Competitive Rational Expectations Equilibria Without Apology *

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Abstract

Consider a standard financial market model with asymmetric information with a finite number N of risk-averse informed traders. In the central scenario where N grows at the same rate as noise trading and there is a well-defined limit economy, competitive rational expectations equilibria provide a good approximation to strategic equilibria as long as N is not too small: equilibrium prices in each situation converge to each other at a rate of $1/N$ as the market becomes large. The approximation is particularly good when the noise trading volume per informed trader is large in relation to his risk-bearing capacity. This is not the case if informed traders are close to risk neutral. Both equilibria converge to the competitive equilibrium of an idealized limit continuum economy as the market becomes large at a slower rate of $1/\sqrt{N}$ and, therefore, the limit equilibrium need not be a good approximation of the strategic equilibrium in moderately large markets. The results extend to endogenous information acquisition and the connections with the Grossman-Stiglitz paradox are highlighted.

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1 Introduction

The aim of this paper is to find out conditions under which we can safely use competitive rational expectations equilibria (REE) as an approximation of the "true" strategic equilibria in a standard financial market context. We would like to bound the error when approximating the equilibrium with a finite number of traders with a price-taking equilibrium, as is often done in applied work.

We find that the competitive approximation basically works, even in a moderately sized market, when competitive traders have incentives to be restrained in their trading in a context where there is no residual market power in a large market.

The concept of competitive REE has been questioned from different quarters. Hellwig (1980) pointed out the "schizophrenia" problem of price-taking behavior in a competitive REE with a finite number of traders. When submitting their demands traders would take into account the information content of the price but not the price impact of their trade. The problem disappears in a large market. Indeed, as the market becomes large, under certain conditions, the strategic equilibria of finite economies converge to the competitive REE of an idealized limit continuum economy (as described, e.g., by Admati (1985) or Vives (1995)).

Kyle (1989) modeled the strategic equilibrium directly where traders are aware of the price impact of their trades and compete in demand schedules (in a REE with imperfect competition) and claimed that the properties of the imperfect competition model were reasonable. He also characterized a mutual convergence condition under which in a large market strategic and competitive equilibria converge to each other. This requires the number of informed speculators to grow unboundedly and the noise trading volume adjusted per capita-unit of informed risk-bearing capacity to grow also unboundedly in relation to the total private precision. This condition ensures that there is no residual market power in the limit.

In this paper we show, in the framework of Kyle's (1989) model, and in the scenario of Kyle's mutual convergence condition where the number of informed traders and the amount of noise trading grow proportionately and there is a well-defined limit economy, that the competitive REE of a large but finite market with risk-averse traders provides a good approximation of the "true" strategic equilibrium. This is particularly the case when the noise trading volume per capita informed trader is large in relation to his risk-bearing capacity. In this context a competitive trader has an incentive to restrain his trading intensity. Then, if the market has a minimum size, we can take the shortcut of assuming competitive behavior of the finite number of agents present as a good enough approximation. That is, we can use competitive REE without apology. However, the shortcut does not work, for example, if informed traders are close to risk neutral. In any case the strategic equilibrium is not well approximated by the competitive REE of the idealized limit continuum economy.

We derive the results first in replica markets where the number of informed traders and the volume of noise trading grow together, and then in a free entry context where the size of the market is parameterized by the volume of noise trading.

We consider thus first the imperfect competition model of Kyle (1989) with N risk-averse informed speculators and focus attention on the case with a competitive risk-neutral market making sector (which is equivalent to letting in the market an infinite number of risk averse uninformed speculators). In this context we study the behavior of strategic and competitive equilibria in replica markets as the market gets large by increasing N and the volume of noise trading σ_z. We obtain different results depending on the race between N and σ_z, that is on whether N grows more or less than proportionately with σ_z. The central scenario is where N grows in proportion to σ_z and there is a well-defined limit economy. The result is that the equilibrium prices in the strategic and competitive cases, as the market becomes large, converge to each other at a rate of $1/N$, while both converge to the competitive equilibrium of the limit economy at a rate of $1/\sqrt{N}$. The same rates of convergence apply for demands, profits and relative utilities of traders. The consequence is that in moderately sized and large markets the assumption of competitive behavior with risk-averse traders turns out to be a good approximation to the "true" strategic (Bayesian) equilibrium. However, thinking in terms of the idealized continuum limit economy will not provide a good approximation for the equilibrium of the finite market. The point is that market power is dissipated quickly, at a rate of $1/N$, while the distance between a finite and the limit economy depends on the rate at which the average error term in the signals of traders vanishes, and this is $1/\sqrt{N}$. We also look at a more refined measure of convergence speed for a given rate of convergence: the asymptotic variance of the price difference in the different regimes. We find that the asymptotic variance of the price difference between the strategic and the competitive regime in a finite economy is small, and the approximation of the strategic equilibrium by the competitive equilibrium is good, when the prior volatility of the asset is low, noise trading is large in relation to the risk bearing capacity of the informed traders, or the signals are very noisy.

We confirm, therefore, the idea that the competitive approximation works, even in a moderately sized market, basically when competitive traders have incentives to be restrained in their trading. As traders become less and less risk averse, the asymptotic variance of the price difference between the strategic and the competitive regime in a finite economy increases unboundedly, reflecting the fact that when traders are close to risk neutral, competitive prices are close to being fully revealing while prices at the strategic equilibrium are far from being so. This points to the fact that the competitive model can be appropriate, as an approximation to the true strategic model, when traders are risk averse but not when they are close to risk neutral.

We also test the boundary of our central result by checking situations where the number of informed traders increases nonproportionally to the size of the market. If the number of informed traders increases faster than the size of the market, then a fully revealing equilibrium is obtained in the limit. If the number of informed traders increases more slowly than the size of the market, then an informationally trivial equilibrium is obtained in the limit. In all cases in which Kyle's mutual convergence condition holds market power is again dissipated faster than the rate at which the finite and the limit economy converge to each other as the market becomes large.

Kyle (1989) also considers a "monopolistic competition case" where residual market power remains in a large market.[1] Then the competitive market need not be a good approximation of strategic trading even in a large market. This occurs when the number of informed traders grows without bound but total noise trading and the total precision of information for the informed are bounded. In this context traders retain some market power even in a large market and convergence to the monopolistically competitive limit as the number of informed traders grows occurs at most at a rate of $1/N$. Therefore, our results on the approximation of strategic equilibria by competitive equilibria have to be qualified as holding in those situations where there is no residual market power in a large market.

Replica markets of the form considered can be rationalized in a model with free entry of speculators parameterizing the size of the market by the volume of noise trading σ_z. As in Verrecchia (1982) and Kyle (1989) speculators can become informed, acquiring a private signal of known precision, by paying a fixed cost, which in our case may depend on the size of the market. The entry of uninformed speculators is free. We show that if the entry cost in a large market is positive but not too large, the equilibrium number of informed speculators N is of the order of σ_z, that is, N grows in proportion to σ_z, in both the strategic and competitive cases. We can then identify increases in the size of the market σ_z with increases in N. This is our central scenario in the replica markets. If the entry cost in a large market is large then the number of informed traders increases, if at all, more slowly than the size of the market, and an informationally trivial equilibrium is obtained in the limit. If the entry cost in a (limit) large market is zero then the number of informed traders increases faster than the size of the market, and a fully revealing equilibrium is obtained in the limit. This result provides a weak resolution of the well-known Grossman and Stiglitz (1980) paradox on the impossibility of (perfectly) informationally efficient markets. Indeed, in the context considered in a large market (i.e. with σ_z large) prices will be close to fully revealing but still there will be incentives to acquire information as long as in the limit information acquisition is free.[2]

When traders are close to (or) risk neutral and information is costly to acquire, no competitive traders would enter into the market and become informed because they would lose money (this is again just a variant of the Grossman-Stiglitz paradox). Indeed, the naive idea that with risk-neutral traders Bertrand competition would push the strategic and competi-

[1] García and Sangiorgi (2011) provide a rationale for this case in an informaiton sales model.

[2] See Milgrom (1981) for a resolution of the Grossman-Stiglitz paradox in the context of a large uniform price auction.

tive regimes closer together does not hold. This is consistent with the results in Biais et al. (2000) according to which adverse selection softens supply schedule competition among risk-neutral market makers in a common value environment. Under risk neutrality the number of strategic informed traders grows less than proportionately than the size of the market σ_z. However, despite this fact prices become fully revealing when $\sigma_z \to \infty$ because then the aggregate response to private information grows faster than σ_z. Strategic trading provides a resolution of the Grossman-Stigliz paradox in this case even if information is costly to acquire in a large (limit) market. Indeed, then prices become fully revealing but the incentives to acquire information are preserved.

The result that market power vanishes quickly at a rate of $1/N$ as the market grows is consistent with the result obtained by Vives (2002) in the context of a Cournot model (which in the financial market would correspond to traders using market orders instead of demand schedules as in Kyle's (1989) model), Cripps and Swinkels (2006) for double auctions in a generalized private value setting, and Vives (2011) with competition in supply schedules with private information.[3]

The rest of the paper is organized as follows. In the next section we present the structure of the model and the equilibria that we are going to consider with a given number of informed speculators. Sections 3 considers replica markets and presents the results on the speed of convergence to price-taking equilibria as the number of informed traders and the amount of noise trading grow, that is, the race between N and σ_z. Section 4 analyzes the monopolistic competition limit. Section 5 endogenizes the number of informed speculators in a free entry model and provides a characterization of equilibria. It considers also the case of risk neutral informed traders and the relationship with the Grossman-Stiglitz paradox. Concluding remarks close the paper. The appendix contains a characterization of equilibrium and proofs of the main results. An online appendix presents complementary results and the rest of proofs.

2 A market with N informed speculators

Consider Kyle's (1989) model (and to facilitate comparison we follow his notation as closely as possible). A single risky asset with random liquidation value $\tilde{v}$ is traded among noise traders, N informed speculators, indexed $n = 1, \ldots, N$, and market makers.[4] The return to trade one unit at a market clearing price $\tilde{p}$ is thus $\tilde{v} - \tilde{p}$. Noise traders trade in the aggregate the random quantity $\tilde{z}$. Each informed speculator $n = 1, \ldots, N$ receives a private signal $\tilde{\imath}_n = \tilde{v} + \tilde{e}_n$, where the random variables $\tilde{v}, \tilde{z}, \tilde{e}_1, \ldots, \tilde{e}_N$ are assumed to be normally and independently distributed with zero means and variances given by $\mathrm{var}\,[\tilde{v}] = \tau_v^{-1}$, $\mathrm{var}\,[\tilde{z}] = \sigma_z^2$,

[3] See also Rustichini et al. (1994) and Hong and Shum (2004) for related work. Reny and Perry (2006) provide a strategic foundation for competitive REE in a double auction context. Yosha (1997) deals with financial intermediation in a large Cournot–Walras economy with i.i.d. shocks and computes the rates of convergence as the economy becomes large for various market parameters.

[4] A tilde distinguishes a random variable from its realization.

$\text{var}[\tilde{e}_n] = \tau_e^{-1}$.[5]

Speculators compete in demand schedules and have constant absolute risk-aversion utility functions with coefficient $\rho > 0$ and a (normalized) zero initial endowment of the risky asset. Speculator n chooses a demand schedule $X_n(\cdot, i_n)$ which depends on his signal i_n and, given the market clearing price p, derives utility

$$U_n(\pi_n) = -\exp(-\rho\pi_n) \quad \text{where } \pi_n = (v-p)x_n \text{ and } x_n = X_n(p, i_n).$$

Market makers are uninformed and make their trade based only on public information transmitted through the price. We assume that there is a competitive risk-neutral market making sector that induces semi-strong efficient pricing:

$$E[\tilde{v} \mid \tilde{p}] = \tilde{p}.$$

This may arise because uninformed traders are risk neutral or because there is costless free entry of uninformed speculators (case $M = \infty$ in Kyle (1989)).

Strategic and competitive equilibria Two types of equilibria are considered. To emphasize the dependence of the market-clearing price on the strategies of speculators, let us write

$$\tilde{p} = \tilde{p}(X), \;\; \tilde{x}_n = \tilde{x}_n(X),$$

where X is a vector of strategies defined by $X = \langle X_1, .., X_N \rangle$ .The first equilibrium is Kyle's REE with imperfect competition. This is simply a Bayesian equilibrium in demand schedules of the game among the N informed traders.

Definition. A *rational expectation equilibrium with imperfect competition* or simply a *strategic equilibrium* is defined as a vector of strategies $X = \langle X_1, .., X_N \rangle$ such that for all $n = 1, .., N$ and for any alternative vector of strategies X' differing from X only in the n^{th} component X_n, the strategy X yields a utility level no less than X' :

$$E\{U_n[(\tilde{v} - \tilde{p}(X))\tilde{x}_n(X)]\} \geq E\left\{U_n\left[(\tilde{v} - \tilde{p}(X'))\tilde{x}_n(X')\right]\right\}.$$

The second equilibrium is also a Bayesian equilibrium, but now each informed trader, when considering what demand schedule to use, does not take into account the impact of his choice on the market price.

Definition. A *competitive rational expectation equilibrium* or simply a *price-taking equilibrium* is defined as a vector of strategies $X = \langle X_1, .., X_N \rangle$ such that for all $n = 1, .., N$ and

[5] The assumption $E[\tilde{v}] = E[\tilde{e}_n] = 0$ is made without loss of generality. The assumption $E[\tilde{z}] = 0$ does not affect the results in this paper.

for any alternative vector of strategies X' differing from X only in the n^{th} component X_n, the strategy X yields a utility level no less than X' :

$$E\left\{U_n\left[(\tilde{v}-\tilde{p}(X))\tilde{x}_n(X)\right]\right\} \geq E\left\{U_n\left[(\tilde{v}-\tilde{p}(X))\tilde{x}_n(X')\right]\right\}.$$

We concentrate attention on symmetric linear equilibria, that is, an equilibrium in which the strategies for each trader X_n, $n = 1, \ldots, N$, are identical affine functions. Thus, there exist constants β, γ, μ such that a strategy X_n can be written

$$X_n(p, i_n) = \mu + \beta i_n - \gamma p.$$

We can solve the market clearing condition for the equilibrium price $\tilde{p}$ and obtain the result that $\tilde{p}$ is informationally equivalent to $(\sum_{n=1}^{N} X_n(\tilde{p}, \tilde{\imath}_n) + \tilde{z})$ and therefore to $(\beta \sum_{n=1}^{N} \tilde{\imath}_n + \tilde{z})$. Thus we can express the price in terms of the parameter β:

$$\tilde{p} = E\left[\tilde{v} \mid \tilde{p}\right] = E\left[\tilde{v} \,\middle|\, \beta \sum_{n=1}^{N} \tilde{\imath}_n + \tilde{z}\right] = \lambda\left(\beta \tilde{v} N + \beta \sum_{n=1}^{N} \tilde{e}_n + \tilde{z}\right).$$

where $\lambda \equiv \frac{\partial p}{\partial z} = \frac{\beta \tau_v^{-1} N}{\beta^2 N\left(\tau_v^{-1} N + \tau_e^{-1}\right) + \sigma_z^2}$. The demand of an informed trader at a strategic equilibrium is given by

$$X_n(\tilde{p}, \tilde{\imath}_n) = \frac{E[\tilde{v} \mid \tilde{p}, \tilde{\imath}_n] - \tilde{p}}{\rho var[\tilde{v} \mid \tilde{p}, \tilde{\imath}_n] + \lambda_{\mathrm{I}}},$$

where $\lambda_{\mathrm{I}} = \left(\lambda^{-1} - \gamma\right)^{-1}$is the slope of inverse supply facing the individual informed trader. In the competitive case $\lambda_{\mathrm{I}} = 0$ since a trader has no market power.

Theorem 5.1 in Kyle (1989) implies that for $\sigma_z^2 > 0$ and $\tau_e > 0$ there exists a unique symmetric linear REE with imperfect competition for any N. That is, there is a symmetric linear equilibrium and this equilibrium is unique in the class of symmetric linear equilibria. Theorem 6.1 in Kyle (1989) provides a corresponding result on the existence and uniqueness of a symmetric linear competitive REE.

We use a superscript c to denote the values associated with a competitive equilibrium; values associated with a strategic equilibrium do not have this superscript. Subscript N (respectively, subscript ∞) corresponds to the values of the market with N (respectively, with an infinite number of) informed traders.

We are interested in how far the competitive equilibrium is from the strategic equilibrium in large markets. We will start our analysis by looking at sequences of replica markets where the number of informed traders N and σ_z increase exogenously. In Section 5 we will endogenize the number of informed traders in both the competitive and the strategic regime and let them vary as the market becomes large.

We say that a market with equilibrium price $\tilde{p}$ is (i) *value revealing* if $\tilde{p} = \tilde{v}$ (almost surely); (ii) *informationally trivial* if $\mathrm{var}^{-1}[\tilde{v} \mid \tilde{p}] = \mathrm{var}^{-1}[\tilde{v}] = \tau_v$. It is easy to see that in case (ii), necessarily, $\tilde{p} = E[\tilde{v}] = 0$ (almost surely).[6]

Remark 1. When $\tau_e = 0$, and there is no asymmetric information, prices are informationally trivial in all equilibria. We have that $\tilde{p}_N^c = \tilde{p}_N = \tilde{p}$ where $\tilde{p} = E[\tilde{v} \mid \tilde{p}] = E[\tilde{v}] = 0$ since $\tilde{p}$ is not informative.

Notation for rates of convergence We use the following notation to make comparisons of the rates of convergence. For two functions $f, g : \mathbf{Z}_+ \to \mathbf{R}$ we have:

(i) $f \sim O(g)$ means that there exist an integer N_0 and a positive constant k such that $|f(N)| \leq k|g(N)|$ for any $N \geq N_0$; that is, $|f|$ grows "at a rate not larger" than $|g|$ as $N \to \infty$;

(ii) $f \sim o(g)$ means that $\lim_{N\to\infty}(f(N)/g(N)) = 0$; that is, $|f|$ grows "at a smaller rate" than $|g|$ as $N \to \infty$;

(iii) $f \propto g$ means that $f \sim O(g)$ and $g \sim O(f)$; that is, $|f|$ grows "at the same rate" as $|g|$ when $N \to \infty$.

Convergence concepts for random variables To compare rates of convergence of random variables we use the *square loss function*. We say that two random variables x, y converge to each other at some rate if $\sqrt{E[(x-y)^2]}$ converges to zero at this rate. Note that

$$E\left[(x-y)^2\right] = (E[x] - E[y])^2 + \mathrm{var}[x-y],$$

and that if $E[x] = E[y]$, then $E\left[(x-y)^2\right] = \mathrm{var}[x-y]$.

A more refined measure of convergence speed for a given convergence rate is provided by the asymptotic standard deviation. Suppose that $\sqrt{E[(x_N - y_N)^2]} = \sqrt{\mathrm{var}[x_N - y_N]}$ converges to zero at a rate of $1/N^\alpha$ for some $\alpha > 0$ (that is, $\sqrt{\mathrm{var}[x_N - y_N]} \propto 1/N^\alpha$), then the asymptotic standard deviation of convergence is given by the constant $\lim_{N\to\infty} N^\alpha \sqrt{\mathrm{var}[x_N - y_N]}$ provided that the limit exists. A higher asymptotic standard deviation means that the speed of convergence is slower.

3 Replica markets: the race between N and σ_z

Let us consider the following sequence of markets indexed by number of the informed agents N, where N and the standard deviation of the noise trade σ_z, a natural measure of the size

[6] Indeed, from $E[\tilde{v} \mid \tilde{p}] = \tilde{p}$ we have that $E[\tilde{p}] = E[E[\tilde{v} \mid \tilde{p}]] = E[\tilde{v}] = 0$. Since $\mathrm{var}[\tilde{v}] = E[\mathrm{var}[\tilde{v} \mid \tilde{p}]] + \mathrm{var}[E[\tilde{v} \mid \tilde{p}]]$, we have that $E[\mathrm{var}[\tilde{v} \mid \tilde{p}]] = E[\mathrm{var}[\tilde{v}]] = \mathrm{var}[\tilde{v}]$ and $\mathrm{var}[E[\tilde{v} \mid \tilde{p}]] = \mathrm{var}[\tilde{p}]$ and therefore it should be that $\mathrm{var}[\tilde{p}] = 0$.

of the market, grow together. In all of the markets $\operatorname{var}[\tilde{v}] = \tau_v^{-1}$ is a fixed constant. and $\operatorname{var}[\tilde{e}_n] = \tau_e^{-1}$ are fixed constants.

Theorem 9.2 in Kyle (1989) provides the conditions under which the strategic and competitive equilibrium tend to each other. In short, we need that prices asymptotically reveal information in the competitive model and that both $\frac{\rho^2\sigma_z^2}{N\tau_e} \to \infty$ and $N \to \infty$. The condition states that the number of informed speculators N tends to infinity and the noise trading volume adjusted per capita-unit of informed risk-bearing capacity $\rho^2\sigma_z^2$ grows also unboundedly in relation to the total private precision $N\tau_e$. This makes sure that there is no residual market power in a large market.

In Sections 3.1-3.3 $\operatorname{var}[\tilde{e}_n] = \tau_e^{-1}$ is a fixed constant. In our central scenario (Section 3.1) the number of informed speculators N is proportional to the standard deviation of the noise trade σ_z. Let $\tilde{z}(N) = N\tilde{z}_0$ and $\operatorname{var}[\tilde{z}_0] = \sigma_{z_0}^2$, where $\sigma_{z_0}^2$ is a constant. We term this case the central scenario because, as we will see in Section 5, this is the case that has as its limit the usual continuum model (e.g. Hellwig (1980)), and Kyle's mutual convergence condition holds. In the other scenarios (Section 3.2) the number of informed speculators does not change proportionally to noise trading and Kyle's mutual convergence condition need not hold.

The race between N and σ_z can also be understood in an economy where noise trading $\tilde{z}$ is the sum of traders' endowments of the asset. That is, trader n would have an endowment $\tilde{z}_n = \tilde{z}_0 + \tilde{\eta}_n$ where $\tilde{z}, \tilde{\eta}_1, \ldots, \tilde{\eta}_N$ are normally and independently distributed with zero means and variances given by $\operatorname{var}[\tilde{z}_0] = \sigma_{z_0}^2$ and $\operatorname{var}[\tilde{\eta}_n] = \sigma_\eta^2$. We have then that $\tilde{z} = \sum_n \tilde{z}_n = N\tilde{z}_0 + \sum_n \tilde{\eta}_n$ and therefore $\operatorname{var}[\tilde{z}] = N^2\sigma_{z_0}^2 + N\sigma_\eta^2$. If $\sigma_{z_0}^2 > 0$ then the endowments are correlated (as in Ganguli and Yang (2009) and Manzano and Vives (2011)) and we have that σ_z is proportional to N, our central scenario. If $\sigma_{z_0}^2 = 0$ then the endowments are independent (as in Diamond and Verrechia (1981)) and we have that σ_z is proportional to $\sqrt{N}$. This scenario is interesting because Kyle's mutual convergence condition does not hold. It must be noted however that with endowment shocks the equilibrium does no longer correspond to the Kyle equilibrium, not even in the competitive case. Indeed in the competitive models of Ganguli and Yang (2009) and Manzano and Vives (2011) with correlated endowments there are multiple linear equilibria (and for some parameter configurations a linear equilibrium does not exist). The reason is that the endowment shock of a trader provides private information about the aggregate endowment shock which affects the price.

In Section 4 we will consider a scenario where Kyle's mutual convergence condition is not fulfilled and the strategic and competitive equilibrium do not tend to each other.

3.1 Convergence when N grows proportionally with σ_z

Let us consider the following sequence of markets indexed by N. At the Nth market there are N informed agents. In all of the markets $\operatorname{var}[\tilde{v}] = \tau_v^{-1}$ and $\operatorname{var}[\tilde{e}_n] = \tau_e^{-1}$ are fixed constants.

Let $\tilde{z}(N) = N\tilde{z}_0$ and $\operatorname{var}[\tilde{z}_0] = \sigma_{z_0}^2$, where $\sigma_{z_0}^2$ is a constant. That is, the standard deviation of the noise trade σ_z grows at a rate of N.

As N grows we know that the strategic and competitive equilibria tend to each other (from Theorem 9.2 in Kyle (1989)) and that they both tend to the competitive equilibrium of the limit continuum economy (as in the static model in Vives (1995)). Indeed, we have that both $\tilde{p}_N$ and $\tilde{p}_N^{\mathrm{c}}$ tend to $\tilde{p}_\infty$ where

$$\tilde{p}_\infty = \frac{\beta_\infty \tau_v^{-1}}{\beta_\infty^2 \tau_v^{-1} + \sigma_{z_0}^2}\left(\beta_\infty \tilde{v} + \tilde{z}_0\right) \text{ with } \beta_\infty = \frac{\tau_e}{\rho}.$$

We now characterize the rate at which strategic and competitive equilibria tend to each other and the rate at which they tend to the competitive equilibrium of the limit economy. We characterize first prices, and then demand and welfare magnitudes.

3.1.1 Prices

In order to compare convergence rates for prices consider the following decomposition:

$$\tilde{p}_N - \tilde{p}_\infty = (\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}) + (\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty).$$

The first term of the decomposition captures the difference between equilibrium prices for the price-taking $\tilde{p}_N^{\mathrm{c}}$ and strategic equilibria $\tilde{p}_N$ in the same finite market. The second term captures the change in the competitive price from the finite to the limit market. We have that

$$E\left[(\tilde{p}_N - \tilde{p}_\infty)^2\right] = E\left[(\tilde{p}_N - \tilde{p}_N^{\mathrm{c}})^2\right] + E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right] + 2\operatorname{Cov}\left[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}, \tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty\right]$$

will be of the order of the higher order term. Using Hölder's inequality (see, e.g., Royden (1968, p. 113)) we obtain that

$$\operatorname{Cov}\left[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}, \tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty\right] \leq \left(E\left[(\tilde{p}_N - \tilde{p}_N^{\mathrm{c}})^2\right]\right)^{1/2} \left(E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right]\right)^{1/2},$$

and therefore the interaction covariance term will be of lower order than the higher-order term of $E\left[(\tilde{p}_N - \tilde{p}_N^{\mathrm{c}})^2\right]$ or $E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right]$. The term $E\left[(\tilde{p}_N - \tilde{p}_N^{\mathrm{c}})^2\right]$ corresponds to the *strategic effect* and the term $E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right]$ corresponds to the *limit effect*.

We show in the next result that $E\left[(\tilde{p}_N - \tilde{p}_N^{\mathrm{c}})^2\right]$ converges to zero faster than $E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right]$ and therefore $E\left[(\tilde{p}_N - \tilde{p}_\infty)^2\right]$ inherits the order of $E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right]$ (they both converge to zero at exactly the same speed).

Given competitive market making, the expectations of the differences in equilibrium prices vanish and we need to compare only the rates of convergence of variances. The following proposition states the result.

Proposition 1. *For a sequence of markets described above:*

1. $\sqrt{E\left[(\tilde{p}_N - \tilde{p}_N^{\mathrm{c}})^2\right]} = \sqrt{\mathrm{var}\left[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}\right]} \propto 1/N$*;*
2. $\sqrt{E\left[(\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty)^2\right]} = \sqrt{\mathrm{var}\left[\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty\right]} \propto 1/\sqrt{N}$*.*

Note that (1) and (2) imply that $\sqrt{E\left[(\tilde{p}_N - \tilde{p}_\infty)^2\right]} = \sqrt{\mathrm{var}\left[\tilde{p}_N - \tilde{p}_\infty\right]} \propto 1/\sqrt{N}$. Let us present here an informal explanation of the result.

An heuristic explanation Let us recall that at a strategic equilibrium $\lambda_{\mathrm{I}} > 0$ is the slope of inverse supply facing the individual informed trader. In the competitive case $\lambda_{\mathrm{I}} = 0$ since a trader has no market power. It is easy to check[7] that λ_{I} is of the order of $1/N$ and this explains why market power vanishes at the rate $1/N$. (Result 1 in Proposition 1.)

In a symmetric linear equilibrium, prices have the following form:

$$\begin{aligned}
\tilde{p}_N^{\mathrm{c}} &= A_N^{\mathrm{c}}\tilde{v} + B_N^{\mathrm{c}}\frac{1}{N}\sum_{n=1}^{N}\tilde{e}_n + C_N^{\mathrm{c}}\tilde{z}_0,\\
\tilde{p}_N &= A_N\tilde{v} + B_N\frac{1}{N}\sum_{n=1}^{N}\tilde{e}_n + C_N\tilde{z}_0,\\
\tilde{p}_\infty &= A_\infty\tilde{v} + C_\infty\tilde{z}_0.
\end{aligned}$$

We have that A_N^{c} and A_N converge to A_∞, B_N^{c} and B_N converge to $B_\infty \neq 0$ and C_N^{c} and C_N converge to C_∞, all at a rate of $1/N$ or faster. Since we have assumed that $E[\tilde{v}] = E[\tilde{e}_n] = E[\tilde{z}] = 0$ expectations of prices are simply zeros (in fact, the expectations of price differences would be zero even if $E[\tilde{v}] > 0$ because $E[\tilde{p}_N^{\mathrm{c}}] = E[\tilde{p}_N] = E[\tilde{v}]$ from $E[\tilde{v} \mid \tilde{p}] = \tilde{p}$ in both cases). Therefore, only variances of the price differences are important:

$$\begin{aligned}
\mathrm{var}\left[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}\right] &= (A_N - A_N^{\mathrm{c}})^2\tau_v^{-1} + (B_N - B_N^{\mathrm{c}})^2\frac{1}{N}\tau_e^{-1} + (C_N - C_N^{\mathrm{c}})^2\sigma_{z_0}^2 \sim O(1/N^2),\\
\mathrm{var}\left[\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty\right] &= (A_N^{\mathrm{c}} - A_\infty)^2\tau_v^{-1} + (B_N^{\mathrm{c}})^2\frac{1}{N}\tau_e^{-1} + (C_N^{\mathrm{c}} - C_\infty)^2\sigma_{z_0}^2 \propto 1/N,\\
\mathrm{var}\left[\tilde{p}_N - \tilde{p}_\infty\right] &= (A_N - A_\infty)^2\tau_v^{-1} + (B_N)^2\frac{1}{N}\tau_e^{-1} + (C_N - C_\infty)^2\sigma_{z_0}^2 \propto 1/N.
\end{aligned}$$

Thus, $\tilde{p}_N^{\mathrm{c}}$ and $\tilde{p}_N$ converge to each other faster than to the limit price $\tilde{p}_\infty$ (at the rate at which market power vanishes) because prices in the finite markets depend in a similar way on the average noise in private information $(1/N)\sum_{n=1}^{N}\tilde{e}_n$, while in the continuum limit market the average noise of information cancels out according to the Strong Law of Large Numbers. The

[7] Conditions (B.8) and (B.9) in Kyle (1989) imply that $\lambda_{\mathrm{I}} = \xi_{\mathrm{I}}/((1-\xi_{\mathrm{I}})\gamma)$ where the parameter ξ_{I} can be interpreted as the marginal market share of an informed trader. We have then that $\lambda_{\mathrm{I}} \propto \xi_{\mathrm{I}} \propto 1/N$ (see Lemma A1 in the online appendix).

distance from $\tilde{p}_N^{\mathrm{c}}$ (or $\tilde{p}_N$) to the limit price $\tilde{p}_\infty$ depends on $(1/N)\sum_{n=1}^{N}\tilde{e}_n$ and this average error term converges to zero at a rate of $1/\sqrt{N}$. (Result 2 in Proposition 1.)

It is worth to remark that the limit effect would disappear if all informed traders were to receive the same signal (indeed, then $\tilde{p}_N^{\mathrm{c}} = \tilde{p}_\infty$).

Asymptotic variances of convergence Note also that in Proposition 1 we actually show slightly more than in the above heuristic argument. Namely, we prove that $\tilde{p}_N$ and $\tilde{p}_N^{\mathrm{c}}$ converge to each other precisely at the rate $1/N$. The asymptotic variances of convergence give us a refined measure of the speed of convergence for a given convergence rate for both the the limit

$$\lim_{N\to\infty}\sqrt{N}\sqrt{\mathrm{var}[\tilde{p}_N^{\mathrm{c}}-\tilde{p}_\infty]} = \lim_{N\to\infty}\sqrt{N}\sqrt{\mathrm{var}[\tilde{p}_N-\tilde{p}_\infty]} = A_{\mathrm{L}} = \frac{\tau_e^{3/2}}{\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2}$$

and the strategic

$$\lim_{N\to\infty} N\sqrt{\mathrm{var}[\tilde{p}_N-\tilde{p}_N^{\mathrm{c}}]} = A_{\mathrm{S}} = \frac{\tau_e^{1/2}}{\sigma_{z_0}\rho}\left(1+\frac{\tau_e\sigma_{z_0}^2\rho^2}{\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2}\right)A_{\mathrm{L}}$$

effects.[8] We find that A_{S} is increasing in the volatility of fundamentals, either decreasing or nonmonotonic in the "risk-bearing adjusted noise trade" $\rho\sigma_{z_0}$, and always nonmonotonic in the noisiness of the signals. We obtain that A_{S} is small, and the approximation of the strategic equilibrium by the competitive equilibrium good, when the prior volatility of the asset is low, noise trading is large in relation to the risk-bearing capacity of the informed traders (i.e., $\rho\sigma_{z_0}$large), or the signals are very noisy. In particular, $A_{\mathrm{S}} \to 0$ if $\tau_v \to +\infty$, or $\rho\sigma_{z_0} \to +\infty$, or $\tau_e \to 0+$. Furthermore, if $\rho\sigma_{z_0} \to 0+$ then $A_{\mathrm{S}} \to +\infty$. If the per capita informed trader noise trading volume is small for his risk-bearing capacity (i.e., $\rho\sigma_{z_0}$small), then the strategic effect vanishes more slowly.

We have that $A_{\mathrm{S}} > A_{\mathrm{L}}\frac{\tau_e^{1/2}}{\sigma_{z_0}\rho}$. For practical purposes the limit effect will dominate the strategic one whenever $A_{\mathrm{L}}/\sqrt{N} > A_{\mathrm{S}}/N$. Therefore for such domination to work we need to have

$$\sqrt{N} > \frac{\tau_e^{1/2}}{\sigma_{z_0}\rho} \quad \text{or} \quad N > \frac{\tau_e}{\sigma_{z_0}^2\rho^2}.$$

When informed traders are risk neutral the result is stark: competitive prices become fully revealing and the strategic and competitive equilibria converge to each other at a slower rate than in Proposition 1 (since $A_{\mathrm{S}} \to +\infty$ as $\rho \to 0$). Indeed, $\{\lim_{\rho\to 0}\sqrt{\mathrm{var}\left[\tilde{p}_N-\tilde{p}_N^{\mathrm{c}}\right]}\} \propto 1/\sqrt{N}$ as $\tilde{p}_N^{\mathrm{c}} \to \tilde{v}$ for $\rho \to 0$. The result should be clear since now the competitive price coincides with the value and therefore the distance with the price with strategic traders is according to the limit effect (of the order $1/\sqrt{N}$). It is worth noting that there is a discon-

[8] See the online appendix for the derivation of the values of A_{S} and A_{L} as well as their comparative static properties.

tinuity in the convergence rate when $\rho = 0$. However, for $\rho > 0$, as $\rho \to 0$ we have that the asymptotic standard deviation $A_{\mathrm{S}} \to +\infty$, implying that as ρ decreases the convergence between strategic and competitive equilibria becomes slower.

We confirm, therefore, the idea that the competitive approximation works even in a moderately sized market when the informationally adjusted risk-bearing capacity of the informed traders ($\frac{\tau_e}{\sigma_{\tilde{z}_0}^2 \rho^2}$) is not very large. This is the situation where competitive traders have incentives to be restrained in their trading. Indeed, it is easily checked (see Theorem 8.2 in Kyle (1989)) that the responsiveness to private information β^{c} is increasing in $\frac{\tau_e}{\sigma_{\tilde{z}_0}^2 \rho^2}$.

A calibration exercise with reasonable values of the parameters for financial markets provides interesing insights (see the online appendix). We find that for typical stock market parameters the standard deviation of the distance between the strategic and competitive price $\sqrt{\mathrm{var}[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}]}$, approximated by A_{S}/N, is quite small even with very few informed traders. The S&P 500 Futures market has high volatility, very noisy information signals and relatively small noise trade. In this context the asymptotic variances are larger than with the stock market parameters. However, since the number of traders in the S&P 500 Futures market is very large, the competitive REE should be a very close approximation of the strategic REE in this market. In the analysis of S&P 500 Futures market one can safely take the shortcut of assuming competitive behavior. Indeed, Cho and Krishnan (2000) find that a competitive rational expectations model provides a reasonable description of this market.

3.1.2 Demands, profits and utilities

We now study the convergence rates for demands, profits and utilities. Those follow from the order of the difference between prices. For all of these we confirm the result that the strategic effect is of the order of $1/N$ while the limit effect is of the order of $1/\sqrt{N}$.

Proposition 2. *We have:*

1. $\sqrt{E\left[(\tilde{x}_N - \tilde{x}_N^{\mathrm{c}})^2\right]} \propto \sqrt{E\left[(\tilde{\pi}_N - \tilde{\pi}_N^{\mathrm{c}})^2\right]} \propto \sqrt{E\left[(U(\tilde{\pi}_N^{\mathrm{c}})/U(\tilde{\pi}_N) - 1)^2\right]} \propto 1/N$;
2. $\sqrt{E\left[(\tilde{x}_N^{\mathrm{c}} - \tilde{x}_\infty)^2\right]} \propto \sqrt{E\left[(\tilde{\pi}_N^{\mathrm{c}} - \tilde{\pi}_\infty)^2\right]} \propto \sqrt{E\left[(U(\tilde{\pi}_N^{\mathrm{c}})/U(\tilde{\pi}_\infty) - 1)^2\right]} \propto 1/\sqrt{N}$;
3. $\sqrt{E\left[(\tilde{x}_N - \tilde{x}_\infty)^2\right]} \propto \sqrt{E\left[(\tilde{\pi}_N - \tilde{\pi}_\infty)^2\right]} \propto \sqrt{E\left[(U(\tilde{\pi}_N)/U(\tilde{\pi}_\infty) - 1)^2\right]} \propto 1/\sqrt{N}$.

As for prices, the expectations of all demands are zero, so these square loss functions simply are the variances of the differences between corresponding demands. From the expression for the demands, $X_n(\tilde{p}_N, \tilde{\imath}_n) = \frac{E[\tilde{v}|\tilde{p}_N, \tilde{\imath}_n] - \tilde{p}_N}{\rho var[\tilde{v}|\tilde{p}_N, \tilde{\imath}_n] + \lambda_{\mathrm{I}}}$ and $X_n(\tilde{p}_N^{\mathrm{c}}, \tilde{\imath}_n) = \frac{E[\tilde{v}|\tilde{p}_N^{\mathrm{c}}, \tilde{\imath}_n] - \tilde{p}_N^{\mathrm{c}}}{\rho var[\tilde{v}|\tilde{p}_N^{\mathrm{c}}, \tilde{\imath}_n]}$, the fact that $var[\tilde{v} \mid \tilde{p}_N, \tilde{\imath}_n]$ and $var[\tilde{v} \mid \tilde{p}_N^{\mathrm{c}}, \tilde{\imath}_n]$ are bounded away from zero for all N (since the equilibria are never fully revealing for $\rho > 0$), and the order of $\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}$, result 1 for the demands in Proposition 2 follows. For results 2 and 3 consider $X_\infty(\tilde{p}_\infty, \tilde{\imath}_n) = \frac{E[\tilde{v}|\tilde{p}_\infty, \tilde{\imath}_n] - \tilde{p}_\infty}{\rho var[\tilde{v}|\tilde{p}_\infty, \tilde{\imath}_n]}$ and the orders of $\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty$ and $\tilde{p}_N - \tilde{p}_\infty$. Indeed, since demands are linear functions of prices, the same effect holding for prices holds in terms of demands. For profits, unlike for prices or demands, expectations of differences do not vanish, so we cannot restrict attention only to the variances but rather we have to consider the entire square loss functions, and the same result holds. The results for profits translate into relative utilities using the (full) Taylor expansion of the exponential function.

3.2 Convergence when N does not grow proportionally with σ_z

In this section we summarize how our results are affected if the number of informed speculators does not change proportionally to noise trading.

We obtain that if the number of informed speculators N grows at a faster rate than noise trading σ_z, a value revealing limit is obtained while if the number of informed speculators N grows at a slower rate than noise trading σ_z an informationally trivial limit obtains and prices converge to zero. In both cases $\sqrt{\text{var}\left[\tilde{p}_N - \tilde{p}_N^{\text{c}}\right]}$ corresponds to the strategic effect and the limit effect is $\sqrt{\text{var}\left[\tilde{p}_N^{\text{c}} - \tilde{v}\right]}$ with a value revealing limit and $\sqrt{\text{var}\left[\tilde{p}_N^{\text{c}}\right]}$ with the zero limit price.

In the first case with a value revealing limit we can distinguish two subcases: First, when σ_z grows slower than a rate N but faster than a rate $\sqrt{N}$; second, when σ_z grows at a rate $\sqrt{N}$ or slower.

Proposition 3. 1. *For a sequence of markets with σ_z growing slower than at the rate N we obtain a value revealing limit and*

(a) *if σ_z grows faster than at the rate $\sqrt{N}$: $\sqrt{\text{var}\left[\tilde{p}_N - \tilde{p}_N^{\text{c}}\right]} \propto 1/\sigma_z$ and $\sqrt{\text{var}\left[\tilde{p}_N^{\text{c}} - \tilde{v}\right]} \propto \sqrt{\text{var}\left[\tilde{p}_N - \tilde{v}\right]} \propto \sigma_z/N$;*

(b) *if σ_z grows not faster than at the rate $\sqrt{N}$: $\sqrt{\text{var}\left[\tilde{p}_N - \tilde{p}_N^{\text{c}}\right]} \propto \sqrt{\text{var}\left[\tilde{p}_N^{\text{c}} - \tilde{v}\right]} \propto \sqrt{\text{var}\left[\tilde{p}_N - \tilde{v}\right]} \propto 1/\sqrt{N}$.*

2. *For a sequence of markets with σ_z growing faster than at the rate N we obtain an informationally trivial limit and*
$\sqrt{\text{var}\left[\tilde{p}_N - \tilde{p}_N^{\text{c}}\right]} \propto N^2/\sigma_z^3$; $\sqrt{\text{var}\left[\tilde{p}_N^{\text{c}}\right]} \propto \sqrt{\text{var}\left[\tilde{p}_N\right]} \propto N/\sigma_z$.

In case 1a we see, since $1/\sigma_z = (\sigma_z/N)(N/\sigma_z^2) \sim o(\sigma_z/N)$, that the strategic effect vanishes faster than the limit effect as in our central scenario. The same obtains in case 2. In case 1b both effects vanish at the same rate. This latter case is a situation where Kyle's mutual convergence condition fails, and market power in the strategic equilibrium does not vanish, since $\frac{\rho^2\sigma_z^2}{N\tau_e} \nrightarrow \infty$ as $N \to \infty$ despite the fact that both equilibria happen to be value revealing (since the noise from the noise traders is too small to prevent it).

3.3 Summary table

Table 1 summarizes the strategic and limit effects for prices in the sequence of markets with τ_e a fixed constant.

	Limit price	Limit effect	Strategic effect
$\sigma_z \sim O(\sqrt{N})$	$\tilde{v}$, value revealing	$1/\sqrt{N}$	$1/\sqrt{N}$
$\sqrt{N} \sim o(\sigma_z)$ and $\sigma_z \sim o(N)$	$\tilde{v}$, value revealing	σ_z/N	$1/\sigma_z$
$\sigma_z \propto N$	$\tilde{p}_\infty$	$1/\sqrt{N}$	$1/N$
$N \sim o(\sigma_z)$	0, informationally trivial	N/σ_z	N^2/σ_z^3

Table 1: Summary of the strategic and limit effects for prices.

One can see that whenever $\sqrt{N} \sim o(\sigma_z)$ the strategic effect is dominated by the limit effect in terms of the speed of convergence. In particular, this happens in our central scenario (the case when $\sigma_z \propto N$). When $\sigma_z \sim O(\sqrt{N})$ the speed of convergence is the same for both effects ($1/\sqrt{N}$).

4 Monopolistic competition

Consider the "monopolistic competition" example of Kyle (1989, Section 9). At the Nth market there are N informed agents and for all of the markets $\text{var}[\tilde{v}] = \tau_v^{-1}$ is a fixed constant. However, now $\text{var}[\tilde{e}_n] = \tau_e^{-1}$ and $N\tau_e = \tau_E$, where τ_E is a given constant. This means that, unlike in the previous sections, the total precision is bounded and does not change with N. A given stock of private information τ_E is divided equally among N speculators. Thus, for large N each speculator's signal contains only a small amount of information. It is convenient to define an average error term $\tilde{e} \equiv (1/N)\sum_{n=1}^{N} \tilde{e}_n$. Obviously, $\tilde{e}$ is normally distributed with zero mean and $\text{var}[\tilde{e}] = (N\tau_e)^{-1} = \tau_E^{-1}$.

It is easy to see that if $\text{var}[\tilde{z}] = \sigma_z^2$ grows with N and τ_E is fixed, then both the competitive and the strategic equilibrium reach the informationally trivial limit and equilibrium prices converge to zero. In this case in a large market a monopolistically competitive limit does not obtain. However, it does obtain if σ_z^2 is fixed as the number of informed traders increases.[9] García and Sangiorgi (2011) derive conditions under which an information seller may want to sell to as many agents as possible very imprecise information, providing a rationale for Kyle's monopolistic competition example. In this case, the competitive and the strategic models yield very different outcomes and the use of the competitive model would be indeed misleading.

Proposition 4. *In the monopolistic competition model:*

[9] It is worth noting, however, that the limit large market economy is not well-defined since as $N \to \infty$, $\tau_e = \tau_E/N \to 0$, and therefore in the limit the signals of traders have zero precision (infinite variance) while the aggregate precision is positive.

1. *If σ_z grows with N then both the competitive and the strategic equilibrium reach the informationally trivial limit and equilibrium prices converge to zero.*

2. *If σ_z is constant then different prices are obtained in the limit as $N \to \infty$ in the price-taking ($\tilde{p}^{\mathrm{c}}_\infty$) and in the strategic case ($\tilde{p}_\infty$), $\tilde{p}_\infty \neq \tilde{p}^{\mathrm{c}}_\infty$. Furthermore,*

   (a) $\sqrt{\operatorname{var}\left[\tilde{p}^{\mathrm{c}}_N - \tilde{p}^{\mathrm{c}}_\infty\right]} \propto 1/N$.

   (b) $\sqrt{\operatorname{var}\left[\tilde{p}_N - \tilde{p}_\infty\right]} \sim O(1/N)$ *and if* $\tau_E > \tau_v$, *then* $\sqrt{\operatorname{var}\left[\tilde{p}_N - \tilde{p}_\infty\right]} \propto 1/N$.

The convergence (in 1) or lack of convergence (in 2) to a common limit follow from Kyle's condition. What we add here are the convergence rates. In result 2 we confirm that the convergence to the respective limit equilibria is fast, at a rate of at least $1/N$, in both cases. The difference in the limits is due to the residual market power traders enjoy in the monopolistic competition case even in a large market. For the competitive model we show that the convergence is always at a rate of $1/N$, while for the monopolistically competitive model we can show that $1/N$ is the exact rate of convergence for the subcase $\tau_E > \tau_v$. Convergence is fast now because all along the sequence of markets, noise trade is constant and the average noise in the signal $\tilde{e}$ has constant variance $((\tau_E)^{-1})$. In contrast, in the central scenario we have that in the Nth market $\operatorname{var}[\tilde{e}]$ is of the order of $1/N$ and this determines the convergence speed to the limit of $1/\sqrt{N}$.

5 Free entry in a large market

Consider the following scenario in which we endogenize the number of informed speculators. There are two stages and a countable infinity of potential traders. In a first stage any trader (except noise traders) can become informed (that is, can receive a signal about the value of the asset) by paying a fixed amount $F > 0$. In a second stage the speculators who have decided to enter compete as in Section 2. Free entry of uninformed speculators, even if they are risk averse, implies that the market at the second stage is semi-strong efficient (Theorem 7.4 in Kyle (1989)).[10] We will look for symmetric subgame-perfect equilibria of the two-stage game.

The size of the market is naturally parameterized by the noise trading volume σ_z. We also index the cost of acquiring information by σ_z: $F(\sigma_z)$ and assume that $F(\infty) \equiv \lim_{\sigma_z \to \infty} F(\sigma_z)$ is well defined. If $F(\cdot)$ is constant we may interpret that there are constant returns to information acquisition. If $F(\cdot)$ is increasing (decreasing) in the size of the market there are decreasing (increasing) returns to scale to information acquisition. Increasing returns to information production may arise because of fixed cost components in information production and decreasing returns may arise because of increased correlation among signals when a

[10] Alternatively, we obtain the same results if at the second stage there is a competitive risk-neutral market making sector and then no risk-averse trader will choose to enter if he does not purchase information.

larger sample is produced. The central case is with constant returns and $F(\cdot) \equiv F > 0$. An alternative interpretation[11] is to think that $F(\sigma_z)$ is the price charged by a financial intermediary to each informed trader to access liquidity (noise) traders. This intermediary owns the the technology and gathers a volume of σ_z liquidity traders. Note that private information is useless without access to the intermediation technology. Assuming that there is a competitive sector of intermediaries pricing will be according to cost and we should expect that $F(\cdot)$ is nondecreasing in σ_z.

The equilibrium number of informed traders is determined as follows. Consider first the strategic case. Let $\tilde{\pi}_N$ be the equilibrium random profits of an informed speculator when N have entered and denote by $\Pi(N)$ the certainty equivalent of profits that makes a trader indifferent between making this payment to be one of the informed speculators or remain uninformed. We have that, because of exponential utility, $\Pi(N)$ is independent of initial wealth and

$$-\exp(-\rho\Pi(N)) = E[-\exp(-\rho\tilde{\pi}_N)].$$

We obtain that $\Pi(N) = \frac{1}{2\rho}\log\left(1+\omega\left(\sigma_z,N\right)\frac{\tau_e}{\tau_v}\right)$ where the function $\omega\left(\cdot\right)$, taking values in $[0,1]$, also depends on τ_e and τ_v. Let $N^*(\sigma_z)$ denote the equilibrium number of informed speculators in the imperfect competition model when noise trading has standard deviation σ_z. Here $N^*(\sigma_z)$ is defined as the largest N such that $\Pi(N) \geq F(\sigma_z)$.

Let us define the competitive equilibrium model $\Pi^{\mathrm{c}}(N)$ and $N^*_{\mathrm{c}}(\sigma_z)$ analogously. We have that $\Pi^{\mathrm{c}}(N) = \frac{1}{2\rho}\log\left(1+\omega^{\mathrm{c}}\left(\sigma_z,N\right)\frac{\tau_e}{\tau_v}\right)$, where the function $\omega^{\mathrm{c}}\left(\cdot\right)$ also takes values in $[0,1]$. Then the equilibrium number of speculators $N^*_{\mathrm{c}}(\sigma_z)$ is defined as the largest N such that $\Pi^{\mathrm{c}}(N) \geq F(\sigma_z)$.

Then there is a unique free-entry equilibrium of the two-stage game in both the strategic and competitive cases (Kyle (1989, Theorems 10.1 and 10.2)).[12]

We are interested in the rate of change of the equilibrium number of entrants in both the competitive and the strategic regime as the market becomes large by increasing $\sqrt{\operatorname{var}\left[\tilde{z}\right]} = \sigma_z$. For all of the markets $\operatorname{var}\left[\tilde{v}\right] = \tau_v^{-1}$ and $\operatorname{var}\left[\tilde{e}_n\right] = \tau_e^{-1}$ are fixed constants. We note that both $\Pi(N)$ and $\Pi^{\mathrm{c}}(N)$ are in $(0,F^*)$ where $F^* \equiv \frac{1}{2\rho}\log\left(1+\frac{\tau_e}{\tau_v}\right)$, which represents the maximal value of information or maximal ex ante certainty equivalent utility. This is so since both $\omega\left(\sigma_z,N\right)$ and $\omega^{\mathrm{c}}\left(\sigma_z,N\right)$ are in $[0,1]$.

Proposition 5. *Let $\rho > 0$ and $F^* \equiv \frac{1}{2\rho}\log\left(1+\frac{\tau_e}{\tau_v}\right)$. In a large market as $\sigma_z \to \infty$ three cases appear (in both the strategic and competitive cases) in a free entry equilibrium.*

1. *If $F(\infty) \geq F^*$, then if the endogenous number of informed speculators grows, it grows at a slower rate than σ_z ($N^*_{\mathrm{c}}(\sigma_z)$ and $N^*(\sigma_z)$ are $\sim o(\sigma_z)$). The limit market is informationally trivial and prices converge to $E\left[\tilde{v}\right] = 0$.*

[11] Suggested by a referee.

[12] In the Kyle theorems an heterogeneous information acquisition cost across traders was used instead of the homogenous $F(\sigma_z)$. The Kyle theorems apply obviously also as a particular case with an homogenous cost.

2. *If $0 < F(\infty) < F^*$, then the endogenous number of informed speculators grows proportionally to σ_z ($N_c^*(\sigma_z) \propto N^*(\sigma_z) \propto \sigma_z$). Moreover, $N_c^*(\sigma_z)/N^*(\sigma_z)$ converges to 1 and the limit market is neither value revealing nor informationally trivial.*

3. *If $F(\infty) = 0$, then the endogenous number of informed speculators grows at a faster rate than σ_z ($\sigma_z \sim o(N_c^*(\sigma_z))$ and $\sigma_z \sim o(N^*(\sigma_z))$. The limit market is value revealing: prices converge to $\tilde{v}$.*

The limit market in case (2) corresponds to the continuum of traders competitive model of Vives (1995) where informed traders co-exist with risk neutral market makers.[13]

The result in case (2) is consistent with related work by García and Urosevic (2008) where information acquisition in large markets is studied. Those authors find a limit of the Vives (1995) type corresponding to their case of "diversifiable noise".

The result in case (3) with $F(\sigma_z) > 0$ for any σ_z and $F(\infty) = 0$ provides a weak resolution of the Grossman-Stiglitz paradox. Indeed, in a large market (i.e. with σ_z large) prices will be close to fully revealing but still there will be incentives to acquire information as long as in the limit information acquisition is free ($F(\infty) = 0$).

In a large market it is worth noting that in case (1) private information would have the largest value (since very few traders are informed) but information is too expensive to obtain, while in case (3) private information would have no value (since many traders are informed and profits are dissipated) and information is free to obtain. If we interpret $F(\sigma_z)$ as the price charged by a for-profit financial intermediary to each informed trader to access a volume σ_z of liquidity trading, then the financial intermediary in a large market will never choose to be in case (1) or (3) since there are no profits to be made in this case. This would again point to case (2) as the central case.[14]

Remark 2. In the *case of constant returns to information acquisition*, $F(\cdot) \equiv F$:
1. if $F \geq F^*$, then $N_c^*(\sigma_z) = N^*(\sigma_z) = 0$;
2. if $0 < F < F^*$, then $N_c^*(\sigma_z) \propto N^*(\sigma_z) \propto \sigma_z$;
3. if $F = 0$, then $N_c^*(\sigma_z) = N^*(\sigma_z) = \infty$.

An heuristic explanation of the proof Consider first the strategic case for an *exogenous* number of speculators N. Then $\Pi(N) \in (0, F^*)$ and $\Pi(N)$ converges to different limits as $\sigma_z \to \infty$ depending on the rate of the growth of N:

(i) if N grows at a slower rate than σ_z, then relatively few informed speculators are in the market, the limit market is informationally trivial, and the certainty equivalent of profits $\Pi(N)$ converges to its upper bound F^* (information is the most valuable);

[13] The competitive models of Hellwig (1980) and Admati (1985) do not have risk neutral market makers.
[14] We thank a referee for suggesting this interpretation.

(ii) if N grows at the same rate as σ_z, then the limit market is neither value revealing nor informationally trivial, and the certainty equivalent of profits $\Pi(N)$ for large N belongs to some interval strictly between 0 and F^* (information is valuable);

(iii) if N grows at a faster rate than σ_z, then relatively many speculators are in the market, the limit market is value revealing and the certainty equivalent of profits $\Pi(N)$ converges to 0 (information is not valuable).

Allow now for an *endogenous* number of speculators N. If N does not grow unboundedly as $\sigma_z \to \infty$, then we should be in the case (i) above and $F(\infty) \geq F^*$ (with no speculators becoming informed when $F(\cdot) \equiv F$). If N grows unboundedly as $\sigma_z \to \infty$ then it should be that $\Pi(N) \to F(\infty)$. (For a given σ_z speculators will enter until $\Pi(N)$ is just above $F(\infty)$.) Depending on the value of $F(\infty)$ we should be in one of the cases (i)–(iii) above. If $F(\infty) \geq F^*$, then we must be in case (i) since eventually it does not pay to become informed. If $F(\infty) = 0$, then eventually all speculators enter and we must be in case (iii). If $0 < F(\infty) < F^*$, then N must grow unboundedly with σ_z and $\Pi(N) \to F(\infty)$ (otherwise, $\Pi(N) \to F^*$ and there would be more entry since $F(\infty) < F^*$). It follows that we must be in case (ii) since if N grows at a faster rate than σ_z, then $\Pi(N) \to 0$, and if N grows at a slower rate than σ_z, then $\Pi(N) \to F^*$. We can show in this case that $\sigma_z / N^*(\sigma_z) \to \sigma_{z0}$ as $\sigma_z \to \infty$ for some constant $\sigma_{z0} \equiv \frac{1}{\rho}\sqrt{\frac{\tau_e}{1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e}}$ which is increasing with $F(\infty)$.

For the price-taking equilibrium the analysis is similar but based on $\Pi^{\mathrm{c}}(N)$. In case (ii) we can show also that $\sigma_z / N_{\mathrm{c}}^*(\sigma_z) \to \sigma_{z0}$ and therefore, that $N_{\mathrm{c}}^*(\sigma_z)/N^*(\sigma_z)$ converges to 1 as $\sigma_z \to \infty$.

Hence, in general, for the whole range of intermediate values of $F(\infty)$ we obtain that the endogenous number of informed speculators is proportional to the standard deviation of the noise trade and this is the case that has as its limit the usual continuum model. Thus, it is natural to consider sequences of replica markets where the numbers of informed speculators are proportional to σ_z (as in Proposition 1) as the central scenario. The replica markets results corresponding to results 1 and 3 in Proposition 5 are in Proposition 3. In result 1 in Proposition 5 conditions are provided for the number of informed speculators to grow (if at all) more slowly than σ_z (corresponding to result 2 in Proposition 3). In result 3 in Proposition 5 conditions are provided for the number of informed speculators to grow faster than σ_z (corresponding to result 1 in Proposition 3).

Remark 3. In the monopolistic competition case where σ_z is fixed and traders can acquire a signal paying a cost $F > 0$ it is easy to see that the endogenous number of informed speculators is finite in both the strategic and competitive cases, and as $F \to 0$, both numbers tend to infinity. The result follows because the certainty equivalent of profits in both the strategic and competitive cases tend to zero as N grows.

Risk neutrality and the Grossman-Stiglitz paradox

We examine the limiting behavior of equilibria in the free-entry set-up when traders are risk neutral ($\rho = 0$). For all of the markets $\mathrm{var}[\tilde{v}] = \tau_v^{-1}$, $\mathrm{var}[\tilde{e}_n] = \tau_e^{-1}$ and $\sqrt{\mathrm{var}[\tilde{z}]} = \sigma_z$. As before we allow traders to become informed by paying a fee $F(\sigma_z) > 0$ and assume that $F(\infty) \geq 0$ is well defined. We have seen before (Section 3.1.1) that the rate of convergence of the strategic and the competitive equilibria in the central scenario when N and σ_z grow at the same rate and informed traders are risk neutral is $1/\sqrt{N}$. Here we see what happens when the number of informed traders is endogenous and how this number depends on the size of the market. Then there is no entry under competitive conditions and "slow" entry when traders are strategic and the market gets large: N^* grows less than proportionately with σ_z.

Proposition 6. *With $\rho = 0$ and a cost of acquiring information $F(\sigma_z) > 0$, we have that:*

1. *$N^*(\sigma_z) \propto (\sigma_z/F(\sigma_z))^{2/3}$ in the strategic model, and as $\sigma_z \to \infty$ the limit market is value revealing: prices converge to $\tilde{v}$; and*

2. *$N_c^*(\sigma_z) = 0$ in the competitive model, and all markets are informationally trivial: prices equal $E[\tilde{v}] = 0$.*

In the central case of constant returns to information acquisition, $F(\cdot) \equiv F > 0$ we obtain $N^*(\sigma_z) \propto (\sigma_z)^{2/3}$, while, indeed, $N_c^*(\sigma_z) = 0$. No traders choose to become informed in the price-taking case because of their risk neutrality. If they chose to become informed they would trade so aggressively that they would reveal their private information and would make zero profits. (This happens also for ρ close to 0.) This is an example of the well-known informational efficiency paradox described by Grossman and Stiglitz (1980). Prices can not be fully informative since then no one would pay to acquire information. In the strategic case the informed traders take into account the effect of their actions on the price and therefore can restrict their trade. So the incentives to acquire information do not disappear as more traders decide to become informed.

It is worth noting that in the risk neutral case, unlike the risk-averse case, the number of informed traders grows less than proportionally with the size of the market σ_z. The reason is that risk neutrality implies fiercer competition among informed traders and there is less room for entry to occur and cover the fixed cost. Note that according to Proposition 5 (2) where $0 < F(\infty) < F^*$, we have that $F^* \to \infty$ as $\rho \to 0$ and $N^*(\sigma_z)$ is proportional to σ_z as long as $\rho > 0$, but according to Proposition 6 (1) this is not true when $\rho = 0$. The reason is that $\sigma_z/N^*(\sigma_z) \to \sigma_{z_0} < \infty$ as $\sigma_z \to \infty$ where $\sigma_{z_0} \to \infty$ as $\rho \to 0$.[15] This means necessarily that when $\rho = 0$, the number of informed traders $N^*(\sigma_z)$ grows less than proportionately than σ_z. However, despite this fact prices become fully revealing when $\sigma_z \to \infty$. The reason is that due to risk neutrality the aggregate response to private information βN grows faster

[15] See Fact and proof before the proof of Proposition 6 in the Appendix.

than the size of the market σ_z. The end result is that strategic trading provides a resolution of the Grossman-Stigliz paradox when $\rho = 0$ and $F(\infty) > 0$. Indeed, in a large market as $\sigma_z \to \infty$ prices become fully revealing but the incentives to acquire information are preserved all along the sequence.

6 Concluding remarks

The basic insight of the paper is that provided there is no residual market power in a large market, competitive and strategic equilibria should be close whenever price-taking traders have incentives to be restrained in their trading. This insight should be robust to dynamic considerations. Think, for example, of the extreme case of risk neutral traders. Then the same logic as in the static model would lead us to conclude that competitive traders in a multiperiod market will have no incentives to acquire information, since profits would be dissipated, while strategic traders would. In any case, an interesing extension of the model would be to consider multiperiod trading.

7 Appendix

7.1 Characterization of the equilibria

For the convenience of the reader we summarize here some characterization results from Kyle (1989). Note that our restriction of the competitive risk-neutral market making sector corresponds to the free entry of uninformed speculators ($M = \infty$) in Kyle (1989).

For any symmetric linear equilibrium with the strategies $X_n = \mu + \beta i_n - \gamma p$, $n = 1, \ldots, N$, as stated in Section 2 we can solve the market clearing condition for the equilibrium price $\tilde{p}$:

$$\tilde{p} = E[\tilde{v} \mid \tilde{p}] = \frac{\beta \tau_v^{-1} N}{\beta^2 \tau_v^{-1} N^2 + \beta^2 \tau_e^{-1} N + \sigma_z^2} \left(\beta \tilde{v} N + \beta \sum_{n=1}^{N} \tilde{e}_n + \tilde{z} \right). \tag{1}$$

Note that $\lambda \equiv \frac{\partial p}{\partial z} = \frac{\beta \tau_v^{-1} N}{\beta^2 \tau_v^{-1} N^2 + \beta^2 \tau_e^{-1} N + \sigma_z^2}$. Regardless of the particular equilibrium concept, the linearity and symmetry assumptions allow useful measures of the informativeness of prices to be obtained. Define τ_{F} as the precision of the forecast of the liquidation value $\tilde{v}$ based on all the information, that is $\tau_{\mathrm{F}} = \mathrm{var}^{-1}[\tilde{v} \mid \tilde{\imath}_1, \ldots, \tilde{\imath}_N]$. The assumptions made on the distributions of $\tilde{v}, \tilde{e}_1, \ldots, \tilde{e}_N$ imply that $\tau_{\mathrm{F}} = \tau_v + N\tau_e$. Now let us define the precision τ_{U} that speculators have basing only on the price and the precision τ_{I} that speculators have basing on the price and their own private signal:

$$\tau_{\mathrm{U}} = \mathrm{var}^{-1}[\tilde{v} \mid \tilde{p}] \quad \text{and} \quad \tau_{\mathrm{I}} = \mathrm{var}^{-1}[\tilde{v} \mid \tilde{p}, \tilde{\imath}_n].$$

Normality makes τ_{U} and τ_{I} constants, while symmetry means that τ_{I} does not depend on

n. Since these precisions are bounded below by the prior precision τ_v, and above by the full-information precision τ_{F}, there exist constants φ_{U} and φ_{I} both in the interval [0,1], such that

$$\tau_{\mathrm{U}} = \tau_v + \varphi_{\mathrm{U}} N \tau_e \quad \text{and} \quad \tau_{\mathrm{I}} = \tau_v + \tau_e + \varphi_{\mathrm{I}}(N-1)\tau_e.$$

The parameters φ_{U} and φ_{I} are convenient indices measuring the "informational efficiency" with which price aggregate private information of informed traders. Theorem 4.1 in Kyle (1989) presents expressions for these indices in terms of the parameter β:

$$\varphi_{\mathrm{I}} = \frac{(N-1)\beta^2}{(N-1)\beta^2 + \sigma_z^2 \tau_e} \quad \text{and} \quad \varphi_{\mathrm{U}} = \frac{N\beta^2}{N\beta^2 + \sigma_z^2 \tau_e}. \tag{2}$$

These formulas imply

$$\varphi_{\mathrm{U}} - \varphi_{\mathrm{I}} = \frac{1}{N}\varphi_{\mathrm{U}}(1 - \varphi_{\mathrm{I}}). \tag{3}$$

Let us turn back now to the two specific equilibria that we are studying. From (1) we obtain expressions for the prices in competitive and strategic equilibria:

$$\begin{aligned}
\tilde{p}^{\mathrm{c}} &= \frac{\beta^{\mathrm{c}} \tau_v^{-1} N}{(\beta^{\mathrm{c}})^2 \tau_v^{-1} N^2 + (\beta^{\mathrm{c}})^2 \tau_e^{-1} N + \sigma_z^2} \left(\beta^{\mathrm{c}} \tilde{v} N + \beta^{\mathrm{c}} \sum_{n=1}^{N} \tilde{e}_n + \tilde{z} \right), \\
\tilde{p} &= \frac{\beta \tau_v^{-1} N}{\beta^2 \tau_v^{-1} N^2 + \beta^2 \tau_e^{-1} N + \sigma_z^2} \left(\beta \tilde{v} N + \beta \sum_{n=1}^{N} \tilde{e}_n + \tilde{z} \right).
\end{aligned} \tag{4}$$

The only difference in the expressions for prices is the difference between the parameters β and β^{c}. Theorem 5.2 and expression (C.5) in Kyle (1989) provide a useful characterization of these parameters:

$$\beta^{\mathrm{c}} = \frac{\tau_e}{\rho}(1 - \varphi_{\mathrm{I}}^{\mathrm{c}}) \quad \text{and} \quad \beta = \frac{\tau_e}{\rho}(1 - \varphi_{\mathrm{I}}) \frac{(1 - 2\zeta)}{(1 - \zeta)}, \tag{5}$$

where $\zeta = \tau_{\mathrm{I}} \beta \lambda / \tau_e \leq 1/2$, and it also holds that

$$(1 - \zeta) = (1 - \xi_{\mathrm{I}})(1 - \varphi_{\mathrm{I}}) \quad \text{and} \quad 0 \leq \xi_{\mathrm{I}} \leq 1/N, \tag{6}$$

where $\xi_{\mathrm{I}} = \gamma\lambda$ can be interpreted as the marginal market share of an informed speculator. Condition (64) in Kyle (1989) will be true for ζ in our case ($M = \infty$):

$$\zeta \tau_{\mathrm{U}} - \varphi_{\mathrm{U}} \tau_{\mathrm{I}} = 0. \tag{7}$$

Finally, the following characterization (Lemma 7.1 of Kyle (1989)) of φ_{I} and $\varphi_{\mathrm{I}}^{\mathrm{c}}$ in the corresponding equilibria is very useful:

$$
\begin{aligned}
\frac{\sigma_z^2\rho^2}{(N-1)\tau_e} &= \frac{(1-\varphi_{\mathrm{I}}^{\mathrm{c}})^3}{\varphi_{\mathrm{I}}^{\mathrm{c}}}, \\
\frac{\sigma_z^2\rho^2}{(N-1)\tau_e} &= \frac{(1-\varphi_{\mathrm{I}})^3}{\varphi_{\mathrm{I}}}\frac{(1-2\zeta)^2}{(1-\zeta)^2} = \frac{(1-\varphi_{\mathrm{I}})}{\varphi_{\mathrm{I}}}\frac{(1-2\zeta)^2}{(1-\xi_{\mathrm{I}})^2}.
\end{aligned}
\tag{8}
$$

Limit marginal market shares One conclusion in Theorem 9.2 in Kyle (1989) requires some qualification for the case of free entry of uninformed speculators. Theorem 9.2 in Kyle (1989) covers the case where the limit is of the Hellwig–Admati type (as in Proposition 1) and concludes that $\xi_{\mathrm{U}}M \to 0$, which is equivalent to $\xi_{\mathrm{I}}N \to 1$. However, if we restrict attention to the case of a competitive risk-neutral market making sector or free entry of uninformed speculators ($M = \infty$), then Lemma A1 in the online appendix proves that for the case where the limit is of the Hellwig–Admati type,

$$\lim_{N\to\infty} \xi_{\mathrm{I}}N = \frac{\tau_e^2}{\tau_v\sigma_{z_0}^2\rho^2 + \tau_e^2} < 1.$$

Note that parameters ξ_{U} and ξ_{I} can be interpreted respectively as marginal market shares of an uninformed speculator and of an informed speculator. Thus, the total marginal market share of all informed speculators ($\xi_{\mathrm{I}}N$) even in the limit is strictly less than one, and it is close to one only if the precision of their signals (τ_e) is large relative to the other parameters of the model.

Similarly, Theorem 9.1 in Kyle (1989) predicts for the case of monopolistic competition that $\xi_{\mathrm{I}}N \to 1$. Again, for $M = \infty$ this does not hold. Lemma A3 in the online appendix proves that in our case with a competitive risk-neutral market making sector, or free entry of uninformed speculators ($M = \infty$), and monopolistic competition

$$\xi_{\mathrm{I}}N = \frac{\tau_v + \tau_E}{(\tau_v/\varphi_{\mathrm{U}}) + \tau_E}.$$

Since $\lim_{N\to\infty}\varphi_{\mathrm{U}} < 1$, we can conclude that the total marginal market share of all informed speculators ($\xi_{\mathrm{I}}N$) even in the limit is strictly less than one.

7.2 Proofs of Propositions 1, 5, and 6

Proof of Proposition 1: From the formulas (4) we get the following expressions for prices in both equilibria in the N-replica market:

$$\tilde{p}_N^c = \frac{\beta_N^c \tau_v^{-1}}{(\beta_N^c)^2 \tau_v^{-1} + \frac{1}{N}(\beta_N^c)^2 \tau_e^{-1} + \sigma_{z_0}^2} \left(\beta_N^c \tilde{v} + \beta_N^c \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + \tilde{z}_0 \right), \tag{9}$$

$$\tilde{p}_N = \frac{\beta_N \tau_v^{-1}}{\beta_N^2 \tau_v^{-1} + \frac{1}{N}\beta_N^2 \tau_e^{-1} + \sigma_{z_0}^2} \left(\beta_N \tilde{v} + \beta_N \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + \tilde{z}_0 \right).$$

The expression for the limiting price in this model is also well known (e.g. Vives (1995); alternatively it is a consequence of our proof below):

$$\tilde{p}_\infty = \frac{\beta_\infty \tau_v^{-1}}{\beta_\infty^2 \tau_v^{-1} + \sigma_{z_0}^2} \left(\beta_\infty \tilde{v} + \tilde{z}_0 \right) \text{ where } \beta_\infty = \frac{\tau_e}{\rho}. \tag{10}$$

Let us prove first two lemmas.

Lemma 1. $\zeta_N \propto (\varphi_I^c)_N \propto (\varphi_I)_N \propto 1/N$ and $(\varphi_I)_N - (\varphi_I^c)_N \propto 1/N^2$.

Proof of Lemma 1: The (equal) left hand sides of (8) grow at the rate N. Thus the right hand sides of both expressions should grow at the rate N. Using (6) this implies that $(\varphi_I^c)_N \propto 1/N$, $(\varphi_I)_N \propto 1/N$. and $\zeta \propto 1/N$. Now from (8) it follows that $\frac{(1-\varphi_I^c)^3}{\varphi_I^c} = \frac{(1-\varphi_I)^3}{\varphi_I} \frac{(1-2\zeta)^2}{(1-\zeta)^2}$. Therefore $\frac{(1-\varphi_I^c)^3}{(1-\varphi_I)^3} \frac{\varphi_I}{\varphi_I^c} - 1 = \frac{(1-2\zeta)^2}{(1-\zeta)^2} - 1$. Then $\frac{(1-2\zeta)^2}{(1-\zeta)^2} - 1 = \frac{\zeta(2-3\zeta)}{(1-\zeta)^2} \propto 1/N$ and $\frac{(1-\varphi_I^c)^3}{(1-\varphi_I)^3} \frac{\varphi_I}{\varphi_I^c} - 1 = \frac{(1-\varphi_I^c)^3}{(1-\varphi_I)^3} \left(\frac{\varphi_I}{\varphi_I^c} - 1 \right) + \left(\frac{(1-\varphi_I^c)^3}{(1-\varphi_I)^3} - 1 \right) \propto N(\varphi_I - \varphi_I^c) + O\left(\varphi_I - \varphi_I^c\right) \propto N(\varphi_I - \varphi_I^c)$. Hence $(\varphi_I)_N - (\varphi_I^c)_N \propto 1/N^2$.■

Lemma 2. $(\beta_N^c - \beta_\infty) \propto (\beta_N - \beta_\infty) \propto (\beta_N - \beta_N^c) \propto 1/N$.

Proof of Lemma 2: Recall from (5) that $\beta^c = \frac{\tau_e}{\rho}(1 - \varphi_I^c)$ and $\beta = \frac{\tau_e}{\rho}(1 - \varphi_I)\frac{(1-2\zeta)}{(1-\zeta)}$. Thus $\beta_N^c - \beta_\infty = \frac{\tau_e}{\rho}\varphi_I^c \propto \frac{1}{N}$ and since $1 - (1 - \varphi_I)\frac{(1-2\zeta)}{(1-\zeta)} = \varphi_I + (1 - \varphi_I)\frac{\zeta}{(1-\zeta)} \propto 1/N$, we have that $\beta_N - \beta_\infty \propto 1/N$. From (2) we have

$$\varphi_I - \varphi_I^c = \frac{(N-1)\sigma_z^2 \tau_e (\beta_N^2 - (\beta_N^c)^2)}{\left((N-1)\beta_N^2 + \sigma_z^2 \tau_e\right)\left((N-1)(\beta_N^c)^2 + \sigma_z^2 \tau_e\right)} \propto \frac{N^3(\beta_N - \beta_N^c)}{N^4}.$$

Thus $\beta_N - \beta_N^c \propto 1/N$.■

Denote by $M^{\infty} := \beta_{\infty}^2 \tau_v^{-1} + \sigma_{z_0}^2$, by $M^c := (\beta_N^c)^2 \tau_v^{-1} + \frac{1}{N}(\beta_N^c)^2 \tau_e^{-1} + \sigma_{z_0}^2$, and by $M := \beta_N^2 \tau_v^{-1} + \frac{1}{N}\beta_N^2 \tau_e^{-1} + \sigma_{z_0}^2$. Then from (9) we get

$$\tilde{p}_N - \tilde{p}_N^c = a\tilde{v} + a\frac{1}{N}\sum_{n=1}^{N}\tilde{e}_n + b\tilde{z}_0,$$

where $a := \frac{\beta_N^2 \tau_v^{-1}}{M} - \frac{(\beta_N^c)^2 \tau_v^{-1}}{M^c}$, and $b := \frac{\beta_N \tau_v^{-1}}{M} - \frac{\beta_N^c \tau_v^{-1}}{M^c}$. We have that $a \propto (\beta_N - \beta_N^c) \propto \frac{1}{N}$ and $b = O(1/N)$. So

$$var\left[\tilde{p}_N - \tilde{p}_N^c\right] = a^2 \tau_v^{-1} + a^2 \frac{1}{N}\tau_e^{-1} + b^2 \sigma_{z_0}^2 \propto 1/N^2.$$

Then from (9) and (10) we obtain

$$\tilde{p}_N^c - \tilde{p}_{\infty} = f\tilde{v} + g\frac{1}{N}\sum_{n=1}^{N}\tilde{e}_n + h\tilde{z}_0,$$

where $f := \frac{(\beta_N^c)^2 \tau_v^{-1}}{M^c} - \frac{\beta_{\infty}^2 \tau_v^{-1}}{M^{\infty}}$, $g := \frac{(\beta_N^c)^2 \tau_v^{-1}}{M^c}$, and $h := \frac{\beta_N^c \tau_v^{-1}}{M^c} - \frac{\beta_{\infty} \tau_v^{-1}}{M^{\infty}}$.
Then $f = O(1/N)$ and $h = O(1/N)$. Finally, g converges to the constant. Thus

$$var\left[\tilde{p}_N^c - \tilde{p}_{\infty}\right] = f^2 \tau_v^{-1} + g^2 \frac{1}{N}\tau_e^{-1} + h^2 \sigma_{z_0}^2 \propto 1/N. \blacksquare$$

Proof of Proposition 5: The proof proceeds in three steps. We deal in some detail with the case of the competitive equilibrium. The case of the strategic equilibrium follows similarly.

Step 1. Let us start with an *exogenously* given number of informed speculators $N > 0$ and let us compare $\Pi^c(N)$ and F^*. Kyle (1989, Theorem 10.1) provides the following expression for the certainty equivalents of profits in the competitive case:

$$\Pi^c(N) = \frac{1}{2\rho}\log\left(1 + \frac{(1-\varphi_I^c)(1-\varphi_U^c)\tau_e}{\tau_U^c}\right).$$

To simplify notation let us define

$$\phi^c(\sigma_z, N) \equiv \frac{(1-\varphi_I^c)(1-\varphi_U^c)\tau_e}{\tau_U^c} = \frac{(1-\varphi_I^c)(1-\varphi_U^c)\tau_e}{\tau_v + \varphi_U^c N \tau_e} = \frac{(1-\varphi_I^c)(1-\varphi_U^c)}{1+\varphi_U^c N \tau_e \tau_v^{-1}}\frac{\tau_e}{\tau_v}.$$

Since $(1-\varphi_I^c)(1-\varphi_U^c) \leqslant 1$ and $1 + \varphi_U^c N \tau_e \tau_v^{-1} \geqslant 1$ we have that $0 \leqslant \phi^c(\sigma_z, N) \leqslant \frac{\tau_e}{\tau_v}$ and therefore

$$0 \leqslant \Pi^c(N) \leqslant \frac{1}{2\rho}\log\left(1 + \frac{\tau_e}{\tau_v}\right) = F^*.$$

Step 2. Now let us consider various cases of possible growth of σ_z and N and their effect on

$\Pi^c(N)$:

Case 1). If N grows slower than at the rate σ_z^2, then the right hand side of characterization (8) increases at the rate σ_z^2/N. This is possible only if φ_I^c converges to 0 at the rate N/σ_z^2. Then it follows from (3) that φ_U^c converges to 0 also at the rate N/σ_z^2. The following subcases of this case should be separated.

Subcase a) of case 1). If N grows slower than at the rate σ_z, then we have that $\tau_U^c = \tau_v + \varphi_U^c N \tau_e$ converges to τ_v. So, the limit model is *informationally trivial.* Moreover $\phi^c(\sigma_z, N)$ converges to $\frac{\tau_e}{\tau_v}$ and $\Pi^c(N)$ converges to F^*.

Subcase b) of case 1). If $\sigma_z \propto N$, then $\varphi_I^c \propto \varphi_U^c \propto N^{-1}$. Hence there exist positive constants k_1 and k_2 such that $k_1 \leqslant \varphi_U^c N \leqslant k_2$ for all large N. Therefore there exist constants F_1 and F_2 such that $0 < F_1 \leqslant \Pi^c(N) \leqslant F_2 < F^*$ for all large N. In this case it is not possible that $\Pi^c(N)$ converges to 0 or F^*.

Notice that $\Pi^c(N)$ would converge to some constant F', $0 < F' < F^*$ and the limit model will be well defined if σ_z/N converges to some positive number. If the limit model is well defined, the limit will be *neither value revealing nor informationally trivial.*

Subcase c) of case 1). Now let us consider the case that N grows faster than at the rate σ_z but slower than at the rate σ_z^2. Then $\tau_U^c = \tau_v + \varphi_U^c N \tau_e$ increases to infinity at the rate N^2/σ_z^2. Therefore $\phi^c(\sigma_z, N)$ and $\Pi^c(N)$ converge to 0. The limit will be *value revealing.* (Recall that $var\,[\tilde{v} - \tilde{p}_N^c] = var\,[\tilde{v} \mid \tilde{p}_N^c] = (\tau_U^c)^{-1}$ and thus converges to zero.)

Case 2). If N grows at the rate σ_z^2 or faster, then the right hand side of characterization (8) is bounded by some number. Therefore φ_I^c and (by (3)) φ_U^c do not converge to 0. Since $\varphi_U^c \leqslant 1$ we get that $(\tau_U^c)_N = \tau_v + \varphi_U^c N \tau_e$ increases to infinity at the rate N. Therefore $\phi^c(\sigma_z, N)$ and $\Pi^c(N)$ converge to 0. The limit will be *value revealing.* (Recall again that $var\,[\tilde{v} - \tilde{p}_N^c] = var\,[\tilde{v} \mid \tilde{p}_N^c] = (\tau_U^c)^{-1}$ and thus converges to zero.)

Summarizing all the cases: $\Pi^c(N)$ converges to F^* if N grows slower than at the rate σ_z (notice that this covers the case when N does not grow at all) - case 1a above; $0 < F_1 \leqslant \Pi^c(N) \leqslant F_2 < F^*$ for all large N if N grows at the same rate as σ_z - case 1b above; and $\Pi^c(N)$ converges to 0 if N grows faster than at the rate σ_z - cases 1c and 2 above.

Moreover notice that $\Pi^c(N+1)$ always converges to the same limit as $\Pi^c(N)$.

<u>*Step 3.*</u> Now let us turn to the *endogenously* determined number of informed speculators $N_c^*(\sigma_z)$. Recall that it is defined so that

$$\Pi^c(N_c^*(\sigma_z)) \geq F(\sigma_z) > \Pi^c(N_c^*(\sigma_z) + 1).$$

Thus, as in Step 2, $\Pi^c(N_c^*(\sigma_z))$ and $\Pi^c(N_c^*(\sigma_z) + 1)$ should converge to the same limit. For $F(\infty) \leq F^*$ we can conclude that $\Pi^c(N_c^*(\sigma_z)) \to F(\infty)$. For $F(\infty) > F^*$ we must have $N_c^*(\sigma_z) = 0$ for all large σ_z. Three cases are possible now.

(I) If $F(\infty) \geq F^*$, then either $F(\infty) > F^*$ and $N_c^*(\sigma_z) = 0$ for all large σ_z or $F(\infty) = F^*$

and by exclusion we must be in the case (1a) above. In either variant $N_c^*(\sigma_z) \sim o(\sigma_z)$ and the limit market is *informationally trivial*. (Note that if $F(\cdot) \equiv F > 0$, $N > 0$ speculators can chose to become informed only if $\Pi^c(N) \geqslant F$. For $F > F^*$ this is not possible since $\Pi^c(N)$ never exceeds $F^*$for any N. For $F = F^*$ this would require $\varphi_I^c = \varphi_U^c = 0$. Then by (5) it will imply that $\beta^c = \frac{\tau_e}{\rho}$. But then our assumption that $N > 0$ makes the second formula of (2) impossible to hold. A contradiction. Hence $N_c^*(\sigma_z) = 0$.)

(II) If $0 < F(\infty) < F^*$, then by exclusion only the case (1b) above is possible. Therefore $N_c^*(\sigma_z) \propto \sigma_z$ and both φ_I^c and φ_U^c converge to 0. Moreover, since $\Pi^c(N_c^*(\sigma_z)) \to F(\infty)$ it should be that $\tau_U^c \to \tau_e/[\exp(2\rho F(\infty)) - 1]$. Therefore, since $\tau_U^c = \tau_v + \varphi_U^c N\tau_e$, we get that $\varphi_U^c N_c^*(\sigma_z) \to 1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e$. Then by (3) $\varphi_I^c N_c^*(\sigma_z) \to 1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e$ and by (8)

$$\frac{\sigma_z}{N_c^*(\sigma_z)} \to \sigma_{z_0} \equiv \frac{1}{\rho}\sqrt{\frac{\tau_e}{1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e}}.$$

So, the limit market is well defined and is *neither value revealing nor informationally trivial*.

(III) If $F(\infty) = 0$, then by exclusion only the cases (1c) or (2) above are possible. Therefore $\sigma_z \sim o(N_c^*(\sigma_z))$ and the limit market is *value revealing*. (Note that if $F(\cdot) \equiv F = 0$, then obviously $N_c^*(\sigma_z) = \infty$.)

Now let us briefly consider the case of the strategic equilibrium. The proof in this case goes along the same lines as in the competitive case. Kyle (1989, Theorem 10.1) provides the following expression for the certainty equivalents of profits in strategic case:

$$\Pi(N) = \frac{1}{2\rho}\log\left(1 + \frac{(1-\varphi_I)(1-\varphi_U)\tau_e}{\tau_U}\frac{(1-2\zeta)}{(1-\zeta)^2}\right).$$

We can similarly define $\phi(\sigma_z, N) \equiv \frac{(1-\varphi_I)(1-\varphi_U)\tau_e}{\tau_U}\frac{(1-2\zeta)}{(1-\zeta)^2} = \frac{(1-\varphi_I)(1-\varphi_U)}{1+\varphi_U N\tau_e\tau_v^{-1}}\frac{(1-2\zeta)}{(1-\zeta)^2}\frac{\tau_e}{\tau_v}$. Since $(1-\varphi_I)(1-\varphi_U) \leqslant 1$, $(1-2\zeta) \leqslant (1-\zeta)^2$ and $1 + \varphi_U N\tau_e\tau_v^{-1} \geqslant 1$ we have $0 \leqslant \Pi(N) \leqslant F^*$ for any N. Now an analysis completely analogous to steps 2 above similarly demonstrates that $\Pi(N)$ converges to F^* if N grows slower than at the rate σ_z; $0 < F_1 \leqslant \Pi(N) \leqslant F_2 < F^*$ for all large N if N grows at the same rate as σ_z; and $\Pi(N)$ converges to 0 if N grows faster than at the rate σ_z. Then again an analysis completely analogous to step 3 above shows that if $F(\infty) \geq F^*$, then $N^*(\sigma_z) \sim o(\sigma_z)$ and the limit market is informationally trivial; and if $F(\infty) = 0$, then $\sigma_z \sim o(N^*(\sigma_z))$ and the limit market is value revealing. If $0 < F(\infty) < F^*$, then by exclusion of other cases $N^*(\sigma_z) \propto \sigma_z$ and both φ_I, φ_U, and ζ converge to 0. Moreover, since $\Pi(N^*(\sigma_z)) \to F(\infty)$, it should be that $\tau_U \to \tau_e/[\exp(2\rho F(\infty)) - 1]$. Therefore, since $\tau_U = \tau_v + \varphi_U N\tau_e$, we get that $\varphi_U N^*(\sigma_z) \to 1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e$. Then by (3)

$\varphi_I N^*(\sigma_z) \to 1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e$ and by (8)

$$\frac{\sigma_z}{N^*(\sigma_z)} \to \sigma_{z_0} \equiv \frac{1}{\rho}\sqrt{\frac{\tau_e}{1/[\exp(2\rho F(\infty)) - 1] - \tau_v/\tau_e}}.$$

So, the limit market is well defined and is neither value revealing nor informationally trivial. Notice that $\sigma_z/N^*(\sigma_z)$ converges to the same constant σ_{z_0}that was a limit for $\sigma_z/N_c^*(\sigma_z)$ in part II of Step 3 above. Therefore, when $0 < F(\infty) < F^*$ we must have that $N_c^*(\sigma_z)/N^*(\sigma_z)$ converges to 1.■

Fact. $\frac{1}{\rho}\sqrt{\frac{\tau_e}{1/[\exp(2\rho F(\infty))-1]-\tau_v/\tau_e}} \to \infty$ as $\rho \to 0$.

Proof: $\frac{1}{\rho}\sqrt{\frac{\tau_e}{1/[\exp(2\rho F(\infty))-1]-\tau_v/\tau_e}} = \sqrt{\frac{\rho^{-2}\tau_e^2[\exp(2\rho F(\infty))-1]}{\tau_e-\tau_v[\exp(2\rho F(\infty))-1]}}$. Then $\tau_e - \tau_v[\exp(2\rho F(\infty)) - 1]$ converges to τ_e as $\rho \to 0$. But by taking the full Taylor expansion for the exponential it is immediate that $\rho^{-2}\tau_e^2[\exp(2\rho F(\infty)) - 1]$ tends to infinity as $\rho \to 0$.■

Proof of Proposition 6: Let us show result 2 first. We consider the model with $\rho > 0$ and let $\rho \to 0$. Consider an exogenously given number of informed speculators N. Expression (8) and (3) imply for ρ converging 0 that both $\varphi_I^c \to 1$ and $\varphi_U^c \to 1$ at the rate $\rho^{2/3}$.

Applying L'Hôpital rule to the expression for the certainty equivalents of profits given by Kyle (1989, Theorem 10.1), we get

$$\Pi^c(N) = \frac{1}{2\rho}\log\left(1 + \frac{(1-\varphi_I^c)(1-\varphi_U^c)\tau_e}{\tau_U^c}\right) \propto \rho^{1/3} \to 0 \text{ as } \rho \to 0.$$

Therefore $N_c^*(\sigma_z) = 0$ for $\rho = 0$.

Let us turn now to the proof of result 1.[16] From Theorem 8.1 in Kyle (1989) we have that for $\rho = 0$

$$1 = \varphi_I\left\{2 + \frac{\tau_v + N\tau_e}{(N-1)(\tau_v + \varphi_I N\tau_e)}\right\}.$$

It follows (see equation (66) in Kyle (1989)) that the equilibrium parameter φ_I is the positive solution to the quadratic equation:

$$2(N-1)\varphi_I^2 N\tau_e + \varphi_I\left(\tau_v(2N-1) + N\tau_e(2-N)\right) - (N-1)\tau_v = 0$$

Letting $y = \frac{\varphi_I}{1-\varphi_I}$, the positive solution to the equation yields

$$y^* = \frac{\frac{N(N-2)\tau_e-\tau_v}{N} + \sqrt{\left(\frac{\tau_v-N(N-2)\tau_e}{N}\right)^2 + 4\left(\tau_v + N\tau_e\right)\frac{N-1}{N}\tau_v}}{2\left(\tau_v + N\tau_e\right)}.$$

[16]We are very grateful to an anonymous referee for pointing out a mistake in the proof of this result and indicating its correction.

This solution pins down as well as $\beta = \sqrt{\frac{\sigma_z^2 \tau_e}{N-1} y^*}$ (from $\varphi_I = \frac{(N-1)\beta^2}{(N-1)\beta^2 + \sigma_z^2 \tau_e}$) and $\varphi_U = \frac{N\beta^2}{N\beta^2 + \sigma_z^2 \tau_e} = \frac{N\beta y^*}{Ny^* + N - 1}$.

From (4) one can easily see that

$$\tilde{p}_N = \lambda N \left(\beta \tilde{v} + \beta \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + \frac{1}{N} \tilde{z} \right) \text{ where } \lambda = \frac{\beta \tau_v^{-1} N}{\beta^2 \tau_v^{-1} N^2 + N\beta^2 \tau_e^{-1} + \sigma_z^2}$$
$$= \frac{\tau_v^{-1}}{\tau_v^{-1} + N^{-1}\tau_e^{-1} + \beta^{-2} N^{-2} \sigma_z^2} \left(\tilde{v} + \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + \beta^{-1} \frac{1}{N} \tilde{z} \right).$$

Since $\beta = \sigma_z \sqrt{\frac{\tau_e}{N-1} y^*}$ for $\rho = 0$ we obtain that for $\rho = 0$,

$$\tilde{p}_N = \frac{\tau_v^{-1}}{\tau_v^{-1} + N^{-1}\tau_e^{-1} + \frac{N-1}{N^2} y^{*-1} \tau_e^{-1}} \left(\tilde{v} + \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + \frac{1}{N \sqrt{\frac{\tau_e}{N-1} y^*}} \frac{\tilde{z}}{\sigma_z} \right).$$

Then we can conclude that $\tilde{p}_N \to \tilde{v}$ if $N \to \infty$ for $\sigma_z \to \infty$ since it can be checked that y^* is of the order of a constant.

Note that λ is of the order of $\left(\sigma_z \sqrt{N}\right)^{-1}$ since β is of the order of $\sigma_z \sqrt{\frac{1}{N-1}}$. This allows us to find the order of magnitude of expected profits in the risk neutral case $\Pi_0(N)$. Indeed, $\Pi_0(N) = \lambda \sigma_z^2 / N$ since they have to equal the expected losses of noise traders. It follows that $\Pi_0(N)$ is of the order of $\sigma_z N^{-3/2}$. We have that $\Pi_0(N) \to 0$ as $N \to \infty$. We know that $\Pi_0(N^*(\sigma_z))$ will equal approximately $F(\sigma_z)$ (and the approximation will be good for σ_z large). It follows that $N^*(\sigma_z) \propto (\frac{\sigma_z}{F(\sigma_z)})^{2/3}$. Note also that $N^*(\sigma_z)\beta^*$ grows faster than the size of the market σ_z (since $\frac{\sigma_z}{N^*(\sigma_z)\beta^*} \propto \left(\frac{F(\sigma_z)}{\sigma_z}\right)^{1/3}$)■

8 Online Appendix

8.1 Asymptotic variances and calibration in the central scenario

We characterize first the asymptotic variances of convergence and their comparative static properties in the central scenario where N grows proportionally with σ_z. We look afterwards at their calibration with parameters from different financial market scenarios.

8.1.1 Asymptotic variances

Proposition A1: Let A_{L} denote the asymptotic standard deviation of the limit effect and A_{S} denote the one of the strategic effect. Then:

1. $\lim_{N\to\infty} N\sqrt{\mathrm{var}[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}]} \equiv A_{\mathrm{S}} = \dfrac{\tau_e^2}{\sigma_{z_0}\rho(\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2)}\left(1+\dfrac{\tau_e\sigma_{z_0}^2\rho^2}{\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2}\right)$;
2. $\lim_{N\to\infty}\sqrt{N}\sqrt{\mathrm{var}[\tilde{p}_N^{\mathrm{c}} - \tilde{p}_\infty]} = \lim_{N\to\infty}\sqrt{N}\sqrt{\mathrm{var}[\tilde{p}_N - \tilde{p}_\infty]} \equiv A_{\mathrm{L}} = \dfrac{\tau_e^{3/2}}{\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2}$.

Proof: We will follow the notation from the proof of Proposition 1.
1). It appears from the proof of Proposition 1 that:

$$\lim_{N\to\infty} N var\left[\tilde{p}_N^c - \tilde{p}_\infty\right] = \lim_{N\to\infty} g^2\tau_e^{-1} = \lim_{N\to\infty}\left(\frac{(\beta_N^c)^2\tau_v^{-1}}{M^c}\right)^2\tau_e^{-1}.$$

Then $\lim_{N\to\infty} M^c = M^\infty = \beta_\infty^2\tau_v^{-1}+\sigma_{z_0}^2$ and $\lim_{N\to\infty}\beta_N^c = \beta_\infty = \frac{\tau_e}{\rho}$. Thus

$$\lim_{N\to\infty} N var\left[\tilde{p}_N^c - \tilde{p}_\infty\right] = \left(\frac{\left(\frac{\tau_e}{\rho}\right)^2\tau_v^{-1}}{\left(\frac{\tau_e}{\rho}\right)^2\tau_v^{-1}+\sigma_{z_0}^2}\right)^2\tau_e^{-1} = \frac{\tau_e^3}{(\tau_e^2+\sigma_{z_0}^2\rho^2\tau_v)^2}.$$

2). It follows from the proof of Proposition 1 that:

$$\lim_{N\to\infty} N^2 var\left[\tilde{p}_N - \tilde{p}_N^c\right] = \lim_{N\to\infty}\left[N^2(a^2\tau_v^{-1}+b^2\sigma_{z_0}^2)\right].$$

Since $a = (MM^c)^{-1}\left[(\beta_N^2-(\beta_N^c)^2)\tau_v^{-1}\sigma_{z_0}^2\right]$ we have

$$\lim_{N\to\infty} Na = \frac{2\beta_\infty\sigma_{z_0}^2}{\tau_v(M^\infty)^2}\lim_{N\to\infty}\left[N(\beta_N-\beta_N^c)\right].$$

Since $b = (\tau_v MM^c)^{-1}\left[\beta_N\beta_N^c(\beta_N^c-\beta_N)(\tau_v^{-1}+\frac{1}{N}\tau_e^{-1})+(\beta_N-\beta_N^c)\sigma_{z_0}^2\right]$, we have

$$\lim_{N\to\infty} Nb = \frac{\beta_\infty^2\tau_v^{-1}-\sigma_{z_0}^2}{\tau_v(M^\infty)^2}\lim_{N\to\infty}\left[N(\beta_N^c-\beta_N)\right].$$

Therefore $\lim_{N\to\infty} N^2 var\left[(\tilde{p}_N - \tilde{p}_N^c)\right] = \lim_{N\to\infty}\left[N^2(a^2\tau_v^{-1} + b^2\sigma_{z_0}^2)\right] =$

$$\begin{aligned}
&= \left[\tau_v (M^\infty)^2\right]^{-2}\left[\left(2\beta_\infty \sigma_{z_0}^2\right)^2 \tau_v^{-1} + \left(\beta_\infty^2 \tau_v^{-1} - \sigma_{z_0}^2\right)^2 \sigma_{z_0}^2\right]\left(\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right]\right)^2 \\
&= \left[\tau_v (M^\infty)^2\right]^{-2}\left[\left(\beta_\infty^2 \tau_v^{-1} + \sigma_{z_0}^2\right)^2 \sigma_{z_0}^2\right]\left(\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right]\right)^2 \\
&= \left[\tau_v M^\infty\right]^{-2}\sigma_{z_0}^2\left(\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right]\right)^2 = \frac{\sigma_{z_0}^2\rho^4}{\left(\tau_e^2 + \sigma_{z_0}^2\rho^2\tau_v\right)^2}\left(\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right]\right)^2.
\end{aligned}$$

Now we have from (5) that $\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right] =$

$$= \frac{\tau_e}{\rho}\lim_{N\to\infty}\left[N(1-\varphi_I^c) - N(1-\varphi_I)\frac{(1-2\zeta)}{(1-\zeta)}\right] = \frac{\tau_e}{\rho}\lim_{N\to\infty}\left[(1-\varphi_I)\frac{\zeta N}{(1-\zeta)}\right],$$

since by Lemma 1 $(\varphi_I - \varphi_I^c) \propto 1/N^2$. Then by (6) $\frac{1-\varphi_I}{1-\zeta} = \frac{1}{1-\xi_I}$ and $\lim_{N\to\infty}\frac{1}{1-\xi_I} = 1$. Hence

$$\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right] = \frac{\tau_e}{\rho}\lim_{N\to\infty}\zeta N.$$

As we mentioned in the appendix to the paper, one conclusion in Theorem 9.2 in Kyle (1989) actually requires a qualification when $M = \infty$. Theorem 9.2 in Kyle (1989) states that $\lim_{N\to\infty}\xi_U M = 0$ which is equivalent to $\lim_{N\to\infty}\xi_I N = 1$. The following lemma corrects that statement when $M = \infty$.

Lemma A1. $\lim_{N\to\infty}\xi_I N = \frac{\tau_e^2}{\tau_v\sigma_{z_0}^2\rho^2 + \tau_e^2}$ and

$$\lim_{N\to\infty}\zeta N = \frac{\tau_e}{\sigma_{z_0}^2\rho^2}\left(1 + \frac{\tau_e}{\tau_v + \frac{\tau_e^2}{\sigma_{z_0}^2\rho^2}}\right) = \frac{\tau_e}{\sigma_{z_0}^2\rho^2}\frac{(\tau_v+\tau_e)\sigma_{z_0}^2\rho^2 + \tau_e^2}{\tau_v\sigma_{z_0}^2\rho^2 + \tau_e^2}.$$

Proof: From formula (7) $\zeta = \varphi_U\frac{\tau_I}{\tau_U} = \varphi_U\frac{\tau_v + \tau_e + \varphi_I(N-1)\tau_e}{\tau_v + \varphi_U N\tau_e}$. Thus $\lim_{N\to\infty}\zeta N = \lim_{N\to\infty}(N\varphi_U\frac{\tau_v+\tau_e+\varphi_I(N-1)\tau_e}{\tau_v+\varphi_U N\tau_e})$ and since $\lim_{N\to\infty}N\varphi_U = \lim_{N\to\infty}N\varphi_I = \frac{\tau_e}{\sigma_{z_0}^2\rho^2}$ (see (3)), we get $\lim_{N\to\infty}\zeta N = \frac{\tau_e}{\sigma_{z_0}^2\rho^2}\frac{\tau_v+\tau_e+\frac{\tau_e}{\sigma_{z_0}^2\rho^2}\tau_e}{\tau_v+\frac{\tau_e}{\sigma_{z_0}^2\rho^2}\tau_e} = \frac{\tau_e}{\sigma_{z_0}^2\rho^2}\left(1+\frac{\tau_e}{\tau_v+\frac{\tau_e^2}{\sigma_{z_0}^2\rho^2}}\right)$. But from (6) we have that $\zeta = \xi_I + \varphi_I - \xi_I\varphi_I$. Hence $\lim_{N\to\infty}\xi_I N = \lim_{N\to\infty}\zeta N - \lim_{N\to\infty}\varphi_I N = \frac{\frac{\tau_e^2}{\sigma_{z_0}^2\rho^2}}{\tau_v+\frac{\tau_e^2}{\sigma_{z_0}^2\rho^2}} = \frac{\tau_e^2}{\tau_v\sigma_{z_0}^2\rho^2+\tau_e^2}$.■

Using Lemma A1, we get

$$\lim_{N\to\infty}\left[N(\beta_N^c - \beta_N)\right] = \frac{\tau_e}{\rho}\lim_{N\to\infty}\zeta N = \frac{\tau_e^2}{\sigma_{z_0}^2\rho^3}\frac{(\tau_v+\tau_e)\sigma_{z_0}^2\rho^2 + \tau_e^2}{\tau_v\sigma_{z_0}^2\rho^2 + \tau_e^2}.$$

Therefore

$$\lim_{N\to\infty} N^2 var\left[\tilde{p}_N - \tilde{p}_N^c\right] = \frac{\sigma_{z_0}^2\rho^4}{\left(\tau_e^2+\sigma_{z_0}^2\rho^2\tau_v\right)^2}\frac{\tau_e^4}{\sigma_{z_0}^4\rho^6}\frac{(\tau_e^2+(\tau_v+\tau_e)\sigma_{z_0}^2\rho^2)^2}{\left(\tau_e^2+\sigma_{z_0}^2\rho^2\tau_v\right)^2} = \frac{\tau_e^4}{\sigma_{z_0}^2\rho^2}\frac{(\tau_e^2+(\tau_v+\tau_e)\sigma_{z_0}^2\rho^2)^2}{\left(\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2\right)^4}. \blacksquare$$

Comparative statics for the limit effect We have that

$$A_{\mathrm{L}} = \frac{\tau_e^{3/2}}{\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2}.$$

The following comparative statics results are immediate.

(1) A_{L} *monotonically decreases* in the precision of the prior τ_v. If $\tau_v \to 0+$, then $A_{\mathrm{L}} \to 1/\sqrt{\tau_e}$. If $\tau_v \to +\infty$, then $A_{\mathrm{L}} \to 0$. This result is quite intuitive. The lower the variance of the value of the asset, the faster equilibrium prices converge to the limit.

(2) A_{L} *monotonically decreases* in "risk-bearing adjusted noise trade" $\rho\sigma_{z_0}$. If $\rho\sigma_{z_0} \to 0+$, then $A_{\mathrm{L}} \to 1/\sqrt{\tau_e}$. If $\rho\sigma_{z_0} \to +\infty$, then $A_{\mathrm{L}} \to 0$. The fact that A_{L} decreases in $\rho\sigma_{z_0}$ may seem surprising. How can it be that more noise trading or a higher degree of risk aversion improve the convergence to the limit price? However, recall that a term corresponding to the average information noise in the finite market $((1/N)\sum_{n=1}^N \tilde{e}_n)$ determines the convergence to the limit price. The more traders are risk averse the less weight they put on their signals and the less information noise is incorporated into the equilibrium price. Similarly, the larger noise trading is the smaller the information noise portion incorporated into the equilibrium price. Thus, an increase in $\rho\sigma_{z_0}$ causes a decrease in the amount of information noise incorporated into the price and therefore decreases the asymptotic variance of the limit effect.

(3) A_{L} is *always nonmonotonic* in the precision of the signal τ_e. If $\tau_e \to 0+$ or if $\tau_e \to +\infty$, then $A_{\mathrm{L}} \to 0$. Actually, A_{L} is *increasing* for $\tau_e < \sigma_{z_0}\rho\sqrt{3\tau_v}$ and *decreasing* for $\tau_e > \sigma_{z_0}\rho\sqrt{3\tau_v}$. At $\tau_e = \sigma_{z_0}\rho\sqrt{3\tau_v}$, A_{L} reaches a maximum. Thus, if signals become very informative or very noisy, then A_{L} becomes arbitrary small and equilibrium prices converge to the limit faster. Indeed, in both cases the average noise in the signals does not matter much in determining the price in a finite market: when signals are very noisy traders put very little weight on them, and when they are very precise signal noise is very small.

Comparative statics for the strategic effect We have that

$$A_{\mathrm{S}} = \frac{\tau_e^2}{\sigma_{z_0}\rho(\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2)}\left(1+\frac{\tau_e\sigma_{z_0}^2\rho^2}{\tau_e^2+\tau_v\sigma_{z_0}^2\rho^2}\right) = A_{\mathrm{L}}\frac{\tau_e^{1/2}}{\sigma_{z_0}\rho}\left(1+\frac{1}{\tau_e/\sigma_{z_0}^2\rho^2+\tau_v/\tau_e}\right).$$

The following comparative statics results are immediate.

(1) A_{S} *monotonically decreases* in τ_v. If $\tau_v \to 0+$ then $A_{\mathrm{S}} \to (\sigma_{z_0}\rho)^{-1} + \tau_e^{-1}(\sigma_{z_0}\rho)$. If $\tau_v \to +\infty$ then $A_{\mathrm{S}} \to 0$. Again, the lower the variance of the value of the asset is, the faster the strategic effect disappears.

(2) A_{S} either *monotonically decreases* or is *nonmonotonic* in $\rho\sigma_{z_0}$ depending on other parameters. (For example, if $\tau_e = 1$ and $\tau_v = 1$, then A_{S} monotonically decreases in $\rho\sigma_{z_0}$. However, if $\tau_e = 1$ and $\tau_v = 0.01$, then for $\rho\sigma_{z_0} = 0.5$ we obtain $A_{\mathrm{S}} = 2.49$; for $\rho\sigma_{z_0} = 1$ we obtain $A_{\mathrm{S}} = 1.97$; and for $\rho\sigma_{z_0} = 2$ we obtain $A_{\mathrm{S}} = 2.33$.) Furthermore, if $\rho\sigma_{z_0} \to 0+$, then $A_{\mathrm{S}} \to +\infty$. If $\rho\sigma_{z_0} \to +\infty$, then $A_{\mathrm{S}} \to 0$. If the noise trading is small for the risk-bearing capacity of the informed traders, then the strategic effect disappears more slowly. In fact, with risk neutrality competitive prices become fully revealing and the strategic and competitive equilibria converge to each other at a slower rate than in Proposition 1 (since $A_{\mathrm{S}} \to +\infty$). In fact, $\{\lim_{\rho\to 0}\sqrt{\operatorname{var}\left[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}\right]}\} \propto 1/\sqrt{N}$ as $\tilde{p}_N^{\mathrm{c}} \to \tilde{v}$ for $\rho \to 0$. If the noise trade is large for the risk-bearing capacity of the informed traders the opposite happens ($A_{\mathrm{S}} \to 0$).

(3) A_{S} is *always nonmonotonic* in τ_e. In general, the behavior of A_{S} is very complicated. However, A_{S} can be presented as the product of two functions: $A_{\mathrm{S}} = QR$, where

$$Q = \frac{\tau_e^2}{\sigma_{z_0}\rho(\tau_e^2 + \tau_v\sigma_{z_0}^2\rho^2)} = \frac{1}{\sigma_{z_0}\rho(1 + \tau_v\sigma_{z_0}^2\rho^2/\tau_e^2)} \quad \text{and}$$
$$R = \left(1 + \frac{\tau_e\sigma_{z_0}^2\rho^2}{\tau_e^2 + \tau_v\sigma_{z_0}^2\rho^2}\right) = \left(1 + \frac{1}{\tau_e/\sigma_{z_0}^2\rho^2 + \tau_v/\tau_e}\right).$$

Then Q is monotonically increasing in τ_e. However, R is increasing in τ_e for $\tau_e < \sigma_{z_0}\rho\sqrt{\tau_v}$ and R is decreasing for $\tau_e > \sigma_{z_0}\rho\sqrt{\tau_v}$. If $\tau_e \to 0+$, then $Q \to 0$ and $R \to 1$, so $A_{\mathrm{S}} \to 0$. Clearly, A_{S} is *increasing* in τ_e for $\tau_e < \sigma_{z_0}\rho\sqrt{\tau_v}$. If $\tau_e \to +\infty$, then Q increases to the value of $(\sigma_{z_0}\rho)^{-1}$ in the limit (and it reaches the limiting value at the rate $1/\tau_e^2$). However, if $\tau_e \to +\infty$, then R is decreasing to the value of 1 in the limit. Function R reaches the limiting value at the rate $1/\tau_e$. Therefore if $\tau_e \to +\infty$, then $A_{\mathrm{S}} \to (\sigma_{z_0}\rho)^{-1}$ and A_{S} is *decreasing* in τ_e. (Alternatively, one can check directly that the sign of the derivative of A_{S} is negative for $\tau_e \to +\infty$.)

At $\tau_e = \sigma_{z_0}\rho\sqrt{\tau_v}$, R reaches a maximum of $(1 + \sigma_{z_0}\rho/\sqrt{\tau_v})$ while $Q = 1/(2\sigma_{z_0}\rho)$. Generally $Q < 1/\sigma_{z_0}\rho$. Therefore, the *maximum value* of A_{S} is between

$$\frac{1}{2\sigma_{z_0}\rho}\left(1 + \frac{\sigma_{z_0}\rho}{\sqrt{\tau_v}}\right) \quad \text{and} \quad \frac{1}{\sigma_{z_0}\rho}\left(1 + \frac{\sigma_{z_0}\rho}{\sqrt{\tau_v}}\right).$$

Thus, if signals are very noisy, then A_{S} (as A_{L}) becomes arbitrary small. For signals that are very informative A_{S} reaches some positive number (while A_{L} becomes arbitrary small). The difference in the behavior of the two asymptotic variances is again a result of the fact that it is not only the information noise that is a factor for the strategic effect.

We have that $A_{\mathrm{S}} > A_{\mathrm{L}} \frac{\tau_e^{1/2}}{\sigma_{z_0}\rho}$. For practical purposes the limit effect will dominate the strategic one whenever $A_{\mathrm{L}}/\sqrt{N} > A_{\mathrm{S}}/N$. Therefore for such domination to work we need to have

$$\sqrt{N} > \frac{\tau_e^{1/2}}{\sigma_{z_0}\rho} \quad \text{or} \quad N > \frac{\tau_e}{\sigma_{z_0}^2 \rho^2}.$$

In summary, we have that A_{S} is small, and the approximation of the strategic equilibrium by the competitive equilibrium good, when the prior volatility of the asset is low, noise trading is large in relation to the risk-bearing capacity of the informed traders, or the signals are very noisy. We confirm, therefore, the idea that the competitive approximation works even in a moderately sized market when the informationally adjusted risk-bearing capacity of the informed traders is not very large (i.e. basically when competitive traders have incentives to be restrained in their trading).

8.1.2 Calibration

Values for stock markets Let us compute the asymptotic variances for some reasonable values of the parameters in a stock market. Let $\rho = 2$ and $\sqrt{\operatorname{var}[\tilde{z}_0]} = \sigma_{z_0} = 0.1$. For the volatility of the fundamentals let us consider two cases: (i) $\sqrt{\operatorname{var}[\tilde{v}]} = \tau_v^{-1/2} = 0.2$ (NYSE type) or (ii) $\sqrt{\operatorname{var}[\tilde{v}]} = \tau_v^{-1/2} = 0.6$ (Nasdaq type). These parameters are chosen to reflect average market data. (See Table 1 in Leland (1992).)

Table 2 presents results for $\sqrt{\operatorname{var}[\tilde{e}_n]} = \tau_e^{-1/2}$ ranging from 0.05 to 20. It is immediate from this table that both A_{S} and A_{L} are nonmonotonic in τ_e. Table 2 is consistent with the statements that A_{S} and A_{L} are decreasing in τ_v and that A_{S} and A_{L} converge to zero as $\tau_e \to 0$. For large τ_e, as predicted, A_{L} converges to zero, while A_{S} converges to $1/(\rho\sigma_{z_0}) = 5$. The standard deviation of the distance between the strategic and competitive price $\sqrt{\operatorname{var}[\tilde{p}_N - \tilde{p}_N^{\mathrm{c}}]}$ is approximated by A_{S}/N. We see that in all scenarios this standard deviation is quite small even with very few informed traders.

		$\tau_e^{-1/2}$						
$\tau_v^{-1/2}$		0.05	0.1	0.5	1	5	10	20
0.2	A_{S}	5.0005	5.0015	4.7502	2.5500	0.0080	0.0005	0.0000
	A_{L}	0.0500	0.1000	0.4706	0.5000	0.0080	0.0009	0.0001
0.6	A_{S}	5.0005	5.0019	5.0148	4.6620	0.0720	0.0045	0.0003
	A_{L}	0.0500	0.1000	0.4966	0.9000	0.0710	0.0090	0.0011

Table 2: Values for A_{S} and A_{L}.

Values for S&P 500 Futures market Cho and Krishnan (2000) address the S&P 500 Futures market. They found that a competitive rational expectations model provides a reasonable description of this market and present estimates of the primitive parameters of the model (in Table 2 of their paper): the standard deviation of the fundamentals at $\sqrt{\mathrm{var}\,[\tilde{v}]} = \tau_v^{-1/2} = 5.495$; and for other parameters ($\sigma_{z_0}\rho$ and $\sqrt{\mathrm{var}\,[\tilde{e}_n]} = \tau_e^{-1/2}$) they presented results, which are summarized in Table 3 together with the corresponding values of the asymptotic standard deviations A_{S} and A_{L}, that differ in the number of the weeks to maturity (from 2 to 7).

	Time to maturity (weeks)					
	2	3	4	5	6	7
$\sigma_{z_0}\rho$	0.021	0.007	0.045	0.038	0.030	0.029
$\tau_e^{-1/2}$	12.541	23.358	10.067	12.925	17.263	20.705
A_{S}	36.76	98.06	14.76	12.44	9.81	5.97
A_{L}	9.21	15.75	5.96	5.54	4.73	3.38

Table 3: Summary of results (Cho and Krishnan (2000)).

The S&P 500 Futures market has high volatility, very noisy information signals and relatively small noise trade. In this context the asymptotic variances are larger than with the stock market parameters. However, since the number of traders in the S&P 500 Futures market is very large, the competitive REE should be a very close approximation of the strategic REE in this market. In the analysis of S&P 500 Futures market one can safely take the shortcut of assuming competitive behavior.

8.2 Proofs of Propositions 2, 3, and 4

Proof of Proposition 2: It follows from Propositions 2A, 2B and 2C.

Proposition 2A. *We have*

1. $\sqrt{E\left[(\tilde{x}_N - \tilde{x}_N^{\mathrm{c}})^2\right]} \propto 1/N$;
2. $\sqrt{E\left[(\tilde{x}_N^{\mathrm{c}} - \tilde{x}_\infty)^2\right]} \propto 1/\sqrt{N}$;
3. $\sqrt{E\left[(\tilde{x}_N - \tilde{x}_\infty)^2\right]} \propto 1/\sqrt{N}$.

Proof: In a symmetric linear equilibrium the demands have the form: $\tilde{x}_N(\tilde{p}, \tilde{\imath}_n) = \mu_N + \beta_N\tilde{\imath}_n - \gamma_N\tilde{p}_N$, $\tilde{x}_N^{\mathrm{c}}(\tilde{p}, \tilde{\imath}_n) = \mu_N^{\mathrm{c}} + \beta_N^{\mathrm{c}}\tilde{\imath}_n - \gamma_N^{\mathrm{c}}\tilde{p}_N^{\mathrm{c}}$, and $\tilde{x}_\infty(\tilde{p}, \tilde{\imath}_n) = \mu_\infty + \beta_\infty\tilde{\imath}_n - \gamma_\infty\tilde{p}_\infty$. Kyle (1989, proofs of theorems 5.1 and 6.1) shows that $\mu_N^{\mathrm{c}} = \mu_N = 0$. Hence $\mu_\infty = 0$ as well. This proves part (I). Recall that we showed in Lemma 2 that $(\beta_N^{\mathrm{c}} - \beta_\infty) \propto (\beta_N - \beta_\infty) \propto (\beta_N - \beta_N^{\mathrm{c}}) \propto 1/N$. Now let us prove an analogous result for γ_N, γ_N^{c}, and γ_∞.

Lemma A2. $\gamma_\infty \neq 0$, $\gamma_N^c - \gamma_\infty = O(1/N)$, $\gamma_N - \gamma_\infty = O(1/N)$, and $\gamma_N - \gamma_N^c = O(1/N)$.

Proof: Kyle (1989, expressions (B.4) and (C.5)) provide the following formulas for γ_N^c and γ_N:

$$\gamma_N^c = \frac{\tau_I}{\rho} - \frac{\varphi_I \tau_e}{\lambda \beta_N \rho} \text{ and } \gamma_N = \frac{\lambda \beta_N \tau_I - \varphi_I \tau_e}{\lambda \beta_N (\lambda_I \tau_I + \rho)} \text{ where } \frac{\lambda}{\lambda_I} = 1 - \xi_I.$$

Corollary 4.1 in Kyle (1989) implies that in our case with the competitive market makers sector we have $\varphi_U \tau_e = \lambda \beta_N \tau_U$. Thus using (3) we get that

$$\begin{aligned}
\gamma_N^c &= \frac{\tau_I}{\rho} - \frac{\varphi_I \tau_U}{\varphi_U \rho} = \frac{\tau_I}{\rho} - \frac{\tau_U}{\rho}(1 - \frac{1}{N}(1 - \varphi_I)) \text{ and that} \\
\gamma_N &= \frac{\tau_I}{\lambda_I \tau_I + \rho} - \frac{\varphi_I \tau_U}{\varphi_U (\lambda_I \tau_I + \rho)} = \frac{\tau_I}{\lambda_I \tau_I + \rho} - \frac{\tau_U}{(\lambda_I \tau_I + \rho)}(1 - \frac{1}{N}(1 - \varphi_I)).
\end{aligned}$$

Then since $(\tau_I)_N = (\tau_I)_\infty + O(1/N)$, $(\tau_U)_N = (\tau_U)_\infty + O(1/N)$, $\varphi_I = O(1/N)$, and $\lambda_I = \frac{\lambda}{1-\xi_I} = O(\lambda) + O(1/N) = O(\varphi_U) + O(1/N) = O(1/N)$ we get that $\gamma_\infty \neq 0$, $\gamma_N^c - \gamma_\infty = O(1/N)$ and $\gamma_N - \gamma_\infty = O(1/N)$. Then as well $\gamma_N - \gamma_N^c = O(1/N)$.
■

Now for simplicity of notation let us rewrite prices expression (9) and (10) in the following way:

$$\begin{aligned}
\tilde{p}_N^c &= A_N^c \tilde{v} + B_N^c \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + C_N^c \tilde{z}_0, \\
\tilde{p}_N &= A_N \tilde{v} + B_N \frac{1}{N} \sum_{n=1}^{N} \tilde{e}_n + C_N \tilde{z}_0, \\
\tilde{p}_\infty &= A_\infty \tilde{v} + C_\infty \tilde{z}_0.
\end{aligned}$$

It follows from the proof of Proposition 1 that A_N^c and A_N converge to A_∞, B_N^c and B_N converge to $B_\infty \neq 0$, and C_N^c and C_N converge to C_∞. All these convergencies also happen at the rate of $1/N$ or faster. Then

$$\tilde{x}_N^c(\tilde{p}, \tilde{\imath}_n) = \beta_N^c \tilde{\imath}_n - \gamma_N^c \tilde{p}_N^c = (\beta_N^c - \gamma_N^c A_N^c)\tilde{v} + (\beta_N^c - \gamma_N^c B_N^c \frac{1}{N})\tilde{e}_n - \gamma_N^c B_N^c \frac{1}{N} \sum_{k \neq n} \tilde{e}_k - \gamma_N^c C_N^c \tilde{z}_0.$$

A similar expression can be written for $\tilde{x}_N(\tilde{p}, \tilde{\imath}_n)$. Now let us define $D_a = (\beta_N^c - \gamma_N^c A_N^c) - (\beta_N - \gamma_N A_N)$, $D_b = (\beta_N^c - \gamma_N^c B_N^c \frac{1}{N}) - (\beta_N - \gamma_N B_N \frac{1}{N})$, $D_c = \gamma_N^c B_N^c - \gamma_N B_N$, and $D_d = \gamma_N^c C_N^c - \gamma_N C_N$. Then we get

$$E\left[\tilde{x}_N^c(\tilde{p}, \tilde{\imath}_n) - \tilde{x}_N(\tilde{p}, \tilde{\imath}_n)\right]^2 = D_a^2 \tau_v^{-1} + D_b^2 \tau_e^{-1} + D_c^2 \frac{N-1}{N^2} \tau_v^{-1} + D_d^2 \sigma_{z_0}^2.$$

It is immediate that $D_a = O(1/N)$, $D_c = O(1/N)$, $D_d = O(1/N)$, while

$$D_b = (\beta^c_N - \beta_N) - \frac{1}{N}(\gamma^c_N B^c_N - \gamma_N B_N) \propto \frac{1}{N} - \frac{1}{N}O(1/N) \propto 1/N.$$

Therefore

$$E\left[\tilde{x}^c_N(\tilde{p}, \tilde{\imath}_n) - \tilde{x}_N(\tilde{p}, \tilde{\imath}_n)\right]^2 \propto 1/N^2.$$

Now let us notice that

$$\tilde{x}_\infty(\tilde{p}, \tilde{\imath}_n) = \beta_\infty \tilde{\imath}_n - \gamma_\infty \tilde{p}_\infty = (\beta_\infty - \gamma_\infty A_\infty)\tilde{v} + \beta_\infty \tilde{e}_n - \gamma_\infty C_\infty \tilde{z}_0.$$

So if we define $K_a = (\beta^c_N - \gamma^c_N A^c_N) - (\beta_\infty - \gamma_\infty A_\infty)$, $K_b = (\beta^c_N - \gamma^c_N B^c_N \frac{1}{N}) - \beta_\infty$, $K_c = \gamma^c_N B^c_N$, and $K_d = \gamma^c_N C^c_N - \gamma_\infty C_\infty$, then we get

$$E\left[\tilde{x}^c_N(\tilde{p}, \tilde{\imath}_n) - \tilde{x}_\infty(\tilde{p}, \tilde{\imath}_n)\right]^2 = K_a^2 \tau_v^{-1} + K_b^2 \tau_e^{-1} + K_c^2 \frac{N-1}{N^2}\tau_v^{-1} + K_d^2 \sigma^2_{z_0}.$$

It is immediate that $K_a = O(1/N)$, $K_b = O(1/N)$, $K_d = O(1/N)$, while $K_c = \gamma^c_N B^c_N$ converges to $\gamma_\infty B_\infty \neq 0$. Hence

$$E\left[\tilde{x}^c_N(\tilde{p}, \tilde{\imath}_n) - \tilde{x}_\infty(\tilde{p}, \tilde{\imath}_n)\right]^2 \propto 1/N. \blacksquare$$

Proposition 2B. *We have*:

1. $\sqrt{E\left[(\tilde{\pi}_N - \tilde{\pi}^c_N)^2\right]} \propto 1/N$;

2. $\sqrt{E\left[(\tilde{\pi}^c_N - \tilde{\pi}_\infty)^2\right]} \propto 1/\sqrt{N}$;

3. $\sqrt{E\left[(\tilde{\pi}_N - \tilde{\pi}_\infty)^2\right]} \propto 1/\sqrt{N}$.

Proof: First for simplicity of notation let us again rewrite the expression for prices (9) as in the proof of Proposition 2A:

$$\tilde{p}^c_N = A^c_N \tilde{v} + B^c_N \frac{1}{N}\sum_{k=1}^N \tilde{e}_k + C^c_N \tilde{z}_0,$$

$$\tilde{p}_N = A_N \tilde{v} + B_N \frac{1}{N}\sum_{k=1}^N \tilde{e}_k + C_N \tilde{z}_0.$$

Then also as in the proof of Proposition 2A let us get an expression for $\tilde{x}^c_N(\tilde{p}, \tilde{\imath}_n)$:

$$\tilde{x}^c_N(\tilde{p}, \tilde{\imath}_n) = (\beta^c_N - \gamma^c_N A^c_N)\tilde{v} + (\beta^c_N - \gamma^c_N B^c_N \frac{1}{N})\tilde{e}_n - \gamma^c_N B^c_N \frac{1}{N}\sum_{k \neq n} \tilde{e}_k - \gamma^c_N C^c_N \tilde{z}_0.$$

Notice that the expression for $\tilde{x}_N(\tilde{p}, \tilde{\imath}_n)$ is just similar (with no superscripts of c). That allows us to get an explicit expression for $\tilde{\pi}^c_n(\tilde{p}, \tilde{\imath}_n) = (\tilde{v} - \tilde{p}^c_N)\tilde{x}^c_N(\tilde{p}, \tilde{\imath}_n)$ as a function of $\tilde{v}$, $\tilde{e}_1$, .., $\tilde{e}_N$, $\tilde{z}_0$. Notice that the expression for $\tilde{\pi}_n$ is again different only in the absence of the superscripts of c. Actually the difference between any corresponding coefficients in $\tilde{\pi}_n$ and $\tilde{\pi}^c_n$ will be of the order $O(1/N)$. So all the coefficients in $(\tilde{\pi}_n - \tilde{\pi}^c_n)$ will be of the order $O(1/N)$. Moreover $(\tilde{\pi}_n - \tilde{\pi}^c_n)$ will be a homogenous polynomials of the second degree. Hence we will get that $(\tilde{\pi}_N - \tilde{\pi}^c_N)^2$ is a homogenous polynomial of the forth degree. Since $\tilde{v}$, $\tilde{e}_1$, .., $\tilde{e}_N$, $\tilde{z}_0$ are all independent with zero expectations, in $E\left[\tilde{\pi}_N - \tilde{\pi}^c_N\right]^2$ only all the fourths moments and all products of the second moments of $\tilde{v}$, $\tilde{e}_1$, .., $\tilde{e}_N$, $\tilde{z}_0$ will be nonzeros. It is easy to notice that all the coefficients in this expression will be of the order $O(1/N)O(1/N) = O(1/N^2)$. Therefore

$$E\left[\tilde{\pi}_N - \tilde{\pi}^c_N\right]^2 = O(1/N^2).$$

To show that the rate of convergence is exactly $\frac{1}{N^2}$ we need to be much more specific describing $E(\tilde{\pi}_N - \tilde{\pi}^c_N)^2$ For some coefficients L_v, L_z, L_{vz}, L_{vn}, L_{kn}, L_{nz}, where $1 \leq k, m \leq N$ we can write:

$$\tilde{\pi}_N(\tilde{p}, \tilde{\imath}_n) - \tilde{\pi}^c_N(\tilde{p}, \tilde{\imath}_n) = (L_v \tilde{v}^2 + \sum_{k=1}^{N} L_{kk}\tilde{e}_k^2 + L_z \tilde{z}_0^2)$$
$$+ (\sum_{k=1}^{N} L_{vk}\tilde{v}\tilde{e}_k + L_{vz}\tilde{v}\tilde{z}_0 + \sum_{k \neq m} L_{km}\tilde{e}_k\tilde{e}_m + \sum_{k=1}^{N} L_{kz}\tilde{e}_k\tilde{z}_0).$$

Notice that all coefficients L_v, L_z, L_{vz}, L_{vn}, L_{km}, L_{kz}, where $1 \leq k, m \leq N$ are of the order $O(\frac{1}{N})$. Since only the fourths moments and all products of the second moments of $\tilde{v}$, $\tilde{e}_1$, .., $\tilde{e}_N$, $\tilde{z}_0$ will be nonzeros in $E\left[\tilde{\pi}_N - \tilde{\pi}^c_N\right]^2$, we get that

$$E\left[\tilde{\pi}_N(\tilde{p}, \tilde{\imath}_n) - \tilde{\pi}^c_N(\tilde{p}, \tilde{\imath}_n)\right]^2 = E\left[L_v \tilde{v}^2 + \sum_{k=1}^{N} L_{kk}\tilde{e}_n^2 + L_z \tilde{z}_0^2\right]^2$$
$$+ \sum_{k=1}^{N} L_{vk}^2 \tau_v^{-1}\tau_e^{-1} + L_{vz}^2 \tau_v^{-1}\sigma_{z_0}^2 + \sum_{k \neq m} L_{km}^2 \tau_e^{-1}\tau_e^{-1} + \sum_{k=1}^{N} L_{kz}^2 \tau_e^{-1}\sigma_{z_0}^2.$$

Since all parts of this expression are positive and of the order of $O(1/N^2)$, it is enough to demonstrate that one part is actually of order of $1/N^2$ to conclude that all expression is order

$1/N^2$. Let us examine more closely L_{kz} for $k = n$. One gets that

$$\begin{aligned}
L_{nz} &= \left[-C_N(\beta_N - \gamma_N B_N \frac{1}{N}) + B_N \frac{1}{N}(-\gamma_N C_N)\right] \\
&- \left[-C_N^c(\beta_N^c - \gamma_N^c B_N^c \frac{1}{N}) + B_N^c \frac{1}{N}(-\gamma_N^c C_N^c)\right] \\
&= -\left[C_N\beta_N - C_N^c\beta_N^c\right] + \frac{1}{N}\left[(\gamma_N B_N - \gamma_N^c B_N^c)\right] - \frac{1}{N}\left[B_N\gamma_N C_N - B_N^c\gamma_N^c C_N^c\right] \\
&= -\left[C_N\beta_N - C_N^c\beta_N^c\right] + \frac{1}{N}O(1/N) + \frac{1}{N}O(1/N) \\
&= -\left[C_N\beta_N - C_N^c\beta_N^c\right] + O(1/N^2)
\end{aligned}$$

The inspection of (9) shows that $C_N\beta_N = A_N$ and $C_N^c\beta_N^c = A_N^c$, while in the proof of Proposition 1 we demonstrated that $(A_N - A_N^c) = a = (MM^c)^{-1}\left[(\beta_N^2 - (\beta_N^c)^2)\tau_v^{-1}\sigma_{z_0}^2\right] \propto (\beta_N - \beta_N^c) \propto 1/N$. Therefore $L_{nz} \propto 1/N$. Hence $L_{nz}^2 \propto 1/N^2$ and

$$E\left[\tilde{\pi}_N(\tilde{p}, \tilde{\imath}_n) - \tilde{\pi}_N^c(\tilde{p}, \tilde{\imath}_n)\right]^2 \propto L_{nz}^2 \propto 1/N^2.$$

This finishes the proof for the strategic effect.

Now let us examine the limit effect. The proof is more straightforward. Recall from the proof of Proposition 2A that

$$\begin{aligned}
&\tilde{x}_\infty(\tilde{p}, \tilde{\imath}_n) = (\beta_\infty - \gamma_\infty A_\infty)\tilde{v} + \beta_\infty \tilde{e}_n - \gamma_\infty C_\infty \tilde{z}_0 \text{ and} \\
&(\tilde{v} - \tilde{p}_\infty) = (1 - A_\infty)\tilde{v} - C_\infty \tilde{z}_0,
\end{aligned}$$

while

$$\begin{aligned}
&\tilde{x}_N^c(\tilde{p}, \tilde{\imath}_n) = (\beta_N^c - \gamma_N^c A_N^c)\tilde{v} + (\beta_N^c - \gamma_N^c B_N^c \frac{1}{N})\tilde{e}_n - \gamma_N^c B_N^c \frac{1}{N}\sum_{k \neq n} \tilde{e}_k - \gamma_N^c C_N^c \tilde{z}_0 \text{ and} \\
&(\tilde{v} - \tilde{p}_N^c) = (1 - A_N^c)\tilde{v} - B_N^c \frac{1}{N}\tilde{e}_n - B_N^c \frac{1}{N}\sum_{k \neq n} \tilde{e}_k - C_N^c \tilde{z}_0.
\end{aligned}$$

By the same argument as the one above for the strategic effect, in $E\left[\tilde{\pi}_n^c - \tilde{\pi}_\infty\right]^2$ only all the fourths moments and all products of the second moments of $\tilde{v}$, $\tilde{e}_1$, .., $\tilde{e}_N$, $\tilde{z}_0$ will be nonzeros. Similarly to the case of strategic effect, all coefficients will converge to zero at the rate $O(1/N^2)$ except for the parts where some moments of $\tilde{e}_k$, $k \neq n$ are included. (Notice that for the rates of convergence we can ignore parts associated with $\tilde{e}_n$ and not some other $\tilde{e}_k$, $k \neq n$.) Hence the only parts of $E\left[\tilde{\pi}_n^c - \tilde{\pi}_\infty\right]^2$ that have to be examined closely involve moments of $\tilde{e}_k$, $k \neq n$. So we have to concentrate on the following remaining difference between $\tilde{\pi}_n^c$ and $\tilde{\pi}_\infty$:

$$\left(B_N^c \frac{1}{N}\sum_{k\neq n}\tilde{e}_k\right)\tilde{x}_N^c(\tilde{p},\tilde{\imath}_n)+\left(\gamma_N^c B_N^c \frac{1}{N}\sum_{k\neq n}\tilde{e}_k\right)(\tilde{v}-\tilde{p}_N^c)$$

Rewriting this expression we obtain:

$$B_N^c\left(\frac{1}{N}\sum_{k\neq n}\tilde{e}_k\right)\left[\tilde{x}_N^c(\tilde{p},\tilde{\imath}_n)+\gamma_N^c(\tilde{v}-\tilde{p}_N^c)\right]$$

Substituting expressions for $\tilde{x}_N^c(\tilde{p},\tilde{\imath}_n)$ and $(\tilde{v}-\tilde{p}_N^c)$ let us exclude parts that will be $O(1/N^2)$ once $E\left[\tilde{\pi}_n^c-\tilde{\pi}_\infty\right]^2$ is computed. Then we get the following remaining difference between $\tilde{\pi}_n^c$ and $\tilde{\pi}_\infty$:

$$\begin{aligned}&B_N^c\tfrac{1}{N}\textstyle\sum_{k\neq n}\tilde{e}_k\left[(\beta_N^c-\gamma_N^c A_N^c)\tilde{v}+(\beta_N^c-\gamma_N^c B_N^c\tfrac{1}{N})\tilde{e}_n-\gamma_N^c C_N^c\tilde{z}_0+\gamma_N^c(1-A_N^c)\tilde{v}-\gamma_N^c C_N^c\tilde{z}_0\right]\\ &\quad=B_N^c\tfrac{1}{N}\textstyle\sum_{k\neq n}\tilde{e}_k\left[(\beta_N^c+\gamma_N^c-2\gamma_N^c A_N^c)\tilde{v}+(\beta_N^c-\gamma_N^c B_N^c\tfrac{1}{N})\tilde{e}_n-2\gamma_N^c C_N^c\tilde{z}_0\right]\end{aligned}$$

Therefore $E\left[\tilde{\pi}_n^c(\tilde{p},\tilde{\imath}_n)-\tilde{\pi}_\infty(\tilde{p},\tilde{\imath}_n)\right]^2=O(1/N^2)+$

$$(B_N^c)^2\tfrac{N-1}{N^2}\tau_v^{-1}\left[(\beta_N^c+\gamma_N^c-2\gamma_N^c A_N^c)^2\tau_v^{-1}+(\beta_N^c-\gamma_N^c B_N^c\tfrac{1}{N})^2\tau_v^{-1}+(2\gamma_N^c C_N^c)^2\sigma_{z_0}^2\right].$$

Since $B_\infty\neq 0$, $C_\infty\neq 0$, $\beta_\infty\neq 0$, and $\gamma_\infty\neq 0$, it is immediate that both of the last two parts are non zero, hence

$$E\left[\tilde{\pi}_n^c(\tilde{p},\tilde{\imath}_n)-\tilde{\pi}_\infty(\tilde{p},\tilde{\imath}_n)\right]^2\propto 1/N.\blacksquare$$

Proposition 2C. *We have*:

1. $\sqrt{E\left[(U(\tilde{\pi}_N^c)/U(\tilde{\pi}_N)-1)^2\right]}\propto 1/N;$
2. $\sqrt{E\left[(U(\tilde{\pi}_N^c)/U(\tilde{\pi}_\infty)-1)^2\right]}\propto 1/\sqrt{N};$
3. $\sqrt{E\left[(U(\tilde{\pi}_N)/U(\tilde{\pi}_\infty)-1)^2\right]}\propto 1/\sqrt{N}.$

Proof: First, notice that using the full Taylor expansion for the exponential function we get for two profit levels π_a and π_b that $U(\pi_a)/U(\pi_b)-1=\exp(\rho(\pi_b-\pi_a))-1=\sum_{k=1}^{\infty}\frac{1}{k!}\rho^k(\pi_b-\pi_a)^k$ and therefore

$$(U(\pi_a)/U(\pi_b)-1)^2=\sum_{k=2}^{\infty}\Lambda_k\rho^k(\pi_b-\pi_a)^k$$

with coefficients $\Lambda_k=\sum_{l=1}^{k-1}\frac{1}{l!}\frac{1}{(k-l)!}$ Hence we get

$$E\left[U(\pi_a)/U(\pi_b)-1\right]^2=\sum_{k=2}^{\infty}\Lambda_k\rho^k E\left[\pi_b-\pi_a\right]^k$$

Now for the strategic effect the argument is straightforward. Recall from the proof of Proposition 2B that all the coefficients in $(\tilde{\pi}_n - \tilde{\pi}_n^c)$ will be of the order $O(1/N)$. Therefore all the coefficients in $E\left[\tilde{\pi}_N - \tilde{\pi}_N^c\right]^k$ will be of the order $O(1/N^k)$, while we have proved in Proposition 2B that $E\left[\tilde{\pi}_N - \tilde{\pi}_N^c\right]^2 \propto 1/N^2$. Thus

$$E\left[U(\tilde{\pi}_N)/U(\tilde{\pi}_N^c) - 1\right]^2 \propto E\left[U(\tilde{\pi}_N^c)/U(\tilde{\pi}_N) - 1\right]^2 \propto 1/N^2.$$

For the limit effect the argument is similar. Recall that in the proof of Proposition 2B we have showed that all the coefficients in $(\tilde{\pi}_n^c - \tilde{\pi}_\infty)$ are similar to the ones in $(\tilde{\pi}_n - \tilde{\pi}_n^c)$ with the exception of those related to of $\tilde{e}_k$, $k \neq n$. The straightforward inspection of this difference reveals that nevertheless $\sum_{k=3}^{\infty} \Lambda_k \rho^k E\left[\pi_b - \pi_a\right]^k$ will be of the order $O(1/N^2)$, while we have proved in Proposition 2B that $E\left[\tilde{\pi}_n^c - \tilde{\pi}_\infty\right]^2 \propto 1/N$ Thus

$$E\left[U(\tilde{\pi}_N^c)/U(\tilde{\pi}_\infty) - 1\right]^2 \propto E\left[U(\tilde{\pi}_\infty)/U(\tilde{\pi}_N^c) - 1\right]^2 \propto 1/N.$$

Finally, for $E\left[U(\tilde{\pi}_N)/U(\tilde{\pi}_\infty) - 1\right]^2$ the argument is exactly the same as it is for $E\left[U(\tilde{\pi}_N^c)/U(\tilde{\pi}_\infty) - 1\right]^2$.
■

Proof of Proposition 3: We shall first establish limits that can be obtained (Step 1), then shall do the proof for case 1a (Step 2), case 1b (Step 3), and finally for case 2 (Step 4).

Step 1 (Different limits). Different scenarios should be considered separately:

A). If σ_z grows faster than at the rate $\sqrt{N}$, then from the characterization (8) we get that φ_I^c and φ_I converge to 0 at the rate N/σ_z^2. Then it follows from (3) that φ_U^c and φ_U also converge to 0 at the same rate as φ_I^c and φ_I. We have the following subcases.

a). If σ_z grows faster than at the rate N, then we have that $\tau_U^c = \tau_v + \varphi_U^c N \tau_e$ and $\tau_U = \tau_v + \varphi_U N \tau_e$ converge to τ_v. So, the limit model is informationally trivial.

b). Now let us consider the case that $\sigma_z(N)$ grows faster than at the rate $\sqrt{N}$ but slower than at the rate N. Recall that $E\{\tilde{v} \mid \tilde{p}\} = \tilde{p}$. This implies that $var(\tilde{v} - \tilde{p}) = var(\tilde{v} \mid \tilde{p}) = \tau_U^{-1}$. But $\tau_U = \tau_v + \varphi_U N \tau_e \propto N^2/\sigma_z^2$ and σ_z^2 grows slower than N^2. Therefore $var(\tilde{v} - \tilde{p})$ converges to 0 at the rate σ_z^2/N^2. The same argument holds for $\tilde{p}^c$. Hence both prices are revealing the value in the limit. Moreover $var(\tilde{p}_N^c - \tilde{v}) \propto var(\tilde{p}_N - \tilde{v}) \propto \sigma_z^2/N^2$.

B). If σ_z grows not faster than at the rate $\sqrt{N}$, then from the characterization (8) we get that φ_I^c and φ_I do not converge to 0. Then it follows from (3) that φ_U^c and φ_U also do not converge to 0. But then $(\tau_U)_N = \tau_v + \varphi_U N \tau_e \propto N$ and, since $E\{\tilde{v} \mid \tilde{p}\} = \tilde{p}$, we have $var(\tilde{v} - \tilde{p}_N) = var(\tilde{v} \mid \tilde{p}_N) = (\tau_U)_N^{-1} \propto 1/N$. The same is true for the competitive price. Thus both prices reveal the value at the rate $1/\sqrt{N}$.

Step 2 (Case 1a). Notice first, that the limit effect result for this case and the fact that φ_I^c, φ_I, φ_U^c, and φ_U converge to 0 at the rate $N/\sigma_z^2(N)$ were proved in Step 1. Thus we need to prove the result for the strategic effect. The proof proceeds in a similar way to the proof of Proposition 1.

Since $(1-\zeta)=(1-\xi_I)(1-\varphi_I)$ and $0\le\xi_I\le\frac{1}{N}$ (see (6)), we have $\zeta_N\propto N/\sigma_z^2(N)$. Then as in the proof of Lemma 1: $\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}\frac{\varphi_I^c}{\varphi_I}-1=\frac{\zeta(2-3\zeta)}{(1-\zeta)^2}\propto\frac{N}{\sigma_z^2}$. And $\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}\frac{\varphi_I^c}{\varphi_I}-1=\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}\left(\frac{\varphi_I^c}{\varphi_I}-1\right)+\left(\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}-1\right)\propto\varphi_I^{-1}(\varphi_I-\varphi_I^c)+O(\varphi_I-\varphi_I^c)\propto\frac{N}{\sigma_z^2}(\varphi_I-\varphi_I^c)$; Hence $(\varphi_I)_N-(\varphi_I^c)_N\propto N^2/\sigma_z^4$. Then β_N and β_N^c converge to $\beta_\infty=\frac{\tau_e}{\rho}$ and from (2) we get

$$\varphi_I-\varphi_I^c=\frac{(N-1)\sigma_z^2\tau_e(\beta_N^2-(\beta_N^c)^2)}{\left((N-1)\beta_N^2+\sigma_z^2\tau_e\right)\left((N-1)(\beta_N^c)^2+\sigma_z^2\tau_e\right)}\propto\frac{N\sigma_z^2(\beta_N-\beta_N^c)}{\sigma_z^4}.$$

Thus $\beta_N-\beta_N^c\propto N/\sigma_z^2$.

Now let $\tilde z_0:=\tilde z/N$. Then $\sigma_{z_0}^2:=var(\tilde z_0)=\sigma_z^2/N^2$. Now let us denote by $M^c:=(\beta_N^c)^2\tau_v^{-1}+\frac{1}{N}(\beta_N^c)^2\tau_e^{-1}+\sigma_{z_0}^2$ and by $M:=\beta_N^2\tau_v^{-1}+\frac{1}{N}\beta_N^2\tau_e^{-1}+\sigma_{z_0}^2$. Then as in the proof of Proposition 1 we get

$$\tilde p_N-\tilde p_N^c=a\tilde v+a\frac{1}{N}\sum_{n=1}^{N}\tilde e_n+b\tilde z_0,$$

where $a:=(MM^c)^{-1}\left[(\beta_N^2-(\beta_N^c)^2)\tau_v^{-1}\sigma_{z_0}^2\right]\propto(\beta_N-\beta_N^c)\sigma_{z_0}^2\propto\frac{N}{\sigma_z^2}\frac{\sigma_z^2}{N^2}\propto\frac{1}{N}$ and $b:=(MM^c)^{-1}\left[\beta_N\beta_N^c(\beta_N^c-\beta_N)(\tau_v^{-1}+\frac{1}{N}\tau_e^{-1})\tau_v^{-1}+(\beta_N-\beta_N^c)\tau_v^{-1}\sigma_{z_0}^2\right]$. So $b\propto(\beta_N-\beta_N^c)\propto\frac{N}{\sigma_z^2}$. Thus

$$var(\tilde p_N-\tilde p_N^c)=a^2\tau_v^{-1}+a^2\frac{1}{N}\tau_e^{-1}+b^2\sigma_{z_0}^2\propto\left(\frac{N^2}{\sigma_z^4}\right)\left(\frac{\sigma_z^2}{N^2}\right)\propto\frac{1}{\sigma_z^2}.$$

Step 3 (Case 1b). It was already proved in Step 1 that $var(\tilde p_N^c-\tilde v)\propto var(\tilde p_N-\tilde v)\propto 1/N$. Therefore, we need to prove only that $var(\tilde p_N-\tilde p_N^c)\propto 1/N$. Let $\tilde z_0:=\tilde z/N$. Then $\sigma_{z_0}^2:=var(\tilde z_0)=\sigma_z^2/N^2$. Now let us denote by $M^c:=(\beta_N^c)^2\tau_v^{-1}+\frac{1}{N}(\beta_N^c)^2\tau_e^{-1}+\sigma_{z_0}^2$ and by $M:=\beta_N^2\tau_v^{-1}+\frac{1}{N}\beta_N^2\tau_e^{-1}+\sigma_{z_0}^2$. Then as in the proof of Proposition 1 we get

$$\tilde p_N-\tilde p_N^c=a\tilde v+a\frac{1}{N}\sum_{n=1}^{N}\tilde e_n+b\tilde z_0,$$

where $a:=(MM^c)^{-1}\left[(\beta_N^2-(\beta_N^c)^2)\tau_v^{-1}\sigma_{z_0}^2\right]$ and
$b:=(MM^c)^{-1}\left[\beta_N\beta_N^c(\beta_N^c-\beta_N)(\tau_v^{-1}+\frac{1}{N}\tau_e^{-1})\tau_v^{-1}+(\beta_N-\beta_N^c)\tau_v^{-1}\sigma_{z_0}^2\right]$. Two cases should be considered separately: when $\sigma_z^2\propto N$ and when $\sigma_z^2=o(N)$.

1). Let $\sigma_z^2\propto N$. Then the (equal) left hand sides of (8) converge to a non-zero constant as N grows. Thus the right hand sides also should converge to a non-zero constant. Hence φ_I^c and φ_I converge to two different constants. Moreover $(1-\varphi_I^c)$, $(1-\varphi_I)$, and $(1-2\zeta)$ do not converge to zero. Therefore it follows from (5) that β_N^c and β_N also converge to some non-zero constants. Then from (2) we get

$$\varphi_I - \varphi_I^c = \frac{(N-1)\sigma_z^2\tau_e(\beta_N^2 - (\beta_N^c)^2)}{\left((N-1)\beta_N^2 + \sigma_z^2\tau_e\right)\left((N-1)(\beta_N^c)^2 + \sigma_z^2\tau_e\right)} \propto \frac{N^2(\beta_N - \beta_N^c)}{N^2} \propto (\beta_N - \beta_N^c).$$

Thus $(\beta_N - \beta_N^c)$ also converge to a non-zero constant. Then we have that $a \propto (\beta_N - \beta_N^c)\sigma_{z_0}^2 \propto \frac{1}{N}$ and $b \propto (\beta_N - \beta_N^c)$. That is, b converge to a non-zero constant. Thus $var(\tilde{p}_N - \tilde{p}_N^c) = a^2\tau_v^{-1} + a^2\frac{1}{N}\tau_e^{-1} + b^2\sigma_{z_0}^2 \propto 1/N$.

2). Let us consider the case $\sigma_z^2 = o(N)$. Then analysis of (8) very similar to the one in the part 1 shows that φ_I^c converge to 1 at the rate $(\frac{\sigma_z^2(N)}{N})^{\frac{1}{3}}$, and that φ_I and ζ converge to 1/2 at the rate $(\frac{\sigma_z^2(N)}{N})^{\frac{1}{2}}$. (Since $\zeta \leq 1/2$, by (6) we get that φ_I and ζ converge to 1/2.) Then from (5) we get that $\beta_N^c \propto (\frac{\sigma_z^2(N)}{N})^{\frac{1}{3}}$ and $\beta_N \propto (\frac{\sigma_z^2(N)}{N})^{\frac{1}{2}}$. Now from (2) we have

$$\varphi_I - \varphi_I^c = \frac{(N-1)\sigma_z^2\tau_e(\beta_N^2 - (\beta_N^c)^2)}{\left((N-1)\beta_N^2 + \sigma_z^2\tau_e\right)\left((N-1)(\beta_N^c)^2 + \sigma_z^2\tau_e\right)} \propto (\beta_N - \beta_N^c)(\frac{N}{\sigma_z^2})^{\frac{1}{3}}.$$

Thus $(\beta - \beta^c) \propto (\frac{\sigma_z^2}{N})^{\frac{1}{3}}$. Then $M^c \propto (\beta_N^c)^2 \propto (\frac{\sigma_z^2}{N})^{\frac{2}{3}}$ and $M \propto \beta_N^2 \propto \frac{\sigma_z^2}{N}$. So, we get $a \propto (\frac{N}{\sigma_z^2})^{\frac{5}{3}}\left[(\frac{\sigma_z^2}{N})^{\frac{2}{3}}\frac{\sigma_z^2}{N^2}\right] =\propto (\frac{1}{N})$ and $b \propto (\frac{N}{\sigma_z^2})^{\frac{5}{3}}\left[(\frac{\sigma_z^2}{N})^{(\frac{1}{2}+\frac{2}{3})}\right] \propto (\frac{N}{\sigma_z^2})^{\frac{1}{2}}$. Thus $var(\tilde{p}_N - \tilde{p}_N^c) \propto O(\frac{1}{N^2}) + O(\frac{1}{N^3}) + \frac{N}{\sigma_z^2}\sigma_{z_0}^2 \propto 1/N$.

<u>*Step 4 (Case 2).*</u> It was already proved in Step 1 that it N grows slower than σ_z, then that φ_I^c, φ_I, φ_U^c and φ_U all converge to 0 at the rate N/σ_z^2. Hence $\tau_U^c = \tau_v + \varphi_U^c N\tau_e$ and $\tau_U = \tau_v + \varphi_U N\tau_e$ converge to τ_v, while τ_I^c and τ_I converge to $\tau_v + \tau_e$. Then from (7) we get that ζ converges to 0 at the rate N/σ_z^2.

Then as in the proof of Lemma 1: $\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}\frac{\varphi_I^c}{\varphi_I} - 1 = \frac{\zeta(2-3\zeta)}{(1-\zeta)^2} \propto \frac{N}{\sigma_z^2}$. And $\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}\frac{\varphi_I^c}{\varphi_I} - 1 = \frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3}\left(\frac{\varphi_I^c}{\varphi_I} - 1\right) + \left(\frac{(1-\varphi_I)^3}{(1-\varphi_I^c)^3} - 1\right) \propto \varphi_I^{-1}(\varphi_I - \varphi_I^c) + O(\varphi_I - \varphi_I^c) \propto \frac{\sigma_z^2}{N}(\varphi_I - \varphi_I^c)$; Hence $(\varphi_I)_N - (\varphi_I^c)_N \propto N^2/\sigma_z^4$. Then β_N and β_N^c converge to $\beta_\infty = \frac{\tau_e}{\rho}$ and from (2) we get

$$\varphi_I - \varphi_I^c = \frac{(N-1)\sigma_z^2\tau_e(\beta_N^2 - (\beta_N^c)^2)}{\left((N-1)\beta_N^2 + \sigma_z^2\tau_e\right)\left((N-1)(\beta_N^c)^2 + \sigma_z^2\tau_e\right)} \propto \frac{N\sigma_z^2(\beta_N - \beta_N^c)}{\sigma_z^4}.$$

Thus $\beta_N - \beta_N^c \propto N/\sigma_z^2$.

Let us define $\tilde{z}_0 := \tilde{z}/N$. Then $\sigma_{z_0}^2 := var(\tilde{z}_0) = \sigma_z^2/N^2$ is growing with N. Now let us denote by $M^c := (\beta_N^c)^2\tau_v^{-1} + \frac{1}{N}(\beta_N^c)^2\tau_e^{-1} + \sigma_{z_0}^2$ and by $M := \beta_N^2\tau_v^{-1} + \frac{1}{N}\beta_N^2\tau_e^{-1} + \sigma_{z_0}^2$. Notice that $M^c \propto M \propto \sigma_{z_0}^2$. Then as in the proof of Proposition 1 we get

$$\tilde{p}_N - \tilde{p}_N^c = a\tilde{v} + a\frac{1}{N}\sum_{n=1}^{N}\tilde{e}_n + b\tilde{z}_0,$$

where $a := (MM^c)^{-1}\left[(\beta_N^2 - (\beta_N^c)^2)\tau_v^{-1}\sigma_{z_0}^2\right] \propto (\beta_N - \beta_N^c)/\sigma_{z_0}^2 \propto \frac{N}{\sigma_z^2}\frac{N^2}{\sigma_z^2} \propto \frac{N^3}{\sigma_z^4}$ and $b :=$

$(MM^c)^{-1}\left[\beta_N\beta_N^c(\beta_N^c-\beta_N)(\tau_v^{-1}+\frac{1}{N}\tau_e^{-1})\tau_v^{-1}+(\beta_N-\beta_N^c)\tau_v^{-1}\sigma_{z_0}^2\right]$. So $b\propto(\beta_N-\beta_N^c)/\sigma_{z_0}^2\propto\frac{N^3}{\sigma_z^4}$. Thus

$$var(\tilde{p}_N-\tilde{p}_N^c)=a^2\tau_v^{-1}+a^2\frac{1}{N}\tau_e^{-1}+b^2\sigma_{z_0}^2\propto\left(\frac{N^3}{\sigma_z^4}\right)^2\left(\frac{\sigma_z^2}{N^2}\right)\propto\frac{N^4}{\sigma_z^6}.$$

This proves the statement for the strategic effect.

The inspection of expressions (9) shows that in the case described in the current Proposition $var(\tilde{p}_N^c)\propto var(\tilde{p}_N)\propto var(\frac{\tilde{z}_0}{\sigma_{z_0}^2})\propto\frac{1}{\sigma_z^2}\propto\frac{N^2}{\sigma_z^2}$. This completes the proof of the proposition.■

Proof of Proposition 4: Part 1 of the proposition is straightforward. Let us state a lemma before proving part 2.

Lemma A3: For the monopolistic competition limit:

$$\xi_I N=\frac{\tau_v+\tau_E}{(\tau_v/\varphi_U)+\tau_E}.$$

Proof: Condition (B.22) in Kyle (1989) states that

$$\zeta\tau_U-\varphi_U\tau_I=(1-\varphi_I)\left[(\xi_I-\frac{\varphi_U}{N})\tau_v-\varphi_U(1-N\xi_I)\tau_e\right].$$

Since the left hand side of this expression is zero by (7) and $\varphi_I<1$, we should get $(\xi_I-\frac{\varphi_U}{N})\tau_v-\varphi_U(1-N\xi_I)\tau_e=0$. Therefore $(N\xi_I-\varphi_U)\tau_v-\varphi_U(1-N\xi_I)\tau_E=0$. But this is equivalent to $N\xi_I(\tau_v+\varphi_U\tau_E)=\varphi_U(\tau_v+\tau_E)$. Thus $N\xi_I=\frac{\varphi_U(\tau_v+\tau_E)}{(\tau_v+\varphi_U\tau_E)}=\frac{\tau_v+\tau_E}{(\tau_v/\varphi_U)+\tau_E}$.■

Let us deal now with part 2. From formulae (4) we get the following expressions for prices in the two equilibria in the N^{th} market:

$$\tilde{p}_N^c=\frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_E^{-1}+(\frac{1}{\beta_N^c N})^2\sigma_z^2}\left(\tilde{v}+\tilde{e}+\frac{1}{\beta_N^c N}\tilde{z}\right),$$

$$\tilde{p}_N=\frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_E^{-1}+(\frac{1}{\beta_N N})^2\sigma_z^2}\left(\tilde{v}+\tilde{e}+\frac{1}{\beta_N N}\tilde{z}\right).$$

The proof will go in four steps.

1). Let us prove that there exist different non-zero constants d and d^c such that:

$$\tilde{p}_\infty^c=\frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_E^{-1}+(d^c)^2\sigma_z^2}\left(\tilde{v}+\tilde{e}+d^c\tilde{z}\right)\text{ and}$$

$$\tilde{p}_\infty=\frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_E^{-1}+d^2\sigma_z^2}\left(\tilde{v}+\tilde{e}+d\tilde{z}\right).$$

This will imply that $\tilde{p}^c_\infty \neq \tilde{p}_\infty$. There exist two different non-zero constants $(\varphi^c_I)_\infty$ and $(\varphi_I)_\infty$, such that $(\varphi^c_I)_N \to (\varphi^c_I)_\infty$ and $(\varphi_I)_N \to (\varphi_I)_\infty$ as $N \to \infty$. Moreover $(1-(\varphi^c_I)_\infty)$, $(1-(\varphi_I)_\infty)$, and $(1-2\zeta_\infty)$ are all non-zero constants. Thus it follows from (5) that there exist non-zero constants d and d^c such that $\frac{1}{\beta^c_N N} \to d^c$ and $\frac{1}{\beta_N N} \to d$. So we proved formulae for the limit prices. Now from (2) we get

$$\varphi_I - \varphi^c_I = \frac{(N-1)\sigma_z^2\tau_e(\beta_N^2 - (\beta^c_N)^2)}{\left((N-1)\beta_N^2 + \sigma_z^2\tau_e\right)\left((N-1)(\beta^c_N)^2 + \sigma_z^2\tau_e\right)} \propto \frac{(\beta_N - \beta^c_N)\frac{1}{N}}{\frac{1}{N}\frac{1}{N}} \propto (\beta_N N - \beta^c_N N).$$

Since $(\varphi^c_I)_\infty \neq (\varphi_I)_\infty$, we have that $d \neq d^c$.

2). Now let us prove the statement of the proposition for competitive equilibrium. Let us first introduce some notation. Let

$$k^c_N := \frac{\tau_v^{-1}}{\tau_v^{-1} + \tau_E^{-1} + (\frac{1}{\beta^c_N N})^2\sigma_z^2} \text{ and } k^c_\infty := \frac{\tau_v^{-1}}{\tau_v^{-1} + \tau_E^{-1} + (d^c)^2\sigma_z^2}.$$

Then $k^c_N - k^c_\infty \propto (\frac{1}{\beta^c_N N})^2 - (d^c)^2 \propto \frac{1}{\beta^c_N N} - d^c$. Since $\tilde{p}^c_N = k^c_N\left(\tilde{v} + \tilde{e} + \frac{1}{\beta^c_N N}\tilde{z}\right)$ and $\tilde{p}^c_\infty = k^c_\infty\left(\tilde{v} + \tilde{e} + d^c\tilde{z}\right)$ we get $\tilde{p}^c_N - \tilde{p}^c_\infty = (k^c_N - k^c_\infty)(\tilde{v} + \tilde{e} + \frac{1}{\beta^c_N N}\tilde{z}) + k^c_\infty(\frac{1}{\beta^c_N N} - d^c)\tilde{z}$ and thus

$$var\left[\tilde{p}^c_N - \tilde{p}^c_\infty\right] \propto (\frac{1}{\beta^c_N N} - d^c)^2 \propto (\beta^c_N N - \frac{1}{d^c})^2.$$

Then by (5) we have

$$\beta^c_N N - \frac{1}{d^c} \propto \left[\frac{\tau_E}{\rho}(1-(\varphi^c_I)_N) - \frac{\tau_E}{\rho}(1-(\varphi^c_I)_\infty)\right] \propto (\varphi^c_I)_N - (\varphi^c_I)_\infty.$$

And now we have from (8) that $\frac{\sigma_z^2\rho^2}{\tau_E}\frac{N}{N-1} = \frac{(1-(\varphi^c_I)_N)^3}{(\varphi^c_I)_N}$ and $\frac{\sigma_z^2\rho^2}{\tau_E} = \frac{(1-(\varphi^c_I)_\infty)^3}{(\varphi^c_I)_\infty}$. Therefore

$$\frac{(1-(\varphi^c_I)_\infty)^3}{(1-(\varphi^c_I)_N)^3}\frac{(\varphi^c_I)_N}{(\varphi^c_I)_\infty} - 1 \propto 1/N.$$

But on the other side

$$\begin{aligned}\frac{(1-(\varphi^c_I)_\infty)^3}{(1-(\varphi^c_I)_N)^3}\frac{(\varphi^c_I)_N}{(\varphi^c_I)_\infty} - 1 &= \frac{(1-(\varphi^c_I)_\infty)^3}{(1-(\varphi^c_I)_N)^3}\left(\frac{(\varphi^c_I)_N}{(\varphi^c_I)_\infty} - 1\right) + \left(\frac{(1-(\varphi^c_I)_\infty)^3}{(1-(\varphi^c_I)_N)^3} - 1\right)\\ &\propto (\varphi^c_I)_N - (\varphi^c_I)_\infty\end{aligned}$$

Thus $(\varphi^c_I)_N - (\varphi^c_I)_\infty \propto 1/N$ and therefore $var\left[\tilde{p}^c_N - \tilde{p}^c_\infty\right] \propto 1/N^2$.

3). Now let us look at the equilibrium with imperfect competition. In the same way as above

$$var\left[\tilde{p}_N - \tilde{p}_\infty\right] \propto (\frac{1}{\beta_N N} - d)^2 \propto (\beta_N N - \frac{1}{d})^2.$$

Then by (5) $\beta_N N = \frac{\tau_E}{\rho}(1-(\varphi_I)_N)\frac{(1-2\zeta_N)}{(1-\zeta_N)}$ and $\frac{1}{d} = \frac{\tau_E}{\rho}(1-(\varphi_I)_\infty)\frac{(1-2\zeta_\infty)}{(1-\zeta_\infty)}$. Then by (6) $(1-\zeta_N) = (1-(\varphi_I)_N)(1-\xi_I)$ and $\zeta_\infty = (\varphi_I)_\infty$. Thus

$$\begin{aligned}\beta_N N - \frac{1}{d} &= \frac{\tau_E}{\rho}\left[\frac{(1-2\zeta_N)}{(1-\xi_I)} - (1-2\zeta_\infty)\right] \\ &\propto \frac{\xi_I}{(1-\xi_I)}(1-2\zeta_N) + 2(\zeta_\infty - \zeta_N).\end{aligned}$$

Then $2(\zeta_\infty - \zeta_N) = 2((\varphi_I)_\infty - (\varphi_I)_N) - 2\xi_I(1-(\varphi_I)_N)$. By Lemma 4, since $0 < \lim_{N\to\infty}\varphi_U = (\varphi_I)_\infty < 1$, we can conclude $\xi_I \propto 1/N$, and $\frac{\xi_I}{(1-\xi_I)}(1-2\zeta_N) = \xi_I(1-2(\varphi_I)_N) + o(1/N)$. Therefore we get

$$\beta_N N - \frac{1}{d} \propto 2((\varphi_I)_\infty - (\varphi_I)_N) - \xi_I + o(\frac{1}{N}).$$

Now let us show that it is impossible to have the case where $1/N = o((\varphi_I)_\infty - (\varphi_I)_N)$. Let us first prove that if $1/N = o((\varphi_I)_\infty - (\varphi_I)_N)$, then

$$\frac{(1-(\varphi_I)_N)^3}{(\varphi_I)_N}\frac{(1-2\zeta_N)^2}{(1-\zeta_N)^2} - \frac{(1-(\varphi_I)_\infty)^3}{(\varphi_I)_\infty}\frac{(1-2\zeta_\infty)^2}{(1-\zeta_\infty)^2} \propto (\varphi_I)_\infty - (\varphi_I)_N. \tag{11}$$

Since $\zeta_N = (\varphi_I)_N + O(1/N)$ we get $\frac{(1-(\varphi_I)_N)^3}{(\varphi_I)_N}\frac{(1-2\zeta_N)^2}{(1-\zeta_N)^2} = \frac{(1-(\varphi_I)_N)(1-2(\varphi_I)_N)^2}{(\varphi_I)_N} + O(1/N)$, and since $\zeta_\infty = (\varphi_I)_\infty$ we have $\frac{(1-(\varphi_I)_\infty)^3}{(\varphi_I)_\infty}\frac{(1-2\zeta_\infty)^2}{(1-\zeta_\infty)^2} = \frac{(1-(\varphi_I)_\infty)(1-2(\varphi_I)_\infty)^2}{(\varphi_I)_\infty}$.

Then $\frac{(1-(\varphi_I)_N)(1-2(\varphi_I)_N)^2}{(\varphi_I)_N} - \frac{(1-(\varphi_I)_\infty)(1-2(\varphi_I)_\infty)^2}{(\varphi_I)_\infty} =$

$$= \frac{((\varphi_I)_\infty - (\varphi_I)_N)}{(\varphi_I)_N(\varphi_I)_\infty}\left[1 - 8(\varphi_I)_N(\varphi_I)_\infty + 4(\varphi_I)_N(\varphi_I)_\infty((\varphi_I)_N + (\varphi_I)_\infty)\right]$$

But $\lim_{N\to\infty}\left[1 - 8(\varphi_I)_N(\varphi_I)_\infty + 4(\varphi_I)_N(\varphi_I)_\infty((\varphi_I)_N + (\varphi_I)_\infty)\right] = 1 - 8\left((\varphi_I)_\infty\right)^2 + 8\left((\varphi_I)_\infty\right)^3 \neq 0$, since $0 < (\varphi_I)_\infty < \frac{1}{2}$. Therefore (11) is proved. But by (8) the left-hand side of (11) must be of order $\frac{1}{N}$. This proves that the case $1/N = o((\varphi_I)_\infty - (\varphi_I)_N)$ is impossible.

Therefore $(\varphi_I)_\infty - (\varphi_I)_N = O(1/N)$ and we get

$$\sqrt{var\left[\tilde{p}_N - \tilde{p}_\infty\right]} \propto \beta_N N - \frac{1}{d} = O(\frac{1}{N}).$$

4). By (8) $\frac{\sigma_z^2\rho^2}{\tau_E}\frac{N}{N-1} = \frac{(1-(\varphi_I)_N)^3}{(\varphi_I)_N}\frac{(1-2\zeta_N)^2}{(1-\zeta_N)^2}$ and thus by (5) $(\beta_N N)^2 = \frac{\tau_E^2}{\rho^2}(1-(\varphi_I)_N)^2\frac{(1-2\zeta_N)^2}{(1-\zeta_N)^2} = \frac{\tau_E^2}{\rho^2}\frac{\sigma_z^2\rho^2}{\tau_E}\frac{(\varphi_I)_N}{1-(\varphi_I)_N}\frac{N}{N-1}$. Similarly $\frac{1}{d^2} = \frac{\tau_E^2}{\rho^2}\frac{\sigma_z^2\rho^2}{\tau_E}\frac{(\varphi_I)_\infty}{1-(\varphi_I)_\infty}$. So

$$\begin{aligned}(\beta_N N) - \frac{1}{d} \propto (\beta_N N)^2 - \frac{1}{d^2} &\propto \frac{(\varphi_I)_N}{1-(\varphi_I)_N}\frac{1}{N-1} + \left(\frac{(\varphi_I)_N}{1-(\varphi_I)_N} - \frac{(\varphi_I)_\infty}{1-(\varphi_I)_\infty}\right) \\ &\propto \frac{(1-(\varphi_I)_\infty)(\varphi_I)_N}{N-1} + (\varphi_I)_N - (\varphi_I)_\infty.\end{aligned}$$

Also recall that, as we have shown in part (3) above, it should be that

$$\beta_N N - \frac{1}{d} \propto 2((\varphi_I)_\infty - (\varphi_I)_N) - \xi_I + o(\frac{1}{N}).$$

Let us assume that $\beta_N N - \frac{1}{d} = o(\frac{1}{N})$. Then we should have $\lim_{N\to\infty} N((\varphi_I)_\infty - (\varphi_I)_N) = (1-(\varphi_I)_\infty)(\varphi_I)_\infty$ and $\lim_{N\to\infty} 2N((\varphi_I)_\infty - (\varphi_I)_N) = \lim_{N\to\infty} N\xi_I$. By Lemma 4, $N\xi_I = \frac{\tau_v+\tau_E}{(\tau_v/\varphi_U)+\tau_E}$, therefore $\beta_N N - \frac{1}{d} = o(\frac{1}{N})$ implies an equality

$$2(1-(\varphi_I)_\infty)(\varphi_I)_\infty = \frac{\tau_v+\tau_E}{\tau_v/(\varphi_U)_\infty + \tau_E}.$$

This equality holds only if

$$(1-(\varphi_I)_\infty) = \frac{(\tau_v+\tau_E)/2}{\tau_v + \tau_E(\varphi_U)_\infty}.$$

And this condition can be rewritten as

$$\tau_E(\varphi_U)^2_\infty + (\tau_v - \tau_E)(\varphi_U)_\infty + (\tau_E - \tau_v)/2 = 0.$$

This equation on $(\varphi_U)_\infty$ has a determinant of $(\tau_v - \tau_E)^2 - 2(\tau_E - \tau_v)\tau_E = \tau_v^2 - {\tau_E}^2$. Therefore for the case of $\tau_E > \tau_v$ this equation does not have a solution and $\beta_N N - \frac{1}{d} = o(\frac{1}{N})$ is impossible. Thus we must have $\beta_N N - \frac{1}{d} \propto \frac{1}{N}$ for the case when $\tau_E > \tau_v$. Therefore

$$\sqrt{var\left[\tilde{p}_N - \tilde{p}_\infty\right]} \propto \beta_N N - \frac{1}{d} \propto \frac{1}{N}.\blacksquare$$

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