the F–S_Y framework, consider the case of Spain. The failure rate would select the most aggressive strategy and RAS would do just the opposite and select the most conservative one; shortfall years, in turn, would select the 30/70 allocation, thus implying no clear choice from the F–S_Y framework.

Why does RAS select the most conservative strategy when the failure rate would suggest the most aggressive one in Spain? A straightforward comparison of the two extreme strategies shows that, when going from the 100/0 to the 0/100 allocation, $E(Y_S)$ falls by 47% (from 58.2 to 30.8) and $SD(Y_S)$ falls by more than 51% (from 31.7 to 15.5). The large fall in $SD(Y_S)$ is due to the large fall in the Y_{S-S}–Y_{S-F} spread, which is much larger for the 100/0 allocation than for the 0/100 allocation. Importantly, although Y_{S-F} is higher for the latter strategy than it is for the former, Y_{S-S} is *much* lower. In other words, most of the decrease in the Y_{S-S}–Y_{S-F} spread is driven by a decrease in upside potential.

In short, in the same way that differences in the Sharpe ratio of different strategies are due to differences in the mean return and/or volatility of those strategies, differences in RAS are due to differences in the mean and/or standard deviation of Y_S. Similarly, in the same way that the Sharpe ratio sometimes selects a strategy that a "reasonable" investor would be unlikely to select, so does occasionally the RAS ratio.

Blanchett [2007] proposes to evaluate retirement strategies with a measure somewhat related to the one proposed here, namely, the success-to-variability ratio (SVR), defined as the ratio between the failure rate and the volatility of the portfolio. In his framework, the failure rate, which usually suggests an aggressive strategy, is balanced against the volatility of the portfolio, which usually suggests a conservative strategy. Interestingly, in his simulations, the failure rate would select the most aggressive (100/0) strategy whereas the SVR would select the most conservative (0/100) one.

The reason for this is that very aggressive strategies are also very volatile and therefore are highly penalized by the SVR. Obviously, this is very much related to the earlier discussion about the very conservative strategies selected by RAS in some countries.

Blanchett [2007] concludes that while "standard deviation is the most common definition of risk for investment purposes, investors do not fear making too much money (upside deviation), which is why other definitions of risk (such as downside risk) may prove to be more useful." A similar argument can also be made about the Sharpe ratio and RAS and the unintuitive strategy selections they occasionally make.

A final issue to be considered is how valuable these results, based on long-term historical data, are to making forward-looking retirement decisions. On the one hand, mean reversion has been a powerful force in financial markets, thus lending support to the results discussed. On the other hand, current conditions of historically low interest rates and (some would argue) high valuations, at least in the United States, may make the analysis less relevant for retirees that need to make decisions for the next several years.

That said, it is important to notice that the analysis in this article compares strategies based on different stock–bond proportions. Thus, unless the *relative* return of stocks and bonds changes substantially in the future, the results discussed here should be useful to retirees to guide their asset-allocation decisions.

Some would argue that the U.S. bond and stock markets are expensive and that lower returns should be expected in the next several years. Even if that is the case, as long as both markets deliver lower returns, and their relative return remains roughly as it has been historically, again the results discussed here should prove useful. As Mark Twain is supposed to have said, "history does not repeat itself but it does rhyme."

ASSESSMENT

The failure rate is arguably the variable most widely used to evaluate retirement strategies. Its main shortcoming, determining how often a strategy failed but ignoring by how much it failed, can be overcome by using it together with shortfall years in what is referred to here as the F–S_Y framework.

This framework, however, is not free from shortcomings, particularly because it requires the use of two variables (rather than just one), which do not always point in the same direction. This article introduces a new variable, years sustained, that aims to overcome this deficiency. The ratio between the expected value and the standard deviation of years sustained, risk-adjusted success, is proposed here as the single variable to be used in the evaluation of retirement strategies.

Risk-adjusted success, just like the Sharpe ratio or the success-to-variability ratio, penalizes volatility. For all three variables, this characteristic has positive and negative implications. On the positive side, uncertainty per se is not desirable and should decrease the value of any target variable that aims to be maximized. On the negative side, volatility does not distinguish between upside and downside fluctuations, and most investors would agree that although the latter is harmful, the former is desirable.

The comprehensive evidence discussed here was used both to understand the strategies selected by RAS and to evaluate the long-term performance of retirement strategies in 21 countries and the world market. Unsurprisingly, the optimal strategy varied substantially from country to country, with the most aggressive strategy being the most desirable in some countries and the most conservative being the most desirable in some others.

These very different optimal choices stem from the trade-off between the number of years a strategy sustained withdrawals—both when it failed and when it succeeded—and the variability in those years, largely determined by the spread between the number of years a strategy sustained withdrawals when it failed and when it succeeded. Not in all cases the strategy selected by RAS seems to be the one a "reasonable" investor would choose, particularly when this variable selects a very conservative allocation that has a very high failure rate.

All ratios that consider a strategy's expected return in the numerator and its risk in the denominator are subject to criticism on the way risk is defined, and the RAS ratio suggested here is no exception. That said, this variable seems to overcome the shortcomings of both the failure rate by itself and the F–S_Y framework and is introduced and proposed here as a comprehensive tool to evaluate retirement strategies.

A possible extension of the framework introduced in this article is to assess risk by focusing not on the standard deviation but on the semideviation and therefore on downside volatility relative to a chosen benchmark; work already in progress considers and evaluates this alternative framework. Furthermore, how investors evaluate the trade-off between risk and return is ultimately determined by their individual preferences, which suggests another possible extension to the framework introduced here, focused on utility; further work already in progress considers and evaluates this other alternative framework.

APPENDIX

YEARS SUSTAINED AND RISK-ADJUSTED SUCCESS

Let f be a variable that takes a value of 1 in a retirement period in which a strategy failed and 0 otherwise. Then, the *failure rate* (F) is formally defined as

$$F = \left(\frac{1}{T}\right) \cdot \sum_{t=1}^{T} f_t \tag{A-1}$$

where T is the number of (historical or simulated) retirement periods evaluated and t indexes retirement periods, both typically measured in years.

Let L be the length of the retirement period considered, and N_t the number of years a strategy fell short from L in retirement period t. Hence, a strategy that failed in period t still sustained L–N_t years of withdrawals. Estrada [2017] defines the variable *shortfall years* (S_Y) as the average number of years a strategy fell short from L, over all the retirement periods in which it failed. Formally,

$$S_Y = \frac{\Sigma_{t=1}^{T} f_t \cdot N_t}{\Sigma_{t=1}^{T} f_t} \tag{A-2}$$

where the numerator is the total number of years a strategy fell short from L over all the retirement periods in which it failed, and the denominator is the number of retirement periods in which the strategy failed.

Let g be a variable that takes a value of 1 in a retirement period in which a strategy succeeded and 0 otherwise; hence, by definition, $g_t = 1$–f_t. Also, let B_t be the bequest left in the periods in which a strategy succeeded. Define now a new variable, *years sustained* (Y_S), which can take two values, one over the retirement periods in which a strategy failed (Y_{S-F}), and another over the retirement periods in which it succeeded (Y_{S-S}). Formally,

$$\begin{aligned} Y_S = Y_{S-F} &= \frac{\Sigma_{t=1}^{T} f_t \cdot (L - N_t)}{\Sigma_{t=1}^{T} f_t} \\ &= L - S_Y \quad \text{when the strategy failed} \\ &= Y_{S-S} = \frac{\Sigma_{t=1}^{T} g_t \cdot (L + B_t)}{\Sigma_{t=1}^{T} g_t} \\ &= L + B_Y \quad \text{when the strategy succeeded} \end{aligned} \tag{A-3}$$

where B_Y is the mean bequest over the years in which a strategy succeeded, expressed in years of inflation-adjusted withdrawals. In words, over all the periods in which a strategy failed, it still sustained (on average) L–S_Y years of withdrawals; over all the periods in which the strategy succeeded, in turn, it sustained L years of withdrawals during the retiree's lifetime and left behind a mean bequest, expressed (not in dollars but) in years of real withdrawals.

Given that the failure rate is a proxy for the probability of failure, which implies that a strategy fails with probability F and succeeds with probability 1–F, then the expected value and standard deviation of Y_S are respectively given by

$$\text{E}(Y_S) = F \cdot (L - S_Y) + (1 - F) \cdot (L + B_Y) \tag{A-4}$$

$$\begin{aligned} \text{SD}(Y_S) = \{&F \cdot [(L - S_Y) - E(Y_S)]^2 \\ &+ (1 - F) \cdot [(L + B_Y) - E(Y_S)]^2\}^{1/2} \end{aligned} \tag{A-5}$$

Finally, define *risk-adjusted success* (RAS) as the ratio between the expected value and the standard deviation of Y_S; that is,

$$\text{RAS} = E(Y_S)/SD(Y_S) \tag{A-6}$$

As already mentioned, when evaluating two strategies, that with a higher RAS is the one that had superior performance. This is the case because a higher RAS indicates more years of withdrawals sustained or less uncertainty about the years of withdrawals sustained.

Exhibit A1
Summary Statistics

	AM	GM	SD	SSD	Min	Max
Stocks						
Australia	8.9	7.3	17.9	9.2	–42.5	51.5
Austria	4.6	0.6	30.0	15.6	–60.1	127.1
Belgium	5.4	2.7	23.7	13.0	–48.9	105.1
Canada	7.2	5.8	16.9	8.4	–33.8	55.2
Denmark	7.2	5.3	20.7	8.9	–49.2	107.8
Finland	9.3	5.3	30.0	13.9	–60.8	161.7
France	5.7	3.2	23.1	12.3	–41.5	66.1
Germany	8.2	3.2	31.7	14.7	–90.8	154.6
Ireland	6.8	4.2	22.9	11.9	–65.4	68.4
Italy	5.9	1.9	28.5	15.6	–72.9	120.7
Japan	8.8	4.1	29.6	15.2	–85.5	121.1
Netherlands	7.1	5.0	21.4	10.3	–50.4	101.6
New Zealand	7.8	6.1	19.4	9.0	–54.7	105.3
Norway	7.2	4.2	26.9	11.7	–53.6	166.9
Portugal	8.4	3.4	34.4	15.3	–76.6	151.8
South Africa	9.5	7.4	22.1	9.0	–52.2	102.9
Spain	5.9	3.7	21.9	11.0	–43.3	99.4
Sweden	8.0	5.8	21.2	10.8	–42.5	67.5
Switzerland	6.3	4.5	19.5	10.1	–37.8	59.4
U.K.	7.1	5.3	19.6	9.7	–57.1	96.7
U.S.	8.5	6.5	20.0	10.4	–37.6	56.3
World	6.6	5.2	17.4	9.4	–41.0	68.2
Bonds						
Australia	2.5	1.7	13.2	7.6	–26.6	62.2
Austria	4.9	–3.8	51.2	20.1	–94.4	441.6
Belgium	1.6	0.4	15.0	9.9	–45.6	62.3
Canada	2.8	2.2	10.4	5.4	–25.9	41.7
Denmark	3.9	3.3	11.9	5.1	–18.2	50.1
Finland	1.5	0.2	13.7	10.9	–69.5	30.2
France	1.1	0.2	13.0	9.5	–43.5	35.9
Germany	1.3	–1.4	15.8	12.4	–95.0	62.5
Ireland	2.7	1.6	15.1	8.0	–34.1	61.2
Italy	0.2	–1.2	14.7	11.8	–64.3	35.5
Japan	1.7	–0.9	19.7	14.7	–77.5	69.8
Netherlands	2.2	1.7	9.8	5.2	–18.1	32.8
New Zealand	2.5	2.1	9.0	4.8	–23.7	34.1
Norway	2.6	1.9	12.0	6.8	–48.0	62.1
Portugal	2.5	0.8	18.7	11.2	–49.7	82.4
South Africa	2.4	1.9	10.4	5.9	–32.6	37.1
Spain	2.5	1.8	12.6	7.1	–30.2	53.2
Sweden	3.5	2.8	12.7	5.9	–37.0	68.2
Switzerland	2.7	2.3	9.4	4.3	–21.4	56.1
U.K.	2.4	1.6	13.7	7.1	–30.7	59.4
U.S.	2.5	2.0	10.4	5.3	–18.4	35.1
World	2.5	1.9	11.3	6.0	–32.0	46.7

Notes: This exhibit shows, for the series of annual returns over the 1900–2014 period, the arithmetic (AM) and geometric (GM) mean return, standard deviation (SD), semideviation for a 0% benchmark (SSD), lowest return (Min), and highest return (Max). All returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for capital gains/losses and cash flows (dividends or coupons). All figures in %.

EXHIBIT A2
Risk-Adjusted Success

Stocks/Bonds	100/0	90/10	80/20	70/30	60/40	50/50	40/60	30/70	20/80	10/90	0/100
Australia											
F	3.5	4.7	5.8	7.0	12.8	17.4	30.2	38.4	53.5	62.8	65.1
S_Y	12.0	9.0	7.6	7.5	5.7	6.5	6.5	7.8	7.8	9.3	10.9
$Y_{S\text{-}F}$	18.0	21.0	22.4	22.5	24.3	23.5	23.5	22.2	22.2	20.7	19.1
$Y_{S\text{-}S}$	156.7	138.2	121.3	106.0	95.5	85.0	80.9	74.3	74.4	72.8	65.0
RAS	**6.0**	5.4	5.0	4.7	3.6	3.2	2.4	2.1	1.8	1.6	1.6
Austria											
F	52.3	47.7	44.2	45.3	44.2	45.3	46.5	51.2	48.8	50.0	54.7
S_Y	14.9	14.7	15.2	15.2	16.2	16.6	16.9	16.3	18.0	18.7	18.3
$Y_{S\text{-}F}$	15.1	15.3	14.8	14.8	13.8	13.4	13.1	13.8	12.0	11.3	11.7
$Y_{S\text{-}S}$	73.5	76.0	77.4	80.6	79.3	77.8	74.3	72.4	63.8	57.7	53.1
RAS	1.5	1.6	**1.6**	1.6	1.5	1.5	1.5	1.4	1.5	1.5	1.5
Belgium											
F	50.0	51.2	52.3	54.7	53.5	54.7	55.8	57.0	60.5	64.0	69.8
S_Y	10.2	10.1	10.0	9.8	10.5	10.7	11.0	11.3	11.2	11.3	11.1
$Y_{S\text{-}F}$	19.8	19.9	20.0	20.2	19.5	19.3	19.0	18.7	18.8	18.7	18.9
$Y_{S\text{-}S}$	91.2	91.1	90.1	89.6	84.0	80.5	76.4	71.8	69.1	66.0	65.3
RAS	1.6	1.5	1.5	1.5	1.5	1.5	1.6	**1.6**	1.6	1.6	1.5
Canada											
F	1.2	0.0	0.0	0.0	1.2	14.0	17.4	26.7	44.2	61.6	64.0
S_Y	4.0	N/A	N/A	N/A	2.0	2.8	5.4	5.8	5.8	6.8	8.7
$Y_{S\text{-}F}$	26.0	N/A	N/A	N/A	28.0	27.3	24.6	24.2	24.2	23.2	21.3
$Y_{S\text{-}S}$	100.7	91.1	82.9	75.4	68.9	67.8	63.2	61.5	65.3	75.5	73.4
RAS	12.5	N/A	N/A	N/A	**15.6**	4.4	3.9	3.1	2.3	1.7	1.6
Denmark											
F	3.5	1.2	2.3	3.5	4.7	10.5	19.8	30.2	39.5	45.3	53.5
S_Y	4.0	5.0	2.5	1.7	1.8	2.1	2.8	3.3	4.6	5.9	6.8
$Y_{S\text{-}F}$	26.0	25.0	27.5	28.3	28.3	27.9	27.2	26.7	25.4	24.1	23.2
$Y_{S\text{-}S}$	79.2	78.3	78.5	78.1	77.1	77.9	80.4	83.9	87.5	88.0	91.4
RAS	7.9	**13.6**	10.1	8.4	7.3	4.7	3.3	2.5	2.1	1.9	1.6
Finland											
F	33.7	36.0	37.2	38.4	38.4	40.7	43.0	45.3	47.7	57.0	68.6
S_Y	13.0	13.1	13.5	14.1	14.8	14.6	14.7	14.8	15.0	13.6	12.7
$Y_{S\text{-}F}$	17.0	16.9	16.5	15.9	15.2	15.4	15.3	15.2	15.0	16.4	17.3
$Y_{S\text{-}S}$	172.3	164.1	151.2	136.7	119.4	105.0	90.5	76.5	63.4	55.7	50.4
RAS	1.6	1.6	1.6	1.5	1.6	1.6	1.6	1.6	1.7	1.7	**1.8**
France											
F	53.5	55.8	57.0	59.3	58.1	59.3	59.3	60.5	58.1	57.0	57.0
S_Y	9.0	9.7	10.4	10.6	11.4	11.5	12.0	12.2	13.0	14.0	14.6
$Y_{S\text{-}F}$	21.0	20.3	19.6	19.4	18.6	18.5	18.0	17.8	17.0	16.0	15.4
$Y_{S\text{-}S}$	97.8	105.8	111.1	117.5	114.9	115.5	112.0	109.2	98.9	90.4	83.0
RAS	**1.5**	1.4	1.3	1.2	1.2	1.2	1.2	1.2	1.3	1.3	1.3
Germany											
F	54.7	54.7	54.7	53.5	53.5	53.5	54.7	54.7	54.7	55.8	57.0
S_Y	13.5	13.7	14.3	14.9	15.2	15.5	15.4	15.8	16.1	16.2	16.3
$Y_{S\text{-}F}$	16.5	16.3	15.7	15.1	14.8	14.5	14.6	14.2	13.9	13.8	13.7
$Y_{S\text{-}S}$	151.6	138.7	127.2	114.3	104.2	94.5	86.6	77.4	68.6	61.1	53.8
RAS	1.2	1.2	1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.5	**1.6**

(continued)

EXHIBIT A2 *(continued)*
Risk-Adjusted Success

Stocks/Bonds	100/0	90/10	80/20	70/30	60/40	50/50	40/60	30/70	20/80	10/90	0/100
Ireland											
F	26.7	27.9	29.1	31.4	36.0	40.7	50.0	59.3	64.0	66.3	66.3
S_Y	8.5	8.8	9.1	9.1	8.5	8.7	8.5	8.7	9.8	10.9	12.1
$Y_{S\text{-}F}$	21.5	21.2	20.9	20.9	21.5	21.3	21.5	21.3	20.2	19.1	17.9
$Y_{S\text{-}S}$	105.2	101.1	96.2	91.6	88.3	84.8	86.2	89.5	88.1	83.6	75.4
RAS	**2.2**	2.2	2.2	2.1	2.0	1.9	1.7	1.5	1.4	1.3	1.4
Italy											
F	60.5	65.1	67.4	69.8	70.9	72.1	75.6	77.9	79.1	82.6	88.4
S_Y	12.3	11.8	11.8	12.0	12.1	12.5	12.3	12.3	12.5	12.6	12.3
$Y_{S\text{-}F}$	17.7	18.2	18.2	18.0	17.9	17.5	17.7	17.7	17.5	17.4	17.7
$Y_{S\text{-}S}$	62.8	64.2	64.9	66.9	67.4	67.3	70.0	70.3	67.6	68.3	77.3
RAS	**1.6**	1.6	1.5	1.5	1.4	1.4	1.4	1.3	1.4	1.4	1.3
Japan											
F	37.2	36.0	36.0	36.0	36.0	34.9	36.0	38.4	39.5	41.9	59.3
S_Y	13.6	13.8	13.6	13.5	13.4	13.8	13.7	13.1	13.1	13.0	10.8
$Y_{S\text{-}F}$	16.4	16.2	16.4	16.5	16.6	16.2	16.3	16.9	16.9	17.0	19.2
$Y_{S\text{-}S}$	198.9	170.9	147.9	127.2	109.0	92.2	80.2	70.4	61.2	53.8	55.3
RAS	1.5	1.6	1.6	1.6	1.7	1.8	1.9	1.9	2.0	**2.1**	1.9
Netherlands											
F	19.8	22.1	20.9	20.9	23.3	32.6	41.9	51.2	55.8	61.6	61.6
S_Y	7.1	5.4	5.2	4.7	4.7	4.4	4.8	5.6	6.9	8.0	9.5
$Y_{S\text{-}F}$	22.9	24.6	24.8	25.3	25.3	25.6	25.2	24.4	23.1	22.0	20.5
$Y_{S\text{-}S}$	111.7	105.4	95.6	86.9	79.9	77.5	75.5	74.1	69.2	65.7	57.4
RAS	2.7	2.6	2.8	**3.0**	2.9	2.5	2.2	2.0	1.9	1.8	1.9
New Zealand											
F	0.0	0.0	1.2	1.2	3.5	11.6	22.1	34.9	48.8	51.2	65.1
S_Y	N/A	N/A	1.0	3.0	2.7	3.0	3.8	4.9	5.5	7.3	7.8
$Y_{S\text{-}F}$	N/A	N/A	29.0	27.0	27.3	27.0	26.2	25.1	24.5	22.7	22.2
$Y_{S\text{-}S}$	94.1	87.1	80.8	73.9	68.1	64.5	61.9	60.8	61.2	55.5	58.2
RAS	N/A	N/A	14.4	**14.6**	8.9	5.0	3.6	2.8	2.4	2.4	2.0
Norway											
F	34.9	34.9	39.5	45.3	50.0	52.3	55.8	58.1	64.0	68.6	70.9
S_Y	8.8	8.5	7.6	7.1	6.9	7.2	7.5	7.8	8.2	8.9	9.9
$Y_{S\text{-}F}$	21.2	21.5	22.4	22.9	23.1	22.8	22.5	22.2	21.8	21.1	20.1
$Y_{S\text{-}S}$	91.6	91.9	95.1	98.4	99.1	95.0	91.1	84.5	82.3	79.0	72.6
RAS	2.0	**2.0**	1.9	1.7	1.6	1.6	1.5	1.6	1.5	1.5	1.5
Portugal											
F	46.5	45.3	43.0	46.5	36.0	34.9	44.2	50.0	51.2	52.3	53.5
S_Y	11.9	10.7	9.6	8.0	9.6	10.7	9.9	10.5	11.9	13.1	14.4
$Y_{S\text{-}F}$	18.2	19.3	20.4	22.0	20.4	19.3	20.1	19.5	18.1	16.9	15.6
$Y_{S\text{-}S}$	118.3	106.9	95.6	92.1	76.4	71.0	72.9	72.8	68.6	64.5	60.7
RAS	1.4	1.5	1.7	1.7	2.1	**2.1**	1.9	1.7	1.7	1.7	1.6
South Africa											
F	2.3	2.3	2.3	3.5	5.8	7.0	16.3	33.7	43.0	59.3	67.4
S_Y	3.5	3.5	4.5	3.3	3.2	4.5	4.0	4.6	6.1	6.8	8.5
$Y_{S\text{-}F}$	26.5	26.5	25.5	26.7	26.8	25.5	26.0	25.4	23.9	23.2	21.5
$Y_{S\text{-}S}$	150.2	128.5	109.4	93.5	80.2	68.0	60.3	56.7	50.8	49.0	45.9
RAS	7.9	8.2	**8.5**	7.4	6.2	6.0	4.3	3.1	3.0	2.7	2.6

(continued)

EXHIBIT A2 *(continued)*
Risk-Adjusted Success

Stocks/Bonds	100/0	90/10	80/20	70/30	60/40	50/50	40/60	30/70	20/80	10/90	0/100
Spain											
F	38.4	39.5	40.7	41.9	43.0	48.8	55.8	61.6	65.1	69.8	72.1
S_Y	12.1	11.1	10.6	10.0	9.7	8.7	8.0	7.7	8.0	8.3	8.8
$Y_{S\text{-}F}$	17.9	18.9	19.4	20.0	20.3	21.3	22.0	22.3	22.0	21.7	21.2
$Y_{S\text{-}S}$	83.2	81.1	78.2	74.5	70.5	69.4	69.0	67.6	63.8	61.0	55.8
RAS	1.8	1.9	1.9	1.9	2.0	1.9	1.8	1.8	1.8	1.9	**2.0**
Sweden											
F	24.4	20.9	18.6	17.4	16.3	18.6	16.3	30.2	41.9	45.3	51.2
S_Y	10.2	10.3	9.9	8.7	7.3	5.2	4.6	3.4	4.3	6.4	8.1
$Y_{S\text{-}F}$	19.8	19.7	20.1	21.3	22.7	24.8	25.4	26.6	25.7	23.6	21.9
$Y_{S\text{-}S}$	174.1	152.9	134.4	118.2	102.9	91.4	77.5	74.7	71.9	65.1	61.5
RAS	2.1	2.3	2.5	2.8	3.0	3.0	**3.6**	2.7	2.3	2.2	2.1
Switzerland											
F	31.4	29.1	27.9	26.7	25.6	24.4	24.4	29.1	29.1	36.0	53.5
S_Y	9.7	9.5	8.9	8.6	8.5	8.4	8.0	6.6	6.9	6.1	5.1
$Y_{S\text{-}F}$	20.3	20.5	21.1	21.4	21.5	21.6	22.0	23.4	23.1	23.9	24.9
$Y_{S\text{-}S}$	87.6	81.0	75.3	69.6	64.0	58.7	54.0	50.9	46.5	43.9	44.3
RAS	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.4	3.7	**3.8**	3.5
U.K.											
F	5.8	11.6	18.6	20.9	22.1	23.3	33.7	48.8	61.6	66.3	66.3
S_Y	4.0	3.5	3.9	5.5	7.1	8.4	7.4	7.2	8.0	9.5	11.1
$Y_{S\text{-}F}$	26.0	26.5	26.1	24.5	22.9	21.7	22.6	22.8	22.0	20.5	18.9
$Y_{S\text{-}S}$	101.3	95.9	91.8	84.4	76.8	69.8	68.4	71.9	78.2	78.0	71.7
RAS	**5.5**	3.9	3.1	2.9	2.9	2.9	2.4	2.0	1.6	1.5	1.5
USA											
F	3.5	3.5	3.5	3.5	4.7	8.1	15.1	25.6	40.7	64.0	65.1
S_Y	5.7	3.3	2.0	2.7	3.0	2.9	3.2	3.9	4.9	5.6	7.9
$Y_{S\text{-}F}$	24.3	26.7	28.0	27.3	27.0	27.1	26.8	26.1	25.1	24.4	22.1
$Y_{S\text{-}S}$	113.7	102.2	91.7	82.0	73.7	66.9	61.9	58.5	58.0	67.2	61.0
RAS	6.7	7.2	7.7	**8.0**	7.3	5.9	4.5	3.6	2.8	1.9	1.9
World											
F	14.0	15.1	16.3	20.9	23.3	29.1	36.0	36.0	38.4	45.3	59.3
S_Y	5.8	6.2	7.1	7.4	8.4	8.5	8.6	10.5	11.3	10.6	9.7
$Y_{S\text{-}F}$	24.2	23.8	22.9	22.6	21.7	21.5	21.4	19.5	18.7	19.4	20.3
$Y_{S\text{-}S}$	94.4	87.0	80.4	76.4	71.5	68.9	67.6	62.6	59.0	57.7	61.9
RAS	**3.5**	3.4	3.3	3.0	2.8	2.6	2.3	2.3	2.2	2.1	1.8

Notes: This exhibit shows, for 11 asset allocations with stock–bond proportions ranging 100/0 (all stocks) to 0/100 (all bonds) and for all countries in the sample, the failure rate (F), shortfall years (S_Y), years sustained when a strategy fails (Y_{S-F}), years sustained when a strategy succeeds (Y_{S-S}), and risk-adjusted success (RAS), all as defined in expressions A-1–A-6 in the appendix, over 86 rolling 30-year retirement periods, beginning with 1900–1929 and ending with 1985–2014. All strategies are based on a starting capital of $1,000, a 4% initial withdrawal rate, subsequent annual withdrawals adjusted by inflation, and annual rebalancing to the stock-bond allocations in the first row. F is expressed in %; S_Y, Y_{S-F}, and Y_{S-S} are expressed in years. The data are described in Exhibit A1.

ENDNOTES

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[1]Milevsky [2016] highlights other shortcoming of the failure rate.

[2]See, also, Brunel [2003].

[3]The analysis is based on a 4% IWR but the intuition behind the results is similar for other IWRs. As is standard in the literature, the IWR indicates the proportion of the portfolio that is withdrawn at the beginning of retirement, with the subsequent annual withdrawals adjusted by inflation, thus preserving the purchasing power of the first withdrawal.

[4]$E(Y_S)$ and $SD(Y_S)$ are not reported in Exhibit A2 because they can be calculated from the variables reported and their definitions in expressions (A-4) and (A-5).

[5]A five-year CD with a fixed 0.1% annual interest rate would have 0 volatility and infinite Sharpe ratio, although that would obviously not make it attractive to investors, who are virtually certain to lose purchasing power with it. Estrada [2014] discusses other shortcomings of volatility as a measure or risk.

[6]The volatility of the 80–20 and the 70–30 strategies are 11.7 and 10.0. As stated in Note 3, $E(Y_S)$ and $SD(Y_S)$ are not reported but can be calculated from the data reported.

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