Internet Appendix: Omitted Variable Bias and the Housing Wealth Effect

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Appendix

A - Details of the model

Kogan (2001, 2004) models a two-sector economy, each with a specialized capital input required to produce the two types of consumption goods or services in the economy. Capital in sector H (the housing sector) can only produce housing services. Capital in sector K (the non-housing sector) can either be used to produce the consumption good, C, or converted into housing stock, H. Investment in the housing sector is irreversible; that is, houses cannot be liquidated and turned into the consumption good.

The stock of non-housing capital (K_t) follows the equation of motion:

$$dK_t = (\alpha K_t - C_t)dt + \sigma K_t dW_t - dI_t, \tag{1}$$

where α and σ are, respectively, the mean and volatility of shocks to growth in non-housing capital, and dW_t is an increment of a standard Brownian motion. dI_t is the investment in the housing sector at time t.

Identical, perfectly competitive firms own all of the capital in sector H used to produce the housing service XH_t for consumption, with X representing the productivity of the housing sector; firms rent out the houses that they own. Firms determine the level of investment at each t to solve the maximization problem

$$\max_{\{I\}_{0 \leq t < \infty}} E_0 \left[\int_0^\infty \eta_{0t} S_t X H_t dt - \eta_{0t} dI_t \right], \tag{2}$$

where S_t is the rent for one unit of housing in units of the consumption good C at time t, and is η_{0t} is firms' the stochastic discount factor. The first term in the integral above is the present value of all the rents that firms receive from housing. The second term is the present value of all the investment in housing. Changes in the housing stock are given by

$$dH_t = -\delta H_t dt + dI_t, \tag{3}$$

where δ is the rate of depreciation.

Households maximize their expected lifetime utility:

$$\max_{\{C_t, I_t\}_{0 \leq t < \infty}} E_0 \left[\int_0^\infty e^{-\rho t} U(C_t, XH_t) dt \right], \tag{4}$$

where ρ is the parameter that specifies household impatience. Households, whose coefficient of risk aversion is γ, have utility separable over the consumption good, C_t, and housing services, XH_t, given by[1]

$$U\left(C_t, XH_t\right) = \frac{1}{1-\gamma}\left(C_t\right)^{1-\gamma} + \frac{b}{1-\gamma}\left(XH_t\right)^{1-\gamma}, \gamma > 0, \gamma \neq 1, \tag{5}$$

where b can be interpreted as the parameter that captures the size of sector H as a fraction of the whole economy.

Households also have access to two long-term financial assets. The value of the first asset at time t is v_t and it follows the dynamic $dv_t = \alpha v_t dt + \sigma v_t dW_t$. The second asset is a claim on all housing sector cash flows; in other words, the second claim is equivalent to the stock in the housing sector firms. In addition, households have access to a short-term bond.

Kogan (2001) shows that an equilibrium exists in which the processes for K_t, H_t, C_t, and I_t are equivalent to the solution of a central planner problem that chooses C_t and I_t to solve the maximization in Equation 4, subject to Equations 1 and 3. In fact, the central planner's choice of the control variables depends only on the state variable $\omega_t = \ln(\Omega_t) = \ln(H_t/K_t)$. In equilibrium, the optimal consumption policy is given by the following equation:

$$\widetilde{c}(\omega_t) = \frac{C_t}{K_t} = \left(f(\omega_t) - \frac{1}{1-\gamma}f'(\omega_t)\right)^{-\frac{1}{\gamma}}, \tag{6}$$

where f is the function that satisfies the ordinary differential equation (ODE)

$$p_2 f''(\omega) + p_1 f'(\omega) + p_0 f(\omega) + \gamma\left(f(\omega) - \frac{1}{1-\gamma}f'(\omega)\right)^{1-\frac{1}{\gamma}} = -be^{(1-\gamma)\omega}, \tag{7}$$

subject to the boundary conditions

$$f'(\omega^*)\left(1+\Omega^*\right) = f(\omega^*)\Omega^*(1-\gamma) \tag{8}$$

$$f''(\omega^*)\left(1+\Omega^*\right) = f'(\omega^*)\left(1+(1-\gamma)\Omega^*\right) \tag{9}$$

$$\lim_{\omega\to\infty} f(\omega) = \left(\alpha\frac{\gamma-1}{\gamma} - \frac{\sigma^2}{2}(\gamma-1) + \frac{\rho}{\gamma}\right)^{-\gamma}, \tag{10}$$

[1] Kogan (2001) also considers the case $\gamma = 1$; the qualitative relationships between the variables that we investigate in our study – consumption, investment, and prices – do not change if we use $\gamma = 1$.

and p_0, p_1, p_2 are constants with the following values:

$$p_0 = (1-\gamma)\alpha - \gamma(1-\gamma)\frac{\sigma^2}{2} - \rho \tag{11}$$

$$p_1 = -\alpha - \delta + (2\gamma - 1)\frac{\sigma^2}{2} \tag{12}$$

$$p_2 = \frac{\sigma^2}{2}. \tag{13}$$

The optimal housing investment policy is such that investment in housing only happens if ω is equal to an endogenously determined threshold, ω^*. Formally, the agent chooses $I_t = 0$ at t when $\omega_t > \omega^*$ and $I_t > 0$ when $\omega_t = \omega^*$. The variable ω follows the process

$$\begin{aligned} d\omega_t &= \mu_\omega(\omega_t)dt - \sigma dW_t + dL_t \\ \mu_\omega(\omega_t) &= -\alpha - \delta + \widetilde{c}_t(\omega_t) + \frac{\sigma^2}{2} \end{aligned}$$

where dL_t is zero when $\omega_t > \omega^*$ and is larger than zero when $\omega_t = \omega^*$. Consequently, dL_t is different from zero only when investment in the housing sector occurs. Specifically, $dL_t = (1 + \Omega^*)H_t^{-1}dI_t$ and ω is a process with a reflexive boundary at ω^*.

The market value of one unit of housing is given by

$$P(\omega_t) = \frac{f'(\omega)\Omega^{-1}}{(1-\gamma)\, f(\omega) - f'(\omega)}, \tag{14}$$

which is bounded by the replacement cost. This market value is equal to the housing Tobin's q, since the replacement cost is assumed to be equal to one.

We use Equations 6 and 14 to plot the consumption-to-capital ratio (C/K) and housing price (P) as a function of ω in Panel A of Figure 1 in the paper. For a fixed value of the housing stock, ω is a linear transformation of $\ln K$ (k), and we can rewrite Equations 6 and 14 to make C/K and P functions of k. In Panel B of Figure 1, we set H arbitrarily equal to 10, and plot $\ln P$ (p) and $\ln C$ (c) as functions of k using the parameters α and σ used to calibrate the model to Minnesota. (See Appendix C for details about this calibration.)

B - Simulating the model

We simulate the time series of housing price appreciation, consumption growth, and non-housing capital for each geographical area i following Kogan's model. The model parameters are those in Table 5 of the paper along with $\alpha_i, \sigma_i, \omega_{0,i}$ and Σ (the correlation matrix of shocks in non-housing capital across states). We choose the parameters based on the calibration procedure described in Appendix C. For a given value of the model parameters, we first solve the ODE in Appendix A to obtain the functions $f(\omega), \widetilde{c}(\omega_t), q(\omega_t)$ and ω^* for each i. We then simulate the time series of ω using the following algorithm:

1. For a given $\omega_{0,i}$, obtain the values of $\widetilde{c}(\omega_{0,i})$, and $q(\omega_{0,i})$.

2. Generate a random shock to the growth rate of non-housing capital $\Delta W_{\Delta t,i} \sim \mathcal{N}\left(\mathbf{0}, \sigma^2 \Delta t\right)$. The correlation of these random shocks across states is Σ. We divide the time interval between successive observations into 1000 intervals, i.e., we set $\Delta t = 1/1000$ in all our simulations.

3. Find $\omega_{\Delta t,i}$ with the discrete approximation $\omega_{\Delta t,i} = \omega_{0,i} + \mu_\omega(\omega_{0,i})\Delta t - \sigma_i \Delta W_{\Delta t,i}$

4. If $\omega_{\Delta t,i} > \omega_i^*$ proceed to next point; otherwise, make $\Delta L_{0,i} = \omega_i^* - \omega_{\Delta t,i}$, and $\omega_{\Delta t,i} = \omega_{\Delta t,i} + \Delta L_{0,i}$

5. Calculate $\Delta I_{0,i}$ as $(1 + \Omega_i^*)^{-1} H_{t,i} \Delta L_{0,i}$, $\Delta H_{\Delta t,i}$ and $\Delta K_{\Delta t,i}$ with the Euler discrete approximation of Equations 3 and 1. Without loss of generality, we set $K_{0,i}$ equal to one.

6. Repeat Step 1 with $\omega_{\Delta t,i}$ instead of $\omega_{0,i}$ until a time series with length $T = 30$ for the variables of interest is obtained.

C - Model calibration

We choose the parameters α_i and σ_i to enable the mean and volatility of consumption growth in the simulations to match the data from geographical area i. We choose the parameter $\omega_{0,i}$ to match either the mean, or the volatility of, housing price appreciation shown in Table 2 of paper. We choose the correlation matrix Σ to match the correlations of consumption growth in Table 3.

The starting point of our calibration is the observation that although $\widetilde{c}(\omega_{t,i})$ is a non-linear function (see Figure 1 in the paper), the variation in $\widetilde{c}(\omega_{t,i})$ is small and can be approximated by a constant. In fact, $\widetilde{c}(\omega_{t,i})$ is close to:

$$\widetilde{c}_i = \lim_{\omega_{t,i} \to \infty} \widetilde{c}(\omega_{t,i}) = \frac{\gamma - 1}{\gamma}\alpha_i - \sigma_i^2 \frac{\gamma - 1}{2} + \frac{\rho}{\gamma} \tag{15}$$

As a result, in our calibration, we choose the parameters $\alpha_{i,}$ σ_i and Σ in which the amount of housing is much larger than the amount of non-housing capital ($\omega_{t,i} \to \infty$). In this economy, the investment in housing is zero, and the log of non-housing capital follows an arithmetic Brownian motion:

$$dk_{t,i} = \left(\alpha_i - \widetilde{c}_i - \frac{1}{2}\sigma_i^2\right) dt + \sigma_i dW_{t,i}$$

Because the level of consumption is $C_{t,i} = \widetilde{c}_i K_{t,i}$, the log-consumption process has the same drift and volatility as $k_{t,i}$. We therefore set σ_i equal to the estimated volatility of consumption growth in Table 2 for geographical area i. We set the correlation matrix Σ equal to the matrix in Table 3. We use the parameter α_i to solve the following equation:

$$\left(\alpha_i - \widetilde{c}_i - \frac{1}{2}\sigma_i^2\right) = \bar{c}_i$$

where $\bar{c}_i$ is the mean consumption growth in Table 2 for geographical area i.

Once, the parameters α_i and σ_i are set, we solve the ODE in Appendix A and then simulate the model for each i as described in Appendix B using different starting values of $\omega_{0,i}$. We then search for the $\omega_{0,i}$ that minimizes the distance between simulated mean or volatility of housing price appreciation and that displayed in Table 2 for geographical area i.

D - Robustness of results using lagged variables as instruments

Under the permanent income hypothesis (PIH), current period consumption growth is driven by contemporaneous innovations in permanent income, and is independent of lagged changes in permanent income. Hence, one way to address the common factor hypothesis under the assumption that the PIH holds is to use lagged values of growth in consumption, income, housing, and non-housing wealth as instruments in an IV estimation of Equation 1 in the paper.

In this Appendix we show that our results are robust to using lagged variables as instruments. First, we verify that the results in Table 4 are robust to using the first four lags of the growth of consumption, income, tradeable, and housing wealth, as instruments.

The estimate of the wealth effect, β_{w^H}, reported in Appendix Table 1, is 11.17% for the full model, which is very similar to the corresponding value of 12.66% obtained in fixed-effects regressions reported in Table 4. Second, we verify that our simulation results are not changed with this IV approach. The results of the fixed-effects regressions using the first four lags of the growth of consumption, housing, and non-housing wealth as instruments are shown in Appendix Table 2. Comparing Appendix Table 2 with Table 7, we see no significant differences in the magnitude of β_{w^H} estimated with or without IVs.

References

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Appendix Table 1: **The Housing Wealth Effect in Historical Data Using Instrumental Variables Estimation.** This table presents the results of panel data regressions of annual aggregated non-housing log-consumption growth (Δc) on the growth of log-housing wealth (Δw^H) and log-income growth (Δy). State-level fixed effects are included in all models and one specification controls for the log of non-housing tradable wealth (Δw^{TR}). All independent variables are instrumented by the first four lags of all of Δw^H, Δw^{TR}, and Δy. β_{w^H} and β_y are the coefficients of the terms Δw^H and Δy, respectively. α is the average of the fixed effect. The value in parentheses below the coefficient is the T-statistic . Overall R^2 (in %) values are reported in the last row. The sample contains observations at annual frequency for the period 1976–2012.

	(1)	(2)	(3)	(4)
β_{w^H}	0.3521		0.1473	0.1117
	(9.24)		(3.44)	(2.44)
β_y		1.0368	0.8479	0.8752
		(12.45)	(8.59)	(8.62)
α	0.0116	-0.0001	0.0015	-0.0023
	(9.31)	(-0.05)	(0.93)	(-1.05)
Δw^{TR} control	No	No	No	Yes
R^2	0.0777	0.1509	0.1730	0.1510

Appendix Table 2: **Wealth Effect Regressions Using Instrumental Variables in Data Simulated to Match the Mean Housing Wealth Growth and with Errors in Independent Variables.** This table displays the results of regressions estimated on 500 panels of simulated data, where all independent variables are instrumented by the first four lags of Δw^H, Δk and Δc. For each state in each simulation, 30 years of data are generated using the calibrated theoretical model. The data are simulated with the theoretical model calibrated to match the mean housing wealth growth in each state. Panel A of Table 6 presents details of this calibration. The dependent variable of the panels is the log-growth of non-housing consumption (Δc). The independent variables in these panels are the log-growth of housing wealth (Δw^H) and the log-growth of non-housing capital (Δk). The independent variables in the panels are assumed to have mean-zero normally-distributed measurement error that are independent of each other and independent of the shocks to non-housing capital. Each model and panel displays results with different noise-to-signal ratio in Δw^H and Δk. Each model contains the results when the variance of the measurement error of Δw^H is equal to 5%, 10%, or 30% of the variance of the error-free Δw^H in that simulation. Panel A (B and C) contains the results when the variance of the measurement error of Δk is equal to 5% (10% and 30%) of the variance of the error-free series of Δk obtained in that simulation. All estimated models have state-level fixed effects. β_{w^H} , β_k and α are the average across all simulated panels of the estimated coefficients on Δw^H, Δk and state-level fixed effects. The value in parentheses below the coefficient is the average T-statisic. The last row of each panel contains the average of the overall R^2 (in %) of the simulated regressions.

	Noise-to-Signal ratio in Δw^H								
	5%			10%			30%		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
	Panel A: Noise-to-signal ratio in Δk equals 5%								
β_{w^H}	1.37		0.17	1.30		0.14	1.07		0.10
	(44.42)		(9.75)	(41.39)		(9.08)	(33.62)		(7.35)
β_k		0.91	0.84		0.91	0.85		0.91	0.87
		(152.75)	(95.87)		(152.75)	(99.61)		(152.75)	(110.45)
α	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-5.03)	(3.63)	(-3.37)	(-2.42)	(3.63)	(-2.63)	(5.14)	(3.63)	(-0.76)
R^2	0.58	0.95	0.95	0.55	0.95	0.95	0.45	0.95	0.95
	Panel B: Noise-to-signal ratio in Δk equals 10%								
β_{w^H}	1.37		0.29	1.30		0.26	1.07		0.18
	(44.42)		(13.41)	(41.39)		(12.49)	(33.62)		(10.13)
β_k		0.87	0.76		0.87	0.77		0.87	0.80
		(109.38)	(68.65)		(109.38)	(71.33)		(109.38)	(79.09)
α	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-5.03)	(8.01)	(-3.41)	(-2.42)	(8.01)	(-2.38)	(5.14)	(8.01)	(0.27)
R^2	0.58	0.90	0.92	0.55	0.90	0.91	0.45	0.90	0.91
	Panel C: Noise-to-signal ratio in Δk equals 30%								
β_{w^H}	1.37		0.60	1.30		0.54	1.07		0.39
	(44.42)		(21.32)	(41.39)		(19.86)	(33.62)		(16.12)
β_k		0.74	0.54		0.74	0.56		0.74	0.61
		(63.69)	(39.97)		(63.69)	(41.53)		(63.69)	(46.05)
α	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-5.03)	(17.39)	(-3.85)	(-2.42)	(17.39)	(-2.29)	(5.14)	(17.39)	(1.86)
R^2	0.58	0.76	0.82	0.55	0.76	0.82	0.45	0.76	0.80