Retirement Planning:
From Z to A

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> **KEY FINDINGS**
> - Retirement planning must be tackled early and solved backward.
> - It starts with an estimation of the withdrawals needed to sustain a desired lifestyle in retirement; continues with the determination of the target retirement portfolio needed to sustain those withdrawals; and ends with the contributions needed to build that portfolio.
> - This article introduces a theoretical framework and an empirical illustration designed to help individuals get on the right path.

ABSTRACT: *Retirement planning is an issue that must be tackled early and solved backward. It must be tackled early because little can be done if an individual is not on the right path and has only a few years left to work. It must be solved backward because it makes little sense to aim for a portfolio that may not be able to sustain the desired lifestyle in retirement. This article introduces an approach that integrates the working period and the retirement period; leads individuals to consider all relevant variables at the beginning of their journey; and enables them to start saving early to build a target portfolio designed specifically to sustain a desired retirement. The analytical framework introduced yields closed-form solutions for the target retirement portfolio and the contributions that need to be made during the working years to hit that target. The framework proposed is illustrated with an empirical base case, sensitivity analysis, and Monte Carlo simulations.*

TOPICS: *Long-term/retirement investing, portfolio management, retirement, wealth management**

Retirement planning involves an interesting paradox. Although people really need to make a financial plan early in their working years, they have neither the motivation nor the information to do it; however, when they approach the end of their careers and have both the motivation and more information, it may already be too late. That is but one of many reasons most people fail to plan properly for retirement.

Compounding the previous problem is the fact that more often than not, if they plan at all, many individuals wrongly view retirement planning as two separate steps. First, they focus on how much they would like to retire with and the periodic contributions that would lead to that target portfolio. Only when they are approaching retirement do they focus on how much they will be able to periodically withdraw from the portfolio, trying to avoid spending too little (thus reducing their standard of living

unnecessarily) or too much (thus running out of money too early).

The problem with this artificial separation is that if the target portfolio has not been built with the specific goal of sustaining the desired lifestyle in retirement, it will most likely be the standard of living that would eventually have to adjust to the funds available. A much better strategy is to work backward, first determining the size of the portfolio necessary to sustain the desired lifestyle, and then determining how to build that portfolio during the working years.

This artificial separation is also reflected in the literature, with a strand of articles that focuses on strategies for the accumulation (or working) period and another strand that focuses on strategies for the withdrawal (or retirement) period. However, it should be obvious that these two periods are not at all independent from each other. In fact, the ultimate goal of retirement planning is to build during the working period a portfolio designed specifically to sustain the desired lifestyle during the retirement period.

Stated differently, retirement planning should be done in a sequence not often implemented in practice and hardly ever addressed properly in the literature. It should start with the desired lifestyle in retirement and an estimate of its periodic costs, it should then map that information into a target portfolio, and it should finally map the latter into a set of periodic contributions to be made during the working years.

More precisely, the approach introduced here starts with the withdrawals to be made during the retirement period and the bequest to be left at the end of it, which are considered exogenous variables. Those withdrawals together with an expected return, given by the asset allocation chosen for the retirement period, jointly determine the target retirement portfolio. And the target portfolio together with an expected return, given by the asset allocation chosen for the working period, jointly determine the contributions to be made during the working years.

Following such an approach, this article provides closed-form solutions for the target portfolio and the contributions to be made during the working period, given the withdrawals to be made during the retirement period, the bequest, and the returns expected for the portfolio during the accumulation and the withdrawal periods. These expressions can be used to evaluate most of the relevant trade-offs individuals face, such as the sensitivity of the target portfolio to changes in contributions during the working years or withdrawals during retirement, or the impact of changes in expected returns during the lifecycle.

The expressions derived here are discussed in the context of an example based on the performance of US stocks and bonds, first using a base case and subsequent sensitivity analysis and then using Monte Carlo simulations. Both approaches provide valuable insights that may help individuals and financial planners to plan properly for retirement.

The rest of this article is organized as follows. The next section introduces the general framework proposed and discusses the analytical problems an individual faces during both the withdrawal and the accumulation periods. That framework is then applied to the specific situation of a person planning for his or her retirement as soon as the person starts working, using an empirical base case and a Monte Carlo simulation, as well as a sensitivity analysis of both. After a closing assessment, an appendix with important technical details concludes the article.

THE ISSUE

This section starts with an overview of the approach introduced and continues with a brief review of relevant literature. It then discusses the problems to be solved in their proper order, beginning with the withdrawal period and continuing with the accumulation period. Finally, it brings both periods together by discussing the individual's lifecycle problem.

Overview

It is both understandable and problematic that most people postpone retirement planning when it is necessary and address it when it is probably too late. On the one hand, young people at the beginning of their working years can hardly be blamed for not wanting to think about retirement; on the other hand, retirement planning *must* start from the end. Without an estimate of the withdrawals needed to sustain the desired lifestyle in retirement, there is no way to determine a target portfolio; and without the latter, there is no way to determine the contributions needed to hit a nonexistent target.

The same reasoning applies to a bequest; it is something that most people tend to consider well into

their retirement, not early in their careers. And yet if individuals want to make sure that they will leave something to the people or institutions of their choice, a rough estimate is also needed to factor into the withdrawals to be made from the retirement portfolio. The framework introduced here takes the withdrawals and the bequest as given (exogenous variables).[1]

Individuals also need to consider their portfolio's asset allocation during both the accumulation and the withdrawal periods, beginning with the latter. The reason for beginning with the latter is that it will determine the return of the portfolio during retirement, which in turn will determine how large the retirement portfolio needs to be. In other words, the less (more) aggressive the asset allocation is, the larger (smaller) the portfolio would have to be to sustain a given level of withdrawals. These three variables, the withdrawals, the bequest, and the expected return of the portfolio during retirement, determine the target portfolio, a relationship that is formalized below. That is Step 1 in the approach proposed.

Having determined the target retirement portfolio, the next step is to determine the periodic contributions during the working years necessary to hit that target. These contributions are related to the rate at which the portfolio grows during the accumulation period, which again is determined by the portfolio's asset allocation; for any given target, the less (more) aggressive the asset allocation is, the higher (lower) the periodic contributions would have to be. Thus, these two variables, the target retirement portfolio and the expected return of the portfolio during the working years determine the contributions to be made during the accumulation period. That is Step 2 in the approach proposed.

Related Research

The literature on both the accumulation and the withdrawal periods is vast, far too extensive to summarize here. Regarding the accumulation period, one of the issues that has received the most attention is how to maximize the size of the retirement portfolio, which in turn involves at least two debates, namely, the choice between static or dynamic strategies and, related to the latter, the choice between declining and rising equity glidepaths.

The most popular dynamic strategy is a declining-equity glidepath, typically used by target-date funds, which features an increasingly conservative asset allocation as retirement approaches; see, for example, Donaldson, Kinniry, Scott, Dinucci, and Cilla (2019). However, this strategy has been criticized for being suboptimal in regard to capital accumulation; Basu and Drew (2009), Arnott, Sherrerd, and Wu (2013), and Estrada (2014), among others, show that a rising-equity glidepath leads to a larger retirement portfolio. Others argue in favor of static strategies, typically with a high allocation to stocks; see, for example, Ayres and Nalebuff (2010).

Most of the previous literature focuses on maximizing the size of the retirement portfolio, although that is not the only possible goal. Some investors may be interested in policies designed to hit a target portfolio, an issue considered by Basu, Byrne, and Drew (2011) and Estrada (2019). Other investors may be interested in obtaining increasing downside protection as retirement approaches, which is what target-date funds offer; see Estrada (2020b).

Regarding the withdrawal period, Bengen (1994) is widely considered the seminal article. Part of the subsequent literature focuses on withdrawal strategies, such as Stout (2008), Blanchett and Frank (2009), Jaconetti, Kinniry, and DiJoseph (2013), and Zolt (2014), among others. Another part of this literature focuses on asset allocation strategies, such as Blanchett (2007), Spitzer and Singh (2007), Pfau and Kitces (2014), Kitces and Pfau (2015), Estrada (2016), and Estrada and Kritzman (2019), among others. Estrada (2020a) considers both withdrawal and asset allocation strategies.

One characteristic of all of the previous literature is that it addresses either the accumulation or the withdrawal period. That approach is valid if the topic discussed is either the target portfolio or the sustainability of a given retirement portfolio. However, as already argued, proper retirement planning necessarily implies integrating both periods, as is done here. Pfau (2011) also considers both periods when introducing the concept of a safe savings rate.

Withdrawal or Retirement Period

Consider an individual that at the very beginning of her working years wants to put a proper retirement plan in place. She thinks about her desired retirement,

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[1]Both Eisenberg (2006) and Benz (2020) discuss this issue.

which she expects to last N years, and believes that it can be sustained with N annual withdrawals W_1 through W_N from her retirement portfolio P, made at the beginning of each year in retirement. She plans to keep her purchasing power constant over time, which implies that in real (inflation-adjusted) terms $W_1 = W_2 = \cdots = W_N = W$, where W is the constant annual real withdrawal.

Our individual plans to leave a bequest B, which can be thought of as a final withdrawal that fully exhausts the portfolio, made one year after W_N. Finally, she needs to select an asset allocation for her portfolio, which in turn will determine R, the annualized return she expects to obtain during the retirement period. As shown in the appendix, our individual's problem can be expressed as

$$P \cdot (1+R)^N - W \cdot \sum_{t=1}^{N}(1+R)^t - B = 0 \qquad (1)$$

where t indexes periods, measured in years. Given B, W, and R, this expression can be solved for the target portfolio (P^*); that is,

$$P^* = \frac{B + W \cdot \sum_{t=1}^{N}(1+R)^t}{(1+R)^N} \qquad (2)$$

Finally, simplifying the sum by using the expression for the sum of a geometric series (see the appendix), one obtains

$$P^* = \frac{B - W \cdot (1+R) \cdot \left(\frac{1-(1+R)^N}{R} \right)}{(1+R)^N} \qquad (3)$$

which formalizes the relationship between the target retirement portfolio, the constant annual real withdrawal, the bequest, and the return of the portfolio during retirement.

Accumulation or Working Period

Having determined the target portfolio P^*, and expecting to work for M years, our individual then needs to select an asset allocation for her portfolio during her working years. The selected asset allocation will in turn determine S, the annualized return she expects to obtain from her portfolio during the accumulation period.

Our individual aims to make $M-1$ annual contributions to her portfolio, the first at the end of her first working year, and the last one year before retirement. Thus, at the end of the accumulation period she aims to liquidate her portfolio, implement her new (more conservative) asset allocation, and immediately take the first withdrawal to fund her first year in retirement. Finally, our individual expects to keep her contributions constant in real terms, which implies that $C_1 = C_2 = \cdots = C_{M-1} = C$, where C is the constant annual real (inflation-adjusted) contribution. As shown in the appendix, our individual's problem can be expressed as

$$C \cdot \sum_{t=1}^{M-1}(1+S)^t = P^* \qquad (4)$$

where t again indexes periods, measured in years. Given S and P^*, this expression can be solved for the constant annual real contribution (C^*); that is,

$$C^* = \frac{P^*}{\sum_{t=1}^{M-1}(1+S)^t} \qquad (5)$$

Finally, simplifying the sum by using the expression for the sum of a geometric series (see the appendix), one obtains

$$C^* = -\frac{P^*}{\left(\frac{(1+S)-(1+S)^M}{S} \right)} \qquad (6)$$

which formalizes the relationship between the constant annual real contribution, the target retirement portfolio, and the return of the portfolio during the working years.

Lifecycle

The previous two sections described the problem of our representative individual, considered and solved in proper order, that is, from the end (the retirement period) to the beginning (the working period). Expression 3 maps the withdrawals to be made in retirement, the bequest to be left, and the expected return of the portfolio in retirement to the target portfolio. Expression 6, in turn, maps the target portfolio determined in the previous step and the expected return of the portfolio during the working years to the contributions to be made during the accumulation period.

Those two expressions, 3 and 6, summarize the framework introduced, which in fact can be summarized in just one expression by inserting Expression 2 into Expression 5 or Expression 3 into Expression 6; that is,

$$C^* = \frac{\dfrac{W \cdot \sum_{t=1}^{N}(1+R)^t + B}{(1+R)^N}}{\sum_{t=1}^{M-1}(1+S)^t}$$

$$= -\frac{\dfrac{B - W \cdot (1+R) \cdot \left(\dfrac{1-(1+R)^N}{R} \right)}{(1+R)^N}}{\left(\dfrac{(1+S)-(1+S)^M}{S} \right)} \quad (7)$$

where the only endogenous variable is C, depending on the exogenous values of N, M, R, S, W, and B. In other words, how much our individual needs to periodically contribute to her retirement portfolio during the accumulation period depends on how many years she expects to work (M), the return of her portfolio during those years (S), how many years she expects to be in retirement (N), the return of her portfolio during those years (R), how much she aims to periodically withdraw from her portfolio (W), and the size of the bequest she aims to leave (B).

EVIDENCE

This section discusses an implementation of the plan, first with a base case and sensitivity analysis, and then with Monte Carlo simulations. Data on stocks, bonds, and inflation during the 1928–2019 period are from Damodaran's web page.[2] Stocks are represented by the S&P 500; bonds, by 10-year US Treasury Notes; and inflation, by the Consumer Price Index. All returns are annual, real (adjusted by inflation), and account for capital gains/losses and cash flows paid. During this 92-year period, stocks and bonds delivered annualized returns of 6.5% and 1.9%, with volatility of 19.5% and 8.1%; their correlation during the sample period was 0.05.

Base Case

At the beginning of her career, our representative individual expects to live 30 years in retirement.

[2] http://people.stern.nyu.edu/adamodar/New_Home_Page/datafile/histretSP.html.

She believes the cost of her lifestyle can be sustained by 30 constant annual real withdrawals of $60,000 from her portfolio, made at the beginning of each of her years in retirement. She aims to leave as a bequest the equivalent of five years of annual withdrawals ($300,000). During retirement, she thinks she will feel comfortable with a 40/60 stock/bond allocation, from which she expects an annualized real return of 4.2%, based on the performance of this allocation during the 1928–2019 period. The target retirement portfolio consistent with these conditions follows from Expression 3 and is equal to

$$P^* = \frac{\$300,000 - \$60,000 \cdot (1+0.042) \cdot \left(\dfrac{1-(1+0.042)^{30}}{0.042} \right)}{(1+0.042)^{30}}$$

$$= \$1,142,637$$

At the beginning of her career, our representative individual aims to work for 40 years. During this period she thinks she will feel comfortable with a more aggressive 60/40 stock/bond allocation, from which she expects an annualized real return of 5.2%, based on the performance of this allocation during the 1928–2019 period. Given that she targets a $1,142,637 retirement portfolio, her constant annual real contribution follows from Expression 6 and is equal to

$$C^* = -\frac{\$1,142,637}{\left(\dfrac{(1+0.052)-(1+0.052)^{40}}{0.052} \right)} = \$9,079$$

This annual contribution to the retirement portfolio could also be found directly from Expression 7, without specifying the target portfolio; that is,

$$C^* = -\frac{\dfrac{\$300,000 - \$60,000 \cdot (1+0.042) \cdot \left(\dfrac{1-(1+0.042)^{30}}{0.042} \right)}{(1+0.042)^{30}}}{\left(\dfrac{(1+0.052)-(1+0.052)^{40}}{0.052} \right)}$$

$$= \$9,079$$

Having all this information, our representative individual is now able to determine the expected path of her retirement portfolio during her 70-year lifecycle, shown in Exhibit 1. Note that at the end of the first

Retirement Portfolio— Expected Path

Y	P	Y	P	Y	P	Y	P	Y	P	Y	P	Y	P
1	9,079	13	162,869	25	445,434	37	964,601	49	927,611	61	607,812		
2	18,629	14	180,417	26	477,675	38	1,023,839	50	906,571	62	573,340		
3	28,676	15	198,877	27	511,593	39	1,086,157	51	884,647	63	537,421		
4	39,246	16	218,297	28	547,274	40	1,082,637	52	861,802	64	499,992		
5	50,366	17	238,727	29	584,811	41	1,068,108	53	837,998	65	460,992		
6	62,063	18	260,219	30	624,300	42	1,052,968	54	813,194	66	420,354		
7	74,369	19	282,829	31	665,842	43	1,037,193	55	787,348	67	378,008		
8	87,315	20	306,615	32	709,544	44	1,020,755	56	760,416	68	333,885		
9	100,934	21	331,638	33	755,519	45	1,003,627	57	732,354	69	287,908		
10	115,261	22	357,961	34	803,884	46	985,779	58	703,113	70	300,000		
11	130,333	23	385,654	35	854,765	47	967,182	59	672,644				
12	146,189	24	414,786	36	908,291	48	947,804	60	640,895				

Notes: This exhibit shows the expected path of a portfolio (P) over 70 years (Y), 40 in the working period and 30 in the retirement period. P receives real contributions of $9,079 at the end of Years 1 through 39 and earns a 5.2% real return during the first 40 years. P is also subject to 30 real withdrawals of $60,000 at the beginning of Years 40 through 70, leaves a bequest of $300,000 at year-end 70, and earns a 4.2% real return during the last 30 years. All figures are in dollars.

working year the portfolio contains only the first contribution; from that point on and until the end of the accumulation period, the portfolio receives annual (real) contributions of $9,079 and compounds at 5.2%. Note, also, that after the last withdrawal of $60,000 at the beginning of Year 70 (the end of Year 69 in Exhibit 1), the portfolio's value is $287,908, which compounds over one more period at 4.2% and ends with the intended bequest of $300,000.

Note that the retirement date (Year 40, highlighted) is the breakpoint; until then the portfolio compounds at 5.2%, and from then it compounds at 4.2%. Note, also, that on the retirement date the value of the portfolio ($1,082,637) is not equal to the target portfolio ($1,142,637); the difference between those two values is equal to the first withdrawal, made to fund the expenses of the first year in retirement.

Specifying the expected path of a retirement portfolio is important because it provides markers to determine whether the individual's plan is on track. Some companies aim to guide investors during their working years by specifying how many times their salary they should have saved at different points in time.[3] Exhibit 1

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[3] Fidelity, for example, suggests that individuals should have three (six) [eight] times their annual salary saved by the time they reach 40 (50) [60] and refers to these figures as "saving factors"; see Fidelity (2018).

shows directly the value the retirement portfolio should have at the end of each year during the lifecycle. Should the individual find herself away from this path, she can consider policies to get back on track; Estrada (2019, 2020a) shows that adjusting contributions or withdrawals is a more effective way to do so than adjusting a portfolio's asset allocation.

Sensitivity Analysis

Let's recap. At the beginning of her 40-year career our individual considers her 30-year retirement, makes an estimate of its annual costs ($60,000), decides the bequest she wants to leave ($300,000), and selects the asset allocation she will feel comfortable with (40/60), from which she expects a 4.2% annualized return. This enables her to determine a target retirement portfolio ($1,142,637), and after selecting the asset allocation she will feel comfortable with during her working years (60/40), from which she expects a 5.2% annualized return, she finds the annual real contributions she needs to make to her retirement portfolio ($9,079). The path the portfolio is expected to follow over our individual's 70-year lifecycle is depicted in Exhibit 1.

Expressions 3 and 6, or just Expression 7, summarize the framework introduced here for proper retirement planning. This framework can also be used to

E X H I B I T 2
Sensitivity Analysis

Panel A	10/90	20/80	30/70	40/60	50/50	60/40	70/30
45,000	1,072,677	987,757	912,809	856,978	806,440	760,602	726,962
50,000	1,191,863	1,097,508	1,014,232	952,198	896,045	845,113	807,735
55,000	1,311,049	1,207,258	1,115,655	1,047,417	985,649	929,624	888,509
60,000	1,430,236	1,317,009	1,217,078	1,142,637	1,075,254	1,014,136	969,282
65,000	1,549,422	1,426,760	1,318,501	1,237,857	1,164,858	1,098,647	1,050,056
70,000	1,668,608	1,536,511	1,419,924	1,333,077	1,254,463	1,183,158	1,130,829
75,000	1,787,795	1,646,261	1,521,348	1,428,296	1,344,067	1,267,670	1,211,603

Panel B	30/70	40/60	50/50	60/40	70/30	80/20	90/10
992,637	11,335	10,064	8,918	7,887	7,139	6,456	5,982
1,042,637	11,906	10,571	9,367	8,284	7,499	6,781	6,283
1,092,637	12,477	11,078	9,816	8,681	7,859	7,106	6,585
1,142,637	13,048	11,585	10,265	9,079	8,218	7,431	6,886
1,192,637	13,619	12,092	10,714	9,476	8,578	7,756	7,187
1,242,637	14,190	12,599	11,164	9,873	8,937	8,082	7,489
1,292,637	14,761	13,106	11,613	10,270	9,297	8,407	7,790

Panel C	10/90	20/80	30/70	40/60	50/50	60/40	70/30
30/70	16,332	15,039	13,898	13,048	12,279	11,581	11,069
40/60	14,501	13,353	12,340	11,585	10,902	10,282	9,827
50/50	12,849	11,832	10,934	10,265	9,660	9,111	8,708
60/40	11,364	10,464	9,670	9,079	8,543	8,058	7,701
70/30	10,287	9,472	8,754	8,218	7,734	7,294	6,971
80/20	9,302	8,565	7,915	7,431	6,993	6,595	6,304
90/10	8,619	7,937	7,335	6,886	6,480	6,112	5,841

Notes: This exhibit shows three panels. Panel A, based on Expression 3, shows target retirement portfolios, with asset allocations during retirement in the top row and annual withdrawals in the first column. Panel B, based on Expression 6, shows annual contributions to the retirement portfolio, with asset allocations during the working years in the top row and target retirement portfolios in the first column. Panel C, based on Expression 7, also shows annual contributions to the retirement portfolio, with asset allocations during the retirement period in the top row, and asset allocations during the accumulation period in the first column. All figures are in dollars.

explore the impact of adjusting some variables the individual controls on the size of the retirement portfolio, and ultimately on the contributions to be made during the working period, thus enabling the evaluation of many relevant trade-offs.

To explore that issue consider the three panels of Exhibit 2, each with the base case highlighted. Panel A, based on Expression 3, shows target retirement portfolios; in the top row it shows stock/bond allocations during retirement, and in the first column it shows levels of the constant annual withdrawal.[4] Note that for any given level of withdrawals, the less (more) aggressive the asset allocation is, the larger (smaller) the target portfolio

would need to be; thus, if a target portfolio looks too large and therefore unreachable, the individual could aim for a smaller portfolio and compensate with a more aggressive asset allocation in retirement. Also, given the asset allocation, the higher (lower) the standard of living desired by the individual, reflected in higher (lower) withdrawals, then the larger (smaller) is the retirement portfolio the individual would need. Expression 3 can be used to explore such trade-offs.

Panel B, based on Expression 6, shows levels of the constant annual contribution during the accumulation period; in the top row it shows stock/bond allocations during the working years, and in the first column it shows target retirement portfolios. Note that given the target portfolio, the less (more) aggressive the asset allocation is, the larger (smaller) the annual contributions

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[4] Recall that the individual aims to leave five years of annual withdrawals as a bequest; hence, different levels of withdrawals also imply different levels of bequest.

would need to be; thus, if an asset allocation is too aggressive for the individual, she could select a more conservative one and compensate by increasing her annual contributions. Also, given the asset allocation, if the individual desires a higher (lower) standard of living, reflected in a larger (smaller) retirement portfolio, then she would need to make larger (smaller) contributions during her working years. Expression 4 can be used to explore such trade-offs.

Finally, Panel C, based on Expression 7, also shows levels of the constant annual contribution; in the top row it shows asset allocations during the retirement period, and in the first column it shows asset allocations during the accumulation period. The trade-off between how aggressive an asset allocation is and how much the individual would need to contribute to the portfolio during the working years is clear; all else being equal, the less (more) aggressive an asset allocation during the accumulation or the withdrawal period is, the more (less) the individual would need to contribute to her portfolio. Expression 7 can be used to explore such trade-offs.

Monte Carlo Simulation

Mike Tyson famously said that everybody has a plan until they get punched in the mouth; much the same could be said about individuals planning for retirement. Plans hardly ever go as expected, even if individuals have the ability or the luck to guess correctly the annualized return of their portfolio. As is well-known, when a portfolio is subject to contributions or withdrawals, the annualized return obviously matters but so does the sequence of returns. When an individual is regularly withdrawing funds from a portfolio, an early sequence of bad returns can ruin a retirement plan, a problem known as sequence of returns risk.

For that reason, besides the base case and sensitivity analysis already discussed, it is insightful to perform a Monte Carlo simulation to gain a sense of how the relevant variables behave when things do not go exactly as planned. Exhibit 3 summarizes the results of 10,000 scenarios during an individual's lifecycle, based on the statistical properties of the 60/40 and the 40/60 allocations during the 1928–2019 period.[5] The parameters on which

[5] Kolmogorov–Smirnov tests do not reject the null hypothesis of normality in the distribution of returns of the 60/40 and 40/60 allocations, with p-values of 0.998 and 0.743. Furthermore, the

EXHIBIT 3
Monte Carlo Simulation

Panel A	Retirement Portfolio		Retirement Portfolio
Median	1,170,203		
1st Percentile	365,252	90th Percentile	2,325,213
5th Percentile	508,533	95th Percentile	2,866,944
10th Percentile	613,959	99th Percentile	4,206,180

Panel B	Bequest		Bequest
Median	371,481	Failure	42.9%
1st Percentile	N/A	90th Percentile	5,546,704
5th Percentile	N/A	95th Percentile	8,341,059
10th Percentile	N/A	99th Percentile	15,596,789

Notes: This exhibit summarizes the results of 10,000 scenarios during an individual's lifecycle based on the statistical properties of the 60/40 and the 40/60 stock/bond allocations during the 1928–2019 period. Panel A considers the end of the accumulation period (the retirement portfolio), and Panel B considers the end of the retirement period (the bequest). "Failure" is the percentage of scenarios in which the portfolio was depleted before the end of the retirement period. All figures but "Failure" are in dollars.

the simulations are based are those already discussed for the base case, except for the bequest, which now is whatever is left at the end of the retirement period. Panel A focuses on the retirement portfolio (the end of the accumulation period), and Panel B, on the bequest (the end of the retirement period).

Recall that the target retirement portfolio in the base case is $1,142,637. Panel A shows two interesting things about this target. First, it shows that the median retirement portfolio across the 10,000 scenarios in the simulation ($1,170,203) is indeed very close to the target portfolio in the plan. Second, it shows that there is a very large dispersion, with the 5th- (95th-) percentile target portfolio being less than half of (over twice as large as) the median portfolio.

Panel B shows rather similar results for the bequest. The median across the 10,000 scenarios in the simulation ($371,481) is somewhat larger than the original target bequest ($300,000).[6] In the upper percentiles of the distribution the bequest is many times larger than originally intended, but in the lower percentiles the portfolio fails; that is, it is depleted before the end of

autocorrelation coefficients of the returns series of the 60/40 and the 40/60 allocations are 0.00 and 0.03.

[6] But recall that in these simulations the bequest is whatever is left at the end of the retirement period.

Sensitivity Analysis on the Monte Carlo Simulation

Panel A	Lower *W*	Lower *IWR*	Higher *R*	Higher *S*	Higher *C*
Median	1,164,871	1,183,060	1,181,517	1,307,785	1,286,243
1st Percentile	361,001	373,071	359,312	348,357	396,888
5th Percentile	506,093	508,136	495,709	499,644	555,407
10th Percentile	598,404	607,665	598,791	615,908	663,209
90th Percentile	2,321,623	2,343,782	2,352,519	2,935,031	2,562,628
95th Percentile	2,840,918	2,864,485	2,887,476	3,679,903	3,111,627
99th Percentile	4,096,042	4,151,768	4,230,587	5,489,660	4,599,491
Panel B	**Lower *W***	**Lower *IWR***	**Higher *R***	**Higher *S***	**Higher *C***
Median	710,613	1,279,304	589,950	817,782	756,672
1st Percentile	N/A	N/A	N/A	N/A	N/A
5th Percentile	N/A	N/A	N/A	N/A	N/A
10th Percentile	N/A	N/A	N/A	N/A	N/A
90th Percentile	6,244,139	6,841,990	6,945,560	7,913,488	6,520,700
95th Percentile	9,149,920	9,931,974	10,333,486	11,645,052	9,600,709
99th Percentile	17,299,387	17,058,064	21,006,052	23,469,850	18,173,951
Failure	36.4%	25.7%	40.6%	37.8%	37.3%

Notes: This exhibit summarizes the results of 10,000 scenarios for each of the five cases considered over an individual's lifecycle, based on the statistical properties of the 60/40 and the 40/60 stock/bond allocations during the 1928–2019 period. Panel A considers the end of the accumulation period (the retirement portfolio), and Panel B considers the end of the retirement period (the bequest). The second column (Lower W) shows the result of reducing the constant annual withdrawal by 10%; the third column (Lower IWR), the result of reducing the initial withdrawal rate by 1.3 percentage points; the fourth column (Higher R), the result of increasing the allocation to stocks during retirement by 10 percentage points; the fifth column (Higher S), the result of increasing the allocation to stocks during the working period by 10 percentage points; and the last column (Higher C), the result of increasing the constant annual contribution by 10%. "Failure" is the percentage of scenarios in which the retirement portfolio was depleted before the end of the retirement period. All figures but "Failure" are in dollars.

the retirement period. In fact, the portfolio runs out of money earlier than expected in almost 43% of the scenarios considered.

This large failure rate should not be entirely surprising. The $60,000 initial withdrawal from the $1,142,637 retirement portfolio represents an initial withdrawal rate of 5.3%, substantially higher than what is typically considered a safe initial withdrawal rate. The latter was originally found to be 4% by Bengen (1994), although this figure has been subject to a heated debate since then.[7] Currently, the consensus seems to be that 4% is too high, implying that individuals who withdraw 4% of their portfolio on retirement day, and subsequently adjust their annual withdrawals by inflation, are likely

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[7] To be precise, Bengen (1994) found that a portfolio with a 50/50 stock/bond allocation, subject to an initial withdrawal rate of 4% and subsequent annual withdrawals adjusted by inflation, never lasted less than 30 years, which is what he considered a minimum requirement for portfolio longevity.

to deplete the portfolio in less than 30 years; see, for example, Pfau (2010) and Estrada (2016).

Additional Sensitivity Analysis

The results just discussed are useful to illustrate both the many things that could happen over an individual's lifecycle and ultimately the degree of uncertainty the individual would have to face. This analysis can be further complemented with a sensitivity analysis on some of the parameters underlying the Monte Carlo simulation. Exhibit 4 summarizes the results of 50,000 scenarios, equivalent to 10,000 scenarios for each of the five cases considered. The parameters are changed one at a time, with the rest remaining at their values in the base case.

Panel A focuses on the retirement portfolio (the end of the accumulation period), and Panel B, on the bequest (the end of the retirement period). In both panels, the second column shows the impact of lowering

the constant annual withdrawal by 10%, from $60,000 to $54,000; the third, the impact of lowering the initial withdrawal rate by 1.3 percentage points, from 5.3% to 4%; the fourth, the impact of selecting a more aggressive asset allocation during retirement, from 40/60 to 50/50; the fifth, the impact of selecting a more aggressive asset allocation during the working period, from 60/40 to 70/30; and the last, the impact of increasing the constant annual contribution by 10%, from $9,079 to $9,986.

In the second, third, and fourth columns, all of which consider changes in variables related to the retirement period, there is predictably little impact on the target retirement portfolio and a substantial impact on the bequest, in all cases relative to the values in Exhibit 3. The median bequest increases by almost 60% (from $371,481 to $589,950) when the 40/60 allocation in retirement is replaced by the 50/50 allocation; by more than 90% (from $371,481 to $710,613) when withdrawals are lowered from $60,000 to $54,000; and by more than 240% (from $371,481 to $1,279,304) when the initial withdrawal rate is lowered from 5.3% to 4%. In all of these three cases, the failure rate of 42.9% in Exhibit 3 decreases, in some cases substantially (to under 26% when the initial withdrawal rate is lowered).

The last two columns consider changes in variables related to the accumulation period and their impact on both the retirement portfolio and the bequest. When the 70/30 allocation replaces the 60/40 allocation, the median retirement portfolio increases by more than 11% (from $1,170,203 to $1,307,785) and the median bequest by more than 120% (from $371,481 to $817,782); when annual contributions are increased from $9,079 to $9,986, the median retirement portfolio increases by almost 10% (from $1,170,203 to $1,286,243) and the median bequest by almost 104% (from $371,481 to $756,672). In both cases, the failure rate of 42.9% in Exhibit 3 decreases.

This type of analysis on the parameters of a Monte Carlo simulation makes it possible to evaluate the empirical impact of a change in any of the model's relevant variables. The difference between the sensitivity analyses in Exhibits 2 and 4 is that, in the former, results are obtained from closed-form solutions based on empirical parameters; in the latter, results are obtained from a large number of potential scenarios based on the statistical behavior of the series considered. Both types of analyses are insightful and complement each other.

Further Discussion

The retirement planning model introduced here properly begins from the end (the retirement period and its relevant variables), then it works backward to determine the target retirement portfolio, and then it works backward once again to the beginning (the working period and its relevant variables). Although this is the only plausible way to plan for retirement, the model rests on the rather implausible assumption that young people can accurately foresee their life in retirement, determine a sequence of withdrawals to sustain it, decide the bequest they want to leave, and select an asset allocation for the retirement period. That may be too much to ask from any young person, however foresighted that person may be.

However, that does not invalidate the model or imply that it cannot be plausibly implemented. It is indeed the case that without the inputs (exogenous variables) relevant for the retirement period neither a target portfolio nor the contributions to be made during the working years can be determined. But a realistic way to get around that problem would be for individuals at the beginning of their working years to make their best guess about the values of the exogenous variables, and then to update their guess over time, thus obtaining new solutions for the target portfolio and the contributions to be made during the rest of their working years.

The only difference with the approach previously proposed is that instead of going from no portfolio to a target portfolio during the whole accumulation period, individuals would instead go from the portfolio they have at the time of updating their guess to a different target portfolio during a shorter period of time. That would change Expression 6 slightly, but the rest of the process would remain unchanged.

To illustrate, assume that after working for 25 years and with 15 working years to go an individual now has a different view of the withdrawals he will need during retirement, or the bequest he wants to leave, or the asset allocation he wants to implement. He would then use the updated values of the relevant (exogenous) variables for the retirement period and find his new target portfolio by using Expression 3, just as before. And he would then find the constant annual real contribution over his remaining 15 working years, beginning from the portfolio he has accumulated during the previous 25 years.

That would require a slight modification to Expression 6, which in this case would be

$$C^* = -\frac{P^* - P_{25}(1+S)^{15}}{\left(\dfrac{(1+S)-(1+S)^{15}}{S}\right)}$$

where P_{25} is the value of the portfolio accumulated during the first 25 working years and P^* would be the *new* target portfolio.

Introducing adjustments to a retirement plan should be considered the rule rather than the exception. These adjustments may follow from revised choices about the retirement period, as in the example above, or from the impact of a wide variety of factors affecting the working period, such as changes in income or the introduction of shorter-term goals for which the individual will have to save, to name but two. Estrada (2019, 2020a) highlights the value of flexibility when planning for retirement, reflected in an individual's ability to adjust contributions, withdrawals, or asset allocations when reality disrupts a financial plan.

ASSESSMENT

Retirement planning, like starting a diet or visiting the dentist, is something that many people always find reasons to postpone; dealing with those issues never appears to be imperative, and short-term problems that must be taken care of usually take priority. In particular, at the beginning of their careers young people can hardly be blamed for lacking the motivation and the information to think about their retirement. But the inevitable result of all this is that many individuals approach the end of their careers unprepared, with little time to react and few options to explore.

To be sure, some people do intend to plan, but often at the wrong time or in the wrong way. Seeking advice shortly before retirement on how to hit a target portfolio is doing it too late; not knowing whether the target portfolio will indeed sustain the desired lifestyle in retirement is doing it the wrong way. Proper retirement planning must be done early and solved backward. It must be done early because it is the best way to minimize the savings that need to be made, and to maximize the range of corrective options available; and it must be solved backward because it is the only way

to ensure that the target portfolio is designed to sustain the desired retirement.

The approach introduced here starts from the end; that is, from assessing the desired retirement, the withdrawals from the portfolio necessary to sustain it, and the bequest to be left. These variables, together with an asset allocation for the withdrawal period (hence an expected return for the portfolio in retirement), jointly determine the target portfolio. And the latter, together with an asset allocation for the accumulation period (hence an expected return for the portfolio during the working years), jointly determine the periodic contributions during a person's career that need to be made to hit the target.

The analytical framework introduced here yields one closed-form solution for the target retirement portfolio as a function of the bequest, the periodic withdrawals, and the return expected from the portfolio in retirement; and another for the contributions to be made during the accumulation period as a function of the target retirement portfolio and the return expected from the portfolio during the working years. It also yields just one closed-form solution to determine the contributions to be made during the accumulation period as a function of all the relevant variables of the model, and it provides a tool to perform insightful sensitivity analysis to evaluate many of the relevant trade-offs that individuals planning for retirement have to face.

The example discussed illustrates how to apply the analytical framework, derive the value of the relevant variables, outline the expected path of the portfolio during the lifecycle, perform a sensitivity analysis, and explore the relevant uncertainties by means of Monte Carlo simulations. It also highlights the exogenous variables that need to be decided before the analysis and the endogenous variables determined by the model.

Understanding what needs to be done to plan properly for retirement is not very difficult; actually doing it is the problem. That said, it is in fact critical to have the correct analytical framework to point individuals and financial advisors in the right direction. It is hoped this article will contribute toward that worthy goal.

A P P E N D I X

This appendix discusses the analytical framework behind Expressions 1–7 in the text. For the reasons already discussed, it starts with the retirement period, then continues

Exhibit A1
Timeline
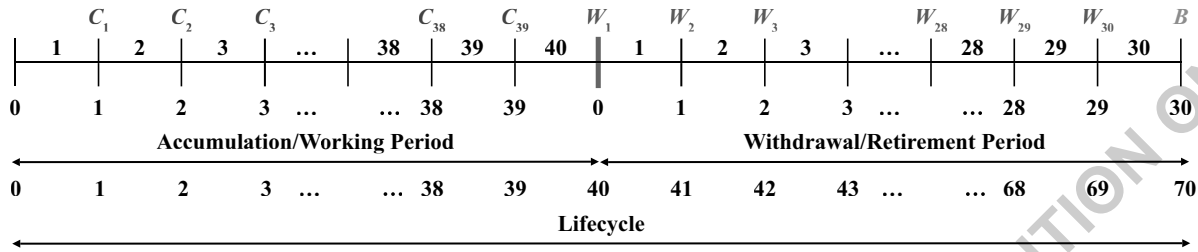
Notes: *This exhibit shows the timeline on which the analysis is based, with a 40-year working period and a 30-year retirement period. The individual makes 39 contributions (C_t) to, and 30 withdrawals (W_t) from, her retirement portfolio. The bequest (B) is what is left after taking the last withdrawal and one more year of compounding.*

with the working period, and concludes with the lifecycle. Exhibit A1 provides the timeline.

The retirement period. Consider a representative individual at the beginning of his working years. He contemplates his retirement, which he expects to last N years, and believes that it can be sustained by N withdrawals (W_1, ..., W_N) from his retirement portfolio P, made at the beginning of each of his years in retirement. He aims to keep his purchasing power constant over time, which implies that $W_1 = \cdots = W_N = W$, where W is the constant annual real (inflation-adjusted) withdrawal. He aims to leave a bequest B and expects an annualized real return R from his portfolio during retirement. At the *beginning* of his first three years in retirement the individual's portfolio will be

Year 1: $P - W$
Year 2: $(P - W)(1 + R) - W = P(1 + R) - W(1 + R) - W$
Year 3: $[P(1 + R) - W(1 + R) - W](1 + R) - W$
$\qquad = P(1 + R)^2 - W(1 + R)^2 - W(1 + R) - W$

It is clear then that at the *beginning* of last year in retirement his portfolio will be

Year N: $P(1 + R)^{N-1} - W(1 + R)^{N-1} - W(1 + R)^{N-2}$
$\qquad - \cdots - W(1 + R) - W$

This portfolio will compound over one more year (the individual's last year in retirement) and become his bequest; hence

$$\{P(1+R)^{N-1} - W(1+R)^{N-1} - W(1+R)^{N-2} - \cdots - W(1+R) - W\}(1+R) = B$$

and, therefore,

$$P(1+R)^N - W(1+R)^N - W(1+R)^{N-1} - \cdots - W(1+R)^2 - W(1+R) = B$$

which can be rewritten as

$$P \cdot (1+R)^N - W \cdot \sum_{t=1}^{N}(1+R)^t - B = 0 \qquad (A1)$$

This expression can be solved for the individual's target portfolio (P^*), which is given by

$$P^* = \frac{B + W \cdot \sum_{t=1}^{N}(1+R)^t}{(1+R)^N} \qquad (A2)$$

Recall that the expression for the sum of a geometric series is given by

$$\sum_{t=0}^{T-1} a \cdot g^t = a \cdot \left(\frac{1 - g^T}{1 - g}\right) \qquad (A4)$$

where a is a constant, g is the periodic rate at which the series grows, and T is the number of compounding periods.

Note that the sum term in (A2) can be rewritten as

$$\sum_{t=1}^{N}(1+R)^t = (1+R) \cdot \sum_{t=0}^{N-1}(1+R)^t \qquad (A5)$$

Using (A4), the sum term on the right-hand side of (A5) can be rewritten as

$$\sum_{t=0}^{N-1}(1+R)^t = \left(\frac{1 - (1+R)^N}{1 - (1+R)}\right) = \left(-\frac{1 - (1+R)^N}{R}\right) \qquad (A6)$$

Thus, inserting (A6) into (A5) we obtain

$$\sum_{t=1}^{N}(1+R)^t = -(1+R)\cdot\left(\frac{1-(1+R)^N}{R}\right) \qquad (A7)$$

Finally, inserting (A7) into (A2) yields

$$P^* = \frac{B - W\cdot(1+R)\cdot\left(\dfrac{1-(1+R)^N}{R}\right)}{(1+R)^N} \qquad (A8)$$

which is the same as Expression 3 in the text; that is, the individual's target portfolio as a function of his bequest, his constant annual real withdrawal, and the expected return of his portfolio over the N years he expects to be retired.

The working period. Having determined his target portfolio and expecting to work for M years, the representative individual needs to determine the annual contributions he needs to make to his retirement portfolio to hit the target P^* on his retirement date. He expects to make $M-1$ contributions ($C_1, \ldots, C_{M-1}$) to his portfolio, the first at the end of his first working year and the last one year before retirement. He aims to keep the contributions constant in real terms, which implies that $C_1 = \cdots = C_{M-1} = C$, where C is the constant annual real (inflation–adjusted) contribution. Finally, during his working years he expects an annualized real return S from his portfolio. Thus, at the *end* of the first three years of his working period the individual's portfolio will be

Year 1: C
Year 2: $C(1 + S) + C = C[(1 + S) + 1]$
Year 3: $C[(1 + S) + 1](1 + S) + C = C[(1 + S)^2 + (1 + S) + 1]$

It is clear then that at the *end* of next-to-last working year his portfolio will be

Year $M-1$: $C[(1 + S)^{M-2} + (1 + S)^{M-3} + \cdots + (1 + S)^2 + (1 + S) + 1]$

This portfolio will compound over one more year (the individual's last working year) and his her target portfolio on the retirement date; that is,

Year M: $\{C[(1 + S)^{M-2} + (1 + S)^{M-3} + \cdots + (1 + S)^2 + (1 + S) + 1]\}(1 + S) = P^*$
$C[(1 + S)^{M-1} + (1 + S)^{M-2} + \cdots + (1 + S)^2 + (1 + S)] = P^*$

which can be rewritten as

$$P^* = C\cdot\sum_{t=1}^{M-1}(1+S)^t \qquad (A9)$$

This expression can be solved for the constant annual real contribution (C^*), which is given by

$$C^* = \frac{P^*}{\sum_{t=1}^{M-1}(1+S)^t} \qquad (A10)$$

Note that the denominator of (A10) can be rewritten as

$$\sum_{t=1}^{M-1}(1+S)^t = (1+S)\cdot\sum_{t=0}^{M-2}(1+S)^t \qquad (A11)$$

Thus, using (A4) again, the sum term on the right-hand side of (A11) can be rewritten as

$$\sum_{t=0}^{M-2}(1+S)^t = \left(\frac{1-(1+S)^{M-1}}{1-(1-S)}\right) = -\frac{1-(1+S)^{M-1}}{S} \qquad (A12)$$

replacing (A12) into (A11) we obtain

$$\sum_{t=1}^{M-1}(1+S)^t = (1+S)\cdot\sum_{t=0}^{M-2}(1+S)^t = -(1+S)\cdot\left(\frac{1-(1+S)^{M-1}}{S}\right) \qquad (A13)$$

and rearranging (A13) we get

$$\sum_{t=1}^{M-1}(1+S)^t = (1+S)\cdot\sum_{t=0}^{M-2}(1+S)^t = -\left(\frac{(1+S)-(1+S)^M}{S}\right) \qquad (A14)$$

Finally, replacing (A14) into (A10) we obtain

$$C^* = -\frac{P^*}{\left(\dfrac{(1+S)-(1+S)^M}{S}\right)} \qquad (A15)$$

which is the same as Expression 6 in the text; that is, the individual's constant annual real contribution as a function of target portfolio and the expected return of his portfolio over the M years he aims to work.

The lifecycle. As a final and unifying step, note that the individual may skip setting a target portfolio and simply consider how much he needs to contribute during his working years to sustain the expected withdrawals during his retirement. Thus, replacing (A8) into (A15) we obtain

$$C^* = -\frac{B - W \cdot (1+R) \cdot \left(\dfrac{1-(1+R)^N}{R}\right)}{\dfrac{(1+R)^N}{\left(\dfrac{(1+S)-(1+S)^M}{S}\right)}} . \qquad \text{(A16)}$$

which is the same as Expression 7 in the text; that is, the individual's constant annual real contribution as a function of his bequest, his constant annual real withdrawal, and the annualized returns of his portfolio during his N years in retirement and his M working years.

ACKNOWLEDGMENTS

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ADDITIONAL READING

The Glidepath Illusion … and Potential Solutions

ROBERT ARNOTT, KATRINA SHERRERD, AND LILLIAN WU
The Journal of Retirement

ABSTRACT: *Target-date investment strategies purport to meet the two primary objectives of any retirement savings program: maximizing the real value of investors' nest eggs while minimizing uncertainty around prospective income in retirement. The authors demonstrate that the classic glidepath approach to retirement investing—moving from equity-centric to bond-centric investing as we age—does not meet these objectives.*

The authors summarize the flaws in traditional glidepath implementation and explore illustrative changes to the rules-based, mechanistic solution for retirement planning that can improve the expected outcome for investors, using simulations to test alternatives. Their findings show that, even with simple rules-based approaches, there are better ways to achieve our financial objectives for retirement.

The Glidepath Illusion: *An International Perspective*

JAVIER ESTRADA
The Journal of Portfolio Management

ABSTRACT: *Target-date funds have become a core product for investors who are saving for retirement. These funds periodically reduce their allocations to stocks and increase their allocations to bonds and cash, becoming more conservative as retirement approaches. This lifecycle strategy implies that investors are aggressive with little capital and conservative with much more capital, which may be suboptimal in terms of wealth accumulation. This article evaluates three alternative types of strategies, including contrarian strategies that follow a glidepath opposite that of target-date funds; that is, they become more aggressive as retirement approaches. The results from a comprehensive sample that spans more than 19 countries, two regions, and 110 years suggest that, relative to lifecycle strategies, the alternative strategies considered here provide investors with higher upside potential, more limited downside potential, and higher uncertainty—although that uncertainty is largely limited to how much higher their terminal wealth is expected to grow with these strategies.*

The Retirement Glidepath: *An International Perspective*

JAVIER ESTRADA
The Journal of Investing

ABSTRACT: *All individuals need to decide how much to save during their working years, how much to spend during retirement, and the asset allocation of their portfolio in both periods. A portfolio's exposure to stocks and bonds affects key variables, such as the probability of portfolio failure, degree of downside protection, and expected bequest. How this exposure should evolve during retirement is the ultimate issue explored in this article. After considering declining-equity, rising-equity, and static glidepaths, the comprehensive international evidence from 19 countries and the world market over 110 years ultimately suggests that both an all-equity portfolio and a 60/40 stock/bond allocation are simple and very effective strategies for retirees to implement.*