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Modeling the Effects of Psychological Pressure on First-Mover Advantage in Competitive Interactions: The Case of Penalty Shoot-Outs

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Abstract

The relationship between psychological pressure and performance outcomes has been studied across a variety of sporting contexts. As an extension and complement to recent empirical studies, we construct a formal model of soccer penalty shoot-outs to determine the links between psychological pressure and first-mover advantage (FMA). Our approach indicates that even seemingly simple competitive interactions may include a rich, complex set of effects. We demonstrate that psychological pressure leads to FMA in shoot-outs; however, we show that this relationship can vary depending on a variety of different factors, such as the nature of the pressure, the magnitude of the pressure, and the specific rules governing the shoot-out. Overall, our work clarifies and extends knowledge of the operation of FMA and of how psychological pressure impacts performance outcomes in competitive interactions.

Keywords

psychological pressure, first-mover advantage, soccer, penalty shoot-outs

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The relationship between timing of moves and performance outcomes in dynamic competitive interactions has drawn interest across a variety of sporting settings, including tennis (e.g., Magnus & Klaassen, 1999), track and field (e.g., Hill, 2014), and soccer (e.g., Krumer, 2013). Sequential interactions are common not only in sporting contests but also in a variety of other areas such as entry into new product or geographic markets, investment in new technologies, promotion tournaments within companies, and research and development (R&D) races between companies (e.g., Breitmoser, Yan, & Zizzo, 2010; Zizzo, 2002).

One potential driver of performance differentials in sequential tournaments is associated with the fact that participants receive interim performance feedback. Participants may experience psychological effects because they are aware of rivals' prior performance and perceive their own performance in comparative terms. Prior research has demonstrated that interim rankings or scores affect performance in sports such as basketball (e.g., Goldman & Rao, 2016), diving (Genakos, Pagliero, & Garbi, 2015), and weight lifting (Genakos & Pagliero, 2012). Baumeister (1984) described one such effect as "choking under pressure" or the fact that if participants are aware that they are lagging behind in performance, this may increase stress leading to decreased performance.

Our aim in this article is to increase understanding of the potential mechanisms that underlie a relationship between task order and performance, more specifically to better understand potential first-mover advantage (FMA) associated with psychological pressure in sequence-based competitive interactions. Although our general topic of interest is consistent with prior studies, we conduct a different type of investigation. Our belief is that while sporting contests may be less complex than interactive contests within and between companies, a good deal of inherent complexity remains. This complexity makes it difficult to identify the operation of specific causal mechanisms, a challenge identified by some prior work in this area (e.g., Kocher, Lenz, & Sutter, 2012). Thus, as a complement to prior empirical research, we construct a mathematical model of a sporting contest and then use that model to investigate the relationship of psychological pressure to FMA. More specifically, we explore how the relationship of psychological pressure to FMA varies depending on (1) different types of pressure, (2) different magnitudes of pressure, and (3) different underlying conditions of the competition, such as rules about shooting sequence. To be clear about our purpose, we do not develop theory to explain why forms of psychological pressure exist (i.e., the antecedents of pressure). Nor do we investigate why psychological pressure might degrade individual performance. Rather, we take different forms of psychological pressure as given (inputs into the model) and determine the consequences of pressure on FMA.

We model penalty shoot-outs in soccer, which have featured in a number of prior studies (e.g., Apesteguia & Palacios-Huerta, 2010; Carrillo, 2007; Kocher et al., 2012; Lenten, Libich, & Stehlík, 2013; Palacios-Huerta, 2014). We elect this setting for two reasons. First, the contest is simple enough to be modeled mathematically while also providing a context subject to a meaningful set of potential psychological effects. This allows us to clearly demonstrate the relationship between several types of psychological pressure and performance outcomes. Second, because prior studies have empirically investigated FMA in penalty shoot-outs, this setting provides a particularly appropriate context to illustrate the complementary value of theoretical modeling. Apesteguia and Palacios-Huerta (2010) found that the first team to shoot in penalty shoot-outs has a clear sequence-based advantage. Kocher et al. (2012) questioned this finding in an alternative analysis of a broader sample; however, Palacios-Huerta (2014) further expanded the sample and confirmed the presence of a significant FMA.

We believe a formal modeling approach provides a natural complement to prior empirical research for several reasons. First, it is difficult to directly measure psychological mechanisms or to manipulate them in empirical studies; instead, the operation of such mechanisms must be inferred from observable outcomes. Second, it can be challenging in empirical analyses to simultaneously consider the operation of multiple types of pressure, some of which might interact or cancel each other out. Third, endogeneity may affect the results and the associated conclusions. The sequence in which competing teams perform a task can be viewed as randomly assigned, but other aspects of the contest—such as order of individual performers within teams who may differ in ability—are not randomly assigned but are rather endogenously determined by the teams. Fourth, recent research has demonstrated the utility of formal models in investigating the design of penalty shoot-outs (Anbarci, Sun, & Ünver, 2015). Finally, empirical studies of psychological pressure in the context of sporting contests may be of limited statistical power because of the relatively small number of recorded contests that have occurred in professional competitions.

Theoretical modeling, in comparison, allows for separation and isolation of specific causal mechanisms. While the prior literature has suggested a relatively straightforward relationship between psychological pressure and FMA, we demonstrate how a rich set of complex relationships can operate in this area. Our model generates a number of important conclusions. The presence of lagging-behind pressure results in FMA, and the magnitude of the advantage is an increasing function of the magnitude of the pressure. We also demonstrate the influence of other forms of psychological pressure, such as the pressure that occurs in situations where scoring a particular shot would result in a win of the shoot-out or where missing a particular shot would result in a loss of the shoot-out. Our model further allows us to investigate how the nature of FMA depends on the choices of decision makers participating in and designing the competition. Specifically, we consider the choice of the manager who selects how to order players, showing first that the decision of where to place more proficient shooters in the order has no influence on outcomes when there is no situation-specific psychological pressure. But, as we show next, this choice does matter when pressure exists, and it matters more for the first team compared to the second. We also demonstrate several ways in which the designer of the rules of the shoot-out could influence the magnitude of FMA. For instance, the task could be made more difficult. We find that, as task difficulty increases, the benefit of going first decreases. Similarly, we show that the magnitude of FMA associated with psychological pressure could be reduced by changing the standard alternating shot format. Overall, our model provides rich insight into the potential role of psychological pressure in competitive situations, a particularly important issue because of the many different ways psychological pressure might become manifest.

Background

Social psychologists have long been interested in the relationship of psychological pressure to performance outcomes. Some early studies in this area (e.g., Zajonc, 1965) linked decreased performance on complex tasks to the mere presence of an audience. Baumeister (1984) similarly showed that a variety of situational sources of pressure, including implicit competition and cash payments, resulted in lower performance outcomes. The psychological mechanisms thought to be associated with choking under pressure can be broadly categorized as follows: (i) drive theories that link arousal and performance, notably the Yerkes–Dodson theory (Yerkes & Dodson, 1908) and social facilitation theory (Zajonc, 1965); (ii) attentional theories that examine the cognitive processes associated with pressure-induced choking, including distraction theory (e.g., Wine, 1971) and explicit monitoring theory (e.g., Beilock & Carr, 2001); and (iii) behavioral theories related to the biomechanical processes associated with performance stress (e.g., Bernstein, 1967).

Sporting competitions have served as a useful context in which to study these relationships because they are characterized by several mechanisms that might influence performance under pressure. As summarized by Ariely, Gneezy, Loewenstein, and Mazar (2009), these characteristics include the presence of an audience, the existence of competition, individuals with traits such as competitiveness, and threats to the participants' egos. Studies have investigated general links between psychological pressure and performance across a variety of sports. Page and Page (2007) argued that motivation and anxiety vary across home versus away settings in two-legged cup competitions in soccer. Follow-on work by Krumer (2013) showed how the existence of a psychological advantage to the winner of the first stage leads to a preference of playing the second stage at one's home field. Dohmen (2008) found that home-team players were more likely to choke under the pressure of performing in front of home crowds when shooting penalty kicks in soccer. Arguing that golfer performance is influenced by loss aversion, Pope and Schweitzer (2011) demonstrated that professional golfers were more likely to convert putts for par relative to similar putts for birdie. González-Díaz, Gossner, and Rogers (2012) confirmed that tennis players differed in their performance abilities during critical situations, when particularly important points in the match were at stake. Toma (2017) showed that free-throw percentages in basketball were lower in the final

30 seconds in tight games; free-throw percentages also declined in play-off and nationally-televised games (Goldman & Rao, 2016).

A particular question of interest in this literature has been whether information on interim rankings or scores might influence performance. Genakos and Pagliero (2012) investigated multiround weight-lifting competitions in which participants receive information about their relative position after each round. They showed that participants trailing in the competition adopted riskier strategies in an attempt to catch up with leaders. Genakos et al. (2015) similarly discovered that interim ranks affect performance achievement in diving competitions. Goldman and Rao (2016) extended these findings to basketball, studying outcomes of hundreds of thousands of free throws in the National Basketball Association and demonstrated that players perform worse when trailing in the score.[1]

Researchers have observed that the presence of the types of psychological pressure described above might lead to performance differentials in competitions where performance has sequential aspects. Prior investigations have established an advantage to moving first across several sporting contexts. Both Kingston (1976) and Anderson (1977) modeled the relationship between moving first and performance outcomes in tennis, while Magnus and Klaassen (1999) provided empirical evidence of an advantage of serving first in tennis due to a "first game effect." The advantage of moving first has also been demonstrated in cognition-based competitions such as chess (González-Díaz & Palacios-Huerta, 2016).

The specific context of interest in our research is penalty kicks in soccer. This context is a high-pressure situation with a simple task and clearly measurable outcomes of success versus failure. Moreover, participants have very strong incentives to perform. An especially high-pressure variant of penalty-kick situations is the penalty shoot-out. Penalty shoot-outs were introduced in 1970 by the world governing body of soccer, the *Fédération Internationale de Football Association* (FIFA), to determine the winner in knockout tournament games in which there is a tie between two teams after regular time and overtime. In a shoot-out, teams take five kicks each in alternating sequence; the team with the highest score wins. If the score is tied after all ten regular kicks have been taken, then both teams each take an additional shot until the tie has been broken. Not surprisingly, pressure appears to affect performance in penalty shoot-outs (e.g., Savage & Torgler, 2012).

Penalty shoot-outs are a particularly appropriate setting to investigate the links between psychological pressure and FMA. Participants take turns shooting, and this sequential performance may create enhanced psychological pressure, leading to choking by later participants. More specifically, when individuals perform a task sequentially, later participants may feel greater pressure to perform if they are aware that opponents have already successfully completed the task. Observation of an opponent's successful performance can increase anxiety about one's ability to match that outcome, leading to worse performance for the second participant (i.e., a lagging-behind effect).

Prior work largely demonstrates an advantage to performing first in a penalty shoot-out. Apesteguia and Palacios-Huerta (2010) (hereafter "APH") proposed that the lagging-behind effect results in an advantage for the first team to shoot because the first team is more likely to impose the lagging-behind effect on its opponent. Their study of 129 shoot-outs indicated a winning advantage from shooting first. Kocher, Lenz, and Sutter (2012) (hereafter "KLS") investigated the same issue but disagreed with APH's conclusions. Specifically, KLS examined 540 shoot-outs (a superset of APH's sample of shoot-outs) and found that the first team's winning proportion was .53. The difference between this proportion and .50 was not statistically significant. In a follow-on study, however, Palacios-Huerta (2014) further increased the underlying data set to include 1,001 shoot-outs. This study's findings confirmed those of APH: In just over 60% of cases, the team leading off was the one to eventually win the shoot-out.

To complement and extend prior work that has largely investigated this issue empirically, we develop a formal mathematical model of a penalty shoot-out. This model elucidates the relationship of lagging-behind pressure and FMA, and it also allows us to address the initial APH–KLS disagreement. As a further extension of work in this area, we derive a number of other results to advance understanding of the role of order-based psychological pressure on performance outcomes. Our examples demonstrate how a number of factors influence the operation of FMA associated with psychological pressure, including the presence of different types of psychological pressure. We show the effects of the choice of how to order participants with differing skills and also demonstrate the implications of alternative rule choices by the designer of the competition. We turn next to an explanation of our modeling approach.

Modeling Approach

Model Details

Our model is set up to mirror the operation of a penalty shoot-out as specified in the official rules of the game designated by FIFA. More specifically, two teams—"Team 1" (which shoots first) and "Team 2" (which shoots second)—compete in accordance with the following rules.

- Teams take turns shooting until five rounds have been completed, with each round consisting of one kick by each team.
- If at any point during the shoot-out one side has accumulated more goals than the other team could possibly accumulate with all its remaining shots, then the shoot-out terminates regardless of the number of shots remaining (e.g., if Team 1 trails 0–3 after each team has taken three shots, then the shoot-out terminates because Team 1 cannot catch up with its remaining two shots).

- If the score remains tied at the end of five rounds, sudden-death rounds of one kick each are added until one side scores and the other does not.

Each penalty kick follows a binomial process: Outcomes of individual kicks are binary (i.e., either "score," 1, or "no score," 0); the probability of a score by player (or kick) i of team j is denoted by p_{ij}, while the probability of not scoring is denoted by its complement, $1 - p_{ij}$, with $i = 1, 2, \ldots n$; $j = 1, 2$, and $0 < p_{ij} < 1$.

Since p_{ij} may vary among kicks, each team's series of kicks in a shoot-out can be described as a Poisson binomial process. Provided both teams' series of kicks (which together constitute the shoot-out) are viewed as being independent, the shoot-out can be represented by a combination of two independent Poisson binomial processes, and the winning probabilities may be derived via the recursive formula for the probability mass function of a Poisson binomial.

However, if the teams' scoring probabilities are not actually independent—owing, for example, to the influence of lagging-behind pressure—then one must revert to more elaborate calculations to derive winning probabilities. For instance, APH employ Markov chains to derive winning probabilities (see Online Appendix B to APH).

We view each kick as a binomial process and as one node within a path linking the applicable nodes. A path of nodes constitutes a possible shoot-out. In a shoot-out consisting of n rounds, each team takes n kicks, and thus $2n$ (regular round) kicks are taken in total. There are, theoretically, 2^{2n} possible paths. There are $n = 5$ rounds in our setting; hence, we have $2^{10} = 1{,}024$ different conceivable paths, each consisting of 10 nodes.[2] Our model encompasses each of these paths. Given its complexity, the extended tree diagram of the 1,024 paths is not provided here but is available upon request.

Several additional assumptions of our modeling approach are worth noting. First, we assume that shot outcomes are primarily determined by kicker performance rather than the performance of the goalkeeper. This is a reasonable assumption in the context of penalty kicks in soccer given that "the task ... to perform (kick a ball once) is one of the simplest [professional soccer players] could possibly be asked to perform" (APH, p. 2549), that "shooters are expected to score" (Jordet, 2009, p. 103), and that penalty shots are successful approximately 75% of the time (Dohmen, 2008). Second, we assume that the choice of effort is not meaningful in this context because of the high incentives for successful performance and the relatively effortless nature of the task (APH).

Model Results

This section provides our core results regarding the relationship between psychological pressure and FMA. In our base model, we consider a version of the shoot-out in which all players have the same penalty-shooting ability and performance is potentially subject to sequence-based psychological pressure. Player ij scores with probability p if not lagging behind in the score and with probability $p - \lambda$ if lagging behind, where $0 < \lambda < p$.

Proposition 1:

(i) $(\text{FMA}|\lambda = 0) = 0$ (i.e., in the absence of lagging-behind pressure, there is no FMA).
(ii) For $0 < p < 1$ and $0 \leq \lambda < p$, $\frac{\partial \text{FMA}}{\partial \lambda} > 0$ (i.e., FMA is an increasing function of the lagging-behind effect).

Proof:

We define FMA as follows:

$$\text{FMA} = P(\text{Team 1 wins}) - P(\text{Team 2 wins}). \tag{1}$$

We first explain the calculation of the probability that Team 1 wins in two simple cases with less than five rounds before moving to the general model. In the first case, we explore a shoot-out that would end immediately after one round unless the score is even. If the score is even, there would be additional, sudden-death rounds until one of the two teams wins. In the second case, we explore a shoot-out with two rounds, that is, the shoot-out will not end after the first round but will end after the second round, unless the score is even. Figure 1 depicts the possible paths and respective probabilities for the first case of a one-round (plus possible sudden-death rounds) shoot-out. The relevant outcome variable at any moment in time is the cumulative score differential s; possible outcomes for each kick taken are 1 (*if goal*) and 0 (*if miss*) for the first-kicking team and, conversely, -1 (*if goal*) and 0 (*if miss*) for the second-kicking team.

For simplicity, we substitute $\mu = p(1 - p + \lambda)$ and $\nu = (1 - p)p$, which are the probability of Team 1 scoring followed by Team 2 missing and the probability of Team 1 missing followed by Team 2 scoring, respectively. The first team may win in the current round (with probability μ) or in one of the ensuing series of sudden-death rounds (shoot-outs enter into sudden-death rounds with a probability of $1 - \mu - \nu$). The probability that the first team will win this simplified one-round shoot-out is calculated as follows:

$$\begin{aligned} P(\text{Team 1 wins} \mid n{=}1) &= \mu + (1-\mu-\nu)\mu + (1-\mu-\nu)^2\mu + \dots \\ &= \frac{\mu}{1-(1-\mu-\nu)} = \frac{\mu}{\mu+\nu} = \frac{1-p+\lambda}{2-2p+\lambda}. \end{aligned} \tag{2}$$

We next consider our second case of a two-round shoot-out to assess whether the probability of Team 1 winning is different from Equation 2. Figure 2 depicts the 16 possible paths that exist in a two-round shoot-out and reports the final score and probability of each path.[3] The outcomes of the tree can be simplified by aggregating those paths that lead to the same cumulative score differential s, as shown in Table 1.

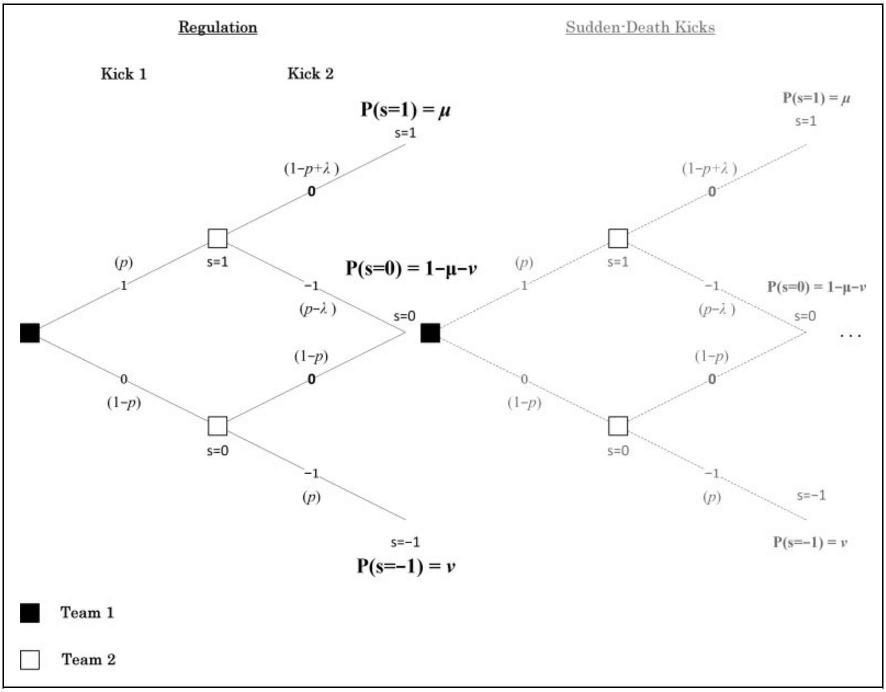

Figure 1. All possible paths (and their respective probabilities) constituting the tree diagram for a one-round shoot-out.

Similarly, for simplicity, we substitute μ' for the probability that Team 1 wins without the need of sudden-death rounds (i.e., $s = 2$ or $s = 1$) and ν' for the probability that Team 1 loses without entering into sudden-death rounds (i.e., $s = -2$ or $s = -1$). The shoot-out advances to additional rounds with probability $1 - \mu' - \nu'$, where Team 1 will win with probability defined in Equation 2. Thus, the probability that the first team will win the two-round shoot-out is as follows:

$$P(\text{Team 1 wins} \mid n{=}2) = \mu' + (1 - \mu' - \nu')\left(\frac{\mu}{\mu + \nu}\right). \tag{3}$$

Equation 3 may be rewritten to express μ, ν, μ', and ν' in terms of p and λ:

$$P(\text{Team 1 wins} \mid n{=}2) = \frac{1 - p + \lambda}{2 - 2p + \lambda}. \tag{4}$$

The above equation demonstrates that the probability that Team 1 wins this two-round shoot-out is equal to the probability of Team 1 winning the one-round shoot-out as shown in Equation 2.

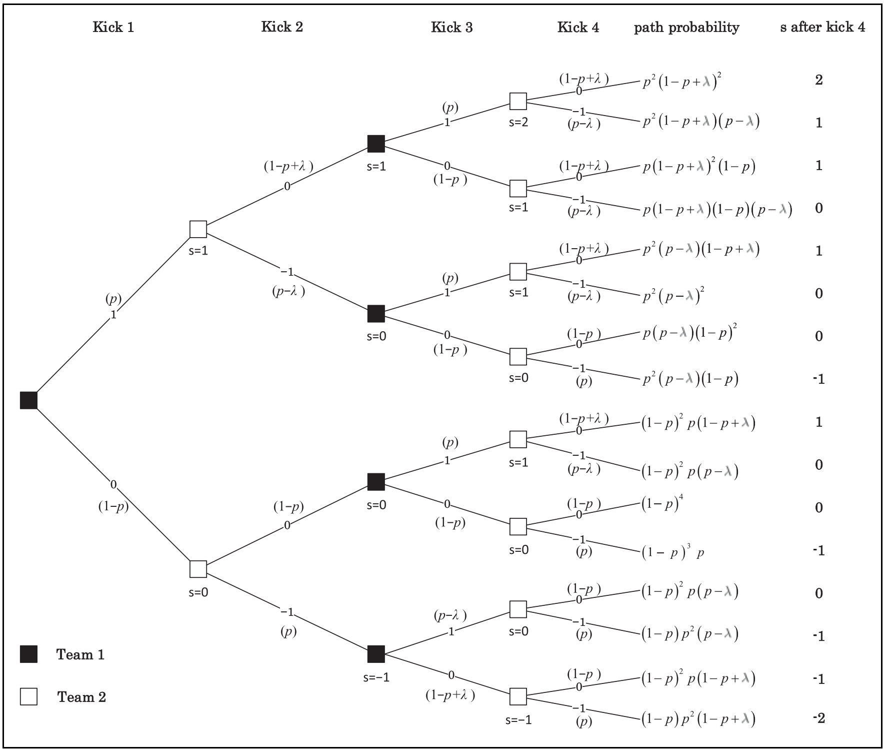

Figure 2. All possible paths (and their respective probabilities) constituting the tree diagram for a two-round shoot-out.

The intuition supporting this result, that is, the equivalence between a one-round and a two-round shoot-out, can be explained as follows. Independent of the number of rounds, the first-round kicks entail FMA, as the probability that Team 1 will lead after Round 1, that is, $p(1 - p + \lambda)$, is larger than the probability that Team 2 will lead after Round 1, that is, $p(1 - p)$. While the one-round shoot-out ends as soon as one team has taken the lead after a complete round of two kicks, in a two-round shoot-out, there would always be an additional round after the first. It is easy to show that once Team 1 has taken the lead in the first round, the probability that Team 1 will maintain a leading position and win the shoot-out is equal to the probability that Team 2 will maintain a leading position and win the shoot-out if Team 2 has taken the lead in the first round. Achieving a leading position in the score is what creates an advantage (once a team is ahead, its opponent's probability of scoring is reduced and the leading team is therefore more likely to win the shoot-out). Although in the presence of a lagging-behind effect, Team 1 has an *ex ante* higher probability of achieving this position (and hence the reason there is FMA), the probability a team will maintain its lead and win in subsequent rounds of a shoot-out once it has

Table 1. Aggregated Probabilities of All Possible Final Cumulative Scoring Differences, Based on a Two-Round Shoot-Out.

Cumulative Score	Aggregated Probability	Outcome
$s = 2$	$p^2(1-p+\lambda)^2$	Team 1 Wins $P(s=2) + P(s=1) = \mu'$
$s = 1$	$p^2(1-p+\lambda)(p-\lambda)$ $+p(1-p+\lambda)^2(1-p)$ $+\left(p(p-\lambda)+(1-p)^2\right)p(1-p+\lambda)$	
$s = 0$	$\left(p(1-p+\lambda)(1-p)\right)(p-\lambda)$ $+\left(p(p-\lambda)+(1-p)^2\right)p(p-\lambda)$ $+2p(p-\lambda)(1-p)^2+(1-p)^4$	Additional Rounds $P(s=0) = (1-\mu'-\nu')$
$s = -1$	$\left(2p(p-\lambda)(1-p)+(1-p)^3\right)p$ $+(1-p)^2p(1-p+\lambda)$	Team 1 Loses $P(s=-1) + P(s=-2) = \nu'$
$s = -2$	$(1-p)p^2(1-p+\lambda)$	

achieved a leading position is the same regardless of which team has taken the lead. Similarly, the probability that the game reverts to a situation of even score is the same regardless of whether Team 1 or Team 2 takes the lead in Round 1. Once the game is back to even, the next round will "recreate" the same FMA as described above. In sum, FMA in a one-round shoot-out is the same as FMA in a two-round shoot-out because only the first round (and potentially subsequent rounds, if the shoot-out reverts to an even score) creates FMA, and subsequent rounds only preserve FMA.

In general, for an n-round model, the probability that Team 1 wins the shoot-out corresponds to the sum of the probability expressions, at the end of Round n, that entail a positive s—including sudden-death wins (with k indicating any possible value of s at the end of Round n with $-n \le k \le n$):

$$P(\text{Team 1 wins}) = \sum_{k=1}^{n} P(s=k) + P(s=0)\left(\frac{1-p+\lambda}{2-2p+\lambda}\right). \tag{5}$$

Following the logic as explained above, this simplifies, for any n, to:

$$P(\text{Team 1 wins}) = \frac{1-p+\lambda}{2-2p+\lambda}. \tag{6}$$

Inserting this expression into Equation 1 together with the fact that P(Team 2 wins) is the complement of P(Team 1 wins) allows us to solve for FMA:

$$\text{FMA} = \frac{\lambda}{2 - 2p + \lambda}. \tag{7}$$

This expression is strictly positive in the domain $0 < \lambda < p < 1$, and is equal to 0 if and only if $\lambda = 0$. It follows from this expression that there is FMA if and only if $\lambda > 0$. Moreover, the magnitude of FMA is independent of the number of rounds. In sum, if lagging-behind pressure exists, it necessarily implies FMA. In the absence of other potential influences, in our setting, there is FMA if and only if there is lagging-behind pressure. This result is consistent with the conclusion of APH's modeling approach (APH, Online Appendix B). A significant benefit of our approach is that it yields a simple formula for calculating FMA.[4] Our approach also provides a foundation for further analysis as described in the remainder of the article.

Differentiating Equation 7 with respect to λ:

$$\frac{\partial \text{FMA}}{\partial \lambda} = \frac{2 - 2p}{(2 - 2p + \lambda)^2}. \tag{8}$$

Equation 8 is strictly positive if $p < 1$, indicating that FMA increases as lagging-behind pressure increases.

Discussion:

As an initial illustration of the results generated by our model, we consider a baseline scoring probability of .75, which most closely approximates the scoring rate reported in prior studies of penalty kicks; representative rates include 74.3% for nonshoot-out penalty kicks reported by Dohmen (2008) and 73.1% for shoot-out penalty kicks reported by APH. Since the prior literature provides no specific guidance with regard to the magnitude of the lagging-behind effect and given the fact that the amount of pressure may vary depending on the stakes of the competition (Genakos & Pagliero, 2012; González-Díaz & Palacios-Huerta, 2016), we consider a variety of values coupled with this base scoring probability. Regardless of the value chosen, the presence of a lagging-behind effect does translate into an advantage for Team 1. Figure 3 shows that FMA increases from 0 when $\lambda = 0$ to more than .286 when $p = .75$ and $\lambda = .20$. The relationship between λ and P(Team 1 wins) is similarly concave and monotonically increasing.[5]

We next consider how this result relates to prior empirical studies of the relationship of lagging-behind pressure to FMA. As a reminder, our purpose here is not to investigate why or if lagging-behind pressure exists. Rather it is to show that if it is present, it creates FMA. KLS and APH disagreed about the existence of lagging-behind pressure because they found conflicting empirical evidence of whether the first team to shoot wins penalty shoot-outs significantly more often. Below, we offer a brief illustration of how a lagging-behind effect can exist as claimed by

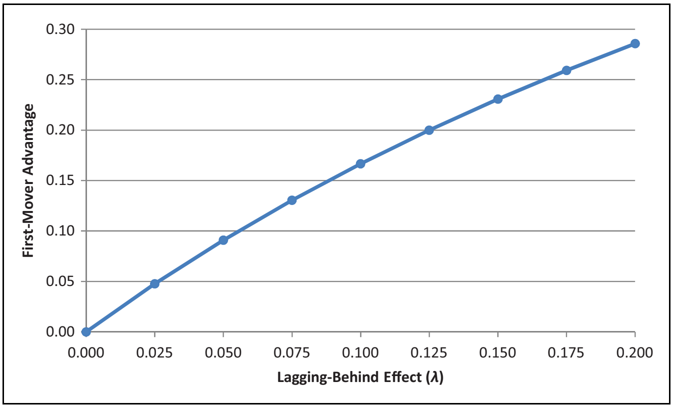

Figure 3. Relationship between lagging-behind effect and first-mover advantage. Scoring probability $p = .75$.

Table 2. First-Mover Advantage at Varying Scoring Probabilities With Lagging-Behind Effects.

		$p = .75$	$p = .35$	Difference
	.00	.0000	.0000	.0000
	.05	.0910	.0370	.0540
Lagging-Behind Effect (λ)	.10	.1666	.0714	.0952
	.15	.2308	.1034	.1274
	.20	.2858	.1334	.1524
	Average	.1548	.0690	

Note. The first column represents a baseline scoring probability $p = .75$. The second column represents a baseline scoring probability $p = .35$. There is no star shooter ($\alpha = 0$).

APH, even as it yields conflicting empirical evidence for the existence of a Team 1 FMA. The key factor to consider is the relatively small sample sizes in both the APH and KLS studies: The APH sample included 129 shoot-outs; the KLS sample 540 shoot-outs.

Consider the case of a relatively small but meaningful lagging-behind effect of .05. Thus, shooters convert at a rate of .75 unless they are shooting while their team trails in the score, in which case they convert at a rate of .70. As indicated by the results from our theoretical model summarized in Table 2, a lagging-behind effect of .05 translates into an FMA of 9.1 percentage points for Team 1 (a winning proportion of .545 for Team 1 vs. .455 for Team 2), which is a meaningful difference. Nevertheless, even if these numbers represent the true population values, the likelihood that a particular empirical study will find a significant Team 1 advantage will be a function of the sample size. Specifically, the power of a statistical test—that is, the probability that the test will reject the null hypothesis when the alternative is

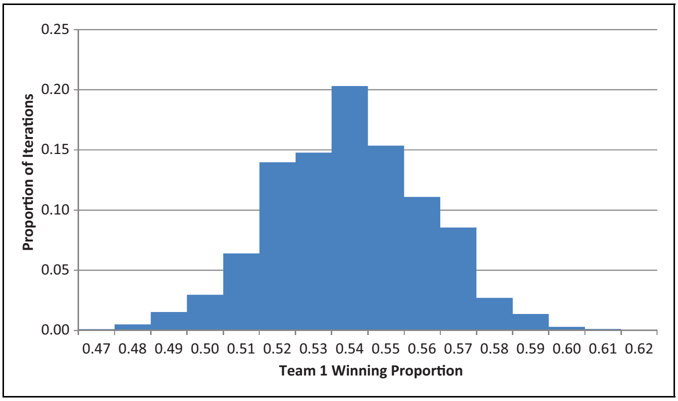

Figure 4. Distribution of Team 1 winning proportions. Scoring probability $p = .75$, lagging-behind effect $\lambda = .05$, sample size is 540, and number of iterations = 10,000.

true—will be weaker in smaller samples. In our case, we are interested in the power of a test to reject the null hypothesis of Team 1's winning probability equaling .50 when Team 1 actually has an FMA. In the specific application we are considering, the population value for the Team 1 winning proportion is .545. With a sample size of 540, as was included in the KLS study, the power of a test of such a proportion is 55.3%. This value is well below the standard accepted power of 80% (Cohen, 1988).

To further illuminate this issue, we devised a simulation program to run multiple instances of a shoot-out. The simulation tool was first constructed using the programming language *R* (version 2.15.0) and subsequently duplicated in Stata (version 12) for purposes of verification. With inputs of $p = .75$ and $\lambda = .05$, our simulations indicate that a sample size of 540 shoot-outs can produce considerable variation in the estimated magnitude of Team 1's advantage. Figure 4 shows the distribution of the estimated Team 1 winning proportions over 10,000 iterations of shoot-out sample sizes of 540. The mean of those values is nearly identical to the value of .545 calculated with our mathematical model; however, the values vary from a low of .468 to a high of .620 (standard deviation = .021). Consistent with the power calculation, in only 56.5% of these cases would the Team 1 winning proportion be found to be statistically greater than .50 at the conventional 5% level of significance.

In sum, given their limited sample size, it is unsurprising that KLS failed to find a significant Team 1 advantage even if the lagging-behind pressure suggested by the APH data actually exists. A straightforward and parsimonious explanation of the inconsistency between these two studies is that the data are simply insufficient to conclusively detect the effects of psychological pressure on FMA. Palacios-Huerta's (2014) study that used a much larger sample size and also found evidence of FMA similar to the earlier APH results is consistent with this explanation.

More generally, the above example is reflective of the complementary role of theoretical modeling to help advance knowledge of empirical phenomena. For example, KLS (p. 1591) concluded that improved understanding of "the relationship of psychological effects with the actual dynamics in a sequential tournament call[s] for much more empirical (experimental) evidence." Yet, we believe that the research described in this article demonstrates that theorizing via formal models can be a valuable complementary tool.

Proposition 2:

(i) For $0 < p < 1$ and $0 < \lambda < p$, $\frac{\partial \text{FMA}}{\partial p} > 0$ (i.e., FMA is an increasing function of the base scoring probability).

Proof:

Differentiating Equation 7 with respect to p:

$$\frac{\partial \text{FMA}}{\partial p} = \frac{2\lambda}{(2 - 2p + \lambda)^2}. \tag{9}$$

Equation 9 is positive for any λ greater than 0.

Discussion:

The responsiveness of FMA to the difficulty of the underlying task is important to consider because it would be relatively easy for the designated body, that is, the International Football Association Board, to make the task more or less difficult (e.g., a simple way to decrease scoring rates would be to force the kick to be taken from a greater distance). For additional illustration, Table 2 compares the Team 1 advantage under more difficult scoring conditions ($p = .35$, for different values of λ). As indicated by the FMA formula given in Equation 7, the results reported in Table 2 show that the relationship between the lagging-behind effect and FMA is a function of the base scoring probability. One might expect FMA to flip into a second-mover advantage once the value of p falls below .50, but the results indicate otherwise: Although FMA is then reduced, it remains positive. In fact, there will be FMA as long as $p > 0$ (if $\lambda > 0$). The intuition behind the fact that FMA exists regardless of the base scoring probability can be seen most easily by considering the first kick of each team. The probability that Team 1 lags behind on its first kick is zero. In contrast, Team 2 will always face an *ex ante* nonzero probability of lagging behind on its first kick as long as the baseline probability of scoring exceeds 0.

We note that this relationship between lagging-behind pressure and the probability of scoring, of course, depends on the particular model we have specified (i.e., shooters feel increased pressure when their team lags behind in the score). Alternative models could produce different results. For example, it may be that if the task is particularly difficult such that shooters are unlikely to score (say, e.g., $p = .25$), shooters may face significantly less pressure to perform. Interestingly, such cases may actually result in pressure on goalkeepers to perform.

The results around this issue are noteworthy for several reasons. First, they suggest that the importance of winning the coin toss that determines the shooting sequence could be reduced by making the task more difficult. Second, our findings have further practical significance because other sports also use shoot-outs, and their conversion rates may be much lower. For example, penalty shots in National Hockey League (NHL) shoot-outs are converted at a rate of approximately .33 per data published by the NHL. In a recent study of NHL shoot-outs, Kolev, Pina, and Todeschini (2015) found no evidence of FMA. Given the lower conversion rates of hockey penalty shots, this finding could be due to the reduced magnitude of FMA. However, it could also be associated with factors such as increased pressure on goaltenders, as briefly mentioned above. Overall, generalizing findings about FMA across settings where task difficulty varies significantly may require additional analysis using different modeling approaches.

We turn next to several extensions of the model that further demonstrate its utility in explicating the relationship between psychological pressure and performance outcomes. The below extensions continue to focus on the role of the shooter. After discussing these extensions, we note that future research focused on the role of the goalkeeper could provide interesting additional extensions.

Model Extensions

Optimal Ordering of Players in a Shoot-Out

Managers choose the order of the team's shooters, raising a natural question of whether and how these choices might matter, given that not all shooters are equally proficient at the task of converting penalties. Thus, our first model extension examines how varying the placement of players of differential ability in the shooting order can affect FMA. We begin by noting that some controversy exists concerning where more proficient shooters should be placed in the shooting order to maximize the likelihood of winning a shoot-out. This controversy received a great deal of attention in the semifinals of the 2012 Euro Championship between Portugal and Spain. When the contest came down to a penalty shoot-out, Portugal elected to hold its best penalty kicker, Cristiano Ronaldo, for the fifth shot. This choice was widely pilloried in the media because the shoot-out ended with a Spanish victory prior to the Portuguese attempting the fifth shot.[6] Conventional wisdom gives conflicting advice about where a "star shooter" should appear in the order—with advice of first, fourth, fifth, and either fourth or fifth depending on whether the team shoots first or second, being provided across different sources.[7] As we demonstrate below, however, the answer to this question depends critically on the existence and the nature of psychological pressure. Moreover, the placement of a star shooter may increase the magnitude of FMA associated with psychological pressure.

To investigate the role of heterogeneity in kicking proficiency and its relationship to optimal sequencing, we define an indicator variable, S_{ij}, denoting that a particular player (the star shooter, for which $S_{ij} = 1$; $S_{ij} = 0$ otherwise) has unique capability, and a parameter, α, reflecting the magnitude of the change in scoring probability associated with that unique shooter (where $0 < \alpha < 1 - p$). As before, λ represents the lagging-behind effect; the indicator variable T_{ij} equals 1 if the shooter's team is lagging behind and equals 0 otherwise (i.e., if the score is tied or the shooter's team is ahead). Then,

$$p_{ij} = p - T_{ij}\lambda + S_{ij}\alpha. \tag{10}$$

Proposition 3:

$P(\text{Team } j \text{ wins})_{xy}$ for $x = 1, \ldots, 5$; $y = 1, \ldots, 5$; and $j = 1, 2$ denotes the probability that Team j wins the shoot-out, given that Team 1 places its star shooter in the x^{th} position and Team 2 places its star shooter in the y^{th} position. Consistent with Equation 1, we define FMA as follows: $FMA_{xy} = P(\text{Team 1 wins})_{xy} - P(\text{Team 2 wins})_{xy}$.

For any $c = 1, \ldots, 5$; $d = 1, \ldots, 5$; $e = 1, \ldots, 5$; and $d < e$:

(i) $\text{FMA}_{dc} > \text{FMA}_{ec}$ (i.e., given the position Team 2 puts its star shooter, the earlier Team 1 puts its star shooter, the larger FMA).
(ii) $\text{FMA}_{cd} < \text{FMA}_{ce}$ (i.e., given the position Team 1 puts its star shooter, the earlier Team 2 puts its star shooter, the smaller FMA).

Proof:

Each team has the option of locating its star shooter from position 1 to 5, creating 25 alternative shot lineup combinations. To determine the relationship of FMA to placement of the star shooter, we calculated the difference in FMA across these alternatives (e.g., $\text{FMA}_{11} - \text{FMA}_{12}$, $\text{FMA}_{12} - \text{FMA}_{13}, \ldots$). We do not report the general results of the calculated differences (detailed formulas available upon request). The results indicate that in all cases, managers should place star shooters earlier in the order. FMA increases as Team 1 places a star shooter earlier, holding constant the placement of Team 2's star shooter. Conversely, FMA decreases as Team 2 places a star shooter earlier, holding constant the placement of Team 1's star shooter. We provide illustrations of the benefit of placing a star shooter earlier using specific parameter values below.

Table 3. First-Mover Advantage Under Alternate Orderings With Lagging-Behind Effects.

A. Conversion rate decreased by $\lambda = .05$, if team is behind when shot taken

		Team 2					
		1	2	3	4	5	Average
Team 1	1	.1002	.1018	.1038	.1060	.1085	.1041
	2	.0949	.0968	.0989	.1011	.1035	.0990
	3	.0912	.0934	.0956	.0978	.1002	.0956
	4	.0884	.0906	.0928	.0952	.0976	.0929
	5	.0858	.0880	.0904	.0928	.0952	.0904
	Average	.0921	.0941	.0963	.0986	.1010	

B. Conversion rate decreased by $\lambda = .20$, if team is behind when shot taken

		Team 2					
		1	2	3	4	5	Average
Team 1	1	.3204	.3286	.3364	.3434	.3498	.3357
	2	.2938	.3030	.3108	.3176	.3240	.3098
	3	.2786	.2892	.2972	.3040	.3104	.2959
	4	.2684	.2790	.2878	.2946	.3012	.2862
	5	.2606	.2710	.2798	.2874	.2936	.2785
	Average	.2844	.2942	.3024	.3094	.3158	

Note. Except for the star shooter, all shooters are equally capable. The star shooter has a conversion rate of .85; all other shooters have a conversion rate of .75 (i.e., $p = .75$ and $\alpha = .10$). Each cell represents the position in the shooting order of the team's star shooter; for example, in cell (2, 3), Team 1's star shooter shoots second and Team 2's star shooter shoots third.

Discussion:

The results shown in Table 3 illustrate how ordering matters if lagging-behind psychological pressure exists (i.e., if $\lambda > 0$): Teams should choose their best kicker to shoot first. The extent to which ordering matters depends on the magnitude of the lagging-behind effect, and at lower levels, the influence of optimal ordering on the magnitude of FMA is relatively minor. For example, Panel A of Table 3 indicates that FMA increases by only 1.36 percentage points (to an average FMA of .1041 in row 1 compared to an average FMA of .0904 in row 5), when Team 1 places its best kicker first rather than fifth under a lagging-behind effect of .05 ($p = .75$, $\lambda = .05$, $\alpha = .10$). Panel B of Table 3 presents the results of differing choices for the location of the star shooter under a larger lagging-behind effect of .20 (i.e., scoring rates when lagging behind decrease from .75 to .55 for average shooters and from .85 to .65 for star shooters). Again the best choice for each team is to place its star shooter first in the order. Moreover, the influence of optimal ordering on the magnitude of FMA is increasing in the magnitude of the lagging-behind effect. Team 1 now increases its winning percentage by 5.72 percentage points (to an average FMA of .3357 from .2785) by placing its best kicker first rather than fifth.

Panel B of Table 3 also reveals another, less obvious result: The benefit of optimal ordering is greater for Team 1 than for Team 2. In contrast to the increase of 5.72 percentage points in FMA from Team 1 engaging in optimal ordering, Team 2 decreases FMA by only 3.14 percentage points on average by placing its best shooter first rather than fifth. This result makes intuitive sense—the benefit of having a star shooter kick earlier in the order is the increased probability of imposing the lagging-behind effect on the opponent. This benefit is clearly available to Team 1 because it begins the shoot-out; yet, because Team 2 is more often trailing in the score, it is less able to impose the lagging-behind effect on its opponent. The implication is that Team 1 benefits twice from starting the shoot-out. First, the lagging-behind effect gives Team 1 an advantage because Team 2 is more often subject to this negative pressure effect, and second, the lagging-behind effect also indirectly contributes to a Team 1 advantage when there is heterogeneity among the kickers and teams order optimally, as the benefits of optimal ordering are higher for Team 1 than for Team 2 if there is a lagging-behind effect.

Finally, we note that if there is no situation-specific psychological pressure (i.e., if $\lambda = 0$), then the choice of where to place a star shooter (i.e., the ordering of the players) has no effect on the outcome. As an illustration, consider the case of only one team having a star shooter (e.g., Team j, where $p = .75$, $\lambda = 0$, and $\alpha = .10$). Although having the more proficient player leads to an overall advantage for Team j, the magnitude of this advantage is not related to the placement of the star shooter in the kicking order. That is, Team j's winning probability increases to .5279 (as calculated by adjusting the scoring formula to allow for different p_{ij}'s, associated with the star shooter, and applying the formula to the underlying tree diagram) regardless of where in the order its star shooter is placed, even when taking into account that kicks 4 and 5 may not actually be taken, due to early termination. This result, which holds in the absence of psychological pressure, makes sense because the shoot-out can then be represented as a series of independent Bernoulli trials, in which the ordering does not affect the distribution of outcomes. Further analysis confirms that ordering also does not matter in the case where both teams have an equally capable star shooter when there is no situation-specific pressure. Regardless of which kick is taken by the star shooter for each team, the overall win probability remains unaffected and is the same for both teams, that is, .5.

In sum, our results demonstrate that the decision of where to place more proficient shooters in the shoot-out depends on assumptions about the presence and nature of psychological pressure in the shoot-out. If there is no pressure, then ordering does not matter. In contrast, the best shooter should be placed early in the order if lagging-behind pressure exists, ceteris paribus. Of course, lagging-behind pressure is just one type of psychological pressure, and one could examine the implications of other types of pressure. Furthermore, it would be interesting to examine the effect of heterogeneity in individual player resilience on lagging-behind pressure, which could lead to different conclusions about the best position for more proficient shooters.

Other Forms of Psychological Pressure

In addition to the above-described lagging-behind effect (λ), prior literature suggests that there may be several other types of psychological pressure that affect performance in our setting. We, therefore, now consider how the advantage of being the first team to shoot is affected by these other forms of pressure. In considering other variants of pressure in penalty shoot-outs, Savage and Torgler (2012) suggested that stress could have either positive or negative effects, depending on the situation. First, in line with the Yerkes–Dodson theory of arousal and performance (Yerkes & Dodson, 1908), Savage and Torgler argued that players face especially high levels of negative stress under a specific lagging-behind scenario. This situation arises when the miss of a player's shot means that the shoot-out will end in a loss for the team (as opposed to cases where the shoot-out will continue for one or more additional rounds even if the player misses). In line with the psychology literature, we label this effect a "negative valence effect (NVE)."[8] Savage and Torgler (2012) also highlighted a special situation, in which a player can end the shoot-out in his or her team's favor by scoring. It is possible that such a situation could lead to negative stress; however, Savage and Torgler (p. 2427) argued that "player performance would be better in this situation (positive stress effect)" because the situation is more comfortable for the kicker. Consistent with this view, their analysis of 325 penalty shoot-outs indicated that the probability of scoring was 17% higher in situations where scoring would terminate the shoot-out in a win. We therefore decided to also investigate the potential effect of what would be described in the psychology literature as a "positive valence effect (PVE),"[9] represented by situations where shooters can win the shoot-out for their team with a successful shot.

We update our scoring formula from Equation 10 by further appending an indicator variable for situations subject to both the negative (B_{ij}, whereby $B_{ij} = 1$ if negative-valence situation; $B_{ij} = 0$ otherwise) and positive (F_{ij}, whereby $F_{ij} = 1$ if positive-valence situation; $F_{ij} = 0$ otherwise) valence effects and parameters for each (γ and ψ, respectively, with $0 \leq \gamma < p - \lambda$ and $0 \leq \psi < 1 - p - \alpha$). To isolate the operation of the NVE and the PVE, we consider the case of $\lambda = 0$ and $\alpha = 0$ (i.e., no general lagging-behind effect and no star shooters). Thus, the discussion below is based on the following scoring formulas:

$$p_{ij} = p - B_{ij}\gamma. \tag{11}$$

$$p_{ij} = p + F_{ij}\psi. \tag{12}$$

Proposition 4:

(i) $\frac{\partial \text{FMA}}{\partial \gamma} > 0$ (i.e., FMA is an increasing function of the NVE).

(ii) $\frac{\partial \text{FMA}}{\partial \psi} < 0$ (i.e., FMA is a decreasing function of the PVE).

Proof:

We follow the same general approach for calculating FMA as described earlier for the case of lagging-behind pressure. That is, we compute the probability of all possible cumulative score outcomes using a tree diagram encompassing all five rounds. The only difference here is that scoring probabilities now reflect the NVE and the PVE rather than the lagging-behind effect. As before, FMA is calculated as P(Team 1 wins) $-$ P(Team 2 wins). The resulting expressions are noticeably more complex than those for the case of the lagging-behind effect.

We first recalculate FMA based on the scoring formula that includes the NVE, that is, Equation 11. The resulting expression for FMA based on the full tree diagram is:

$$\begin{aligned}\mathrm{FMA} = \frac{1}{2-2p+\gamma}\big[\gamma(&10\gamma^2p^8 - 90\gamma p^9 + 80p^{10} - 50\gamma^2p^7 + 450\gamma p^8 - 400p^9 \\ &+ 106\gamma^2p^6 - 1,002\gamma p^7 + 966p^8 - 124\gamma^2p^5 + 1,308\gamma p^6 - 1,464p^7 \\ &+ 86\gamma^2p^4 - 1,104\gamma p^5 + 1,518p^6 - 34\gamma^2p^3 + 624\gamma p^4 - 1,110p^5 + 6\gamma^2p^2 \\ &- 236\gamma p^3 + 576p^4 + 58\gamma p^2 - 210p^3 - 8\gamma \mathrm{p} + 52p^2 - 8p + 1)\big].\end{aligned} \tag{13}$$

Differentiating this expression with respect to γ:

$$\begin{aligned}\frac{\partial \mathrm{FMA}}{\partial \gamma} = \frac{1}{(2-2p+\gamma)^2}[2\big(&10\gamma^3p^8 - 75\gamma^2p^9 + 180\gamma p^{10} - 80p^{11} - 50\gamma^3p^7 + 405\gamma^2p^8 \\ &-1,080\gamma p^9 + 480p^{10} + 106\gamma^3p^6 - 969\gamma^2p^7 + 2,904\gamma p^8 - 1,366p^9 \\ &-124\gamma^3p^5 + 1,344\gamma^2p^6 - 4,620\gamma p^7 + 2,430p^8 + 86\gamma^3p^4 - 1,182\gamma^2p^5 \\ &+4,824\gamma p^6 - 2,982p^7 - 34\gamma^3p^3 + 672\gamma^2p^4 - 3,456\gamma p^5 + 2,628p^6 \\ &+6\gamma^3p^2 - 238\gamma^2p^3 + 1,720\gamma p^4 - 1,686p^5 + 47\gamma^2p^2 - 588\gamma p^3 + 786p^4 \\ &-4\gamma^2p + 132\gamma p^2 - 262p^3 - 16\gamma \mathrm{p} + 60p^2 - 9p + 1)\big].\end{aligned} \tag{14}$$

This expression is positive for all values of p between 0 and 1 and values of γ greater than or equal to zero and less than p.[10]

(ii) Next, we recalculate FMA using the scoring formula of Equation 12, which includes the PVE. The resulting FMA expression is again derived from the full tree diagram:

Table 4. First-Mover Advantage Under Alternate Positive and Negative Valence Effects.

		Positive Valence Effect (ψ)					
		.00	.05	.10	.15	.20	Average
Negative Valence Effect (γ)	.00	.0000	−.0132	−.0242	−.0332	−.0406	−.0222
	.05	.0384	.0236	.0110	.0000	−.0092	.0128
	.10	.0692	.0536	.0398	.0276	.0172	.0415
	.15	.0940	.0782	.0638	.0510	.0396	.0653
	.20	.1144	.0984	.0840	.0708	.0590	.0853
	Average	.0632	.0481	.0349	.0232	.0132	

Note. Each cell represents a combination of each type of psychological pressure. Under the negative valence effect, scoring rates are decreased if a missed shot would result in a loss of the shoot-out. Under the positive valence effect, scoring rates are increased if a made shot would result in a win of the shoot-out. The base scoring rate $p = .75$; there is no lagging-behind effect ($\lambda = 0$) and no star shooter ($\alpha = 0$).

$$\begin{aligned}\mathrm{FMA} = -\frac{1}{2p+\psi}\big[&\psi(80p^{10} + 90p^9\psi + 10p^8\psi^2 - 400p^9 - 360p^8\psi - 30p^7\psi^2 \\ &+ 966p^8 + 642p^7\psi) + 36p^6\psi^2 - 1,464p^7 - 666p^6\psi - 22p^5\psi^2 + 1,518p^6 \\ &+ 438p^5\psi + 6p^4\psi^2 - 1,110p^5 - 186p^4\psi + 576p^4 + 50p^3\psi - 210p^3 \\ &- 8p^2\psi + 52p^2 - 8p + 1\big].\end{aligned} \tag{15}$$

Differentiating this expression with respect to ψ:

$$\begin{aligned}\frac{\partial \mathrm{FMA}}{\partial \psi} = -\frac{1}{(2p+\psi)^2}\big[&2p(80p^{10} + 180p^9\psi + 75p^8\psi^2 - 400p^9 + 10p^7\psi^3 \\ &- 720p^8\psi - 270p^7\psi^2 - 30p^6\psi^3 + 966p^8 + 1,284p^7\psi + 429p^6\psi^2 \\ &+ 36p^5\psi^3 - 1,464p^7 - 1,332p^6\psi - 399p^5\psi^2 - 22p^4\psi^3 + 1,518p^6 \\ &+ 876p^5\psi + 237p^4\psi^2 + 6p^3\psi^3 - 1,110p^5 - 372p^4\psi - 93p^3\psi^2 + 576p^4 \\ &+ 100p^3\psi + 25p^2\psi^2 - 210p^3 - 16p^2\psi - 4p\psi^2 + 52p^2 - 8p + 1)\big].\end{aligned} \tag{16}$$

This expression is negative for all values of p between 0 and 1 and values of ψ greater than or equal to 0 and less than $1 - p$.[11]

Discussion:

To provide an illustration of the above results, Table 4 summarizes FMA across a variety of combinations of the NVE and the PVE with a baseline scoring rate p of .75. Several outcomes are noteworthy. We first consider the effects individually (i.e., when the other effect is set equal to 0). Column 1 of Table 4 clearly shows that an NVE for both teams leads to a Team 1 advantage. For example, if $\gamma = .20$, then Team 1 has an FMA of 11.44%. This result is driven by Team 1's lower probability of being in positions where its shooter's kick can terminate the shoot-out in a loss; hence, Team 1 suffers less often than Team 2 from the lower performance associated with the NVE. For example, in a shoot-out where $p = .75$ and $\gamma = .20$, the probability that Team 1 will face a negative-valence situation at some point in a five-round shoot-out is .29 as compared with a probability of .48 for Team 2. This difference makes intuitive sense: In the third round and in sudden-death rounds, a negative-valence situation can only occur for Team 2. Conversely, there is no round in which Team 1 can face a negative-valence scenario while Team 2 cannot (in the fourth and fifth round, the NVE could occur for both teams). This Team 1 advantage is consistent also with the notion that the NVE is a special case of the more general lagging-behind effect, which also results in a Team 1 advantage. Yet, because the NVE applies in fewer cases, it has less of an impact than does a general lagging-behind effect of similar magnitude.

In contrast to this result, Table 4 also shows that the PVE leads instead to an advantage for Team 2. For example, the Team 2 advantage when $\psi = .20$ and $\gamma = 0$ is .0406. This result is again driven by the fact that Team 2 is more often in position to take decisive kicks—it is more likely to face situations where shots can terminate the shoot-out in a win. Team 2 has a .35 probability of experiencing at least one positive-valence situation in a five-round shoot-out whereas Team 1's such probability is .29 (when $p = .75$, $\psi = .20$, and $\gamma = 0$). We also note that a comparison of the FMA values indicates that the NVE has a stronger influence than the PVE. Effects of similar magnitude (e.g., .20) result in nearly 3 times the increase in advantage for Team 1 (from 0 to .1144 in column 1 of Table 4) under the NVE as compared to Team 2's increase in advantage (from 0 to .0406 in row 1 of Table 4) under the PVE.

We can also consider the relative strength of these two effects in cases where both effects may operate, that is, with a scoring probability of

$$p_{ij} = p - B_{ij}\gamma + F_{ij}\psi. \tag{17}$$

The remaining rows and columns of Table 4 show the magnitude of FMA across a variety of combinations of the two effects. The NVE's greater impact can also be seen here. When $\gamma = \psi$ (i.e., both the NVE and the PVE are present in similar magnitude), FMA is still positive (i.e., Team 1 has an advantage). For instance, FMA is .0590 when $p = .75$, $\gamma = .20$, and $\psi = .20$. This advantage makes intuitive sense in that, under such specifications, Team 2 is 1.67 times more likely than Team 1 to suffer a negative-valence situation (i.e., a probability of .46 vs. .27) but is only 1.22 times more likely to benefit from a positive-valence situation (i.e., a probability of .30 vs. .24).[12]

Finally, we note that unreported results indicate that the relative strength of the two effects depends on the base scoring rate. As long as the base scoring rate p exceeds .50

Table 5. First-Mover Advantage Under Alternate Shot-Sequencing Approaches.

		Shot-Sequencing Approach		
		1212	1221	PTM
Lagging-Behind Effect (λ)	.00	.0000	.0000	.0000
	.05	.0910	.0140	.0076
	.10	.1666	.0306	.0174
	.15	.2308	.0504	.0300
	.20	.2858	.0732	.0458
	Average	.1548	.0336	.0202

Note. The first column represents the standard sequence of 1-2-1-2-1-2-1-2-1-2 . . . , the second column represents an alternative sequence of 1-2-2-1-1-2-2-1-1-2 . . . , and the third column represents a Prouhet–Thue–Morse (PTM) sequence. The base scoring rate $p = .75$, and there is no star shooter ($\alpha = 0$).

(as in the examples discussed so far and shown in Table 4), the NVE outweighs an equivalent PVE, leading to an advantage for Team 1. However, if $p < .50$, then the PVE outweighs an equivalent NVE, and so Team 2 has an advantage (e.g., when $p =$ .35 with $\gamma = .20$ and $\psi = .20$, the Team 2 win proportion is .5149). When $p = .50$, the two effects are exactly balanced; the reason is that, when $p = .50$, for any given kick it is equally likely that the negative or the positive valence situation will apply and so neither team has an advantage. These insights show how our models foster the uncovering of relatively subtle relationships between various forms of psychological pressure, situational elements (such as baseline scoring probabilities), and performance outcomes. Our models could also be further adapted to analyze increasingly complicated situations, such as the effect of interactions between individual-level heterogeneity—for example, resilience to psychological pressure (e.g., Baumeister, 1984; Beilock & Carr, 2001; González-Díaz et al., 2012)—and situation-specific factors, such as the lagging-behind and valence effects.

Rule Variations

Another advantage of our modeling approach is that we are not constrained to examining only those outcomes that are consistent with the current set of rules. That is, we can further experiment with the model to consider the effects of alternative shoot-out structures. One interesting question, also taken up by Anbarci et al. (2015), concerns structural changes that might be made in order to reduce FMA associated with the existence of lagging-behind effects. FMA arises because the first team more often has the opportunity to impose the lagging-behind effect on the second team when Team 1 and Team 2 alternate shots (so that their order is 1-2-1-2-1-2-1-2 . . .). We therefore consider an alternative sequencing where the first team leads off and then each team alternates taking not one but rather two shots (i.e., an ordering of 1-2-2-1-1-2-2-1 . . .).[13] As illustrated in Table 5, this alternative shot-sequencing structure does not totally eliminate FMA; however, the magnitude of that advantage is markedly reduced. For example, for $\lambda = .20$, FMA of this sequencing structure is reduced to .0732, compared to .2858 in the standard sequencing structure. Still another sequence, recently proposed by Palacios-Huerta (2012, 2014) as the "Prouhet–Thue–Morse sequence," further mitigates (but does not entirely eliminate) FMA (for $\lambda = .20$, FMA is further reduced to .0458).[14,15] Another noteworthy difference under these alternative sequencing approaches is the relationship between FMA and the total number of rounds. Under the standard shoot-out rule, FMA associated with lagging-behind pressure is independent of the number of rounds. In contrast, the magnitude of FMA does vary depending on the number of rounds for each of these alternative sequencing rules. For example, under the 1-2-2-1 rule, FMA is gradually reduced as the number of rounds increases.

It is also worth noting that the relative benefit of these alternative shot-sequencing structures is increasing with the magnitude of the lagging-behind effect. In sum, replacing the current sequence with either of these proposed alternatives reduces the importance of winning the coin toss that determines shooting sequence—provided a lagging-behind effect exists.

As noted earlier, the extensions we have discussed here have focused on the role of the shooter. However, we believe a potentially worthwhile set of additional extensions to our work would be to consider the role of the goalkeeper more extensively. For example, what if goalkeepers also experience lagging-behind pressure? What are the implications of fielding a "star goalkeeper?" Would teams prefer to move first if they had such a star? Would this decision depend on the difficulty of the underlying task? These extensions might be particularly relevant in other contexts where outcomes are more dependent on the performance of the goalkeeper.[16]

Conclusion

The motivation for this research was to gain greater insight into the operation of FMA, more specifically the relationship between psychological pressure and FMA in competitive interactions. Our study demonstrates how additional theoretical modeling can provide an important complementary approach to empirical analysis in this area. This approach allows us to model psychological mechanisms directly in order to develop a more precise understanding of their operation. Moreover, we are able to consider multiple types of pressure—both individually and jointly. Our work demonstrates that even apparently simple competitive interactions, such as penalty shoot-outs in soccer, may involve a complex set of relationships. Clearer insight into these relationships is essential to increase understanding of the consequences of psychological pressure to performance outcomes.

Overall, our study yields several important conclusions. First, we demonstrated that lagging-behind pressure results in FMA, and that the magnitude of that advantage is an increasing function of both the amount of pressure and the difficulty of the underlying task. As part of this demonstration, we have also provided a plausible reconciliation of two prior empirical studies that shared a significant amount of data and investigated the same issue of lagging-behind pressure and FMA yet came to opposite conclusions. Second, we have offered several extensions of the model that demonstrate how additional theoretical exploration can provide rich insight into the relationship between psychological pressure and performance outcomes. These extensions probed the role of managerial choice of participation order, other sources of psychological pressure, and potential effects of choices to alter the structure of the rules defining the task. All of these extensions generated theoretical predictions that could be tested by means of empirical investigation. Given the prevalence of competitive situations in which psychological pressure might play a role in creating FMA, such as tournament settings and R&D races between companies, the relation between pressure and performance is an important area of inquiry—and one in which additional theorizing can yield valuable insights.

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Notes

1. We note that as a refinement of the predominant view in the literature that performance deficits create stress, Berger and Pope (2011) argued and provided empirical evidence indicating that performance may improve if the trailing performer is only "slightly behind" in performance. Although our model does not treat falling only slightly behind differently, it could be easily adapted to further investigate the consequences of such an effect.
2. Note that a number of paths (specifically, 352 of the 1,024 in our setting) have a zero probability because early termination of the shoot-out precludes those paths from

occurring. For example, one theoretically possible path is for Team 1 to take and score all five kicks, while Team 2 takes and misses all five kicks. Practically, this result would never occur because the shoot-out would terminate after the third round with the score 3–0 in favor of Team 1 (and with Team 2 having only two kicks left). To streamline presentation of our model, we omit the possibility of early shoot-out termination, as it does not affect any of the conclusions.

3. Two of the paths in Figure 2 include kicks by Team 2 in cases where the outcome is already determined (i.e., when Team 2 trails 2–0 or leads 0–1 prior to its second kick). Whether these kicks are actually taken affects the probability of specific final score outcomes, but it does not affect the probability of which team wins. As such, the results of the FMA analysis are unchanged in this analysis (and in the full tree diagram covering five rounds) irrespective of whether or not the shoot-out terminates prior to these kicks being taken.
4. Note that the formulas given here, although applicable for any n, correspond to the simple model developed in this section while assuming only lagging-behind pressure. The formulas become increasingly complicated when we introduce such additional factors as other forms of psychological pressure and heterogeneity in kicking proficiency—factors included in our model extensions.
5. We also investigated whether a lagging-behind effect is theoretically equivalent to a push-when-ahead effect (i.e., a higher probability of scoring when the shooter's team leads the competition). The two psychological effects are equivalent but only if two specific conditions hold. First, the "push" effect must apply to tied situations as well as situations of being ahead. Second, for a given scoring probability (p) and lagging-behind effect (λ), a "push-when-ahead-or-tied effect" with baseline scoring probability $p' = p - \lambda$ and push effect of size $\lambda' = \lambda$ generates an equivalent FMA. For example, a lagging-behind case of $p = .75$ and $\lambda = .05$ produces the same FMA as a push-when-ahead-or-tied case with $p' = .70$ and $\lambda' = .05$. In cases where the parameter values are equal ($p = p'$ and $\lambda = \lambda'$), a push-when-ahead-or-tied effect produces the largest FMA, followed by the lagging-behind effect. A push-when-ahead effect produces the smallest FMA of the three alternatives.
6. When Portugal's Euro 2016 quarterfinals against Poland ended in a shoot-out, Cristiano Ronaldo this time took the first shot for Portugal.
7. Arguing that the team's best kicker should shoot first is Kastrinakis, "Ron Call Makes Fab Spot King," The *Sun Online*, June 28, 2012. The best kicker should shoot fourth according to Bialik, "Why Ronaldo Watched and Waited," *Wall Street Journal* blogs, June 28, 2012. Advocating the fifth position are McGarry and Franks (2000), and arguing for the fourth (or fifth) position is MacAree, "Cristiano Ronaldo, Paulo Bento, and the Optimal Penalty Shootout Order," *SB Nation*, June 28, 2012.
8. In the formulas, corresponding to our particular setting, the negative valence effect (NVE) will be symbolized by B, for "Baggio," after one of the most famous cases of a player's miss resulting in his team's loss, which occurred in the 1994 World Cup final. The fact that many remember Roberto Baggio's role in the shoot-out—and not the roles of Franco Baresi and Daniele Massaro who also missed penalties for Italy earlier in the

shoot-out—lends credence to the idea that pressure might be higher for shooters whose result is more proximal to the outcome.

9. With reference to the previously discussed Euro 2012 semifinal shoot-out between Portugal and Spain, which was won with a final successful shot by Cesc Fàbregas of Spain, in the formulas corresponding to our particular setting, we indicate the positive valence effect (PVE) by F for "Fàbregas."
10. We determined the minimum value of this expression with the assistance of Maple software (Maple version 17.02) to confirm that it was positive (minimum value of expression $=$ .0227).
11. We determined the maximum value of this expression with the assistance of Maple software to confirm that it was negative (maximum value of expression $= -.0227$).
12. Note that in this example, both effects (NVE and PVE) could occur in the same shoot-out. Hence, the percentages here differ somewhat from those that apply when only one of these two effects is present.
13. This ordering is how the tiebreaker proceeds in tennis: The first player serves once and then the two players alternate serving twice each.
14. This sequence is obtained by starting with 0 and successively appending the Boolean complement of the sequence obtained thus far. The first 0 is followed by 1. The 01 is then followed by 10. The 0110 is then followed by 1001 and so on. For teams labeled 1 and 2, this would lead to a sequence of 1-2-2-1-2-1-1-2-2-1 for the first 10 kicks.
15. Anbarci, Sun, and Ünver (2015) consider further alternatives, including a variant of the Prouhet–Thue–Morse sequence whereby a lagging team always goes first in the next round; in case of a tie, the Prouhet–Thue–Morse sequence is applied. These authors also suggest an exogenous mechanism in which a coin flip decides the shooting order in each round.
16. We thank an anonymous reviewer for the suggestions to more deeply consider the role of the goalkeeper and how that might vary in importance depending on context.

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