Sequence Risk: Is It Really a Big Deal?

Javier Estrada

Javier Estrada
is a professor of finance at IESE Business School in Barcelona, Spain.
jestrada@iese.edu

KEY FINDINGS

- Sequence risk, or the possibility of facing a sequence of low returns early in retirement that forces retirees to scale down their plans significantly, is a scary scenario.
- The evidence suggests that, however scary it might be, sequence risk is not very likely to hit retirees, who should certainly be informed, but not obsess about, this type of risk.
- Three ways of assessing sequence risk are proposed here, among them one that enables retirees to monitor the sustainability of a withdrawal strategy periodically and to introduce adjustments when necessary.

ABSTRACT

Financial planners are keenly aware of, and routinely warn clients about, sequence risk; that is, the possibility of facing a sequence of low returns early in retirement that may force retirees to scale down significantly the plans they had made. This really is a scary scenario, but one that the evidence here shows that retirees are not very likely to encounter. A new and refined definition of sequence risk is advanced in this article, linking this type of risk to the sustainability of a withdrawal strategy. Furthermore, three ways of assessing sequence risk are proposed, among them one that enables a retiree to monitor the sustainability of his withdrawal strategy periodically and to introduce adjustments when necessary. The ultimate message of this article is that retirees should be informed, but not obsess, about sequence risk.

TOPICS

*Retirement, risk management, portfolio management/multi-asset allocation**

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Sequence of returns risk, or sequence risk for short, is often highlighted as one of the critical issues faced by individuals toward the end of their working years and the beginning of their retirement. In fact, Harlow and Brown (2016) call it "the most significant investment risk that a retiree faces." To be sure, there is little doubt that being hit by a sequence of very low returns early in retirement, thus being forced to cut back significantly whatever plans an individual may have had, is a scary prospect.

That said, the likelihood of having to deal with such a scenario should play a role in whatever a retiree does (if anything) to deal with this problem. Being attacked by a shark is a scary prospect, but the fact that shark attacks rarely happen should be a factor in an individual's decision about whether or not to go for a swim. One of the main points raised in this article is that focusing on the impact of sequence risk,

EXHIBIT 1

Sequence Risk—An Example

Year	S&P 500	Observed	Reversed	High to Low	Low to High
1989		$960	$960	$960	$960
1990	–8.7%	$837	$1,194	$1,248	$564
1991	26.6%	$1,019	$1,078	$1,597	$389
1992	4.6%	$1,026	$1,246	$2,042	$298
1993	7.1%	$1,059	$1,327	$2,586	$222
1994	–1.3%	$1,005	$1,296	$3,233	$163
1995	34.2%	$1,309	$1,422	$4,052	$112
1996	19.0%	$1,517	$1,814	$5,078	$71
1997	31.1%	$1,950	$2,029	$6,211	$30
1998	26.5%	$2,427	$1,972	$7,371	–$10
1999	17.9%	$2,821	$2,196	$8,732	
2000	–12.1%	$2,440	$2,663	$10,253	
2001	–13.2%	$2,077	$1,636	$11,650	
2002	–23.9%	$1,540	$1,619	$13,167	
2003	26.3%	$1,906	$1,788	$14,829	
2004	7.4%	$2,007	$1,774	$16,693	
2005	1.4%	$1,996	$1,865	$18,269	
2006	12.9%	$2,214	$2,315	$19,579	
2007	1.4%	$2,204	$1,722	$20,936	
2008	–37.1%	$1,347	$1,454	$21,856	
2009	23.1%	$1,618	$1,238	$22,132	
2010	13.4%	$1,795	$1,419	$22,392	
2011	–0.8%	$1,740	$1,756	$22,498	
2012	14.0%	$1,944	$2,263	$22,272	
2013	30.4%	$2,495	$2,653	$21,938	
2014	12.8%	$2,776	$3,520	$20,496	
2015	0.6%	$2,754	$3,433	$18,677	
2016	9.7%	$2,980	$3,638	$16,381	
2017	19.3%	$3,516	$3,765	$14,173	
2018	–6.4%	$3,251	$4,726	$10,744	
2019	28.6%	$4,180	$4,316	$6,763	

NOTES: This exhibit shows the evolution of four retirement portfolios fully invested in the S&P 500 over the 1990–2019 period. All portfolios start the 30-year retirement period with $1,000 and are subject to 30 withdrawals of $40 at the beginning of each year. The third column is based on the chronological returns as observed; the fourth column on the same returns, reversed; the fifth column on the same returns, ranked from high to low; and the sixth column on the same returns, ranked from low to high. All returns and dollar amounts in real (inflation-adjusted) terms.

but ignoring the likelihood of being hit by it, is likely to result in suboptimal portfolio decisions.

Examples of sequence risk (such as that in Exhibit 1 in the next section) are a good way to highlight the problem and illustrate its potential consequences. However, such discussions are often misleading because they fail to raise two critical questions. First, how likely is an individual to actually encounter a sequence of returns as detrimental as the one discussed in the example? And second, how likely is the set of returns considered to be sequenced in such a way as to lead to portfolio failure? Both questions are addressed in this article.

One of the problems with sequence risk is that it typically is very loosely defined. Most definitions highlight a sequence of low returns early in the retirement period, but that of course leaves plenty of room for interpretation. After reviewing many of the definitions previously proposed in the literature, a new one is proposed here. The definition proposed links the sequence of returns experienced by an individual, the sustainability of his withdrawal strategy, and the impact on his expected lifestyle in retirement.

Some important issues addressed in this article are often downplayed in the literature. These include a reality check on a typical example of sequence risk, addressing how likely is an individual to be hit by a sequence of returns like the one discussed in the example (spoiler alert: very unlikely); a determination of how early into the retirement period sequence risk may be a concerning issue (spoiler alert: longer than usually suggested); and a consideration of the right tail, meaning scenarios with remarkable outcomes that are just as likely to occur as those emphasized by the sequence risk literature.

Finally, three ways to assess sequence risk are proposed here. The first is an ex-post approach that compares annualized returns over the first few years of a given retirement period to the analogous averages across all the retirement periods considered. The second is another, and superior, ex-post approach that reshuffles the returns of a given retirement period a large number of times in order to determine how likely are those same returns to lead to failure. And finally, an ex-ante approach that periodically evaluates the sustainability of a withdrawal strategy.

This last approach has the advantage that it can be used to determine not only whether sequence risk has been a problem but also whether it is a problem, thus enabling an individual to dynamically adjust his withdrawal strategy. It involves a periodic sustainability test of the withdrawal strategy implemented, and the determination of a new level of sustainable withdrawals when necessary. And it makes it possible to assess not just whether a retiree has been hit by sequence risk but also how much he has been affected by it.

The rest of the article is organized as follows. The second section discusses an example of sequence risk; some definitions previously proposed in the literature as well as a new one proposed here; and existing metrics devised to assess this type of risk. The third section discusses the evidence, including the likelihood of having to deal with sequence risk; the number of years a retiree may have to worry about it; the remarkable outcomes that are just as likely to occur as the harmful ones that result from sequence risk; and three ways to assess this type of risk. Finally, the fourth section concludes with an assessment.

THE ISSUE

This section first introduces sequence risk by way of an example; then reviews different definitions of sequence risk that have been suggested in the literature and advances a new one; and finally discusses some approaches that have been previously proposed to assess sequence risk.

An Example

More often than not, perhaps following from the lack of a precise definition, the issue of sequence risk is introduced with an example that highlights its main features. Exhibit 1 shows in the second column the real (after-inflation) returns of the S&P 500 between 1990 and 2019; during this 30-year period its annualized return was 7.4%. The last four columns show the evolution of four retirement portfolios, all of them: 1) fully invested in stocks, 2) starting with $1,000, and 3) subject to 30 withdrawals of $40 at the beginning of each year. All dollar amounts are expressed in real terms.

The third column (Observed) shows the evolution of a portfolio based on the returns of the S&P 500 as they chronologically happened. As the last row shows, the strategy of withdrawing $40 at the beginning of each of the 30 years in retirement results in a bequest of $4,180, over 4.1 times larger than the initial portfolio.

The fourth column (Reversed) shows the evolution of a portfolio based on the same withdrawal strategy but with the 30 returns in the second column in reverse order. When cash flows (contributions or withdrawals) are involved, the order of the returns does matter, which is confirmed by the fact that the ending value of the portfolio ($4,316) is different from that in the third column. Still, there is a substantial bequest left, over 4.3 times larger than the initial portfolio and larger than in the previous case.

The fifth column (High to Low) shows the evolution of a portfolio based again on the same withdrawal strategy but with the 30 returns in the second column ranked from the highest to the lowest. Early high returns are beneficial for a retiree, which is confirmed by the fact that the bequest in this case is over 6.7 times larger than the initial portfolio.

Finally, the sixth column (Low to High) shows the evolution of a portfolio based again on the same withdrawal strategy but with the 30 returns in the second column ranked from the lowest to the highest. Early low returns, combined with fixed withdrawals, have a devastating impact on the value of the portfolio; by the end of the second (third) year in retirement the portfolio is down by more than 60% (70%). If the retiree continues with the strategy of withdrawing $40 from his portfolio every year, he will run out of money less than 10 years into his retirement. This last case is what sequence risk is all about, namely, an early sequence of low returns that, combined with a fixed withdrawal strategy, leads to portfolio depletion earlier than initially expected.

Importantly, what leads to failure is *the combination* of early low returns and inflexible withdrawals. Portfolio depletion can obviously be avoided if the retiree lowers his withdrawals enough to make it through the end of his retirement. Furthermore, a strategy of withdrawing *x*% of the value of the portfolio at the beginning of each year in retirement would never end in failure, however low the early returns.[1]

Although not the focus of this article, there is a vast literature on how to lessen the impact of sequence risk, such as implementing adaptive withdrawals or derivatives-based strategies. Examples of the former include Jaconetti et al. (2013), Zolt (2014), Kitces and Pfau (2015), and Clare et al. (2017); examples of the latter include Harlow and Brown (2016).

Examples such as that in Exhibit 1 are routinely shown to illustrate the relevance of sequence risk and should not be dismissed. This type of risk is real; investors should be aware of it, and perhaps do something about it. That said, such examples highlight the problem and its potential magnitude but fail to say anything about its probability of occurring; more on this in the "Reality Check" section.

A Definition

It is difficult to assess a problem if it has not been defined precisely. In fact, part of the reason why it is not trivial to assess how much sequence risk affects investors is that, unlike volatility or beta, this type of risk typically is, at best, very loosely defined. Many definitions seem to agree, explicitly or implicitly, on some of the components that need to be present, such as low returns early in the retirement period, but that still leaves ample room for different interpretations.

Basu et al. (2013) define sequence risk first as "the risk of experiencing returns in an unfavorable order" and later as "the worst returns in their worst order." Cotton (2013) acknowledges the different characterizations of sequence risk and defines it as "the observation that two investors who withdraw or invest periodically from their

[1] On the other hand, this strategy would yield a sequence of withdrawals just as volatile as the returns of the portfolio, something that most retirees would not consider desirable.

retirement portfolio can experience the same average stock market returns and have very different outcomes when the order of those returns differs."

Minor (2014) defines sequence risk as having to do with "the (bad) risk of needing to pull money out of a portfolio during a particularly poor performance year and the (good) risk of being able to add money during a down year." Kitces (2016) defines it as "the fact that even if markets average out to long-term returns eventually, if early returns are too low for too long, ongoing withdrawals can deplete the portfolio before the 'good' returns finally arrive." Harlow and Brown (2016) define it more briefly as relating to "the timing or sequence of a series of adverse investment returns."

Raskie (2017) defines sequence risk as "withdrawing regularly from a portfolio when the assets have declined in value," whereas Clare et al. (2017) define it as "the risk of experiencing bad investment outcomes at the 'wrong' time." Burns (2019) defines it as "the risk that unfavorable returns during the first 10 to 15 years of retirement might result in losses from which a retirement portfolio never recovers," and Clare et al. (2020), echoing Basu et al. (2013), define it as "the *worst* possible investment returns in the *worst* possible *order*." (Emphasis in original.)

Note that some of these definitions are somewhat extreme, to the point of being essentially wrong as a matter of course. Defining sequence risk as experiencing the worst returns in the worst order may be catchy but sets up the definition for failure; arguably, returns can always be worse and so can be the order in which they occur.

Similarly, arguing that sequence risk depletes the portfolio before the good returns arrive seems to go one step beyond what is necessary. A retiree that was hit with an early sequence of low returns may still make it through the end of his retirement if he lowers his withdrawals, hence his standard of living, enough. And yet, if in order to avoid failure the retiree has to (say) halve his withdrawals, he can hardly be said not to have been the victim of a bad sequence of returns. In other words, portfolio failure should not be a necessary component of sequence risk.

What are the necessary elements of a proper definition of sequence risk then? First, it is clear that it is a type of risk that affects investors 'early' in their retirement. Second, it is also clear that a sequence of 'low' returns is involved; and although this is typically not specified (see the definitions above), those returns are thought of as being not just low but both large and negative. And third, the low returns early in the retirement period imply such a 'large' decrease in the size of a retirement portfolio that it is put on a path to failure, *unless* the withdrawals that had been planned are 'substantially' decreased.[2]

Given these elements, and the fact that there is some inherent ambiguity about how 'low' returns would have to be or how 'early' they must affect retirees, just as there is ambiguity about the meaning of 'low' volatility or 'high' beta, the following definition of sequence risk is proposed here: *A sequence of low (typically large and negative) returns, early in (typically in the first half of) the retirement period, that leads to portfolio failure unless the withdrawal strategy is reconsidered (typically implying a substantial decrease in withdrawals)*.

The last part is important and often omitted from most definitions, but as already mentioned, it should be clear that portfolio failure can always be avoided by lowering the withdrawals enough. The relevant characteristic of sequence risk is that it derails a financial plan in the sense of making a retiree reconsider the withdrawals he expected, forcing him to decrease them in a substantial way.

[2] Although sequence risk is typically thought of as a relevant issue during the early years in retirement, it is nearly as important toward the end of the working years. At both times individuals have similar reasons for being concerned about the impact of low returns on their portfolios, which is one of the reasons why target-date funds gradually switch their focus from capital accumulation to downside protection as the retirement date approaches.

Assessing Sequence Risk—Previous Approaches

The literature on sequence risk has focused much more on highlighting the problem, and on proposing strategies to deal with it, than on discussing the extent and likelihood of the problem. This focus is unfortunate because, to highlight the obvious, the magnitude of a problem matters and so does the probability that it occurs. Furthermore, this focus has implied that little attention has been devoted to developing ways to assess sequence risk.

Frank and Blanchett (2010) consider a framework in which the probability of failure is estimated periodically and used as an indirect measure of sequence risk. Given the withdrawal rate and the years remaining in the retirement period, they estimate the probability of failure on an annual basis in order to ensure that the withdrawal rate remains prudent throughout the retirement period.

Suarez et al. (2015) define the perfect withdrawal amount, a variable very similar to the maximum withdrawal rate discussed in "The Maximum Withdrawal Rate" section below and propose to assess sequence risk with the inverse of the denominator of this variable; that is,

$$\{(1+R_1)\cdots(1+R_T)+(1+R_2)\cdots(1+R_T)+(1+R_3)\cdots(1+R_T)+\cdots+(1+R_T)\}^{-1}$$

where R_t is the return of the retirement portfolio in period t and T is the length of the retirement period. They argue that, because late returns appear in the denominator more often than early returns, the lower this expression is, the worse the sequence of returns is. Note that if low returns occur early and high returns late (a bad sequence of returns), the denominator will be higher, and the whole expression lower, than if low and high returns occurred in the opposite order. Clare et al. (2017) and Clare et al. (2020) also discuss and support this measure of sequence risk.

Note, however, that the expression above does not really capture sequence risk; rather, it captures whether a sequence of returns is better or worse than another. Two retirees may have obtained a spectacular performance from their portfolios, but if one retiree's sequence of returns was better than the other's, the expression above would yield a higher value for that retiree, although sequence risk was not an issue for either individual. In other words, this metric discriminates between better and worse sequences of returns, but does not really determine whether or not sequence risk has been a problem.

Clare et al. (2020) propose three approaches to measure sequence risk. The first is the 20-year decumulation risk ratio, defined as the ratio between the average perfect withdrawal rate over 20 years and the probability of obtaining a perfect withdrawal rate lower than 5%. The second is a score calculated from the distribution of perfect withdrawal rates, based on the probability of success and the average perfect withdrawal rate failure outcome. And the third, which they consider a more intuitive approach, is to simply rank portfolios by their historical failure rate. As is the case with the measure proposed by Suarez et al. (2015), these three metrics are useful to distinguish between better and worse sequences of returns, but less useful to determine the existence of sequence risk.

Finally, Kitces (2008) emphasizes the correlation between safe withdrawal rates and returns over the first 15 years in retirement, and the correlation between CAPEs (cyclically-adjusted price-earnings ratios) at a given point in time and returns over the subsequent 15 years. Then, given that both correlations are high, he argues that a high CAPE at the beginning of retirement forecasts low (hence a bad sequence of) early returns, which should lead a retiree to be cautious when selecting his initial withdrawal rate. Put differently, he uses (a high) CAPE as an indirect measure of (a bad sequence of) expected returns.

EVIDENCE

This section starts by discussing the data and methodology used in the empirical analysis; then introduces an important variable in the inquiry, the maximum withdrawal rate; then highlights the importance of assessing the likelihood of being a victim of sequence risk, the part of the retirement period that should concern retirees, and the relevance of keeping in mind that distributions have two tails, not just one; and finally discusses three different ways of assessing sequence risk, two ex-post approaches and a dynamic ex-ante approach, as well as an alternative use of the framework proposed.

Data and Methodology

The sample consists of annual returns of the S&P 500 over the 1900–2019 period. All returns are annual, real (adjusted by inflation as measured by the Consumer Price Index), and account for capital gains/losses and dividends. During this 120-year period the S&P 500 delivered an annualized real return of 6.5% with volatility of 20.0%. (In nominal terms, the annualized return and volatility were 9.7% and 19.6%.) Because sequence risk tends to have a more severe impact on more aggressive asset allocations, to fully assess the consequences of exposure to this type of risk the portfolio considered is 100% allocated to stocks.

The results discussed in this section stem from two approaches, a historical perspective and a Monte Carlo simulation. Unless otherwise stated, in both cases the analysis is based on a 30-year retirement period, an initial portfolio of $1,000, an initial withdrawal rate of 4%, and subsequent withdrawals adjusted by inflation. All 30 withdrawals are made at the beginning of each year in retirement. After the last withdrawal, one year before the end of the retirement period, the portfolio compounds for one more year and the end result is the bequest. As usual in most of this literature, transaction costs and taxes are not considered.

The historical analysis is based on rolling 30-year retirement periods between 1900 and 2019; this yields 91 overlapping retirement periods beginning with 1900–1929 and ending with 1990–2019. The Monte Carlo simulation is based on the statistical properties of the distribution of S&P 500 returns over the whole sample period; it generates returns for 10,000 30-year retirement periods with the initial portfolio and withdrawal strategy mentioned in the previous paragraph.[3]

The Maximum Withdrawal Rate

An important variable in the discussion is the maximum withdrawal rate (MWR), which is given by the expression

$$\text{MWR} = \frac{(1+R_1)(1+R_2)\cdots(1+R_T)-(B/P_0)}{(1+R_1)\cdots(1+R_T)+(1+R_2)\cdots(1+R_T)+(1+R_3)\cdots(1+R_T)+\cdots+(1+R_T)} \quad (1)$$

where R_t is the return of the portfolio in period t, T is the number of years in the retirement period, B is the bequest, and P_0 is the portfolio at the beginning of retirement. By definition, the MWR is the highest initial withdrawal rate such that, given the returns and subsequent inflation-adjusted withdrawals, the terminal value of the

[3] A Kolmogorov-Smirnov test does not reject the null hypothesis of normality in the distribution of returns, yielding a p-value of 0.759. The autocorrelation coefficient of the returns series is –0.03 and the null hypothesis that it is not significantly different from 0 has a p-value of 0.726.

portfolio is equal to the intended bequest; see Estrada (2018 *a*,*b*).[4] A few remarks about this variable are in order.

First, the MWR is the proportion of the portfolio withdrawn at the beginning of retirement; hence, it is an *initial* withdrawal rate. Second, the MWR (measured in percent), together with the value of the portfolio at the beginning of retirement (P_0), determine the initial withdrawal W (measured in dollars); that is, $W = \text{MWR}\cdot P_0$. Third, W remains constant, in real terms, throughout the retirement period. Finally, the MWR can only be calculated ex-post, once the returns of the portfolio during the retirement period are known; hence it summarizes the best a retiree could have done *had he known* the future returns of his portfolio.

The MWR is used in retirement planning in at least two ways. First, it can be used to evaluate retirement strategies; this is the case because higher values of this variable lead to higher inflation-adjusted withdrawals, and therefore to a higher standard of living. Second, it can be used to estimate the probability of success or failure of a given withdrawal strategy; this can be done from a historical or simulated distribution of MWRs.[5]

For perspective, note that over the 91 historical retirement periods considered here, the mean and median MWRs are 7.2% and 6.8%, with a minimum of 3.6% (for the 1929–1958 period) and a maximum of 13.0% (for the 1949–1978 period). By definition, given the withdrawal strategy considered in the base case, with an initial withdrawal rate of 4%, any retirement period with a MWR lower than 4% ends in failure. As discussed in more detail later, there are four such failing periods of the 91 considered in this article, for a 4.4% failure rate.

Reality Check

Examples such as that in Exhibit 1 are designed not just to illustrate what sequence risk is about but also to emphasize (or perhaps overemphasize) the importance of the problem. However, consider the following facts hidden behind the figures in the last column of Exhibit 1. First, ordering the returns of the S&P 500 over the 1990–2019 period from low to high leads to annualized real returns over the first 3, 5, and 10 years of –25.4%, –19.7%, and –11.0%. However, historically, no 30-year retirement period has *ever* started with such low returns.

Exhibit 2 shows in the third row (Observed) the worst two beginnings of a portfolio fully invested in stocks over the 120-year period between 1900 and 2019, measured by the annualized real return over the first 3, 5, and 10 years. It also shows in the fourth row (Low to High) the annualized real returns over the first 3, 5, and 10 years for the 30-year period between 1990 and 2019 when returns are ranked from low to high, as is done in the last column of Exhibit 1. Finally, the last two rows show the probability of observing returns equal to or lower than those in the fourth row, based on the distributions of annualized 3-year, 5-year, and 10-year historical returns over the 1900–2019 period (P_H) and based on 10,000 scenarios from the Monte Carlo simulation (P_S).

As the exhibit clearly shows, no historical three-year period ever had annualized returns as low as –25.4%; the worst two three-year periods were 1929–1931 (–23.5%) and 1930–1932 (–20.5%). Similarly, no historical five-year period ever had close to

[4] Blanchett et al. (2012) derive an expression similar to (1) and call it the *sustainable spending rate*. Suarez et al. (2015) and Clare et al. (2017) do the same and call it the *perfect withdrawal rate*. Miller (2016) discusses this metric without a formal framework and calls it the *maximum withdrawal rate*.

[5] Most of the results discussed in this article using the MWR are consistent with those that would be obtained using the coverage ratio, a different performance metric introduced by Estrada and Kritzman (2019), which seeks to overcome the most serious shortcomings of the failure rate.

EXHIBIT 2
Reality Check

	3 Years		5 Years		10 Years	
Period	1929–1931	1930–1932	1916–1920	1937–1941	1911–1920	1965–1974
Observed	–23.5%	–20.5%	–12.1%	–9.3%	–4.4%	–3.8%
Low to High	–25.4%		–19.7%		–11.0%	
P_H	0.16%		0.10%		0.10%	
P_S	0.31%		0.24%		0.38%	

NOTES: This exhibit shows in the third row (Observed) the two lowest annualized returns observed over the first 3, 5, and 10 years of a portfolio fully invested in the S&P 500 over 1900–2019 period. The fourth row (Low to High) shows the annualized return over the first 3, 5, and 10 years of the S&P 500 over the 1990–2019 period when returns are ranked from low to high. The last two rows show the probability of observing returns equal to or lower than those in the fourth row, based on the distributions of annualized 3-year, 5-year, and 10-year historical returns of the S&P 500 over the 1900–2019 period (P_H) and based on 10,000 scenarios from the Monte Carlo simulation (P_S). All returns in real (inflation-adjusted) terms.

annualized returns as low as –19.7%; the worst two five-year periods were 1916–1920 (–12.1%) and 1937–1941 (–9.3%). Finally, no historical 10-year period ever had even close to annualized returns as low as –11.0%; the worst two 10-year periods were 1911–1920 (–4.4%) and 1965–1974 (–3.8%).[6]

Interestingly, of the 30-year retirement periods that started in the first year of the six periods mentioned in the previous paragraph (1911, 1916, 1929, 1930, 1937, and 1965), only the one beginning in 1929 led to portfolio failure based on the withdrawal strategy considered here; the other five periods left an average bequest of $602, or just over 60% of the size of the initial retirement portfolio, in real terms.

Needless to say, the fact that a peculiar sequence, with such low returns over 3, 5, and 10 years has never happened does not necessarily imply that it may not happen in the future, which leads to two additional perspectives. First, based on the distributions of annualized 3-year, 5-year, and 10-year historical returns of the S&P 500 over the 1900–2019 period, the fifth row of Exhibit 2 shows that the probability of observing annualized returns equal to or lower than those in the fourth row of the same exhibit is negligible, namely, 0.16%, 0.10%, and 0.10%. Furthermore, based on 10,000 scenarios of the Monte Carlo simulation, the last row of Exhibit 2 shows the probability of observing annualized returns equal to or lower than those in the fourth row of the same exhibit; these probabilities (0.31%, 0.24%, and 0.38%) are nearly as negligible as those in the fifth row.

In short, a peculiar sequence of returns such as that considered in the last column of Exhibit 1 may lead to shocking results, but a reality check should substantially mitigate the unaware investor's concerns. Rational individuals consider both the impact of an event and the probability of its occurrence; hence advisors that highlight the dangers of sequence risk but do not discuss its probability of occurring are not really helping retirees. Obviously, before deciding what (if anything) to do about it, retirees should consider both the impact *and the probability* of being hit by a sequence of low returns.

To summarize, Exhibit 1 or similar examples are used to illustrate, and perhaps scare investors about, sequence risk. Again, this risk is real, and investors should be aware of it. However, the message from Exhibit 2 is just as important as that from Exhibit 1; that is, most extreme examples used to illustrate sequence risk have never happened and have a negligible probability of ever happening. Should advisors

[6]The 1965–1974 period's annualized return was –3.83%, just marginally lower than the –3.81% annualized return over the 1999–2008 period.

EXHIBIT 3

What Does 'Early' Mean?

	3	27	5	25	10	20	15	15	20	10	30
Historical											
Rho	0.60	0.14	0.73	–0.05	0.85	–0.33	0.91	–0.54	0.80	–0.51	0.54
p-value	0.00	0.19	0.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Monte Carlo											
Rho	0.63	0.60	0.76	0.51	0.88	0.32	0.90	0.18	0.87	0.10	0.77
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

NOTES: This exhibit shows correlations (Rho), across all the historical and simulated retirement periods considered, between each period's maximum withdrawal rate (MWR) and annualized returns over the first *N* years and the subsequent *M* years, for *N* = 3, 5, 10, 15, 20, and 30; and for *M* = 27, 25, 20, 15, 10, and 0. It also shows the *p*-values for the null hypothesis that each correlation coefficient is equal to 0.

highlight only the magnitude of a potential problem without any reference to its probability of occurring? Should retirees focus just on the magnitude of sequence risk and ignore the probability of having to deal with it? (More on this later.)

'Early' Is Not That Early

Most definitions of sequence risk agree that the sequence of low returns occurs 'early' in the retirement period, but what does 'early' really mean? To explore this issue, Exhibit 3 shows correlations, across all the historical and simulated retirement periods considered, between each retirement period's MWR and the annualized returns over two subperiods, the first *N* years and the subsequent *M* years. To illustrate, the second and third columns show the correlation between MWRs and the annualized returns over the first 3 years of retirement, and the correlation between the same MWRs and the annualized returns over the subsequent 27 years.

The top half of the exhibit focuses on the 91 historical retirement periods considered and highlights two important results. First, the correlations for the first part of the retirement periods are, in all cases, higher than those for the second part of the periods; to illustrate, the correlation between MWRs and the annualized return over the first 3 years of retirement (0.60) is higher than that between MWRs and the annualized return over the subsequent 27 years (0.14). In other words, the degree of success or failure of a withdrawal strategy, as measured by its MWR, is more tightly linked to the returns over the first 3 years than to those of the remaining 27 years. And the same is the case for all the first parts and the second parts of the periods considered.

Second, of all the correlations displayed in the top half, the highest is that between MWRs and the annualized return over the first 15 years of retirement (0.91), with the second highest being that between MWRs and the annualized return over the first 10 years of retirement (0.85). This shows that the bad sequence that really matters is not just over the first 'few' years, as often implied in discussions of sequence risk, but rather over a fairly longer period of time. In other words, if a retiree is concerned with sequence risk, he should worry not just over the first three to five years of retirement but over a substantially longer period of time.

The bottom half of Exhibit 3 focuses on 10,000 scenarios of the Monte Carlo simulation and confirms and strengthens the previous two results, namely, the first part of a retirement period matters more than the second, and the period that matters the

EXHIBIT 4
Two Tails, Not Just One

	Panel A: Historical		Panel B: Monte Carlo	
Percentile	B	MWR	B	MWR
P1	N/A	3.6%	N/A	1.7%
P5	$67	4.1%	N/A	2.8%
P10	$202	4.2%	N/A	3.5%
P90	$7,021	10.7%	$15,569	11.8%
P95	$8,092	11.2%	$23,988	13.4%
P99	$10,860	13.0%	$50,691	16.4%

NOTES: This exhibit shows some percentiles (P) for the distributions of the bequest (*B*) and the maximum withdrawal rate (MWR). Panel A is based on the 91 historical retirement periods between 1900–1929 and 2000–2019, and panel B on 10,000 scenarios from the Monte Carlo simulation. All returns and dollar amounts in real (inflation-adjusted) terms.

most is the first 15 years.[7] The relevance of the first 15 years is in fact consistent with results discussed by Kitces (2008), as well as with Burns' (2019) definition of sequence risk discussed earlier.

The fact that the most relevant period, the one over which retirees should be concerned, is fairly longer than often hinted or discussed has at least two important implications. First, if retirees consider implementing some strategy to mitigate the impact of sequence risk, the strategy should be applied over a fairly long period, roughly over the first half of the standard 30-year retirement period. In other words, protecting the portfolio over the first three to five years would be not nearly enough. Second, and following from the first, whatever the cost of the mitigating strategy, it will be borne not just over the first few years of retirement but over a much longer period of time.

Distributions Have Two Tails

A rational individual offered a gamble with 50-50 odds and an outcome of –$100 if the coin lands tails would not reject it right away; obviously, he would ask what he stands to gain if the coin lands heads. Similarly, an individual assessing the potential impact of sequence risk should consider the impact and likelihood of experiencing a bad sequence *or a good sequence* of returns. In that spirit, Exhibit 4 shows some percentiles of the distributions of the bequest (the value of the portfolio at the end of the retirement period) and the MWR, across the 91 historical retirement periods and the 10,000 simulated retirement periods considered.

Panel A shows some percentiles for the distributions of the bequest and the MWR, both based on the 91 historical retirement periods considered. These figures show that, if history is any guide, there is 5% (10%) probability of leaving a bequest lower than $67 ($202). These figures complement the 4.4% failure rate already mentioned, which indicates the probability of leaving no bequest. Although this historical record would most likely not deter retirees from implementing the withdrawal strategy considered here, it is only part of the story a sensible retiree should consider; there is another tail that cannot (should not) be ignored.

Panel A suggests that the probability of leaving a bequest smaller than $67 ($202) is the same as that of leaving a bequest larger than $8,092 ($7,021), which amounts to more than eight (seven) times the size of the initial retirement portfolio, in real terms. Should retirees focus only on the probability of leaving a very small bequest and ignore *the same probability* of leaving a remarkable bequest? Should retirees focus on the probability of facing MWRs lower than 4.2% and ignore *the same probability* of facing MWRs higher than 10.7%? More generally, should retirees focus only on their exposure to sequences of bad returns that may lead to failure or a small bequest and ignore their exposure to sequences of good returns that may lead to a dream retirement?

Panel B complements Panel A by showing some percentiles for the distributions of the bequest and the MWR, in this case based on 10,000 scenarios of the Monte Carlo simulation. As is often the case, the simulation yields a distribution with longer

[7]As is the case in the historical analysis, the second highest correlation is for the first 10 years. In both analyses, historical and simulated retirement periods, the third highest correlation is for the first 20 years.

tails than the historical record. This means that the probability of leaving no bequest is higher than in Panel A (at least 10% according to the figures in the exhibit) but so is the probability of leaving a very large bequest.[8] In this case, there is a 10% (5%) probability of leaving a bequest more than 15 times (almost 24 times) larger than the initial retirement portfolio; and a 10% (5%) probability of facing a MWR almost three (more than three) times higher than the 4% considered in the base case of the financial plan.

Kitces (2019) makes a similar analysis and highlights that it is important to notice that sequence risk cuts both ways; that is, a bad sequence creates the risk of depleting a portfolio too early, but a good sequence increases a retiree's chances of improving his standard of living substantially. In fact, even assuming that individuals dislike losses twice as much as they like gains of the same magnitude (Kahneman and Tversky 1979), it would be very difficult to justify focusing on potential failures or very low bequests and ignoring the equally likely possibility of having a dream retirement.

Assessing Sequence Risk Ex-Post

Discussions of sequence risk typically focus on the impact of an early sequence of low returns, but is it really (or just) the returns that lead to portfolio failure? Note that given the withdrawal strategy considered here, a portfolio that earns a 0% real return *in every period* would sustain withdrawals for 25 years, falling 5 years short of the 30-year goal. But importantly, this strategy would not fail because of an early sequence of low returns; it would fail simply because the withdrawals are too high relative to the returns obtained.

To elaborate, note that given the 91 retirement periods considered, a strategy with an initial withdrawal rate of 4% and subsequent withdrawals adjusted by inflation (the base case considered here) would have failed four times, for a failure rate of 4.4%. For an initial withdrawal rate of 6.9%, the failure rate would have been 50.5%. And for an initial withdrawal rate of 13%, every retirement period would have ended in failure. In other words, all else equal, higher withdrawals lead to higher failure rates.

Consider this issue from a different perspective. Exhibit 5 shows in its first column the four historical retirement periods, out of the 91 considered, in which the withdrawal strategy considered here failed; it also shows, in its second column, each period's MWR.[9] The third to fifth columns show the annualized returns of these four historical periods over their first 3, 5, and 10 years; and the sixth column shows the annualized return over the entire 30-year periods. The last row of the exhibit shows averages across all the historical retirement periods considered.

It is clear from the exhibit that the annualized return over the first 3, 5, and 10 years of these four periods was far lower than the historical averages across all periods; to illustrate, the 1929–1958 period had annualized returns of –23.5%, –7.2%, and 0.7% over its first 3, 5, and 10 years, compared with historical averages of 6.9%, 6.6%, and 6.3%. Furthermore, note that the 30-year annualized return of these four periods was not substantially lower than that across all historical periods, which suggests that overall, these were not retirement periods of particularly low returns; to illustrate, the 6.3% annualized return over the 1929–1958 period was not substantially lower than that across all 30-year periods (6.7%). Both of these facts suggest that these four periods ended in failure because of their early sequences of low returns.

[8] The actual failure rate in the simulation is 14.1%, substantially higher than the 4.4% failure rate observed historically. On the other hand, the median bequest in the historical and simulated retirement periods are fairly similar, at $2,868 and $2,735.

[9] By definition, the MWR of a failing period is lower than the initial withdrawal rate, which explains why the four MWRs reported are all lower than 4%.

EXHIBIT 5
Portfolio Failures

	Panel A: Historical					Panel B: Reshuffling	
Period	**MWR**	**3Y**	**5Y**	**10Y**	**30Y**	**Failures**	**Rate**
1929–1958	3.6%	–23.5%	–7.2%	0.7%	6.3%	880	8.8%
1966–1995	3.8%	3.5%	–1.2%	–2.4%	5.0%	612	6.1%
1968–1997	3.9%	–3.5%	2.8%	–2.6%	6.4%	195	2.0%
1969–1998	3.8%	–2.0%	–3.3%	–3.4%	7.0%	121	1.2%
Hist Avg	7.2%	6.9%	6.6%	6.3%	6.7%	N/A	N/A

NOTES: This exhibit focuses on the four retirement periods in which the withdrawal strategy considered failed; Panel A focuses on the historical record and Panel B on 10,000 reshufflings of the 30 returns of each failing period. The first column shows the failing periods; the second column their maximum withdrawal rate (MWR); and the next four columns the annualized return over the first 3 (3Y), 5 (5Y), 10 (10Y), and 30 (30Y) years. Panel B shows in its first column the number of failures and in its second column the failure rate. All returns in real (inflation-adjusted) terms.

That said, it is important to also notice that with an initial withdrawal rate of 3.5%, none of these periods would have ended in failure, although *the returns experienced by the retirees would have been exactly the same*. In other words, it is not *only* the sequence of returns that determines whether a strategy succeeds or fails; it is the sequence of returns *and* the withdrawal strategy implemented. As already mentioned, withdrawing an *x*% of the portfolio at the beginning of each year in retirement would never end in failure, however low the early returns.

In the same way that a retiree's concern with sequence risk should increase with the aggressiveness of his asset allocation, it should also increase with the size of his withdrawals. In fact, even if a retiree has a fixed withdrawal strategy in inflation-adjusted terms, the strategy can always succeed if the initial withdrawal rate is low enough. For this reason, it is critical to discuss sequence risk *given* the intended withdrawal strategy. It is not just the sequence of returns that matters for success or failure; it is *both* the sequence of returns and the intended withdrawals.

Assessing Sequence Risk Ex-Post—A Better Way

One way to assess whether sequence risk was an issue over a given retirement period, as suggested in the previous section, is to compare the annualized returns over the initial (say, 3, 5, and 10) years of a retirement period to the analogous returns averaged across all retirement periods. The lower the former are relative to the latter, the more likely is sequence risk to have played a role in a failure. However, the problem with this approach is that it focuses on the returns and ignores the withdrawal strategy; as already discussed, the same sequence of early returns may lead to success or failure depending on the intended withdrawals.

A better way to determine the importance of a specific sequence of returns, *for a given retirement period and withdrawal strategy*, is to assess the impact of different rearrangements of the same returns. More precisely, this process involves first taking the 30 returns of a failing period and reshuffling them in order to create a large number of alternative histories or scenarios *of the same 30 returns*; then tracking the evolution of a portfolio in each scenario; and finally determining how often the withdrawal strategy failed across all the scenarios considered. The higher the proportion of failures, the more likely it is that the strategy failed because the withdrawals were too high relative to the returns obtained; on the contrary, the lower the proportion of failures, the more likely it is that the strategy failed because of a very peculiar sequence of returns, unlikely to have occurred.[10]

Panel B of Exhibit 5 shows, for the four failing retirement periods in the exhibit, the results of reshuffling 10,000 times the 30 returns of each period. As the figures show, reshuffling the 30 returns of the 1929–1958 period led to failure 8.8% of the time; for the other three periods, the failure rates were 6.1%, 2.0%, and 1.2%. These figures show that the retirees of each of these four failing periods were victims of very unlikely sequences of returns; most other rearrangements of the same 30 returns these retirees were exposed to would have not ended in failure. Should an investor implement a very conservative asset allocation, or pay a substantial cost to protect his portfolio, given this likelihood of being a victim of sequence risk?

To summarize, there is no question that an early sequence of low returns may derail a retirement plan, a prospect that scares many retirees, as it probably should. But the impact of this type of risk must be considered together with its probability of occurring. The evidence here shows that the withdrawal strategy considered has failed less than 5% of the time. Furthermore, and more importantly, the evidence also shows that the specific returns observed in those failing periods were unlikely to have led to failure, with the probability ranging between just over 1% and just under 9%. At the very least, these two facts should be considered together with the potential impact of sequence risk when deciding what (if anything) to do about it.

Assessing Sequence Risk Ex-Ante

The previous two sections discuss, among other things, two ways of assessing whether a retiree was a victim of sequence risk. One way is by comparing the annualized returns experienced by the retiree in the first few (say, 3, 5, and 10) years of retirement relative to the analogous averages over all historical retirement periods; the lower the returns experienced relative to the broader averages, the more likely is sequence risk to have harmed the retiree. A problem with this approach is that early returns that may lead to failure for one withdrawal strategy may lead to success for another, which highlights an important insight: It is not just the sequence of returns that matters; both the sequence of returns and the withdrawal strategy play a role in a strategy's failure.

The other way to assess sequence risk is to reshuffle the returns experienced by a retiree a large number of times to determine how often the same (rearranged) returns would have led to failure. The lower the proportion of times those returns led to failure, the more likely that sequence risk was the culprit of the observed failure. The advantage of this approach over the previous one is that it fixes both the returns experienced and the withdrawal strategy considered, thus truly isolating the impact of the sequence of those returns.

These two approaches are a clear improvement over simplistic arguments that, instead of assessing sequence risk in a comprehensive way, simply highlight its existence and dangers and address neither its likelihood nor the impact of other variables that may lead to portfolio failure. That said, these two approaches share a common shortcoming, namely, they can only be applied ex-post, after observing all the returns of a given retirement period. Put differently, ex-post approaches are useful to assess whether sequence risk *has been* a problem, but they do not help retirees to introduce dynamic adjustments to their withdrawal strategy once they have been hit by sequence risk.

[10] Note that because the 30 returns are the same across all scenarios, so is the annualized return over the whole period. Furthermore, given that the withdrawal strategy is also fixed, the only difference across scenarios comes from the sequence of returns.

EXHIBIT 6
Sustainability Test

	Panel A: 1928–1957			Panel B: 1929–1958			
Period	Return	PBW	Test	Return	PBW	Test	Check
0		$1,000	N/A		$1,000	N/A	N/A
1	45.0%	$1,392	$5,198	–9.4%	$869	$1,969	N/A
2	–9.4%	$1,224	$3,949	–20.2%	$662	$687	N/A
3	–20.2%	$946	$2,230	–38.1%	$385	–$825	$357
4	–38.1%	$561	$164	1.6%			$351
5	1.6%	$529	$41	51.7%			$345
6	51.7%	$742	$1,040	–3.8%			$339
7	–3.8%	$675	$734	42.9%			$332
8	42.9%	$908	$1,656	30.9%			$325
9	30.9%	$1,136	$2,448	–37.1%			$317
10	–37.1%	$690	$772	37.0%			$309
11	37.0%	$890	$1,427	–0.9%			$300
12	–0.9%	$843	$1,232	–10.7%			$291
13	–10.7%	$717	$830	–19.7%			$281
14	–19.7%	$543	$346	11.0%			$270
15	11.0%	$559	$405	22.1%			$259
16	22.1%	$634	$600	17.0%			$247
17	17.0%	$694	$742	33.5%			$235
18	33.5%	$873	$1,117	–22.3%			$221
19	–22.3%	$648	$639	–3.3%			$207
20	–3.3%	$588	$527	2.0%			$191
21	2.0%	$559	$484	20.6%			$175
22	20.6%	$626	$605	23.3%			$158
23	23.3%	$722	$758	17.5%			$139
24	17.5%	$801	$867	17.6%			$119
25	17.6%	$895	$983	–1.8%			$98
26	–1.8%	$840	$892	53.5%			$76
27	53.5%	$1,228	$1,346	30.9%			$52
28	30.9%	$1,555	$1,676	3.5%			$27
29	3.5%	$1,569	$1,628	–13.4%			$0
30	–13.4%	$1,325	$1,325	40.9%			$0

NOTES: This exhibit shows, over two historical retirement periods, the annual return, the value of a portfolio before withdrawals (PBW), the sustainability test (Test) given by expression (2), and a sustainability check (Check) that confirms that the calculated sustainable withdrawal depletes the portfolio by the end of the retirement period. All returns and dollar amounts in real (inflation-adjusted) terms.

There is, however, a third option with several attractive features, including the fact that it is an ex-ante approach that does not require knowledge of all the returns over a retirement period. This approach is consistent with the definition proposed earlier, and in particular with the sustainability of the withdrawal strategy considered. Importantly, it makes it possible to assess not just whether a retiree has been the victim of sequence risk but also how much he has been affected by it. And finally, it enables a retiree who has been hit by a sequence of low returns to introduce dynamic adjustments to his withdrawal strategy in order to avoid portfolio failure.

In a nutshell, the process involves asking, at the beginning of each year in retirement, if the withdrawal strategy implemented is sustainable given the *expected* return of the portfolio; or, put differently, whether a portfolio that delivers its expected return every year is able to sustain the inflation-adjusted withdrawals contemplated in a financial plan. Exhibit 6 provides an illustration with a good (1928–1957) and a bad (1929–1958) retirement period.

Consider the 1928–1957 period in Panel A first. Consistent with the base case scenario discussed so far, the individual retires with $1,000 and immediately takes 4% of his portfolio to spend during his first year in retirement. The $960 at the beginning of period 1 compound at 45.0% during the first year in retirement, which implies that the value of the portfolio before taking the second annual withdrawal is the $1,392 shown in the PBW (portfolio before withdrawal) column.

At this point the retiree tests the sustainability of his strategy. More precisely, he asks whether withdrawing $40 every year, and obtaining a 6.5% return every year, will enable him to make it through the end of his retirement period.[11] In order to answer this question the retiree calculates his expected bequest, which at any time t can be done with the expression

$$B_t = P_t \cdot (1+R)^{T-t} + W \cdot (1+R) \cdot \left(\frac{1-(1+R)^{T-t}}{R} \right) \tag{2}$$

where B is the bequest, or terminal value of the portfolio; P is the value of the portfolio at the time the test is run; R is the expected return of the portfolio; W is the annual withdrawal; and T is the length of the retirement period.[12] To illustrate, at the end of his first year in retirement, an individual that from that point on plans to withdraw $40 every year, and expects to obtain a return of 6.5% every year, would expect to leave a bequest equal to

$$\$1,392 \cdot (1+0.065)^{29} + \$40 \cdot (1+0.065) \cdot \left(\frac{1-(1+0.065)^{29}}{0.065} \right) = \$5,198$$

which is the first figure in the 'Test' column. In words, at the end of his first year in retirement the individual's withdrawal strategy is sustainable in the sense that the portfolio is not *expected* to be depleted before the end of the retirement period. Formally, the withdrawal strategy is sustainable as long as $B_t \geq 0$.

Given this result, at the beginning of his second year in retirement the individual withdraws $40 to spend over the coming year and leaves his portfolio with $1,352. The rest of the columns in Panel A show that every year the retiree runs the same test, finds that his withdrawal strategy is sustainable ($B_t \geq 0$), and therefore makes all the $40 annual withdrawals as initially planned. In other words, sequence risk has not affected this retiree.

Consider now the 1929–1958 period in Panel B. As before, the individual retires with $1,000 and takes 4% of that amount right away, but now his remaining $960 decrease by 9.4% during his first year in retirement, leaving him with $869 before taking the second withdrawal. The retiree then tests the sustainability of his strategy, finds that it is sustainable (B_1 = $1,969 > 0), and withdraws $40 to spend over the coming year.[13]

The following year the retiree goes over the same process, finds that his strategy remains sustainable (B_2 = $687 > 0), and withdraws another $40. But, one year

[11] In this example the expected return of the portfolio is the same as the asset allocation's historical annualized (real) return. Needless to say, this figure could be replaced by any other the retiree or his advisor believes is the more likely annualized return of the portfolio over the rest of the retirement period.

[12] This expression assumes that the expected return of the portfolio is constant over time. This assumption could easily be replaced by a sequence of expected returns (for example, from a Monte Carlo simulation) at the cost of making the expression more cumbersome.

[13] In both panels the sustainability test at time 0 is omitted because it simply tests the sustainability of 30 withdrawals of $40, yielding a figure that would be the same ($2,914) for all retirement periods. Only after the first year in retirement the test starts producing different figures for different retirement periods.

after that, at the beginning of his third year in retirement, after his portfolio went down 38.1%, the retiree finds that his withdrawal strategy is no longer sustainable ($B_3 = -\$825 < 0$). In words, if the retiree earns an annual return of 6.5%, his current portfolio is not large enough to sustain $40 annual withdrawals through the end of the retirement period. This retiree has been hit by sequence risk.

Needless to say, not all bad sequences of return are equally bad, which begs the question how severely this retiree has been affected by sequence risk. This can be determined by finding the new level of inflation-adjusted withdrawals that would make the retiree's strategy sustainable, which can be obtained from the expression

$$W_t = \frac{B - P_t \cdot (1+R)^{T-t}}{(1+R) \cdot \left(\frac{1-(1+R)^{T-t}}{R} \right)} \tag{3}$$

given any target bequest B, the portfolio's size at time t, and the portfolio's expected return. To illustrate, having determined at the beginning of his third year in retirement that the strategy of withdrawing $40 every year is no longer sustainable, and assuming he intends to leave no bequest ($B = 0$), the retiree can calculate that the new sustainable level of withdrawals is

$$\frac{-\$385 \cdot (1+0.065)^{27}}{(1+0.065) \cdot \left(\frac{1-(1+0.065)^{27}}{0.065} \right)} = \$28.7$$

thus having to reduce his standard of living by over 28% with respect to the previous $40.

The last column of Panel B (Check) simply shows that the new level of withdrawals is in fact sustainable. If $28.7 are withdrawn at the beginning of the third year in retirement from the $385 portfolio, and every year after that the portfolio grows at 6.5% and $28.7 are withdrawn, by the end of the retirement period the portfolio is (by design) left with $0.

Drawing a Tentative Line

The ex-ante approach proposed here has several advantages over others previously proposed in the literature. First, it does not require knowledge of all the returns of a given retirement period, which implies that it can be used to assess whether sequence risk has been *or is* a problem. This feature, in turn, enables a retiree that has experienced an adverse sequence of returns to incorporate dynamic adjustments to his withdrawal strategy in order to avoid early depletion of his retirement portfolio.

Second, this ex-ante approach is consistent with the definition of sequence risk proposed earlier, and in particular with the sustainability of a withdrawal strategy, which is essential to assess the impact of this type of risk. And third, it makes it possible to assess not just whether a retiree has been (or is) a victim of sequence risk but also *by how much* he has been (or is) affected by it. This third point is elaborated further in Exhibit 7.

The first and fifth columns of Exhibit 7 show the 22 historical retirement periods in which the withdrawal strategy considered here became unsustainable; in the other 69 historical periods considered the strategy was deemed sustainable every year. The second and sixth columns show the year of each retirement period in which the strategy became unsustainable. The third and seventh columns show the revised, sustainable level of inflation-adjusted withdrawals, as calculated from Equation 3.

EXHIBIT 7
Withdrawal Adjustments

Period	Year	W	Decrease	Period	Year	W	Decrease
1903–1932	18	$39.5	–1.4%	1973–2002	2	$34.3	–14.3%
1972–2001	3	$38.7	–3.2%	1968–1997	7	$33.4	–16.4%
1909–1938	9	$38.7	–3.3%	1930–1959	2	$33.1	–17.3%
1970–1999	12	$37.8	–5.4%	1966–1995	9	$32.7	–18.3%
1907–1936	11	$37.5	–6.1%	1969–1998	6	$32.1	–19.9%
1914–1943	7	$37.5	–6.3%	1917–1946	4	$32.0	–19.9%
1910–1939	8	$37.4	–6.6%	1911–1940	10	$31.5	–21.2%
1906–1935	12	$37.2	–7.1%	1912–1941	9	$31.1	–22.3%
1965–1994	10	$36.5	–8.7%	1913–1942	8	$30.1	–24.6%
1937–1966	5	$35.3	–11.7%	1916–1945	5	$29.6	–26.1%
1905–1934	16	$35.1	–12.3%	1929–1958	3	$28.7	–28.2%

NOTES: This exhibit shows all the periods in which a sustainability test determined a reduction in withdrawals, including how many years into the retirement period the sustainability test failed (Year), the revised sustainable level of withdrawals (*W*), and the decrease relative to the original $40 withdrawal (Decrease). All returns and dollar amounts in real (inflation-adjusted) terms.

And the fourth and last columns show the decrease in withdrawals relative to the $40 originally planned.

The definition proposed here links sequence risk to a reassessment of the original withdrawal strategy, and in particular to a 'substantial' decrease in withdrawals. The reason this last part is relevant is that if a sequence of returns rendered a strategy unsustainable, and the new sustainable strategy implied a reduction in withdrawals of (say) 1%, then it could hardly be argued that sequence risk has affected the retiree in a significant way. To be sure, any definition of 'substantial' is arbitrary and open to discussion, but in order to draw a line, however tentative, think of it as a decrease in inflation-adjusted withdrawals (hence in purchasing power) larger than 10%–15%.

Exhibit 7 shows that of the 22 times the withdrawal strategy considered became unsustainable, in 9 (12) occasions the new sustainable level of withdrawals implied a decrease of less than 10% (15%). In other words, and based on the tentative line suggested above, sequence risk has not been a critical problem for retirees in those periods. Of the rest of the retirement periods in the exhibit, in five the reduction in withdrawals was 15%–20%, and in the other five the reduction was larger than 20%.[14] In 10 to 13 cases, then, it could be plausibly argued that sequence risk did have a considerable impact on the standard of living of retirees in those periods.

These figures lead to yet another perspective that suggests that sequence risk is a serious problem but one that is rather unlikely to affect retirees: In only 10 to 13 of the 91 historical periods considered (11% to 14%) sequence risk has forced retirees to a substantial downward adjustment in their standard of living. Importantly, this figure does not account for the fact that, later in these retirement periods, portfolio returns may have been such that they enabled a retiree to eventually increase his withdrawals. (More on this in the next section.)

In the Monte Carlo simulation that complements the historical analysis, the withdrawal strategy considered becomes unsustainable in 2,209 of the 10,000 scenarios. In more than half of those scenarios (1,164), the return to sustainability implied a reduction in withdrawals of less than 10%; of the 1,045 scenarios (10.5% of the

[14] Four of the five retirement periods in which the reduction in withdrawals was larger than 20% are tightly concentrated, all ending between 1940 and 1945.

total) in which the reduction in withdrawals was larger than 10%, in less than half of those (696, or 7% of the total) the reduction in withdrawals was larger than 15%. In short, and based again on the tentative line suggested above, sequence risk was a problem in only 7% to 10.5% of the simulated scenarios considered.

Needless to say, the figures underlying this discussion are relevant for the US, which is the focus of this article. Estrada (2021) discusses the global evidence on the sustainability of withdrawal strategies. He finds that withdrawal strategies generally fail more often, and become unsustainable more often, in most other countries than they do in the US. To illustrate, across 20 countries (excluding the US) over the 1900–2019 period, a strategy based on a 4% initial withdrawal rate and a 40–60 asset allocation failed (became unsustainable) on average 40.1% (62.2%) of the time; the corresponding figures for the US are 14.3% and 38.5%.

An Extension

The ex-ante approach advanced here can be used, as done in Exhibit 6, to test the sustainability of a withdrawal strategy. Panel A of that exhibit shows that as long as the test proposed determines that a strategy is sustainable, the retiree sticks to his withdrawal plan, which in our case implies withdrawing $40 every year. On the contrary, if the test deems a strategy to be unsustainable, as in Panel B of the same exhibit, the retiree calculates the new, sustainable level of withdrawals, and sticks to those new withdrawals as long as they remain sustainable.

The reason for implementing the methodology proposed in this way is that the main goal of this article is assessing sequence risk, not finding an ideal withdrawal strategy. That said, Expression (3) can also be used in a dynamic way in order to determine, year after year, the highest level of sustainable withdrawals, given the size and expected return of the portfolio (and the intended bequest). Exhibit 8 shows an example for the 1990–2019 retirement period.

Panel A of Exhibit 8 shows the value of a retirement portfolio before the withdrawals are made (PBW), the sustainability test proposed (Test), and the withdrawal made each year (*W*). As in Exhibit 6, this panel shows that as long as the intended strategy remains sustainable, the retiree sticks to his plan of withdrawing $40 every year, which is the main use of the methodology proposed, given the goal of this article.

Panel B of Exhibit 8 shows an alternative use for Equation 3, which is to determine every year the sustainable level of withdrawals, given the portfolio's size and expected return at each point in time. To illustrate, the value in the last column for the year 2005 ($83.8) shows that given an $805 portfolio, 15 years until the end of the retirement period, a 6.5% return every year, and an intended bequest of $0, the retiree can withdraw $83.8 every year.[15] Note that, on the one hand, this figure is substantially higher than the $40 in Panel A; but, on the other hand, the bequest in Panel A is $4,180 whereas that in Panel B is (by design) $0.

This alternative use of Equation 3 enables a retiree to spend as much as is sustainable at each point in time. However, note that in this case the annual withdrawals are very volatile, with a minimum of $50 (in 2008) and a maximum of $152.9 (in 1999), hardly a fluctuation in purchasing power a retiree would be willing to tolerate. This volatility in withdrawals is given by the volatility of the portfolio returns, which in turn is increasing in the aggressiveness of the portfolio's asset allocation.

[15] As mentioned before, 6.5% is the annualized return of a portfolio fully invested in stocks over the 1900–2019 period, and an estimate of the portfolio's expected return in the example. This figure can be easily replaced by any other estimate that a retiree or his advisor considers more plausible. Similarly, an intended bequest of $0 is an arbitrary assumption that can be easily replaced by any other target desired by a retiree.

EXHIBIT 8
Two Withdrawal Strategies

		Panel A			Panel B	
Year	Return	PBW	Test	W	PBW	W
1989		$1,000	$2,914	$40.0	$1,000	$71.8
1990	–8.7%	$877	$2,014	$40.0	$848	$61.6
1991	26.6%	$1,059	$2,991	$40.0	$995	$73.2
1992	4.6%	$1,066	$2,885	$40.0	$964	$71.9
1993	7.1%	$1,099	$2,920	$40.0	$956	$72.3
1994	–1.3%	$1,045	$2,522	$40.0	$872	$67.0
1995	34.2%	$1,349	$3,779	$40.0	$1,080	$84.5
1996	19.0%	$1,557	$4,474	$40.0	$1,185	$94.4
1997	31.1%	$1,990	$5,963	$40.0	$1,430	$116.2
1998	26.5%	$2,467	$7,425	$40.0	$1,662	$138.1
1999	17.9%	$2,861	$8,396	$40.0	$1,797	$152.9
2000	–12.1%	$2,480	$6,669	$40.0	$1,445	$126.2
2001	–13.2%	$2,117	$5,179	$40.0	$1,144	$102.9
2002	–23.9%	$1,580	$3,343	$40.0	$792	$73.5
2003	26.3%	$1,946	$4,177	$40.0	$908	$87.2
2004	7.4%	$2,047	$4,222	$40.0	$882	$87.9
2005	1.4%	$2,036	$3,978	$40.0	$805	$83.8
2006	12.9%	$2,254	$4,269	$40.0	$814	$88.9
2007	1.4%	$2,244	$4,028	$40.0	$735	$84.6
2008	–37.1%	$1,387	$2,114	$40.0	$410	$50.0
2009	23.1%	$1,658	$2,534	$40.0	$443	$57.8
2010	13.4%	$1,835	$2,730	$40.0	$437	$61.5
2011	–0.8%	$1,780	$2,513	$40.0	$372	$57.3
2012	14.0%	$1,984	$2,716	$40.0	$359	$61.4
2013	30.4%	$2,535	$3,395	$40.0	$388	$75.2
2014	12.8%	$2,816	$3,612	$40.0	$353	$79.7
2015	0.6%	$2,794	$3,404	$40.0	$275	$75.3
2016	9.7%	$3,020	$3,510	$40.0	$219	$77.6
2017	19.3%	$3,556	$3,944	$40.0	$169	$86.9
2018	–6.4%	$3,291	$3,462	$40.0	$76	$76.4
2019	28.6%	$4,180	N/A	N/A	$0	N/A

NOTES: This exhibit shows the portfolio before the annual withdrawal (PBW) and the annual withdrawal (*W*) for two strategies over the 1990–2019 period. The strategy in Panel A makes annual withdrawals of $40 as long as the strategy is sustainable; that in Panel B calculates and withdraws every year the maximum sustainable amount given a target bequest of $0. 'Test' is the expected bequest as calculated from expression (2). All returns and dollar amounts in real (inflation-adjusted) terms.

This alternative use of the framework proposed is in no way suggested here as an optimal withdrawal strategy, which in any case is not the issue discussed in this article. In order to discuss optimal withdrawal strategies an evaluation metric needs to be defined, something explored for the retirement period in much more detail by Estrada (2020).[16] The main reason for briefly addressing this issue here is because Panel B of Exhibit 6 begs the question of why the analysis ends as soon as a strategy becomes unsustainable. The answer is that the analysis does not really end there.

[16] Estrada (2019) considers a similar analysis but for the accumulation period.

In fact, as the discussion of Exhibit 6 suggests, the retiree calculates a new sustainable level of withdrawals and sticks to it *as long as it remains sustainable*. The point highlighted by Exhibit 8 is that, alternatively, a retiree could use the same methodology to determine a new sustainable level of withdrawals *every year*.

ASSESSMENT

Unlike volatility, factor exposure, or other more widely discussed sources of risk, sequence risk is far less known and has received far less attention in the financial literature. However, financial planners and more generally the retirement planning literature are aware and have often highlighted the importance of considering this type of risk when setting a withdrawal strategy for retirement.

This article does not have the goal of downplaying the relevance of sequence risk. As already mentioned, this type of risk is real, and investors should be aware of it and perhaps do something about it. That said, this article does raise the point that focusing on the impact of, but ignoring the probability of having to deal with, sequence risk may result in suboptimal portfolio decisions, such as implementing an unnecessarily conservative asset allocation or bearing the cost of protecting a portfolio.

It is trivial to come up with impressive examples of portfolio depletion early in retirement given a sequence of low returns, but it is essential to ask how likely are those sequences to happen. Just as it is easy to scare individuals about shark attacks, and to come up with some graphic and shocking examples, it is important to be equally candid about the probability of such attacks occurring. Only by taking into account both the impact and the probability of being hit by sequence risk can individuals make optimal portfolio decisions.

Sequence risk has been previously defined in many ways, typically highlighting the impact of a sequence of low returns early in retirement. The definition proposed here aims to be more precise than existing definitions, and to link this type of risk to the sustainability of a withdrawal strategy. When an early sequence of low returns hits a retiree and leads him to lower his future withdrawals, and therefore his standard of living in retirement, a tentative line is suggested in order to distinguish those cases in which sequence risk did have a significant impact on a retiree's lifestyle from those in which it did not. The tentative line suggested is a decrease in inflation-adjusted withdrawals larger than 10%–15% relative to those outlined in a financial plan.

In a nutshell, the evidence discussed here shows that many examples of sequence risk are intimidating but very unlikely to be faced by investors. It also shows that if an individual is going to worry about sequence risk, and somehow protect his portfolio, the cost of the protection will have to be borne not just over the first few years of retirement but over a period not shorter than 10 to 15 years. And it shows that having to deal with the impact of sequence risk is just as likely as enjoying remarkable outcomes that amount to a dream retirement or an extraordinary bequest.

Three ways to assess sequence risk are proposed here. An ex-post approach suggests to compare the annualized returns over the first few (say, 3, 5, and 10) years of a retirement period to the analogous averages across all the retirement periods considered; the lower the former is relative to the latter, the more likely that a retiree was a victim of sequence risk. Another ex-post approach suggests to reshuffle the returns of a given retirement period a large number of times to determine how often those same returns lead to failure; the lower the resulting figure, the more likely that a retiree was a victim of sequence risk.

The ex-ante approach links sequence risk to the sustainability of a withdrawal strategy. If an early sequence of low returns leads a retiree to lower his future inflation-adjusted withdrawals so that they remain sustainable, the next issue to be addressed is by how much; if the decrease in withdrawals is larger than a tentative 10%–15%, then the retiree is said to have been the victim of sequence risk. The advantage of this framework is that it enables a retiree to monitor in real time the sustainability of his withdrawal strategy and to dynamically introduce changes that lower, and in the limit eliminate, the probability of portfolio depletion.

The evidence discussed here is based on the impact of sequence risk on a portfolio fully invested in stocks, which is an asset allocation far more aggressive than that selected by most retirees. The reason for focusing on this asset allocation is to assess the impact of this type of risk in a situation in which it would hurt the most, with the implicit understanding that the impact would be less dramatic for more conservative portfolios. And yet, even in this rather extreme set up, sequence risk is found to be an issue that individuals should be aware of, but probably not lose their sleep about.

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