The Race for Product Renewal

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Abstract

In many product categories, life-cycles have been shortening in recent years. Competitive forces seem to have a strong influence on new product decisions, but it is not clear how they impact the pace at which new items are introduced, as well as their price, quality and durability. While consumers appreciate novelty in settings where product utility decays over time, it also involves higher product launch costs, so it is important to understand what the net effect on welfare is. In this paper, we build an analytical model where consumers decide to renew their products as a function of product launches, and firms optimize product features, pricing, and introduction times. We use an optimal control framework to derive the consumer's optimal time to change products. We then analyze the firm's problem both for the monopoly, using dynamic programming, and under competition, characterizing the subgame-perfect equilibrium of the firms' dynamic launch game. We find that the firm's problem for the monopoly is fundamentally different from that of competition: a monopoly internalizes inter-product cannibalization. As a result, it sets efficient launch decisions at optimality, while competition results in an oversupply of new products. Our results suggest that indeed competitive outcomes generate an increase in new product releases.

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1 Introduction

We live in an era of hyperconsumption (Lipovetsky 2006, Pardo 2007), where products and experiences have lost their durability (Lipovetsky 2002). Bauman (2013) even suggests that we are experiencing *liquid times* where changes happen so fast, that we cannot hold onto solid elements anymore. This evolution is clearly visible is in the consumption of durable goods, where products that in the past lasted for decades, are now renewed in a matter of years. For example, smartphones, one of the fastest growing consumer items since Apple introduced the iPhone in 2007, have grown to a global market size of 1.5 billion units sold in 2017, up from 122 million in 2007 (Gartner 2020). Interestingly, Apple has maintained its place among the best sellers because it releases an improved iPhone version every year. The company announces the product's technical specifications and physical characteristics before its official launch date, and with this strategy possible buyers compare the version that they own versus the one that is coming to see if it is worthy to switch. On December 21 2017, news broke that Apple was purposefully slowing down the performance of old phones to prevent them from malfunction (The Verge 2017, Geekbench 2017), and resulted in significant consumer backlash (Fortune 2017). This example suggests that companies actively

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manage product durability by coordinating times between product releases, product functionality, and product performance evolution, in addition to price.

The phenomenon of shortening of product lifespans is thus a complex one, where firms take an active role and compete with each other over time, and consumers react to changes in the market. Indeed, what we observe should be the equilibrium of a dynamic game between firms, and it is important to understand their underlying economic motives to evaluate whether the observed outcomes are good ones. The aim of this paper is precisely to model this game of planned obsolescence. We answer the following questions: (1) When should a consumer acquire a new product? (2) How does the firm decide the attributes for every product? and (3) When should the firm release a new product?

In order to answer these questions we develop a theoretical model where firms are first movers that compete, and consumers react to their product launch decisions. Specifically, we first look at a single consumer's replacement decision, in an optimal control framework that determines the optimal time to change products. We find that, in any equilibrium, the consumer adopts a new product at the time of launch. We then consider the firm's problem. We start analyzing the problem of a monopolist that has to decide the product's optimal features and release dates, and characterize the optimal policy. We then analyze the effect of competition, with two or more firms offering sequential product launches. We characterize the equilibrium of this dynamic game and show that it is inefficient. Indeed, under competition, we observe an arms race, where firms develop products that will last shorter and have lower quality. This result highlights that firms should internalize the cost of new product launches. Finally, we extend our results to the case where consumers are heterogeneous and the firm's decisions are influenced by different combinations of consumer types. In these scenarios, consumers that are more patient might decide to skip a generation if the expected utility from the new product does not create a positive value. Thus, the firm is forced to reduce its renewal speed to capture patient and impatient consumers.

Our work thus contributes to the literature by considering endogenous product features, in addition to optimal launch times (Fishman and Rob 2000, Lim and Tang 2006, Plambeck and Wang 2009, Caro et al. 2014, Lobel et al. 2015). It thus complements the findings of Lobel et al. (2015) in a dynamic setting. Our model is grounded on a consumer model with optimal product switching, so that consumers do not necessarily buy every new product, given that in our model the product's utility is not always increasing in value, in contrast with Fishman and Rob (2000), and Plambeck and Wang (2009). Furthermore, our model considers both quality and durability (in the form of a utility decay), which means that the evolution of product value is also endogenous, in contrast with Lobel et al. (2015). These model differences lead to new insights. First, because of the larger decision space for the firms, they have more instruments to capture consumer surplus and are able to keep all welfare to themselves, when consumers are homogeneous. Second, endogenous durability implies that the competitive setting affects the incentive to invest in longer-lasting products, and specifically competition leads to lower product quality and lower durability. This is a short-term blessing for a firm, because it captures market share sooner with a lower-cost product, but a longer-term disaster, because its own product is quickly replaced by a competitor's. Our paper thus advances our understanding of the competitive pressures behind product introductions and specifically connects them to consumer intertemporal optimal choices and the use of the multiple dimensions of new products: price, quality, and decay rate. We hope that our framework can be used as a building block for future studies on dynamic competition and planned obsolescence.

The rest of the paper is organized as follows: §2 reviews the relevant literature. We present the consumer's model in §3 and the firm's problem in §4. We incorporate heterogeneous consumers in §5. Finally, §6 concludes and presents limitations and future avenues for research.

2 Literature Review

Our work is related to multiple fields from economics and more specifically to industrial organization.

The first relevant theme is the theory of durable goods and planned obsolescence. Levhari and Srinivasan (1969) characterize the conditions for a monopolist to benefit from a shorter durability strategy. Coase (1972) shows that in a monopoly where there is no possible resale, the firm should reduce the lifespan of a durable good so as to sustain over time the incentive to limit supply and charge monopoly prices as a result. The key takeaway is that the firm has an incentive to make short-lived durable goods in order to increase sales. The Coase conjecture suggests that monopoly prices cannot be sustained when consumers anticipate that more supply of durable goods will be brought to the market in the future. Bulow (1982) analyzes the problem of a monopolist seller and shows that if the durability of a product is reduced, the firm offers renting as opposed to selling, and obtains larger profits. Bulow (1986) considers obsolescence factors in a two-period model, and shows that it is optimal for a monopolist to select a shorter lifespan strategy. In his model, competition provides an incentive to extend product durability, which is in opposition with our findings. The reason for this difference is that in our model firms also decide on product features and introduction times, which allows them to capture all consumer surplus even under competition. We point the reader to Waldman (2003) for a review of the durable goods literature. Questions about revenue aspects (selling vs. leasing), consumer preferences, and operational options (remanufacturing and closed-loop supply chains) have also been considered by Desai and Purohit (1999), Savaskan et al. (2004), Agrawal et al. (2012), Gowrisankaran and Rysman (2012) or Agrawal and Bellos (2016), among others. Guide Jr and Van Wassenhove (2009) and Agrawal et al. (2019) review the recent literature on the topic.

Empirical papers on planned obsolescence are scarce. Purohit (1992) analyzes the automobile industry and shows that as new models are introduced, the price of used cars drops. Esteban and Shum (2007) develop a structural model of dynamic oligopoly to understand the relationship between new products and secondary markets. They use data from the automobile industry to estimate their parameters, and they characterize the equilibrium conditions for every firm. Iizuka (2007) uses data from the textbook market to show that publishers revise editions at a faster pace when competition from used versions rises. Gordon (2009) develops and estimates a dynamic model of consumer demand using data from the PC processor industry. He shows that there is a positive correlation between quality and replacement decisions, and a negative relationship between prices and replacements. Gavazza (2011) analyzes the trade-off between leasing and selling in the commercial aircraft market. He determines that leased planes are traded more frequently and have a higher capacity utilization than owned planes.

New product introduction is another relevant area. Fishman and Rob (2000) show that a monopolist should design each model to last until a new version is introduced: with this strategy the firm obtains the full social value and keeps its motivation to innovate. We find the same result in our model, which incorporates a decay rate to capture progressive obsolescence over time. Lobel et al. (2015) show that the release of a new product should take place as soon as the new technology surpasses the one that is currently available in the market, even in the case of strategic consumers that are fully informed about future product launches. Plambeck and Wang (2009) analyze the effect of regulations alongside new product introduction. They show that a fee-upon-sale would reduce the number of items produced and disposed, and that a fee-upon-disposal would reduce manufacturer's profits. Furthermore, they also analyze what would happen in a duopoly: firms release products faster as in our model, but they do not consider how these items decay over time and how obsolescence influences decision making. In contrast with these papers, we introduce endogenous decay rates, which can be controlled in addition to quality levels: this additional degree of freedom leads firms to alter the quality-durability trade-off and is strongly influenced by competition structure. Bernstein and Martínez-de-Albéniz (2017) also consider sequential releases, but they focus on the inter-temporal substitution aspect by allowing consumers to buy just once over the horizon. In reality, multiple products may coexist in the market, hence substitution patterns and demand diffusions need to be considered. Krider and Weinberg (1998) consider competitive dynamics and optimize movie introduction times. Swami et al. (2001), Krankel et al. (2006), Bradley and Guerrero (2008), Martínez-de-Albéniz and Valdivia (2019), and Kash et al. (2021) among others optimize product replacements. Caro et al. (2014) optimize product launch decisions of a given assortment taking into account inter-product cannibalization over time.

We contribute to this literature by developing a model that calculates the optimal purchase time for the consumer and shows how the firm should select its release schedule in order to maximize its profits. Additionally, we incorporate competition and through a dynamic game setting we find the equilibrium to compare the outcomes in both cases. Moreover, we endogenize the decision of the technology adoption, in contrast with the previous literature. In particular, we do not adopt the standard assumption that quality is increasing over time, and propose a different view: product starting quality can be freely adjusted, but reduces over time. While increasing quality and discounting (Lobel et al. 2015), vs. stable quality and decay (here) are qualitatively similar for consumer behavior, we allow firms to select freely among quality and decay rates, which leads to new insights arising from the larger strategy space for firms.

3 A Model of Dynamic Consumer Choice

Consider a representative consumer (she) that purchases items in a given category over time. Even when there is a continuum of consumers, we assume that they have the same preferences: they all start empty-handed and thus face exactly the same product choices over time.

Items are provided by one or multiple firms (we analyze the firm's problem below), and are differentiated. For the purpose of consumer choices, the characteristics of these items are given in advance and can be taken as fixed in the consumer's decision process.

Each item i is introduced at a time τ_i and is always available afterwards, i.e., items are not removed from the choice set. Item i offers a level of utility $\alpha_i \geq 0$ at the moment of introduction τ_i. This can be interpreted as the level of technology that product i has at launch. Because there is innovation in the category, the utility received by the consumer at $t > \tau_i$ might vary over its launch value. Indeed, consumers appreciate novelty, either because of intrinsic reasons such as preference for variety (Caro and Martínez-de-Albéniz 2012) or because of extrinsic variations in product value (Caro et al. 2014, Lobel et al. 2015). To account for her preference for novelty, we introduce a second characteristic of product i: a decay rate, denoted $\beta_i \geq 0$, which exponentially reduces utility of item i over time. This can be considered as an obsolescence factor. Given τ_i, α_i and β_i, the utility that a consumer derives from product i at t is equal to:

$$u_i(t) = \begin{cases} 0, & \text{if } t < \tau_i; \\ \alpha_i e^{-\beta_i(t-\tau_i)}, & \text{otherwise.} \end{cases} \tag{1}$$

This is an infinitesimal utility that the consumer perceives as long as she uses product i. In addition, at the moment of acquisition of i, the consumer must incur in a one-time expense of p_i, which represents the price of product i.

Note that this formulation is similar in spirit to Lobel et al. (2015) with two main differences. First, in our model utility is decreased over time through a decay, while in Lobel et al. (2015) utility is stable but alternative technology keeps growing over time through a random walk. Second, in our model product-specific decays are endogenous values decided by the firm, while in Lobel et al. (2015) they are given exogenously; this introduces a richer competitive space which turns out to generate different outcomes.

We are now ready to formulate the consumer's dynamic choice problem, which follows the lines of Dagsvik (2002). At the beginning of the horizon $t = 0$, the consumer has no product in her hands, which we denote by item $i = 0$ with characteristics $\tau_0 = -\infty, \alpha_0 = 0, \beta_0 = 0$, and $p_0 = 0$. After that, she may purchase one product, with the condition that if she wants to buy another one, she will dispose of the old one. This is the expected behavior in consumer electronics, such as consumption of smartphones, computers, or refrigerators. It also applies for other types of purchases such as automobiles or certain categories within fast-fashion apparel (Caro and Martínez-de-Albéniz 2015). Hence, in every moment t, she will compare the current item i that she owns, versus the available set of products in the market at that time, which we denote by $I_t = \{j | t \geq \tau_j\}$.

The objective of the consumer is to maximize her discounted cumulative utility. Utility is discounted at an exogenous rate $r \geq 0$. This is an optimal control problem, which can be defined through a value function $U(i, t)$, which is the optimal discounted cumulative utility achieved in $[t, +\infty)$, starting from a state where the consumer holds product i in her hands.

At time t, the optimal choice condition is that

$$U(i,t) = \max \begin{cases} U(i,t), & \text{if she keeps product i} \\ U(i',t) - p_{i'}, & \text{if she acquires product } i' \neq i, i' \in I_t. \end{cases} \tag{2}$$

Moreover, $U(i, t)$ satisfies the Hamilton-Jacobi-Bellman equation, which can be expressed here as[3]

$$\frac{dU}{dt}(i,t) = -u_i(t) + rU(i,t). \tag{3}$$

This implies that the structure of the value function must be of the form

$$U(i,t) = \frac{u_i(t)}{r + \beta_i} + Ke^{r(t-\tau_i)}, \tag{4}$$

where $K \geq 0$ is a constant that depends on the future optimal choices. This constant is derived from a border condition at the moment of product change, and must be non-negative to guarantee that it is indeed optimal to switch products (otherwise it would be best not to switch).

The problem is to characterize the value of the constant K in every state. For this purpose, assume that in the optimal solution, products $i_1, i_2, \ldots$ are purchased at times $t_1, t_2, \ldots$. Let K_n be the constant such that $U(i_n, t) = \frac{u_{i_n}(t)}{r+\beta_{i_n}} + K_n e^{r(t-\tau_{i_n})}$. From (2), we must have that

$$\frac{u_{i_n}(t_n)}{r + \beta_{i_n}} + K_n e^{r(t_n - \tau_{i_n})} - p_{i_n} = \frac{u_{i_{n-1}}(t_n)}{r + \beta_{i_{n-1}}} + K_{n-1} e^{r(t_n - \tau_{i_{n-1}})}. \tag{5}$$

Using this condition, we can vary t_n (for any n) and determine the optimal purchase time for the consumer, as shown in our first theorem.

Theorem 1 *It is optimal for a consumer to purchase product i:*

(a) either at $t = \tau_i$;

[3] The development of the optimal control function can be found in the Appendix.

(b) or at the unique time t satisfying $\alpha_{i_n} e^{-\beta_{i_n}(t-\tau_{i_n})} = \alpha_{i_{n-1}} e^{-\beta_{i_{n-1}}(t-\tau_{i_{n-1}})} + rp_{i_n}$;

(c) or not at all.

As formally shown later on, it will never be optimal for the firm to have consumers choosing optimal purchase time in (b), because the firm can launch a less attractive product (hence cheaper) without changing the corresponding revenues.

4 The Firm's Problem

4.1 Preliminary results

We now turn to the firm's optimization problem. This involves setting the launch date τ_i, the values of the product's characteristics α_i and β_i, and the price of purchase p_i, for each product $i \in S$, the set of products launched by the firm. These products will compete with competitor products $i' \in S'$. We consider that strategic interactions between firms are closed-loop, i.e., firms select at time t whether they want to launch a product. This means that their actions may affect future decisions of competitors, and hence our equilibrium concept is based on subgame perfection. Once these strategies are fixed, the consumer decides on a consumption path, which means that we assume that the consumer is a passive player that provides a static response to firms' actions (because she optimizes her inter-temporal choices). This approach is similar to the pre-announced strategies of Lobel et al. (2015) but with multiple firms. Such strategies are reminiscent of the dynamics around smartphone manufacturers, where Apple releases a new version of the iPhone every Fall, Samsung responds by launching a new S model every Spring, and consumers can expect this cycle to occur.

The firm incurs in a launch cost $\hat{c}(\alpha_i, \beta_i) \geq 0$ that depends on product i's characteristics, with $\hat{c}(0, \infty) = 0$. For tractability (see derivations below), we define $h_i = \frac{\alpha_i}{r+\beta_i}$ as the quality-decay ratio. Moreover, we let $c(h_i, \beta_i) = \hat{c}\Big(h_i(r + \beta_i), \beta_i\Big)$, and we assume that c is strictly increasing in h_i and decreasing in β_i. It is also jointly convex in (h_i, β_i), to reflect on the one hand the increasing efforts required to push the technology frontier, and to make products more durable. Similar assumptions have been made before in the literature (Plambeck and Wang 2009, Druehl et al. 2009, Chao and Kavadias 2008). We also define $prev(i)$ as the immediately preceding product, and $next(i)$ the immediately following one.

Given the strategies of all firms, the solution of the consumer's problem indicates whether item i was purchased. We denote this event by B_i, and when B_i is true, then t_i denotes the finite purchase time, described in Theorem 1. As a result, letting θ be the firm's discount factor, and assuming that the market size is constant over time and normalized to one, the firm's discounted profit can be written as

$$\Pi = \sum_{i\in S} \left[-e^{-\theta\tau_i}\hat{c}(\alpha_i, \beta_i) + e^{-\theta t_i} p_i 1_{B_i} \right]. \tag{6}$$

It turns out that we can simplify this formulation by showing that the firm never finds it optimal to launch a product that is not bought or bought later than its introduction time (otherwise it would be strictly better to postpone launch and reduce α_i).

Lemma 1 *Given fixed competitor's introduction times, it is optimal for a firm to launch product i so that the consumer purchases it at the moment of launch τ_i.*

This implies that, from a firm's perspective, it will set τ_i, α_i, and β_i so that the consumer always opts for option (a) in Theorem 1. Therefore, using (5), we can rewrite (6) as:

$$\Pi = \sum_{i\in S} e^{-\theta\tau_i} \left(h_i + K_i - h_{prev(i)} e^{-\beta_{prev(i)}(\tau_i - \tau_{prev(i)})} - K_{prev(i)} e^{r(\tau_i - \tau_{prev(i)})} - c(h_i, \beta_i) \right). \tag{7}$$

Given the strategies of the competitors, the firm seeks to maximize its discounted profit Π. As shown in §3, the consumer's utility increases in $K_i \geq 0$, the constant that captures the utility derived from a product change. Since this constant is directly affected by p_i, it is immediate that given τ_i, α_i and β_i, it is optimal for the firm to increase p_i until $K_{prev(i)} = 0$. This means that the firm captures all consumer utility until she hits her reservation value of zero utility (the same argument has been used in Fishman and Rob 2000). This allows us to characterize the optimal price that the firm can charge for product i.

Lemma 2 *The firm will set the optimal price for product i, p_i, as follows:*

$$p_i = h_i - h_{prev(i)} e^{-\beta_{prev(i)}(\tau_i - \tau_{prev(i)})}. \tag{8}$$

Note that if this value is negative, then it is better for the firm not to launch the product at all.

4.2 Monopoly

We begin by analyzing a monopoly, to make comparisons with the competitive market later on. Moreover, in our case we are not focusing on the monopolist's profit like in Fishman and Rob (2000) and Plambeck and Wang (2009). Our analysis is centered on the selection of the optimal characteristics of every product that the firm manufactures: τ^*, h^*, and β^*. By letting the firm select these parameters, we consider a broader decision space, in contrast with the previous literature.

In the monopoly scenario, all the products launched are adopted, and hence $prev(i) = i - 1$. The firm must decide when to launch the product, τ_i, and what specific characteristics to offer, h_i and β_i. According to Lemma 1, the firm will begin to capture sales at the moment of introduction, τ_i. We can define the discounted profit of the firm as follows:

$$\Pi = \sum_{i=1}^{\infty} e^{-\theta\tau_i}[p_i - c(h_i, \beta_i)] \tag{9}$$

We define Δ_i as the amount of time that has passed between the introduction of two consecutive products. Thus, $\Delta_i = \tau_i - \tau_{i-1}$. From the previous section, we know that the price of the product will be given by the expected difference in the consumer's utility function. In order to maximize profits the firm must choose the optimal values of the variables it controls.

We define the value function $V_i(h_{i-1}, \beta_{i-1})$ as the optimal discounted value after the $i-1$th product has been released at τ_{i-1} (therefore not including the revenue collected by product $i-1$). This value function can be recursively expressed as a dynamic program (DP), as follows:

$$V_i(h_{i-1}, \beta_{i-1}) = \max_{h_i,\beta_i,\Delta_i \geq 0} e^{-\theta\Delta_i} \left\{ h_i - h_{i-1}e^{-\beta_{i-1}\Delta_i} - c(h_i, \beta_i) + V_{i+1}(h_i, \beta_i) \right\}. \tag{10}$$

We define $d_i = e^{-\Delta_i}$, and we know that the value of this parameter is going to be bounded between zero and one. Hence, the optimization problem can be rewritten as:

$$V_i(h_{i-1}, \beta_{i-1}) = \max_{h_i,\beta_i \geq 0, d_i \in [0,1]} d_i^{\theta} \left\{ h_i - h_{i-1}d_i^{\beta_{i-1}} - c(h_i, \beta_i) + V_{i+1}(h_i, \beta_i) \right\}. \tag{11}$$

Notice that the maximum for (h_i, β_i) is independent of (h_{i-1}, β_{i-1}). This decision echoes the insight of Lobel et al. (2015), where there is no uncertainty in R&D. This implies that:

$$\bar{V}_i := \max \{h_i - c(h_i, \beta_i) + V_{i+1}(h_i, \beta_i)\} \text{ and } (h_i^*, \beta_i^*) = \operatorname{argmax} \{h_i - c(h_i, \beta_i) + V_{i+1}(h_i, \beta_i)\}. \tag{12}$$

Moreover, optimizing over d_i yields $\theta d_i^{\theta-1}\bar{V}_i = (\theta + \beta_{i-1})h_{i-1}d_i^{\theta+\beta_{i-1}-1}$ so that

$$d_i^* = \left(\frac{\theta \bar{V}_i}{(\theta + \beta_{i-1})h_{i-1}} \right)^{\frac{1}{\beta_{i-1}}} \tag{13}$$

Equations (12) and (13) allow us to rewrite (11) as

$$V_i(h_{i-1}, \beta_{i-1}) = \frac{\bar{V}_i^{1+\frac{\theta}{\beta_{i-1}}}}{h_{i-1}^{\frac{\theta}{\beta_{i-1}}}} \frac{\beta_{i-1}\theta^{\frac{\theta}{\beta_{i-1}}}}{(\theta + \beta_{i-1})^{1+\frac{\theta}{\beta_{i-1}}}} \tag{14}$$

Hence, in infinite horizon, we must have that $V_i(\cdot) = V_{i+1}(\cdot)$ which implies that

$$\bar{V} := \max_{h,\beta \geq 0} \left\{ h - c(h, \beta) + \frac{\bar{V}^{1+\frac{\theta}{\beta}}}{h^{\frac{\theta}{\beta}}} \frac{\beta\theta^{\frac{\theta}{\beta}}}{(\theta + \beta)^{1+\frac{\theta}{\beta}}} \right\} \tag{15}$$

which is a one-dimensional fixed point equation, and admits a largest solution $\bar{V}$. This leads to the optimal release strategy, shown in the next theorem.

Theorem 2 *Let $\bar{V}$ be the largest solution to Equation* (15). *Then it is optimal for a monopoly firm to launch products with stationary characteristics*

$$(h^*, \beta^*) := \underset{h,\beta \geq 0}{argmax} \left\{ h - c(h,\beta) + \frac{\bar{V}^{1+\frac{\theta}{\beta}}}{h^{\frac{\theta}{\beta}}} \frac{\beta\theta^{\frac{\theta}{\beta}}}{(\theta+\beta)^{1+\frac{\theta}{\beta}}} \right\}, \tag{16}$$

with a fixed time between releases given by

$$\Delta^* = -\frac{1}{\beta^*}\left[\log(\theta) + \log(\bar{V}) - \log(\theta + \beta^*) - \log(h^*)\right]. \tag{17}$$

Theorem 2 fully characterizes the monopoly's launch strategy. It establishes that inter-product interactions are a key driver of launch decisions: the firm internalizes the effect of a new product in the future revenue, which takes the form of $\frac{\bar{V}^{1+\frac{\theta}{\beta}}}{h^{\frac{\theta}{\beta}}} \frac{\beta\theta^{\frac{\theta}{\beta}}}{(\theta+\beta)^{1+\frac{\theta}{\beta}}}$. In this expression, $\bar{V}$ accounts for the future market potential, and future revenue decreases with h and increases with β, i.e., if the current product's value is lower or drops faster, then the comparative value (and hence the price paid) of the future product increases. Moreover, the time between product launches decreases with $\bar{V}$, suggesting that when future market value is higher, then the firm has an incentive to hasten the pace of renewal. Note that this finding is contrary to Bulow (1986). This is because product features as well as the related costs are also determined endogenously in our model. We are also not making any assumptions with regard to the quality level of the product, while in previous papers it needs to be increasing with respect to time (Fishman and Rob 2000, Plambeck and Wang 2009). In fact, future revenue of the firm is decreasing with respect to the product's quality and increases with the product's decay, suggesting that high quality might be an undesired choice for the monopoly. As an illustration, a similar strategy was followed by the classic case of Dupont in the 1940s. Since the firm held the technology that made possible the manufacturing of nylon stockings, it decided to release a new version with lower quality in order to induce higher replacement. This decision allowed the firm to obtain larger levels of profits and it increased its market position because prices were also lowered (Slade 2009).

4.3 Oligopoly

In the case of multiple firms, we contain the analysis to cyclic launch policies. That is, we assume that there are $n \geq 2$ firms, in the set $\{1, \ldots, n\}$, which launch products one after the other. We denote product (i, j) as the i-th product launched by firm j. Hence, the sequence of products in the market is $(1,1) \to (1,2) \to \ldots \to (1,n) \to (2,1) \to \ldots$. We now denote $\Delta_{(i,j)} = \tau_{(i,j)} - \tau_{(i,j-1)}$ for $j \geq 2$ and $\Delta_{(i,1)} = \tau_{(i,1)} - \tau_{(i-1,n)}$. As before $d_{(i,j)} = e^{-\Delta_{(i,j)}}$.

The strategy of firm j is to set $(h_{(1,j)}, \beta_{(1,j)}, d_{(1,j)}), (h_{(2,j)}, \beta_{(2,j)}, d_{(2,j)})$, and so on. We are interested in subgame-perfect equilibrium strategies, so an optimal strategy must satisfy for $j \geq 2$ (the expression is similar for $j = 1$),

$$
\begin{aligned}
V_{(i,j)}\left(h_{(i-1,j)},\beta_{(i-1,j)}\right) = \max_{h_{(i,j)},\beta_{(i,j)},d_{(i,j)}} d_{(i,j)}^{\theta_j}\Big[h_{(i,j)} - h_{(i,j-1)} d_{(i,j)}^{\beta_{(i,j-1)}} - c_j(h_{(i,j)},\beta_{(i,j)}) \\
+ \left(d_{(i,j+1)}\dots d_{(i,n)} d_{(i+1,1)}\dots d_{(i+1,j-1)}\right)^{\theta_j} V_{(i+1,j)}\left(h_{(i,j)},\beta_{(i,j)}\right)\Big]
\end{aligned}
\tag{18}
$$

$V_{(i,j)}$ in Equation (18) now represents the discounted profit of firm j at time $t = \tau_{(i-1,j)}$, i.e., just after firm j launched its last product, assuming that after t all firms play in equilibrium. Firm j's discount factor is now θ_j. In comparison with (10), we see that now the discounted revenue for products $(i+1,j),(i+2,j)$, etc. now depends on the difference between launch dates of all the items $i+1$ and i that are released in the market, because there is going to be competition among all the firms that will release products during that period.

As in the case of monopoly, we can see that the choice of $\left(h_{(i,j)},\beta_{(i,j)}\right)$ is independent of $\left(h_{(i-1,j)},\beta_{(i-1,j)}\right)$ so, letting $e_{(i,j)} := \left(d_{(i,j+1)}\dots d_{(i,n)} d_{(i+1,1)}\dots d_{(i+1,j-1)}\right)^{\theta_j}$ – an input for firm j at this point, which will need to satisfy an equilibrium condition – the optimal policy of firm j is given by

$$
\bar{V}_{(i,j)} := \max_{h_{(i,j)},\beta_{(i,j)}} \left\{h_{(i,j)} - c_j(h_{(i,j)},\beta_{(i,j)}) + e_{(i,j)} V_{(i+1,j)}(h_{(i,j)},\beta_{(i,j)})\right\} \tag{19}
$$

which again implies that product features are stationary in an equilibrium policy. Note that the difference with Equation (12) is the presence of $e_{(i,j)} \leq 1$, which suggests that under competition a firm will discount future revenues and put more emphasis on income from the current product. As before, we can now optimize over $d_{(i,j)}$ and obtain

$$
d^*_{(i,j)} = \left(\frac{\theta_j \bar{V}_{(i,j)}}{(\theta_j + \beta_{(i-1,j)}) h_{(i-1,j)}}\right)^{\frac{1}{\beta_{(i-1,j)}}}. \tag{20}
$$

Thus, similarly as in (14), we have

$$
V_{(i,j)}(h_{(i-1,j)},\beta_{(i-1,j)}) = \frac{\bar{V}_{(i,j)}^{1+\frac{\theta_j}{\beta_{(i-1,j)}}}}{h_{(i-1,j)}^{\frac{\theta_j}{\beta_{(i-1,j)}}}} \frac{\beta_{(i-1,j)} \theta_j^{\frac{\theta_j}{\beta_{(i-1,j)}}}}{(\theta_j + \beta_{(i-1,j)})^{1+\frac{\theta_j}{\beta_{(i-1,j)}}}} \tag{21}
$$

Because competitor policies are stationary too, then it follows that the optimal launch strategy given by $d^*_{(i,j)}$ is also fixed when $\bar{V}_{(i,j)}$ is independent of i, i.e., $\bar{V}_{(i,j)} = \bar{V}_j$. Then a sufficient and necessary condition for equilibrium is that:

$$
(h_j^{eq},\beta_j^{eq}) := \underset{h_j,\beta_j}{argmax} \left\{h_j - c_j(h_j,\beta_j) + e_j^{eq} \frac{\left(\bar{V}_j^{eq}\right)^{1+\frac{\theta_j}{\beta_j}}}{h_j^{\frac{\theta_j}{\beta_j}}} \frac{\beta_j \theta_j^{\frac{\theta_j}{\beta_j}}}{(\theta_j+\beta_j)^{1+\frac{\theta_j}{\beta_j}}}\right\}; \tag{22}
$$

$$d_j^{eq} = \left(\frac{\theta_j \bar{V}_j^{eq}}{(\theta_j + \beta_j^{eq}) h_j^{eq}} \right)^{\frac{1}{\beta_j^{eq}}} ; \tag{23}$$

$$\bar{V}_j^{eq} := \max_{h_j, \beta_j \geq 0} \left\{ h_j - c_j(h_j, \beta_j) + e_j^{eq} \frac{\left(\bar{V}_j^{eq}\right)^{1+\frac{\theta_j}{\beta_j}}}{h_j^{\frac{\theta_j}{\beta_j}}} \frac{\beta_j \theta_j^{\frac{\theta_j}{\beta_j}}}{(\theta_j + \beta_j)^{1+\frac{\theta_j}{\beta_j}}} \right\} \tag{24}$$

and

$$e_j^{eq} = \prod_{j' \neq j} (d_{j'}^{eq})^{\theta_j}. \tag{25}$$

Theorem 3 *There exists an equilibrium of the launch decisions for an oligopoly. It is stationary and defined by Equations* (22)-(25).

Note that while Theorem 3 guarantees existence of equilibrium, it does not imply uniqueness. It is possible to guarantee uniqueness as well by imposing conditions on the function c. Specifically, when c is such that it induces a contraction mapping, then we can prove uniqueness.

Theorem 4 *When firms are symmetric, i.e.,* $c_j = c$ *and* $\theta_j = \theta$*, then equilibrium of the launch decisions is determined by*

$$\bar{V}^{eq} = \underset{h, \beta \geq 0}{max} \left\{ h - c(h, \beta) + e^{eq} \frac{\left(\bar{V}^{eq}\right)^{1+\frac{\theta}{\beta}}}{h^{\frac{\theta}{\beta}}} \frac{\beta \theta^{\frac{\theta}{\beta}}}{(\theta + \beta)^{1+\frac{\theta}{\beta}}} \right\}, \tag{26}$$

$$(h^{eq}, \beta^{eq}) := \underset{h, \beta \geq 0}{argmax} \left\{ h - c(h, \beta) + e^{eq} \frac{\left(\bar{V}^{eq}\right)^{1+\frac{\theta}{\beta}}}{h^{\frac{\theta}{\beta}}} \frac{\beta \theta^{\frac{\theta}{\beta}}}{(\theta + \beta)^{1+\frac{\theta}{\beta}}} \right\}, \tag{27}$$

$$d^{eq} := \left(\frac{\theta \bar{V}^{eq}}{(\theta + \beta^{eq}) h^{eq}} \right)^{\frac{1}{\beta^{eq}}} \tag{28}$$

and

$$e^{eq} = (d^{eq})^{\theta(n-1)}. \tag{29}$$

The solution to this system of equations can be compared with the monopoly solution from Theorem (2) *as follows:* $h^{eq} \leq h^*, \beta^{eq} \geq \beta^*, \bar{V}^{eq} \leq \bar{V}, \Delta^{eq} \leq \Delta^*$.

Theorem 4 establishes that in a symmetric competitive market, where $e_j^{eq} < 1$, firms are going to launch products with more haste because they have an incentive to capture the market rapidly, and discount the negative impact on future revenues. Additionally, we know that $\bar{V}^{eq}$ accounts for the value that a firm is going to capture in the future, this parameter decreases with h^{eq}, and it increases with β^{eq} and d^{eq}. Confirming the notion that for a firm in a competitive market it makes economical sense to release products that are worse at a faster pace, because this type of new product has the potential of obtaining quick positive returns even though quality does not last as long.

This finding advances the discussion in the new product development literature about the interplay among quality, release times, and durability. Previous papers focused on the first two dimensions but failed to incorporate the role of the third. In particular, Plambeck and Wang (2009) consider a duopoly scenario and show that firms are going to release products faster. We generalize the competitive market to allow n firms to enter and we show that we witness an arms race where every firm decides to release products faster as n grows. Thus, we confirm the notion of Plambeck and Wang (2009) but provide further details about product outcomes: as more firms decide to enter the market, the quality-decay ratio goes down, and prices also go down because firms are in a hurry to be the one offering the latest item. Hence, competitive forces are making firms more impatient and they are being pushed to released worst products at a faster pace.

We can now compare monopoly vs. oligopoly strategies graphically, in Figures 1-3, to illustrate the findings of Theorem 4. In these illustrations, we vary the firms' discount factor θ and depict the equilibrium values of h, β, Δ, and p, using the cost function: $c(h, \beta) = c_h h^2 - c_\beta \beta$, and parameters $r = 0.05, c_h = 1$, and $c_\beta = 1$. Note that we plot the monopoly decision vs. the oligopoly equilibrium with $n = 2$, and $n = 3$. Recall that h represents the quality-decay ratio of the product, β shows how the item's utility is going to decay over time because of novelty and technological obsolescence. Δ is the amount of time that passes between the release of two consecutive products. And finally, $\alpha = h(r + \beta)$ represents the quality of the item. These four parameters are relevant because they are what firms decide when they are designing and releasing a new product.

If we take a look at Figure 1, the product's quality is higher in a monopoly situation, and hence the consumer receives higher utility from owning this product and is willing to pay a higher price. Whereas in Figure 2, we observe that β is smaller. Furthermore, firms under competition tend to launch products more often, because they take turns at selling their products, see Figure 3. This is consistent with a faster deterioration of the product. In Figure 4, we see that there is a negative connection between quality and release dates. This suggests that items that have more quality last longer on the market. However, there is a positive relationship between quality and decay, which is driven by increases in technology. This is the principal trade-off that we are analyzing in this paper. Quality is driven by exogenous forces, and the decision to update product lines to stay on top makes firms release more new items (Lobel et al. 2015).

Furthermore, the figures show how firm impatience, through the discount factor θ, affects outcomes. We see that as a firm becomes more impatient, i.e., larger θ, it will release more products, with better quality (higher α) initially, but lasting less (higher decay β): this maximizes the immediate profits generated by a given product launch.

Overall, our results imply that competition generates worse products that deteriorate faster,

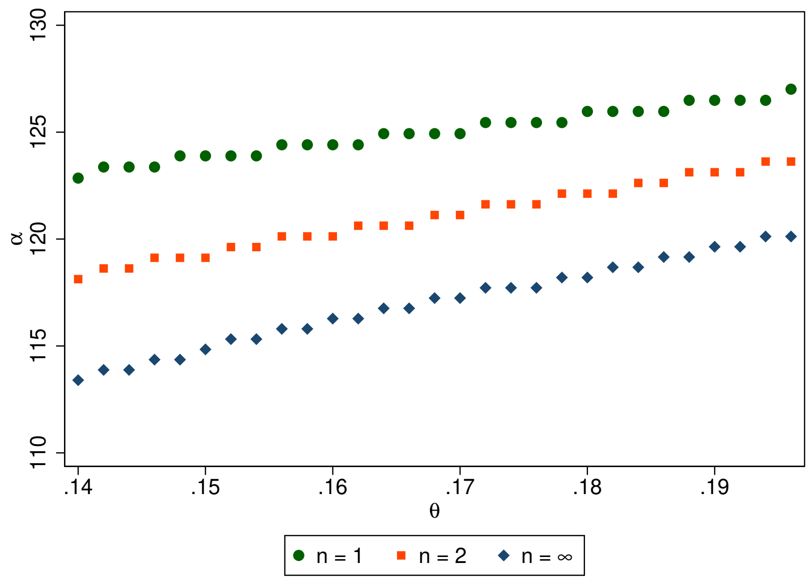


Figure 1: Comparison between the product's quality in monopoly and in competition, as a function of θ, the firms' common discount factor.

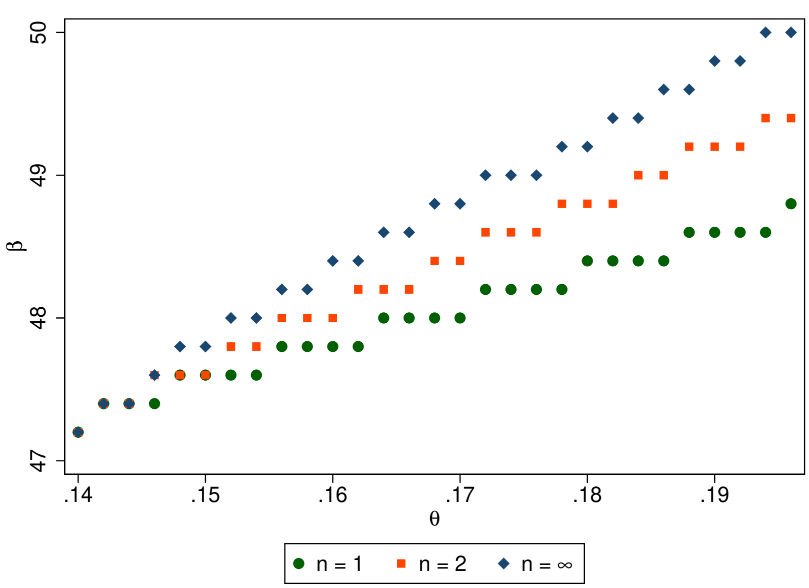


Figure 2: Comparison between the product's decay in monopoly and in competition, as a function of θ, the firms' common discount factor.

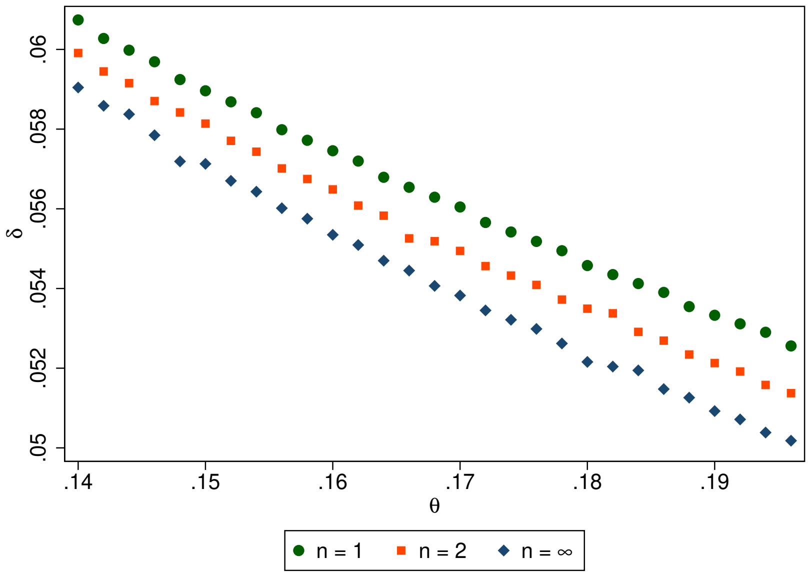
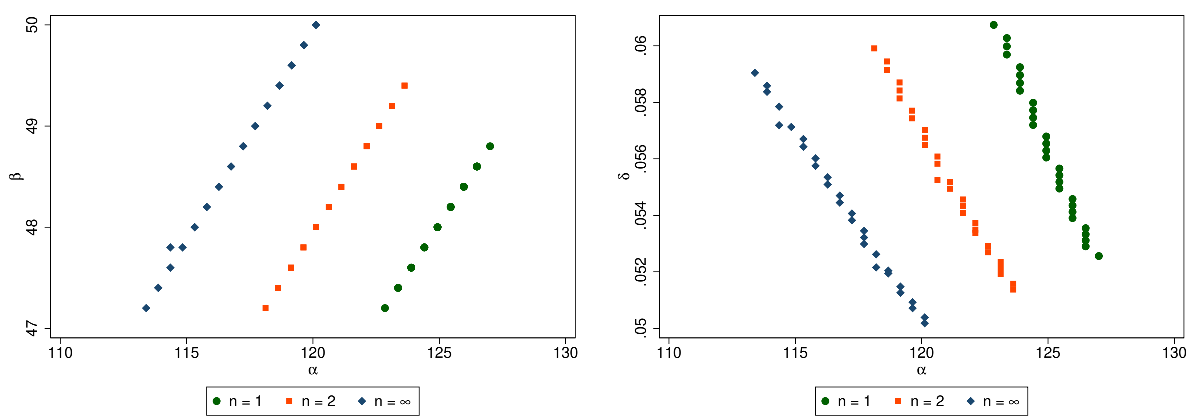


Figure 3: Comparison between the product's release dates in monopoly and in competition, as a function of θ, the firms' common discount factor.



(a) Quality and Decay

(b) Quality and Release Date

Figure 4: Comparison between the product's quality, its decay and its release dates in monopoly and in competition.

because this approach maximizes short-term profits at the expense of generating more aggressive competition that reduces profits for the entire industry. Because both in monopoly and oligopoly consumer surplus is fully appropriated by the firms, this also means that social welfare is lower under competition. In a way, competitive pressures in our model lead firms to an arms race that leaves them worse off, compared to a monopoly that internalizes the effects of a new product (which generates future profits) on past products (an investment that has to be recouped).

Additionally, our findings contribute to the literature in new product development. In particular, we extend the findings of Plambeck and Wang (2009) by incorporating the decay parameter β. It is important to consider the effect of obsolescence because it is one of the main drivers of product renewal. In other words, a consumer decides to change a product that she owns after it has become obsolete. If this decay is larger, consumers will choose to rotate their items at a faster pace. This behavior allows firms to increase their revenues from new product releases. Moreover, by analyzing the firm's problem in a competitive setting we are contributing to the findings of Fishman and Rob (2000). In their paper, obsolescence is implicit because a monopolist designs a product to last until a new one is launched, their assumption is that the technological growth between consecutive releases will create a large incentive for consumers to make a purchase. In our case, we are modeling obsolescence as a decay factor, which gives the firm more flexibility in order to design its products, and we see that the quality-decay ratio is an important characteristic when we analyze consumer welfare and the firm's profit. Moreover, we contribute to the discussion of the interplay between quality, decay, and releases in a competitive setting. In this market, launches are independent and competitors are not paying attention to what the other players do. They just plan for their turn and they release the product which will bring them the largest possible utility.

5 Heterogeneous Consumers

So far, our analysis has rested on the assumption of having a representative consumer. In this section, we extend the model to the case of heterogeneous consumers. We thus consider multiple types of consumers, denoted by $j = 1, \ldots, J$, that are present in the market and evaluate the utility that a product i will give over its lifespan, considering again Equation (1). Since every consumer type is different, we now need to calculate the value of the constant K in every state using Equation (5), which may not be zero anymore. To do this, we assume that K in the last period is equal to zero, and we compute the rest of the values for K recursively. As discussed in Section 3, the consumer's choice is driven by the value of K. If it is negative, the consumer will decide not to purchase and she will continue to use the previous item that she had. Since we obtain the value of K for every period, we can also calculate the proportion of products that the consumer purchases, which we define as s_j. For example an $s_j = 1$ would mean that the consumer purchased every generation of the product, whereas an $s_j = 0.2$ means that the consumer bought one out of five products released.

We can now explore the decision that different consumer types make assuming that there is a stationary policy and a total of 100 products launched, for the monopoly scenario. We know that the firm needs to carefully select the time between releases (Δ) and the price (p) as well as the product characteristics (α and β) to maximize its profits. The firm now faces a trade-off between volume and profitability. It can choose to release items at a faster pace with low prices to capture consumers that are impatient (Liang et al. 2018). On the other hand, it can take more time to launch products and set a higher price to capture consumers that are more patient. We numerically optimize the decision for different levels of heterogeneity on the consumers' discount factor, which indicates the level of patience for each consumer, while keeping the average discount factor unchanged. Specifically, a consumer type is defined by the discount factor r (higher r is related to more impatient consumers). The results are shown in Table 1.

Table 1: Optimal Price Selection Under Different Consumer Types.

Consumer Types (r)	$\bar{r}$	σ_r	Firm's Profit	s	p	Δ	p_l	p_h	Δ_l	Δ_h
(0.5)	0.5	0	102.716	1.000	2.283	0.0345				
(0.4,0.6)	0.5	0.1	101.312	0.895	2.280	0.0348	2.310	2.260	0.0354	0.0337
(0.3,0.7)	0.5	0.2	100.014	0.883	2.287	0.0351	2.344	2.239	0.0367	0.0331
(0.2,0.8)	0.5	0.3	96.813	0.852	2.304	0.0359	2.395	2.220	0.0389	0.0325
(0.1,0.9)	0.5	0.4	86.076	0.750	2.359	0.0391	2.518	2.201	0.0460	0.0319

Notes: $\bar{r}$ represents the average consumer's discount factor. σ_r is the standard deviation for the consumer's discount factor. s represents the proportion of items released by the firm that consumers acquire. p is the optimal price set by the firm. Δ is the optimal time between product releases set by the firm. p_l is the optimal price if there were only consumers with the low r in the market. p_h is the optimal price if there were only consumers with the high r in the market. Δ_l is the time between releases if there were only consumers with the low r in the market. Δ_h is the time between releases if there were only consumers with the high r in the market.

The starting point is to examine the market with homogeneous consumers ($\sigma_r = 0$). In this case, in concordance with our results –Lemma 1 specifically–, all the items are bought and the firm optimizes the launch cycle and the price for that specific consumer group. When we allow two consumer types to be present in the market, we see that the firm adjusts its strategy to maximize its profits accepting that there is a portion of the market that will not make a purchase. As the standard deviation grows, the firm reduces its profit because the proportion of items that are acquired goes down. This reduction is led by the patient consumer type, who might decide to skip a generation if the expected future utility from the new product does not create a positive value (see Lobel et al. 2015 for a similar insight). We know that in these scenarios the firm faces the dilemma of setting lower prices with faster releases, which captures the impatient consumer type

(Liang et al. 2018), or setting a larger price with longer renewal times to create an incentive for patient consumers to purchase.

The first strategy is beneficial for impatient consumers because they look forward to the jump in utility that the acquisition of a new product brings. This is driven by the rapid decay of the used item on the consumer's hand. Hence, when they analyze the optimal choice condition given in Equation (2), they are most likely to make the switch to the new item. However, the firm might want to slow down the pace of product introduction when there are patient consumers on the market. For these consumers, decay is slower and they will take more time to replace the product that they own for a new one.

Following Lemma 2, we can calculate the price and the time between releases that the firm would set if only one consumer type was present in the market. In Table 1, we present the results using the subscript l for the low discount factor, and the subscript h for the high discount factor. This allows us to compare the strategic change that the firm has to make when two consumer types are present in the market. First of all, we observe that there is a negative relationship between the discount factor and the price, and between the discount factor and the time between releases. Therefore, the firm would set lower prices and it would speed up its introduction times if only impatient consumers were present. But in this context, patient consumers coexist with their impatient counterparts, and the firm must make a single decision to accommodate both: we find that the sensitivity with respect to the discount factor is higher when r is low. As a result, the firm's optimal decision is more heavily influenced by these patient consumers. Hence, the firm prefers to reduce its renewal speed to be more attractive to these patient consumers.

6 Conclusions

This paper presents a model where consumers decide the optimal time to renew a durable product, taking into consideration future product launches (in particular updates between generations) and obsolescence. We analyze the firm's problem, where it has to optimize product features, pricing, and introduction times. We derive closed-form expressions for optimal policies, which can provide testable predictions. In a monopoly, optimal launch decisions balance short-term profits generated by the current product with the negative price pressures that great present products put on future launches. In other words, the firm internalizes inter-product cannibalization because it optimally chooses the release schedule for the entire portfolio. In contrast, under competition, firms discount future revenues more, and hence in equilibrium tend to launch worse products that decay faster, more often. Moreover, our results corroborate previous findings in the new product development literature (Fishman and Rob 2000, Plambeck and Wang 2009, Lobel et al. 2015), and we contribute by introducing endogenous obsolescence, which is a key factor for durable goods. Hence, we see our model as a first attempt to understand the relationship between product launch economics (ultimately driven by consumer decisions) and broad societal problems related to consumerism. We hope that our research can be used as a framework that sparks future work on this topic.

Our results open a number of questions for future research. First and foremost, given that competitive outcomes are inefficient, one obvious question is how to remedy the situation. Our model suggests that, to improve outcomes, firms should internalize the negative externalities of current product decisions on future launches, and in particular avoid "pulling the trigger" too early, which reduces the profits made by previous products in the market. One way to mitigate this behavior is to slow down the launch cycle. This can be accomplished by adding taxes to the costs of new product launches, in the form of homologation or disposal processes. For example, following Plambeck and Wang (2009) there could be a fee upon the sale of a product that takes into consideration its future disposal. Another approach would be to change the revenue model from a per-purchase fee into a per-use charge, i.e., through leasing or renting. By doing this, the immediate income generated by a new product launch will be lower and hence firms will be forced to increase durability, as a way to increase their revenues. Studying this alternative revenue models complicates the analysis of the dynamic game and is a promising yet challenging effort.

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Appendix

Optimal control function for the consumer

We can write the utility-to-go function as the following: $U_i(t) = u_i(t)dt + e^{-rdt}U_i(t+dt)$. If we develop this equation in order to find the control function of this case that comes from the Hamilton-Jacobi-Bellman (HJB) equation (Bertsekas 1995), we obtain: $\frac{dU}{dt}(j,t) = -u_i(t) + rU(j,t)$

In order to continue analyzing the problem, we must define the accrued utility as $U(j,t) = e^{r(t-\tau_i)}V_i(t)$. We also know that $u_i(t) = \alpha_i e^{-\beta_i(t-\tau_i)}$. If we input these functions into the HJB equation, we will obtain: $V_i(t) = \frac{u_i(t)e^{-r(t-\tau_i)}}{r+\beta_i} + K_i$, where $K_i \geq 0$ is a constant that comes from the antiderivative solution. With the value of $V_i(t)$, we can input it into the accrued utility function in order to get: $U(j,t) = e^{r(t-\tau_i)}V_i(t) = e^{r(t-\tau_i)}(\frac{u_i(t)e^{-r(t-\tau_i)}}{r+\beta_i} + K)$.

Proof of Theorem 1

Consider a given solution where products $i_1, i_2, \ldots$ are purchased at times $t_1, t_2, \ldots$. $U(i_{n-1}, t_{n-1})$ is increasing in K_{n-1}, which is the only element that depends on t_n. We show that K_{n-1} is unimodal in t_n. Equation (5) yields

$$\begin{aligned} K_{n-1} = & \left(\frac{u_{i_n}(t_n)}{r+\beta_{i_n}} + K_n e^{-r(t_n-\tau_{i_n})} - p_{i_n} - \frac{u_{i_{n-1}}(t_n)}{r+\beta_{i_{n-1}}}\right) e^{-r(t_n-\tau_{i_{n-1}})} \\ = & \; K_n e^{-r(\tau_{i_n}-\tau_{i_{n-1}})} + \left(\frac{u_{i_n}(t_n)}{r+\beta_{i_n}} - p_{i_n} - \frac{u_{i_{n-1}}(t_n)}{r+\beta_{i_{n-1}}}\right) e^{-r(t_n-\tau_{i_{n-1}})}. \end{aligned}$$

Since this is an optimal solution, then t_n must be a maximizer of this expression in $[t_{n-1}, t_{n+1}]$, every other parameter being fixed (particularly K_n which comes from optimal future choices). Because this is a differentiable function, we must have that

$$\frac{dK_{n-1}}{dt_n} = e^{-r(t_n-\tau_{i_{n-1}})}\left(\alpha_{i_{n-1}} e^{-\beta_{i_{n-1}}(t_n-\tau_{i_{n-1}})} - \alpha_{i_n} e^{-\beta_{i_n}(t_n-\tau_{i_n})} + rp_{i_n}\right) = 0$$

Since the function $\alpha_{i_{n-1}} e^{-\beta_{i_{n-1}}(t_n-\tau_{i_{n-1}})} - \alpha_{i_n} e^{-\beta_{i_n}(t_n-\tau_{i_n})}$ is unimodal in t_n, then there is at most one interior value that satisfies $\frac{dK_{n-1}}{dt_n} = 0$, which leads to the optimal purchase time in (b) - in case this is a maximum of K_{n-1}. Otherwise K_{n-1} is maximized at an extreme feasible value, i.e., $t_n^* = \tau_{i_n}$ – case (a): product i_n is purchased at the time of launch – or $t_n^* = t_{n+1}^*$ – case (c): product i_n is never purchased. ∎

Proof of Lemma 1

Consider that product i was introduced at the optimal moment of launch $t = \tau_i$. Assume that the product was consumed at $t_i = \tau_i + \Delta > \tau_i$. Consider an alternative launch strategy, with $\tilde{\tau}_i = t_i$, $\tilde{h}_i = h_i e^{-\Delta} < h_i$ and $\tilde{\beta}_i = \beta_i$. This product launch is less costly to the firm because the absolute cost is lower – $c(\tilde{h}_i, \tilde{\beta}_i) \leq c(h_i, \beta_i)$ – and the expense is incurred at a later time. Hence it generates strictly higher profits to the firm without changing the consumer's purchasing action. This contradicts optimality of the firm's behavior, hence $t_i = \tau_i$ must be optimal. ■

Proof of Lemma 2

Following Equation (5), we obtain:

$$\frac{\alpha_i}{r+\beta_i} + K_i e^{r(t_i-\tau_i)} - p_i = \frac{\alpha_{prev(i)} e^{-\beta_{prev(i)}(\tau_i - \tau_{prev(i)})}}{r+\beta_{prev(i)}} + K_{prev(i)} e^{r(t_n - \tau_{prev(i)})}.$$

We know that the firm needs to set the value of K_i and $K_{prev(i)}$ equal to zero in order to charge the maximum price, and we know that $h = \frac{\alpha}{r+\beta}$. Therefore, we must have $p_i = h_i - h_{prev(i)} e^{-\beta_{prev(i)}(\tau_i - \tau_{prev(i)})}$. ■

Proof of Theorem 2

Equations (10)-(14) provide the optimal values of $\bar{V}, h^*, \beta^*$ and d^*. Specifically, the optimal value of d^* is:

$$d^* = \left(\frac{\theta \bar{V}}{(\theta + \beta^*) h^*} \right)^{\frac{1}{\beta^*}}.$$

Now, we can change the variable $d^* = e^{-\Delta^*}$ in order to obtain the time between releases $\Delta^* = -\frac{1}{\beta^*} \left[\log(\theta) + \log(\bar{V}) - \log(\theta + \beta^*) - \log(h^*) \right]$. ■

Proof of Theorem 3

Equilibrium (if it exists) is made of the $4n$-dimensional vector $(h_j^{eq}, \beta_j^{eq}, d_j^{eq}, \bar{V}_j^{eq})_{j=1,\ldots,n}$, which belongs to a compact set, because the only unbounded variable is h, which, due to the convexity of c, cannot take values higher than the solution to $h - c(h, \beta) = 0$. The best-response mapping from Equations (22)-(25) are continuous. Hence from Kakutani's fixed point theorem, there exists an equilibrium to the game, which is a fixed point of the best-response mapping (Kakutani et al. 1941).

■

Proof of Theorem 4

Equations (26)-(29) provide the equilibrium conditions for $\bar{V}^{eq}, h^{eq}, \beta^{eq}, d^{eq}$ and e^{eq} in the case of symmetric firms. Following Equation (28), we have that:

$$d^{eq} = \left(\frac{\theta \bar{V}^{eq}}{h^{eq}(\beta^{eq} + \theta)} \right)^{1/\beta^{eq}}$$

If we reconvert the variable $d = e^{-\Delta}$ to obtain the equilibrium time between releases, we obtain: $\Delta^{eq} = -\frac{1}{\beta^{eq}} \left[\log(\theta) + \log(\bar{V}^{eq}) - \log(\theta + \beta^{eq}) - \log(h^{eq}) \right]$. ■