No holdup in dynamic markets

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Abstract

In many markets, heterogenous agents make non-contractible investments before bargaining over both who matches with whom and the terms of trade. In static markets, the holdup problem—that is, inefficient investments caused by agents receiving only a fraction of the returns from their investments—is ubiquitous. However, markets are often dynamic, with agents entering over time. Taking a general non-cooperative investment and bargaining approach, we show that the holdup problem vanishes in markets with dynamic entry as agents become patient.

1 Introduction

In many markets, crucial investments are sunk by the time agents bargain over prices and allocations. For example, workers and employers invest in human and physical capital well before bargaining over who will match with whom and for what wages. This can lead to holdup problems—that is, agents underinvesting because they do not expect to fully appropriate the returns from their investments (e.g., Williamson 1975; Grout 1984; Grossman and Hart 1986; Tirole 1986; Hart and Moore 1990) and severely limit the efficiency of these markets (e.g., Hosios 1990; Acemoglu 1996, 1997; Cole, Mailath, and Postlewaite 2001a; de Meza and Lockwood 2010; Elliott 2015; Felli and Roberts 2016).
The objective of this paper is to investigate the extent to which holdup is a problem in matching markets featuring dynamic entry. While the holdup problem in static markets has been extensively studied, relatively little attention has been given to markets featuring dynamic entry—particularly in the case of thin markets, where the holdup problem is often most problematic.

The extent to which dynamic entry generates sufficient competition to ameliorate the holdup problem in matching markets is an important open question. On the one hand, it is well-known that, in certain settings, the shadow of the future can create substantial competition. For example, in the Coase conjecture, a durable-good monopolist competes against her future selves (Coase 1972). On the other hand, it is also well-known that dynamic markets are not necessarily more competitive than their static equivalents. For example, in the chain store paradox (Diamond 1971), the number of stores that a customer can sequentially visit does not affect the price that she is quoted at each store.

This paper shows that the holdup problem can be generally recast as a problem of impatience: When agents are sufficiently patient or, equivalently, when markets are sufficiently fluid, the holdup problem disappears. This has important practical implications. For example, Davis and Haltiwanger (2014) document how US labor market fluidity—as measured by flows of jobs and workers across employers—has fallen over the last few decades, and they argue that this has significantly reduced productivity. Our findings suggest an investment channel by which lower market fluidity exacerbates holdup problems, which can lead to less investment and hence lower productivity.

Our framework is a general non-cooperative investment and bargaining game with dynamic entry. There are finitely many types of agents. In order to focus on markets that appear thin at every point in time, we impose an (arbitrarily small) upper bound on the number of agents of each type that can be in the market at any point in time. Agents enter stochastically over time as a function of the market conditions, and leave upon matching. Agents invest before entering the market, and these investments shape their matching surpluses.

Once in the market, agents bargain non-cooperatively over whom to match with and their

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1The Coase conjecture can fail for a number of reasons; see Gul, Sonnenschein, and Wilson 1986, Gul (1987), Ausubel and Deneckere (1989), Board and Pycia (2014), Strulovici (2017) and Nava and Schiraldi (2019).

2See also Molloy et al. (2016) and Decker et al. (2018), for example.

3This is in contrast to the standard approach in the literature that studies non-cooperative bargaining in non-stationary markets with dynamic entry. Indeed, this literature typically considers economies with a continuum of agents of each type; see for example Binmore and Herrero (1988a, 1988b) and Manea (2017).
terms trade. Our bargaining protocol encompasses many standard protocols from the existing literature. Consistent with this literature, the equilibrium payoffs are sensitive to the details of the protocol. Nevertheless, we show that—in the limit as agents become patient—the continuation value of each agent’s potential trade partners are unaffected by her investment choice. In other words, agents are price takers, and so everyone receives the full change in value in her realized match surplus generated by her investment. Hence, there is no holdup problem.

A natural benchmark for our work is the literature on the assignment game with ex ante non-contractible investments. There is a large literature—dating back at least to Becker (1975)—that investigates the holdup problem in this setting. An important message from this literature is that only under special circumstances can everyone fully appropriate the social value of their investments. To illustrate this point, assume that, once investments are sunk, a Walrasian outcome of the resulting matching market is selected. In such markets, there are typically many Walrasian equilibria, and these support a continuum of possible payoffs for each agent. Leonard (1983) (see also Demange 1982) shows that an agent fully appropriates the social value of her investments if and only if she receives her highest possible payoff among all the Walrasian equilibria. Hence, a natural way to obtain investment efficiency is to give each agent her highest possible Walrasian payoff. Unfortunately, however, this is usually impossible. Outcomes that are better for workers are generally worse for firms, and giving all workers and firms their best possible outcome simultaneously is only possible when there is a unique Walrasian equilibrium that pins down all prices—a situation that, as shown by Gretsky, Ostroy, and Zame (1999), is not generic in finite markets.

In the case of unidimensional attributes and complementarities in these attributes, Cole, Mailath, and Postlewaite (2001a) provide a condition, called “doubly overlapping attributes”, that guarantees that there is an essentially unique stable outcome, and that the associated prices continue to clear the market after any unilateral investment deviation. Under these conditions, agents are price takers—in the sense that no unilateral change in attributes affects the market prices—and, as a result, efficient non-contractible investments can be supported in equilibrium. We take a dynamic approach to address similar questions, and we find that

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4The assignment game is a static two-sided one-to-one matching market with transferable utility. See for example Shapley and Shubik (1972).

5It is possible to simultaneously give everyone on one side of the market her maximum possible Walrasian equilibrium payoff, but this requires also giving everyone on the other side of the market her minimum possible Walrasian equilibrium payoff (Shapley and Shubik 1972). Nevertheless, if only one side of the market has investment opportunities, this selection can support efficient investments (e.g., Kranton and Minehart 2001; Hatfield, Kojima, and Kominers 2014; Felli and Roberts 2016).
essentially no restrictions on the nature of the investments and resulting matching surpluses are required to preclude holdup problems when agents are sufficiently patient.

A literature in search also investigates investment incentives in dynamic economies. This literature focuses on thick markets, however, with a continuum of agents on both sides of the market and random meetings determined by a matching function that is homogeneous of degree 1. For example, Acemoglu and Shimer (1999) show that holdup is not a problem in directed search environments where firms form submarkets by committing to a posted wage. Relatedly, Bester (2013) uses Nash bargaining to model wage negotiations, and shows that there is no holdup in the frictionless limit of his economy. In contrast, in this paper we show that holdup need not be a problem in markets that appear arbitrarily thin at every point in time.

The literature investigating the efficiency of investments under competitive matching (e.g., Cole, Mailath, and Postlewaite 2001b; Peters and Siow 2002; Mailath, Postlewaite, and Samuelson 2013 and 2017; Nöldeke and Samuelson 2015; Chiappori, Salanié, and Weiss 2017; Chiappori, Dias, and Meghir 2018; Dizdar 2018) focuses on markets featuring a continuum of price-taking agents on each side to turn off the holdup problem and investigate other sources of investment inefficiencies—like coordination failures, participation constraints, and imperfect information. Our results provide non-cooperative foundations in finite markets for this widely used price-taking assumption, and suggest that dynamic entry can be an important force behind price-taking behavior.

Makowski and Ostroy (1995) generalize the First Theorem of Welfare Economics by relaxing the price-taking and market-making assumptions: They consider a finite population model in which individuals choose occupations, and those occupations determine the goods that can be consumed. They show that a version of the First Theorem holds in their environment under two conditions. The first condition requires that everyone fully appropriates the social value of her actions, and the second condition precludes coordination problems. In this paper, we show that—when investment opportunities do not depend on the state of the economy—the full appropriability condition is endogenously satisfied in our dynamic non-cooperative bargaining game in the limit as agents become arbitrarily patient: In every Markov perfect equilibrium, everyone bears the full consequences of her investment deviations.

This paper is also related to the literature investigating the extent to which dynamic matching and bargaining games become Walrasian in the limit as frictions vanish (e.g., Rubinstein and Wolinsky 1985 and 1990, Gale 1987, de Fraja and Sákovics 2001, Lauermann
2013). This literature typically compares the equilibrium outcome of dynamic games to the Walrasian outcome of an associated static economy. However, in non-stationary settings like the one considered in this paper, it is not always clear which static economy is the appropriate one for identifying the relevant Walrasian outcome. To the extent that Walrasian equilibria are of interest because of their efficiency properties, in this paper we sidestep this issue: We show that the full appropriability condition of Makowski and Ostroy (1995) holds in our economy under relatively mild conditions in the limit as the frictions vanish, which, by the general insights of their approach (e.g., Makowski and Ostroy 2001), is a key ingredient for establishing an analog of the First Theorem of Welfare Economics in this setting.

The rest of this paper is organized as follows. We begin in section 2 with an example that illustrates our result in the simplest possible setting. In section 3, we describe the general model, in section 4 we state and prove our main result, and we conclude in section 5. For brevity, we relegate a (fairly standard) equilibrium existence proof to Appendix A.

2 Example

There are two types of agents, buyers and sellers, and a sequence of agents of each type. Bargaining occurs in discrete periods \( t = 1, 2, \ldots \). There is a common discount factor \( 0 \leq \delta < 1 \). At each point in time, there is one buyer and one seller active. Any time an agent leaves the market, the agent behind her in the sequence replaces her, and becomes active. Hence, there is always exactly one buyer and one seller active, and each agent enters the market at most once. Buyers can invest upon entering. Investing costs \( 1/2 < c < 1 \). Investments are not contractible. The matching surplus \( x \) of an active buyer with an active seller is 2 if the buyer has invested, and 1 otherwise. Hence, as \( \delta \) goes to 1, the social surplus of each investment is \( 1 - c > 0 \), so efficiency requires that all buyers invest.

The sharing rule in each match is determined by a standard generalization of the Nash bargaining solution, as follows: Let \( V_b(x) \) and \( V_s(x) \) denote the expected payoffs of buyers and sellers at the start of each period in which their matching surplus is \( x \). Each period, the two agents meet with probability \( p < 1 \), in which case they match and exit the market with shares \( \delta V_b(x) + \frac{1}{2} (x - \delta V_b(x) - \delta V_s(x)) \) and \( \delta V_s(x) + \frac{1}{2} (x - \delta V_b(x) - \delta V_s(x)) \), respectively.
Otherwise, they wait in the market for the next period. Hence, these payoffs satisfy

\[ V_b(x) = p \left( \delta V_b(x) + \frac{x - \delta V_b(x) - \delta V_s(x)}{2} \right) + (1 - p) \delta V_b(x), \]

\[ V_s(x) = p \left( \delta V_s(x) + \frac{x - \delta V_b(x) - \delta V_s(x)}{2} \right) + (1 - p) \delta V_s(x). \]

Solving these equations yields \( V_b(x) = V_s(x) = \frac{px}{2(1 - \delta(1 - p))} \), which converges to \( \frac{x}{2} \) as \( \delta \) goes to 1. Hence, the classical holdup problem prevents efficient investments: Buyers have to pay the full costs of their investments, but they recoup only half of the resulting gains. As a result, buyers do not have incentives to invest.

While this shows that holdup can be a problem in markets with dynamic entry and patient agents, we argue that this is a knife-edge case. For this, we slightly modify the example to capture the fact that agents sometimes receive outside offers that cause them to exit the market (e.g., Fuchs and Skrzypacz 2010). Suppose that, at the beginning of each period, there is a probability \( q > 0 \) that the buyer receives an outside offer that gives her exactly her continuation value, which she accepts. If this occurs, the seller moves to the next period, where she faces the next buyer in sequence. Otherwise, the buyer and seller match with probability \( p/(1 - q) \) (so the unconditional probability of a buyer-seller match in any given period is \( p \), as before), or they move to the next period with the remaining probability.

Focusing on symmetric situations in which either all buyers invest or no buyer invests, it is easy to see that this modification does not change agents’ payoffs. Indeed, the expected payoffs at the start of each period are now

\[ V_b(x) = \begin{cases} q \delta V_b(x) + p \left( \delta V_b(x) + \frac{2 - \delta V_b(x) - \delta V_s(x)}{2} \right) + (1 - p - q) \delta V_b(x), \\ \text{outside offer} \\ \text{match with } s \\ \text{no trade} \end{cases} \]

\[ V_s(x) = \begin{cases} q \delta V_s(x) + p \left( \delta V_s(x) + \frac{2 - \delta V_b(x) - \delta V_s(x)}{2} \right) + (1 - p - q) \delta V_s(x), \\ \text{outside offer} \\ \text{match with } b \\ \text{no trade} \end{cases} \]

Note that these are the same value functions as above. Hence, the solution is also the same. But considering such outside offers dramatically affects the investment incentives: While we have seen above that there is a severe holdup problem without them (so buyers do not have incentives to invest), we now show that this problem vanishes as \( \delta \) goes to 1 when outside offers are forthcoming (so buyers have incentives to invest).

To see this, consider the case in which buyers invest. We show that, if buyer \( b \) unilaterally deviates by not investing, she obtains a limit payoff of 0. Indeed, the continuation values of
the deviating buyer $b$ and the seller $s$ that she faces satisfy

\[
\hat{V}_b = q \delta \hat{V}_b + p \left( \delta \hat{V}_b + \frac{1 - \delta \hat{V}_b - \delta \hat{V}_s}{2} \right) + (1 - p - q) \delta \hat{V}_b ,
\]

\[
\hat{V}_s = q \delta \hat{V}_s(2) + p \left( \delta \hat{V}_s + \frac{1 - \delta \hat{V}_b - \delta \hat{V}_s}{2} \right) + (1 - p - q) \delta \hat{V}_s ,
\]

where we have used the fact that, when the buyer receives an outside offer, the seller faces a new buyer that has invested, and so receives her on-path continuation value $V_s(2)$. Solving the above system of equations, $\hat{V}_s$ converges to 1 as $\delta$ goes to 1. Hence, each buyer faces a limit matching price of 1 regardless of whether she invests or not. As a result, when $\delta$ is close enough to 1, no buyer has an incentive to deviate to not investing. Indeed, the limit net payoff from investing is $1 - c > 0$, which is bigger than the limit net payoff from not investing, which is 0.

This example is extremely stylized in various dimensions. First, we have assumed an exogenous split of the matching surpluses; a general non-cooperative bargaining approach might be more compelling. Second, we have assumed that there are only two types of agents (buyers and sellers), and one active agent of each type; it would be worthwhile to allow arbitrarily many different types, and to let there be stochastic entry of different types of agents over time, possibly as a function of the market conditions. Third, we have assumed that only buyers can invest upon entering; in practice, everyone is likely to have some investment opportunities, and the investment opportunities are likely to be considerably richer than the binary alternatives considered in this example. Finally, the matching opportunities of different pairs of agents might also be a function of the market conditions. We now turn to our general model featuring all of these generalizations, and we show that the holdup problem still vanishes as agents become patient.

3 Model

Bargaining occurs in discrete periods $t = 1, 2, \ldots$. There is a common discount factor $0 \leq \delta < 1$, complete information, and common knowledge of the game. There is a finite set $I$ of types of agents, and a sequence of agents of each type. For each type $i$, there are $n_i \geq 1$ bargaining slots. In any given period, each slot of a given type can be occupied by one agent of that type, or be empty. We let $S$ denote the set of all slots, and we refer to the agents
occupying the slots in any given period as the active agents in that period.

**Remark 3.1.** Since the number of slots of each type can be arbitrarily small, this framework encompasses markets that are always extremely thin (on and off the equilibrium path).

There is a finite set $K_i$ of possible investments associated with each type $i$. Without loss of generality, we assume that the sets $\{K_i\}_{i \in I}$ do not overlap. Two agents with investments $k_i$ and $k_j$ produce $y(k_i, k_j) > 0$ units of surplus when they match. The cost of investment $k$ is denoted by $c(k)$. We slightly abuse terminology by referring to the set of all possible investments $K_i$ associated with a slot $s$ of type $i$ by $K_s$. Also, we let $K$ denote the set $\bigcup_{i \in I} K_i$ of all possible investments.

**Remark 3.2.** Given that the function $y$ determines the surplus of each match only as a function of the investment profiles of its members, this formulation encodes all the heterogeneities among types via their investment opportunities. Because the sets $\{K_i\}_{i \in I}$ do not overlap, this can capture arbitrary heterogeneity among different types of agents. For example, suppose that there are two seller types, $i'$ and $i''$, and two buyer types, $j'$ and $j''$, and further that $i'$ is a much better fit for type $j'$ than $j''$ is, while $i''$ is a much better fit for type $j''$ than $j'$ is. To capture this situation, we can simply take the surplus $y(k_i, k_j)$ associated with any investment profile $(k_i, k_j) \in (K_{i'} \times K_{j'}) \cup (K_{i''} \times K_{j''})$ to be high relative to the associated investment costs, and the surplus $y(k_i, k_j)$ associated with any investment profile $(k_i, k_j) \in (K_{i''} \times K_{j'}) \cup (K_{i'} \times K_{j''})$ to be low relative to the associated investment costs.

A (Markov) state $\theta$ specifies the investment of the agent occupying each slot. Since each agent can choose among a finite set of investments, the set $\Theta$ of states is finite. We let $\Theta_s \subset \Theta$ denote the set of states in which the slot $s$ is filled, and $\Theta_{ks} \subset \Theta_s$ denote the set of states in which the slot $s$ is filled with an agent with investment $k$.

Each period consists of three stages. The first stage—the entry stage—captures the idea that agents enter the market over time, possibly as a function of the state of the economy, with an investment that can also depend on the state of the economy. In this stage, at most one slot $s$ is selected at random, together with a nonempty investment set $Q \subseteq K_s$; in state $\theta$, for each slot $s$ and each $Q \subseteq K_s$ the pair $(s, Q)$ is selected with probability $\alpha^{\theta, s, Q} > 0$. If the selected slot is empty, it is filled with the first agent (of the corresponding type) in sequence that is yet to enter the market. When an agent is selected to enter into slot $s$ with investment set $Q$, she chooses any one of the investments $k$ in $Q$ and pays the associated cost $c(k)$.

**Remark 3.3.** The assumption that $\alpha^{\theta, s, Q}$ is strictly positive implies that agents enter with positive probability even in periods in which the entry conditions (investment opportunities and/or the state
of the market) are not the most favorable. These probabilities can be arbitrarily small, however, and can be interpreted as capturing the possibility that some agents have incentives to enter the market that are not captured by the model, or that some agents have an arbitrarily small probability of making entry or investment mistakes.

The second stage—the outside offer stage—captures the idea that agents sometimes receive outside offers: In this stage, at most one slot is selected at random; slot $s$ is selected with probability $\gamma_{\theta,s} > 0$. When slot $s$ is selected, a short-run trader associated with it enters the market, and makes the agent occupying the associated slot (if any) a take-it-or-leave-it offer specifying how to share the maximum possible surplus $Y := \max_{(k_i,k_j) \in K \times K} y(k_i,k_j)$. The receiver of this offer can either accept it (in which case she matches with the short-run trader, and both leave with their agreed upon payoffs) or reject it (in which case she waits in the market for the next period, while the short-run trader leaves with a payoff of 0).

**Remark 3.4.** The assumption that all short-run traders generate $Y$ units of surplus is for simplicity only. Our results hold in the more general case in which short run traders draw their matching surpluses from a distribution whose support includes values equal or higher than $Y$.

Finally, the third stage—the bargaining stage—specifies how the agents currently in the market bargain over which matches to form and how to share the resulting surplus. In this stage, at most one slot $s$ is selected at random to be the proposer, together with an offer set $S \subset S - \{s\}$ of slots that she can make offers to. We denote the probability that this happens when the state is $\theta$ by $p^{s,S,\theta}$, and we assume that $\sum_{s,S} p^{s,S,\theta} < 1$. If the slot $s$ is empty, or no slot is selected to be the proposer, we move directly to the next stage. Otherwise, its occupant chooses a slot $s'$ in the offer set $S$ and makes the agent occupying it (if any) a take-it-or-leave-it offer specifying a split of their surplus. The receiver of this offer can then accept or reject. If she accepts, the proposer and receiver exit the market with the agreed shares, vacating their respective bargaining slots. Otherwise, no trade occurs in this stage.

**Remark 3.5.** The assumption that at most one agent enters the market in the entry stage, and that at most one agent leaves the market in the outside offer stage is for simplicity only; our assumption that $\sum_{s,S} p^{s,S,\theta} < 1$ ensures that arbitrarily many agents can enter or exit in these stages before any bargaining occurs.

**Remark 3.6.** Two special cases of the protocol in our bargaining stage are standard in the literature on non-cooperative bargaining in matching markets. On the one hand, when the offer set $S$ is always the set of all slots, the proposer can always strategically choose whom to make offers to in an unrestricted fashion, as in Elliott and Nava (2019) and Talamàs (2019), for example. On the other
hand, when the offer set $S$ is always a singleton, the proposer cannot choose whom to make an offer to, as in Rubinstein and Wolinsky (1985), Manea (2011) and Nguyen (2015), for example. Further, the possible state dependence of the proposer probability distribution supports a rich set of alternative bargaining protocols.

A strategy $\sigma_i$ for an agent of type $i$ specifies the probability that she chooses each available investment when she is selected to enter in each slot with each investment set, the probability that she makes each possible offer for each offer set when she is selected to be the proposer, and the probability that she accepts an offer for each amount offered. A strategy $\beta_s$ for a short-run trader associated with slot $s$ specifies the probability that she makes each offer. A strategy profile $\{\sigma_i\}_{i \in I} \cup \{\beta_s\}_{s \in S}$ is a Markov perfect equilibrium if it conditions only on the Markov state and constitutes a Nash equilibrium of every bargaining subgame.

4 No holdup in equilibrium

After showing that a Markov-perfect equilibrium always exists (Proposition 4.1), we argue that holdup is not a problem in any such equilibrium when agents are sufficiently patient—in the sense that the matching prices that an agent faces are independent of her investment. This follows from Theorem 4.2, which states that, in the limit as $\delta$ goes to 1, the equilibrium payoffs are independent of the Markov state.

**Proposition 4.1.** There exists a Markov-perfect equilibrium.

**Proof.** Proposition 4.1 follows from a fairly standard fixed point argument that uses Kakutani’s fixed point theorem. See Appendix A.

For each state $\theta$ in $\Theta_{k_s}$, let $V_{k_s}(\theta, \delta)$ be the gross expected equilibrium payoff of an agent with investment $k$ occupying slot $s$ at the beginning of a bargaining stage that starts with state $\theta$ (without taking into account the investment costs).

**Theorem 4.2.** Consider a sequence $\mathcal{D}$ of discount factors converging to 1, together with an associated sequence of Markov-perfect equilibria whose payoffs $\{V_{k_s}(\theta, \delta)\}$ converge to $\{V^*_{k_s}(\theta)\}$ as $\delta$ goes to 1. For every investment $k$ and slot $s$, there exists $V^*_{k_s}$ such that $V^*_{k_s}(\theta) = V^*_{k_s}$ for every state $\theta \in \Theta_{k_s}$.

**Proof.** For any investment $k$, any slot $s$, and any two states $\theta$ and $\theta'$ in $\Theta_{k_s}$, an agent $a$ with investment $k$ occupying slot $s$ can, starting at state $\theta$, keep on delaying until the market moves to $\theta'$. It follows that $V_{k_s}(\theta, \delta) \geq \sum_{t \in \mathbb{N}} \omega_{\theta, \theta'}(\delta, t)\delta^t V_{k_s}(\theta', \delta)$, where $\omega_{\theta, \theta'}(\delta, t)$ denotes the
probability that it takes exactly \( t \) periods to reach (for the first time) state \( \theta' \) from state \( \theta \) when \( a \) follows this waiting strategy and everyone else follows her equilibrium strategy. Hence, it is enough to show that, for each \( \epsilon > 0 \), there exists \( T \) such that the probability that it takes at most \( T \) periods to reach \( \theta' \) from state \( \theta \) is bounded below by \( 1 - \epsilon \) for all \( \delta \in D \).

For this, in the next paragraph we argue that, for every state \( \theta'' \in \Theta_{k_s} \), there is a strictly positive number \( \ell_{\theta''} \) and a finite number \( t_{\theta''} \) such that, for all \( \delta \in D \), the probability \( \omega_{\theta'', \theta'}(t_{\theta''}, \delta) \) that it takes exactly \( t_{\theta''} \) periods to reach \( \theta' \) from \( \theta'' \) is bounded below by \( \ell_{\theta''} \). Hence, letting \( t' = \max_{\theta'' \in \Theta_{k_s}} t_{\theta''} \), and \( \ell = \min_{\theta'' \in \Theta_{k_s}} \ell_{\theta''} \), we have that, for all \( \delta \in D \), the probability that it takes at most \( t' \) periods to reach \( \theta' \) from any state \( \theta'' \) is bounded below by \( \ell \). We obtain that, for any \( M > 0 \) and for any \( \delta \in D \), \( \sum_{t \leq t' M} \omega_{\theta, \theta'}(\delta, t) \geq \sum_{m \in \{1, 2, \ldots, M\}} \ell (1 - \ell)^m \) which converges to 1 as \( M \) goes to infinity.

Naturally, the payoff of each active agent at the start of each period is bounded above by the maximum possible matching surplus \( Y \). Hence, short-run traders always have gains from trade, so they always make offers that are accepted with probability one in equilibrium. As a result, the outside offer stage ensures that, in each period, every active agent has a strictly positive probability of leaving. This, combined with the fact that in every state every empty slot has a positive probability of being filled with an agent with every possible investment, and that there is always a positive probability that the bargaining stage is mute (\( \sum_{s,S} D^{s,S,\theta} < 1 \)) implies that, for each state \( \theta'' \in \Theta_{k_s} \), we can find a finite number of periods \( t_{\theta''} \) and a strictly positive number \( \ell_{\theta''} \) such that \( \omega_{\theta'', \theta'}(t_{\theta''}, \delta) > \ell_{\theta''} \) irrespective of agents’ behavior, and hence irrespective of the discount factor \( \delta \).

\[ \text{Theorem 4.2 implies that, when } \delta \text{ is close enough to } 1, \text{ agents are essentially price takers, and a law of one price holds: All the agents of a given type that make the same investment and occupy the same bargaining slot obtain the same payoff.} \]

In particular, when agents choose their investments, they take as given the payoffs of the other agents they could match to. Corollary 4.3 formalizes how this translates into investment choices that maximize the joint surplus of the investor with her potential partners—in other words, investment choices that never exhibit holdup.

Let \( S_s \) denote the set of slots that a given slot \( s \) has a strictly positive probability of making an offer to in some state in \( \Theta_s \).

\[ \text{Moreover, it follows from the argument in the proof of Corollary 4.3 below that, if we further assume that all entering agents of the same type get the opportunity to make offers to the same other slots regardless of the slot they occupy (even if at possibly different rates), the following stronger law of one price holds: All the agents of a given type that make the same investment obtain the same payoff.} \]
Corollary 4.3. Consider a sequence $\mathcal{D}$ of discount factors converging to 1, together with an associated sequence of Markov-perfect equilibria whose payoffs $\{V_{k,s}(\theta, \delta)\}$ converge to $\{V^*_k\}$ as $\delta$ goes to 1. There exists $\bar{\delta} < 1$ such that, in all Markov perfect equilibria associated with $\delta > \bar{\delta}$, an agent that enters slot $s$ with investment set $Q \subseteq K_s$ chooses an investment $\ell \in Q$ that maximizes

$$\max_{s' \in S_s, k \in K_{s'}} \left[ y(\ell, k) - V_{k,s'}^* \right] - c(\ell).$$

(1)

Proof. The limit equilibrium payoff of an agent in slot $s$ after making investment $\ell$ is given by (1), because she can always wait to make an offer to an agent occupying any slot $s' \in S_s$ with any investment $k \in K_{s'}$. Thus, because the sets of slots and investments are finite, for all sufficiently large discount factors $\delta$, an agent that enters slot $s$ with investment opportunities $Q$ chooses an investment $\ell \in Q$ that maximizes (1).

Two final remarks are in order. Remark 4.4 clarifies the relationship between Theorem 4.2 and Corollary 4.3, and the full appropriability condition from Makowski and Ostroy (1995). Remark 4.5 discusses Pareto efficiency.

**Remark 4.4.** Theorem 4.2 shows that an agent’s investment decision does not affect the limit payoffs of others conditional on their investments. However, as investment opportunities are state dependent, one’s investment decision can affect the Markov state, and thus the investment opportunities of the agents that enter the market after her. As a result, in general, agents do not bear the full consequences of their investment decisions—that is, full appropriability does not hold in general. This is, however, the only impediment to full appropriability. If the realization of the investment sets do not depend on the state $\theta$—that is, for each slot $s$ and each investment set $Q \subseteq K_s$, the probability $\alpha^{\theta,s,Q}$ that slot $s$ draws the investment set $Q$ in the entry stage is independent of the state $\theta$—then full appropriability holds in the limit as $\delta$ goes to 1.

**Remark 4.5.** The fact that holdup is not a problem in any equilibrium does not mean that the equilibrium is necessarily Pareto efficient. For example, in general, we cannot rule out the possibility that agents (mis)coordinate on a Pareto-dominated equilibrium. There could also be participation constraints that prevent efficient investments. For instance, if we modify the example in section 2 by giving buyers a much lower bargaining power than sellers (instead of equal bargaining powers) then, were all the buyers to invest, they would all receive negative net payoffs (i.e., they would not recoup the costs of their investments); even though the surplus that a buyer must give to a seller to match does not change if she deviates unilaterally by not investing, there would be a profitable deviation to not invest and then not match with any seller.

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7In a related model without investment, Manea (2017) shows how multiple self-fulfilling expectations about the trajectory of the economy can generate multiple equilibria.
5 Conclusion

In this paper, we recast the holdup problem in matching markets as a problem of impatience: In the context of a general non-cooperative investment and bargaining game, we show that holdup is not a problem in markets featuring dynamic entry when agents are sufficiently patient. In particular, even markets that appear extremely thin at every point in time can be sufficiently competitive to sustain efficient investments. More generally, this paper shows that dynamic entry is a powerful source of competition when agents are patient or match sufficiently quickly. This provides non-cooperative foundations for the standard price taking assumption in matching markets.

Two features of our approach make the analysis particularly transparent. First, in each period, there is a positive but potentially small probability that agents in the market receive acceptable outside offers. Second, for each possible market position (or bargaining slot) and each possible investment compatible with it, in each period there is a positive but potentially small probability that an agent will enter that position with that investment.

The combination of these two assumptions guarantees that the Markov process governing the transition between the possible states of the economy is irreducible. This substantially simplifies what would otherwise be an extremely complicated economy to analyze. Indeed, even with these assumptions we are not able to characterize the equilibrium payoffs in our general model, or to provide conditions that guarantee equilibrium uniqueness. Nevertheless, we can show that a Markov perfect equilibrium always exists and that, in the limit as \( \delta \) goes to 1, holdup is not a problem in any such equilibrium. While we think that the two assumptions mentioned above are fairly realistic, investigating the extent to which holdup is a problem in settings in which one or both of these assumptions fail is a worthwhile avenue for future research.
A Proof of Proposition 4.1

The following argument generalizes the Markov-perfect equilibrium existence proof in Elliott and Nava (2019) to the setting of this paper.

A value function specifies, for every $\theta \in \Theta_s$, the gross expected payoff $V_s(\theta)$ of slot $s$ at the start of a period that begins with state $\theta$ (without taking into account investment costs). For every $\theta \notin \Theta_s$, we normalize $V_s(\theta)$ to be 0. We start by describing all the restrictions on behavior imposed by Markov perfection conditional on an arbitrary value function $V$. Then, we use these restrictions to construct a correspondence (Equation 2) from the set of all value functions to itself whose fixed points correspond to equilibrium value functions, and we show that one such fixed point exists using Kakutani’s fixed point theorem.

Let $\theta + k_s$ denote the state $\theta$ after filling the slot $s$ (in case it is empty) with an agent with investment $k$. Also, let $\theta_{ss'}$ denote the state $\theta$ after $s$ and $s'$ are vacated, and $y_{ss'}(\theta)$ denote the surplus of a match between the agent in slot $s$ and the agent in slot $s'$ in state $\theta$ when both slots are filled under $\theta$, and 0 otherwise.

First, we describe the equilibrium restrictions on the investment behavior. The probability that the state transitions from $\theta$ to $\theta + k_s$ in the entry stage is $\alpha_{\theta,s,Q}^\theta k_s$, where recall that $\alpha_{\theta,s,Q}^\theta$ denotes the probability that slot $s$ is chosen to be filled with investment set $Q$ in an entry stage that begins with state $\theta$, and $\beta_{\theta,s,Q}^\theta k_s$ denotes the probability that the agent filling slot $s$ in state $\theta$ with investment set $Q$ chooses the investment $k \in Q$. Note that $\alpha_{\theta,s,Q}^\theta$ is independent of $V$. The profile $\beta_{\theta,s,Q}^\theta$ is consistent with $V$ if it is in the set

$$B_{\theta,s,Q}^\theta(V) = \left\{ \beta_{\theta,s,Q}^\theta \in \Delta(Q) \mid \beta_{\theta,s,Q}^\theta k_s = 0 \text{ if } k \notin \text{argmax}_{k' \in Q} (V_s(\theta + k'_s) - c(k')) \right\}.$$

Second, we describe how the above restrictions on the investment behavior translate into restrictions on the transition probabilities from one bargaining stage to the next one. For each $\theta \in \Theta_s$, denote by $\phi_{\theta,\theta'}^s$ the probability that the state transitions from $\theta$ to $\theta'$ from the end of one bargaining stage to the beginning of the next bargaining stage conditional on the agent in slot $s$ not leaving in this transition. Also, let $\psi_{\theta,\theta'}^s$ denote the probability that the state transitions from $\theta$ to $\theta'$ in the outside offer stage conditional on the agent in slot $s$ not exiting in this stage. (For each state $\theta \notin \Theta_s$, let $\phi_{\theta,\theta'}^s = \psi_{\theta,\theta'}^s = 0$ for all $\theta'$). Note that $\phi_{\theta,s}^s$ is...
independent of $V$. All transition functions $\Phi^s(V)$ that are consistent with $V$ are in

$$
\Phi^s(V) = \left[ \phi^s_\theta \in \Delta(\theta) \mid \phi^s_\theta = \sum_{s' \in S, Q \subseteq K_s} \alpha^{s',Q} \sum_{k \in Q} \beta^{s',Q} \psi_{k,\theta,\theta'}^s \text{ for some } \beta^{s',Q} \in B^{s',Q}(V) \right]
$$

Third, we describe the equilibrium restrictions on the bargaining behavior. When slot $s$ is the proposer with offer set $S$ in state $\theta$, if there exists $s' \in S$ such that $\delta \sum_{s'} \phi^s_{\theta,\theta'}(V_s(\theta')+V_{s'}(\theta')) < y_{ss'}(\theta)$, then she makes offers only to slots $s' \in S$ for which $y_{ss'}(\theta)-\delta \sum_{s'} \phi^s_{\theta,\theta'}(V_s(\theta')+V_{s'}(\theta'))$ is maximum (offering the amount $\delta \sum_{s'} \phi^s_{\theta,\theta'}(V_s(\theta'))$ to her counterparty in $s'$) and agreement obtains with probability one. Otherwise, she delays—in the sense that she makes offers that are not accepted in equilibrium. For each slot $s$ and each offer set $S$, let $\pi^s_{s,S}(\theta)$ denote the probability that $s$ and $s'$ trade when $s$ is the proposer and $S$ is the offer set if $\theta \in \Theta_s$, and $0$ otherwise. By assumption, the offer set $S$ never contains the proposer $s$; we slightly abuse terminology by denoting by $\pi^s_{s,S}(\theta)$ the probability that slot $s$ delays. Any agreement probability profile that is consistent with the value function $V$ must be in $\Pi^{s,s}(V)$, defined to be the set of all $\pi^{s,s} \in \Delta(S)$ such that there exists $\{\phi^s_\theta \in \Phi^s(V)\}_{s \in S}$ such that

$$
\pi^{s,s} = 0 \text{ if } \delta \sum_{s'} \phi^s_{\theta,\theta'} V_s(\theta') < \max_{s' \in S} \left( y_{ss'}(\theta) - \delta \sum_{s'} \phi^s_{\theta,\theta'} V_{s'}(\theta') \right)
$$

$$
\pi^{s,s} = 0 \text{ if } y_{ss'}(\theta) - \delta \sum_{s'} \phi^s_{\theta,\theta'} V_{s'}(\theta') < \max_{s' \in S} \left( y_{ss'}(\theta) - \delta \sum_{s'} \phi^s_{\theta,\theta'} V_{s'}(\theta') \right) \quad \forall s' \neq S.
$$

Finally, we describe the set $W^{s,s}(V)$ of expected payoffs in a bargaining stage in which $s$ is the proposer, the offer set is $S$, and the state is $\theta$, that are consistent with $V$. These are all the profiles $W^{s,s}$ such that there exists $\{\phi^s_\theta \in \Phi^s(V)\}_{s \in S}$ and $\pi^{s,s} \in \Pi^{s,s}(V)$ such that

$$
W^{s,s} = \pi^{s,s} \delta \sum_{s' \in S} \phi^s_{\theta,\theta'} V_s(\theta') + (1-\pi^{s,s}) \max_{s' \in S} \left( y_{ss'}(\theta) - \delta \sum_{s'} \phi^s_{\theta,\theta'} V_{s'}(\theta') \right)
$$

$$
W^{s,s} = (\pi^{s,s} + \pi^{s,s}) \delta \sum_{s' \in S} \phi^s_{\theta,\theta'} V_s(\theta') + \sum_{s' \neq s,s} \pi^{s,s} \delta \sum_{s'} \phi^s_{\theta,\theta'} V_{s'}(\theta') \quad \forall s' \neq s.
$$

We are now ready to show that a Markov perfect equilibrium exists. Letting $V$ denote the set of value functions $V : \Theta \rightarrow \mathbb{R}^S$, consider the correspondence $F : V \rightarrow V$ defined by

$$
F(V)(\theta) = \left\{ \sum_{s,S} \phi^{s,s} W^{s,s} \mid W^{s,s} \in W^{s,s}(V) \right\},
$$

where recall that $p^{s,s}$ denotes the probability that, in a bargaining stage with state $\theta$, slot $s$ is selected to be the proposer with offer set $S$. By construction, if $V \in F(V)$, then there exist equilibrium strategies that are both optimal given the continuation values $V$, and consistent

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8The fact that the short-run traders generate a surplus larger than $Y$ with the agent occupying their associated slot implies their offers are always accepted in equilibrium regardless of $V$. 

15
with the value function $V$. It is therefore enough that the correspondence $F$ has a fixed point. This follows from Kakutani’s fixed point theorem. Indeed, the domain $\mathcal{V}$ of $F$ is a non-empty, compact and convex subset of an Euclidean space. Moreover, since, for any state $\theta$, any slot $s$ and any offer set $S$, the correspondences $\Phi^s_\theta$ and $\Pi^{s,S,\theta}$ are upper-hemicontinuous with non-empty convex images, so is the correspondence $\mathcal{W}^{s,S,\theta}$, and hence so is $F$. \qed
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