Target-Date Funds, Glidepaths, and Risk Aversion

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Abstract

Target-date funds feature asset allocations that become increasingly conservative as investors approach retirement. An important shortcoming of this strategy is that it is suboptimal in terms of capital accumulation, which begs the question of why these funds are so popular. A possible answer is that investors become more risk averse as they age, gradually favoring more downside protection as they approach retirement. The main issue explored in this article is how much more risk averse would investors need to become during their working years to select asset allocations similar to those in target-date funds; the evidence here shows that investors would have to roughly double their risk aversion during the last 25 years of their working period. An intuitive interpretation of this result, based on how much an individual would pay to avoid a gamble, is also discussed.

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1. Introduction

Target-date funds (TDFs) are, and have been for some time, the most popular default option in defined contribution retirement plans. Although they were introduced by Barclays Global Investors in 1993, when the company launched its LifePath Portfolios, interest in TDFs took off when they became a Qualified Default Investment Alternative (QDIA) with the passage of the Pension Protection Act (PPA) of 2006.1

Despite their seemingly-plausible underlying strategy, with an asset allocation that becomes increasingly conservative as retirement approaches, TDFs have not been free from criticism. One of their main drawbacks is that their declining-equity glidepath is suboptimal in terms of capital accumulation.2 In fact, both theory and evidence suggest that if an investor aims to maximize the size of his retirement portfolio, his asset allocation should become more (rather than less) aggressive over time; see Basu and Drew (2009), Arnott et al (2013), and Estrada (2014), among others.

Why do TDFs feature a declining-equity glidepath, and why are these products so popular with investors, despite this important shortcoming? The most likely answer is that investors

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* I would like to thank Jack Rader for his comments. David Vila provided research assistance. The views expressed below and any errors that may remain are entirely my own.
1 Although TDFs were one of the three QDIAs identified in the PPA, it was the one that plan sponsors and individuals overwhelmingly gravitated to.
2 A glidepath is the relationship between a fund’s asset allocation and the number of years away from retirement investors in the fund are. A declining-equity glidepath refers to an asset allocation that gradually reduces the proportion of stocks, and increases the proportion of bonds, thus making the portfolio increasingly conservative over time.
prioritize capital accumulation early in their working years, and gradually switch to prioritize capital protection as they approach retirement. This is consistent with survey evidence showing that in the last few years before retirement most TDF investors want the fund to provide strong protection against losses; see ING (2012).

This survey evidence seems to suggest that investors become more risk averse as they age, a hypothesis that has some empirical support. In fact, Albert and Duffy (2012) report experimental evidence revealing that older individuals are more risk averse, and have higher discount rates, than younger individuals. Viewed through the lens of this evidence, asset management companies seem to be offering just what investors are demanding; that is, a gradual switch from upside potential to downside protection as retirement approaches.

The declining-equity glidepath featured by TDFs, the preference for strong protection against losses when approaching retirement, and the positive relationship between risk aversion and age, all come together in the ultimate question posed in this article: How would an investor’s attitude toward risk have to evolve over time so that he would choose a glidepath similar to that featured in TDFs?

Answering this question requires, first, specifying a benchmark glidepath; second, characterizing the preferences of a representative investor; and third, asking how the investor’s risk aversion would have to evolve over time so that he would choose the asset allocations in the benchmark glidepath. That is precisely what is done in this article, resulting in the following main finding: To choose a glidepath similar to that featured in TDFs, an investor's risk aversion would have to roughly double during the last 25 years of his accumulation period.

To provide some intuition for this somewhat abstract result, an individual is faced with a gamble in order to determine how much he would paid to avoid it, at different times during his accumulation period, or similarly, at different levels of risk aversion. To avoid the gamble, an individual whose risk aversion increases from the lowest to the highest value of those estimated here would be willing to pay 131% more on the retirement date than he would pay when being 25 years away from retirement.

The rest of the article is organized as follows. Section 2 discusses declining-equity glidepaths, risk aversion, and the main research questions. Section 3 discusses the benchmark glidepath, the representative investor, the data, the methodology and its implementation, and the main results of the inquiry. Finally, section 4 concludes with an assessment.

2. The Issue

This section discusses three issues that set the stage for the empirical analysis in the next section. First, it discusses the main shortcoming of declining-equity glidepaths; then, it discusses
some evidence on risk aversion; and finally, it discusses the two main research questions posed in this article.

2.1. Declining-Equity Glidepaths

TDFs have been criticized in a number of ways, among them that they are too aggressive; that they are too conservative; that they typically exclude alternative assets; that they focus on just one variable (years away from retirement); and that all the funds within a TDF typically are from the same asset management company that offers the TDF.

The relevant criticism for the purpose of this article is that the declining-equity glidepath featured in TDFs is suboptimal in terms of capital accumulation. In fact, it is trivial to show that a strategy that exposes a small portfolio (what investors typically have early in their working years) to high returns (the result of an aggressive asset allocation), and a large portfolio (what investors typically have approaching retirement, after years of contributions and compounding) to low returns (the result of a conservative asset allocation), will almost certainly be outperformed by a strategy that does the opposite.

Basu and Drew (2009), Arnott et al (2013) and Estrada (2014) consider declining-equity glidepaths and their mirrors (strategies that remain invested in stocks and bonds the same amount of time but evolve in the opposite direction, from less aggressive to more aggressive), and find that declining-equity glidepaths underperform rising-equity glidepaths in terms of capital accumulation, both on average as well as in bad scenarios. In fact, declining-equity glidepaths also underperform constant-equity glidepaths, as long as the underlying asset allocation of the latter is aggressive enough.

2.2. Risk Aversion

Why do TDFs feature an asset allocation that becomes gradually more conservative over time, and why are these products so popular with investors, if the resulting retirement portfolio is smaller than it could be with the opposite strategy? The most plausible answer seems to be that asset management companies are simply providing what investors demand, which is to prioritize upside potential early in the working period and gradually switch the prioritize downside protection as investors approach retirement.

This strategy is also consistent with the theory of time diversification, which suggests that risk is a function of the holding period. In practice, this usually translates into the idea that stocks are risky in the short term, but much less so in the long term; and that stocks are riskier than bonds in the short term, but the opposite is the case in the long term. A corollary of time
diversification is that young investors should have a high (low) exposure to stocks (bonds), and older investors should have the opposite.\(^3\)

The strategy also seems to be consistent with investor preferences. In a survey of over 500 defined contribution plan participants, ING (2012) finds that 93% of TDF investors expect the fund to provide strong protection against losses when approaching and in retirement. The survey also finds that, when approaching and in retirement, if asked to choose between stronger protection against losses or a stronger growth potential, 80% of TDF investors (and 66% of non-TDF investors) prefer the former.

This survey evidence is consistent with experimental evidence reported by Albert and Duffy (2012). They ask two groups of individuals, younger and older, to perform a paired-lottery choice task commonly used in experimental economics to elicit risk aversion, and find the older group to be more risk averse, and to have a higher discount rate, than the younger group. They also discuss other studies that report results consistent with their findings.

### 2.3. The Research Questions

Three facts are worth highlighting at this point. First, all TDFs feature a declining-equity glidepath; second, survey evidence shows that as retirement approaches investors clearly prefer protection against losses over potential for growth; and third, experimental evidence suggests that risk aversion increases with age. These three facts lead directly to the first and main question posed in this article, namely, how would an investor’s attitude toward risk have to evolve over time so that he would choose a glidepath similar to that featured in TDFs?

Answering this question requires three steps. First, to specify a benchmark glidepath; then, to characterize the preferences of a representative investor; and third, to ask how the investor’s risk aversion would have to evolve over time so that he would choose the asset allocations in the benchmark glidepath. All these steps are discussed in the next section.

The second research question posed here aims to complement the first. Having established how risk aversion would have to evolve over time for a representative investor to choose the asset allocations in the benchmark glidepath, the next step is to ask how those levels of risk aversion can be characterized in a practical way. Most practitioners find the value of a risk aversion coefficient to be an abstract figure without much intuition. However, having an individual face a gamble, and exploring how much he would be willing to pay to avoid it, yields a useful perspective, particularly when applied to the range of risk aversion coefficients obtained from the first question.

\(^3\) For an in-depth discussion of time diversification, see Estrada (2013).
3. Evidence

This section discusses the results of the inquiry. First it discusses the benchmark glidepath and the representative investor; then it discusses the data and methodology; and finally it discusses how the methodology is applied, the main results in terms of risk aversion, and a practical interpretation of the results.

3.1. The Benchmark Glidepath

The first of the three steps of the analysis consists of specifying the glidepath that will be faced by the representative investor. Different asset management companies offer somewhat different glidepaths, diverging in the assets included in the funds, as well as in how aggressive or conservative the asset allocation is at different points in time, including on the retirement date.

The glidepath used here as a benchmark is the one featured by Vanguard, the leading provider of TDFs. The first column of Exhibit 1 shows the number of years away from retirement (YAR) investors in Vanguard’s TDFs are. The second and third columns show the proportion of stocks (S) and bonds (B) at different points along the accumulation period. And the next four columns show the breakdown between U.S. and international (world excluding U.S.) stocks and bonds, with the first two columns showing the split between U.S. stocks (S-US) and international stocks (S-exUS), and the last two showing the split between U.S. bonds (B-US) and international bonds (B-exUS). This glidepath is based on Donaldson et al (2019), who discuss Vanguard’s approach to TDFs, as well as on the asset allocation of Vanguard’s TDFs at year-end 2019.

Exhibit 1: The Benchmark Glidepath

This exhibit shows Vanguard’s asset allocation for its TDFs at different times during the accumulation period. For each number of years away from retirement (YAR), the exhibit shows the proportion of stocks (S) and bonds (B), as well as the split between U.S. stocks (S-US) and international stocks (S-exUS), and that between U.S. bonds (B-US) and international bonds (B-exUS). Figures rounded.

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<td>51%</td>
<td>34%</td>
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</tr>
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As the exhibit shows, as long as the investor is at least 25 years away from retirement, the asset allocation is an aggressive 90-10 stock-bond split. From that point on, the allocation to stocks gradually decreases, and that to bonds gradually increases, so that on the retirement date

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4 At the beginning of 2020, the four leading providers of TDFs were Vanguard (38% of assets under management), Fidelity (19%), T. Rowe Price (11%), and American Funds (11%). The rest of the companies all had less than 5% of assets under management. John Rekenthaler from Morningstar kindly provided this information.
the investor holds an equally-weighted portfolio of stocks and bonds. Throughout the glidepath, Vanguard maintains a 60-40 split between U.S. and international stocks, and a 70-30 split between U.S. and international bonds.

3.2. The Representative Investor

The second of the three steps of the analysis consists of characterizing the representative investor. To that purpose, we will consider an investor described by the widely-used mean-variance utility function given by the expression

\[ E(U) = \mu_p - \lambda \cdot \sigma_p^2 \]

where \( E(U) \) denotes expected utility; \( \mu_p \) and \( \sigma_p^2 \) denote a portfolio’s expected return and risk, the latter measured by its variance; and \( \lambda \) is the coefficient of (relative) risk aversion.

This representative investor will be tasked with choosing, at different times during his accumulation period, the asset allocation that maximizes his utility, given different levels of risk aversion. His choices will then be compared to those in the benchmark glidepath in order to determine the level of risk aversion that equates both allocations. (More on this below.)

3.3. Data

Vanguard’s TDFs are essentially built with four assets, U.S. stocks and bonds, and international (world excluding U.S.) stocks and bonds. To assess the long-term performance of these four assets, the sample used here is the Dimson-Marsh-Staunton (DMS) database, described in detail in Dimson, Marsh, and Staunton (2002), as well as in the annual Credit Suisse Global Investment Returns Yearbooks.

The sample period for the computation of all relevant summary statistics is 1900-2014. All returns are annual, real (adjusted by inflation), in dollars, and account for both capital gains/losses and cash flows (dividends or coupons). Exhibit 2 summarizes some characteristics of the four series of stock and bond returns in the sample, as well as the characteristics of two portfolios, a 60-40 split between U.S. and international stocks, and a 70-30 split between U.S. and international bonds. As already mentioned, these are the splits used by Vanguard in its TDFs.

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5 After the retirement date, Vanguard continues to reduce the allocation to stocks, and increase the allocation to bonds, until the fund reaches a 30-70 stock-bond split seven years into the retirement period. From that point on, the asset allocation is kept constant.

6 When a TDF is five years away from the target retirement date, Vanguard introduces a small allocation to short-term TIPS (Treasury Inflation Protected Securities), which is omitted here due to the much shorter sample period available for this asset.
**Exhibit 2: Data – Summary Statistics**

This exhibit shows summary statistics for the annual returns of four assets, U.S. stocks (S-US) and bonds (B-US), and international stocks (S-exUS) and bonds (B-exUS), as well as for two portfolios, a 60-40 split between U.S. and international stocks, and a 70-30 split between U.S. and international bonds. The summary statistics include the arithmetic (AM) and geometric (GM) mean return, standard deviation of returns (SD), and the correlation between the 60-40 and the 70-30 portfolios (Rho). All returns are real, in dollars, and account for capital gains/losses and cash flows paid. The sample period is 1900-2014.

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<td>GM</td>
<td>6.5%</td>
<td>2.0%</td>
<td>4.4%</td>
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</tr>
<tr>
<td>SD</td>
<td>20.0%</td>
<td>10.4%</td>
<td>19.0%</td>
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<td>Rho</td>
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<td></td>
<td>0.31</td>
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As the exhibit shows, in terms of compound returns (GM), U.S. stocks and bonds outperformed international stocks and bonds over the full sample period. Although U.S. stocks were slightly more volatile than international stocks, U.S. bonds were less volatile than international bonds. Unsurprisingly, the 60-40 portfolio of stocks had both a higher return and higher volatility than the 70-30 portfolio of bonds; their correlation over the whole sample period was a relatively low 0.31.

### 3.4. Methodology

The third of the three steps of the analysis consists of determining how the representative investor’s risk aversion would have to evolve over time so that he would choose asset allocations similar to those in the benchmark glidepath. To that purpose, the following steps are taken:

1. For a holding period of 25 years (when the representative investor is 25 years away from retirement), the expected return and risk of 21 portfolios are calculated, ranging from 100% to 0% in the 60-40 portfolio of stocks, with the rest invested in the 70-30 portfolio of bonds, in steps of 5 percentage points (100%, 95%, 90%, ..., 10%, 5%, 0%).
2. For a risk aversion coefficient (λ) equal to 0, the representative investor’s expected utility is calculated using expression (1) for each of the 21 portfolios in the previous step.
3. The highest of the 21 expected utility levels from the previous step determines the asset allocation the representative investor would select.
4. The previous two steps above are repeated for λ = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 5.0, and 10.0, which yields a matrix with 189 expected utility levels (21 portfolios × 9 risk aversion coefficients), and 9 highest expected utility levels (one for each level of risk aversion), each determining an asset allocation selected by the representative investor.
5. These 9 optimal asset allocations are then compared to the asset allocation in the benchmark glidepath with 25 years to the target retirement date; the relevant risk
6. The five steps above are repeated for holding periods of 20, 15, 10, 5, and 0, which yields 6 risk aversion coefficients, one for each of the 6 holding periods considered.

Two points are worth noticing about this methodology. First, accumulation (or working) periods are usually thought of as being 40 years long. However, given that the asset allocation in the benchmark glidepath is constant until an individual is 25 years away from retirement, only the 25-year holding period is relevant; longer ones are redundant. Second, the methodology above omits one step that is easier to explain when discussing the evidence, which is done in the next section. For now it suffices to say that the omitted step implies that it is possible to determine more than one level of risk aversion for each of the holding periods considered.

3.5. Application of the Methodology

Exhibit 3 illustrates the implementation of the steps described in the previous section, as well as the missing step mentioned, when the representative investor is 10 years away from retirement. The first column (Stocks) shows the allocation to the 60-40 portfolio of stocks, with the rest being allocated to the 70-30 portfolio of bonds. The next two columns show the expected return ($\mu_p$) and risk ($\sigma_p$) of 21 portfolios based on the weights in the first column, as well as on the parameters in Exhibit 2 for the 60-40 portfolio of stocks and the 70-30 portfolio of bonds.

For an investor 10 years away from retirement, the benchmark glidepath in Exhibit 1 indicates a 70% allocation to stocks (and 30% to bonds), which is highlighted for reference in the first column of Exhibit 3. The fourth column of the exhibit considers a risk neutral investor ($\lambda=0.0$); of the 21 portfolios he can choose from, his highest utility (highlighted) corresponds to a 100% allocation to stocks, which is not consistent with the 70% allocation to stocks in the benchmark glidepath. The next two columns show that the investor would make the same choice (100% stocks) if he had risk aversion coefficients of 0.5 and 1.0.

Moving to the right then, for $\lambda=1.5$ the investor finds his highest utility (highlighted) by allocating 70% to stocks (hence 30% to bonds), which in this case is in fact the same allocation as in the benchmark glidepath for investors 10 years away from retirement. For $\lambda=2.0$ or higher, the investor finds his highest utility for portfolios with allocations to stocks between 55% (for $\lambda=2.0$) and 25% (for $\lambda=10.0$), all of them different from the allocation in the benchmark glidepath for investors 10 years away from retirement.
Exhibit 3: Risk Aversion and Optimal Choice – 10 Years Away from Retirement

This exhibit shows the expected utility of a representative investor for different portfolios and levels of risk aversion. The first column (Stocks) shows the allocation to the 60-40 portfolio of U.S. and international stocks, with the rest invested in the 70-30 portfolio of U.S. and international bonds. The next two columns show the expected return ($\mu_p$) and risk ($\sigma_p$) of 21 portfolios (in %) based on the weights in the first column and the parameters in Exhibit 2. The numbers on the top row indicate different levels of risk aversion ($\lambda$).

Expected utilities are calculated using expression (1).

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<td>-0.010</td>
<td>-0.060</td>
</tr>
<tr>
<td>25</td>
<td>3.8</td>
<td>9.9</td>
<td>0.038</td>
<td>0.033</td>
<td>0.028</td>
<td>0.024</td>
<td>0.019</td>
<td>0.014</td>
<td>0.009</td>
<td>-0.010</td>
<td>0.059</td>
</tr>
<tr>
<td>20</td>
<td>3.6</td>
<td>9.7</td>
<td>0.036</td>
<td>0.031</td>
<td>0.026</td>
<td>0.021</td>
<td>0.017</td>
<td>0.012</td>
<td>0.007</td>
<td>-0.012</td>
<td>-0.059</td>
</tr>
<tr>
<td>15</td>
<td>3.3</td>
<td>9.7</td>
<td>0.033</td>
<td>0.028</td>
<td>0.024</td>
<td>0.019</td>
<td>0.014</td>
<td>0.010</td>
<td>0.005</td>
<td>-0.014</td>
<td>-0.061</td>
</tr>
<tr>
<td>10</td>
<td>3.1</td>
<td>9.8</td>
<td>0.031</td>
<td>0.026</td>
<td>0.021</td>
<td>0.016</td>
<td>0.012</td>
<td>0.007</td>
<td>0.002</td>
<td>-0.017</td>
<td>-0.065</td>
</tr>
<tr>
<td>5</td>
<td>2.8</td>
<td>9.9</td>
<td>0.028</td>
<td>0.023</td>
<td>0.018</td>
<td>0.013</td>
<td>0.009</td>
<td>0.004</td>
<td>-0.001</td>
<td>-0.021</td>
<td>-0.069</td>
</tr>
<tr>
<td>0</td>
<td>2.6</td>
<td>10.1</td>
<td>0.026</td>
<td>0.021</td>
<td>0.015</td>
<td>0.010</td>
<td>0.005</td>
<td>0.000</td>
<td>-0.005</td>
<td>-0.025</td>
<td>-0.076</td>
</tr>
</tbody>
</table>

Here is where the missing step in the methodology described in the previous section comes in. Having established that an investor with $\lambda=1.5$ would choose an allocation consistent with that in the benchmark glidepath, the missing step involves a deeper search, in steps of 0.05, in the vicinity of this coefficient. In this case, the search (not shown in Exhibit 4) suggests that the investor would maximize his utility at the same 70% allocation to stocks if he had risk aversion coefficients between 1.45 and 1.55; for $\lambda<1.45$ ($\lambda>1.55$), he would choose a more aggressive (conservative) allocation.

### 3.6. Results

Repeating the same process just discussed at different points in time along the investor’s accumulation period results in the risk aversion coefficients shown in Exhibit 4. In those cases in which a range is shown, the lower (higher) number is the lowest (highest) risk aversion coefficient consistent with the allocations in the benchmark glidepath.\(^7\)

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\(^7\) As already mentioned, the search for other $\lambda$s is done in steps of 0.05.
Exhibit 4: Risk Aversion Along the Accumulation Period

This exhibit shows, at each number of years away from retirement (YAR), the level of risk aversion ($\lambda$) that leads the representative investor to choose the allocation to stocks ($S$) and bonds ($B$) featured in the benchmark glidepath. $S$ ($B$) is the allocation to the 60-40 (70-30) portfolio of U.S. and international stocks (bonds).

<table>
<thead>
<tr>
<th>YAR</th>
<th>$S$</th>
<th>$B$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 25</td>
<td>90%</td>
<td>10%</td>
<td>1.10</td>
</tr>
<tr>
<td>20</td>
<td>85%</td>
<td>15%</td>
<td>1.15 – 1.20</td>
</tr>
<tr>
<td>15</td>
<td>75%</td>
<td>25%</td>
<td>1.35 – 1.40</td>
</tr>
<tr>
<td>10</td>
<td>70%</td>
<td>30%</td>
<td>1.45 – 1.55</td>
</tr>
<tr>
<td>5</td>
<td>60%</td>
<td>40%</td>
<td>1.75 – 1.95</td>
</tr>
<tr>
<td>0</td>
<td>50%</td>
<td>50%</td>
<td>2.25 – 2.55</td>
</tr>
</tbody>
</table>

This exhibit shows at least two interesting results. First, in order to choose the allocations in the benchmark glidepath, the representative investor would have to become gradually more risk averse as he gets older. Combining this result with Albert and Duffy’s (2012) finding that individuals do in fact get more risk averse as they age seems to validate the decision of asset management companies to offer TDFs with declining-equity glidepaths; they just seem to be offering what investors demand.

Second, the figures in the exhibit show that in order to choose the allocations in the benchmark glidepath, the representative investor’s risk aversion would not just have to marginally increase over time; it would have to more than double, from 1.10 to the 2.25-2.55 range, during the last 25 years of his accumulation period. (The next section provides some intuition behind this rather abstract result.)

Importantly, note that the risk aversion coefficients in Exhibit 4 are consistent with other estimates in the literature. Gandelman and Hernández-Murillo (2014) estimate risk aversion at the aggregate level for 75 countries and obtain a range between 0 and 3; a value in the vicinity of 1 for most of the countries; and a cross-sectional average of 0.98. Thomas (2016) reports that the UK Treasury recommends the use of a risk-aversion coefficient equal to 1. Baz and Guo (2017) argue that the most widely accepted values for asset allocation are in the 1 to 5 range. And Estrada and Kritzman (2019) find no differences in the asset allocations selected by a representative investor for risk aversion coefficients higher than 3.0. In short, the range of risk aversion coefficients in Exhibit 4 is consistent with other estimates and results reported in the literature.

3.7. An Intuitive Perspective of Risk Aversion

Utility and risk aversion are two concepts that many practitioners find abstract and obscure. In order to provide some meaning to the risk aversion coefficients in Exhibit 4 we can resort to one of the utility functions most widely used in finance and economics, the power utility function, which is given by the expression
where \( U, W, \) and \( \lambda \) denote utility, wealth, and the coefficient of risk aversion, which is 0 for risk neutral individuals, and positive for risk averse ones. When \( \lambda = 1 \), the individual has log utility.\(^8\)

Let’s consider first an individual with \( \lambda = 1.10 \), which is the lowest level of risk aversion reported in Exhibit 4. Assume that this individual has $1 million and faces a gamble with a 50-50 chance to win or lose 10% of his capital. How much money would he accept to have with certainty in order to avoid the gamble? The answer is given by the certainty equivalent of wealth, which is equal to $994,487.\(^9\) Put differently, this individual would be willing to pay up to $5,513 to avoid the gamble.

How would an individual with risk aversion in the 2.25-2.55 range behave? One with \( \lambda = 2.25 \) would have a certainty equivalent of wealth of $988,763, which means that he would be willing to pay up to $11,237 to avoid the gamble. On the other hand, an individual with \( \lambda = 2.55 \) would have a certainty equivalent of wealth of $987,286, and would therefore pay up to $12,714 to avoid the gamble.

This analysis suggests that an individual implementing an asset allocation similar to the declining-equity glidepaths featured in TDFs would pay, in order to avoid a 50-50 chance of winning or losing 10% of his capital, between 104% and 131% more on the retirement date than he would pay when being 25 years away from retirement.\(^10\) That is, perhaps, the most intuitive way to interpret the range of risk aversion coefficients reported in Exhibit 4.

### 3.8. Other Glidepaths

The benchmark glidepath in Exhibit 1 is based on the TDFs offered by Vanguard, the largest provider of these products in early 2020. Other providers offer different glidepaths, some more aggressive than others, but all sharing the same underlying strategy; that is, an asset allocation that becomes gradually more conservative as retirement approaches. Exhibit 5 illustrates these points.\(^11\)

As the exhibit shows, the TDFs offered by these three companies all feature declining-equity glidepaths, thus providing increasing downside protection as investors approach retirement; that is the most distinctive characteristic of all these products. Importantly, the exhibit also shows that, at each point along glidepath, and particularly over the last 25 years

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\(^8\) Levy and Markowitz (1979) show that mean-variance behavior as formalized in expression (1) is approximately equal to expected utility for several utility functions, including the power utility function.

\(^9\) The certainty equivalent of wealth \( (W_c) \) is given by \( W_c = \{1+(1-\lambda)\cdot E[U(W)]\}^{1/(1-\lambda)} \), where \( E[U(W)] \) denotes the expected utility of wealth.

\(^10\) For \( \lambda = 2.25 \), \( $11,237/5,513−1 = 104\% \); similarly, for \( \lambda = 2.55 \), \( $12,714/5,513−1 = 131\% \).

\(^11\) This exhibit is based on information provided by Fidelity Investments (2020).
before retirement, the allocation to stocks offered by these companies are not dramatically different from each other, nor are they dramatically different from those in Exhibit 1.12

**Exhibit 5: Other Glidepaths – Equity Exposure**

This exhibit shows the glidepath of TDFs offered by BlackRock (LifePath Index Funds), Fidelity (Freedom Index Funds), and State Street Global Advisors (Target Date Retirement Pools) as of the end of 2019. The numbers in the top row indicate years away from retirement; the rest of the numbers in the table indicate the allocation to stocks (in %) in the TDFs offered by each company. Based on Fidelity Investments (2020).

<table>
<thead>
<tr>
<th>Company</th>
<th>45</th>
<th>40</th>
<th>35</th>
<th>30</th>
<th>25</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>BlackRock</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>98</td>
<td>95</td>
<td>88</td>
<td>78</td>
<td>68</td>
<td>56</td>
</tr>
<tr>
<td>Fidelity</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>83</td>
<td>68</td>
<td>59</td>
</tr>
<tr>
<td>State Street</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>88</td>
<td>84</td>
<td>78</td>
<td>71</td>
<td>62</td>
</tr>
</tbody>
</table>

This last point has an important implication: Although the results in Exhibit 4 were determined relative to the benchmark glidepath, based on Vanguard’s TDFs, they apply more generally to the TDFs offered by other companies. In other words, the analysis here suggests that the somewhat different glidepaths offered by TDF providers satisfy the demand for increasing downside protection as investors approach retirement, which stems from investors’ increase in risk aversion, particularly during the last 25 years of their accumulation period.

**4. Assessment**

Target-date funds (TDFs) are by far the most popular default option in defined contribution retirement plans. Their main characteristic is a declining-equity glidepath that exposes investors to an increasingly conservative asset allocation as they approach retirement. Given that it is clear that this strategy is suboptimal in terms of capital accumulation, it is interesting to explore why investors are attracted to these funds.

Survey evidence shows that investors approaching retirement have a strong preference for protection against losses over potential for growth. Furthermore, experimental evidence reveals that individuals get more risk averse as they age. These two closely-intertwined facts seem to justify the declining-equity glidepaths that asset management companies have selected for their TDFs. They also beg the question of how much more risk averse investors would need to become during their accumulation period to select asset allocations similar to those featured in TDFs. That is the ultimate issue explored in this article.

The main finding of the study is that, in order to select asset allocations similar to those in standard glidepaths, investors would have to roughly double their risk aversion during the last 25 years of their accumulation period. More precisely, their risk aversion would have to increase from 1.10 when being 25 years away from retirement to the 2.25-2.55 range at the time of

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12 Interestingly, relative to the other companies in the exhibit, as well as relative to Vanguard, BlackRock’s TDFs start out as more aggressive and end up as more conservative.
retirement. All these risk aversion levels are consistent with previous estimates in the literature, as well as with commonly-accepted values for asset allocation.

Finally, because most practitioners view utility and risk aversion as somewhat abstract and obscure concepts, some intuition is provided for risk aversion coefficients, as well as for the increase over the last 25 years of an individual's accumulation period found here. The results show that to avoid a 50-50 chance of winning or losing 10% of his capital, an individual would be willing to pay 131% more when his risk aversion is 2.55 (the value obtained when the individual retires) than when it is 1.10 (the value obtained when he is 25 years away from retirement).

In short, although it is clear that TDFs are not optimal in terms of capital accumulation, it seems clear why they are so popular with investors. They seem to be consistent with an increase in risk aversion as investors age, and its corresponding demand for increasing downside protection as investors approach retirement.
References

Albert, Steven, and John Duffy (2012). "Differences in Risk Aversion Between Young and Older Adults." Neuroscience and Neuroeconomics, 1, 3-9.


Fidelity Investments (2020). "Target Date Evaluation. How Fidelity Freedom Index Funds Compare on Glidepath, Performance, and Fees."


