Retirement Planning: The Volatility-Adjusted Coverage Ratio

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Abstract

The important decisions that retirees have to make to try to achieve their financial goals during retirement often stem from models used by financial planners. Despite the important role it plays in many of those models, the failure rate has several limitations and many alternatives have been proposed. This article introduces a new metric, the volatility-adjusted coverage ratio, which incorporates the benefit (the coverage ratio) and the cost (the volatility of the portfolio) of the strategies considered. Application of this new metric is illustrated by determining the optimal asset allocation, for several initial withdrawal rates, for 22 global markets.

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1. Introduction

Successful retirement planning involves setting retirees on a path that enables them to make all the withdrawals they planned, including the bequest they decided to leave behind. This is typically achieved by first selecting a methodology with a target variable to maximize or minimize, from which an optimal decision follows. The failure rate often plays an important role in these models, but some shortcomings of this target variable led academics and practitioners to propose alternative metrics.

Estrada and Kritzman (2019) introduced the coverage ratio (the number of years of withdrawals supported by a strategy, relative to the length of the retirement period) to overcome two serious shortcomings of the failure rate, namely, its inability to distinguish between failing early or late into the retirement period, and between leaving a small or a large bequest. They also proposed a kinked utility function that depends on the coverage ratio and suggest to select the strategy that maximizes a retiree's expected utility.

Without disagreeing with the previous approach, Estrada (2023) argues that its main limitation is that most retirees are not familiar with utility functions; hence they may not be able to fully understand an advisor's explanation of the framework that determines some important financial decisions they have to make. To avoid this limitation he proposes to consider the whole distribution of coverage ratios, or some particularly relevant percentiles of it, so that retirees can

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make these important financial decisions by carefully balancing the relevant trade-offs between upside potential and downside protection.

Although the previous framework would not be difficult for retirees to understand, it is not as straightforward as considering a methodology that determines the optimal retirement strategy directly. This is precisely the contribution of the metric proposed in this article, *the volatility-adjusted coverage ratio* (VAC), which integrates the coverage ratio and the volatility of the retirement portfolio. The framework proposed here selects the strategy with the highest VAC, thus yielding the optimal decision directly rather than several trade-offs for a retiree to consider.

The VAC introduced in this article has several attractive characteristics. First, it is based on the coverage ratio, which embeds all the information contained in, plus some additional information ignored by the failure rate. Second, it accounts for the volatility of the strategies considered, which is a cost that retirees have to live with throughout their retirement. And third, it is similar to some widely-used measures of risk-adjusted return, such as the Sharpe ratio, in the sense that it accounts for the benefit of a strategy in the numerator and its cost in the denominator.

The approach proposed is illustrated by highlighting the optimal asset allocation, across several initial withdrawal rates, for 22 global markets and well over a century of data. The overall results show that maximizing the volatility-adjusted coverage ratio typically yields optimal asset allocations that are, first, more conservative than those stemming from maximizing the coverage ratio or the expected utility of the coverage ratio; and second, more consistent with those featured by target-date funds. Put differently, it yields strategies that seem to be more aligned with retirees' preference for downside protection over upside potential.

The rest of the article is organized as follows. Section 2 briefly reviews the literature on the evaluation of retirement strategies and introduces the metric proposed in this article, the volatility-adjusted coverage ratio; section 3 discusses the evidence, based on an extensive database of 22 markets over 120 years, and over a longer (151-year) period for the U.S. market; and section 4 provides an assessment. An appendix with tables concludes the article.

2. The Issue

2.1. A Brief Literature Review¹

It is widely acknowledged that the literature on the evaluation of retirement strategies begins with Bengen's (1994) seminal article, in which he aims to determine a safe withdrawal rate. This pioneering article did more than just give birth to the widely-used '4% rule' as a

¹ This section borrows heavily from Estrada (2023).

withdrawal strategy; in fact, it pioneered the failure rate as a metric to evaluate retirement strategies and inspired a huge literature on the topic.

The failure rate has a neat intuition and is very widely used but, as noted by Milevsky (2016) and others, it is also badly flawed. It suffices to highlight here that this metric is indifferent between two strategies that fail (say) 5 and 25 years into a retirement period, as well as indifferent between two strategies that leave (say) a \$1 million or a \$10 million bequest. Obviously, no retiree would be indifferent between those two failing strategies and between those two successful strategies.

These and other shortcomings of the failure rate led academics and practitioners to propose many alternative metrics to evaluate retirement strategies. To illustrate, Blanchett (2007) proposed the success-to-variability ratio; Frank and Blanchett (2010) proposed the probability of failure; Blanchett et al (2012) proposed the sustainable spending rate; Suarez et al (2015) and Clare et al (2017) proposed the perfect withdrawal amount; Estrada (2017, 2018*a*, 2018*b*, 2018*c*) proposed shortfall years, risk-adjusted success, the maximum withdrawal rate, and downside risk-adjusted success; Estrada and Kritzman (2019) proposed the coverage ratio and an associated utility function; and Estrada (2023) proposed to consider the whole distribution of coverage ratios.

Importantly, the coverage ratio improves upon the failure rate by overcoming the two main limitations of the latter. Hence it distinguishes, first, between a strategy that fails early and another that fails late into a retirement period; and second, between a strategy that leaves a small bequest and another that leaves a large bequest. Formally, for any strategy *i* and any retirement period *t*, let Y_{it} be the number of years of inflation-adjusted withdrawals sustained by the strategy, both during and after the retirement period; and *L* be the length (in years) of the retirement period considered. Then the *coverage ratio* of strategy *i* in retirement period *t* (C_{it}) is defined as

$$C_{it} = \frac{Y_{it}}{L} \tag{1}$$

By definition, $C_{it} < 1$ indicates that the strategy depleted the portfolio before the end of the retirement period (it failed); $C_{it} > 1$ indicates that the strategy sustained the planned withdrawals through the whole retirement period and left a bequest; and $C_{it} = 1$ indicates that the strategy sustained the planned withdrawals through the end of a retirement period and left no bequest.²

Furthermore, the *coverage ratio* of strategy $i(C_i)$ is defined as

² To illustrate, consider a 30-year retirement period, a \$1,000 retirement portfolio, annual inflationadjusted withdrawals of \$40, and three strategies. The first strategy depleted a portfolio in 24 years, the second did so in exactly 30 years, and the third sustained withdrawals for 30 years and left a bequest of \$240 (which can support another six years of \$40 withdrawals). Then, *Y* would be 24, 30, and 36, for the first, second, and third strategies; and *C* would respectively be 0.8, 1.0, and 1.2.

$$C_i = (1/T) \cdot \sum_{t=1}^T C_{it}$$

where *T* is the number of retirement periods in the sample. In words, calculating a strategy's coverage ratio involves two steps: First, calculating the strategy's coverage ratio for each retirement period in the sample; and second, calculating the average of those coverage ratios across all the retirement periods considered.

2.2. The Volatility-Adjusted Coverage Ratio

Unlike the failure rate, the coverage ratio counts the number of withdrawals sustained by a given strategy, both during and after a retirement period, and then puts the resulting figure in relation to the length of the retirement period considered. By doing so it makes it possible to distinguish between strategies that fail early or late into the retirement period, and between those that leave a small or a large bequest. That said, for any strategy that does not fail, the coverage ratio is always increasing in the size of the bequest, thus favoring strategies with low withdrawals, or those in which the portfolio grows aggressively, during retirement. Exhibit 1 illustrates the second point, which is essential to motivate the new metric proposed in this article.

The exhibit is based on a \$1,000 retirement portfolio, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, an initial withdrawal rate of 4%, annual rebalancing, and the most recent retirement period in the sample (1993-2022) for the U.S. market.³ The second, third, and fourth columns show the real (inflation-adjusted) returns of three strategies with an allocation to stocks (*S*) of 90%, 50%, and 10%, with the rest invested in bonds. As the third row from the bottom shows, the volatility of returns (*V*) of these three strategies is 14.8%, 9.2%, and 8.3%.

The last three columns of the exhibit show the evolution of three \$1,000 portfolios based on the returns on the previous three columns and subject to annual withdrawals of \$40. Note that all three strategies sustain 30 years of withdrawals during retirement. The bequests they leave, given by the terminal wealth at year-end 2022, enable 87.9 (=\$3,517/\$40), 50.2 (=\$2,006/\$40), and 19.6 (=\$786/\$40) additional years of withdrawals, for a total of 117.9, 80.2, and 49.6 years of withdrawals for each strategy. Dividing these figures by 30 (the length of the retirement period considered) yields the 3.9, 2.7, and 1.7 coverage ratios shown in the next-to-last row.

(2)

³ The choice of a 4% initial withdrawal rate is solely motivated by the fact that it is the one that has received the most attention from both academics and practitioners; any other figure would have been just as useful for this example. That said, some argue that a 4% initial withdrawal rate may be too optimistic, particularly in a low interest rate environment such as that after Covid; see, for example, Webb (2021).

This exhibit shows the real returns of three strategies with different proportion of stocks (*S*) and bonds (100-S) over the last retirement period in the sample. It also shows the evolution of three \$1,000 portfolios subject to 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, an initial withdrawal rate of 4%, and annual rebalancing. For each strategy it also shows the volatility of its returns (*V*), coverage ratio (*C*), and volatility-adjusted coverage ratio (VAC). Returns are in percent and portfolio values in dollars. The (Shiller) data is described in Exhibit A1 in the appendix.

		Returns			Portfolio	
Year $ S \rightarrow$	90	50	10	90	50	10
1992				960	960	960
1993	7.3	9.0	10.7	990	1,006	1,022
1994	-2.9	-6.0	-9.1	922	906	889
1995	33.3	27.9	22.5	1,189	1,119	1,049
1996	17.3	9.0	0.7	1,354	1,180	1,017
1997	27.3	19.0	10.7	1,684	1,364	1,086
1998	22.4	18.4	14.4	2,021	1,574	1,202
1999	15.5	4.5	-6.4	2,294	1,606	1,085
2000	-6.8	1.1	9.1	2,098	1,584	1,143
2001	-12.2	-4.7	2.8	1,801	1,469	1,135
2002	-18.6	-5.3	7.9	1,426	1,351	1,185
2003	17.9	10.0	2.0	1,641	1,445	1,169
2004	8.4	5.3	2.1	1,740	1,481	1,154
2005	3.1	1.2	-0.6	1,753	1,460	1,107
2006	10.4	6.5	2.5	1,896	1,514	1,095
2007	2.4	3.3	4.2	1,902	1,524	1,100
2008	-33.0	-9.8	13.5	1,234	1,335	1,208
2009	22.2	8.2	-5.7	1,468	1,405	1,100
2010	11.4	8.2	5.0	1,595	1,481	1,115
2011	0.6	5.6	10.6	1,564	1,524	1,193
2012	13.7	8.7	3.7	1,738	1,616	1,197
2013	24.0	9.2	-5.6	2,114	1,724	1,090
2014	14.3	11.6	8.8	2,376	1,883	1,147
2015	1.3	1.2	1.1	2,368	1,866	1,119
2016	8.1	3.4	-1.3	2,521	1,890	1,065
2017	16.6	9.6	2.7	2,898	2,032	1,054
2018	-3.4	-3.1	-2.8	2,759	1,929	984
2019	21.8	16.0	10.2	3,320	2,197	1,045
2020	15.8	12.5	9.3	3,803	2,432	1,101
2021	16.7	4.9	-7.0	4,399	2,511	984
2022	-20.1	-20.1	-20.1	3,517	2,006	786
V	14.8	9.2	8.3			
С				3.9	2.7	1.7
VAC				26.5	29.1	19.8

As this row shows, the coverage ratio (*C*) increases with the allocation to stocks in the portfolios, essentially due to the fact that the bequest (the value of the portfolio at the end of 2022) is also increasing in the aggressiveness of the portfolios. However, as the allocation to stocks increases (from 10% to 90%), so does the volatility of the portfolios (from 8.3% to 14.8%), which is a cost that retirees have to bear throughout their retirement. Enter then the metric introduced in this article, the *volatility-adjusted coverage ratio* of strategy *i* in retirement period *t* (VAC_{*it*}), which is defined as

$$VAC_{it} = \frac{c_{it}}{v_{it}}$$
(3)

where C_{it} and V_{it} denote strategy *i*'s coverage ratio and volatility of returns, both during retirement period *t*. Furthermore, the *volatility-adjusted coverage ratio* of strategy *i* (VAC_{*i*}) is defined as

$$\operatorname{VAC}_{i} = (1/T) \cdot \sum_{t=1}^{T} \operatorname{VAC}_{it} = (1/T) \cdot \sum_{t=1}^{T} \left(\frac{C_{it}}{V_{it}} \right)$$
(4)

where *T* is as before the number of retirement periods in the sample. In words, calculating a strategy's VAC involves two steps: First, calculating the strategy's VAC for each retirement period in the sample (which in turn requires calculating the strategy's coverage ratio, volatility of returns, and the ratio between the former and the latter for each retirement period in the sample); and second, calculating the average of those VACs across all the retirement periods considered.

Recall that the coverage ratios of the three strategies considered in Exhibit 1 are 3.9, 2.7, and 1.7. Then, dividing these figures by the volatilities in the third line from the bottom yields the 26.5 (=3.9/0.148), 29.1 (=2.7/0.092), and 19.8 (=1.7/0.083) volatility-adjusted coverage ratios shown in the last line. Importantly, note that although *C* is monotonically increasing in the allocation to stocks in the portfolios, VAC is not. In fact, the strategy with 90% in stocks is highly penalized by its volatility, thus making the strategy with 50% in stocks the best of the three considered in the exhibit, for this specific retirement period.

In short, the volatility-adjusted coverage ratio introduced here considers the coverage ratio, thus improving upon the failure rate by overcoming the main two limitations of the latter; and considers the volatility of the portfolios underlying each strategy, thus accounting for the cost of those strategies. The next section discusses the evidence from 22 global markets, with special focus on the U.S. market, and highlights the optimal asset allocations for different initial withdrawal rates for all of those markets.

3. Evidence

3.1. Data and Methodology

The data used in this article consists of two samples. One is the Dimson-Marsh-Staunton (DMS) database, described in detail in Dimson et al (2002) and in the annual updates of the database documentation. It contains annual returns for stocks and government bonds over the 1900-2019 period for 21 countries and the World market. All returns are real (adjusted by each country's inflation rate), in local currency, and account for both capital gains/losses and cash flows (dividends or coupons). Real returns for the World market are in dollars and adjusted by the U.S. inflation rate. The 1900-2019 sample period enables the consideration of 91 rolling 30-year retirement periods, beginning with 1900-1929 and ending with 1990-2019.

Although the DMS database includes the U.S., a somewhat longer perspective for this market can be obtained from the data provided by Robert Shiller on his web page, which covers the 1872-2022 period for both stocks and government bonds.⁴ The focus here is on real returns that account for capital gains/losses and cash flows. This longer sample period enables the consideration of 122 rolling 30-year retirement periods, beginning with 1872-1901 and ending with 1993-2022. Exhibit A1 in the appendix reports some summary statistics for all the series of stock and bond returns in both samples.

The analysis is based on a portfolio of 1,000 units of local currency at the beginning of retirement, a 30-year retirement period, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, and annual rebalancing to each of the 11 asset allocations considered. If a strategy does not fail, after the last withdrawal (at the beginning of the last year in retirement) the portfolio compounds for one more year and its terminal value becomes the bequest.

3.2. U.S. Evidence

Exhibit 2, based on the Shiller data over the 1872-2022 period, shows the coverage ratio and volatility-adjusted coverage ratio for 11 strategies with different allocations to stocks (*S*) and bonds (100–*S*), for nine initial withdrawal rates (IWR) between 2% and 6%, across the 122 retirement periods in the sample. For each IWR, the strategy highlighted is the asset allocation that maximizes *C* or VAC. As the exhibit clearly shows, a framework of maximizing *C* leads to portfolios fully invested in stocks for all the IWRs considered. Needless to say, this strategy may be far too aggressive for many retirees to live with.

A framework that selects the optimal strategy by maximizing VAC, on the other hand, yields far more plausible results. Across all the IWRs considered, the highest allocation to stocks is 60% (for IWR=2.0%) and the lowest is 20% (for IWR=6.0%). For the ubiquitous 4% IWR, the methodology proposed here selects and optimal allocation of 50% to stocks; for perspective, this is the allocation that Vanguard's target-date funds have by the retirement date.⁵

Adjusting coverage ratios by volatility, then, does make a substantial difference in the selection of an optimal retirement strategy. The volatility of a portfolio is indeed a cost that retirees have to bear during retirement, and whether or not this cost is accounted for leads (and more generally, is likely to lead) to very different choices. The next section explores whether the more comprehensive global evidence confirms this result.

⁴ <u>http://www.econ.yale.edu/~shiller/data.htm</u>.

⁵ According to Vanguard's glidepath, this 50-50 stock-bond allocation gets gradually more conservative during the first seven years in retirement, and then settles on a 30-70 allocation from that point on.

Exhibit 2: C vs. VAC - Optimal Asset Allocations - USA - 1872-2022

This exhibit shows the coverage ratio (*C*) and volatility-adjusted coverage ratio (VAC) for asset allocations with different proportion of stocks (*S*) and bonds (100-S), for different initial withdrawal rates (IWR), across 122 rolling 30-year retirement periods. The analysis is based on a \$1,000 retirement portfolio, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, and annual rebalancing. For each IWR, the figures highlighted indicate the optimal asset allocation as selected by maximizing *C* and VAC. The (Shiller) data is described in Exhibit A1 in the appendix.

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$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
IWR=2.0%											
С	10.00	9.09	8.21	7.39	6.61	5.89	5.23	4.62	4.07	3.58	3.14
VAC	58.38	58.42	58.55	58.73	<mark>58.88</mark>	58.78	58.06	56.14	52.42	46.60	39.10
<u>IWR=2.5%</u>											
С	<mark>7.51</mark>	6.82	6.17	5.56	4.98	4.44	3.95	3.50	3.09	2.73	2.41
VAC	43.86	43.93	44.07	44.25	44.42	<mark>44.43</mark>	43.99	42.66	40.00	35.75	30.22
<u>IWR=3.0%</u>											
С	<mark>5.84</mark>	5.32	4.81	4.34	3.89	3.48	3.10	2.75	2.44	2.17	1.92
VAC	34.18	34.27	34.41	34.60	34.79	<mark>34.86</mark>	34.60	33.68	31.72	28.52	24.22
<u>IWR=3.5%</u>											
С	<mark>4.65</mark>	4.24	3.84	3.46	3.11	2.79	2.49	2.22	1.98	1.76	1.57
VAC	27.27	27.37	27.51	27.70	27.90	28.03	27.90	27.26	25.81	23.30	20.19
<u>IWR=4.0%</u>											
С	<mark>3.77</mark>	3.43	3.11	2.81	2.53	2.27	2.03	1.82	1.63	1.46	1.32
VAC	22.10	22.20	22.34	22.53	22.74	22.90	22.88	22.44	21.34	19.62	17.25
<u>IWR=4.5%</u>											
С	<mark>3.08</mark>	2.81	2.55	2.31	2.08	1.87	1.68	1.52	1.37	1.24	1.13
VAC	18.14	18.22	18.36	18.55	18.77	18.96	19.01	18.80	18.17	16.98	15.10
<u>IWR=5.0%</u>											
С	<mark>2.56</mark>	2.33	2.11	1.92	1.73	1.57	1.42	1.29	1.18	1.08	0.99
VAC	15.05	15.14	15.26	15.46	15.71	15.97	16.12	<mark>16.13</mark>	15.83	14.95	13.46
<u>IWR=5.5%</u>	0.4.4	4.05	4 50			1.00	4.00	4.4.0	4.00	0 0 -	
C	<mark>2.14</mark>	1.95	1.78	1.61	1.47	1.33	1.22	1.12	1.03	0.95	0.87
VAC	12.64	12.71	12.86	13.06	13.30	13.62	13.93	14.03	13.94	13.50	12.01
<u>IWR=6.0%</u>	4.04	4 6 -		4.05	1.0.6		1.0.6				
C	1.81	1.65	1.51	1.37	1.26	1.15	1.06	0.98	0.90	0.84	0.78
VAC	10.68	10.76	10.92	11.14	11.44	11.78	12.17	12.48	12.63	12.09	10.82

3.3. Global Evidence

The performance of stocks and bonds in the rest of the countries in the sample, which as Exhibit A1 shows has been in many cases very different from that of the U.S., may lead retirees in those countries to make different choices from those made by U.S. retirees. In order to explore this issue, Exhibit A2 in the appendix reports, for several asset allocations and IWRs, the optimal strategy selected by maximizing *C* and VAC for the 21 countries in the DMS sample. Exhibit 3 below shows averages across all those 21 countries, as well as results for the World market.⁶

The results in this exhibit are different from, but consistent with, those already discussed for the longer sample period for the U.S. in Exhibit 2. To elaborate, maximizing the coverage ratio leads to a portfolio fully invested in stocks, both in the average country in the sample as well as in the World market; as already mentioned, this strategy may be difficult for many retirees to live

⁶ The difference between them is that in panel A all countries have the same weight whereas in panel B countries are weighted by their market cap (as done by construction in the World index).

with. Maximizing the volatility-adjusted coverage ratio, on the other hand, leads to far more conservative portfolios, again both in the average country and in the World market. In the latter case, optimal strategies allocate between 30% and 40% to stocks, and 40% for the ubiquitous 4% IWR. This last figure is not too different from the 50% allocation for the U.S. market shown in Exhibit 2.

for the World	for the World market. For each IWR, the figures highlighted indicate the optimal asset allocation as selected by maximizing <i>C</i> and VAC. The (DMS) data is described in Exhibit A1 in the appendix.										
by maximizi	ng C and	VAC. The	e (DMS)	data is de	escribed	in Exhib	it A1 in t	he appei	ndix.		
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
A: Avg											
<u>IWR=3.0%</u>											
С	<mark>4.58</mark>	4.30	4.00	3.71	3.41	3.12	2.84	2.56	2.30	2.06	1.84
VAC	21.20	21.48	21.78	22.11	22.45	22.78	23.02	<mark>23.05</mark>	22.63	21.45	19.15
<u>IWR=3.5%</u>											
С	<mark>3.66</mark>	3.43	3.20	2.96	2.73	2.50	2.28	2.06	1.86	1.67	1.50
VAC	16.97	17.19	17.44	17.72	18.01	18.30	18.52	<mark>18.60</mark>	18.34	17.50	15.78
<u>IWR=4.0%</u>											
С	<mark>2.98</mark>	2.79	2.60	2.42	2.23	2.04	1.86	1.69	1.53	1.39	1.25
VAC	13.85	14.04	14.26	14.51	14.77	15.03	15.25	15.37	15.23	14.66	13.39
<u>IWR=4.5%</u>											
С	<mark>2.46</mark>	2.31	2.15	2.00	1.84	1.69	1.55	1.41	1.29	1.17	1.07
VAC	11.50	11.66	11.85	12.08	12.32	12.57	12.80	<mark>12.98</mark>	12.96	12.55	11.60
<u>IWR=5.0%</u>											
С	<mark>2.05</mark>	1.93	1.80	1.67	1.55	1.42	1.31	1.20	1.10	1.01	0.93
VAC	9.68	9.83	10.01	10.21	10.46	10.71	10.97	11.18	11.23	10.96	10.33
<u>B: World</u>											
<u>IWR=3.0%</u>											
С	<mark>4.29</mark>	3.98	3.69	3.42	3.17	2.94	2.72	2.52	2.33	2.15	1.98
VAC	24.99	24.96	25.06	25.28	25.58	25.89	<mark>26.07</mark>	25.90	24.92	23.01	20.42
<u>IWR=3.5%</u>											
С	<mark>3.41</mark>	3.16	2.93	2.72	2.52	2.34	2.18	2.02	1.88	1.74	1.61
VAC	19.80	19.79	19.88	20.10	20.41	20.73	<mark>20.92</mark>	20.76	20.04	18.58	16.57
<u>IWR=4.0%</u>		~	-				. = 0				
С	<mark>2.74</mark>	2.55	2.37	2.20	2.04	1.90	1.78	1.66	1.54	1.44	1.34
VAC	15.97	16.01	16.11	16.32	16.57	16.83	17.04	16.96	16.45	15.34	13.95
<u>IWR=4.5%</u>	2.2.4	2.00	1.00	1.00	1 (0	1 69	1 4 7	1 20	1 20	1.01	1 1 0
C	2.24	2.08	1.93	1.80	1.68	1.57	1.47	1.38	1.29	1.21	1.13
VAC	13.07	13.12	13.21	13.37	13.61	13.89	14.12	14.12	13.77	13.24	12.24
<u>IWR=5.0%</u> C	1.84	1.71	1.60	1 50	1.40	1 0 1	1.24	116	1 1 0	1.02	0.98
<i>C</i> VAC	1.84 10.82	1.71	1.60 10.92	1.50 11.08	1.40 11.36	1.31		1.16 <mark>12.25</mark>	1.10 12.16	1.03	
VAL	10.82	10.03	10.92	11.00	11.30	11.61	11.93	12.23	12.10	11.71	10.84

Exhibit 3: C vs. VAC - Optimal Asset Allocations - Global - 1900-2019

This exhibit shows the coverage ratio (*C*) and volatility-adjusted coverage ratio (VAC) for asset allocations with different proportion of stocks (*S*) and bonds (100-S), for different initial withdrawal rates (IWR), across 91 rolling 30-year retirement periods. The analysis is based on a 1,000 retirement portfolio, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, and annual rebalancing. Panel A shows averages across the 21 countries in Exhibit A2 in the appendix, and panel B shows results for the World market. For each IWR, the figures highlighted indicate the optimal asset allocation as selected by maximizing *C* and VAC. The (DMS) data is described in Exhibit A1 in the appendix

Exhibit A2 unsurprisingly shows a wide variety of results across the 21 markets considered. In a few countries, such as Australia, Japan, and the UK, maximizing *C* or VAC leads to the same optimal strategy (100% stocks). In some other countries, such as Belgium, Germany, or Switzerland, maximizing *C* or VAC leads to substantially different optimal strategies. Finally, in *all*

countries the optimal strategy selected by maximizing VAC is more conservative, or at least not more aggressive, than that selected by maximizing *C*.

In short, the results from this comprehensive global evidence do confirm and reinforce the previous results already discussed for the U.S. market. The volatility of a portfolio is indeed a cost that retirees have to bear during retirement, and whether or not this cost is accounted for does generally have a significant impact on the retirement strategy ultimately selected.

3.4. Some Final Thoughts

A few comments and caveats may be a useful way to round up the discussion on retirement planning, competing methodologies, and optimal strategies. Regarding caveats, as is typically the case in this literature, the analysis here does not account for taxes and transaction costs. Both are substantially different across countries, and have also changed substantially over time, which is part of the reason why they are often ignored.

Maximizing the volatility-adjusted coverage ratio is obviously related to the coverage ratio introduced by Estrada and Kritzman (2019), as well as to the consideration of the distributions of coverage ratios advanced by Estrada (2023). In addition, relative to other metrics proposed in the literature, the volatility-adjusted coverage ratio is most closely related to the success-to-variability ratio proposed by Blanchett (2007); this is the case because both metrics consider the benefit of a strategy in the numerator, the (same) cost in the denominator, and a ultimately a ratio between the former and the latter.⁷

The VAC introduced in this article, just as the coverage ratio, the failure rate, and most other metrics used to select an optimal withdrawal strategy is an ad-hoc metric and therefore not necessarily consistent with expected utility maximization. That said, this last criterion, used by Estrada and Kritzman (2019) among others, is not devoid of criticism; behavioral economists reject it as not representing the way that 'normal' (as opposed to rational) individuals actually make decisions.

The analysis implicitly assumes that retirees in any given country are only exposed to (can only invest in) their own country. Although such an assumption may have been plausible some decades ago, it is clearly unrealistic nowadays. In fact, it is becoming increasingly possible for investors to diversify their portfolios globally at a very low cost; for this reason, the results for

⁷ Most measures of risk-adjusted return, such as the Sharpe ratio, the Treynor ratio, or the Sortino ratio have the benefit of a strategy in the numerator and its cost in the denominator. The cost of a strategy, in turn, is typically represented by a measure of risk, such as volatility, beta, or the semideviation, which is explained by the fact that risk is ultimately a cost that investors have to bear. The VAC introduced here can be viewed in a similar way. That said, unlike the Sharpe, Treynor, and Sortino ratios, which aim to select an optimal *investment* strategy, the VAC aims to select an optimal *retirement* strategy.

the World market may be particularly informative.⁸ Interestingly, a comparison between panel A (for the average country in the sample) and panel B (for the world market) of Exhibit 3 shows that, for each IWR and asset allocation considered, the VAC for the globally-diversified investor is always higher than that for the investor in the average closed or segmented market; put differently, global diversification enhances the prospects of retirees.

That said, investors in different countries have obtained different benefits from global diversification, and for some global diversification may have even been detrimental. A comparison of Exhibit 2 and panel B of Exhibit 3 shows that the VAC for U.S. investors is higher than that for the World market for each IWR and asset allocation considered in both cases. Put differently, at least as far as retirement strategies is concerned, it could be argued that U.S. investors would have been better off investing just in their own country.⁹

The analysis for global markets spans over a very long period, beginning in 1900, and that for the U.S. begins even further back, in 1872. It may be argued, then, that the sample captures a long period in which the world and the investing landscape was very different from what it has been in more recent times. That is of course true; what is arguable, however, is whether the 'old' data should be discarded in order to focus only on a more recent period. Plausible arguments can be made to defend both viewpoints, but that discussion is beyond the scope of this article. That said, Exhibit A3 in the appendix shows, for the case of the U.S., that using the full (1872-2022) sample period or a post-war (1945-2022) sample period does not imply a dramatic change in the optimal asset allocations for the IWRs considered.

Finally, recall that Exhibit A2 presents a comprehensive analysis of optimal choices given several asset allocations and IWRs across 21 countries, and Exhibit 3 summarizes those results with averages across all those countries. In order to assess better the differences between the optimal strategies suggested by maximizing the coverage ratio and the volatility-adjusted coverage ratio, Exhibit 4 isolates the results for the widely-used 4% IWR. The exhibit also shows, for additional perspective, the optimal strategies that result from maximizing the expected utility of coverage ratios, as reported in Exhibit 2 in Estrada and Kritzman (2019).¹⁰ All the figures in the exhibit show the optimal allocation to stocks, with the rest invested in bonds.

⁸ To clarify, the analysis does not really *assume* that retirees can only invest in their own country. The analysis is based on the performance of stocks and bonds in each individual country, although that does not necessarily imply either that the performance of stocks and bonds in any given country affected only the retirees of that country, or that the retirees of that country were not able to invest in other countries. ⁹ The reasons for this could be many and varied and are likely to be related to the exceptionalism of the U.S. as place to do business and invest. I thank a co-editor, Anthony Webb, for highlighting this issue. ¹⁰ Both Exhibit A2 in this article and Exhibit 2 in Estrada and Kritzman (2019) are based on the same DMS

data, but the sample in the latter article is five years shorter (between 1900 and 2014). Furthermore, their Exhibit 2 refers to the base case they consider; they also perform a subsequent sensitivity analysis.

Exhibit 4: Optimal Asset Allocations Across Methodologies

This exhibit shows the optimal proportion of stocks, with the rest invested in bonds, as selected by maximizing the expected utility of coverage ratios (Max EU), maximizing the coverage ratio (Max *C*), and maximizing the volatility-adjusted coverage ratio (Max VAC). The analysis is based on a 1,000 retirement portfolio, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, an initial withdrawal rate of 4%, and annual rebalancing. The (DMS) data is described in Exhibit A1 in the appendix.

			0	<u> </u>			
Country	Max EU	Max C	Max VAC	Country	Max EU	Max C	Max VAC
Australia	100	100	100	Netherlands	90	100	30
Austria	80	60	10	New Zealand	100	100	100
Belgium	100	100	20	Norway	90	90	20
Canada	100	100	40	Portugal	60	100	100
Denmark	90	100	50	South Africa	100	100	100
Finland	100	100	20	Spain	70	100	40
France	100	70	10	Sweden	60	100	60
Germany	100	100	10	Switzerland	70	100	10
Ireland	100	100	60	UK	100	100	100
Italy	100	100	10	USA	100	100	60
Japan	90	100	100	Average	90.5	96.2	50.0

Maximizing the coverage ratio yields the most aggressive strategies in most countries, with a 96.2% average allocation to stocks across all countries. Maximizing the expected utility of coverage ratios also yields very aggressive strategies in most countries, with a slightly lower (90.5%) average allocation to stocks across all countries. Maximizing the volatility-adjusted coverage ratio, on the other hand, yields far more conservative strategies in most countries, with an average allocation to stocks of 50%. Therefore, given that the discussion here focuses on strategies to be implemented during the retirement period, the methodology proposed in this article seems to suggest the most plausible recommendations.

4. Assessment

After finishing their working period most retirees move to a phase in which they periodically spend more than they receive, a situation that calls for paying special attention to two critical decisions they have to make. The first is the asset allocation of their portfolio, for which they need to balance the growth prospects of their nest egg against their tolerance for risk; the second is their periodic withdrawals, for which they need to balance the risk of spending too much and running out of money too early, against the risk of spending too little and foregoing a higher standard of living in retirement.

Regardless of whether retirees make these decisions by themselves or with the help of a financial planner, the standard methodology calls for choosing a variable to maximize or minimize, perhaps subject to some relevant constraints, and let the model determine the optimal strategy to be implemented. In many of these models the failure rate plays a prominent role, but several limitations of this variable call for a better metric.

The coverage ratio is one of those metrics and improves upon the failure rate both in failure and in success. This is the case because when a strategy fails, the coverage ratio accounts

for when it fails; and when a strategy succeeds, it accounts for the size of the bequest left. For the failure rate, failing early or late into the retirement period, or leaving a small or a large bequest, makes no difference at all, which is clearly unrealistic.

The coverage ratio can be used together with a utility function, as proposed by Estrada and Kritzman (2019); or together with its whole distribution, as proposed by Estrada (2023). Although both approaches improve upon selecting a retirement strategy by minimizing the (or settling for an acceptable) failure rate, each comes with a drawback. Using utility functions may leave retirees without fully understanding the framework used to make important financial decisions; and using distributions provides retirees with trade-offs to balance themselves rather than with a clear path to follow.

This article introduces a new metric, the volatility-adjusted coverage ratio, which aims to overcome these two deficiencies. In fact, it avoids the use of utility functions, which most retirees are not familiar with; and avoids offering trade-offs to balance, which is more cumbersome than suggesting a specific policy to implement. The metric proposed here is based on the coverage ratio, thus improving upon the failure rate; accounts for the volatility of the strategies considered, which is a cost that retirees have to live with; and has a similar construction to that of other widely-used measures of risk-adjusted return.

The comprehensive evidence from 22 global markets and well over a century of data considered yields additional support to the metric introduced in this article. In fact, the volatility-adjusted coverage ratio determines more plausible asset allocations than the coverage ratio (either used by itself or together with a utility function). Part of that plausibility stems from the fact that the strategies suggested by the metric advanced here are fairly consistent with those implemented by widely-used target-date funds.

Finding the best methodology to help retirees with the important financial decisions they have to make is an ongoing endeavor. Our understanding of metrics and frameworks has been evolving over time, a process that is likely to continue. It would be implausible for any academic or practitioner to claim that the holy grail has been found and no further evolution could be expected; needless to say, that claim is not made here. That said, the new metric introduced in this article is hopefully considered another step forward in the evolution of the tools we have to help retirees enjoy a higher standard of living in retirement.

Appendix

Exhibit A1: Summary Statistics

This exhibit shows, for the series of annual returns, the geometric mean return (MR), standard deviation (SD), lowest return (Min), highest return (Max), and equity risk premium (ERP) defined as the difference between the mean returns reported for stocks and bonds. All returns are real (adjusted by each country's inflation rate), in local currency (except for the World market, in dollars), account for capital gains/losses and cash flows (dividends or coupons), and over the 1900-2019 period (with the exception of the Shiller data on the first line, which is over the 1872-2022 period). All figures in %.

		Sto	ocks			Во	nds		ERP
	MR	SD	Min	Max	MR	SD	Min	Max	
USA-Shiller	6.8	17.6	-38.8	51.5	2.5	8.7	-20.2	31.4	4.3
Australia	6.8	17.4	-42.5	51.5	1.8	13.0	-26.6	62.2	5.0
Austria	1.0	30.4	-59.6	132.7	-3.5	53.1	-94.7	484.8	4.5
Belgium	2.6	23.2	-48.9	105.1	0.5	14.8	-45.6	62.3	2.1
Canada	5.7	16.8	-33.8	55.2	2.2	10.2	-25.9	41.7	3.5
Denmark	5.6	20.6	-49.2	107.8	2.1	12.8	-27.6	63.6	3.5
Finland	5.5	29.4	-60.8	161.7	0.3	13.4	-69.5	30.2	5.2
France	3.4	22.8	-41.5	66.1	0.3	12.8	-43.5	35.9	3.1
Germany	3.3	31.2	-90.8	154.6	-1.2	15.4	-95.0	62.5	4.5
Ireland	4.4	22.8	-65.4	68.4	1.7	14.8	-34.1	61.2	2.6
Italy	2.2	28.2	-72.9	120.7	-0.9	14.6	-64.3	35.5	3.1
Japan	4.2	29.1	-85.5	121.1	-0.8	19.3	-77.5	69.8	5.0
Netherlands	5.1	21.1	-50.4	101.6	1.8	9.6	-18.1	32.8	3.3
N. Zealand	6.4	19.1	-54.7	105.3	2.3	8.9	-23.7	34.1	4.2
Norway	4.4	26.4	-53.6	166.9	1.8	11.8	-48.0	62.1	2.6
Portugal	3.6	33.8	-76.6	151.8	-1.3	18.0	-45.1	90.6	5.0
S. Africa	7.1	21.8	-52.2	102.9	1.9	10.3	-32.6	37.1	5.2
Spain	3.6	21.6	-43.3	99.4	2.0	12.4	-30.2	53.2	1.6
Sweden	6.0	20.9	-42.5	67.5	2.7	12.5	-37.0	68.2	3.2
Switzerland	4.6	19.3	-37.8	59.4	2.4	9.3	-21.4	56.1	2.3
UK	5.5	19.5	-56.6	99.3	1.9	13.4	-29.9	59.4	3.6
USA	6.5	19.8	-38.6	55.8	2.0	10.3	-18.1	35.2	4.5
World	5.2	17.3	-41.5	67.6	2.0	10.9	-31.6	46.0	3.2

Exhibit A2: C vs. VAC - Optimal Asset Allocations - Global - 1900-2019

This exhibit shows the coverage ratio (*C*) and volatility-adjusted coverage ratio (VAC) for asset allocations with different proportion of stocks (*S*) and bonds (100-S), for different initial withdrawal rates (IWR), across 91 rolling 30-year retirement periods. The analysis is based on a 1,000 retirement portfolio, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, and annual rebalancing. For each IWR, the figures highlighted indicate the optimal asset allocation as selected by maximizing *C* and VAC. The (DMS) data is described in Exhibit A1.

$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
Australia											
<u>IWR=3.0%</u>											
С	<mark>6.34</mark>	5.71	5.14	4.61	4.11	3.66	3.24	2.86	2.52	2.21	1.93
VAC	<mark>39.76</mark>	37.24	34.97	32.97	31.22	29.63	27.94	25.83	22.95	19.52	16.11
<u>IWR=3.5%</u>											
С	5.08	4.57	4.10	3.67	3.28	2.91	2.58	2.28	2.01	1.77	1.58
VAC	31.89	29.81	27.96	26.33	24.91	23.64	22.34	20.65	18.41	15.80	13.28
<u>IWR=4.0%</u>											
С	4.14	3.72	3.33	2.98	2.65	2.36	2.09	1.85	1.65	1.47	1.33
VAC	<mark>25.99</mark>	24.26	22.70	21.35	20.22	19.24	18.18	16.84	15.14	13.24	11.21
<u>IWR=4.5%</u>	2 4 1	2.07	274	2.4.4	2 1 0	1.0.4	1 70	1 6 4	1 20	1.25	1 1 2
C	3.41	3.06	2.74	2.44	2.18	1.94	1.72	1.54	1.38	1.25	1.13
VAC	21.42	19.95	18.66	17.58	16.65	15.84	15.04	14.05	12.81	11.24	9.61
<u>IWR=5.0%</u> C	<mark>2.83</mark>	2.54	2.27	2.02	1.81	1.61	1.44	1.30	1.19	1.08	0.99
VAC	2.83 17.80	2.34 16.58	15.50	2.02 14.61	13.85	13.21	12.66	1.50	10.96	1.08 9.68	0.99 8.45
Austria	17.00	10.00	10.00	1 1.01	10.00	10.21	12.00	11.70	10.70	2.00	0.10
<u>IWR=3.0%</u>											
С	2.14	2.36	2.50	2.58	<mark>2.58</mark>	2.52	2.40	2.25	2.07	1.87	1.66
VAC	7.17	8.74	10.13	11.47	12.80	14.42	16.51	19.22	22.34	24.34	22.04
<u>IWR=3.5%</u>											
С	1.71	1.89	2.01	2.07	2.08	2.03	1.95	1.83	1.68	1.52	1.36
VAC	5.89	7.09	8.21	9.23	10.35	11.61	13.32	15.46	17.94	19.53	17.69
<u>IWR=4.0%</u>											
С	1.40	1.55	1.65	1.70	<mark>1.71</mark>	1.67	1.60	1.51	1.39	1.27	1.13
VAC	4.89	5.91	6.85	7.78	8.66	9.66	10.98	12.66	14.65	<mark>15.92</mark>	14.42
<u>IWR=4.5%</u>											
С	1.17	1.28	1.37	1.42	1.43	1.40	1.34	1.26	1.17	1.06	0.95
VAC	4.18	5.02	5.84	6.57	7.36	8.26	9.33	10.65	12.20	<mark>13.14</mark>	12.25
<u>IWR=5.0%</u>					1.0.1						
C	1.00	1.09	1.16	1.20	1.21	1.19	1.14	1.08	0.99	0.91	0.81
VAC	3.74	4.36	5.03	5.64	6.29	7.06	7.98	9.23	10.56	11.33	<mark>11.88</mark>
<u>Belgium</u>											
<u>IWR=3.0%</u>	2.07	2.07	2.01	274	264	ງ⊏ງ	2 40		2 1 0	105	1 70
C VAC	<mark>2.87</mark> 13.64	2.86 14.74	2.81 15.84	2.74 16.97	2.64 18.10	2.53 19.24	2.40 20.32	2.25 21.27	2.10 <mark>21.84</mark>	1.95 21.65	1.79 20.38
VAC <u>IWR=3.5%</u>		14./4	15.04	10.97	10.10	17.24	20.32	L1.L/	21.04	21.05	20.30
<u>тик-з.э</u> С	<mark>2.29</mark>	2.27	2.24	2.19	2.11	2.03	1.93	1.81	1.70	1.58	1.46
VAC	10.84	11.71	12.60	13.50	14.43	15.34	16.26	17.04	17.52	17.36	16.39
<u>IWR=4.0%</u>	10.01	11./1	12.00	10.00	1 1.15	10.01	10.20	17.01	17:04	17.00	10.07
<u>с по ж</u>	1.86	1.85	1.83	1.79	1.73	1.66	1.58	1.49	1.40	1.31	1.21
VĂC	8.82	9.53	10.26	11.02	11.81	12.58	13.31	14.01	14.40	14.26	13.40
<u>IWR=4.5%</u>	-		-	_	_ `		-			-	-
С	<mark>1.54</mark>	1.54	1.52	1.49	1.44	1.38	1.32	1.25	1.18	1.10	1.03
VAC	7.35	7.97	8.59	9.27	9.90	10.58	11.22	11.73	12.03	11.92	11.23
<u>IWR=5.0%</u>											
С	<mark>1.30</mark>	1.30	1.29	1.26	1.22	1.18	1.13	1.07	1.01	0.95	0.89
VAC	6.27	6.80	7.34	7.90	8.47	9.01	9.53	9.98	10.25	10.15	9.64
										(C_{0})	ntinues

Exhibit $A_2 \cdot C_{VS} V A C = 0$	Optimal Asset Allocations -	- Global - 1900-20)19 (Cont)
$\mathbf{L}\mathbf{X}\mathbf{H}\mathbf{D}\mathbf{H}\mathbf{L}\mathbf{A}\mathbf{L}\mathbf{L}\mathbf{U}\mathbf{V}\mathbf{S}\mathbf{V}\mathbf{A}\mathbf{U}=\mathbf{U}$	ipumai Assel Anocations -	- GIUDAI - 1900-20	JIJ (CUIIL)

Exhibit A2: C vs. VAC – Optimal Asset Allocations – Global – 1900-2019 (Cont.)											
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
<u>Canada</u>											
<u>IWR=3.0%</u>											
С	<mark>5.12</mark>	4.71	4.34	3.99	3.67	3.37	3.10	2.84	2.61	2.40	2.21
VAC	29.53	29.96	30.57	31.31	32.10	32.76	<mark>32.97</mark>	32.30	30.40	27.31	23.53
<u>IWR=3.5%</u>	1.00	- 	o 4 =		0.04	0.40	0.44	2.24		4.00	4 = 0
C	4.08	3.75	3.45	3.17	2.91	2.68	2.46	2.26	2.08	1.92	1.78
VAC	23.54	23.87	24.34	24.92	25.55	26.09	<mark>26.28</mark>	25.81	24.35	22.03	19.29
<u>IWR=4.0%</u> C	3.30	3.03	2.79	2.56	2.35	2.16	1.99	1.83	1.70	1.58	1.47
VAC	19.05	3.03 19.30	19.66	2.50	2.55	2.10	21.39 21.30	21.02	20.11	18.47	16.26
<u>IWR=4.5%</u>	17.05	17.50	17.00	20.12	20.03	21.11	<mark>21.30</mark>	21.02	20.11	10.47	10.20
<u>rwn=4.570</u> C	<mark>2.69</mark>	2.47	2.27	2.08	1.92	1.77	1.64	1.52	1.42	1.33	1.25
VĂC	15.57	15.74	16.02	16.44	16.89	17.33	17.64	17.62	17.05	15.82	14.07
IWR=5.0%	10107	1017 1	10.01	10.11	10107	17100	1/101	17102	17100	10102	1 1107
С	2.21	2.03	1.86	1.72	1.59	1.47	1.37	1.29	1.21	1.14	1.08
VAC	12.82	13.00	13.26	13.63	14.10	14.57	14.92	15.08	14.73	13.72	12.34
<u>Denmark</u>											
IWR=3.0%											
С	<mark>4.32</mark>	4.18	4.00	3.80	3.57	3.32	3.06	2.80	2.54	2.28	2.04
VAC	22.35	23.16	23.82	24.28	<mark>24.49</mark>	24.45	24.13	23.45	22.19	20.37	18.00
<u>IWR=3.5%</u>											
С	<mark>3.44</mark>	3.32	3.18	3.02	2.84	2.65	2.44	2.23	2.03	1.84	1.66
VAC	17.83	18.49	19.04	19.43	<mark>19.63</mark>	19.63	19.39	19.04	18.26	16.85	14.89
<u>IWR=4.0%</u>	_										
С	2.77	2.68	2.57	2.44	2.29	2.14	1.98	1.82	1.67	1.53	1.39
VAC	14.42	14.99	15.44	15.78	15.94	<u>16.17</u>	16.08	15.99	15.36	14.09	12.58
<u>IWR=4.5%</u> C	<mark>2.26</mark>	2.18	2.09	1.00	1.87	1.75	1 6 4	1 5 2	1 4 0	1 20	1.19
VAC	<mark>2.20</mark> 11.78	2.18 12.20	2.09 12.60	1.99 13.05	13.40	1.75	1.64 <mark>13.70</mark>	1.52 13.50	1.40 12.98	1.29 12.21	1.19
<u>IWR=5.0%</u>	11.70	12.20	12.00	15.05	13.40	15.00	15.70	15.50	12.70	12.21	11.07
<u>тик=5.0 %</u> С	1.85	1.79	1.72	1.64	1.56	1.47	1.38	1.29	1.20	1.11	1.03
VĂC	9.70	10.19	10.76	11.13	11.49	11.65	11.65	11.58	11.36	10.74	9.85
Finland											
<u>IWR=3.0%</u>											
С	<mark>6.34</mark>	5.96	5.52	5.02	4.49	3.93	3.38	2.85	2.36	1.93	1.55
VAC	19.28	19.76	20.13	20.42	20.66	20.91	21.22	21.63	22.01	21.49	17.92
<u>IWR=3.5%</u>											
С	<mark>5.03</mark>	4.74	4.39	3.99	3.57	3.14	2.70	2.29	1.90	1.56	1.27
VAC	15.31	15.70	16.00	16.28	16.49	16.71	16.97	17.32	<mark>17.67</mark>	17.28	14.49
<u>IWR=4.0%</u>											
С	<mark>4.06</mark>	3.83	3.55	3.23	2.89	2.55	2.20	1.87	1.56	1.29	1.07
VAC	12.41	12.74	13.01	13.21	13.40	13.58	13.80	14.10	<mark>14.40</mark>	14.15	12.31
<u>IWR=4.5%</u>	2 2 2	0.40	2.04	2.45	0.05	2.00	1.04	4 66	1.04	1 4 0	0.00
C	3.32	3.13	2.91	2.65	2.37	2.09	1.81	1.55	1.31	1.10	0.92
VAC	10.19	10.46	10.69	10.88	11.04	11.20	11.41	11.69	12.06	12.13	10.56
<u>IWR=5.0%</u> C	2 74	250	2 4 0	2 10	1.07	1 70	1 5 1	1 20	1 1 1	0.05	0.01
<i>C</i> VAC	<mark>2.74</mark> 8.45	2.59	2.40 8.88	2.19 9.04	1.97 9.19	1.73 9.34	1.51 9.58	1.30 0.01	1.11	0.95 <mark>10.40</mark>	0.81 0.10
VAL	8.45	8.69	0.00	7.04	7.17	7.34	7.50	9.91	10.38		9.19
										ננס	ntinues)

Exhibit A2									-	-	
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
<u>France</u> <u>IWR=3.0%</u>											
С	3.07	3.16	3.22	<mark>3.24</mark>	3.23	3.18	3.09	2.97	2.82	2.64	2.45
VAC	12.57	14.14	15.83	17.73	19.83	22.10	24.61	26.96	28.67	29.14	27.84
<u>IWR=3.5%</u>											
С	2.45	2.53	2.58	<mark>2.60</mark>	2.59	2.56	2.49	2.40	2.28	2.14	1.98
VAC	10.07	11.33	12.74	14.28	16.09	17.97	19.88	21.70	23.03	23.40	22.34
<u>IWR=4.0%</u> C	2.00	2.06	2.10	2 1 2	2 1 2	2 1 0	2.05	1.07	1 00	176	1 6 2
VAC	2.00 8.28	2.06 9.39	10.62	<mark>2.13</mark> 11.95	2.13 13.33	2.10 14.81	2.05 16.32	1.97 17.71	1.88 18.85	1.76 <mark>19.15</mark>	1.63 18.36
<u>IWR=4.5%</u>	0.20).5)	10.02	11.75	15.55	14.01	10.52	1/./1	10.05	17.15	10.50
<u>тин 1.5 %</u> С	1.67	1.72	1.76	1.78	1.78	1.75	1.71	1.64	1.56	1.46	1.36
VĂC	7.10	8.02	9.01	10.07	11.23	12.46	13.76	15.05	15.96	16.21	15.46
<u>IWR=5.0%</u>											
С	1.42	1.46	1.49	1.51	1.50	1.48	1.44	1.39	1.32	1.24	1.15
VAC	6.14	6.89	7.71	8.63	9.64	10.75	11.88	12.99	13.82	<mark>14.13</mark>	13.70
<u>Germany</u> IWR=3.0%											
С	<mark>3.89</mark>	3.59	3.34	3.10	2.88	2.66	2.45	2.24	2.03	1.83	1.63
VAC	13.57	14.17	14.99	16.00	17.25	18.75	20.47	22.20	23.31	23.13	20.82
<u>IWR=3.5%</u>											
С	<mark>3.13</mark>	2.91	2.71	2.52	2.35	2.17	2.00	1.83	1.66	1.50	1.35
VAC	11.08	11.58	12.26	13.07	14.05	15.28	16.66	18.01	<mark>19.07</mark>	18.68	16.86
<u>IWR=4.0%</u>		2 4 1	2.25	2 1 0	105	1 01	1 (7	1 5 3	1 20	1.25	1 1 2
C VAC	<mark>2.58</mark> 9.27	2.41 9.74	2.25 10.30	2.10 10.98	1.95 11.78	1.81 12.68	1.67 13.77	1.53 15.01	1.39 15.68	1.25 <mark>15.89</mark>	1.12 15.05
VAC <u>IWR=4.5%</u>	9.27	9.74	10.50	10.90	11.70	12.00	15.//	15.01	15.00	13.09	15.05
<u>түүк=4.570</u> С	2.18	2.04	1.90	1.78	1.65	1.53	1.41	1.29	1.17	1.06	0.95
VĂC	7.98	8.35	8.85	9.38	10.03	10.88	11.80	12.85	13.70	13.71	12.89
IWR=5.0%											
С	<mark>1.87</mark>	1.75	1.64	1.52	1.41	1.30	1.20	1.10	1.00	0.90	0.82
VAC	7.01	7.33	7.74	8.15	8.86	9.55	10.38	11.20	11.88	<mark>11.91</mark>	11.37
<u>Ireland</u>											
<u>IWR=3.0%</u>											
С	<mark>4.46</mark>	4.29	4.09	3.86	3.60	3.33	3.05	2.77	2.50	2.24	2.01
VAC	19.37	19.86	20.18	20.33	20.28	20.01	19.53	18.76	17.71	16.26	14.58
<u>IWR=3.5%</u>		2.20	2.24	2.07	2.07	2.65	2.42	2.22	2.01	1.00	1.65
C	3.52	3.39	3.24	3.06 16.18	2.86	2.65	2.43 15.83	2.22 15.30	2.01 14.48	1.83	1.65
VAC <i>IWR=4.0%</i>	15.29	15.70	16.04	10.10	<mark>16.22</mark>	16.14	15.05	15.50	14.40	13.52	12.28
<u>тик=4.090</u> С	2.83	2.73	2.61	2.47	2.32	2.15	1.99	1.82	1.67	1.53	1.40
VĂC	12.48	12.83	13.18	13.44	13.52	13.50	13.29	12.90	12.28	11.50	10.50
<u>IWR=4.5%</u>	12.10	12.00	10.10	10.11	10102	10.00	10.27	12.70	12:20	11.00	10.00
С	2.32	2.25	2.15	2.04	1.91	1.79	1.66	1.54	1.42	1.31	1.20
VAC	10.54	10.91	11.19	11.40	<mark>11.51</mark>	11.46	11.28	10.97	10.59	9.96	9.18
<u>IWR=5.0%</u>											
С	<mark>1.93</mark>	1.87	1.79	1.71	1.61	1.51	1.42	1.32	1.23	1.14	1.05
VAC	9.00	9.33	9.63	9.84	<mark>9.91</mark>	9.88	9.80	9.59	9.28	8.77	8.08
										(C_{α})	ntinues

Exhibit A2: C vs. VAC - Optimal Asset Allocations - Global - 1900-2019 (Cont.)

Exhibit A2: C vs. VAC – Optimal Asset Allocations – Global – 1900-2019 (Cont.)											
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
<u>Italy</u>											
IWR=3.0%											
С	1.72	1.66	1.63	1.61	1.60	1.59	1.58	1.55	1.51	1.46	1.40
VAC	5.87	6.26	6.80	7.50	8.38	9.44	10.65	11.90	12.97	13.53	13.19
<u>IWR=3.5%</u>											
С	1.41	1.36	1.34	1.33	1.33	1.32	1.31	1.29	1.26	1.22	1.17
VAC	4.83	5.15	5.63	6.22	6.97	7.87	8.85	9.90	10.87	11.41	11.14
<u>IWR=4.0%</u>											
С	<mark>1.18</mark>	1.15	1.14	1.14	1.14	1.13	1.12	1.10	1.08	1.04	1.01
VAC	4.14	4.45	4.85	5.38	6.06	6.83	7.71	8.63	9.41	<mark>9.86</mark>	9.65
<u>IWR=4.5%</u>											
С	1.02	1.00	0.99	0.99	0.99	0.98	0.98	0.96	0.94	0.91	0.88
VAC	3.68	3.98	4.38	4.85	5.42	6.06	6.81	7.57	8.24	8.58	<mark>8.70</mark>
<u>IWR=5.0%</u>											
С	<mark>0.89</mark>	0.88	0.88	0.88	0.88	0.87	0.86	0.85	0.83	0.80	0.78
VAC	3.40	3.70	4.05	4.47	4.95	5.52	6.12	6.73	7.28	7.69	<mark>8.12</mark>
<u>Japan</u>											
<u>IWR=3.0%</u>											
С	5.97	5.29	4.66	4.09	3.59	3.16	2.79	2.48	2.22	1.99	1.79
VAC	<mark>20.95</mark>	20.45	19.95	19.49	19.14	18.97	19.07	19.52	20.20	20.36	18.84
<u>IWR=3.5%</u>				~ ~ =			-				
С	4.94	4.36	3.83	3.35	2.94	2.58	2.27	2.02	1.80	1.61	1.45
VAC	17.32	16.89	16.43	16.00	15.64	15.44	15.46	15.78	16.29	16.38	15.11
<u>IWR=4.0%</u>	110	2 (7	2.24	2.00	2.45	2.1.4	1.00	1 (7	1 40	1 2 2	1 20
C	4.16	3.67	3.21	2.80	2.45	2.14	1.88	1.67	1.48	1.33	1.20
VAC <i>IWR=4.5%</i>	<mark>14.59</mark>	14.19	13.75	13.34	13.01	12.80	12.79	13.02	13.39	13.43	12.38
<u>түк=4.5%</u> С	3.55	3.12	2.73	2.37	2.06	1.80	1.58	1.39	1.24	1.11	1.01
VAC	5.55 12.57	5.12 12.07	2.75 11.68	2.37 11.29	2.06 10.95	1.80	1.58	1.59	1.24	1.11	10.1
<u>IWR=5.0%</u>	12.37	12.07	11.00	11.29	10.95	10.72	10.00	10.01	11.00	11.00	10.20
<u>1WK-5.0%</u> C	3.07	2.69	2.34	2.03	1.76	1.53	1.33	1.17	1.04	0.94	0.87
VAC	10.98	10.60	10.10	2.03 9.63	9.30	9.06	8.98	9.04	9.24	9.29	8.72
Netherland		10.00	10.10	7.05	7.50	5.00	0.70	7.04	7.24).2)	0.72
<u>IWR=3.0%</u>	<u>5</u>										
<u>тик-з.0%</u> С	5.08	4.76	4.41	4.05	3.69	3.34	2.99	2.66	2.35	2.07	1.81
VAC	23.31	24.12	24.95	25.81	26.69	27.55	28.21	28.27	27.10	24.21	19.93
IWR=3.5%	23.31	27.12	24.75	25.01	20.07	27.55	20.21	20.27	27.10	27.21	17.75
<u>тик=5.5 %</u> С	4.01	3.75	3.49	3.21	2.93	2.65	2.38	2.13	1.89	1.67	1.47
VĂC	18.54	19.16	19.77	20.44	21.18	21.89	22.46	22.65	21.86	19.72	16.48
<u>IWR=4.0%</u>	10.01	17.10	17.77	20.11	21.10	21.07	22.10	22.00	21.00	17.72	10.10
<u>C</u>	3.21	3.01	2.80	2.58	2.36	2.14	1.93	1.73	1.55	1.38	1.23
VĂC	14.96	15.52	16.09	16.68	17.31	17.90	18.43	18.64	18.11	16.43	13.92
<u>IWR=4.5%</u>	0		,								
С	2.62	2.46	2.29	2.12	1.94	1.77	1.61	1.45	1.31	1.17	1.05
VAC	12.36	12.79	13.30	13.82	14.34	14.86	15.35	15.65	15.29	14.01	11.97
<u>IWR=5.0%</u>		-		_	-				-	-	
С	2.17	2.04	1.90	1.76	1.63	1.49	1.36	1.23	1.11	1.01	0.91
VAC	10.34	10.78	11.12	11.59	12.04	12.52	12.98	13.29	13.09	12.22	10.58
											ntinups

Exhibit A2: C vs. VAC - Optimal Asset Allocations - Global - 1900-2019 (Cont.)

Exhibit A2										-	
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
<u>New Zealan</u> <u>IWR=3.0%</u>											
С	4.74	4.44	4.15	3.85	3.56	3.26	2.98	2.70	2.44	2.19	1.97
VAC	31.27	30.60	30.04	29.56	29.17	28.88	28.64	28.35	27.81	26.86	25.53
<u>IWR=3.5%</u>											
С	3.79	3.55	3.31	3.07	2.84	2.60	2.38	2.16	1.96	1.77	1.61
VAC	25.05	24.50	24.03	23.65	23.36	23.14	22.99	22.86	22.64	22.44	21.65
<u>IWR=4.0%</u>	2.00	2.00	2.0	2.40	2.20	2 1 1	1.02	170	1 (1	1 47	1.25
C VAC	<mark>3.08</mark> 20.39	2.88 19.93	2.68 19.53	2.49 19.22	2.30 18.99	2.11 18.85	1.93 18.79	1.76 18.82	1.61 19.05	1.47 19.20	1.35 18.54
VAC <u>IWR=4.5%</u>	20.39	19.95	19.55	19.22	10.99	10.05	10.79	10.02	19.05	19.20	10.34
<u>түүк-4.5%</u> С	2.53	2.36	2.20	2.03	1.88	1.73	1.59	1.47	1.35	1.24	1.15
VĂC	16.76	16.37	16.04	15.77	15.61	15.59	15.75	16.30	16.47	16.43	16.30
<u>IWR=5.0%</u>	10170	10.07	10.01	10177	10.01	10.07	101/0	10.00	10.17	10110	10100
С	<mark>2.09</mark>	1.95	1.81	1.68	1.56	1.44	1.33	1.24	1.15	1.07	1.00
VAC	13.87	13.56	13.29	13.16	13.11	13.39	13.68	14.07	14.35	14.64	14.78
<u>Norway</u> <u>IWR=3.0%</u>											
С	3.73	3.77	3.75	3.65	3.49	3.28	3.02	2.74	2.45	2.16	1.89
VAC	16.68	18.30	19.87	21.33	22.62	23.71	24.63	25.24	<mark>25.40</mark>	24.57	21.76
<u>IWR=3.5%</u>											
С	2.98	<mark>3.01</mark>	2.99	2.91	2.78	2.61	2.41	2.19	1.97	1.75	1.56
VAC	13.52	14.81	16.06	17.35	18.40	19.30	20.12	20.88	21.02	20.38	18.30
<u>IWR=4.0%</u>		0.44	~	-	-						
С	2.43	<mark>2.46</mark>	2.44	2.37	2.27	2.13	1.97	1.80	1.63	1.46	1.31
VAC	11.27	12.36	13.41	14.50	15.49	16.36	17.00	17.52	17.62	17.26	15.82
<u>IWR=4.5%</u> C	2.02	<mark>2.04</mark>	2.02	1.97	1.89	1.78	1.65	1.51	1.38	1.25	1.13
VAC	2.02 9.60	2.04 10.55	2.02 11.54	12.41	13.17	13.83	14.39	1.51	1.58 15.13	1.25	13.89
IWR=5.0%	9.00	10.55	11.54	12.41	13.17	13.03	14.59	14.04	15.15	13.11	13.09
<u>тик=э.о ж</u> С	1.69	1.71	1.70	1.66	1.59	1.50	1.40	1.29	1.19	1.08	0.99
VĂC	8.35	9.14	9.93	10.69	11.37	11.98	12.60	13.14	13.41	13.25	12.31
Portugal											
<u>IWR=3.0%</u>											
С	<mark>3.53</mark>	3.27	3.00	2.72	2.44	2.18	1.94	1.71	1.50	1.30	1.13
VAC	<mark>12.80</mark>	12.54	12.31	12.09	11.93	11.76	11.53	11.09	10.39	9.39	8.17
<u>IWR=3.5%</u>											
С	<mark>2.84</mark>	2.64	2.42	2.21	2.01	1.82	1.62	1.44	1.27	1.12	0.97
VAC	<mark>10.29</mark>	10.14	10.02	9.95	9.86	9.80	9.65	9.37	8.87	8.14	7.15
<u>IWR=4.0%</u>											
С	2.35	2.18	2.02	1.86	1.70	1.54	1.39	1.24	1.10	0.97	0.85
VAC	<mark>8.53</mark>	8.45	8.39	8.36	8.35	8.36	8.30	8.15	7.79	7.16	6.59
<u>IWR=4.5%</u>	1.00	1.05	1 70	1 50	1 4 (1 77	1 20	1.00	0.07	0.05	0.75
C VAC	<mark>1.98</mark> 7.21	1.85 7.16	1.72 7.15	1.59 7.18	1.46 7.24	1.33 7.29	1.20 <mark>7.30</mark>	1.08 7.20	0.96 6.91	0.85 6.51	0.75
VAC <u>IWR=5.0%</u>	1.41	/.10	/.15	/.10	1.24	1.29	1.50	1.20	0.91	0.31	6.11
<u>түк-3.0%</u> С	1.70	1.59	1.49	1.38	1.27	1.16	1.06	0.95	0.85	0.75	0.67
VAC	6.18	6.17	6.22	6.28	6.39	6.46	6.52	6.45	6.28	6.10	5.82
	0.10	0.17	0.22	0.20	0.07	0.10	0.01	0.10	0.20		ntinues)

Exhibit A2: C VS. VAC – Optimal Asset Allocations – Global – 1900-2019 (Cont.)											
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
South Africe	<u>a</u>										
С	<mark>7.59</mark>	6.57	5.67	4.87	4.17	3.57	3.04	2.58	2.19	1.86	1.59
VAC	<mark>33.62</mark>	31.63	29.83	28.19	26.67	25.23	23.78	22.25	20.56	18.82	16.92
<u>IWR=3.5%</u>											
С	<mark>6.03</mark>	5.21	4.49	3.86	3.30	2.82	2.40	2.04	1.74	1.50	1.30
VAC	<mark>26.70</mark>	25.10	23.65	22.33	21.13	19.99	18.87	17.75	16.61	15.42	14.13
<u>IWR=4.0%</u>											
С	4.86	4.19	3.61	3.09	2.65	2.26	1.93	1.65	1.42	1.23	1.08
VAC	21.52	20.20	19.00	17.94	16.97	16.09	15.29	14.57	13.83	13.11	12.32
<u>IWR=4.5%</u>	<mark>3.95</mark>	2 / 1	2.02	2 5 0	214	1 0 2	1 57	1.25	1 1 0	1.0.4	0.02
C VAC	3.95 17.53	3.41 16.43	2.92 15.44	2.50 14.58	2.14 13.83	1.83 13.25	1.57 12.66	1.35 12.18	1.18 11.68	1.04 11.28	0.93 10.76
<u>IWR=5.0%</u>	17.55	10.45	15.44	14.50	13.03	15.25	12.00	12.10	11.00	11.20	10.70
<u>1WK-5.0%</u> C	3.24	2.78	2.39	2.04	1.75	1.50	1.30	1.13	1.00	0.90	0.81
VĂC	14.36	13.47	12.69	12.02	11.46	10.99	10.62	10.36	10.12	9.88	9.57
<u>Spain</u>											
IWR=3.0%											
С	3.04	2.97	2.87	2.75	2.61	2.46	2.30	2.14	1.97	1.81	1.65
VAC	13.31	14.19	15.04	15.82	16.47	16.95	17.15	16.96	16.40	15.40	14.03
<u>IWR=3.5%</u>											
С	<mark>2.46</mark>	2.41	2.33	2.23	2.12	2.00	1.88	1.75	1.62	1.49	1.36
VAC	10.82	11.56	12.28	12.93	13.50	13.90	14.10	14.00	13.56	12.91	11.88
<u>IWR=4.0%</u>											
С	<mark>2.05</mark>	2.00	1.94	1.86	1.77	1.67	1.57	1.46	1.36	1.25	1.15
VAC	9.04	9.68	10.29	10.85	11.34	11.74	<mark>11.95</mark>	11.89	11.60	10.97	10.16
<u>IWR=4.5%</u> C	1.74	1.70	1.65	1.58	1.50	1.42	1.33	1.25	1.16	1.07	0.99
VAC	7.72	8.26	8.78	1.38 9.27	9.70	10.04	10.20	1.25 10.21	9.97	9.46	0.99 8.83
<u>IWR=5.0%</u>	1.72	0.20	0.70).27	5.70	10.04	10.20	10.21).)/	7.40	0.05
<u>тин=5.0 %</u> С	1.49	1.46	1.41	1.36	1.29	1.22	1.15	1.08	1.00	0.93	0.86
VĂC	6.69	7.17	7.61	8.06	8.43	8.71	8.85	8.83	8.68	8.33	7.76
Sweden											
<u>IWR=3.0%</u>											
С	<mark>7.32</mark>	6.81	6.26	5.69	5.12	4.56	4.03	3.53	3.09	2.69	2.35
VAC	31.92	32.36	32.73	33.02	<mark>33.16</mark>	33.08	32.57	31.33	28.98	25.45	21.24
<u>IWR=3.5%</u>											
С	<mark>5.83</mark>	5.42	4.98	4.53	4.07	3.63	3.20	2.81	2.45	2.14	1.87
VAC	25.46	25.79	26.07	26.29	<mark>26.40</mark>	26.34	25.95	24.98	23.13	20.58	17.73
<u>IWR=4.0%</u>			4.00	0.55	0.00	0.00	0 = 0	0.01	4.00		
C	4.72	4.39	4.03	3.66	3.29	2.92	2.58	2.26	1.98	1.74	1.53
VAC	20.59	20.88	21.12	21.27	21.37	21.29	20.97	20.26	19.02	17.36	15.11
<u>IWR=4.5%</u> C	<mark>3.86</mark>	3.58	3.29	2.99	260	2.38	2.10	1.85	167	1 1 1	1.28
C VAC	<mark>3.86</mark> 16.85	3.58 17.08	3.29 17.25	2.99 17.40	2.68 <mark>17.45</mark>	2.38 17.41	2.10 17.21	1.85 16.85	1.62 16.01	1.44 14.78	1.28
VAC <u>IWR=5.0%</u>	10.00	17.00	17.20	17.40	17.45	17.41	17.41	10.00	10.01	14./0	13.00
<u>1WK-5.0%</u> C	<mark>3.18</mark>	2.95	2.71	2.46	2.21	1.97	1.75	1.55	1.37	1.22	1.09
VAC	13.86	14.07	14.23	14.36	14.47	14.48	14.53	14.33	13.78	12.83	11.56
	20100	1.107	2.180	1.00				1.00	2000		ntinues)
										ູເວບ	icinacsj

Exhibit A2: C vs. VAC - O	ptimal Asset Allocations – Glo	hal – 1900-2019 (Cont.)
$\mathbf{LAIIIDIU A2. U V3. VAU = 0}$	pullial Asset Allocations - ulo	$Dai = 1700^{-2}017 (COIIC)$

Exhibit A2: C vs. VAC – Optimal Asset Allocations – Global – 1900-2019 (Cont.)											
$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
Switzerland	<u>d</u>										
<u>IWR=3.0%</u>											
С	<mark>3.65</mark>	3.48	3.29	3.10	2.89	2.68	2.48	2.28	2.08	1.90	1.73
VAC	18.67	19.46	20.29	21.19	22.17	23.24	24.40	25.56	26.46	<mark>26.59</mark>	25.34
<u>IWR=3.5%</u>											
С	<mark>2.88</mark>	2.75	2.61	2.46	2.30	2.14	1.98	1.82	1.67	1.53	1.40
VAC	14.74	15.39	16.06	16.79	17.58	18.47	19.44	20.45	21.27	21.51	20.60
<u>IWR=4.0%</u>											
С	<mark>2.32</mark>	2.22	2.11	1.98	1.86	1.73	1.61	1.48	1.37	1.26	1.16
VAC	11.91	12.41	12.94	13.54	14.19	14.93	15.78	16.69	17.44	17.69	17.10
<u>IWR=4.5%</u>											
С	<mark>1.91</mark>	1.82	1.73	1.63	1.53	1.43	1.33	1.23	1.14	1.06	0.98
VAC	9.81	10.25	10.71	11.23	11.83	12.49	13.25	14.11	14.91	15.19	<mark>15.29</mark>
<u>IWR=5.0%</u>											
С	<mark>1.59</mark>	1.52	1.44	1.36	1.28	1.20	1.12	1.04	0.97	0.91	0.85
VAC	8.30	8.68	9.09	9.57	10.13	10.76	11.45	12.19	12.90	13.45	<mark>13.69</mark>
<u>UK</u>											
<u>IWR=3.0%</u>											
С	<mark>5.60</mark>	5.14	4.69	4.28	3.89	3.53	3.19	2.88	2.60	2.35	2.12
VAC	<mark>29.36</mark>	29.09	28.75	28.33	27.75	26.92	25.74	24.17	22.15	19.87	17.36
<u>IWR=3.5%</u>											
С	<mark>4.45</mark>	4.08	3.73	3.39	3.09	2.80	2.54	2.30	2.08	1.89	1.72
VAC	23.40	23.19	22.93	22.60	22.16	21.50	20.62	19.46	17.95	16.10	14.31
<u>IWR=4.0%</u>											
С	3.58	3.28	3.00	2.74	2.49	2.27	2.06	1.88	1.71	1.57	1.44
VAC	<mark>18.91</mark>	18.76	18.57	18.34	18.02	17.62	17.02	16.10	14.86	13.58	12.23
<u>IWR=4.5%</u>	0.00	0.40	o 4 =		0.0 -	4.05	4 = 0			4.00	4.00
С	2.92	2.68	2.45	2.24	2.05	1.87	1.70	1.56	1.44	1.33	1.23
VAC	<mark>15.51</mark>	15.44	15.33	15.20	15.00	14.74	14.28	13.61	12.68	11.72	10.62
<u>IWR=5.0%</u>	2 40	2.24	2.02	1.07	1 70	1 50	1 4 4	1.2.4	1.2.4	115	1.07
C	2.40	2.21	2.03	1.86	1.70	1.56	1.44	1.34	1.24	1.15	1.07
VAC	<mark>12.92</mark>	12.87	12.86	12.84	12.75	12.57	12.20	11.69	11.01	10.22	9.32
<u>USA</u>											
<u>IWR=3.0%</u>		F 22	4 75	4.20	2.07	246	2.00	275	2 4 2	215	1.00
C	5.74	5.23	4.75	4.29	3.86	3.46	3.09	2.75	2.43	2.15	1.89
VAC	30.28	30.30	30.39	30.52	<u>30.57</u>	30.31	29.47	27.80	25.30	22.12	18.66
<u>IWR=3.5%</u>		415	2 77	2 / 1	2.07	2.75	216	2 1 0	1.0.4	1 7 2	1 5 2
<i>C</i> VAC	<mark>4.55</mark> 24.04	4.15 24.06	3.77 24.15	3.41 24.27	3.07	2.75 24.17	2.46 23.54	2.19 22.27	1.94 20.36	1.72 17.98	1.53
VAC <u>IWR=4.0%</u>	24.04	24.00	24.13	24.27	<mark>24.33</mark>	24.17	23.34	LL.L1	20.30	17.90	15.41
<u>тик=4.0%</u> С	<mark>3.67</mark>	3.34	3.04	2.75	2.48	2.22	1.99	1.78	1.59	1.42	1.27
VAC	<mark>5.07</mark> 19.40	5.54 19.40	5.04 19.48	2.75 19.60	2.48 19.67	2.22 19.58	1.99	1.78	1.59	1.42	1.27
VAC <u>IWR=4.5%</u>	17.40	17.40	1 7.40	17.00	17.07	17.30	17.14	10.10	10.02	13.14	13.10
<u>тик-4.5%</u> С	3.00	2.73	2.47	2.24	2.02	1.82	1.64	1.47	1.33	1.20	1.09
VAC	15.84	15.84	15.91	16.02	16.12	16.09	15.85	15.21	14.36	13.11	11.55
<u>IWR=5.0%</u>	13.04	13.04	13.71	10.02	10.12	10.07	10.00	13.41	14.30	13.11	11.33
<u>1WK-5.0%</u> C	<mark>2.48</mark>	2.25	2.04	1.85	1.68	1.52	1.38	1.25	1.14	1.03	0.94
VAC	13.11	13.10	13.16	13.27	13.42	13.50	13.43	13.09	12.44	11.48	10.21
v110	10.11	10.10	15.10	10.47	15.74	10.00	10.70	15.07	14.77	11.70	10.21

Exhibit A3: USA - Optimal Asset Allocations - Full Sample vs. Post-1944 Sample

This exhibit shows the volatility-adjusted coverage ratio (VAC) for asset allocations with different proportion of stocks (*S*) and bonds (100–*S*), for different initial withdrawal rates (IWR), across 122 (49) rolling 30-year retirement periods for the full (post-1944) sample. The analysis is based on a 1,000 retirement portfolio, 30 inflation-adjusted withdrawals made at the beginning of each year in retirement, and annual rebalancing. Panel A shows data for the full (1872-2022) sample and panel B for the post-1944 (1945-2022) sample. For each IWR, the figures highlighted indicate the optimal asset allocation as selected by maximizing the VAC. The (Shiller) data is described in Exhibit A1.

$S \rightarrow$	100	90	80	70	60	50	40	30	20	10	0
<u>A: 1872-2022</u>											
IWR=2.0%	58.38	58.42	58.55	58.73	<mark>58.88</mark>	58.78	58.06	56.14	52.42	46.60	39.10
IWR=2.5%	43.86	43.93	44.07	44.25	44.42	<mark>44.43</mark>	43.99	42.66	40.00	35.75	30.22
IWR=3.0%	34.18	34.27	34.41	34.60	34.79	<mark>34.86</mark>	34.60	33.68	31.72	28.52	24.22
IWR=3.5%	27.27	27.37	27.51	27.70	27.90	<mark>28.03</mark>	27.90	27.26	25.81	23.30	20.19
IWR=4.0%	22.10	22.20	22.34	22.53	22.74	<mark>22.90</mark>	22.88	22.44	21.34	19.62	17.25
IWR=4.5%	18.14	18.22	18.36	18.55	18.77	18.96	19.01	18.80	18.17	16.98	15.10
IWR=5.0%	15.05	15.14	15.26	15.46	15.71	15.97	16.12	<mark>16.13</mark>	15.83	14.95	13.46
IWR=5.5%	12.64	12.71	12.86	13.06	13.30	13.62	13.93	14.03	13.94	13.50	12.01
IWR=6.0%	10.68	10.76	10.92	11.14	11.44	11.78	12.17	12.48	<mark>12.63</mark>	12.09	10.82
<u>B: 1945-202</u>	22										
IWR=2.0%	58.69	60.08	61.31	62.32	<mark>62.95</mark>	62.87	61.51	58.14	52.43	45.00	37.05
IWR=2.5%	44.10	45.16	46.10	46.88	<mark>47.39</mark>	47.38	46.42	43.95	39.73	34.19	28.24
IWR=3.0%	34.38	35.22	35.96	36.59	37.02	<mark>37.06</mark>	36.36	34.50	31.26	26.98	22.34
IWR=3.5%	27.44	28.11	28.72	29.25	29.62	<mark>29.68</mark>	29.18	27.74	25.20	21.83	18.66
IWR=4.0%	22.26	22.81	23.30	23.74	24.07	<mark>24.16</mark>	23.80	22.69	20.77	18.58	16.07
IWR=4.5%	18.34	18.77	19.18	19.56	19.85	<mark>19.99</mark>	19.76	19.17	17.95	16.41	14.27
IWR=5.0%	15.34	15.71	16.04	16.40	16.69	<mark>16.96</mark>	16.96	16.68	15.86	14.53	12.65
IWR=5.5%	13.02	13.31	13.63	13.96	14.26	14.57	<mark>14.66</mark>	14.50	14.05	13.19	11.24
IWR=6.0%	11.15	11.43	11.72	12.02	12.33	12.62	<mark>12.78</mark>	12.72	12.61	11.87	10.23

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