The Gain-Pain Index: Asset Allocation for Individual (and Other?) Investors

Javier Estrada

Javier Estrada
is a professor in the Department of Finance at IESE Business School in Barcelona, Spain.
jestrada@iese.edu

KEY FINDINGS

- Asset allocation is a critical lever of portfolio construction, but the main tool available to individual investors to determine it—the investor questionnaire—is a black box that does not help investors understand the important trade-offs of different portfolios.
- This article introduces a tool designed to overcome this shortcoming—the gain-pain index—which makes these trade-offs explicit while considering the investor's holding period and risk tolerance, as well as a broader view of risk than does mean-variance optimization.
- The gain-pain index is used to determine optimal asset allocations between stocks and bonds in 21 countries and the world market for different holding periods and levels or investor risk aversion.

ABSTRACT

Individual investors typically determine their asset allocation using investor questionnaires, which can be viewed as black boxes that generate a result without highlighting the benefits and costs of the portfolios considered. This article introduces an asset allocation tool—the gain-pain index—that overcomes this shortcoming. The proposed tool incorporates two critical variables found in investor questionnaires—the portfolio holding period and the investor's risk tolerance—and broadens the definition of risk beyond volatility by considering the probability of suffering a loss and the magnitude of the loss. The model is used to determine optimal asset allocations for 21 countries and the world market for different holding periods and levels of risk aversion.

As is widely acknowledged, determining a portfolio's asset allocation is one of the most important decisions investors make. Yet the typical way financial advisors address this issue with individual investors (i.e., giving them an investor questionnaire) is rather problematic on at least two counts. First, investor questionnaires are at best a mix of art and science; second, investors perceive only a black box between their answers and the suggested asset allocation, which clearly does not contribute to their understanding of the recommendation.

In principle there seems to be nothing wrong with a black box; after all, most people drive without fully understanding the mechanics of a car. That said, and typically, the better that investors understand their portfolios, the more likely they are to stick with them, as opposed to bailing out at the first sign of trouble. For this reason, the tool that advisors use to make an asset allocation recommendation to individual investors should ideally help clients understand the relevant trade-offs involved; this is precisely the ultimate purpose of the methodology introduced in this article.

The gain-pain index (GPI) proposed here highlights the gain and pain expected from the asset allocations considered, the latter accounting for an investor's risk tolerance. In the model, the gain is given by the return expected from each portfolio and the pain by three sources of risk, namely, volatility, the probability of suffering a loss, and the magnitude of the loss. The second and third sources combine into one variable—the expected loss.

The model explicitly accounts for two variables that are invariably found in investor questionnaires: the portfolio's holding period and the investor's risk tolerance. The former is taken into account by considering the four relevant variables of the GPI over holding periods of the same length as that of the portfolio's intended holding period; the latter is considered by incorporating a risk aversion coefficient estimated for each individual investor.

The tool proposed here is first illustrated with an example in which all relevant variables are calculated and the relevant trade-offs are discussed. Then it is used to determine optimal asset allocations for the US for several portfolio holding periods and levels of investor risk aversion, which are subsequently compared with those that stem from the standard mean-variance optimization framework. Next, using an extensive data set of 21 countries over 120 years, optimal asset allocations are determined for each country for several portfolio holding periods and levels of investor risk aversion. Finally, the technique suggested to infer an investor's risk aversion coefficient is discussed.

The rest of the article is organized as follows. The next section discusses the intuition behind the GPI model introduced in this article and briefly reviews the relevant asset allocation literature. The Evidence section discusses the data, methodology, a concrete example of the model's application, optimal asset allocations for all countries in the sample, and the estimation of the risk aversion coefficient. The final section presents an assessment. An appendix with tables concludes the article.

THE ISSUE

First Things First

All portfolios have an asset allocation, but not all of them are created equal. Some are *residual* asset allocations, or simply the result of aggregating the assets bought to build a portfolio without first setting specific targets for each relevant asset class; others are *targeted* asset allocations, or the result of first determining the ideal proportions for each relevant asset class and then buying the assets to build the target portfolio. Obviously, only the latter is the correct approach. Proper portfolio construction requires *first* determining the right asset allocation and *only then* continuing with security selection; the opposite is simply akin to shooting in the dark.

This argument goes beyond the debate started by Brinson, Hood, and Beebower (1986) regarding whether asset allocation or security selection is the main determinant of portfolio performance.[1] Both have an impact on performance, and both have a critical role to play in portfolio construction. Clearly, different proportions of stocks and bonds result in different patterns of risk and return, which makes asset allocation a critical step in portfolio construction.

[1] On this contentious issue, see also Brinson, Singer, and Beebower (1991); Ibbotson and Kaplan (2000); Statman (2000); and Kritzman and Page (2002), among many others.

And just as clearly, a careful selection of securities is important for many reasons, among them that it may help reduce the cost of a portfolio, which is also a critical determinant of performance, particularly for long-term investors. In fact, Bogle (2017) shows that low-cost funds outperformed high-cost funds in terms of both return and risk-adjusted return over the 1991–2016 period. Furthermore, Johnson (2020) shows that funds in the cheapest quintile experienced positive net flows over the 2015–2019 period, whereas the remaining 80% of funds experienced negative net flows over the same period.

In short, the debate about whether asset allocation is more important than security selection, or the other way around, is not relevant for this discussion; both are important. That said, the order in which these two levers of portfolio construction is implemented *is* in fact important, and it is clear that asset allocation must precede security selection.

Asset Allocation Methodologies

To determine a target asset allocation, institutional investors tend to rely on portfolio optimization and individual investors on investor questionnaires. As is well known, portfolio optimization was pioneered by Markowitz (1952, 1959) and evolved over time. Part of that evolution involved the consideration of different ways of assessing risk beyond volatility, such as downside volatility, beta, downside beta, value at risk, expected loss, expected absolute deviation, probability of loss, and maximum loss, to name but a few; Arnott (2003) and Markowitz (2010) discuss this issue in more detail.

Furthermore, the long-term excess return that stems from several factors, such as the traditional value and size, or the more recent momentum and low volatility, has also been linked to the risk that these factors add to portfolios. To be sure, whether the long-term excess return from these and other factors stems from exposure to additional risk or from market inefficiencies, including behavioral biases, is a controversial subject.

Besides Markowitz's (1952, 1959) pioneering insight of optimizing portfolios on the basis of mean and variance, several other methodologies have been proposed, including full-scale optimization (Adler and Kritzman 2007), mean-semivariance optimization (Estrada 2008), maximum diversification (Choueifaty and Coignard 2008), optimization with higher moments (Harvey et al. 2010), geometric mean maximization (Estrada 2010), and maximally rho-representative portfolios (Froidure, Jalalzai, and Choueifaty 2019).

All the methodologies have pros and cons, but analyzing them is beyond the scope of this article; that said, they are overwhelmingly used by institutional (rather than individual) investors, but only the latter are the main focus of this article. Kim et al. (2021) discuss mean-variance optimization, including its limitations and applicability to asset allocation, as well as some methods to improve its robustness.

Just as the literature on asset allocation is vast, literature on the effectiveness of investor questionnaires is scarce. Pan and Statman (2012) argue that the typical investor questionnaires are deficient for five reasons. First, each investor has a multitude of risk tolerances; second, questionnaires offer no clear link between risk tolerance scores and asset allocations; third, risk tolerance varies by circumstances and associated emotions; fourth, risk tolerance assessed in foresight is likely to differ from that assessed in hindsight; and fifth, investor propensities other than risk tolerance also matter. For these reasons, they propose a new type of investor questionnaire that probes not just for risk tolerance but also for overconfidence, maximization, regret, trust, and life-satisfaction.

Pan and Statman (2013) follow up their previous research by exploring the links between personality and risk tolerance, overconfidence, maximization, regret, trust, attributing success to luck or skill, and life-satisfaction. Their results of a survey of more than 2,500 individuals suggest that knowledge about these links are important to financial advisors when guiding their clients' investment decisions.

Finally, Packin (2020) finds that when making consumer finance decisions, including significant investment decisions, individuals prefer to follow the recommendations of an algorithm than those of human experts, thus viewing algorithms as a superior authority. She also finds that increasing deference to algorithms is concerning because individuals avoid obtaining a second opinion, even when the first opinion comes from an algorithm that has made mistakes in the past. Importantly, she concludes that advisors should nudge individuals to look for second opinions.

The GPI—Motivation

The GPI, introduced here, aims to provide an asset allocation tool that accounts for risk variables that investors consider important and that makes the relevant trade-offs explicit and clear. The *gain* term of the index is given by the expected return of the portfolios considered. The *pain* term is given by two variables that summarize the risk (or pain) that stems from those portfolios; the risk variables are volatility and the expected loss—which is itself the product of two variables: the probability of suffering a loss and the magnitude of the loss.[2] Finally, the index also accounts for an investor's risk aversion. (The formal expression of the GPI is introduced in the next section.)

The pain term of the GPI is partially inspired by Arthur Okun's misery index. Okun was chairman of the Council of Economic Advisers during part of the Lyndon Johnson administration, and he proposed to add the inflation rate and the seasonally adjusted unemployment rate to measure how badly the average individual was doing.[3] Similarly, the pain term of the GPI adds two variables that, all else equal, make investors worse off.[4]

The underlying idea of the GPI is not entirely different from Markowitz's (1959) approximation to expected utility, which depends on expected return, variance, and a coefficient of risk aversion. One difference is that Markowitz's (1959) framework is a one-period model, whereas the GPI explicitly accounts for the holding period selected by the investor. Another difference is the definition of risk; expected utility is based on variance, whereas the GPI is based on volatility, the probability of suffering a loss, and the magnitude of the loss.

Regarding the first risk variable, Markowitz (1999) highlights that although a mean-variance optimal portfolio is also a mean-volatility optimal portfolio, volatility is actually the "intuitively meaningful" measure of dispersion. Regarding the other two risk variables—the probability of suffering a loss and the magnitude of the loss—they are very much in investors' minds when building and evaluating a portfolio, as any financial advisor would attest.

Finally, the GPI accounts for two variables that are essential when determining an investor's asset allocation, which is why they are invariably found in investor

[2] Not too differently from what is done in this article, Shepherd (2017) relates risk to discomfort. In fact, bearing pain or discomfort can be thought of as a more general way of thinking about risk.

[3] Barro (1996) subsequently added long-term interest rates and gross domestic product growth to Okun's measure so that misery is created when inflation, unemployment, or interest rates go up, or when gross domestic product growth goes down.

[4] Combining volatility and expected loss is partially motivated by an attempt to balance a short-term and a longer-term perspective of risk. Furthermore, adding these two variables is motivated largely by the goal of keeping the parallel to the misery index. (That said, as a reviewer rightfully pointed out, the way these two variables are combined has an impact on both the utility function and the optimal portfolio ultimately selected.)

questionnaires: the portfolio's holding period and the investor's risk tolerance. The former is considered implicitly by calculating the inputs of the GPI over periods of the same length as that of the portfolio's intended holding period; the latter is taken into account explicitly by incorporating a specific risk aversion coefficient for the investor building the portfolio.

EVIDENCE

Data

The sample consists of the Dimson-Marsh-Staunton database, described in detail in Dimson, Marsh, and Staunton (2002) and in the annual updates of the database documentation. The database contains annual returns for stocks and government bonds over the 1900–2019 period for 21 countries and the world market. Returns are real (adjusted by each country's inflation rate), in local currency, and account for both capital gains/losses and cash flows (dividends and coupons). Real returns for the world market are in dollars and adjusted by the US inflation rate. Exhibit A1 in the appendix reports summary statistics for all the series of stock and bond real returns in the sample.

Importantly, because the holding period is an essential variable when selecting an asset allocation, and because multiperiod arithmetic returns tend to be positively skewed because of compounding, unless otherwise stated the analysis is performed with logarithmic returns. To illustrate, the distributions of 20-year arithmetic and log US stock returns have coefficients of standardized skewness of 4.28 and 0.43, clearly indicating significant positive skewness in the first case and no significant skewness in the second case.[5]

Methodology

The GPI considers the gain and pain of different portfolios (asset allocations), as well as an investor's risk tolerance. More precisely, the GPI is defined as

$$\text{GPI}_T = G_T - \lambda \cdot P_T, \tag{1}$$

where G and P denote gain and pain, λ denotes the investor's risk aversion coefficient, and the subscript T denotes the portfolio's expected holding period. G and P, in turn, are given by

$$G_T = M(R_T) \tag{2}$$

$$P_T = V(R_T) + |L(R_T)|, \tag{3}$$

where M, V, and L denote mean return, volatility, and expected loss (measured in absolute value), and R_T denotes T-year log returns. Finally, L is given by

$$L(R_T) = q(R_T) \cdot A(R_T), \tag{4}$$

[5] Furthermore, a Kolmogorov-Smirnov test clearly rejects the null hypothesis of normality for 20-year arithmetic stock returns but is not able to reject it for 20-year log stock returns.

where q denotes the probability of suffering a loss over T-year periods, and A denotes the average loss over T-year periods with losses.[6]

Combining Equations (1) through (4), the GPI can be restated as

$$\text{GPI}_T = G_T - \lambda \cdot P_T = M(R_T) - \lambda \cdot \{V(R_T) + |L(R_T)|\} = M(R_T) - \lambda \cdot \{\{V(R_T) + |q(R_T) \cdot A(R_T)|\}, \quad (5)$$

which highlights the trade-offs involved between the return expected from different portfolios, on one hand, and the volatility, probability of suffering a loss, and average loss of those portfolios, on the other hand. Thus, the determination of the optimal portfolio (asset allocation) using the GPI involves the following steps:

1. Determine λ for the specific investor building a portfolio. (More on this below.)
2. Determine the investor's holding period (T) for the portfolio.
3. Calculate historical T-year log returns for all relevant asset classes.
4. Calculate M (hence, G), V, q, and A (hence, L and P) for all relevant asset allocations.
5. Calculate the GPI for those asset allocations and choose the one with the highest GPI.

A few comments are in order. First, although the preceding steps involve estimating all inputs on the basis of historical data, other approaches are obviously possible, such as estimating them from Monte Carlo simulations based on the characteristics of the relevant return distributions. Second, the main advantage of the steps just outlined is that an investor/client can observe the value of all the variables and can therefore evaluate the relevant trade-offs. (An example is discussed in the next section.)

Third, steps 4 and 5 can be collapsed into one by simply finding the asset allocation that maximizes the GPI through numerical optimization. This process may be particularly useful when more than two asset classes are involved, at the cost of not enabling the investor to assess the gain and pain of different portfolios. That said, it is obviously possible to calculate the value of all the relevant variables for the optimal asset allocation and discuss with the investor the relevant trade-offs of that specific portfolio.

An Example

Exhibit 1 illustrates the implementation of steps 1 through 5 described in the previous section for a portfolio of two asset classes, US stocks and bonds, a 20-year holding period, and a risk aversion coefficient of 1. Exhibit 1 shows, for 21 asset allocations with different proportions of stocks (S) and bonds (100–S), the value of all the components of the GPI, the GPI, and the optimal asset allocation (highlighted), which in this case is 95% stocks and 5% bonds.

The main benefit of a table such as Exhibit 1 is that an investor/client could evaluate the expected gain (return) and pain (volatility, probability of suffering a loss, and average loss) of different asset allocations. An investor could observe, for example, that portfolios with at least 70% in stocks have never lost purchasing power over 20-year periods; if history is any guide, this implies that the expected (inflation-adjusted) loss for these relatively-aggressive portfolios is zero.

Interestingly, the investor could also observe that although returns decrease monotonically as the allocation to stocks decreases, volatility is U-shaped; that is,

[6] A reviewer rightfully pointed out that the expected loss variable may fail to fully capture extreme risks that a value-at-risk measure may capture. This is particularly likely to happen when the underlying distribution of returns is negatively skewed and/or fat-tailed.

EXHIBIT 1
Motivation

S	100	95	90	85	80	75	70
G	123.0	120.8	118.4	115.8	113.0	110.0	106.7
V	58.1	55.9	54.0	52.3	50.9	49.7	48.8
q	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
P	58.1	55.9	54.0	52.3	50.9	49.7	48.8
GPI	0.648	0.649	0.644	0.635	0.621	0.603	0.579
S	**65**	**60**	**55**	**50**	**45**	**40**	**35**
G	103.2	99.6	95.7	91.6	87.3	82.8	78.0
V	48.2	47.8	47.7	47.9	48.4	49.0	49.9
q	1.0	2.0	2.0	2.0	3.0	4.0	5.9
A	–1.6	–3.5	–7.2	–11.1	–11.0	–11.2	–10.7
L	0.0	–0.1	–0.1	–0.2	–0.3	–0.4	–0.6
P	48.2	47.9	47.9	48.1	48.7	49.5	50.6
GPI	0.551	0.517	0.478	0.435	0.386	0.333	0.274
S	**30**	**25**	**20**	**15**	**10**	**5**	**0**
G	73.1	67.9	62.6	57.0	51.2	45.1	38.9
V	51.1	52.4	53.9	55.6	57.4	59.4	61.5
q	7.9	8.9	13.9	14.9	21.8	31.7	41.6
A	–12.1	–15.6	–13.8	–18.0	–18.0	–18.9	–22.5
L	–1.0	–1.4	–1.9	–2.7	–3.9	–6.0	–9.4
P	52.0	53.8	55.8	58.3	61.4	65.4	70.9
GPI	0.211	0.141	0.067	–0.013	–0.102	–0.203	–0.320

NOTES: This exhibit shows, for 21 asset allocations with different proportions of US stocks (*S*) and bonds (100–*S*), the gain (*G*), volatility (*V*), probability of suffering a loss (*q*), average loss (*A*), expected loss (*L*), pain (*P*), and gain-pain index (GPI) for a risk aversion coefficient of 1, all as defined in Equations (1) through (5) and calculated over 20-year holding periods. The data are described in Exhibit A1 in the appendix.

it first falls and then rises as the allocation to stocks decreases, with the minimum for a portfolio invested 55% in stocks. Similarly, pain is also U-shaped, first decreasing and then increasing as the allocation to stocks decreases, with a minimum for a portfolio invested 55% to 60% in stocks. Also of interest, over 20-year periods, stocks are *less* volatile than bonds, as reflected by their relative volatilities of 58.1% for stocks and 61.5% for bonds.

In short, one way of implementing the methodology introduced here is to calculate the value of the relevant variables for several portfolios and let the investor evaluate the trade-offs. Although strictly speaking there is an optimal asset allocation, which is the one that maximizes the GPI, it is conceivable that an investor may consider some variables more important than others (e.g., by focusing more on the expected loss than on volatility) and select an allocation that does not necessarily maximize the GPI as stated in Equation (5). In fact, although this issue is not pursued any further here, the pain variable could alternatively be defined as

$$P_T = x_V \cdot V(R_T) + x_L \cdot |L(R_T)|, \qquad (6)$$

where x_V and x_L would be the weight an investor chooses to give to volatility and the expected loss, and $x_V + x_L = 1$. This alternative definition of P_T would obviously result in an alternative definition of the GPI.

EXHIBIT 2
Optimal Allocations, US

	λ										
T	**0.25**	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.50**	**3.00**	**5.00**
Panel A											
1	54.0	36.6	32.1	28.6	26.7	25.5	24.4	23.6	22.4	21.6	19.9
5	95.5	74.3	63.7	56.4	52.0	48.1	46.5	44.3	41.0	38.7	34.5
10	100.0	97.1	86.8	80.4	75.9	71.6	68.7	66.0	61.9	59.0	53.0
15	100.0	100.0	97.0	90.4	85.7	82.0	78.8	76.2	72.3	69.6	63.6
20	100.0	100.0	100.0	97.1	92.0	88.0	84.8	82.1	78.1	75.2	68.5
25	100.0	100.0	100.0	100.0	98.4	94.4	91.2	88.6	84.6	81.7	75.3
30	100.0	100.0	100.0	100.0	100.0	99.5	96.4	93.9	90.1	87.3	81.2
Panel B											
1	100.0	84.2	71.5	62.6	56.2	51.2	47.4	44.3	39.6	36.2	28.8
5	100.0	92.4	81.7	74.0	68.1	63.4	59.7	56.6	51.7	48.1	39.8
10	100.0	100.0	91.6	85.3	80.3	76.3	73.0	70.2	65.8	62.4	54.3
15	100.0	100.0	94.9	88.9	84.4	80.7	77.7	75.3	71.3	68.4	61.4
20	100.0	100.0	100.0	94.6	90.2	86.8	84.0	81.7	78.0	75.2	68.8
25	100.0	100.0	100.0	100.0	97.6	94.2	91.4	89.1	85.5	82.8	76.4
30	100.0	100.0	100.0	100.0	100.0	100.0	97.7	95.5	92.0	89.3	83.0

NOTES: This exhibit shows the optimal allocation to US stocks, with the rest allocated to US bonds, as selected by maximizing the gain-pain index (Panel A) and mean-variance expected utility (Panel B), for several holding periods (T) and risk aversion coefficients (λ). The data are described in Exhibit A1 in the appendix.

Optimal Asset Allocations

An alternative way of implementing the methodology introduced here is to simply find the optimal asset allocation by maximizing the GPI through numerical optimization, given the investor's portfolio holding period and risk aversion coefficient. Exhibit 2 shows, for the US market, the optimal allocation to stocks (with the rest allocated to bonds) for several values of T and λ. Panel A shows the optimal allocations that result from maximizing Equation (5) and Panel B those that result from mean-variance optimization defined as

$$EU_T = M(R_T) - \lambda \cdot V(R_T)^2, \tag{7}$$

where EU denotes Markowitz's (1959) mean-variance approximation to expected utility.[7]

As expected, Panel A shows that given the holding period, the higher the investor's risk aversion, the lower the proportion of stocks in the optimal portfolio. Similarly, given the investor's risk aversion, the longer the holding period, the higher the proportion of stocks in the optimal portfolio. Both of these patterns hold in Panel B; the difference between the two panels is that, in general, the GPI tends to select more conservative allocations, with the largest differences largely concentrated over short holding periods of 10 years or less.

A similar (but somewhat less comprehensive) analysis to that in Panel A of Exhibit 2 was performed for the other 20 countries in the sample; the results for

[7] Although utility theory is mainstream in economics and finance, it is not devoid of controversy or criticism; some consider it "too abstract and nebulous for use in practice," as a reviewer pointed out. Part of this practical shortcoming is also related to the difficulty of properly estimating a risk aversion coefficient. Furthermore, Markowitz's (1959) framework has the particular limitation of being a one-period model, although most investors explicitly or implicitly use a multiperiod framework.

EXHIBIT 3
Optimal Allocations, All Countries

	λ				
T	0.5	1.0	1.5	2.0	3.0
Panel A					
1	38.1	26.4	24.3	22.4	21.4
5	87.1	70.5	62.3	60.1	54.8
10	100.0	97.8	90.8	87.5	83.5
20	100.0	100.0	97.5	93.1	87.2
30	100.0	100.0	100.0	98.4	92.0
Panel B					
1	38.1	29.1	26.1	24.7	23.2
5	75.2	61.9	55.3	52.0	47.9
10	89.7	83.3	79.4	76.3	72.7
20	95.2	91.6	89.1	87.3	85.0
30	98.0	95.9	93.7	91.9	89.7

NOTES: This exhibit shows the optimal allocation to stocks, with the rest allocated to bonds, as selected by maximizing the gain-pain index for the world market (Panel A), and for each individual country and then averaging across countries (Panel B), for several holding periods (*T*) and risk aversion coefficients (λ). The data are described in Exhibit A1 in the appendix.

each individual country are shown in Exhibit A2 in the appendix. Exhibit 3 shows results for the world market (Panel A) and for the average country in the sample (Panel B). The two panels are different because, although both consider the 21 countries in the sample, Panel A weighs the countries by market capitalization and Panel B weighs them equally.

Again as expected, both panels show that given the holding period, the higher the investor's risk aversion, the lower the proportion of stocks in the optimal portfolio; given the investor's risk aversion, the longer the holding period, the higher the proportion of stocks in the optimal portfolio. Across all 21 countries, holding periods, and risk aversion coefficients considered, the average allocation to stocks is 70.1%.

As for the results in Exhibit A2, they unsurprisingly vary markedly across markets. In countries such as Australia, Denmark, South Africa, and the UK, the optimal allocations tend to be more aggressive, whereas in countries such as Spain, Sweden, and Switzerland, they tend to be more conservative. Interestingly, there does not seem to be a close relationship between the average allocation to stocks in each country and each country's equity risk premium; the correlation between these two variables is a rather weak 0.25.

Estimating the Risk Aversion Coefficient

To accurately calculate the GPI of different allocations, financial advisors need two critical pieces of information from an investor/client: the portfolio's intended holding period and the investor's risk aversion coefficient. Obviously, no investor knows their own coefficient, which implies that advisors need to estimate it; many methodologies exist to do so, such as that proposed by Holt and Laury (2002) in their influential work. Ideally, the process used by the advisor to ascertain a client's risk aversion coefficient should be as simple as possible so that the client has no difficulty going through the motions.[8]

In his work attempting to rationalize the glidepath used by target-date funds, Estrada (2020) finds that investors whose risk aversion roughly doubles (from 1.10 to the 2.25–2.55 range) during the last 25 years of their working period would select asset allocations similar to those featured by target-date funds. A similar methodology is implemented here, which essentially involves three steps: First, face an investor with a gamble; then ask the investor how much he would pay to avoid it; and finally, infer the risk aversion coefficient from his answer.

An example helps to illustrate the methodology. Consider an investor whose preferences can be summarized by the widely used power utility function

$$U(W) = \frac{W^{1-\lambda} - 1}{1-\lambda}, \quad \text{for } \lambda \geq 0, \lambda \neq 1, \tag{8}$$

[8] Barsky et al. (1997) propose a measure of risk tolerance that follows from a set of questions and find that, according to their measure, risk tolerance is higher among people younger than 55 and older than 70 than among those in the 55–70 range and higher for men than for women. Furthermore, Beckmann and Menkhoff (2008) find that not even expertise eliminates gender differences in risk tolerance.

EXHIBIT 4

Estimating the Risk Aversion Coefficient

λ	W_c	π	λ	W_c	π	λ	W_c	π
0.00	10,000	0	1.50	9,925	75	3.00	9,851	149
0.25	9,987	13	1.75	9,912	88	3.50	9,827	173
0.50	9,975	25	2.00	9,900	100	4.00	9,803	197
0.75	9,962	38	2.25	9,888	112	4.50	9,762	238
1.00	9,950	50	2.50	9,875	125	5.00	9,742	258
1.25	9,937	63	2.75	9,863	137			

NOTES: This exhibit shows, for the power utility function in Equation (8) and several values of the risk aversion coefficient (λ), the certainty equivalent of wealth (W_c), and the risk premium (π) as defined in Equations (9) and (10). All figures except λ are in dollars.

where U and W denote utility and wealth. As before, λ denotes the risk aversion coefficient, with 0 indicating risk neutrality and 1 indicating log utility. Assume this investor has a $10,000 portfolio (a number easily scalable), and the advisor faces him with a gamble in which with equal probability the investor can gain or lose 10% of his portfolio ($1,000). How much would the investor accept to have with certainty to avoid this gamble?

The answer is given by the certainty equivalent of wealth (W_c), which is given by a level of wealth such that $U(W_c) = E[U(W)]$, where E denotes the expectation operator. In the specific case of a power utility function, the certainty equivalent of wealth is given by

$$W_c = \{1 + (1-\lambda) \cdot E[U(W)]\}^{\frac{1}{1-\lambda}}. \quad (9)$$

Furthermore, the Markowitz's risk premium (π), which represents how much an investor is willing to pay to avoid the gamble, is given by

$$\Pi = E(W) - W_c, \quad (10)$$

where E(W) denotes the expected wealth if the gamble is played.

Exhibit 4 shows several values of the risk aversion coefficient and the corresponding values of W_c and π. Obviously, a risk neutral investor is not willing to pay anything to avoid risk, and therefore his certainty equivalent of wealth and risk premium are $10,000 and $0. An investor with log utility ($\lambda = 1$), on the other hand, has a certainty equivalent of wealth and risk premium of $9,950 and $50. Finally, an investor with $\lambda = 3$ has a certainty equivalent of wealth and risk premium of $9,851 and $149.

The range of risk aversion coefficients shown in Exhibit 4 is wide enough to encompass the range of values most often used in practice. In fact, Gandelman and Hernández-Murillo (2015) estimate risk aversion at the aggregate level for 75 countries and obtain a range between 0 and 3, a value around 1 for most countries, and a cross-sectional average of 0.98; Baz and Guo (2017) argue that the most widely accepted values for asset allocation are in the 1–5 range; and Estrada and Kritzman (2019) find no differences in the asset allocations selected by an investor for risk aversion coefficients higher than 3.0.

Because an exact estimation of λ is unlikely to be possible, financial advisors could use a table such as Exhibit 4 to infer a client's coefficient of risk aversion. After facing the client with the gamble previously described, perhaps using as the starting portfolio the actual size of the client's portfolio (rather than $10,000), and after obtaining the client's answer (a certainty equivalent of wealth or a risk premium), the advisor could match such an answer to the appropriate risk aversion coefficient in the table. Interpolation between the figures in the exhibit is also possible and unlikely to have a substantial impact on the asset allocation recommended.

Further Discussion

The tool proposed here to determine the asset allocation of individual (and perhaps other) investors has the ultimate goal of helping an investor understand a portfolio recommendation, thus avoiding the black-box nature of the typical investors questionnaire. Understanding a recommendation, as well as the relevant trade-offs involved, in turn helps investors to stick with their portfolios during the inevitable bad times they will have to go through.

Admittedly, the previous claim is not readily testable. Gathering evidence on how much the clients stick to their portfolios depending on the asset allocation tool used by their advisors is all but impossible. And yet, although it cannot really be tested at the 5% level of significance, most advisors are likely to agree with the statement that clients that do not show much interest in understanding a recommendation are less likely to stick with their portfolios during turbulent markets than those who do understand their portfolios. This is precisely where the GPI introduced in this article comes in.

Also admittedly, implementing the GPI may be far from trivial for individual investors, but it should not be difficult for financial advisors. The tool proposed here does not require more information than that needed for standard portfolio optimization; it only requires the calculation of two additional variables, the probability of suffering a loss and the magnitude of the loss, both of which should be trivial for any advisor to calculate.

Furthermore, most clients understand these two variables intuitively (and associate them with risk) much better than they understand volatility, which in turn helps them understand the relevant pain-gain (or risk-return) trade-offs. In this regard, this article may be viewed as a stepping stone to the next round of research on intuitive measures of risk and intuitive asset allocation methodologies.

ASSESSMENT

Asset allocation is a critical lever of portfolio construction. Institutional investors typically deal with it by using optimizers, which, to be sure, are far from flawless. Individual investors, on the other hand, typically resort to investor questionnaires, which introduce a black box between an investor's answers and the recommended asset allocation. This black box hides the relative benefits and costs—hence the relevant trade-offs—of the portfolios considered, thus preventing investors from understanding why a given asset allocation has been suggested.

This article introduces a tool to overcome this shortcoming. The GPI makes both the expected benefits (gain) and expected costs (pain) of the portfolios considered explicit. The proposed tool goes beyond viewing volatility as the only source of risk and considers two additional variables: the probability of suffering a loss and the magnitude of the loss. In addition, it accounts for two variables invariably found in investor questionnaires: a portfolio's holding period and an investor's risk tolerance. The latter must be ascertained for each individual investor, which can be done with a gamble such as the one discussed here.

After illustrating its implementation, the GPI was used to determine optimal asset allocations for the US, for different portfolio holding periods and levels of investor risk aversion. These allocations were then compared with those that stem from traditional mean-variance optimization, the former in general more conservative than the latter. The GPI was also used to determine optimal asset allocations for 20 other countries and for the world market, with the results varying markedly across countries, holding periods, and risk aversion coefficients.

In short, this article introduces a novel tool that can be used by financial advisors to determine a client's optimal asset allocation, at the same time making it possible for the client to understand both the suggested portfolio and the relevant trade-offs involved. Given the importance of asset allocation when building a portfolio, any tool that helps advisors and investors make the right decision is a tool worth considering.

APPENDIX

EXHIBIT A1

Summary Statistics, 1900–2019

	Stocks (%)				Bonds (%)				
	AR	SD	Min	Max	AR	SD	Min	Max	ERP (%)
Australia	6.8	17.4	–42.5	51.5	1.8	13.0	–26.6	62.2	5.0
Austria	1.0	30.4	–59.6	132.7	–3.5	53.1	–94.7	484.8	4.5
Belgium	2.6	23.2	–48.9	105.1	0.5	14.8	–45.6	62.3	2.1
Canada	5.7	16.8	–33.8	55.2	2.2	10.2	–25.9	41.7	3.5
Denmark	5.6	20.6	–49.2	107.8	2.1	12.8	–27.6	63.6	3.5
Finland	5.5	29.4	–60.8	161.7	0.3	13.4	–69.5	30.2	5.2
France	3.4	22.8	–41.5	66.1	0.3	12.8	–43.5	35.9	3.1
Germany	3.3	31.2	–90.8	154.6	–1.2	15.4	–95.0	62.5	4.5
Ireland	4.4	22.8	–65.4	68.4	1.7	14.8	–34.1	61.2	2.6
Italy	2.2	28.2	–72.9	120.7	–0.9	14.6	–64.3	35.5	3.1
Japan	4.2	29.1	–85.5	121.1	–0.8	19.3	–77.5	69.8	5.0
Netherlands	5.1	21.1	–50.4	101.6	1.8	9.6	–18.1	32.8	3.3
N. Zealand	6.4	19.1	–54.7	105.3	2.3	8.9	–23.7	34.1	4.2
Norway	4.4	26.4	–53.6	166.9	1.8	11.8	–48.0	62.1	2.6
Portugal	3.6	33.8	–76.6	151.8	–1.3	18.0	–45.1	90.6	5.0
S. Africa	7.1	21.8	–52.2	102.9	1.9	10.3	–32.6	37.1	5.2
Spain	3.6	21.6	–43.3	99.4	2.0	12.4	–30.2	53.2	1.6
Sweden	6.0	20.9	–42.5	67.5	2.7	12.5	–37.0	68.2	3.2
Switzerland	4.6	19.3	–37.8	59.4	2.4	9.3	–21.4	56.1	2.3
UK	5.5	19.5	–56.6	99.3	1.9	13.4	–29.9	59.4	3.6
US	6.5	19.8	–38.6	55.8	2.0	10.3	–18.1	35.2	4.5
World	5.2	17.3	–41.5	67.6	2.0	10.9	–31.6	46.0	3.2

NOTES: This exhibit shows, for the series of annual arithmetic returns, the annualized return (AR), standard deviation (SD), lowest return (Min), highest return (Max), and equity risk premium (ERP) defined as the difference between the annualized returns reported for stocks and bonds. All returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for capital gains/losses and cash flows (dividends and coupons).

EXHIBIT A2

Optimal Allocations, Individual Countries

λ →	0.5	1.0	1.5	2.0	3.0		0.5	1.0	1.5	2.0	3.0
Australia						Austria					
1	58.6	43.8	38.6	36.3	35.1	1	65.7	65.7	65.7	65.6	65.4
5	100.0	94.0	83.1	78.7	72.9	5	75.5	76.5	77.0	77.3	77.6
10	100.0	100.0	100.0	100.0	100.0	10	78.5	80.8	81.4	81.5	82.1
20	100.0	100.0	100.0	100.0	100.0	20	77.0	79.0	80.0	80.7	81.5
30	100.0	100.0	100.0	100.0	100.0	30	71.0	71.1	71.1	71.1	71.2
Belgium						Canada					
1	31.5	25.4	23.3	21.7	20.6	1	39.9	31.7	29.1	27.6	26.4
5	64.4	54.2	50.0	48.8	46.2	5	84.7	66.4	59.9	55.6	51.6
10	86.5	79.3	75.3	73.6	71.7	10	100.0	99.0	92.8	88.7	84.2
20	96.7	92.5	90.2	89.0	86.8	20	100.0	100.0	96.1	92.0	87.4
30	100.0	100.0	100.0	100.0	100.0	30	100.0	100.0	94.3	89.6	84.6
Denmark						Finland					
1	42.3	32.4	29.4	27.2	24.8	1	37.3	26.8	24.0	23.1	21.0
5	89.9	76.2	70.6	66.2	63.1	5	81.0	64.6	57.3	52.4	47.1
10	100.0	100.0	100.0	100.0	100.0	10	100.0	100.0	99.8	96.7	93.4
20	100.0	100.0	100.0	100.0	100.0	20	100.0	100.0	100.0	100.0	100.0
30	100.0	100.0	100.0	100.0	100.0	30	100.0	100.0	100.0	100.0	100.0
France						Germany					
1	33.8	26.3	23.2	21.4	20.7	1	53.8	44.0	39.1	36.3	33.1
5	78.6	65.9	59.9	57.7	54.0	5	91.8	84.6	80.2	77.5	74.8
10	100.0	98.7	97.2	95.7	94.0	10	100.0	99.0	97.0	96.2	95.3
20	100.0	100.0	100.0	100.0	100.0	20	100.0	100.0	99.8	98.0	95.9
30	100.0	100.0	100.0	100.0	100.0	30	100.0	99.8	95.3	92.3	88.6
Ireland						Italy					
1	32.2	22.6	19.6	19.5	18.5	1	28.1	19.8	17.4	16.2	14.4
5	68.9	55.1	49.2	45.3	40.0	5	72.0	65.1	61.4	59.5	57.1
10	86.2	78.3	73.2	69.6	65.3	10	87.8	83.0	80.8	80.3	79.5
20	100.0	100.0	100.0	100.0	97.4	20	99.4	96.5	95.1	94.6	93.6
30	100.0	100.0	100.0	100.0	100.0	30	100.0	98.2	96.7	95.7	94.8
Japan						Netherlands					
1	47.7	37.2	33.0	31.3	29.8	1	30.0	23.7	21.5	21.0	19.8
5	78.1	66.2	60.2	56.9	53.0	5	56.9	43.3	37.6	33.9	30.4
10	96.1	88.3	83.8	81.7	79.3	10	75.8	61.5	54.9	50.3	45.5
20	100.0	100.0	100.0	100.0	100.0	20	91.4	78.4	71.9	66.6	61.9
30	100.0	100.0	100.0	100.0	100.0	30	100.0	95.2	87.5	82.1	75.1
New Zealand						Norway					
1	36.0	23.0	18.6	17.9	15.9	1	24.8	18.4	15.8	15.7	15.0
5	81.9	61.7	55.3	51.2	48.6	5	60.7	48.5	43.4	41.2	37.3
10	100.0	95.8	90.6	87.0	83.6	10	81.5	74.5	69.8	66.5	62.1
20	100.0	100.0	100.0	100.0	100.0	20	87.0	85.4	83.9	83.2	81.0
30	100.0	100.0	100.0	100.0	98.7	30	91.0	92.7	94.8	97.4	100.0
Portugal						South Africa					
1	37.9	26.2	22.8	21.5	19.6	1	38.0	24.2	20.2	18.6	16.8
5	78.0	66.9	60.9	58.1	54.3	5	100.0	77.9	60.5	53.9	45.3
10	96.2	91.0	87.9	86.5	84.7	10	100.0	100.0	98.1	87.2	75.1
20	100.0	100.0	100.0	100.0	100.0	20	100.0	100.0	100.0	100.0	100.0
30	100.0	100.0	100.0	100.0	99.1	30	100.0	100.0	100.0	100.0	100.0

(continued)

EXHIBIT A2 *(continued)*

Optimal Allocations, Individual Countries

λ →	0.5	1.0	1.5	2.0	3.0		0.5	1.0	1.5	2.0	3.0
Spain						Sweden					
1	26.9	21.9	21.2	20.2	19.5	1	34.3	27.6	24.4	23.1	21.9
5	33.1	18.4	12.7	10.4	7.7	5	61.4	47.2	42.0	39.0	34.6
10	55.0	36.2	29.4	26.5	22.6	10	75.0	59.2	52.1	47.9	42.3
20	82.7	68.6	58.8	51.8	44.5	20	82.2	67.4	61.2	57.6	51.8
30	99.0	94.5	90.9	88.6	84.4	30	97.7	84.7	74.6	67.9	59.8
Switzerland						UK					
1	19.6	12.5	10.1	8.5	7.7	1	44.4	30.2	25.6	22.9	20.6
5	47.0	26.2	17.6	14.2	9.2	5	100.0	84.4	74.7	69.2	61.9
10	66.8	44.5	32.7	25.5	18.8	10	100.0	100.0	100.0	94.6	88.3
20	82.7	59.7	45.4	37.1	27.4	20	100.0	100.0	100.0	100.0	99.7
30	99.7	78.3	62.3	51.6	39.9	30	100.0	100.0	100.0	100.0	100.0

NOTES: This exhibit shows the optimal allocation to stocks, with the rest allocated to bonds, as selected by maximizing the gain-pain index for several holding periods (1, 5, 10, 20, and 30 years) and risk aversion coefficients (λ = 0.5, 1.0, 1.5, 2.0, and 3.0). The data are described in Exhibit A1 in the appendix.

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REFERENCES

Adler, T., and M. Kritzman. 2007. "Mean-Variance Versus Full-Scale Optimization: In and Out of Sample." *Journal of Asset Management* 7 (5): 302–311.

Arnott, R. 2003. "What Risk Matters? A Call for Papers!" *Financial Analysts Journal* 59 (3): 6–8.

Barro, R. *Getting It Right: Markets and Choices in a Free Society*. Cambridge, MA: MIT Press, 1996.

Barsky, R., T. Juster, M. Kimball, and M. Shapiro. 1997. "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study." *Quarterly Journal of Economics* 112 (2): 537–579.

Baz, J., and H. Guo. 2017. "An Asset Allocation Primer: Connecting Markowitz, Kelly and Risk Parity." *PIMCO Quantitative Research*, October 2017.

Beckmann, D., and L. Menkhoff. 2008. "Will Women Be Women? Analyzing the Gender Difference Among Financial Experts." *Kyklos* 61 (3): 364–384.

Bogle, J. *The Little Book of Common Sense Investing. The Only Way to Guarantee Your Fair Share of Stock Market Returns*. Hoboken, NJ: Wiley. 2017.

Brinson, G., R. Hood, and G. Beebower. 1986. "Determinants of Portfolio Performance." *Financial Analysts Journal* 42 (4): 39–48.

Brinson, G., B. Singer, and G. Beebower. 1991. "Determinants of Portfolio Performance II: An Update." *Financial Analysts Journal* 47 (3): 40–48.

Choueifaty, Y., and Y. Coignard. 2008. "Toward Maximum Diversification." *The Journal of Portfolio Management* 35 (1): 40–51.

Dimson, E., P. Marsh, and M. Staunton. *Triumph of the Optimists. 101 Years of Investment Returns*. Princeton, NJ: Princeton University Press. 2002.

Estrada, J. 2008. "Mean-Semivariance Optimization: A Heuristic Approach." *Journal of Applied Finance* 18 (1): 57–72.

——. 2010. "Geometric Mean Maximization: An Overlooked Portfolio Approach?" *The Journal of Investing* 19 (4): 134–147.

——. 2020. "Target-Date Funds, Glidepaths, and Risk Aversion." *The Journal of Wealth Management* 23 (3): 50–60.

Estrada, J., and M. Kritzman. 2019. "Toward Determining the Optimal Investment Strategy for Retirement." *The Journal of Retirement* 7 (1): 35–42.

Froidure, T., K. Jalalzai, and Y. Choueifaty. 2019. "Portfolio Rho-Presentativity." *International Journal of Theoretical and Applied Finance* 22 (7): 1950034-1-52.

Gandelman, N., and R. Hernández-Murillo. 2015. "Risk Aversion at the Country Level." *Federal Reserve Bank of St. Louis Review* 97 (1): 53–66.

Harvey, C., J. Liechty, M. Liechty, and P. Muller. 2010. "Portfolio Selection with Higher Moments." *Quantitative Finance* 10 (5): 469–485.

Holt, C., and S. Laury. 2002. "Risk Aversion and Incentive Effects." *American Economic Review* 92 (5): 1644–1655.

Ibbotson, R., and P. Kaplan. 2000. "Does Asset Allocation Policy Explain 40, 90, or 100 Percent of Performance?" *Financial Analysts Journal* 56 (1): 26–33.

Johnson, B. 2020. "2019 U.S. Fund Fee Study." *Morningstar*, June 2020.

Kim, J. H., Y. Lee, W. C. Kim, and F. Fabozzi. 2021. "Mean-Variance Optimization for Asset Allocation." *The Journal of Portfolio Management* 47 (5): 24–40.

Kritzman, M., and S. Page. 2002. "Asset Allocation versus Security Selection: Evidence from Global Markets." *Journal of Asset Management* 3 (3): 202–212.

Markowitz, H. 1952. "Portfolio Selection." *The Journal of Finance* 7 (1): 77–91.

——. *Portfolio Selection. Efficient Diversification of Investments*. New York: Wiley & Sons. 1959.

——. 1999. "The Early History of Portfolio Theory: 1600–1960." *Financial Analysts Journal* 55 (4): 5–16.

——. 2010. "Portfolio Theory: As I Still See It." *Annual Review of Financial Economics* 2: 1–23.

Packin, N. G. 2020. "Consumer Finance and AI: The Death of Second Opinions?" *NYU Journal of Legislation & Public Policy* 22 (2): 319–374.

Pan, C., and M. Statman. 2012. "Questionnaires of Risk Tolerance, Regret, Overconfidence, and Other Investor Propensities." *Journal of Investment Consulting* 13 (1): 54–63.

——. 2013. "Investor Personality in Investor Questionnaires." *Journal of Investment Consulting* 14 (1): 48–56.

Shepherd, S. 2017. "Risk: Preparing Clients for an Uncertain Journey." *Research Affiliates Advisor Essentials*, October 2017.

Statman, M. 2000. "The 93.6% Question of Financial Advisors." *The Journal of Investing* 9 (1): 16–20.