Retirement Planning:
A Comprehensive Approach

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Abstract
This article puts together all the steps an individual or financial planner must consider when designing a retirement plan, a process that must be worked out backward. These steps include considering the lifestyle desired in retirement, the withdrawals needed to sustain that lifestyle, the portfolio needed to support those withdrawals, and the contributions needed to build that portfolio. The steps also include considering the portfolio's asset allocation and expected return over the lifecycle, performing sensitivity analysis, and evaluating the feasibility of the plan.

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1. Introduction

A retirement plan should ideally be put in place as early as possible, giving an individual plenty of time to make the contributions needed to build a target retirement portfolio over a long period of time. That said, the target portfolio cannot be any aspirational number; it must be consistent with the withdrawals needed to sustain the lifestyle desired in retirement, which is something that most individuals are reluctant to consider early in their working years. And yet it is still the case that the process must be worked out backward, from the lifestyle desired in retirement, to the withdrawals needed to sustain that lifestyle, to the portfolio needed to support those withdrawals, to the contributions needed to build that portfolio.

Proper retirement planning also requires determining a target bequest, which can be essentially thought of as the final withdrawal from the portfolio. It requires choosing an asset allocation for the portfolio, considering how (or if) it should change over time, and estimating the portfolio's expected return. And when evaluating the plan's feasibility on the basis of historical or simulated working and retirement periods, it requires determining maximum acceptable failure rates for both the withdrawal plan and the bequest.

The framework discussed in this article brings together all the steps that must be followed when designing a retirement plan. As already mentioned, the analysis must be worked out backward, from the lifestyle desired in retirement, to the withdrawals needed to sustain that lifestyle, to the portfolio needed to support those withdrawals, to the contributions needed to build that portfolio.

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To elaborate briefly, the framework proposed starts with the sequence of withdrawals needed to sustain the lifestyle desired in retirement, including a target bequest. These withdrawals and bequest, together with the portfolio's asset allocation (hence expected return) chosen for the retirement period, determine the target retirement portfolio. This target portfolio, together with the asset allocation (hence expected return) chosen for the working period, determine the contributions to be made during the working years.

The withdrawals and bequest are exogenous variables, only to be adjusted if the initial plan is deemed likely to fail, or if circumstances along the way make it essential to do so. The portfolio's asset allocation (hence its expected return), which is partly determined by the individual's risk tolerance, is also exogenous and may, and likely will, vary over time. The endogenous variables are the target retirement portfolio and the contributions to be made to it during the working years; closed-form solutions for both exist and are discussed here.

The ultimate goal of this article is to provide step-by-step guidelines to elaborate a financial plan for retirement. The discussion includes the analytical framework and highlights the key mathematical expressions needed for the relevant calculations; it also includes an example that considers a base case plan, sensitivity analysis on it, and an evaluation of the plan's feasibility based on both historical data and Monte Carlo simulations.

The rest of the article is organized as follows. Section 2 discusses the intuition behind the methodology proposed here, provides a 10-step framework for retirement planning, and highlights the relevant expressions of the model; section 3 discusses an example based on the long-term performance of U.S. stocks and bonds, including a base case, sensitivity analysis, and an evaluation of the plan's feasibility over historical and simulated periods; and section 4 provides an assessment. An appendix with the analytical framework concludes the article.

2. The Basics

2.1. The Big Picture

Planning for retirement should ideally start as early as possible during the working years; the size of the annual contributions needed to build a target retirement portfolio is clearly negatively related to the number of years over which the portfolio is built. In other words, given the target portfolio, the longer an individual waits to start building the portfolio, the larger must be the periodic contributions to it, and so must be the saving rate; see Pfau (2011).

Granted, at the beginning of a career nobody wants to think about life in retirement, or the size and beneficiaries of a bequest, let alone how long one is going to live; but unfortunately, the alternative is to shoot in the dark. It is far better to make a feeble attempt at quantifying tentative withdrawals (and bequest) over a tentative retirement period, and adjust the estimates over time if need there be, than just picking an arbitrary number for the size of the retirement portfolio; Eisenberg (2006) discusses this issue at length.[1]

Similarly, most individuals are likely to have very little idea about how high or low their risk tolerance in retirement will be, thus making it difficult to determine their portfolio's asset allocation (hence its expected return) during that period. Then again, a crude idea is better than no idea. In this regard, a possible broad guideline is the asset allocation of target-date funds during retirement. Vanguard's target-date funds, for example, hold a 50-50 stock-bond allocation on the retirement date; they gradually decrease (increase) the proportion of stocks (bonds) during the first seven years in retirement; and finally settle on a 30-70 stock-bond allocation, which is kept constant from that point on; see Donaldson et al (2019).

Given a sequence of withdrawals (including a bequest) over a specified period of time, and given the expected annualized return for the portfolio over that period, the size of the target portfolio can be formally determined; see Estrada (2020*b*). But that is only half of the work to be done. In order to determine the periodic contributions to the target portfolio, the length of the working period and the portfolio's asset allocation (hence its expected return) during that period also need to be determined.[2]

It is likely to be the case that the individual's risk tolerance during the working years will be different from (and in general higher than) that during retirement, which implies that the portfolio's asset allocation for the working period may be different from that for the retirement period.[3] The asset allocation during the working years (which may or may not vary over time) will determine the portfolio's expected return (which may or may not vary over time) over that period. Then, given the target portfolio, the expected number of working years, and the expected annualized return for the portfolio over those years, the size of the periodic contributions can be formally determined; see Estrada (2020*b*).

Having reached this point, the individual has a financial plan. A possible next step, not strictly necessary but very useful, is to outline the path of the retirement portfolio over the life cycle; that is, from the beginning of the working period to the end of the retirement period. This may be valuable when considering how (or if) to adjust the plan when reality does not conform to its underlying assumptions, which is almost certain to happen at some point.

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[1] Although ex-post it may vary dramatically across individuals, and life expectancy statistics may help each individual in this regard, the length of the retirement period most widely used for the purpose of retirement planning is 30 years.

[2] Although ex-post it may vary substantially across individuals, the length of the working period most widely used for the purpose of retirement planning is 40 years.

[3] Albert and Duffy (2012) report experimental evidence showing that older individuals are more risk averse than younger individuals. ING (2012) reports survey evidence showing that approaching retirement and after retirement most investors prioritize protection from losses over potential for growth.

In any case, regardless of whether the plan's expected path is outlined, the analysis should not end at this point. An important next step is performing sensitivity analysis; that is, asking how the size of the retirement portfolio and the periodic contributions to it would change if some of the framework's parameters (the exogenous variables) change. Another important next step is evaluating the feasibility of the plan, which can be done on the basis of historical or simulated periods, and ideally both ways. These two additional steps may lead the individual or financial planner to conclude that the plan is feasible and implement it, or alternatively that the plan is unlikely to happen as outlined and therefore that some adjustments need to be considered.

The feasibility of a plan should be evaluated across a large number of historical or simulated periods. A plan may fail because it depletes the portfolio before the end of the retirement period, or because it (delivered the withdrawals planned but) fell short from delivering the target bequest. Given that it seems reasonable to assume that most individuals would care more about the first failure than about the second, these two should ideally be treated as separate failures.

Importantly, any given plan is extremely unlikely to succeed across all the historical or simulated periods over which it is evaluated; in other words, the plan should be expected to fail (in one or both of the two dimensions just discussed) over some of those periods. For this reason, it is important to determine, ex-ante, a maximum acceptable failure rate. For example, should a plan be implemented if history or simulations show that it would have failed in 10% of the periods considered? What if it had failed in 20% of those periods? Or, more generally, what is the failure rate beyond which the plan becomes unacceptable? Ideally, a maximum acceptable failure rate should be determined ex-ante and for both the withdrawal plan and the bequest.

2.2. A 10-Step Framework

The discussion in the previous section can be summarized in a sequence of steps that an individual or financial planner need to follow when designing a financial plan for retirement. The ten essential steps, beginning from the retirement (or withdrawal) period and continuing with the working (or accumulation) period, are:

1. Estimate the expected number of years in retirement; absent better information, use 30 years as the retirement period.
2. Estimate the expected annual cost of the lifestyle desired during retirement, summarizing it in a sequence of annual withdrawals over the period determined in step 1.
3. Determine the target bequest, which will be the last withdrawal from the retirement portfolio.
4. Determine the portfolio's (fixed or time-varying) asset allocation during the retirement period, and estimate its expected (fixed or time-varying) return.
5. Solve for the retirement portfolio needed to sustain the withdrawals in step 2 and the bequest in step 3, given the portfolio's expected return in step 4.

6. Estimate the expected number of working years; absent better information, use 40 years as the working period.
7. Determine the portfolio's (fixed or time-varying) asset allocation during the working period, and estimate its expected (fixed or time-varying) return.
8. Solve for the annual contributions over the period in step 6 needed to build the target retirement portfolio in step 5, given the portfolio's expected return in step 7.
   - ☞ If deemed necessary, outline the plan's expected path over the life cycle, from the beginning of the working period through the end of the retirement period.
9. Perform sensitivity analysis to assess how much the endogenous variables change given changes in the main parameters of the model.
10. Determine maximum acceptable failure rates for the withdrawal plan and the bequest, and evaluate the feasibility of the plan on the basis of historical and/or simulated periods.

Steps 5 and 8 require the individual or financial planner to solve for the endogenous variables (the retirement portfolio and the contributions needed to build that portfolio during the working years) given the assumptions made on the other steps of the planning process. The solution requires an underlying analytical framework, the basics of which are discussed in the appendix; the next section highlights the set up and essential expressions of that framework.

2.3. The Analytical Framework

Consider an individual at the very beginning of her career. She considers her desired lifestyle in retirement, which she expects it to last N years; estimates its annual cost; and summarizes it in a sequence of annual real (inflation-adjusted) withdrawals W_1, ..., W_N to be made from her retirement portfolio (P) at the beginning of each year in retirement. She also plans to leave a bequest (B), which will be the final withdrawal from her portfolio at the end of the retirement period. Finally, she selects an asset allocation for her portfolio during retirement and estimates a corresponding sequence of expected annual real returns R_1, ..., R_N. As shown in the appendix, the target portfolio (P^*) needed to sustain this individual's retirement plans is given by

$$P^* = \frac{B + \sum_{t=1}^{N} W_t \cdot \prod_{j=t}^{N}(1+R_j)}{\prod_{t=1}^{N}(1+R_t)} \tag{1}$$

The expression above can be simplified if the individual aims to keep her purchasing power constant over time, in which case $W_1 = ... = W_N = W$, where W is the constant annual real withdrawal. In addition, if the sequence of expected annual real returns R_1, ..., R_N is replaced by the expected annualized real return during retirement (R), then (1) can be rewritten as

$$P^* = \frac{B - W \cdot (1+R) \cdot \left(\frac{1-(1+R)^N}{R}\right)}{(1+R)^N} \tag{2}$$

which expresses the target portfolio as a function of the constant annual real withdrawal, the target bequest, the expected annualized real return of the portfolio during retirement, and the length of the retirement period.

Our individual expects to work M years and make M–1 constant annual real contributions (C) to her portfolio at the end of each of her working years, with the exception of the last one (when she retires and makes her first withdrawal instead). She also selects an asset allocation for her portfolio during her working years and estimates a corresponding sequence of expected annual real returns S_1, …, S_{M-1}. As shown in the appendix, the annual real contribution that follows from this framework (C^*) is given by

$$C^* = \frac{P^*}{\sum_{t=1}^{M-1} \prod_{j=t}^{M-1} (1+S_j)} \quad (3)$$

This expression can be simplified if the expected annual real returns S_1, …, S_{M-1} are replaced by the expected annualized real return during the working period (S), in which case (3) can be rewritten as

$$C^* = \frac{P^*}{\left(\frac{(1+S)^M - (1+S)}{S}\right)} \quad (4)$$

which expresses the constant annual real contribution to the retirement portfolio as a function of the target portfolio that satisfies the assumptions made for the retirement period, the portfolio's expected annualized real return during the working years, and the length of the working period.

Finally, although it is not strictly necessary to do so, expression (1) can be inserted into (3), thus obtaining

$$C^* = \frac{\frac{B + \sum_{t=1}^{N} W_t \cdot \prod_{j=t}^{N} (1+R_j)}{\prod_{t=1}^{N} (1+R_t)}}{\sum_{t=1}^{M-1} \prod_{j=t}^{M-1} (1+S_j)} \quad (5)$$

Similarly, expression (2) can be inserted into (4) thus obtaining

$$C^* = \frac{\frac{B - W \cdot (1+R) \cdot \left(\frac{1-(1+R)^N}{R}\right)}{(1+R)^N}}{\left(\frac{(1+S)^M - (1+S)}{S}\right)} \quad (6)$$

Expression (5) yields the constant annual real contributions to be made during the working years in the more general case, with time-varying real withdrawals and returns; expression (6), on the other hand, yields the same variable in the more restricted case, with constant real withdrawals and annualized returns.

Expressions (5)-(6) highlight one of the critical messages of the framework proposed in this article; that is, *properly calculated* the contributions to be made to a retirement portfolio during the working years ultimately depend on 1) the expected withdrawals during retirement, which in turn depend on the lifestyle desired for that period; 2) the target bequest; 3) how long an individual expects the working period and the retirement period to be; and 4) the expected returns of the portfolio during those two periods, which in turn depend on the asset allocations selected for each period.

Unlike the unified framework proposed here, which ties the retirement and the working periods together, most of the previous literature focuses on just one of the two periods. Exceptions to this are Pfau (2011), who considers both periods when introducing the concept of safe saving rates; and Estrada (2020*b*), who also considers both periods and advances the unified approach discussed and generalized in this article.[4]

The research that focuses only on the working period usually discusses strategies aimed at maximizing the size of the retirement portfolio; see, for example, Basu and Drew (2009), Ayres and Nalebuff (2010), Arnott et al (2013), and Estrada (2014). The research that focuses only on the retirement period, on the other hand, usually discusses optimal withdrawal strategies or metrics that can be used to decide between different withdrawal strategies; see, for example, the pioneering work of Bengen (1994), as well as Blanchett (2007), Stout (2008), Blanchett and Frank (2009), Pfau and Kitces (2014), Zolt (2014), Kitces and Pfau (2015), Estrada (2016), Estrada and Kritzman (2019), and Blanchett (2022).[5]

3. Example – A Retirement Plan

3.1. The Base Case

Having discussed the intuition and analytical framework, as well as the ten essential steps that must be followed when designing a retirement plan, it may be useful to go over a specific example. Consider an individual that at the beginning of her working years expects to live 30 years in retirement (N). She contemplates her desired lifestyle during that period, aims to keep her purchasing power constant over time, and concludes that 30 annual real withdrawals (W) of $100,000, made at the beginning of each year in retirement, would do the job. At the same time, she decides to leave a bequest (B) equal to five years of her annual withdrawals ($500,000).

She knows that Vanguard keeps a 30-70 stock-bond allocation on its target-date funds during most of the retirement period and decides to implement that allocation, expecting an annualized real return (R) of 4.1%, based on the long-term performance of U.S. stocks and bonds. The target portfolio (P^*) consistent with these assumptions follows from (2) and is equal to $1,928,222; more precisely,

[4] The present article borrows from but also generalizes the analytical framework advanced in Estrada (2020*b*). It also introduces and highlights a 10-step framework for retirement planning; features a different sensitivity analysis; and discusses a feasibility analysis based on a historical perspective and a on Monte Carlo perspective.

[5] Some additional general references that may be useful to consider when putting together a retirement plan are Zwecher (2010), Gadenne (2013), Idzorek and Kaplan (2024), and Pfau (2023).

$$P^* = \frac{\$500{,}000 - \$100{,}000 \cdot (1+0.041) \cdot \left(\frac{1-(1+0.041)^{30}}{0.041}\right)}{(1+0.041)^{30}} = \$1{,}928{,}222$$

Having her target portfolio, the next step is to determine the annual contributions (C) she needs to make in order to build it during her working years. She thinks she will work 40 years (M) and decides to keep the contributions to her portfolio constant over time in inflation-adjusted terms. She also decides to make those contributions at the end of each of her working years, with the exception of the last one; at the end of that last year, which will mark the beginning of her retirement, she will make the first annual withdrawal from her portfolio.

She thinks that the 30-70 stock-bond allocation she will implement during retirement is too conservative for her working years, so she decides to implement a 70-30 allocation instead. Based on the long-term performance of U.S. stocks and bonds, she expects an annualized real return (S) of 5.8% from her portfolio during her working years. The constant annual real contribution (C^*) consistent with these assumptions follows from (4) and is equal to $13,189; more precisely,

$$C^* = \frac{\$1{,}928{,}222}{\left(\frac{(1+0.058)^{40}-(1+0.058)}{0.058}\right)} = \$13{,}189$$

To summarize, our individual expects to live 30 years in retirement, plans to spend $100,000 in real terms in each of those years, aims to leave a bequest of $500,000, and expects an annualized real return of 4.1% from her portfolio during those years. To be consistent with these expectations she needs to start her retirement with a $1,928,222 portfolio. To build that portfolio over the 40 years she expects to work, given an expected annualized real return of 5.8% during those years, she needs to make 39 annual real contributions of $13,189.

Our individual knows that this (or any other) plan is very unlikely to evolve as expected and that more likely than not adjustments will need to be made along the way. In order to make the proper adjustments she thinks it will be useful to know periodically whether her plan is on the right track, and if not, by how much. This leads her to outline the plan's expected path shown in Exhibit 1; the portfolio values shown are at the end of each year and in thousands of real dollars.

Exhibit 1: A Retirement Plan – Base Case – Expected Portfolio Path

This exhibit shows the expected path of a portfolio (*P*) over 70 years (*Y*), 40 during the working period and 30 during the retirement period. *P* receives real contributions of $13.2 at the end of years 1 through 39 and earns a 5.8% real return during the first 39 years. *P* is also subject to 30 real withdrawals of $100 at the beginning of years 40 through 70, leaves a bequest of $500 at year-end 70, and earns a 4.1% real return during the last 30 years. *Y* in years and *P* in thousands of inflation-adjusted dollars.

Y	*P*	*Y*	*P*	*Y*	*P*	*Y*	*P*	*Y*	*P*	*Y*	*P*
1	13.2	13	245.9	25	703.6	37	1,603.9	49	1,562.1	61	1,018.8
2	27.1	14	273.3	26	757.6	38	1,710.1	50	1,526.2	62	960.5
3	41.9	15	302.4	27	814.7	39	1,822.5	51	1,488.7	63	899.9
4	57.5	16	333.1	28	875.1	40	1,828.2	52	1,449.8	64	836.8
5	74.1	17	365.6	29	939.1	41	1,803.2	53	1,409.2	65	771.1
6	91.5	18	400.0	30	1,006.7	42	1,777.1	54	1,367.0	66	702.7
7	110.0	19	436.4	31	1,078.3	43	1,750.0	55	1,323.0	67	631.6
8	129.6	20	474.9	32	1,154.1	44	1,721.7	56	1,277.3	68	557.5
9	150.3	21	515.6	33	1,234.2	45	1,692.3	57	1,229.6	69	480.3
10	172.2	22	558.7	34	1,318.9	46	1,661.7	58	1,180.1	70	500.0
11	195.4	23	604.3	35	1,408.6	47	1,629.8	59	1,128.4		
12	219.9	24	652.5	36	1,503.5	48	1,596.6	60	1,074.7		

As the exhibit shows, our individual makes the first $13,189 contribution to her retirement portfolio at the end of her first working year. She keeps contributing the same amount in real terms at the end of each of the following 38 years, for a total of 39 annual contributions. During the working period her portfolio grows at the annualized real rate of 5.8%. At the end of year 40 our individual retires with her $1,928,222 target portfolio, and immediately takes her first $100,000 withdrawal; hence the $1,828,222 shown in the exhibit. From that point on she takes another 29 real withdrawals of $100,000 at the beginning of each of the following years in retirement, for a total of 30 withdrawals. Then the portfolio compounds for one more year and leaves the $500,000 target bequest. During the retirement period her portfolio grows at the annualized real rate of 4.1%.

That is the retirement plan. At this point our individual does not only know her target portfolio but also the annual contributions she needs to make to build it. And importantly, she also knows where she needs to be at the end of each year; should the plan go off track, as will sooner or later most likely be the case, she would know how far she is from where she needs to be and consider the best way to get back on track.

3.2. Sensitivity Analysis

Mike Tyson has presumably said that everybody has a plan until they get punched in the mouth; much the same could be said about retirement planning. It is critical to make a plan, but it would be naive to believe that is going to happen exactly as expected. For this reason, it is always important to consider how the endogenous variables (the size of the retirement portfolio and the contributions needed to build it) would change if the values assumed for the model's parameters change; Exhibit 2 addresses this issue.

Exhibit 2: A Retirement Plan – Sensitivity Analysis

This exhibit shows a sensitivity analysis on the base case retirement plan. It shows the target retirement portfolio (P^*) and the annual real contribution to the portfolio (C^*) for different values of the number of years in the retirement (N) and the working (M) periods; the annualized real return obtained during the retirement (R) and the working (S) periods; the annual real withdrawal (W); and the bequest (B). N and M in years; P^*, C^*, W, and B in thousands of inflation-adjusted dollars; R and S in %.

N	P^*	C^*	***M***	P^*	C^*	***R***	P^*	C^*
20	1,626.2	11.1	30	1,928.2	25.6	3.1	2,195.0	15.0
25	1,792.3	12.3	35	1,928.2	18.2	3.6	2,054.8	14.1
30	**1,928.2**	**13.2**	**40**	**1,928.2**	**13.2**	**4.1**	**1,928.2**	**13.2**
35	2,039.4	13.9	45	1,928.2	9.7	4.6	1,813.7	12.4
40	2,130.3	14.6	50	1,928.2	7.1	5.1	1,709.8	11.7
S	P^*	C^*	***W***	P^*	C^*	***B***	P^*	C^*
4.8	1,928.2	16.9	90	1,750.4	12.0	400	1,898.3	13.0
5.3	1,928.2	14.9	95	1,839.3	12.6	450	1,913.2	13.1
5.8	**1,928.2**	**13.2**	**100**	**1,928.2**	**13.2**	**500**	**1,928.2**	**13.2**
6.3	1,928.2	11.6	105	2,017.1	13.8	550	1,943.2	13.3
6.8	1,928.2	10.2	110	2,106.1	14.4	600	1,958.2	13.4

The exhibit shows the target portfolio and the annual contributions needed to build it for different values of the length of the working and retirement periods, the annualized real return of the portfolio during both periods, the annual withdrawals, and the bequest. Interestingly, living in retirement five years more or less than expected has a moderate impact (less than 8% in absolute value) on the target portfolio and the annual contributions to be made during the working years. Similarly, if the portfolio's annualized return during retirement is half a percentage point higher or lower than expected, the impact on the target portfolio and annual contributions is again rather moderate (less than 7% in absolute value).

On the flip side, working five years more or less than expected does have a very substantial impact on the annual contributions, +38.2% when working less and –26.8% when working more. Similarly, if the portfolio's annualized return during the working period is half a percentage point higher or lower than expected, the impact on the annual contributions is again sizeable, +13.3% with lower returns and –11.9% with higher returns.[6]

Increasing or decreasing the annual withdrawals by 5% has a symmetric impact on the target portfolio (±4.6%), and the same (and also symmetric) impact on the annual contributions to the portfolio (±4.6%). Finally, increasing or decreasing the bequest by 5% has a very small impact on the target portfolio, and therefore on the annual contributions to it; in both cases the impact is less than 1% in absolute value.[7]

[6] The size of the target portfolio is fully determined by the parameters for the retirement period (its length, the annual withdrawals, the bequest, and the returns obtained during that period), which explains why it is not affected by the length of the working period or the returns obtained during that period.

[7] The base-case assumes that the bequest is equal to five years of annual withdrawals, an assumption that is relaxed in these last two scenarios. This is because in the standard sensitivity analysis one parameter is changed and the rest remain fixed at their base-case values. Thus, in our case, when the level of withdrawals is changed, the bequest remains fixed at $500,000; and when the bequest is changed, the level of withdrawals remains fixed at $100,000.

This type of sensitivity analysis is critical in order to determine which parameters of the analytical framework have a larger impact on the endogenous variables. In this regard, it is clear from Exhibit 2 that the parameters for the working period, and in particular the number of working years, have a much larger impact on the size of the retirement portfolio, and therefore on the contributions needed to build it, than the parameters for the retirement period.

3.3. Feasibility – Methodology

Once the target portfolio and the contributions needed to build it over the working years have been determined, the plan's expected path has been outlined (if deemed necessary), and a proper sensitivity analysis has been performed, the next important step is to evaluate the feasibility of the plan. This can be done by assessing the plan's success or failure over a large number of historical or simulated periods, and ideally both ways.

The feasibility analysis essentially asks, for each historical or simulated period considered, whether the retirement plan has succeeded or failed. As already discussed, a plan may fail because it depleted the portfolio before the end of the retirement period, or because it delivered the withdrawals planned but fell short from delivering the target bequest.[8] And, as already mentioned, maximum acceptable failure rates should ideally be determined ex-ante and separately for both situations.

The data used in what follows consists of annual returns of U.S. stocks and government bonds over the 1872-2022 period, downloaded from Robert Shiller's web page.[9] All returns are real (adjusted by inflation) and account for capital gains/losses and cash flows. Unless otherwise stated, the analysis is performed on the base case retirement plan discussed above; that is,

- ☞ 30 annual real withdrawals of $100,000 from the retirement portfolio, made at the beginning of each year in retirement;
- ☞ a 30-70 stock-bond allocation during retirement;
- ☞ 39 annual real contributions of $13,189 to the retirement portfolio, made at the end of each working year, with the exception of the last one;
- ☞ a 70-30 stock-bond allocation during the working years;
- ☞ annual rebalancing to the target asset allocations during both the retirement and the working periods.

To complete this set up, assume that our individual has decided that a plan is unacceptable if it fails to deliver the withdrawals planned in more than 25%, and the target bequest in more than 30%, of the periods over which it is evaluated. Put differently, assume that our individual

[8] For the feasibility analysis the bequest is whatever is left at the end of the retirement period; hence, part of the analysis involves asking whether the goal of leaving a $500,000 bequest has been achieved.
[9] http://www.econ.yale.edu/~shiller/data.htm.

has determined, ex-ante, maximum acceptable failure rates of 25% for the withdrawal plan and 30% for the bequest.

The metric most widely used to evaluate the feasibility of a retirement plan is the failure rate. However, as is well known, this metric has serious shortcomings, including that it ignores how far into a retirement period a strategy fails, and the size of the bequest left; see Milevsky (2016), among others. The coverage ratio introduced by Estrada and Kritzman (2019) overcomes both limitations; that is, it does distinguish between a strategy that fails early and another that fails late into a retirement period, as well as between a strategy that leaves a small bequest and another that leaves a large bequest.

Formally, for any strategy i and any retirement period t, let Y_{it} be the number of years of inflation-adjusted withdrawals sustained by the strategy, both during and after the retirement period; and L be the length (in years) of the retirement period considered. Then the coverage ratio of strategy i in retirement period t (CR_{it}) is defined as

$$CR_{it} = \frac{Y_{it}}{L} \tag{7}$$

By definition, $CR_{it} < 1$ indicates that the strategy depleted the portfolio before the end of the retirement period (it failed); $CR_{it} > 1$ indicates that the strategy sustained the withdrawals planned through the whole retirement period and left a bequest; and $CR_{it} = 1$ indicates that the strategy sustained the withdrawals planned exactly through the end of a retirement period (thus leaving no bequest).[10]

Furthermore, the coverage ratio of strategy i (CR_i) is defined as

$$CR_i = (1/T) \cdot \sum_{t=1}^{T} CR_{it} \tag{8}$$

where T is the number of retirement periods considered. In words, calculating a strategy's coverage ratio involves two steps: First, calculating the strategy's coverage ratio for each retirement period in the sample; and second, calculating the average of those coverage ratios across all the retirement periods considered.[11]

Note that for any given retirement period, a coverage ratio lower than 1 indicates a strategy that failed; therefore, by definition, the proportion of retirement periods considered for which the coverage ratio is lower than 1 is equal to the widely-used failure rate. Thus, as far as

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[10] To illustrate, consider a 30-year retirement period, a $1 million retirement portfolio, annual inflation-adjusted withdrawals of $40,000 and three strategies. The first strategy depleted a portfolio in 24 years, the second did so in exactly 30 years, and the third sustained withdrawals for 30 years and left a bequest of $240,000 (which can support another six years of $40,000 withdrawals). Then, for the first, second, and third strategies Y would respectively be 24, 30, and 36; and CR would respectively be 0.8, 1.0, and 1.2.

[11] Estrada (2023) discusses how the whole distribution of coverage ratios can be used to decide between competing retirement strategies.

the withdrawal plan is concerned, the *target coverage ratio* is a value of 1; the *target bequest*, on the other hand, is the already-mentioned $500,000. Note, also, that in the base case plan the $500,000 target bequest is equal to five years of annual real withdrawals; therefore, a strategy that sustains the target withdrawal plan *and* the target bequest would have a coverage ratio of 1.17 (=35/30).

3.4. Feasibility – Historical Analysis

The 1872-2022 history of annual U.S. stock and bond returns enables the consideration of 83 rolling 70-year life cycle periods (40-year working periods followed by 30-year retirement periods), beginning with 1871-1940 and ending with 1953-2022. Exhibit 3 summarizes the feasibility of the retirement plan discussed, as well as that of three modifications to it, from a historical perspective. The probability of being short of the target values (P_S) is actually a historical frequency, which may be viewed as a forward-looking probability as long as history can be reliably extrapolated into the future.

Exhibit 3: Feasibility – Historical Perspective
This exhibit summarizes the feasibility of the base case retirement plan from a historical perspective, as well as the feasibility of three variations of the base case. It shows the coverage ratio (*CR*) and the bequest (*B*) for different stock-bond asset allocations during the working (AA_W) and the retirement (AA_R) periods, as well as for different annual real contributions (*C*), for given values of the number of years in the retirement (*N*) and working (*M*) periods and the annual real withdrawal (*W*). P_S denotes the probability of being short of the target coverage ratio and bequest. The target coverage ratio and bequest are 1 and $500,000. *N* and *M* in years; *C*, *W*, and *B* in thousands of inflation-adjusted dollars.

Parameters	Base Case		Scenario 2		Scenario 3		Scenario 4	
M	40		40		40		40	
N	30		30		30		30	
AA_W	70-30		**80-20**		**80-20**		**80-20**	
AA_R	30-70		30-70		**40-60**		**40-60**	
C	13.2		13.2		13.2		**15.8**	
W	100.0		100.0		100.0		100.0	
Results	*CR*	*B*	*CR*	*B*	*CR*	*B*	*CR*	*B*
P_S	73.5	84.3	50.6	59.0	42.2	51.8	12.0	27.7
Mean	0.83	0	1.05	131.5	1.16	464.5	1.63	1,882.0
Median	0.80	0	0.97	0	1.08	245.0	1.54	1,609.6

The exhibit shows that, across the 83 historical lifecycle periods considered, in 73.5% the target withdrawal plan (30 annual withdrawals of $100,000) was not sustained, and in 84.3% the target bequest ($500,000) was not left, both clearly above the 25% and 30% maximum acceptable failure rates set by our individual. Both the mean (0.83) and the median (0.80) coverage ratios are below 1, indicating that the retirement plan sustained 24-25 years of withdrawals in the average historical retirement period, hence falling short from delivering on the target withdrawal plan, and therefore on the target bequest.

Assume that given this rather gloomy assessment of the retirement plan outlined, our individual decides, or her financial planner recommends, to be somewhat more aggressive with the portfolio's asset allocation during the working years; in particular, assume that she considers an 80-20 stock-bond allocation for this period (Scenario 2). In that case, the failure rates of the target withdrawal plan and target bequest would have fallen considerably, to 50.6% and 59%, but would still be higher than the maximum acceptable failure rates. The mean (1.05) and median (0.97) coverage ratios would have been both close to 1, indicating that in the average historical retirement period the portfolio would have sustained the withdrawals planned (and would have left just over 26% of the target bequest in the mean period, and no bequest in the median period).

What if *besides* implementing an 80-20 allocation during the working years our individual considers to *also* implement a more aggressive (40-60) allocation during retirement (Scenario 3)? In that case the failure rates of the target withdrawal plan and the target bequest would have again fallen considerably, to 42.2% and 51.8%, but would still remain higher than the maximum acceptable failure rates. The mean and median coverage ratios would have both been above 1 (that is, in the average period the withdrawal plan would have been sustained), and the mean bequest would have been close to the target bequest.

Finally, what if *besides* being more aggressive with the portfolio's asset allocation during both the working period and the retirement period our individual considers to *also* increase the annual contribution to her portfolio by 20%, from $13,189 to $15,827 (Scenario 4)? In this case the decrease in the failure rates of the target withdrawal plan and target bequest would have been even more dramatic, to 12% and 27.7%. In the average historical period a sizeable bequest (over three times its target value) would have been left, thus obviously implying that the withdrawal plan would have been sustained. And importantly, in this scenario both failure rates (12% and 27.7%) are below the maximum acceptable failure rates (25% and 30%).

This type of analysis is critical to determine whether a base case plan should be put in place, or alternatively adjustments to it need to be made. A plan may look ideal when its expected path is outlined, but the historical evidence may show that it would have hardly ever been successful, as was largely the case, in fact, with the base case plan considered here. The feasibility analysis called for changes to the plan, albeit maintaining the lifestyle desired during retirement. Adjustments to the portfolio's asset allocation (making it more aggressive during both the retirement and the working periods) and to the annual contributions to the portfolio (a 20% increase) turned an unacceptable plan into an acceptable one.

3.5. Feasibility – Monte Carlo Analysis

The feasibility of a retirement plan can be evaluated on the basis of historical life cycle periods (as done in the previous section), or on the basis of simulated life cycle periods (as done in this section), and ideally both ways. The late Paul Samuelson has famously said that we have only one sample of history, which is why it is useful to assess how the retirement plan considered would have fared in many alternative life cycle periods that *could* have happened. Exhibit 4 summarizes the feasibility of the base case retirement plan discussed, as well as that of three adjustments to it, based on 10,000 simulated life cycle periods using Monte Carlo analysis.[12]

Exhibit 4: Feasibility – Monte Carlo Perspective

This exhibit summarizes the feasibility of the base case retirement plan from a Monte Carlo perspective, as well as the feasibility of three variations of the base case. It shows the coverage ratio (*CR*) and the bequest (*B*) for different stock-bond asset allocations during the working (AA_W) and the retirement (AA_R) periods, as well as for different annual real contributions (*C*), for given values of the number of years in the retirement (*N*) and working (*M*) periods and the annual real withdrawal (*W*). P_S denotes the probability of being short of the target coverage ratio and bequest. The target coverage ratio and bequest are 1 and $500,000. *N* and *M* in years; *C*, *W*, and *B* in thousands of inflation-adjusted dollars.

Parameters	Base Case		Scenario 2		Scenario 3		Scenario 4	
M	40		40		40		40	
N	30		30		30		30	
AA_W	70-30		**80-20**		**80-20**		**80-20**	
AA_R	30-70		30-70		**40-60**		**40-60**	
C	13.2		13.2		13.2		**15.8**	
W	100.0		100.0		100.0		100.0	
Results	*CR*	*B*	*CR*	*B*	*CR*	*B*	*CR*	*B*
P_S	42.2	47.7	38.2	42.9	34.6	39.3	24.8	28.7
Mean	1.93	2,552.3	2.30	3,680.4	2.75	4,989.1	3.55	7,498.1
Median	1.24	727.1	1.43	1,288.7	1.66	1,973.0	2.27	3,807.9

As is more often that not the case when comparing historical and Monte Carlo perspectives, the latter reveals more extreme results; that is, relative to a historical analysis, a Monte Carlo analysis tends to show a larger variance, with better results on the upper tail and worse results on the lower tail. Exhibit 4 shows that in the average (mean or median) simulated period, coverage ratios are well above 1 and bequests are well above $500,000; in other words, in the average period, the base case plan and all the alternative scenarios considered were successful. The three alternative scenarios in the exhibit are the same as those considered in the previous section; that is, first setting an 80-20 asset allocation during the working period (Scenario 2), then adding a 40-60 asset allocation during the retirement period (Scenario 3), and finally adding a 20% increase in the annual contributions (Scenario 4).

Interestingly, for the base case plan both the probability of failing to sustain the target withdrawal plan (42.2%) and that of failing to leave the target bequest (47.7%) are lower over the simulated periods than over the historical periods. That said, in the Monte Carlo analysis, as the scenarios become more aggressive these two probabilities decline more slowly, to the point that in Scenario 4 both are higher (24.8% and 28.7%) than they were in the historical perspective. Note, also, that in this final scenario both probabilities are just below the maximum acceptable failure rates for the withdrawal plan and the bequest; thus, once again, a few adjustments turned an unacceptable plan into an acceptable one.

[12] Kolmogorov-Smirnov tests do not reject the normality of the distributions of annual returns of the 70-30 and 30-70 allocations, with *p*-values of 0.764 and 0.987. The autocorrelation coefficients of the return series of these two allocations respectively are 0.028 and 0.126, neither of which is statistically significant.

As already mentioned, this type of analysis is critical to determine whether a plan should be put in place, or adjustments to it need to be made. Needless to say, the historical and Monte Carlo perspectives are not expected to provide identical results or even insights; but far from being a matter of right or wrong, these two perspectives should be considered as complementing each other. In short, after outlining a base case plan and performing sensitivity analysis on it, a critical next step is to assess its feasibility over historical or simulated periods, and ideally both ways.[13]

3.6. Some Final Thoughts

The framework proposed here is comprehensive, but as usual some comments and caveats are in order. First, and most obviously, this (or any other) framework requires the individual or financial planner to make guesses about the value of variables in a highly uncertain environment, which in turn implies that the plan will almost never come to happen as outlined; far more often than not the plan will require adjustments along the way. In that case, Estrada (2019) shows that adjusting the contributions to the portfolio is a far more effective way to get the plan back on track than adjusting the portfolio's asset allocation.[14]

Second, the steps above assume away other sources of income that could be received during retirement. However, most individuals have social security in addition to the retirement portfolio that is the focus of this discussion. In that case, the withdrawals taken from the portfolio considered here can be thought of as the annual cost of the lifestyle desired in retirement *net* of whatever can be funded by other sources of income received during that period.

Third, taxes and transaction costs are not considered in the analysis. Transaction costs have been decreasing over time, both in the U.S. and globally, and can be reduced in many ways; investing in a few broadly-diversified, passively-managed funds and minimizing portfolio turnover are two obvious ways to do so. Taxes, on the other hand, vary greatly across individuals (and across countries) and focusing on specific tax-related circumstances would run counter to the general analysis that this article aims to focus on.

And fourth, the focus of the discussion is on planning for retirement, not on portfolio construction. The asset allocations considered here could be implemented in many different ways, with individual stocks and bonds in one extreme, or with two broadly-diversified funds in the other extreme, and with countless alternatives in between. Furthermore, because many of the

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[13] The historical and Monte Carlo perspectives discussed here can also be used to evaluate tail risks, thus incorporating in the analysis the consideration of unlikely but relevant scenarios, which would yield a more comprehensive evaluation of the risk of different strategies.

[14] Similarly, if adjustments need to be made during the retirement period, Estrada (2020*a*) shows that adjusting withdrawals is far more effective than adjusting the portfolio's asset allocation.

pros and cons of different types of retirement accounts (such as traditional IRAs, Roth IRAs, etc.) are tax related, the choice of the ideal type of account is beyond the scope of this article.[15]

4. Assessment

Admittedly, retirement planning can be viewed as a nuisance. It requires individuals to consider issues they would rather not think about, particularly at a young age, such as how long they are going to live after they retire, what kind of life they would like to have in retirement, and how large a bequest they would like to leave behind. But unfortunately, the alternative to doing that is to shoot in the dark.

Contributing periodically to a retirement portfolio without having a specific target in mind makes planning impossible; and having a target retirement portfolio that may not sustain the lifestyle desired in retirement makes little sense. Therefore, retirement planning must be done from the lifestyle desired in retirement, to the withdrawals needed to sustain such lifestyle, to the portfolio needed to support those withdrawals, to the contributions needed to build that portfolio. The framework proposed in this article is designed to do just that.

Unfortunately, that does not solve all the problems that retirement planning entails. Individuals still have to face the many uncertainties that will affect their decisions, such as the number of years they will work, how much they will be able to contribute periodically to a retirement portfolio, the size of that portfolio, the number of years in retirement, the lifestyle desired during that period, the withdrawals needed to support it, and the portfolio returns over the lifecycle.

And yet, having an uncertain plan clearly beats having no plan. A plan that needs to be adjusted when circumstances change is a far better option than improvising contributions and withdrawals along the way. A plan provides the destination and the directions to get there; enables the periodic evaluation of whether an individual is on the right track to reach his or her goals; and if that is not the case, the changes that need to be made to get back on track.

The comprehensive framework proposed here consists of ten steps that start from the end; that is, from the variables that affect the retirement period, which determine the size of the target portfolio, which in turn determines, together with the variables that affect the working period, the periodic contributions that need to be made to build that portfolio. Importantly, a base case plan needs to be complemented by sensitivity analysis; and perhaps more importantly, by an evaluation of the feasibility of the plan on the basis of historical and simulated lifecycle periods.

[15] Other issues that may be important when planning for retirement, such as disability risk and long-term health expenditures, are also beyond the scope of this article.

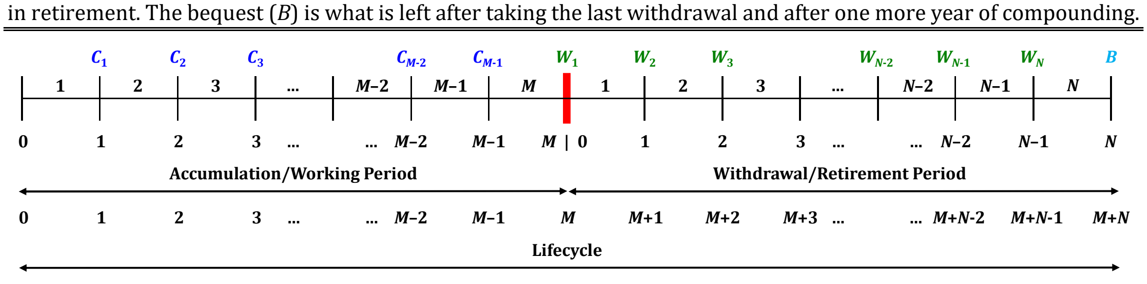
Retirement planning is too important to tackle only when an individual feels that it can no longer be postponed; it needs to be tackled as early and as thoroughly as possible. The step-by-step methodology proposed in this article, including the relevant analytical framework and essential mathematical expressions, and illustrated with a comprehensive example, will hopefully give individuals the incentive to confront, by themselves or with the help of a financial planner, a critical problem they will have to deal with sooner rather than later.

Appendix

This appendix discusses the analytical framework behind expressions (1) through (6) borrowing from and generalizing Estrada (2020*b*). As in the text, and for the reasons already discussed, the discussion starts with the retirement period and then continues with the working period. Because the exact mathematical expressions depend on the timing of the relevant cash flows (contributions and withdrawals), Exhibit A1 shows the specific timeline behind the framework presented in this article.

Exhibit A1: Timeline

This exhibit shows the timeline on which the analysis is based, with an M-year working period followed by an N-year retirement period. The individual makes M–1 contributions (C_t) to, and N withdrawals (W_t) from, the retirement portfolio, the former at the end of each working year (except the last one) and the latter at the beginning of each year in retirement. The bequest (B) is what is left after taking the last withdrawal and after one more year of compounding.



The Retirement (or Withdrawal) Period. Consider an individual at the beginning of her working years. She expects to live N years in retirement and estimates the cost of the lifestyle she desires during that period, which she summarizes in a sequence of real (inflation-adjusted) withdrawals W_1, …, W_N to be made from her retirement portfolio (P) at the beginning of each year in retirement. She also plans to leave a bequest (B), which will be the final withdrawal from her portfolio, at the end of the retirement period. Finally, she selects her portfolio's asset allocation for the retirement period and estimates a sequence of (expected) annual real returns R_1, …, R_N. Then, *at the beginning* of her first, second, third, and last year in retirement, the value of her portfolio will be

Year 1: $P–W_1$
Year 2: $(P–W_1)(1+R_1)–W_2 = P(1+R_1)–W_1(1+R_1)–W_2$
Year 3: $[P(1+R_1)–W_1(1+R_1)–W_2](1+R_2)–W_3 = P(1+R_1)(1+R_2)–W_1(1+R_1)(1+R_2)–W_2(1+R_2)–W_3$
…
Year N: $P(1+R_1)…(1+R_{N-1})–W_1(1+R_1)…(1+R_{N-1})–W_2(1+R_2)…(1+R_{N-1})–…–W_N$

After compounding for one final year the portfolio becomes the bequest; hence

$$[P(1+R_1)…(1+R_{N-1})–W_1(1+R_1)…(1+R_{N-1})–W_2(1+R_2)…(1+R_{N-1})–…– W_N](1+R_N) = B$$
$$P(1+R_1)…(1+R_N)–W_1(1+R_1)…(1+R_N)–W_2(1+R_2)…(1+R_N)–…– W_N(1+R_N) = B$$

and solving for P yields the target retirement portfolio (P^*), which is given by

$$P^* = \frac{B + \sum_{t=1}^{N} W_t \cdot \prod_{j=t}^{N}(1+R_j)}{\prod_{t=1}^{N}(1+R_t)} \quad \text{(A1)}$$

which is expression (1) in the text. If the individual aims to keep her purchasing power constant over time, then $W_1 = W_2 = … = W_N = W$, where W is the constant annual real withdrawal. If, in addition, the annual real returns are replaced with the expected annualized real return during the retirement period (R), then $R_1 = R_2 = … = R_N = R$, and then (A1) becomes

$$P^* = \frac{B + W \cdot \sum_{t=1}^{N}(1+R)^t}{(1+R)^N} \tag{A2}$$

which, using the expression for the sum of a geometric series, can be written as

$$P^* = \frac{B - W \cdot (1+R) \cdot \left(\frac{1-(1+R)^N}{R}\right)}{(1+R)^N} \tag{A3}$$

which is expression (2) in the text.

The Working (or Accumulation) Period. Having determined her target retirement portfolio (P^*), and expecting to work M years, our individual needs to determine the annual contributions that will enable her to retire with P^*. She expects to make M–1 annual contributions to her portfolio (C_1, …, C_{M-1}), the first at the end of her first working year and the last one year before retirement, and she aims to keep them constant in real terms; hence $C_1 = \ldots = C_{M-1} = C$, where C is the constant annual real contribution. She selects her portfolio's asset allocation for her working years and estimates a sequence of expected annual real returns S_1, …, S_{M-1}. Then, *at the end* of her first, second, third, and next-to-last working years the value of her portfolio will be

Year 1: C
Year 2: $C(1+S_1) + C$
Year 3: $[C(1+S_1) + C](1+S_2) + C = C(1+S_1)(1+S_2) + C(1+S_2) + C$
…
Year M–1: $C(1+S_1)\ldots(1+S_{M-2}) + C(1+S_2)\ldots(1+S_{M-2}) + C(1+S_3)\ldots(1+S_{M-2}) + \ldots + C(1+S_{M-2}) + C$

After compounding for one more year, during our individual's last working year, the portfolio becomes her target retirement portfolio; that is,

Year M: $[C(1+S_1)\ldots(1+S_{M-2}) + C(1+S_2)\ldots(1+S_{M-2}) + C(1+S_3)\ldots(1+S_{M-2}) + \ldots + C(1+S_{M-2}) + C](1+S_{M-1}) = P^*$
$C(1+S_1)\ldots(1+S_{M-1}) + C(1+S_2)\ldots(1+S_{M-1}) + C(1+S_3)\ldots(1+S_{M-1}) + \ldots + C(1+S_{M-1}) = P^*$
$C[(1+S_1)\ldots(1+S_{M-1}) + (1+S_2)\ldots(1+S_{M-1}) + (1+S_3)\ldots(1+S_{M-1}) + \ldots + (1+S_{M-1})] = P^*$

and solving for C yields the constant annual real contribution (C^*), which is given by

$$C^* = \frac{P^*}{\sum_{t=1}^{M-1}\prod_{j=t}^{M-1}(1+S_j)} \tag{A4}$$

which is expression (3) in the text. If the annual real returns are replaced with the expected annualized real return during the working period (S), then $S_1 = \ldots = S_N = S$, and then (A4) becomes

$$C^* = \frac{P^*}{\sum_{t=1}^{M-1}(1+S)^t} \tag{A5}$$

which, using the expression for the sum of a geometric series, can be written as

$$C^* = \frac{P^*}{\left(\frac{(1+S)^M-(1+S)}{S}\right)} \tag{A6}$$

which is expression (4) in the text.

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