Oligopoly, Macroeconomic Performance, and Competition Policy*

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Abstract

We develop a macroeconomic framework in which firms are large and have market power with respect to both products and factors. Each firm maximizes a share-weighted average of shareholder utilities, which makes the equilibrium independent of price normalization. In a one sector economy, if returns to scale are non-increasing, then an increase in "effective" market concentration (which accounts for overlapping ownership) leads to declines in employment, real wages, and the labor share. Moreover, if the goal is to foster employment then (i) controlling common ownership and reducing concentration are complements and (ii) government jobs are a substitute for either policy. Yet when there are multiple sectors, due to an intersectoral pecuniary externality, an increase in common ownership can stimulate the economy when labor market oligopsony power is low relative to product market oligopoly power. We find that neither the monopolistically competitive limit of Dixit and Stiglitz nor the oligopolistic one of Neary (when firms become small relative to the economy) are attained *unless* there is incomplete portfolio diversification.

Keywords: ownership, portfolio diversification, labor share, market power, oligopsony, antitrust policy

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1 Introduction

Oligopoly is widespread and seems to be on the rise. Many industries are characterized by oligopolistic conditions—including, but not limited to, the digital ones dominated by FAMGA: Facebook, Apple, Microsoft, Google (now Alphabet), and Amazon. Yet oligopoly is seldom considered by macroeconomic models, which focus on monopolistic competition because of its analytical tractability.[1] In this paper we build a tractable general equilibrium model of oligopoly, characterize its equilibrium, and then use it to derive welfare-improving policies.

Recent empirical research has renewed interest in the issue of aggregate market power and its consequences for macroeconomic outcomes. Grullon et al. (2016) claim that concentration has increased in more than 75% of US industries over the last two decades and also that firms in industries with larger increases in product market concentration have enjoyed higher profit margins and positive abnormal stock returns, which suggests that market power is the driver of these outcomes. Barkai (2016) and De Loecker and Eeckhout (2017) document an increase in economic profits and markups in the economy overall and use cross-industry regressions to show that increases in market concentration are correlated with declines in the labor share.[2] In addition to increases in concentration as traditionally measured, recent research has shown that increased overlapping ownership of firms by financial institutions (in particular, funds)—what we refer to as *common ownership*—has led to substantial increases in effective concentration indices in the airline and banking industries, and that this greater concentration is associated with higher prices (Azar et al., 2016, Forthcoming). Gutiérrez and Philippon (2016) suggest that the increase in index and quasi-index fund ownership has played a role in declining aggregate investment.

The concern over market power is now a subject of policy debate. For example, the Council of Economic Advisers has produced two reports (CEA, 2016a,b) on the issue of market power. The first one presents evidence of increasing concentration in most product markets, and the second presents evidence of substantial monopsony power in the labor market.[3] The increase in common ownership has also raised antitrust concerns (Baker, 2016; Elhauge, 2016) and some bold proposals for remedies (Posner et al., 2016; Scott Morton and Hovenkamp, 2017) as well as calls for caution (O'Brien and Waehrer, 2017; Rock and Rubinfeld, 2017).

Our paper contributes to this growing literature by developing a model of oligopoly in general equilibrium. The difficulties of incorporating oligopoly into a general equilibrium framework have hindered the modeling of market power in macroeconomics. With price-taking firms and complete markets, a firm's shareholders agree unanimously that the objective of the firm should be to maximize its own

[1] This statement applies also in international trade theory; the few exceptions include Neary (2003a,b, 2010) and Head and Spencer (2017).

[2] Blonigen and Pierce (2016) attribute the US increase in markups to increased merger activity. Autor et al. (2017) argue that globalization and technological change lead to concentration and the rise of what they call "superstar" firms, which have high profits and a low labor share. As the importance of superstar firms rises (with the increase in concentration), the aggregate labor share falls.

[3] For instance, Samsung and Hyundai are large relative to Korea's economy (Gabaix, 2011). Although even General Motors and Walmart have never employed more than 1% of the US workforce, those firms may figure prominently in *local* labor markets.

profits. This result is called the "Fisher Separation Theorem" (Ekern and Wilson, 1974; Radner, 1974; Leland, 1974; Grossman and Hart, 1979; DeAngelo, 1981). Under imperfect competition, however, the Fisher Separation Theorem does not apply, and therefore there is no simple objective function for the firm (Hart, 1982a). The presence of firms that are large in relation to the economy raise questions about whether the firm's objective truly is to maximize profits—since, for example, high prices may harm shareholders as consumers (Farrell, 1985). In addition, in general equilibrium, a firm with pricing power will influence not only its own profits but also the wealth of consumers and therefore demand (these feedback effects are sometimes called Ford effects). Firms that are large relative to factor markets also have to take into account their impact on factor prices. At the same time, if a firms' shareholders have holdings in competing firms, they would benefit from high prices through their effect not only on their own profits, but also on the profits of rival firms, as well as internalizing other externalities between firms (Rotemberg, 1984; Gordon, 1990; Hansen and Lott, 1996).

In the literatures on monopolistic competition (e.g., Hart, 1982b) and oligopoly (Neary, 2003a), it is widely assumed that firms are small in relation to the economy; this approach makes for a tractable model, but it is not appropriate in the presence of monopsony power. Large firms will influence their own local labor markets at least and, as we will see, common ownership may amplify the effect of large firms on product and factor prices. Gabszewicz and Vial (1972) originally proposed the concept of a Cournot–Walras equilibrium to deal with oligopoly in general equilibrium while assuming that firms seek to maximize profits. One problem related to general equilibrium feedback effects is that the Cournot–Walras equilibrium, when it exists, depends on the choice of numéraire.[4] This problem is often sidestepped by assuming that firms' owners care about only one good (an outside good or numéraire); see, for example, Mas-Colell (1982).

We build a tractable model of oligopoly under general equilibrium, allowing firms to be large in relation to the economy, and then examine the effect of oligopoly on macroeconomic performance. We assume that firms maximize a weighted average of shareholder utilities in Cournot–Walras equilibrium. The weights in a firm's objective function are given by the influence or "control weight" of each shareholder.[5] This solves the numéraire problem because *indirect* utilities depend only on relative prices and not on the choice of numéraire. Firms are assumed to make strategic decisions that account for the effect of their actions on prices and wages. When making decisions about hiring, for instance, a firm realizes that increasing employment could put upward pricing pressure on real wages—reducing not only its own profits but also the profits of all other firms in its shareholders' portfolios. The model is parsimonious and identifies the key parameters driving equilibrium: the elasticity of substitution across industries, the elasticity of labor supply, the market concentration of each industry, and the ownership structure (i.e., extent of diversification) of investors.

We seek answers to a number of key questions. How do output, labor demand, prices, and wages depend on market concentration and the degree of common ownership? To what extent are markups

[4] The existence problem was highlighted early on by Roberts and Sonnenschein (1977).

[5] This objective function can be "microfounded" with competition among managers to run the firm (Azar, 2012; Brito et al., 2017). Azar (2017) shows that, even in uncontested elections, this objective function of the firm can be microfounded if one assumes that dissenting votes by shareholders are costly for the incumbent directors.

in product markets, and markdowns in the labor market, affected by how much the firm internalizes other firms' profits? Can common ownership be pro-competitive in a general equilibrium framework? How do common ownership effects change when the number of industries increases? In the presence of common ownership, is the monopolistically competitive limit (as described by Dixit and Stiglitz, 1977) attained when firms become small relative to the market?—and, more generally, how does ownership structure affect this limit? Is antitrust policy a complement or rather a substitute with respect to other government policies aimed at boosting employment?

In the base model we develop here, there is one good in addition to leisure; also, the model assumes both oligopoly in the product market and oligopsony in the labor market. Firms compete by setting their labor demands à la Cournot and thus have market power. There is a continuum of risk-neutral owners, who have a proportion of their respective shares invested in one firm and have the balance invested in the market portfolio (say, an index fund). This formulation is numéraire-free and allows us to characterize the equilibrium. The extent to which firms internalize competiting firms' profits depends on market concentration and investor diversification. We demonstrate the existence and uniqueness of a symmetric equilibrium, and then characterize its comparative static properties, under the assumption that labor supply is upward sloping (while allowing for some economies of scale). Our results show that, in the one-sector model, the markdown of real wages with respect to the marginal product of labor is driven by the common ownership–modified Herfindahl–Hirschman index (MHHI) for the labor market and also by labor supply elasticity (but *not* by product market power). We perform comparative statics on the equilibrium (employment and real wages) with respect to market concentration and degree of common ownership, and we develop an example featuring Cobb–Douglas firms and consumers with constant elasticity of substitution (CES). We find that increased market concentration—due either to fewer firms or to more common ownership—depresses the economy by *reducing* employment, output, real wages, and the labor share (if one assumes non-increasing returns to scale). The model determines the interest rate by incorporating both investment in productive capital and household savings; our results indicate that market concentration depresses both real interest rates and investment levels.

We also extend our base model to allow for multiple sectors and differentiated products across sectors (with CES aggregators as in Dixit and Stiglitz, 1977). The firms supplying each industry's product are finite in number and engage in Cournot competition. In this extension, a firm deciding whether to marginally increase its employment must consider the effect of that increase on three relative prices: (i) the increase would reduce the relative price of the firm's own products, (ii) it would boost real wages, and (iii) it would increase the relative price of products in other industries—that is, because overall consumption would increase. This third effect, referred to as *inter-sector pecuniary externality*, is internalized only if there is common ownership involving the firm and firms in other industries. In this case, the markdown of real wages relative to the marginal product of labor increases with the MHHI values for the labor market and product markets but decreases with the pecuniary externality (weighted by the extent of competitor profit internalization due to common ownership). We find that common ownership can have a pro-competitive effect when there is low labor market power (i.e., high elasticity of labor supply) and/or when there is high product market power (low elasticity of substitution among product varieties, few firms in each sector). We discover that increased profit internalization has two competing effects on the equilibrium markdown of the real wage: a positive effect because of the increased monopsony power in the labor market; and an overall negative product market effect because of the pecuniary externality. Under the conditions we stipulate (low labor market power and/or high product market power), the effect of the labor market monopsony is dominated by the effect of the total product market. Indeed, under those conditions the increased labor market power due to increased common ownership is more than compensated by the negative effect on product markets due to the intersector pecuniary externality, which encourages competition. It is worth to remark that when the labor market is competitive, common ownership has always a pro-competitive effect.

We then consider the limiting case when the number of sectors tends to infinity.[6] This formulation allows us to check for whether—and, if so, under what circumstances—the monopolistically competitive limit of Dixit and Stiglitz (1977) is attained, in the presence of common ownership, when firms become small relative to the market; it also enables a determination of how ownership structure affects that competitive limit. If portfolios are incompletely diversified then, regardless of how many sectors the economy has, the Dixit and Stiglitz (1977) monopolistically competitive (wage-taking) limit is attained if there is one firm per sector *or* the oligopolistic limit of Neary (2003b) is reached if sectors comprise multiple firms. Note that, under sufficiently rapid convergence to full diversification as the economy expands, in the limit there is monoposony power.

Competition policy in the one-sector economy can foster employment and increase real wages by reducing market concentration (with nonincreasing returns) and/or the level of common ownership, which serve as complementary tools. We also find that government employment can have an expansionary effect on the economy by reducing firms' monopsonistic labor market power, which reduces the markdown of wages relative to marginal product of labor and thereby induces upward movement along the labor supply curve. This mechanism has a "Kaleckian" flavor and differs from that of government spending's Keynesian multiplier effect. When there are multiple sectors, it is optimal for worker-consumers to have full diversification (common ownership) when product market power exceeds labor market power. In this case, competition policy should not seek to alter ownership structure.

The rest of our paper proceeds as follows. Section 2 reviews the related literature. Section 3 develops a one-sector model of general equilibrium oligopoly with labor as the only factor of production; this is where we derive comparative statics results with respect to the effect of market concentration on employment, wages, and the labor share. In Section 4 we extend the model to allow for multiple sectors with differentiated products, and we then derive results that characterize the limit economy as the number of sectors approaches infinity. Section 5 discusses the implications for competition and government jobs policies, and we conclude in Section 6 with a summary and suggestions for further research. The proofs of most results are given in Appendix A. An online appendix provides charts with empirics, an extension of the model that incorporates both capital and labor to derive additional results for investment and real interest rates, and a formalization of our equilibrium concept's numéraire-free property.

[6] See d'Aspremont et al. (1996) for rigorous formulations of those large economies.

2 Literature review

2.1 Observed trends: Increased concentration and common ownership

There is substantial evidence that market concentration is high in many product markets and has increased over time. Autor et al. (2017) use US Census data to calculate 4-firm and 20-firm concentration ratios based on revenue shares; these authors find that the ratios have increased (on average) between 1982 and 2012, and especially since the early 1990s, for 4-digit industries in manufacturing, finance, services, utilities and transportation, retail trade, and wholesale trade (see Figure A.1 in online appendix). De Loecker and Eeckhout (2017) document sharp increases in markups, dividends, and stock market valuations since the early 1980s. Using a different methodology, Hall (2018) also finds a large increase in markups over the period 1988-2015. Head and Spencer (2017) report that, starting in the mid-2000s, many industries have become more dominated by oligopolies (a notable exception is mobile phones; see Figure 4 in their paper). According to Giandrea and Sprague (2017), this period has also seen a secular decline in the labor share (Figure A.2). Barkai (2016) and Autor et al. (2017) show that increases over time in industry-level concentration are correlated with declines in the industry-level labor share.

There is also considerable evidence that large firms have market power not just in product markets but also in labor markets. A thriving literature in labor economics documents that individual firms face labor supply curves that are imperfectly inelastic, which is indicative of substantial labor market power (Falch, 2010; Ransom and Sims, 2010; Staiger et al., 2010; Matsudaira, 2013). In more recent work, Azar et al. (2017, 2018) provide labor market Herfindahl-Hirschman indices for commuting zones covering most of the United States and 6-digit occupational codes (e.g., Registered Nurses) capturing the most populated occupations; these authors find that, with the exception of big cities, labor markets are generally concentrated. This research finds also that commuting zones and occupations characterized by higher labor market concentration have significantly lower real wages. Benmelech et al. (2018) likewise report high levels of local market concentration; their approach employs US Census data and defines markets by industry instead of by occupation. The authors show that labor market concentration has increased over the period 1977–2009 (Figure A.3 in online appendix).

In addition to the recently increasing average market share of the top firms in each industry, common ownership has risen to prominence following an increase in the ownership of firms by institutional investors and especially by index and quasi-index funds. Gutiérrez and Philippon (2016) report that these quasi-index funds' fraction of US stock shareholding increased from less than a fifth in 1980 to nearly two fifths in 2015 (Figure A.4). These authors also examine private fixed investment in the United States since the early 2000s and report underinvestment relative to standard valuation measures such as Tobin's Q. Using proxies for competition and ownership, they argue that this investment gap is driven by firms owned by quasi-indexers and belonging to industries that have high concentration and high common ownership. Under those circumstances, firms spend a disproportionate amount of free cash flow on share buybacks. Brun and González (2017) also document an increase in Tobin's Q; in the model they develop, an increase in Q due to product market power (which they assume to be determined exogenously by a constant elasticity parameter) is associated with lower investment.

Gutiérrez and Philippon (2017) calculate MHHIs, which account for common ownership by institutional investors, for publicly traded firms in various US industries. They find that the average industry is highly concentrated (in terms of MHHI) and also document an upward trend of both the average MHHI and the average markup (Figure A.5 in the online appendix). Azar et al. (2016, Forthcoming) offer evidence that higher market concentration is associated with higher prices in geographically defined airline and retail banking markets. Anton et al. (2018) and Liang (2016) provide evidence on the transmission mechanism of common institutional ownership on managers' incentives; they find that relative performance evaluation is lower in industries with more common ownership.

2.2 Theoretical developments

The most closely related theoretical papers are perhaps Hart (1982b), d'Aspremont et al. (1990), and Neary (2003a).[7] In Hart (1982b), each firm (which is small relative to the economy as a whole) is assumed to maximize profits as a Cournot oligopolist in its product market (or "island") and to take the wage rate as given; unions are assumed to have market power in labor markets. No firm can affect the income of consumers in its market, and firms are small in relation to their labor market. The resulting general equilibrium model can generate underemployment and has Keynesian features.[8] Hart's work differs from ours in assuming that firms are small relative to the overall economy and have separate owners—and thus have no market power. Unions have the labor market power in his model and so equilibrium real wages are higher than the marginal product of labor; in our model's equilibrium, real wages are lower than that marginal product. Finally, Hart (1982b) assumes that firms maximize profits in terms of money, which is the numéraire in his model.

d'Aspremont et al. (1990) explain the emergence of involuntary unemployment in terms of imperfect price competition while accounting for general equilibrium feedback effects. Here firms are large relative to the economy, but it is still assumed that firms maximize profits in terms of an arbitrary numéraire and that they compete in prices while taking wages as given. The authors also assume that labor supply is completely inelastic, which means that market power reduces employment in their model by driving real wages to zero. We consider instead the more realistic case of a strictly increasing labor supply, which yields a positive equilibrium real wage even when market power reduces employment to below the competitive level. Our focus differs from theirs also in that we derive measures of market concentration, discuss competition policy in general equilibrium, and consider effects on the labor share. Furthermore, we explore what happens in a sequence of economies as the number of sectors becomes large.

[7] See Silvestre (1993) for a survey of the market power foundations of macroeconomic policy. Gabaix (2011) also considers firms that are large in relation to the economy but with no strategic interaction among them; his aim is to show how microeconomic shocks to large firms can create meaningful aggregate fluctuations. Acemoglu et al. (2012) pursue a similar goal but assume that firms are price takers.

[8] Silvestre (1990) relaxes an assumption in Hart's model to show that unemployment is possible when there is product market power but not labor market power. See also Mankiw (1988) and Rotemberg and Woodford (1992) for models in which firms take factor prices as given and have little effect on macroeconomic variables.

Neary (2003a) considers a continuum of industries with Cournot competition in each industry, taking the marginal utility of wealth (instead of the wage) as given. Workers supply labor inelastically and firms maximize profits. He finds a negative relationship between the labor share and market concentration. Our work differs by considering the case in which firms are large relative to the economy and therefore have market power in both product and labor markets. We also consider the effects of firms' ownership structure and do not assume that firms maximize profits. He also assumes a perfectly inelastic labor supply, so that changes in market power can affect neither employment nor output in equilibrium. In contrast, we allow for an increasing labor supply function and examine more possible effects of competition policy.[9] Suppose we take, as a starting point, our framework with multiple sectors and then allow the number of sectors to approach infinity. Then our sequence of equilibria converges to the same equilibrium as in Neary (2003a) when firms are separately owned—or when there is some common ownership but the sequence of portfolios is such that firms in the limit economy do not put positive weights on the profits of other firms. In the general case, however, there can be sequences of economies in which the economy does *not* converge to the Neary equilibrium (as when all shareholders hold market portfolios).

There is another crucial difference between our paper and those cited here. All of them assume the existence of a set of identical consumer-worker-owners, whereas we follow Kalecki (1954) and distinguish between two groups: worker-consumers and owner-consumers. Absent this heterogeneity, the firm would completely internalize its effects on consumers and workers—because they would be its owners—with the result that price would equal marginal cost. If firms are assumed to maximize profits, as in the literature, then their market power will be used to set prices above marginal cost *despite* the objections of owners. Our model has Kaleckian flavor also in relating product market power to the labor share; recall that, in Kalecki (1938), the labor share is determined by the economy's average Lerner index.

Some of the macroeconomic papers already mentioned, in addition to documenting the facts that motivated our paper, also develop theoretical frameworks that link changes in market power to the labor share (Barkai, 2016; De Loecker and Eeckhout, 2017; Eggertsson et al., 2018) and to investment and interest rates (Brun and González, 2017; Gutiérrez and Philippon, 2017; Eggertsson et al., 2018). The models described by Barkai (2016), Brun and González (2017), Gutiérrez and Philippon (2017), and Eggertsson et al. (2018) are based on the monopolistic competition framework of Dixit and Stiglitz (1977); hence markups are determined exogenously by the parameter reflecting elasticity of substitution preferences, which cannot be affected by competition policy. In the partial equilibrium model of De Loecker and Eeckhout (2017), the level of market power is exogenously determined by a "conduct" parameter. In all cases, only product market power is considered and the firms are assumed to have *no* market power in labor or capital markets. Our theoretical framework differs from these because we explicitly model oligopoly and strategic interaction between firms in general equilibrium, which enables our study of how competition policy affects the macroeconomy. We also allow for market power in both product and

[9] Shleifer (1986) and Murphy et al. (1989) also consider models involving more than one firm per sector and some strategic interaction.

factor markets.

3 One-sector macroeconomic model with large firms

In this section we first describe the model in detail. We then characterize the equilibrium and comparative static properties before providing a constant elasticity example.

3.1 Model setup

We consider an economy with (a) a finite number of firms, each of them large relative to the economy as a whole, and (b) an infinite number (a continuum) of people, each of them infinitesimal relative to the economy as a whole. There are two types of people: workers and owners. Workers and owners both consume the good produced by firms. The workers obtain income to pay for their consumption by offering their time to a firm in exchange for wages. The owners do not work for the firms. Instead, an owner's income derives from ownership of the firm's shares, which entitles the owner to control the firm as well as a share of its profits. There is a unit mass of workers and a unit mass of owners, and we use I_W and I_O to denote (respectively) the set of workers and the set of owners. There are a total of J firms in the economy.

There are two goods: a consumer good, with price p; and leisure, with price w. Each worker has a time endowment of T hours but owns no other assets. Workers have preferences over consumption and leisure; this is represented by the utility function $U(C_i, L_i)$, where C_i is worker i's level of consumption and L_i is i's labor supply. We assume that the utility function is twice continuously differentiable and satisfies $U_C > 0$, $U_L < 0$, $U_{CC} < 0$, $U_{LL} < 0$, and $U_{CL} \leq 0$.[10] The last of these expressions implies that the marginal utility of consumption is decreasing in labor supply.

The owners hold all of the firms' shares. We assume that the owners are divided uniformly into J groups, one per firm, with owners in group j owning $1-\phi$ of firm j and ϕ of an index fund representing the market portfolio; here $\phi \in [0,1]$. Thus ϕ can be interpreted as representing the level of portfolio diversification, or (quasi-)indexation, in the economy.[11]

[10] In the notation used here, U_x is the first derivative of U with respect to variable x, and U_{xy} is the second derivative of U with respect to x and y.

[11] An alternative interpretation of the ownership structure is as follows. Assume that each owner in group j is endowed with a fraction $(1-\phi+\phi/J)/(1/J)$ of firm j and a fraction $(\phi/J)/(1/J) = \phi$ of each of the other firms. Since the mass of the group is $1/J$, it follows that the combined ownership in firm j of all the owners in group j is $1-\phi+\phi/J$ and that their combined ownership in each of the other firms is ϕ/J. The combined ownership shares of all shareholders sum to 1 for every firm:

$$\underbrace{\frac{1-\phi+\phi/J}{1/J}}_{\substack{\text{Ownership of firm } j \\ \text{by an owner in group } j}} \times \underbrace{1/J}_{\text{Mass of group } j} + (J-1) \times \underbrace{\frac{\phi/J}{1/J}}_{\substack{\text{Ownership of firm } j \\ \text{by an owner in group } k \neq j}} \times \underbrace{1/J}_{\text{Mass of group } k} = 1.$$

If we use π_k to denote the profits of firm k, then the financial wealth of owner i in group j is given by

$$W_i = \frac{1-\phi+\phi/J}{1/J}\pi_j + \sum_{k\neq j}\phi\pi_k. \tag{3.1.1}$$

Total financial wealth is $\sum_{k=1}^{J}\pi_k$, the sum of the profits of all firms. The owners obtain utility from consumption only, and for simplicity we assume that their utility function is $U^O(C_i) = C_i$. A firm produces using only labor as a resource, and it has a twice continuously differentiable production function $F(L)$ with $F' > 0$ and $F(0) \geq 0$. We use L_j to denote the amount of labor employed by firm j. Firm j's profits are $\pi_j = pF(L_j) - wL_j$.

We assume that the objective function of firm j is to maximize a weighted average of the (indirect) utilities of its owners, where the weights are proportional to the number of shares. That is, we suppose that ownership confers control in proportion to the shares owned.[12] In this simple case, because shareholders do not work and there is only one consumption good, their indirect utility (as a function of prices, wages, and their wealth level) is $V^O(p,w;W_i) = W_i/p$. Hence the objective function of the firm's manager is

$$\underbrace{\left(1-\phi+\frac{\phi}{J}\right)}_{\substack{\text{Control share of}\\ \text{group } j \text{ in firm } j}} \underbrace{\frac{(1-\phi+\frac{\phi}{J})\pi_j + \frac{\phi}{J}\sum_{k\neq j}\pi_k}{p}}_{\text{Indirect utility of shareholder group } j} + \sum_{k\neq j} \underbrace{\frac{\phi}{J}}_{\substack{\text{Control share of}\\ \text{group } k \text{ in firm } j}} \underbrace{\frac{(1-\phi+\frac{\phi}{J})\pi_k + \frac{\phi}{J}\sum_{s\neq k}\pi_s}{p}}_{\text{Indirect utility of shareholder group } k}. \tag{3.1.2}$$

After regrouping terms, we can write the objective function as

$$\left[\left(1-\phi+\frac{\phi}{J}\right)^2 + (J-1)\left(\frac{\phi}{J}\right)^2\right]\frac{\pi_j}{p} + \left[2\left(1-\phi+\frac{\phi}{J}\right)\frac{\phi}{J} + (J-2)\left(\frac{\phi}{J}\right)^2\right]\sum_{k\neq j}\frac{\pi_k}{p}. \tag{3.1.3}$$

After some algebra we obtain that, for firms' managers, the objective function simplifies to maximizing (in terms of the consumption good) the sum of own profits and the profits of other firms—discounted by a coefficient λ. Formally, we have

$$\frac{\pi_j}{p} + \lambda\sum_{k\neq j}\frac{\pi_k}{p}, \tag{3.1.4}$$

where

$$\lambda = \frac{(2-\phi)\phi}{(1-\phi)^2 J + (2-\phi)\phi}.$$

We interpret λ as the weight—due to common ownership—that each firm's objective function assigns to the profits of other firms relative to its own profits. This term was called the coefficient of "effective sympathy" between firms by Edgeworth. It increases with ϕ, the level of portfolio diversification in the economy, and also with market concentration $1/J$. We remark that $\lambda = 0$ if $\phi = 0$ and $\lambda = 1$ if $\phi = 1$, so all firms behave "as one" when portfolios are fully diversified.

[12] See O'Brien and Salop (2000) for other possibilities that allow for cash flow and control rights to differ.

Next we define our concept of equilibrium. We provide a more general version of this concept (and prove that the equilibrium is independent of price normalization) in the online appendix.

3.2 Equilibrium concept

An *imperfectly competitive equilibrium with shareholder representation* consists of (a) a price function that assigns consumption good prices to the production plans of firms, (b) an allocation of consumption goods, and (c) a set of production plans for firms such that the following statements hold.

(1) The prices and allocation of consumption goods are a competitive equilibrium relative to the production plans of firms.

(2) Production plans constitute a Cournot–Nash equilibrium when the objective function of each firm is a weighted average of shareholders' indirect utilities.

It follows then that if a price function, an allocation of consumption goods, and a set of production plans for firms is an imperfectly competitive equilibrium with shareholder representation, then also a scalar multiple of prices will be an equilibrium with the same allocation of goods and productions. The reason is that the indirect utility function is homogenous of degree zero in prices and income and if a consumption and production allocation satisfies (1) and (2) with the original price function then it will continue to do so when prices are scaled. (See the online appendix for additional details.)

We start by defining a competitive equilibrium relative to the firms' production plans—in the particular model of this section, a Walrasian equilibrium conditional on the quantities of output announced by the firms. To simplify notation, we proxy firm j's production plan by the quantity L_j of labor demanded, leaving the planned *production quantity* implicitly equal to $F(L_j)$.

Definition 1 (Competitive equilibrium relative to production plans)**.** *A competitive equilibrium relative to* $(L_1, \ldots, L_J)$ *is a price system and allocation* $[\{w, p\}; \{C_i, L_i\}_{i \in I_W}, \{C_i\}_{i \in I_O}]$ *such that the following statements hold.*

(i) For $i \in I_W$, (C_i, L_i) *maximizes* $U(C_i, L_i)$ *subject to* $pC_i \leq wL_i$*; for* $i \in I_O$, $C_i = W_i / p$.

(ii) Labor supply equals labor demand by the firms: $\int_{i \in I_W} L_i \, di = \sum_{j=1}^{J} L_j$.

(iii) Total consumption equals total production: $\int_{i \in I_W \cup I_O} C_i \, di = \sum_{j=1}^{J} F(L_j)$.

A price function $\mathbb{W}(\mathbf{L})$ and $\mathbb{P}(\mathbf{L})$ assigns prices $\{w, p\}$ to each labor (production) plan vector $\mathbf{L} \equiv (L_1, \ldots, L_J)$, such that for any $\mathbf{L}$, $[\mathbb{W}(\mathbf{L}), \mathbb{P}(\mathbf{L}); \{C_i, L_i\}_{i \in I_W}, \{C_i\}_{i \in I_O}]$ is a competitive equilibrium for some allocation $\{\{C_i, L_i\}_{i \in I_W}, \{C_i\}_{i \in I_O}\}$. A given firm makes employment and production plans conditional on the price function, which captures how the firm expects prices will react to its plans as well as its expectations regarding the employment and production plans of other firms. The economy is in equilibrium when every firm's employment and production plans coincide with the expectations of all the other firms.

Definition 2 (Cournot–Walras equilibrium with shareholder representation)**.** *A Cournot–Walras equilibrium with shareholder representation is a price function* $(\mathbb{W}(\cdot), \mathbb{P}(\cdot))$, *an allocation* $(\{C_i^*, L_i\}_{i \in I_W}, \{C_i^*\}_{i \in I_O})$, *and a set of production plans* $\mathbf{L}^*$ *such that the next two statements hold.*

(i) $[\mathbb{W}(\mathbf{L}^*), \mathbb{P}(\mathbf{L}^*); \{C_i^*, L_i\}_{i \in I_W}, \{C_i^*\}_{i \in I_O}]$ *is a competitive equilibrium relative to* $\mathbf{L}^*$.

(ii) The production plan vector $\mathbf{L}^*$ *is a pure-strategy Nash equilibrium of a game in which players are the* J *firms, the strategy space of firm* j *is* $[0, T]$, *and the firm's payoff function is*

$$\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p};$$

here $p = \mathbb{P}(\mathbf{L})$, $w = \mathbb{W}(\mathbf{L})$, *and* $\pi_j = pF(L_j) - wL_j$ *for* $j = 1, \ldots, J$.

Note that the objective function of firm j depends only on the real wage $\omega = w/p$, which is invariant to normalizations of prices.

3.3 Characterization of equilibrium

Given firms' production plans, we derive the real wage—under a competitive equilibrium—by assuming that workers maximize their utility $U(C_i, L_i)$ subject to the budget constraint $C_i \leq \omega L_i$. This constraint is always binding because utility is increasing in consumption but decreasing in labor. Substituting the budget constraint into the utility function of the representative worker yields the following equivalent maximization problem:

$$\max_{L_i \in [0,T]} U(\omega L_i, L_i). \tag{3.3.1}$$

Our assumptions on the utility function guarantee that the second-order condition holds. Thus the first-order condition for an interior solution implicitly defines a labor supply function $h(\omega)$ for worker i such that labor supply is given by $L_i = \min\{h(\omega), T\}$. Aggregate (average) labor supply is then $\int_{i \in I} L_i \, di = \min\{h(\omega), T\}$. Let η denote the elasticity of labor supply. We assume that preferences are such that $h(\cdot)$ is strictly increasing.[13]

Assumption 1. *The labor supply function* $h'(\omega) > 0$ *for* $\omega \in [0, \infty)$.

This assumption is consistent with a wide range of empirical studies that show that the elasticity of labor supply with respect to wages is positive. A meta-analysis of empirical studies based on different methodologies (Chetty et al., 2011) concludes that the long-run elasticity of aggregate hours worked with respect to the real wage is about 0.59. We also assume that the range of the labor supply function is $[0, T]$. This together with Assumption 1, guarantees the existence of an increasing inverse labor supply

[13] We can obtain the slope of h by taking the derivative with respect to the real wage in the first-order condition. This procedure yields

$$\text{sgn}\{h'\} = \text{sgn}\{U_C + (U_{CC}\omega + U_{CL}) \int_{i \in I} L_i \, di\}.$$

function h^{-1} that assigns a real wage to every possible labor supply level on $[0, T]$. In a competitive equilibrium relative to the vector of labor demands by the firms, labor demand has to equal labor supply:

$$\sum_{j=1}^{J} L_j = \int_{i \in I} L_i di. \tag{3.3.2}$$

Any competitive equilibrium relative to firms' production plans $\mathbf{L}$ must satisfy $\omega = h^{-1}(L)$ if $L = \sum_{j=1}^{J} L_j < T$ or $\omega \geq h^{-1}(T)$ if $L = T$.[14] In what follows we will use the price function that assigns $\omega = h^{-1}(T)$ if $L = T$. Given that the relative price depends only on L, we can define (with some abuse of notation) the competitive equilibrium real-wage function $\omega(L) = h^{-1}(L)$.

3.4 Cournot–Walras equilibrium: Existence and characterization

Here we identify the conditions under which symmetric equilibria exist. We shall also provide a characterization that relates the markdown of wages relative to the marginal product of labor to the level of market concentration in the economy.

The objective of the manager of firm j is to choose L_j so that the following expression is maximized:

$$F(L_j) - \omega(L)L_j + \lambda \sum_{k \neq j} [F(L_k) - \omega(L)L_k]. \tag{3.4.1}$$

First of all, note that firm j's best response depends only on the aggregate response of its rivals: $\sum_{k \neq j} L_k$. This claim follows because the marginal return to firm j is $F'(L_j) - \omega(L) - \big(L_j + \lambda \sum_{k \neq j} L_k\big)\omega'(L)$. Let $E_{\omega'} \equiv -\omega'' L / \omega'$ denote the elasticity of the inverse labor supply's slope. Then a sufficient condition for the game (among firms) to be of the "strategic substitutes" variety is that $E_{\omega'} < 1$. In this case, one firm's increase in labor demand is met by reductions in labor demand by the other firms and so there is an equilibrium (Vives, 1999, Thm. 2.7). Furthermore, if $F'' \leq 0$ and $E_{\omega'} < 1$, then the objective of the firm is strictly concave and the slope of its best response to a rival's change in labor demand is greater than -1. In that event, the equilibrium is unique (Vives, 1999, Thm. 2.8).

Proposition 1. *Let $E_{\omega'} < 1$. Then the game among firms is one of strategic substitutes and an equilibrium exists. Moreover, if returns are non-increasing (i.e., if $F'' \leq 0$), then the equilibrium is both unique and locally stable under continuous best-reply dynamics (unless $F'' = 0$ and $\lambda = 1$); it is also symmetric. In an interior symmetric equilibrium with $L^* \in (0, T)$, the following statements hold.*

(a) *The markdown of real wages is given by*

$$\mu \equiv \frac{F'(L^*/J) - \omega(L^*)}{\omega(L^*)} = \frac{H}{\eta(L^*)}, \tag{3.4.2}$$

where $H = (1 + \lambda(J-1))/J$ is the MHHI.

[14] The implication here is that the competitive equilibrium real wage as a function of $(L_1, \ldots, L_J)$ depends on firms' individual labor demands *only* through their effect on aggregate labor demand L.

(b) *The total employment level L^* and the real wage ω^* are each increasing in J and decreasing in ϕ.*

(c) *The share of income going to workers, $(\omega(L^*)L^*)/(JF(L^*/J))$, decreases with ϕ.*

Remark. To ensure a unique equilibrium it is enough that $-F''(L_j) + (1-\lambda)\omega'(L) > 0$ if the second-order condition holds. In this case we may have a unique (and symmetric) equilibrium with moderate increasing returns. Note that $F'' < 0$ is required if the condition is to hold for all λ.

Remark. If $F'' = 0$ (constant returns) and $\lambda = 1$ ($\phi = 1$, firm cartel), then there is a unique symmetric equilibrium and also multiple asymmetric equilibria, with each firm employing an arbitrary amount between zero and the monopoly level of employment *provided* that the total employment by firms is equal to that under monopoly. The reason is that the shareholders in this case are indifferent over which firm engages in the actual production.

The Lerner-type misalignment of the marginal product of labor and the real wage (i.e., the markdown μ of real wages) is equal to the MHHI divided by the elasticity η of labor supply. The question then arises: Why does there seem to be no effect of product market power? The reason is that, when there is a single good, this effect (equal to product market MHHI divided by demand elasticity) is exactly compensated by the effect of owners internalizing their consumption—that is, since they are also consumers of the product that the oligopolistic firms produce.[15] To see this, consider the objective function of a firm explicitly in terms of nominal wages and prices. Then the objective function of firm j's manager is

$$\frac{1}{p}\left\{pF(L_j) - wL_j + \lambda \sum_{k\neq j}[pF(L_k) - wL_k]\right\}, \tag{3.4.3}$$

where $p = \mathbb{P}(\mathbf{L})$, $w = \mathbb{W}(\mathbf{L})$, and $\mathbf{L}$ is the J-dimensional vector of firms' employment plans. The first-order condition (without simplifying the expression, as we did before to put it in terms of the real wage) with respect to L_j yields

$$\underbrace{\frac{\partial\mathbb{P}/\partial L_j}{p}\left(F(L_j) + \lambda\sum_{k\neq j}F(L_k)\right) + F'(L_j)}_{\text{Marginal revenue effect}} - \underbrace{\frac{\partial\mathbb{W}/\partial L_j}{p}\left(L_j + \lambda\sum_{k\neq j}L_k\right) - \frac{w}{p}}_{\text{Marginal cost effect}}$$
$$\underbrace{-\frac{\partial\mathbb{P}/\partial L_j}{p^2}\left(\pi_j + \lambda\sum_{k\neq j}\pi_k\right)}_{\text{Purchasing power of profits effect}} = 0. \tag{3.4.4}$$

Thus, a marginal increase in employment by firm j has three effects on the utility of the owner-consumers. First, it changes revenues by increasing output at the margin and changing prices (relative to the numéraire), which changes the revenues of firm j and also of the other firms; here the latter is

[15]Note that, unlike in the partial equilibrium model of Farrell (1985), in our model the equilibrium markdown is not zero even when ownership is proportional to consumption because of the labor market power effect. If the labor market is competitive, i.e., $\eta = \infty$, then the labor market power effect is zero and the equilibrium markdown would be zero.

internalized by the owners of firm j according to λ. The second effect is that this marginal increase changes costs by increasing employment and altering nominal wages (relative to the numéraire). Note that the change in (nominal) prices and wages is subject to renormalization and so could go in either direction. Third, the increase affects the purchasing power of profits, which in turn affects utility because it changes the price of the consumption good purchased by the owners with their profits.

Absent the "purchasing power of profits" effect—for example, in a model with quasi-linear utility where owners consume an outside good instead of the oligopolistic good—the equilibrium markdown would reflect both labor market power and product market power. Yet since there is only one good and since owners use their profits only to purchase that good, they fully internalize the effect of product market power and so the markdown reflects only labor market power (because, although owners are consumers, they are not workers). To see how the purchasing power of profits effect cancels out the effect of product market power, we can regroup and cancel terms in the preceding equation to obtain the first-order condition in terms of ω as follows:

$$F'(L_j) - \underbrace{\frac{w}{p}}_{\omega} = \underbrace{\left(\frac{\partial \mathbb{W}/\partial L_j}{p} + \frac{\partial \mathbb{P}/\partial L_j}{p}\frac{w}{p}\right)}_{\omega'} \left(L_j + \lambda \sum_{k \neq j} L_k\right);$$

this equality yields the expression (3.4.2) at the symmetric solution. Hence, as previously explained, the equilibrium in (3.4.2) can be characterized strictly in terms of the elasticity of labor supply and without any reference to the effect of product market power.

3.5 CES preferences and Cobb–Douglas production

We now consider a special case of the model, one in which consumer-workers have separable CES preferences over consumption and leisure:

$$U(C_i, L_i) = \frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{L_i^{1+\xi}}{1+\xi}, \tag{3.5.1}$$

where $\sigma \in (0,1)$ and $\chi, \xi > 0$. The elasticity of labor supply is $\eta = (1-\sigma)/(\xi+\sigma) > 0$, and the equilibrium real wage in the competitive equilibrium—given firms' aggregate labor demand—can be written as

$$\omega(L) = \chi^{1/(1-\sigma)} L^{1/\eta} \tag{3.5.2}$$

with elasticities

$$\frac{\omega' L}{\omega} = \frac{1}{\eta} \quad \text{and} \quad E_{\omega'} = 1 - \frac{1}{\eta} < 1. \tag{3.5.3}$$

Because $E_{\omega'} < 1$, firms' decisions are strategic substitutes. The production function is $F(L_j) = AL_j^{\alpha}$, where $A > 0$ and $\alpha > 0$.

Nonincreasing returns to scale ($\alpha \leq 1$) In this case, the objective function of each firm is strictly concave and Proposition 1 applies. It is easily checked that total employment under the unique symmetric equilibrium is

$$L^* = \left(\chi^{-1/(1-\sigma)} J^{1-\alpha} \frac{A\alpha}{1 + H/\eta} \right)^{1/(1-\alpha+1/\eta)}. \tag{3.5.4}$$

Figure 1 illustrates that an increase in common ownership (or a decrease in the number of firms) reduces equilibrium employment and real wages. With increasing returns to scale, however, reducing the number of firms involves a trade-off between market power and efficiency. In that case, a decline in the number of firms can increase real wages under some conditions.

[[INSERT Figure 1 about Here]]

The symmetric equilibrium is locally stable if $\alpha - 1 < (1-\lambda)(J\eta)^{-1}(1 + H/\eta)^{-1}$, so that a range of increasing returns may be allowed provided that it exists.

Increasing returns to scale If $\alpha > 1$, then neither the inequality $-F'' + (1-\lambda)\omega' > 0$ nor the payoff global concavity condition need hold. We characterize the situation where $\alpha \in (1, 2)$ and $\eta \leq 1$. Then, with respect to L_j, firm j's objective function has a convex region below a certain threshold and a concave region above that threshold. Hence we conclude that there are no more than two candidate maxima for L_j, when given the other firms' decisions, at a *symmetric* equilibrium: $L_j = 0$; and the critical point in the concave region (if there is any). We identify the following necessary and sufficient condition for the candidate interior solution to be a symmetric equilibrium: $\alpha \leq (1 + H/\eta)\{1 + \lambda(J-1)[1 - (1 - 1/J)^{1/\eta}]\}^{-1}$ (see Claim in the Appendix).[16] For small λ we have that when an equilibrium exists it is stable. Here L^* is strictly decreasing in ϕ, but it may either increase or decrease with J:

$$\frac{\partial \log L^*}{\partial J} = \frac{1}{1-\alpha+1/\eta} \frac{1}{J} \left(\underbrace{(1-\lambda)\frac{H/\eta}{1+H/\eta}}_{\text{Markdown effect}} - \underbrace{(\alpha - 1)}_{\text{Economies of scale effect}} \right). \tag{3.5.5}$$

Increasing the number of firms has two effects on a symmetric equilibrium with increasing returns to scale: a positive effect from fewer markdowns, and a negative effect from diminished economies of scale. That is, a merger between two firms (decreasing J) would involve a so-called Williamson trade-off between higher market power and efficiencies from a larger scale of production. In our example, a merger would increase equilibrium employment if α were high enough to dominate the markdown effect.

[[INSERT Figure 2 about Here]]

A higher MHHI (the H in our formulation) makes it more difficult for the scale effect to dominate.

[16] The symmetric equilibrium is locally stable under continuous adjustment provided that $\alpha - 1 \leq (1-\lambda)(J\eta)^{-1}(1 + H/\eta)^{-1}$.

Yet for a given H, a higher internalization λ makes it easier for that effect to dominate because if λ is high enough then firms will act jointly irrespective of the total number J of firms. In fact, if they act fully as one firm i.e., in the case of $\lambda = 1$, the condition is always fulfilled. Indeed, reducing J then improves scale yet does not affect the markdown because it is already at the monopoly level. It is easy to generate examples where, under increasing returns, some firms do not produce (see Figure 2).

3.6 Summary and investment extension

To summarize our results so far, the simple model developed in this section can help make sense of some recent macroeconomic stylized facts, including persistently low output, employment and wages in the presence of high corporate profits and financial wealth, as a response to a permanent increase in market concentration (due either to common ownership or to a reduced number of competitors). Because we have yet to incorporate investment decisions into the model, there is no real interest rate and so we have nothing to say about how it is affected. However, the model can easily be extended to include savers, capital, investment, and the real interest rate. In the online appendix we show that—for a Cobb–Douglas CES specification, and investors who are not fully diversified—either a fall in the number J of firms or a rise in ϕ, the common ownership parameter, will lead to an equilibrium with *lower* levels of capital stock, employment, real interest rate, real wages, output, and labor share of income.

When firms are large relative to the economy, an increase in market power implies that firms have an incentive to reduce both their employment and investment below the competitive level; this follows because, even though such firms sacrifice in terms of output, they benefit from lower wages and lower interest rates on every unit of labor and capital that they employ. The effect described here is present only when firms' shareholders perceive that they can affect the economy's equilibrium level of real wages and real interest rates by changing their production plans. Thus, when oligopolistic firms have market power over the economy as a whole, their owners can extract rents from both workers and savers.[17]

4 Multiple industries

In this section we extend the model to multiple sectors in a Cobb–Douglas CES environment. We characterize the equilibrium, uncover new and richer comparative static results, and have a look at large markets.

4.1 Model setup

Consider now the case in which there are N sectors, each offering a different consumer product. We assume that both the mass of workers and the mass of owners are equal to N. So as we scale the economy by increasing the number of sectors, the number of people in the economy scales proportionally. The utility function of worker i is as in the CES model: $U(C_i, L_i) = C_i^{1-\sigma}/(1-\sigma) - \chi L_i^{1+\xi}/(1+\xi)$ for $\sigma \in$

[17] Our model does not account for possible technological spillovers due to investment among firms. López and Vives (2017) show that if spillovers are high enough then increasing common ownership may increase R&D investment as well.

$(0, 1)$ and $\chi, \xi > 0$, where

$$C_i = \left[\left(\frac{1}{N} \right)^{1/\theta} \sum_{n=1}^{N} c_{ni}^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}; \tag{4.1.1}$$

where c_{ni} is the consumption of worker i in sector n, and $\theta > 1$ is the elasticity of substitution indicating a preference for variety.[18]

For each product, there are J firms that can produce it using labor as input. The profits of firm j in sector n are given by

$$\pi_{nj} = p_n F(L_{nj}) - wL_{nj}; \tag{4.1.2}$$

here, as before, the production function is $F(L_{nj}) = AL_{nj}^{\alpha}$ for $A > 0$ and $\alpha > 0$.

The ownership structure is the same as in the single-sector case, except now there are $J \times N$ groups of shareholders. Group nj owns a fraction $1 - \phi + \phi/JN$ of firm j in industry n and a fraction ϕ/JN of every other firm. The owners' utility is simply their consumption of the composite good C_i. Solving the owners' utility maximization problem yields the indirect utility function of shareholder i, or $V(P, w; W_i) = W_i/P$, when: prices are $\{p_n\}_{n=1}^{N}$, the level of wages is w, the shareholder's wealth is W_i, and $P \equiv \left(\frac{1}{N} \sum_{n=1}^{N} p_n^{1-\theta} \right)^{1/(1-\theta)}$ is the price index.

The objective function of the manager of firm j in sector n is to choose the firm's level of employment, L_{nj}, that maximizes a weighted average of shareholder (indirect) utilities. By rearranging coefficients so that the coefficient $= 1$ for own profits, we obtain the following objective function:

$$\underbrace{\frac{\pi_{nj}}{P}}_{\text{Own profits}} + \lambda \left(\underbrace{\sum_{k \neq j} \frac{\pi_{nk}}{P}}_{\text{Industry } n \text{ profits from other firms}} + \underbrace{\sum_{m \neq n} \sum_{k=1}^{J} \frac{\pi_{mk}}{P}}_{\text{Profits from other industries}} \right). \tag{4.1.3}$$

Here λ is the resulting Edgeworth sympathy coefficient, given (as before) by replacing J with JN:

$$\lambda = \frac{(2-\phi)\phi}{(1-\phi)^2 JN + (2-\phi)\phi}. \tag{4.1.4}$$

Thus the firm accounts for the effects of its actions not only on same-sector rivals but also on firms in other sectors. Note that the manager's objective function depends on $N + 1$ relative prices: w/P in addition to $\{p_n/P\}_{n=1}^{N}$ for $N > 1$.

4.2 Cournot–Walras equilibrium with N sectors

Given the production plans of the J firms operating in the N sectors, $\mathbf{L} \equiv \{\mathbf{L}_1, ..., \mathbf{L}_J\}$ where $\mathbf{L}_j \equiv (L_{1j}, ..., L_{Nj})$, we characterize first the competitive equilibrium in terms of w/P, and $\{p_n/P\}_{n=1}^{N}$. Second,

[18] The form of C_i is the one used by Allen and Arkolakis (2015). The weight $(1/N)^{1/\theta}$ in C_i implies that, as N grows, the indirect utility derived from C_i does not grow unboundedly but is consistent with a continuum formulation for the sectors (replacing the summation with an integral) of unit mass. More precisely: if the equilibrium is symmetric then, regardless of N, C_i is equal to the consumer's income divided by the price.

we characterize the equilibrium in the plans of the firms.

4.2.1 Relative prices in a competitive equilibrium given firms' production plans

Because the function that aggregates the consumption of all sectors is homothetic, workers face a two-stage budgeting problem. First, workers choose their consumption across sectors (conditional on their aggregate level of consumption) to minimize expenditures; second, they choose labor supply L_i and consumption level C_i to maximize their utility $U(C_i, L_i)$ subject to $PC_i = wL_i$, where P is the aggregate price level.

We can therefore write the first-stage problem as

$$\min_{\{c_{ni}\}_{n=1}^N} \sum_{n=1}^{N} p_n c_{ni} \tag{4.2.1}$$

subject to

$$\left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{1/\theta} c_{ni}^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} = C_i. \tag{4.2.2}$$

The solution to this problem yields the standard demand for each consumer product conditional on aggregate consumption:

$$c_{ni} = \frac{1}{N}\left(\frac{p_n}{P}\right)^{-\theta} C_i. \tag{4.2.3}$$

It follows from homotheticity that, for every consumer, total expenditure equals the price index multiplied by their respective level of consumption:

$$\sum_{n=1}^{N} p_n c_{ni} = PC_i. \tag{4.2.4}$$

In the second stage, the first-order condition for an interior solution is

$$\frac{w}{P} = -\frac{U_L\left(\frac{w}{P}L_i, L_i\right)}{U_C\left(\frac{w}{P}L_i, L_i\right)}. \tag{4.2.5}$$

Since workers are homogeneous, it follows that total labor supply $\int_{i\in I} L_i \, di$ is simply N times the individual labor supply L_i; moreover, because total labor demand L must equal total labor supply, equation (4.2.5) implicitly defines the equilibrium real wage (now relative to the price of the composite good) as a function $\omega(L)$ of the firms' total employment plans. We retain the assumptions for increasing labor supply that ensure $\omega' > 0$. When elasticity is constant we have $\omega(L) = \chi^{1/(1-\sigma)}(L/N)^{1/\eta}$; once again, $\eta = (1-\sigma)/(\xi+\sigma)$ is the elasticity of labor supply.

Shareholders maximize their aggregate consumption level conditional on their income. Their consumer demands, conditional on their respective levels of consumption, are identical to those of workers.

Adding up the demands across both owners and workers, we obtain

$$\underbrace{\int_{i\in I_W\cup I_O} c_{ni}\,di}_{c_n} = \frac{1}{N}\left(\frac{p_n}{P}\right)^{-\theta}\underbrace{\int_{i\in I_W\cup I_O} C_i\,di}_{C}. \tag{4.2.6}$$

In a competitive equilibrium, consumption demand must equal the sum of all firms' production of each product:

$$c_n = \sum_{j=1}^{J} F(L_{nj}). \tag{4.2.7}$$

Using equation (4.2.3) and integrating across consumers, we have that $c_n = \frac{1}{N}\left(\frac{p_n}{P}\right)^{-\theta}C$. So given firms' production plans, the following equality holds in a competitive equilibrium:

$$\frac{p_n}{P} = \left(\frac{1}{N}\right)^{1/\theta}\left(\frac{c_n}{C}\right)^{-1/\theta}. \tag{4.2.8}$$

The elasticity of the relative price of sector n, p_n/P, in relation to the aggregate production of the sector c_n for given productions in the other sectors, c_m for $m \neq n$, evaluated at a symmetric equilibrium, is given by $-(1-1/N)/\theta$. Its absolute value is decreasing in the elasticity of substitution of the varieties θ and increasing in the number of sectors N. Increasing c_n has a direct negative impact on p_n/P of $-1/\theta$ for a given C, and an indirect positive impact on p_n/P by increasing aggregate real income C, yielding $1/\theta N$. When there is only one sector ($N = 1$) there is obviously no impact on the relative price. Furthermore, the overall impact increases in the number of sectors N since then the indirect effect diminishes.

We can now use equations (4.2.7) and (4.2.8) to obtain an expression for p_n/P in a competitive equilibrium (ρ_n) conditional on firms' production plans $\mathbf{L}$:

$$\rho_n(\mathbf{L}) = \left(\frac{1}{N}\right)^{1/\theta}\left\{\frac{\sum_{j=1}^{J} F(L_{nj})}{\left[\sum_{m=1}^{N}\left(\frac{1}{N}\right)^{1/\theta}\left(\sum_{j=1}^{J} F(L_{mj})\right)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)}}\right\}^{-1/\theta}. \tag{4.2.9}$$

Observe that—unlike the previous case of a real-wage function, where the dependence was only through total employment plans—relative prices under a competitive equilibrium depend directly on the employment plans of each individual firm.

Proposition 2. *Given the production plans $\mathbf{L} \equiv \{L_{mj}\}$ of firms with aggregate labor demand L, the competitive equilibrium depends on the real wage $\omega(L)$ and on the relative price in sector n: $\rho_n(\mathbf{L})$ for $n = 1, \ldots, N$. If firm j in sector n expands its employment plans, then ω increases; in addition, ρ_n decreases ($\partial\rho_n/\partial L_{nj} < 0$) while ρ_m, $m \neq n$, increases ($\partial\rho_m/\partial L_{nj} > 0$).*

An increase in employment by a firm in sector n increases the relative supply of the consumption good of that sector relative to other sectors, thereby reducing the relative price of that sector's good. Since the increased employment increases overall supply of the aggregate consumption good while leaving supply of the other sectors unchanged, the relative prices of goods in the other sectors increase.

4.2.2 Cournot-Walras equilibrium

The maximization problem of firm j in sector n is given by

$$\max_{L_{nj}} \frac{\pi_{nj}}{P} + \lambda \left\{ \sum_{k \neq j} \frac{\pi_{nk}}{P} + \sum_{m \neq n} \sum_{k=1}^{J} \frac{\pi_{mk}}{P} \right\}, \tag{4.2.10}$$

where $\pi_{nj}/P = \rho_n F(L_{nj}) - \omega(L) L_{nj}$. The first-order condition for the firm is

$$\underbrace{\rho_n (\mathbf{L}) F'(L_{nj})}_{\text{VMPL}} - \underbrace{\omega(L)}_{\text{real wage}} \underbrace{- \frac{\partial \omega}{\partial L_{nj}} \left[L_{nj} + \lambda \left(\sum_{k \neq j} L_{nk} + \sum_{m \neq n} \sum_{k=1}^{J} L_{mk} \right) \right]}_{\text{(i) wage effect}}$$
$$\underbrace{+ \frac{\partial \rho_n}{\partial L_{nj}} \left[F(L_{nj}) + \lambda \sum_{k \neq j} F(L_{nk}) \right]}_{\text{(ii) own-industry relative price effect}} + \underbrace{\lambda \left\{ \sum_{m \neq n} \frac{\partial \rho_m}{\partial L_{nj}} \left[\sum_{k=1}^{J} F(L_{mk}) \right] \right\}}_{\text{(iii) other industries' relative price effect}} = 0. \tag{4.2.11}$$

When a firm in a given sector considers expanding employment, it faces the following trade-offs. On the one hand, expanding employment increases profits by the value of the marginal product of labor (VMPL), which the shareholders can consume after paying the new workers the real wage. On the other hand, expanding employment will increase real wages for all workers because the labor supply is upward sloping. So when there is common ownership, the owners will take into account the wage effect not only for firms that expand employment (or just for the firms in the same industry) but for firms in all industries. Furthermore, expanding employment increases output in the firm's sector and thereby reduces relative prices in that sector, which again the owners internalize not just for the firm itself but for all firms in the sector in which they have common ownership. Finally, expanding output in the firm's sector increases overall consumption and thus increases relative prices in all the other sectors; the owners of the firm, if they have common ownership involving other sectors, internalize these increased relative prices as a positive pecuniary externality. However, we will show that the own-sector negative price effect always dominates the effect of increased demand in other sectors.

As we show in the Appendix, a firm's objective function is strictly concave if $\alpha \leq 1$. We can thus establish the following existence and characterization result.[19]

Proposition 3. *Consider a multi-sector economy with CES preferences and a Cobb–Douglas production function under non-increasing returns to scale ($\alpha \leq 1$). Then there exists a unique symmetric equilibrium, and equilibrium*

[19] As in the one-sector case, if $\phi = 1$ and $\alpha = 1$ then there is a unique symmetric equilibrium and there also exist asymmetric equilibria, since shareholders are indifferent to *which* firms employ the workers as long as total employment is at the monopoly level.

employment is given by

$$L^* = N\left(J^{1-\alpha}\frac{\chi^{-1/(1-\sigma)}A\alpha}{1+\mu^*}\right)^{1/(1-\alpha+1/\eta)}. \tag{4.2.12}$$

The equilibrium markdown of real wages relative to the value of the marginal product of labor is

$$\mu^* = \frac{1+H_{JN}/\eta}{1-(H_J-\lambda)(1-1/N)/\theta} - 1, \tag{4.2.13}$$

where $H_{JN} = (1+\lambda(JN-1))/JN$ *is the labor market MHHI and* $H_J = (1+\lambda(J-1))/J$ *is each sector's product market MHHI.*

The markdown μ^* *decreases with* J, η, *and* θ, *and we have* $\text{sgn}\{\frac{\partial\mu^*}{\partial\phi}\} = \text{sgn}\{\theta(JN-1)-(1+\eta)(N-1)\}$.

The markdown of wages below the marginal product of labor depends on two components. This markdown is increasing in H_{JN}/η, which reflects the level of labor market power (and so is decreasing in JN and η), and it is also increasing in $H_J(1-1/N)/\theta$ (which reflects the level of product market power). Recall that, when evaluated at a symmetric equilibrium, the (absolute value of the) elasticity of "inverse demand" p_n/P with respect to c_n is $(1-1/N)/\theta$; this explains why $H_J(1-1/N)/\theta$ is the usual indicator of product market power (note that the indicator decreases with J and θ but increases with N).

When $\lambda > 0$, the effect of expanding employment by firm j in sector n on the profits of other firms must be taken into account. Expanding employment in one sector benefits firms in other sectors by increasing the relative prices in those sectors (pecuniary externality) via the increase in overall consumption generated by firm nj's expanded employment plans. The result is that the market power indicator is diminished then by the term $\lambda(1-1/N)/\theta$. The net effect is that an increase in λ (due to an increase in ϕ) will *more than* compensate for the product market power's effect on the equilibrium markdown. To see this, note that $(H_J-\lambda)(1-1/N)/\theta = (1-\lambda)(1-1/N)/\theta J$. In the limit, when $\lambda = 1$ (or $N = 1$), we have a cartel or monopoly and the two product market effects cancel each other out exactly. The $N = 1$ case is the one-sector model developed in Section 3. We explained that, in this case, the effect on profits from increased prices is exactly balanced by the lower purchasing power of profits for the owner-consumers; hence the product market effect on the markdown is nil, which leaves only the labor market effect. Here $\lambda = 1$ can be understood in similar terms, except that in this case we have an aggregate good C. When portfolios are perfectly diversified ($\phi = 1$), we can view the economy as consisting of a single large firm that produces the composite good. Since the owner-consumers own shares in each of the components of the composite good in the same proportion, and since they use profits only to purchase that good, these owner-consumers are to the same extent shareholders and consumers of the composite good. So just as in the single-sector economy, the effects cancel out exactly.

It is worth noting that μ^* may either increase or decrease with portfolio diversification ϕ depending on whether labor market effects or rather product market effects prevail. The markdown will be decreasing in ϕ when labor market power is low (high η) *and/or* product market power is high (low θ and J). In this case, the increased labor market power due to the higher ϕ is more than compensated by the effect on product markets due to the pro-competitive intersector pecuniary externality. When the labor market is competitive ($\eta \to \infty$), common ownership has always a pro-competitive effect.

As the elasticity of substitution parameter θ tends to infinity, the products of the different sectors become close to perfect substitutes; then the equilibrium is as in the one-industry case but with JN firms instead of J firms. This outcome should not be surprising given that, in the case of perfect substitutes, all firms produce the same good and so—for all intents and purposes—there is but a single industry in the economy.

In the multiple-industry case we find that the equilibrium real wage, employment, and output are analogous—as a function of the markdown—to those in the single-industry case. The only difference is that the markdown is now more complicated owing to the existence of multiple sectors and of product differentiation across firms in different sectors. An important result that contrasts with the single-sector case is that employment, output, and the real wage may all be increasing in ϕ.

4.3 Large economies

Most of the literature on oligopoly in general equilibrium considers the case of an infinite number of sectors such that each sector, and therefore each firm, is small relative to the economy. Monopolistic competition can be considered a special case of a model with infinite sectors in which there is only one firm per industry. Here we consider what happens when the number of sectors, N, tends to infinity. We show that, when the sequence of economies is such that the sympathy coefficient tends to zero as N tends to infinity, our model converges to Neary's general oligopolistic equilibrium model (when there is more than one firm) or to the Dixit–Stiglitz monopolistic competition model (when there is one firm per industry). However, if the sequence of economies is such that the limit of the sympathy coefficient is positive, then our model does not converge to Neary's or the Dixit–Stiglitz model. In this case of positive sympathy, we provide a characterization of the limit economy.

Consider a sequence of economies (ϕ_N, N) with an increasing number N of sectors, increasing masses of workers and owners in proportion to N, and possibly varying degrees ϕ_N of common ownership. We use μ_N to denote the markdown of economy N in the sequence. Let

$$\lambda_N = \frac{(2-\phi_N)\phi_N}{(1-\phi_N)^2 JN + (2-\phi_N)\phi_N} \to \lambda \text{ as } N \to \infty.$$

If $\phi_N \to \phi < 1$ then $\lambda_N \to 0$—as, for example, when $\phi_N = \phi < 1$ for all N. To have $\lambda_N \to \lambda \in (0, 1]$, we need for ϕ_N to approach unity at least as rapidly as $1/\sqrt{N}$ (i.e., $\sqrt{N}(1-\phi_N) \to k$ for $k \in [0, \infty)$). If the convergence rate is faster than $1/\sqrt{N}$ with $k = 0$, then the limiting λ is always equal to 1. For sequences $1 - \phi_N$ with convergence rates equal to $1/\sqrt{N}$, the value of λ in the limit is determined by k, the constant of convergence. Therefore, if $\sqrt{N}(1-\phi_N) \to k$ then

$$\lim_{N\to\infty} \lambda_N = \frac{1}{1+Jk^2}.$$

This result follows from the expression for λ_N by noting that $(1-\phi_N)^2 N$ is of order k^2 and that $\phi_N \to 1$ as $N \to \infty$. Note that the limit sympathy coefficient λ is increasing in concentration $1/J$ and also in the speed of convergence of $\phi_N \to 1$, as measured by the constant $1/k$. When diversification increases faster (k smaller), profit internalization is larger. Hence, in order for λ to be positive, the limiting portfolio must be fully diversified: $\phi_N \to 1$. Indeed, if $\phi_N = 1$ for all N then also $\lambda_N = 1$ for all N.[20]

If $\lambda_N \to \lambda$, then the limit markdown is:

$$\mu^*_\infty \equiv \lim_{N\to\infty} \mu^*_N = \frac{1+\lambda/\eta}{1-(1-\lambda)/\theta J} - 1. \tag{4.3.1}$$

An increase in λ affects the equilibrium markdown as follows. Higher λ increases labor market power, which has a positive effect on the markdown. It also has two effects on the product market markup: an increase in the market power of each sector's firms; and an increase in the internalization of the effects of a given sector's firm on the relative prices of firms in other sectors. The latter effect dominates, so the overall product market effect is negative. The labor market monopsony effect dominates the total product market effect if and only if the elasticity η of labor supply is *lower* than $\theta J - 1$. These results are summarized in our next proposition.

Proposition 4. *Consider a sequence of economies (ϕ_N, N). In order to have $\lambda_N \to \lambda_\infty \in (0,1]$ as $N \to \infty$, we need that $\phi_N \to 1$ at least as fast as $1/\sqrt{N}$ (i.e., $\sqrt{N}(1-\phi_N) \to k$ for $k \in [0,\infty)$). Now we have that λ_∞ is increasing in concentration $1/J$ and in the speed of convergence of $\phi_N \to 1$ as measured by the constant $1/k$. The limit markdown is $\mu^*_\infty = \frac{1+\lambda_\infty/\eta}{1-(1-\lambda_\infty)/\theta J} - 1$, which is increasing in λ_∞ if and only if $\theta J - 1 > \eta$.*

The market power friction at a symmetric equilibrium can also be expressed in terms of the markup of product prices over effective marginal cost of labor $\left(\text{mc} \equiv \frac{w}{F'(L/JN)}\right)$,

$$\tilde{\mu} \equiv \frac{p - \text{mc}}{p} = \frac{\mu}{1+\mu},$$

rather than in terms of the markdown $\mu = (p - \text{mc})/\text{mc}$; thus we obtain

$$\tilde{\mu}^*_\infty = \lim_{N\to\infty} \tilde{\mu}^*_N = \frac{\lambda/\eta + (1-\lambda)/\theta J}{1+\lambda/\eta}. \tag{4.3.2}$$

If $\lambda_\infty = 0$ then the labor market is competitive, $\mu^*_\infty = 1/(J\theta - 1)$, and $\tilde{\mu}^*_\infty = 1/\theta J$; these equalities yield the Dixit–Stiglitz monopolistic competition result when $J = 1$ or Neary's (2003a) oligopolistic result, with constant elasticity, when $J > 1$. When $\lambda_\infty > 0$, however, the labor market is oligopsonistic. In this case, if $J \to \infty$ then there is no product market power and so the markdown λ_∞/η is due only to labor market power. When $\lambda_\infty = 1$, we obtain the monopoly solution $\mu^*_\infty = 1/\eta$. Hence the case of monopolistic competition can be viewed as a limit of oligopolistic economies when (i) there is one firm per variety, and, crucially, (ii) the firms' sympathy coefficient λ tends to zero as the number of product

[20] For λ to be constant for all N, we need the sequence $\phi_N = 1 - \sqrt{(1-\lambda)/(\lambda J N + (1-\lambda))}$.

varieties becomes large. Without this second condition, the markup does not converge to $1/\theta$ even when the number of firms becomes large.

5 Government policy

In this section, we show that equilibrium outcomes in oligopolistic economies are suboptimal from a social welfare perspective. We then consider the effects of government policies that could have a positive effect on aggregate equilibrium outcomes. Our model is static and should therefore be interpreted as capturing only long-run phenomena. In this model, then, the low levels of output and employment are of a long-run nature and so would not be affected by monetary policy. Hence we consider instead the effects of competition policy and government employment.

Competition policy can influence aggregate outcomes by directly affecting product and labor market concentration—that is, by affecting the number of firms and also the extent of their ownership overlap. Alternatively, the government could tax the oligopolistic firms' profits and then use those revenues to employ people, thereby also changing the labor market's effective level of concentration.[21] Because profits in our model are based solely on rents from market power, there is no distortion in the model from taxation. While this is useful to gain intuition within a relatively simple framework, it would not be hard to think of models in which taxation is distortionary. The fact that government employment has a "Kaleckian" effect–driving down markdowns–is in contrast to the monopolistic competition model, where markups are exogenous, and firms take wages as given, and therefore government expenditure has zero impact on markdowns.

We illustrate the analysis with the one-sector model constant elasticity specification. We look in turn at the social planner allocation (first best), then competition policy and government jobs (second best).

5.1 Social planner's solution

Here we characterize, in the one-sector Cobb–Douglas CES model, the allocation that would be chosen by a benevolent social planner who maximizes a weighted sum of the utilities of all worker-consumers with weight $1-\kappa$ and the utilities of all owner-consumers with weight $\kappa \in [0,1]$.[22] We assume that the social planner can choose the allocation of labor and consumption as well as the number of firms (with access to a large number $J^{\max}$). Let (C, L) be the consumption and labor supply of a representative worker, and let C_O be the consumption of a representative owner; then the social planner's problem is constrained by $C + C_O \leq JA(L/J)^{\alpha} = AL^{\alpha}(1/J)^{\alpha-1}$. This constraint will always hold with equality, since otherwise it would be possible to increase welfare by increasing workers' consumption until the

[21] The model assumes that government expenditures are useless so we can focus on the government's labor market impact. Of course, public good provision would lead to a higher desired level of government employment.

[22] One can interpret κ as determining the welfare standard used by society. Thus $\kappa = 0$ represents the case of a "worker-consumer welfare standard" in which owners' utilities are assigned zero weight; this case is analogous—in our general equilibrium oligopoly model—to that of the usual *partial* equilibrium *consumer* welfare standard. The case $\kappa = 1/2$ corresponds to a "total welfare standard" in which all agents' utilities are equally weighted.

constraint binds. Therefore, the problem can be rewritten as

$$\max_{C,L,J}(1-\kappa)\left(\frac{C^{1-\sigma}}{1-\sigma}-\chi\frac{L^{1+\xi}}{1+\xi}\right)+\kappa[AL^{\alpha}J^{1-\alpha}-C]. \tag{5.1.1}$$

We solve this problem in two steps. First we choose the welfare-maximizing C and L conditional on the number J of firms that are used (symmetrically) in production. Second, we maximize over J to obtain the optimal number of firms from the social planner's perspective.

The first-order conditions (which are sufficient under non-increasing returns to scale) for the first maximization problem ensure that, in an interior solution, $C^{-\sigma}$, the marginal utility of workers' consumption, is equal to $\kappa/(1-\kappa)$ multiplied by the owners' marginal utility of consumption (which is constant and equals 1) and that it is equal also to the marginal disutility from working divided by the marginal product of labor: $\chi L^{\xi}/(A\alpha(L/J)^{\alpha-1})$.[23] This condition cannot hold in an oligopolistic equilibrium because the markdown of wages relative to the marginal product of labor is positive; that outcome follows, in turn, because worker-consumers equalize the marginal utility of labor to the ratio of the marginal disutility of work and the real wage. Thus a positive markdown introduces a wedge between the marginal product of labor and the real wage:

$$\underbrace{C^{-\sigma}=\frac{\chi L^{\xi}}{w/p}=\frac{\chi L^{\xi}}{A\alpha(L/J)^{\alpha-1}}(1+\mu)}_{\text{Oligopoly equilibrium condition}}>\frac{\chi L^{\xi}}{A\alpha(L/J)^{\alpha-1}}.$$

How many firms will the social planner choose to use in the production process? If there are *decreasing* returns to scale, then social benefits are increasing in J and so the optimal choice is $J^{\max}$. With *constant* returns to scale, the number of firms in operation is irrelevant. Under *increasing* returns to scale, the social planner would choose to produce using only one firm; however, the planner would still set—contra the monopolistic outcome—the marginal product of labor equal to the marginal rate of substitution between consumption and labor.[24] Thus, from the viewpoint of a social planner, there is no Williamson trade-off because the planner can set the "shadow" markdown to zero and still benefit fully from the economies of scale due to producing with only one firm. Next we address the second-best allocation, where the planner can affect the oligopoly equilibrium only by controlling the variables J and ϕ.

5.2 Competition policy

The models developed so far illustrate how the level of competition in the economy has macroeconomic consequences, from which it seems reasonable to conclude that competition policy may stimulate the economy by boosting output and inducing a more egalitarian distribution of income. We showed that if

[23] It is possible, however, for low enough values of κ, to have a corner solution, such that all the output is assigned to the workers, and the consumption of the owners is zero, i.e., $C = AL^{\alpha}$ and $C_O = 0$.

[24] With increasing returns to scale, and $\alpha < 1+\xi$, the objective of the social planner is convex in L below a threshold, and concave in L above that threshold. This guarantees that the optimal L is strictly positive (however, just like in the non-increasing returns case, there can be a corner solution for the consumption of the workers and the owners, that is $C = AL^{\alpha}$ and $C_O = 0$). If $\alpha > 1+\xi$, in some cases there could be a corner solution with $L = 0$.

returns to scale are non-increasing then employment, output, real wages, and the labor share all decrease under higher market concentration and more common ownership.

In the one-sector case, the equilibrium MHHI (our H) was the same for the product and labor markets and also was proportional to the markdown of wages relative to the marginal product of labor in the economy. In the multi-sector case, the markdown was a function of both the within-industry and the economy-wide MHHIs, of which the latter are most relevant for the labor market. (In practice, labor markets are segmented and so the labor market MHHI would differ from the economy-wide MHHI; however, the insight would be similar.)

5.2.1 Worker-consumer welfare

We can think of the competition policy in our model as setting a policy environment that affects—in a symmetric equilibrium—the number of firms per industry and/or the extent of common ownership. We start by showing that $1-\phi$ and J are complements as policy tools. Then common ownership mitigates the effect of "traditional" competition policy on employment because increasing the number of firms has a diminished effect on concentration when firms have more similar shareholders.

Proposition 5. *Let $\alpha < 1+1/\eta$ and let L^* be a symmetric equilibrium. Then reducing common ownership $($increasing $1-\phi)$ and reducing concentration $($increasing $J)$ are complements as policy tools for affecting equilibrium employment.*

The proposition follows because $\operatorname{sgn}\{\frac{\partial^2 \log L^*}{\partial(1-\phi)\partial J}\} = \operatorname{sgn}\{-(J-1)(1-\lambda)\frac{\partial\lambda}{\partial(1-\phi)}\} > 0$ for $J>1$, $\eta < \infty$ and $\frac{\partial\lambda}{\partial(1-\phi)} < 0$. We remark that this proposition holds under decreasing returns and also in our increasing returns example with $\eta \leq 1$ and $\alpha \in (1,2)$.

We claim that, under either constant or decreasing returns to scale, it is always welfare-increasing for worker-consumers if the planner's policy decreases common ownership and increases the number of firms—although the latter claim need not apply under increasing returns. Under non-increasing returns, the result follows because L^* increases with both $1-\phi$ and J, equilibrium real wages increase with employment, and worker-consumer utility increases with real wages. Under increasing returns, however, there is a trade-off between market power and efficiency; in this scenario, the optimal number of firms (from the perspective of worker-consumer welfare) is limited.[25] In short: if returns to scale are increasing, then a decrease in the equilibrium markdown does not always translate into an increase in worker-consumer welfare. The following proposition presents these results formally.

Proposition 6. *Employment, real wages, and the welfare of worker-consumers are maximized by setting $\phi = 0$ and*:

(a) *$J = J^{\max}$ with non-increasing returns $(\alpha \leq 1)$; and*

(b) *J equal to the greatest integer less than $\frac{2-\alpha}{\alpha-1}\eta^{-1}$ when returns are increasing, $\alpha \in (1,2)$ and $\eta \leq 1$.*[26]

[25] One can easily check that, under our assumptions, L^* is increasing in $1-\phi$ and that it peaks for J (when considered as a continuous variable) at $\eta^{-1}(2-\alpha)/(\alpha-1)$.

[26] If $J > \eta^{-1}(2-\alpha)/(\alpha-1)$, then $\alpha - 1 > (\eta J)^{-1}(1+(\eta J)^{-1})^{-1}$ and the equilibrium would be unstable (see Section 2.5).

In the case of non-increasing returns, competition policy can lead to equilibria arbitrarily close to the social planner's as $J^{\max}$ becomes large. This is because the markdown then becomes arbitrarily close to zero.

5.2.2 Positive weight on owner-consumer welfare

The polar case of $\kappa = 1$, when the social planner maximizes the utility of the owner-consumers only, implies, under the assumption that $\eta \leq 1$, setting $\phi = 1$ to have a completely concentrated economy in terms of the MHHI, while choosing the number of firms to produce as efficiently as possible, which implies setting $J = J^{\max}$ in the case of decreasing returns, $J = 1$ in the case of increasing returns, and any $J \in \{1, \ldots, J^{\max}\}$ in the case of constant returns. (This claim is proved in the Appendix).

[[INSERT Figure 3 about Here]]

For intermediate values of κ, there is no simple analytic solution to the problem of choosing a competition policy that maximizes social welfare. Yet we do know that, as κ increases, owner-consumer welfare increases while worker-consumer welfare declines; this implies that equilibrium employment and wages are both lower, in equilibrium, when κ is higher. In the figures that follow, we present results of simulations from which we derive the optimal policy—and the resulting employment and welfare of each type of agent—as a function of κ. Figure 3 shows the results when $\alpha = 0.8$. The optimal policy always sets $J = J^{\max}$ ($= 100$ in this simulation). The parameter ϕ starts at 0 and remains there for an interval corresponding to κ values between 0 and about 0.4; thereafter, ϕ increases rapidly and reaches $\phi = 1$ when $\kappa = 1$. Employment and worker welfare are highest in the range of κ for which $\phi = 0$, after which they both decrease monotonically and achieve their lowest value at $\kappa = 1$. Owner-consumer welfare is lowest for low values of κ; then it increases because larger κ values result in larger values of ϕ, the common ownership parameter.

[[INSERT Figure 4 about Here]]

In Figure 4, $\alpha = 1.2$. The optimal policy always sets $\phi = 0$. We have that $J = 4$ when $\kappa = 0$, which implies that it is optimal—even from the worker-consumers' standpoint—if some market power is allowed so as to exploit economies of scale. The number of firms declines as κ increases, and the optimal policy is a single firm when κ is slightly above one-half. Employment and worker welfare are largest for low values of κ, and both decrease monotonically as κ increases.[27]

[27] Although ϕ is always zero in this example and market power increases when there are fewer firms, this is not true in general with increasing returns to scale. For example, if $\alpha = 1.001$ then, as κ increases, at first the policy achieves a higher H by reducing the number of firms. Yet as κ continues to increase, in some parts of the range it becomes optimal (since the number of firms is a discrete quantity) for the policy to increase concentration—say, by increasing ϕ above zero—before decrementing the number J again. The implication is that ϕ here increases monotonically with κ but then "jumps back" to zero when κ reaches the level that leads to the next decline in the number of firms J.

5.2.3 Competition policy with multiple sectors

In the one-sector case, with the worker-consumer welfare standard ($\kappa = 0$) it is always efficient to force completely separate ownership of firms—that is, regardless of how many firms there are—because there are no efficiencies associated with common ownership. In the multi-sector case, however, common ownership is associated with internalization of demand effects in other sectors; this means that—depending on the elasticity of substitution, the elasticity of labor supply, and the number of firms per industry—worker-consumers could be better-off under complete indexation of the economy. In any case, it is better to reach the maximum number of firms if the goal is to maximize employment. Along these lines, our next result is a corollary of Proposition 5.

Proposition 7. *Suppose the economy has N sectors and non-increasing returns to scale. Then employment, real wages, and the welfare of worker-consumers are maximized when $J = J^{\max}$ and when $\phi = 0$ (resp., $\phi = 1$) if $\theta(J^{\max} - 1/N) > (1+\eta)(1-1/N)$ (resp., if that inequality is reversed).*[28]

So if labor market power is low (i.e., high η) and if product market power is high (low $\theta J^{\max}$), then allowing full common ownership increases equilibrium employment. Conversely, if labor market power is high and product market power is low then the optimal policy is no common ownership, as in the one-sector case.

For large economies, the following analogous proposition holds. There is a $\hat{N}$ such that, for economies with $N > \hat{N}$: (i) the number of firms that maximizes employment under non-increasing returns to scale is $J^{\max}$; and (ii) $\phi = 0$ is optimal if $\theta J - 1 > \eta$, but $\phi = 1$ is optimal if $\theta J - 1 < \eta$. (Note that the inequality $\theta J - 1 > \eta$ is the limit of $\theta(J^{\max} - 1/N) > (1+\eta)(1-1/N)$ as $N \to \infty$.) It is noteworthy that, even under Neary's (2003b) assumption of no common ownership ($\lambda = 0$), competition policy has an effect when firms across all sectors employ the same constant-returns technology. This result follows because we have an elastic supply of labor (and so changes in the real wage affect both employment and output) and because we have two types of agents. If our model included only worker-owner-consumers, then the representative agent would always choose the optimal level of employment.

5.3 Government employment policy

Suppose that the government decides to hire a given number of workers for some purpose and that this scheme is financed by a proportional tax on profits. We show that government hiring would compete with hiring by oligopolistic firms and, in so doing, would tend to reduce the markdown of real wages relative to the marginal product of labor and/or the markup over marginal costs. This result is in stark contrast to the standard monopolistic competition model, where the markup is exogenously given (by the elasticity of substitution parameter) and is not affected by government policy; however, it is consistent with the idea that government hiring competes with private-sector hiring and thereby reduces the private sector's labor market power.[29]

[28] When the inequality becomes an equality, the employment-population ratio, real wages, and worker-consumer welfare are maximized (in a large economy) by $J = J^{\max}$ for any $\phi \in [0, 1]$.

[29] Related ideas were presented by Kalecki (1943).

Given private employment plans $\mathbf{L}$ and public employment plans L_G, we now have a competitive equilibrium. Hence the overall equilibrium must be redefined accordingly, as follows.

Definition 3 (Cournot–Walras equilibrium with shareholder representation and fiscal policy)**.** *A Cournot–Walras equilibrium with shareholder representation and government policy (L_G, τ) consists of a price function $(\mathbb{W}(\cdot), \mathbb{P}(\cdot))$, an allocation $\{C_i^*, L_i^*\}_{i \in I_W}$, $\{C_i^*\}_{i \in I_O}$, and a set of production plans $(\mathbf{L}^*; L_G)$ such that the following statements hold.*

(i) $[\mathbb{W}(\mathbf{L}^*; L_G), \mathbb{P}(\mathbf{L}^*; L_G); \{C_i^*, L_i^*\}_{i \in I_W}, \{C_i^*\}_{i \in I_O}]$ *is a competitive equilibrium relative to* $(\mathbf{L}^*; L_G)$.

(ii) The vector $\mathbf{L}^*$ is a pure-strategy Nash equilibrium of the game in which players are the J firms, the strategy space of firm j is $[0, T]$, and the firm's payoff function is

$$(1 - \tau)\left(\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p}\right);$$

here $p = \mathbb{P}(\mathbf{L}; L_G)$, $w = \mathbb{W}(\mathbf{L}; L_G)$, and $\pi_j = pF(L_j) - wL_j$ for $j = 1, \ldots, J$.

(iii) The policy (L_G, τ) satisfies the government budget constraint at the equilibrium prices and employment plans of the firms:

$$\mathbb{W}(\mathbf{L}^*; L_G) L_G \leq \tau \left(\sum_{j=1}^{J} \pi_j^* \right).$$

The worker-consumer's problem is unchanged from that in the model with one sector and no government policy. Because the firm's objective function is multiplied by the constant $(1 - \tau)$, the first-order condition for firm j is the same as before. Overall labor demand L is now the sum of demand by firms $(\sum_{k=1}^{J} L_k)$ and demand by the government (L_G). It follows as an immediate consequence that the equilibrium markdown in a symmetric equilibrium for firms is

$$\mu = \frac{H}{\eta}(1 - s_G); \tag{5.3.1}$$

here $s_G = L_G / L$ is the government's share of total employment and, once again, $H = (1 + \lambda(J - 1))/J$ denotes the MHHI of the private sector.

In what follows, we solve the case of constant returns ($\alpha = 1$) and constant elasticity so we can focus on the model's main insights while keeping the analysis simple. We show that there is an upper bound $\overline{L}_G$ on how much government employment can be compatible with an equilibrium.

Proposition 8. *Let*

$$\alpha = 1 \quad \text{and} \quad \overline{L}_G = \bar{s} \left[\frac{A \chi^{-1/(1-\sigma)}}{1 + \frac{H}{\eta}(1 - \bar{s})} \right]^{\eta},$$

where $\bar{s} \equiv 1 + \eta/(2H) - \sqrt{[1 + \eta/(2H)]^2 - 1}$. If $L_G < \overline{L}_G$, then there is a $\tau < 1$ such that a symmetric equilibrium exists under government policy (L_G, τ). The equilibrium level of total employment L^* is implicitly *determined by*

$$L^* = \left[\frac{A\chi^{-1/(1-\sigma)}}{1+\frac{H}{\eta}(1-L_G/L^*)}\right]^{\eta}. \tag{5.3.2}$$

The government can increase its share of employment by taxing corporate profits—but only up to a point, because (as we will see) government employment reduces private-sector employment and hence the amount of profits subject to taxation. A zero markdown under government policy would require that $s_G \to 1$ as the tax rate increases. However, this limit is impossible to reach because even a tax rate that approaches unity can add only so much s_G to the economy. More specifically: the limit of s_G as the tax rate $\tau \to 1$ is given by $\bar{s}$, and the limit of government employment is $\overline{L}_G$. We are now in a position to characterize the (balanced-budget) multiplier of government employment.

Proposition 9. *Let $\alpha = 1$ and $L_G < \overline{L}_G$. Then the multiplier of government employment is*

$$\frac{\partial L^*}{\partial L_G} = \frac{1}{H^{-1} + (1-s_G)\eta^{-1} + s_G} < 1. \tag{5.3.3}$$

Consider the following heuristic dynamic. An increase in government employment increases s_G; that increase reduces the markdown, which is proportional to $H(1-s_G)$. The reduced markdown, in turn, increases real wages and also (because labor supply is increasing) total employment. The consequence is a reduction of s_G, which attenuates the initial increase in overall employment that resulted from the government policy. These higher-round effects end up mitigating the initial effects, multiplying them by a factor

$$\left(1 - \frac{\partial \log L^*}{\partial \log L_G}\right) = \left(1 + \frac{1+\frac{H}{\eta}(1-s_G)}{s_G H}\right)^{-1} < 1.$$

Employment decisions are strategic substitutes: firm j's desired employment is reduced if the other firms increase employment. The same effect operates when the government increases employment. Namely, it reduces the desired employment level of private-sector firms and generates a "crowding out" effect under which the multiplier becomes less than 1.

While the existence of a multiplier in the model is in a way similar to Keynesian models of fiscal policy under imperfect competition (see e.g., Hart, 1982b; Mankiw, 1988; Startz, 1989; Silvestre, 1993; Matsuyama, 1995), the mechanism through which government employment increases overall employment in the Keynesian models is different from the one developed in this paper. In those models, the multiplier does not operate by reducing firms's market power. Instead of competing with the firms in either the labor market or the product market, the government purchases consumption goods from the monopolistic firms, financed through lump-sum taxes. This fiscal policy shifts demand from a non-produced good (as in Hart) or from leisure (as in Startz and in Mankiw) to the produced-goods sector, increasing demand for those goods; in turn, this shift increases income and generates higher-round effects that end up increasing overall demand in the produced-goods sector by *more* than the shortfall resulting from taxation. In our model, then, government spending—rather than increasing demand for the oligopolistic firms' products—increases competition for workers in the labor market and thereby reduces the market power of those oligopolistic firms. Hence wages increase, which leads to upward movement along the labor supply curve.[30]

We can also show that, as policy tools, competition policy and government employment are substitutes. Note that if $\eta = 1$, then an immediate consequence of Proposition 9 is that $\partial L^*/\partial L_G$ increases with concentration H. Our final proposition generalizes this result.

Proposition 10. *Let $\alpha = 1$ and $L_G < \overline{L}_G$. With respect to the policy tool of increasing government employment L_G, both reducing common ownership (i.e., increasing $1-\phi$) and reducing concentration (increasing J) are substitutes if the goal is to affect total equilibrium employment L^*.*

Thus a failure of competition policy, which implies a higher H, should *increase* the effectiveness of government employment policy. The converse also holds: a successful competition policy makes government employment policy less effective. It is intuitive that, when firms have oligopsony power, government employment policy can increase overall employment by reducing the equilibrium markdown of real wages relative to the marginal product of labor. Yet if the markdown is already low because of competition policy, then there is less scope for government employment policy to reduce the markdown further.[31]

6 Conclusion

In our macroeconomic oligopoly model, firms' employment decisions affect prices in both product and factor markets; furthermore, a higher *effective* market concentration (which accounts for common ownership) can reduce both real wages and employment. When there are multiple industries, common ownership can have a positive or negative effect on the equilibrium markup: the sign depends on the elasticity of substitution, the elasticity of labor supply, and the extent of within-industry concentration. The common ownership effect will be positive when labor market power is less important than product market power.

Competition policy can increase employment and improve welfare. In the one-sector economy we find that controlling common ownership and reducing concentration are complements with respect to fostering employment, whereas government employment is a substitute for those policies. One finding of particular interest is that, when there are multiple sectors, increases in common ownership may actually improve the welfare of worker-consumers. This result may have to be qualified when there is more intra-industry than inter-industry common ownership. Similarly, vertical relations should also be taken into account since products in one sector may be inputs for another sector. Then common ownership may lead to partial internalization of double marginalization and decrease markups.[32] In general,

[30] In this case, government spending is assumed to be unproductive. It therefore also introduces an inefficiency even if worker-consumers are better-off thanks to higher wages.

[31] Nonconstant returns to scale complicate the analysis significantly. Even so, a similar result holds as long as the equilibrium is such that competition policy still has the effect of increasing employment and reducing markdowns.

[32] Azar (2012) finds that common ownerhsip links across industries are associated to lower markups.

our results indicate a need to go beyond the traditional partial equilibrium analyses of competition policy, where consumer surplus is king. Because of political economy considerations, competition policy should be the advocate of worker-consumers since owner-consumers will have already a voice in the regulatory process. If this is so then traditional competition policy (e.g., lowering market concentration) is fully valid while policy towards diversification and common ownership should depend on the relative levels of market power in product and labor markets.

The models presented here are extremely stylized. Although we take care to distinguish between owners and workers, we consider neither the benefits of diversification in an uncertain world nor the effects of unions' market power on the labor market. In other words, there is ample room in future research for extensions and generalizations of our approach.

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A Proofs

PROOF OF PROPOSITION 1: The objective of the manager of firm j is to maximize

$$\zeta\left(\mathbf{L}\right) = F(L_j) - \omega(L)L_j + \lambda \sum_{k \neq j} [F(L_k) - \omega(L)L_k]. \tag{A.1}$$

The first derivative $\partial\zeta/\partial L_j$ is given by $F' - \omega - \omega'\left(L_j + \lambda \sum_{k\neq j} L_k\right)$ and therefore the best response of firm j depends only on $\sum_{k\neq j} L_k$. The cross derivative $(\partial^2\zeta//\partial L_j \partial L_m)$ equals

$$-\omega'\left(1+\lambda\right) - \left(L_j + \lambda \sum_{k\neq j} L_k\right)\omega'' = -\omega'\left(1+\lambda\right) - (s_j + \lambda s_{-j})\omega'' L,$$

where $s_j = L_j/L$ and $s_{-j} = \sum_{k \neq j} L_k/L$. If $E_{\omega'} \equiv -\omega'' L/\omega' < 1$, it follows that the cross derivative is negative since $s_j + \lambda s_{-j} \leq 1$ and

$$-(1+\lambda) - (s_j + \lambda s_{-j})\omega'' L/\omega' < -(1+\lambda) + (s_j + \lambda s_{-j}) < -\lambda.$$

In this case Thm. 2.7 in Vives (1999) guarantees the existence of equilibrium. The second derivative $(\partial^2\zeta/\left(\partial L_j\right)^2)$ equals $F'' - 2\omega' - \left(L_j + \lambda \sum_{k\neq j} L_k\right)\omega''$, and is negative provided that $F'' \leq 0$ also. Let $L_{-j} \equiv \sum_{k\neq j} L_k$ and $R\left(L_{-j}\right)$ denote the best response of firm j. Under the assumptions,

$$R' = -\frac{-\left((1+\lambda)\,\omega' + \left(L_j + \lambda \sum_{k\neq j} L_k\right)\omega''\right)}{F'' - \left(2\omega' + \left(L_j + \lambda \sum_{k\neq j} L_k\right)\omega''\right)}.$$

If the SOC holds, then $R' > -1$ whenever $-F'' + (1-\lambda)\omega' > 0$ and indeed when $F'' \leq 0$ (except when $F'' = 0$ and $\lambda = 1$). When $R' > -1$, Thm. 2.8 in Vives (1999) guarantees that the equilibrium is unique.

The equilibrium is locally stable under continuous best-reply adjustment dynamics if the Jacobian matrix of the profitability gradient $(\partial\zeta/\partial L_1, ..., \partial\zeta/\partial L_J)$ is negative definite (at the equilibrium, see Section 4.3 in Vives (1999)). The Jacobian is a square $J \times J$ matrix with jth diagonal element $F''(L_j) - 2\omega'(L) - (L_j + \lambda L_{-j})\omega''(L)$, and off-diagonal elements in the jth row given by $-(1+\lambda)\omega'(L) - (L_j + \lambda L_{-j})\omega''(L)$. Evaluated at a symmetric equilibrium, this is a matrix with jth diagonal element $F''(L/J) - 2\omega'(L) - H\omega''(L)L$ and off-diagonal elements $-(1+\lambda)\omega'(L) - H\omega''(L)L$. Defining $x \equiv -(1+\lambda)\omega'(L) - H\omega''(L)L$ and $\Delta \equiv F''(L/J) - (1-\lambda)\omega'(L)$, we can write the Jacobian matrix as $D + x\mathbf{e}\prime\mathbf{e}$, $D + x\mathbf{e}\prime\mathbf{e}$ where D is a diagonal matrix with diagonal elements Δ, and $\mathbf{e}$ is a $1 \times J$ vector of ones. The characteristic polynomial of the Jacobian matrix is $\det(D + x\mathbf{e}\prime\mathbf{e} - \epsilon I) = 0$, where ϵ represents an eigenvalue of the Jacobian matrix. Its determinant is $\det(D + x\mathbf{e}\prime\mathbf{e} - \epsilon I) = (\Delta - \epsilon)^J + Jx(\Delta - \epsilon)^{J-1}$. Therefore, we have

$$\det(D + x\mathbf{e}\prime\mathbf{e} - \epsilon I) = (\Delta - \epsilon)^{J-1}(\Delta - \epsilon + Jx) = 0.$$

Thus, the characteristic polynomial has $J-1$ roots equal to Δ and one root equal to $\Delta + Jx$. The Jacobian matrix is negative definite if it has eigenvalues all negative. Since (under the assumption that $E_{\omega'} < 1$), x is negative, this requires $\Delta = F''(L/J) - (1-\lambda)\omega'(L) < 0$, which always holds if $F'' \leq 0$ and $\lambda < 1$ or $F'' < 0$ and $\lambda = 1$.

(a) Dividing the FOC by $\omega(L)$, and letting $s_j = L_j/L$, we have that:

$$\frac{F'\left(L_j\right) - \omega(L)}{\omega(L)} = \frac{\omega'(L)L}{\omega(L)}\left(s_j + \lambda \sum_{k\neq j} s_k\right). \tag{A.2}$$

In a symmetric equilibrium $s_j = 1/J$ for every j. Thus,

$$\frac{F'\left(\frac{L}{J}\right) - \omega(L)}{\omega(L)} = \frac{\omega'(L)L}{\omega(L)}\left(\frac{1}{J} + \lambda\frac{J-1}{J}\right). \tag{A.3}$$

(b) The symmetric equilibrium is given by the fixed point of $L_{-j}/(J-1) = R\left(L_{-j}\right)$. Total employment is $L = L_{-j} + R\left(L_{-j}\right)$, which is increasing in L_{-j} since $R' > -1$. Furthermore, R is decreasing in λ since the first derivative of the objective function is decreasing in λ. This implies that L_{-j} and therefore L and $\omega(L)$ are also decreasing in λ (and in ϕ). We have also that L_{-j} is increasing in J since $R' < 0$ and R itself is increasing in J (since R is decreasing in λ and λ is decreasing in J). It follows then that, in equilibrium, L and $\omega(L)$ are increasing in J.

(c) The labor share is $\frac{\omega(L)L}{JF(L/J)}$. The derivative with respect to total employment L is

$$\frac{\omega'(L)L + \omega(L)\left[F\left(\frac{L}{J}\right) - \frac{L}{J}F'\left(\frac{L}{J}\right)\right]}{J(F(\frac{L}{J}))^2} > 0$$

given that returns to scale are nonincreasing, $F\left(\frac{L}{J}\right) - \frac{L}{J}F'\left(\frac{L}{J}\right) \geq 0$.[33] Since employment is decreasing in ϕ, that implies the labor share is decreasing in ϕ as well. □

Claim: *In the CES-Cobb-Douglas model with increasing returns $\alpha \in (1,2)$ and $\eta \leq 1$, a necessary and sufficient condition for the candidate interior solution to be a symmetric equilibrium is that*

$$\alpha \leq \left(1 + \frac{H}{\eta}\right)\left\{1 + \lambda(J-1)\left[1 - \left(\frac{J-1}{J}\right)^{\frac{1}{\eta}}\right]\right\}^{-1}.$$

PROOF: For $\alpha \in (1,2)$ *and* $\eta \leq 1$, we can check that the third-order derivative of the objective function of firm j with respect to L_j is negative,

$$A\alpha(\alpha-1)(\alpha-2)L_j^{\alpha-3} - 3\omega''(L) - \omega'''(L)(L_j + \lambda L_{-j}) < 0.$$

It follows that the second derivative is strictly decreasing in L_j conditional on L_{-j}. Since the second derivative tends to $+\infty$ as $L_j \to 0$, and to $-\infty$ as $L_j \to \infty$ and is continuous, then it must be zero at some $L_j > 0$ (which can depend on L_{-j}). The objective function is strictly convex below that value, and strictly concave above that value. Therefore, the global maximum must be either at the local maximum of the concave region (assuming that T is larger than the candidate symmetric equilibrium) or at zero. We obtain a condition for the existence of symmetric equilibria by comparing the objective function of the firm at the candidate for a symmetric equilibrium $\widetilde{l} = \left(\frac{A\alpha J^{1-\alpha}\chi^{-\frac{1}{1-\sigma}}}{1+H/\eta}\right)^{\frac{1}{\frac{1}{\eta}+1-\alpha}}/J$ and the corner solution in which the firm produces zero, when the other firms produce at $\widetilde{l}$. On the revenue side, when the firm

[33] If $F(x)$ is increasing and concave for $x \geq 0$, with $F(0) \geq 0$, then $F(x)/x \geq F'(x)$.

produces at $\widetilde{l}$, it obtains $A\left(\widetilde{l}\right)^{\alpha}$. On the cost side, if the firm produces, the real wage is $\chi^{\frac{1}{1-\sigma}}\left(\widetilde{l}J\right)^{\frac{1}{\eta}}$, while if it doesn't produce and the other firms produce at $\widetilde{l}$, the real wage is $\chi^{\frac{1}{1-\sigma}}\left[\widetilde{l}(J-1)\right]^{\frac{1}{\eta}}$, which is lower. If the firm produces, it must pay wages for its employees, plus it internalizes partially the effect of the higher wage on the cost of the competing firms. If the firm employs zero, then it doesn't pay wages, and it internalizes the lower wage that competing firms pay. Firm j wants to produce at $\widetilde{l}$ if its objective function evaluated at $\widetilde{l}$

$$A\left(\widetilde{l}\right)^{\alpha}+\lambda(J-1)A\left(\widetilde{l}\right)^{\alpha}-\chi^{\frac{1}{1-\sigma}}\left(\widetilde{l}J\right)^{\frac{1}{\eta}}_{.}\left[\widetilde{l}+\lambda(J-1)\widetilde{l}\right] \tag{A.4}$$

is greater than its objective function evaluated when other firms produce at $\widetilde{l}$ and it produces zero:

$$\lambda(J-1)A\left(\widetilde{l}\right)^{\alpha}-\chi^{\frac{1}{1-\sigma}}\left[\widetilde{l}(J-1)\right]^{\frac{1}{\eta}}\lambda(J-1)\widetilde{l}. \tag{A.5}$$

After some simplifications and substituting in this condition the expression for $\widetilde{l}$, we obtain that the condition is equivalent to the following:

$$\alpha\leq\left(1+\frac{H}{\eta}\right)\left\{1+\lambda(J-1)\left[1-\left(\frac{J-1}{J}\right)^{\frac{1}{\eta}}\right]\right\}^{-1}.$$

If the condition doesn't hold, then $\widetilde{l}$ cannot be an equilibrium, since every firm would (unilaterally) choose to produce zero instead. Therefore, the condition is necessary for a symmetric equilibrium to exist. If the condition holds it ensures that producing at $\widetilde{l}$ is a global maximum for every firm, conditional on the other firms producing at the candidate symmetric equilibrium, and therefore it is also sufficient for a symmetric equilibrium to exist. □

PROOF OF PROPOSITION 2:

The change in the relative price of the firm's own sector is:

$$\begin{aligned}\frac{\partial\rho_n}{\partial L_{nj}}&=-\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}-1}\frac{F'(L_{nj})C-c_n\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\frac{\theta-1}{\theta}c_n^{\frac{\theta-1}{\theta}-1}F'(L_{nj})}{C^2}\\&=-\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\left[1-\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{\frac{\theta-1}{\theta}}\right]\frac{F'(L_{nj})}{c_n}\\&=-\frac{1}{\theta}\rho_n\left[1-\left(\frac{p_nc_n}{PC}\right)\right]\frac{F'(L_{nj})}{c_n}<0.\end{aligned} \tag{A.6}$$

The change in the relative price of the other sectors ($m \neq n$) is:

$$\begin{aligned}
\frac{\partial \rho_m}{\partial L_{nj}} &= -\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_m}{C}\right)^{-\frac{1}{\theta}-1}\frac{c_m}{C}\frac{-1}{C}\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\frac{\theta-1}{\theta}c_n^{\frac{\theta-1}{\theta}-1}F'(L_{nj}) \\
&= \frac{1}{\theta}\left(\frac{1}{N^2}\right)^{\frac{1}{\theta}}\left(\frac{c_m}{C}\right)^{1-\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\frac{F'(L_{nj})}{c_m} \\
&= \frac{1}{\theta}\left(\frac{p_m c_m}{PC}\right)\rho_n\frac{F'(L_{nj})}{c_m} > 0.\square
\end{aligned} \tag{A.7}$$

PROOF OF PROPOSITION 3:

The expressions in the proof of 2 imply the following relationship between the change in the relative price of sector n and the changes in the relative prices of the other sectors:

$$\frac{\partial \rho_n}{\partial L_{nj}}c_n = -\sum_{m\neq n}\frac{\partial \rho_m}{\partial L_{nj}}c_m. \tag{A.8}$$

Multiplying and dividing by L in the wage effect term, by c_n in the own-industry relative price effect term, and by c_m in the other industry relative price terms, and using equation (A.8), the first order condition simplifies to:

$$\rho_n F'\left(L_{nj}\right) - \omega(L) - \omega'(L)L\left[s^L_{nj} + \lambda\left(1 - s^L_{nj}\right)\right] + \frac{\partial \rho_n}{\partial L_{nj}}c_n\left[s_{nj} + \lambda(1-s_{nj}) - \lambda\right] = 0, \tag{A.9}$$

where $s_{nj} = \frac{F(L_{nj})}{c_n}$ is the share of firm j in the total production of sector n, and $s^L_{nj} = \frac{L_{nj}}{L}$ is the share of firm nj in the labor market.

This can be simplified further, since $s_{nj} + \lambda(1-s_{nj}) - \lambda = s_{nj}(1-\lambda)$:

$$\rho_n F'\left(L_{nj}\right) - \omega(L) - \omega'(L)\left[L_{nj} + \lambda\left(L - L_{nj}\right)\right] + \frac{\partial \rho_n}{\partial L_{nj}}F\left(L_{nj}\right)(1-\lambda) = 0. \tag{A.10}$$

The second derivative of the objective function of firm j in sector n is:

$$\begin{aligned}
&\frac{\partial \rho_n}{\partial L_{nj}}F'(L_{nj}) + \rho_n F''(L_{nj}) - 2\omega'(L) - \omega''(L)\left[L_{nj} + \lambda\left(\sum_{k\neq j}L_{nk} + \sum_{m\neq n}\sum_{k=1}^{J}L_{mk}\right)\right] \\
&+ \frac{\partial \rho_n}{\partial L_{nj}}F'(L_{nj})(1-\lambda) + \frac{\partial^2 \rho_n}{(\partial L_{nj})^2}F(L_{nj})(1-\lambda).
\end{aligned} \tag{A.11}$$

The second derivative of the relative price of firm j in sector n with respect to its own employment is

$$\frac{\partial^2 \rho_n}{(\partial L_{nj})^2} = \frac{\partial \rho_n}{\partial L_{nj}}\left[\frac{\partial \rho_n}{\partial L_{nj}}\frac{1}{\rho_n}\left(1 + (\theta-1)\frac{\frac{p_n c_n}{PC}}{1-\frac{p_n c_n}{PC}}\right) + \frac{F''(L_{nj})}{F'(L_{nj})} - \frac{F'(L_{nj})}{c_n}\right]. \tag{A.12}$$

Replacing this in the second derivative and grouping terms yields

$$
\begin{aligned}
&\frac{\partial \rho_n}{\partial L_{nj}} F'(L_{nj}) \left\{ 1 - \lambda - (1-\lambda) \frac{F(L_{nj})}{c_n} \left[\frac{1}{\theta} \left(1 - \frac{p_n c_n}{PC} \right) + \left(1 - \frac{1}{\theta} \right) \frac{p_n c_n}{PC} \right] \right\} \\
&+ \frac{\partial \rho_n}{\partial L_{nj}} F'(L_{nj}) \left[1 - (1-\lambda) \frac{F(L_{nj})}{c_n} \right] + \rho_n F''(L_{nj}) + \frac{\partial \rho_n}{\partial L_{nj}} F'(L_{nj}) (1-\lambda) \frac{F''(L_{nj}) F(L_{nj})}{\left[F'(L_{nj}) \right]^2} \\
&- 2\omega'(L) - \omega''(L) \left[L_{nj} + \lambda \left(\sum_{k \neq j} L_{nk} + \sum_{m \neq n} \sum_{k=1}^{J} L_{mk} \right) \right].
\end{aligned}
$$

The first row of this expression is negative because $\frac{\partial \rho_n}{\partial L_{nj}}$ is negative, F' is positive, and the expression in curly brackets is positive because $\left[\frac{1}{\theta} \left(1 - \frac{p_n c_n}{PC} \right) + \left(1 - \frac{1}{\theta} \right) \frac{p_n c_n}{PC} \right] < 1$. The first term of the second row is clearly negative. The second term is non-positive, but the third term is non-negative. The two combined, however, can be rewritten as $\frac{\partial \rho_n}{\partial L_{nj}} \frac{F''(L_{nj}) F(L_{nj})}{F'(L_{nj})} \left[-\frac{\theta}{s_{nj}} + 1 - \lambda \right]$, which is the product of three nonpositive factors and therefore the whole expression is nonpositive. The third row is strictly negative because, with the constant-elasticity utility functional form, it is equal to $-\frac{\omega}{L} \frac{1}{\eta} \left\{ 2 + \left(\frac{1}{\eta} - 1 \right) \left[s_{nj} + \lambda (1 - s_{nj}) \right] \right\}$. The expression $\left\{ 2 + \left(\frac{1}{\eta} - 1 \right) \left[s_{nj} + \lambda (1 - s_{nj}) \right] \right\}$ is greater than one, and it is multiplying a factor $-\frac{\omega}{L} \frac{1}{\eta}$ that is negative, and therefore the third-row of the second-order condition is negative.

The objective function of each firm is thus globally strictly concave, and therefore any solution to the system of equation implied by the first-order conditions is an equilibrium. To find the symmetric equilibria, we thus start by simplifying the first-order condition of firm nj when it is evaluated at a symmetric equilibrium, with $c_n = c$ for all n, and $p_n = p$ for all n. Note first that in the symmetric case $c_n / C = c / C = 1/N$.

In a symmetric equilibrium, the marginal product of labor is equal to $F'(\frac{L}{JN})$. Using that fact and replacing $\frac{c_n}{C} = \frac{c_m}{C} = \frac{1}{N}$ in the expression for the change in the relative price of the firm's industry when the firm expands employment plans it simplifies to

$$
\frac{\partial \rho_n}{\partial L_{nj}} = -\frac{1}{\theta} \left(1 - \frac{1}{N} \right) \frac{F'(\frac{L}{JN})}{c}. \tag{A.13}
$$

Dividing the first-order condition by the real wage and substituting the derivatives of the relative price that we just derived in equation (A.13) yields:

$$
\frac{F'\left(\frac{L}{JN}\right) - \omega(L)}{\omega(L)} = \frac{\omega'(L) L}{\omega(L)} \left[s^L_{nj} + \lambda \left(1 - s^L_{nj} \right) \right] + \frac{1}{\theta} \left(1 - \frac{1}{N} \right) \frac{F'\left(\frac{L}{JN}\right)}{\omega(L)} s_{nj} (1 - \lambda). \tag{A.14}
$$

In a symmetric equilibrium, the employment share of firm j in sector n is equal to $\frac{L_{nj}}{L} = \frac{1}{JN}$ for all sectors n and all firms j within the sector, since the employment shares of all firms are the same. Similarly, the product market share of firm j in sector n is $\frac{F(L_{nj})}{c} = \frac{1}{J}$.

Using these expressions for the employment and product market shares, and defining, as before,

μ as the markdown of real wages relative to the marginal product of labor $\mu = \frac{F'\left(\frac{L}{JN}\right)-\omega(L)}{\omega(L)}$, and $\frac{1}{\eta}$ as the elasticity of the competitive equilibrium real wage with respect to employment $\frac{1}{\eta} = \frac{\omega'(L)L}{\omega(L)}$, we can rewrite equation (A.14) as:

$$\mu = \frac{1}{\eta}\left[\frac{1}{JN} + \lambda\left(1 - \frac{1}{JN}\right)\right] + \frac{1+\mu}{\theta}\frac{1}{J}(1-\lambda)\left(1-\frac{1}{N}\right). \tag{A.15}$$

We can then express this in terms of MHHIs for the labor market and product markets as follows:

$$\mu = \frac{1}{\eta}\underbrace{\left[\frac{1}{JN} + \lambda\left(1 - \frac{1}{JN}\right)\right]}_{H_{JN}} + \frac{1+\mu}{\theta}\left\{\underbrace{\left[\frac{1}{J} + \lambda\left(1-\frac{1}{J}\right)\right]}_{H_J} - \lambda\right\}\left(1-\frac{1}{N}\right). \tag{A.16}$$

In this expression, H_{JN} is the MHHI for the labor market, which equals $\frac{1}{JN} + \lambda\left(1 - \frac{1}{JN}\right)$, and H_J is the MHHI for the product market of one industry, which equals $\frac{1}{J} + \lambda\left(1-\frac{1}{J}\right)$. $\square$

The expression for the markup provides an equation in L:

$$\omega(L) = \frac{F'\left(\frac{L}{JN}\right)}{\frac{1+\frac{H_{JN}}{\eta}}{1-\frac{1}{\theta}\left(H_J-\lambda\right)\left(1-\frac{1}{N}\right)}}. \tag{A.17}$$

Combining this equation in L and w/P with the inverse labor supply and imposing labor market clearing yields an equation for the equilibrium level of employment L:

$$-\frac{U_L\left(\frac{w}{P}\frac{L}{N}, \frac{L}{N}\right)}{U_C\left(\frac{w}{P}\frac{L}{N}, \frac{L}{N}\right)} = \frac{F'\left(\frac{L}{JN}\right)}{\frac{1+\frac{H_{JN}}{\eta}}{1-\frac{1}{\theta}\left(H_J-\lambda\right)\left(1-\frac{1}{N}\right)}}. \tag{A.18}$$

We can obtain a closed-form solution for the constant-elasticity labor supply and Cobb-Douglas production function case. In this case, the equation for equilibrium total employment level is:

$$\chi^{\frac{1}{1-\sigma}}\left(\frac{L}{N}\right)^{\frac{\xi+\sigma}{1-\sigma}} = \frac{A\alpha\left(\frac{L}{JN}\right)^{\alpha-1}}{\frac{1+\frac{H_{JN}}{\eta}}{1-\frac{1}{\theta}\left(H_J-\lambda\right)\left(1-\frac{1}{N}\right)}}. \tag{A.19}$$

This equation has a unique solution for L:

$$L^* = N\left(\frac{\chi^{-\frac{1}{1-\sigma}}A\alpha}{1+\mu^*}\right)^{\frac{1}{\frac{1}{\eta}-(\alpha-1)}} J^{-\frac{\alpha-1}{\frac{1}{\eta}-(\alpha-1)}}, \tag{A.20}$$

where $1+\mu^*$ is

$$1+\mu^* = \frac{1+\frac{H_{JN}}{\eta}}{1-\frac{1}{\theta}\left(H_J-\lambda\right)\left(1-\frac{1}{N}\right)}. \tag{A.21}$$

The markdown is clearly decreasing with η and θ. We check that it is also decreasing in J and increasing or decreasing with ϕ and N.

We have that

$$\begin{aligned}
\frac{\partial \log\left(1+\mu^*\right)}{\partial \log J} &= \frac{\frac{H_{JN}}{\eta}}{1+\frac{H_{JN}}{\eta}}\frac{\partial \log H_{JN}}{\partial \log J} + \frac{\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}{1-\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}\frac{\partial \log \frac{1-\lambda}{J}}{\partial \log J} \\
&= -(1-\lambda)\left\{\frac{\frac{H_{JN}}{\eta}}{1+\frac{H_{JN}}{\eta}} + \frac{\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}{1-\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}\right\} < 0.
\end{aligned}$$

Here, we have used that $\frac{\partial \log H_{JN}}{\partial \log J} = \frac{\partial \log \frac{1-\lambda}{J}}{\partial \log J} = -(1-\lambda)$.

Now,

$$\frac{\partial \log\left(1+\mu^*\right)}{\partial \phi} = \left\{\frac{\frac{1}{\eta}\left(1-\frac{1}{JN}\right)}{1+\frac{1}{\eta}\left[\frac{1}{JN}+\lambda\left(1-\frac{1}{JN}\right)\right]} - \frac{\frac{1}{\theta J}\left(1-\frac{1}{N}\right)}{1-\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}\right\}\frac{\partial \lambda}{\partial \phi}.$$

This is negative whenever

$$\frac{\theta J}{1-\frac{1}{N}} - 1 < \frac{\eta}{1-\frac{1}{JN}} + \frac{1}{JN-1} \text{ or } \theta\left(JN-1\right) < \left(1+\eta\right)\left(N-1\right)$$

Finally,

$$\begin{aligned}
\frac{\partial \log\left(1+\mu^*\right)}{\partial \log N} &= \frac{\frac{H_{JN}}{\eta}}{1+\frac{H_{JN}}{\eta}}\frac{\partial \log H_{JN}}{\partial \log N} + \frac{\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}{1-\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}\left[\frac{\partial \log(1-\lambda)}{\partial \log N} + \frac{\partial \log(1-1/N)}{\partial \log N}\right] \\
&= -\frac{\frac{H_{JN}}{\eta}}{1+\frac{H_{JN}}{\eta}}(1-\lambda) + \frac{\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}{1-\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}\left(\lambda+\frac{1}{N-1}\right).
\end{aligned}$$

Thus, the sign depends on parameter values. □

PROOF OF PROPOSITION 5: We have that

$$\frac{\partial^2 \log L^*}{\partial(1-\phi)\partial J} = \frac{1}{\frac{1}{\eta}-(\alpha-1)}\frac{\frac{1}{\eta}\frac{\partial^2 H}{\partial(1-\phi)\partial J}\left(1+\frac{H}{\eta}\right) - \frac{1}{\eta^2}\frac{\partial H}{\partial(1-\phi)}\frac{\partial H}{\partial J}}{\left(1+\frac{H}{\eta}\right)^2} > 0 \tag{A.22}$$

since $sign\left\{\frac{\partial^2 H}{\partial(1-\phi)\partial J}\left(1+\frac{H}{\eta}\right) - \frac{1}{\eta}\frac{\partial H}{\partial(1-\phi)}\frac{\partial H}{\partial J}\right\} = sign\left\{-\left(1-\frac{1}{J}\right)(1-\lambda)\frac{\partial \lambda}{\partial(1-\phi)}\right\}$, which is positive for $J>1$ since $\frac{\partial \lambda}{\partial(1-\phi)} < 0$. □

PROOF OF PROPOSITION 8:The government's budget constraint is given by

$$\omega L_G = \tau \sum_{j=1}^{J} \frac{\pi_j}{p} \tag{A.23}$$

and the sum of profits in a symmetric equilibrium are:

$$\sum_{j=1}^{J} \frac{\pi_j}{p} = \omega \frac{H}{\eta}(1 - s_G)(L - L_G) = \omega \frac{H}{\eta}(1 - s_G)^2 L. \tag{A.24}$$

Combining this with the government's budget constraint, we obtain

$$s_G = \tau \frac{H}{\eta}(1 - s_G)^2 < \frac{H}{\eta}(1 - s_G)^2, \tag{A.25}$$

since τ has to be less than one. This is a quadratic inequality that implies an upper bound for the equilibrium share of government employment:

$$s_G < \underbrace{1 + \frac{\eta}{2H} - \sqrt{\left(1 + \frac{\eta}{2H}\right)^2 - 1}}_{\bar{s}}. \tag{A.26}$$

The first-order condition of firm j evaluated at a symmetric equilibrium implies:

$$\omega(L) = \frac{A}{1 + \frac{H}{\eta}(1 - s_G)}. \tag{A.27}$$

Combining this with the expression for the inverse labor supply $\omega(L)$ and imposing labor market clearing we obtain

$$L^* = \left[\frac{A\chi^{-\frac{1}{1-\sigma}}}{1 + \frac{H}{\eta}\left(1 - \frac{L_G}{L^*}\right)}\right]^{\eta}. \tag{A.28}$$

Any equilibrium also needs to satisfy condition (A.26). We obtain:

$$s_G = L_G \left[\frac{1 + \frac{H}{\eta}(1 - s_G)}{A}\right]^{\eta} \chi^{\frac{\eta}{1-\sigma}}. \tag{A.29}$$

The right-hand side of this equation is strictly decreasing in s_G. Therefore, it will cross the 45 degree line at an $s_G < \bar{s}$ if and only if $L_G < \underbrace{\bar{s}\left[\frac{A\chi^{-\frac{1}{1-\sigma}}}{1 + \frac{H}{\eta}(1 - \bar{s})}\right]^{\eta}}_{\overline{L_G}}$. □

PROOF OF PROPOSITION 9: We have that

$$\frac{\partial \log L^*}{\partial \log L_G} = \underbrace{-\eta}_{\frac{\partial \log L^*}{\partial \log(1+\mu)}} \underbrace{\frac{-\frac{H}{\eta}}{1+\frac{H}{\eta}(1-s_G)}}_{\frac{\partial \log(1+\mu)}{\partial s_G}} \underbrace{s_G\left(1-\frac{\partial \log L^*}{\partial \log L_G}\right)}_{\frac{\partial s_G}{\partial \log L_G}}. \tag{A.30}$$

Solving for $\frac{\partial \log L^*}{\partial \log L_G}$ yields:

$$\frac{\partial \log L^*}{\partial \log L_G} = \frac{\frac{Hs_G}{1+\frac{H}{\eta}(1-s_G)}}{1+\frac{Hs_G}{1+\frac{H}{\eta}(1-s_G)}}, \tag{A.31}$$

and noting that $\frac{\partial \log L^*}{\partial \log L_G} = \frac{\partial L^*}{\partial L_G} s_G$, it follows that

$$\frac{\partial L^*}{\partial L_G} = \frac{H}{1+\frac{H}{\eta}(1-s_G)+Hs_G} < 1 \tag{A.32}$$

since $H \leq 1$ and $1+\frac{H}{\eta}(1-s_G)+Hs_G > 1$. $\square$

PROOF OF PROPOSITION 10: Using $\frac{\partial \log L^*}{\partial H} = -\frac{1-s_G}{1+\frac{H}{\eta}(1-s_G)}$, we have that

$$\frac{\partial^2 L^*}{\partial L_G \partial J} = \frac{\frac{1}{H^2}+\left(\frac{1}{\eta}-1\right)\frac{s_G(1-s_G)}{1+\frac{H}{\eta}(1-s_G)}}{\left[\frac{1}{H}+\frac{1}{\eta}(1-s_G)+s_G\right]^2}\frac{\partial H}{\partial J} < 0 \tag{A.33}$$

$$\frac{\partial^2 L^*}{\partial L_G \partial(1-\phi)} = \frac{\frac{1}{H^2}+\left(\frac{1}{\eta}-1\right)\frac{s_G(1-s_G)}{1+\frac{H}{\eta}(1-s_G)}}{\left[\frac{1}{H}+\frac{1}{\eta}(1-s_G)+s_G\right]^2}\frac{\partial H}{\partial(1-\phi)} < 0 \tag{A.34}$$

since $\frac{1}{H^2} \geq 1$, $\left(\frac{1}{\eta}-1\right)\frac{s_G(1-s_G)}{1+\frac{H}{\eta}(1-s_G)} > -1$, $\frac{\partial H}{\partial J} < 0$, and $\frac{\partial H}{\partial(1-\phi)} < 0$. $\square$

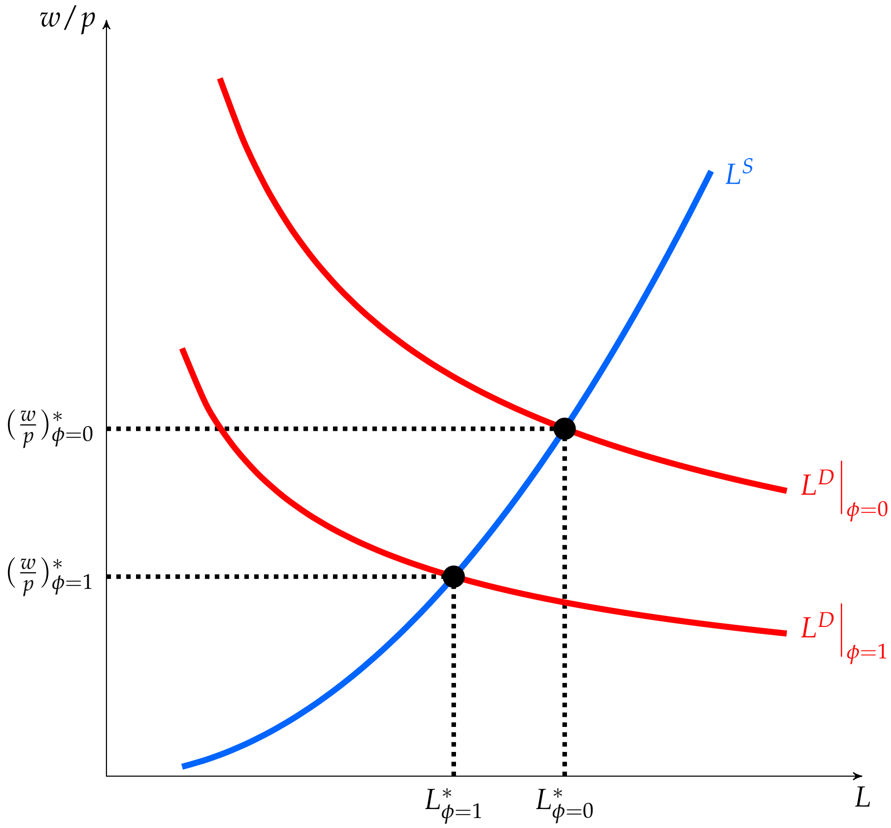


Figure 1. Effect of an increase in market concentration on equilibrium real wages and employment in the one-sector model. The model parameters for the plot are: $A = 6$, $J = 4$, $\alpha = 0.5$, $\xi = 0.5$, $\sigma = 0.5$, $\chi = 0.5$. In the case of $\phi = 0$, the MHHI is $H = 0.25$. In the case of $\phi = 1$, the MHHI is $H = 1$. L^S refers to the labor supply curve. L^D refers to the labor demand curve.

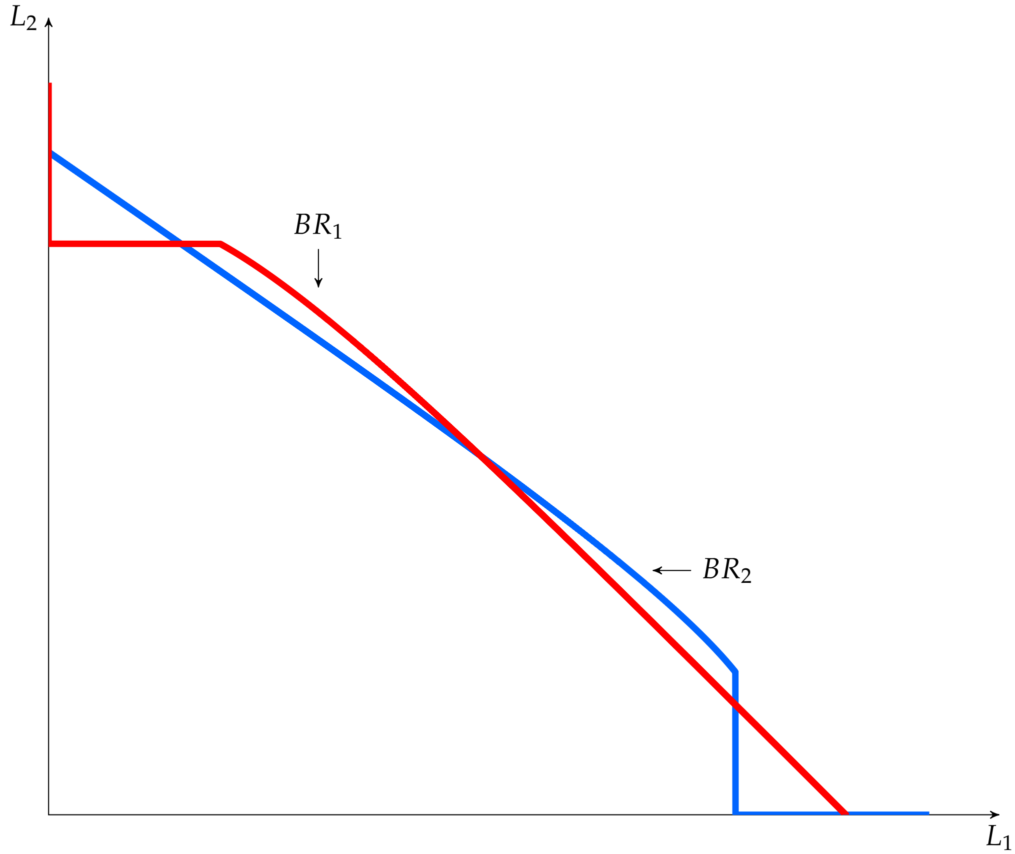


Figure 2. Best-response functions in a two-firm example with increasing returns and multiple equilibria ($\alpha = 1.4$). The model parameters for the plot are: $A = 6$, $J = 2$, $\phi = 0$, $\xi = 0.5$, $\sigma = 0.5$, $\chi = 0.5$.

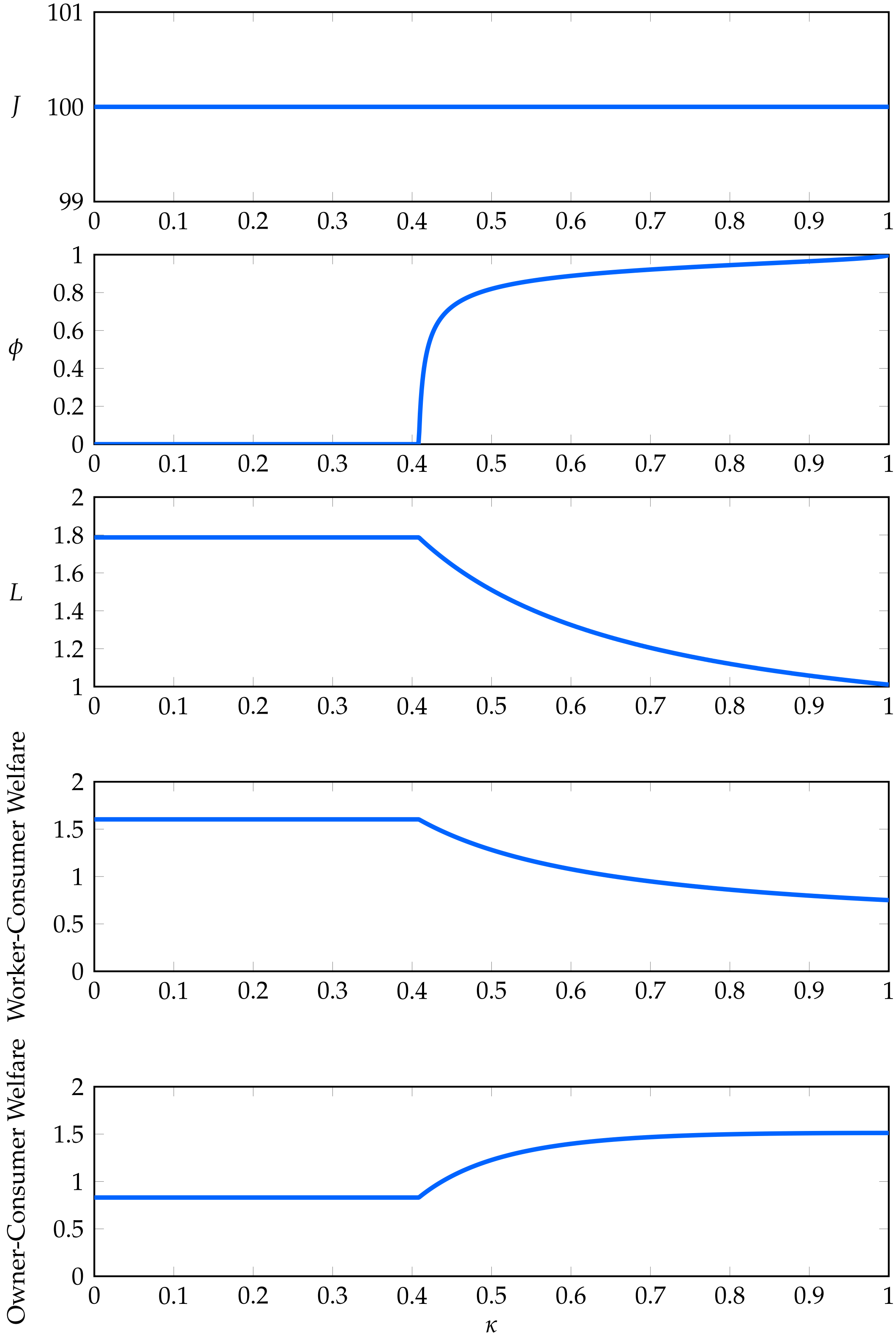


Figure 3. Optimal Competition Policy with Decreasing Returns to Scale ($\alpha = 0.8$). The model parameters for the plot are: $A = 1$, $\chi = 1$, $\sigma = 1/3$, $\xi = 1/3$, $J^{\max} = 100$.

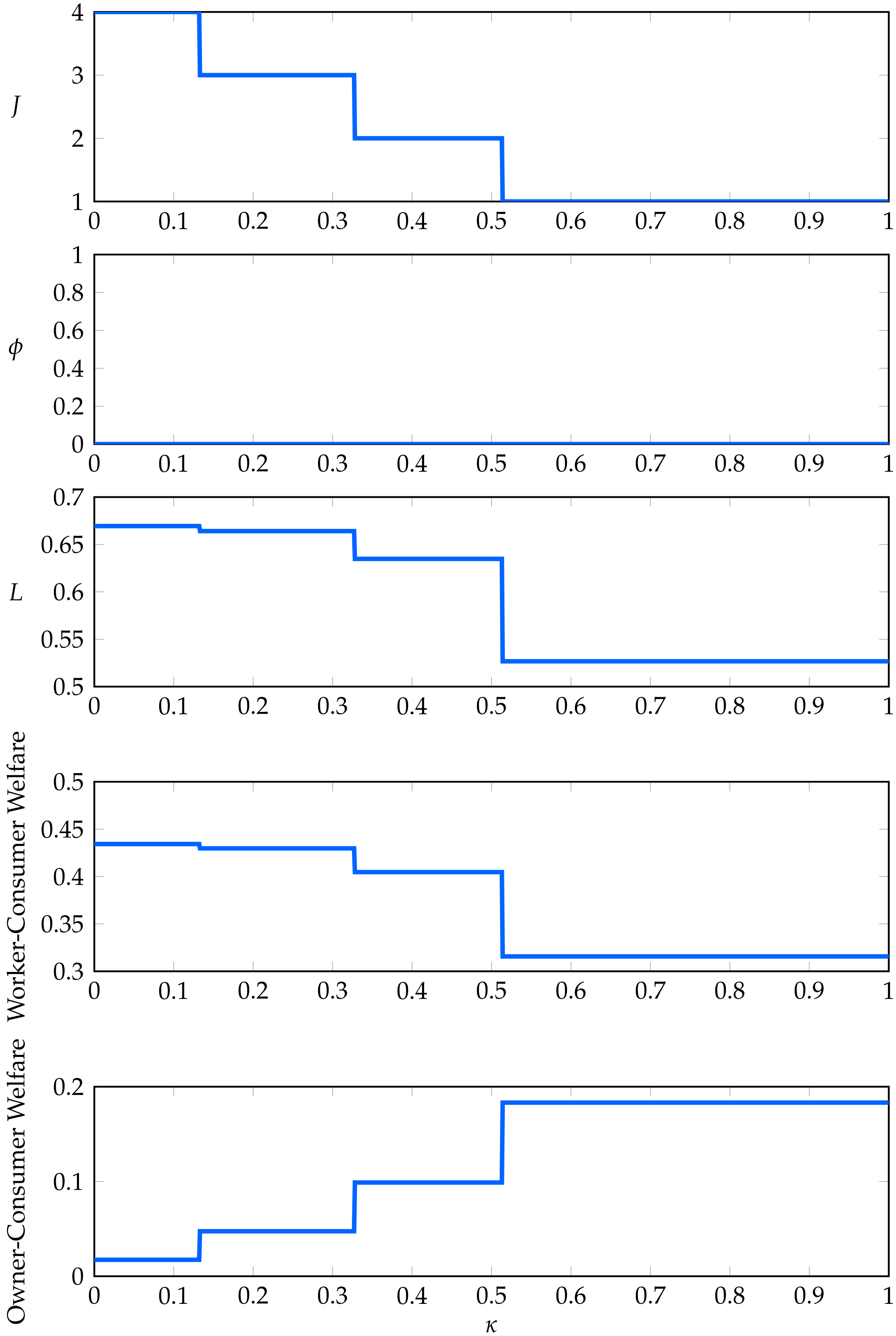


Figure 4. Optimal Competition Policy with Increasing Returns to Scale ($\alpha = 1.2$). The model parameters for the plot are: $A = 1$, $\chi = 1$, $\sigma = 1/3$, $\xi = 1/3$, $J^{\max} = 100$.

Online Appendix for "Oligopoly, Macroeconomic Performance, and Competition Policy"

by José Azar and Xavier Vives

In this appendix we provide some trends for concentration, market power, labor share and common ownership for the US economy in Section A.1; an extension of the one-sector model to include capital in Section A.2; a general equilibrium oligopoly framework with invariance to price normalization in Section A.3; and a mapping of the general model to the different models in the paper in Section A.4.

A.1 Trends in the US economy

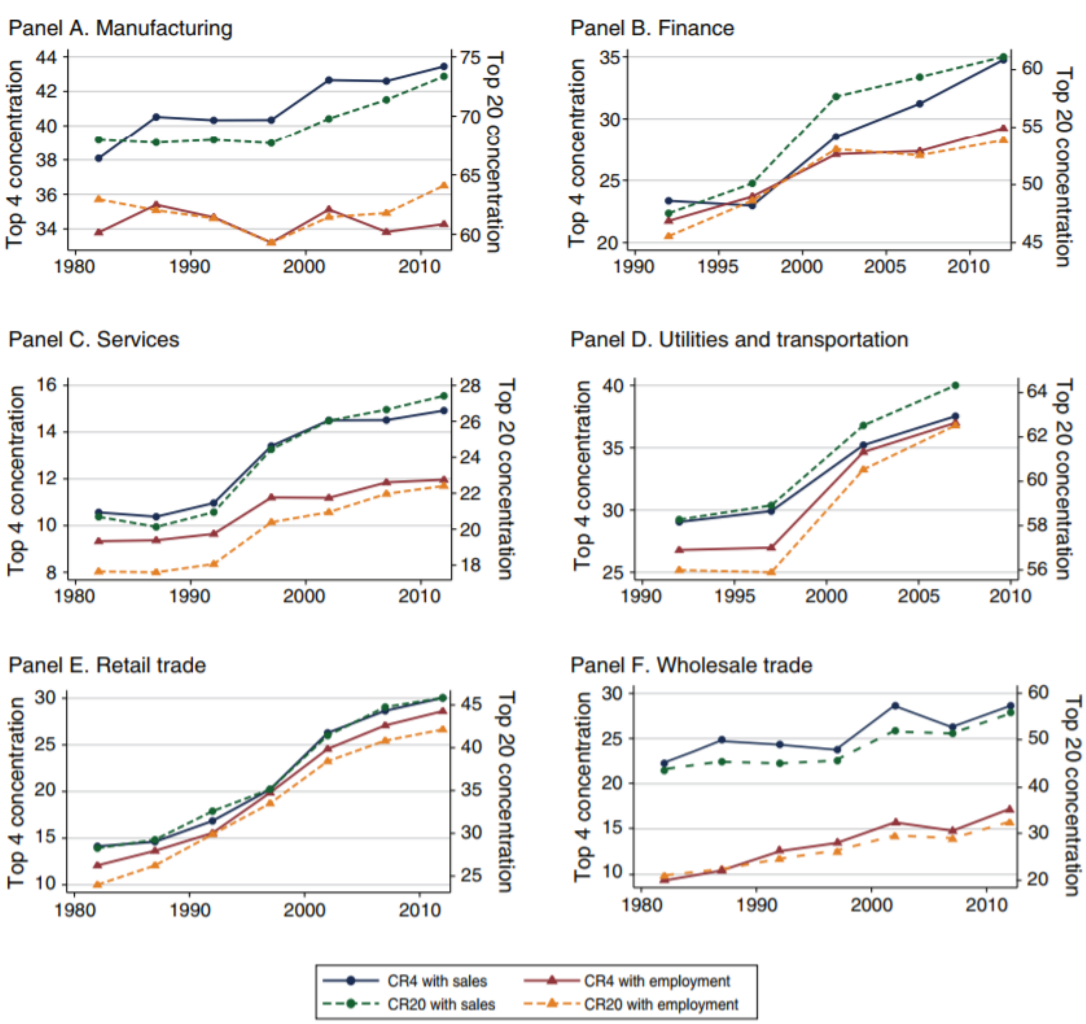
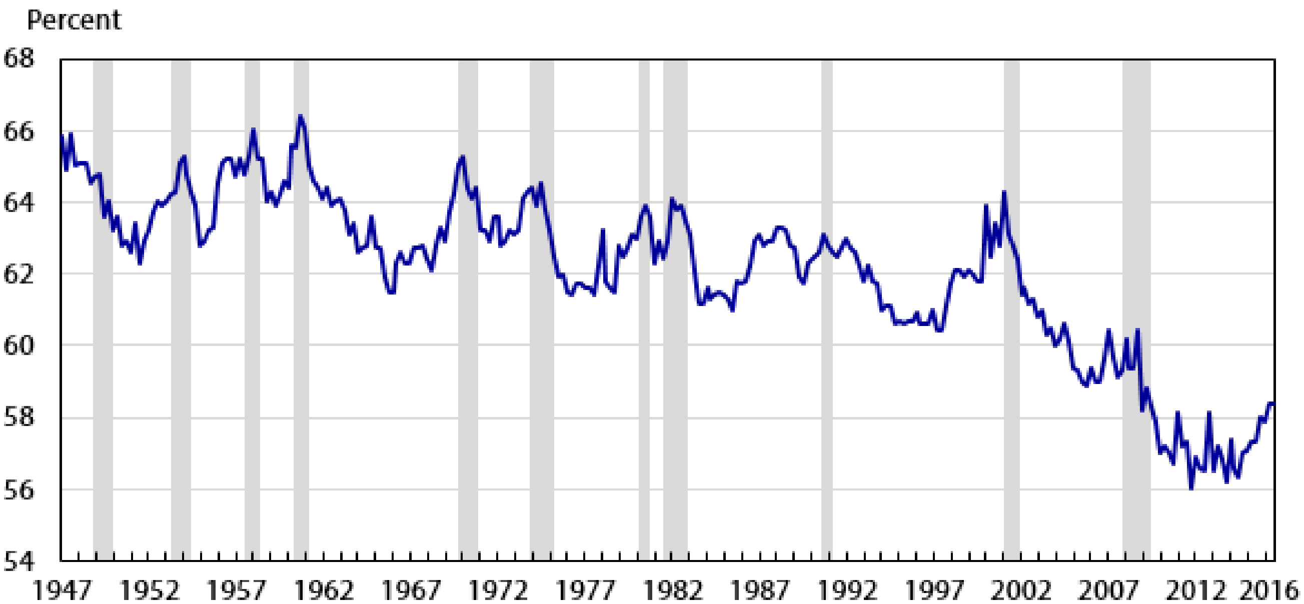


Figure A.1. Average CR4 across industries (Census). Source: Autor et al. (2017).



Note: Shaded areas indicate recessions, as determined by the National Bureau of Economic Research.
Source: U.S. Bureau of Labor Statistics.

Figure A.2. Labor Share of Nonfarm Business Sector Output. Source: Giandrea and Sprague (2017).

This figure plots trends in the employment-weighted average of the Herfindahl-Hirschman Index (HHI) of employment by firms computed at the county-three-digit industry-year level. The computed HHI is averaged across county-three-digit industry-year cells within each of the five-year periods (the last period includes eight years, 2002–2009) using the number of employees in each cell as the weight. Thus, the average HHI represents the degree of employer concentration the average worker faces in the labor market.

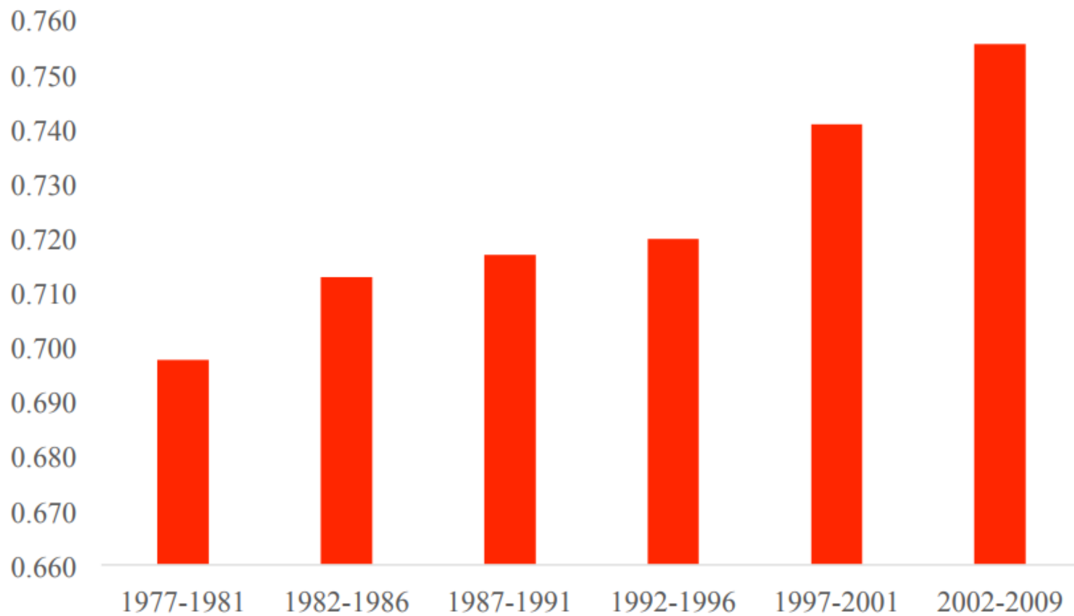


Figure A.3. Trends in Average Local-Level Employment Concentration, 1977–2009. Source: Benmelech et al. (2018).

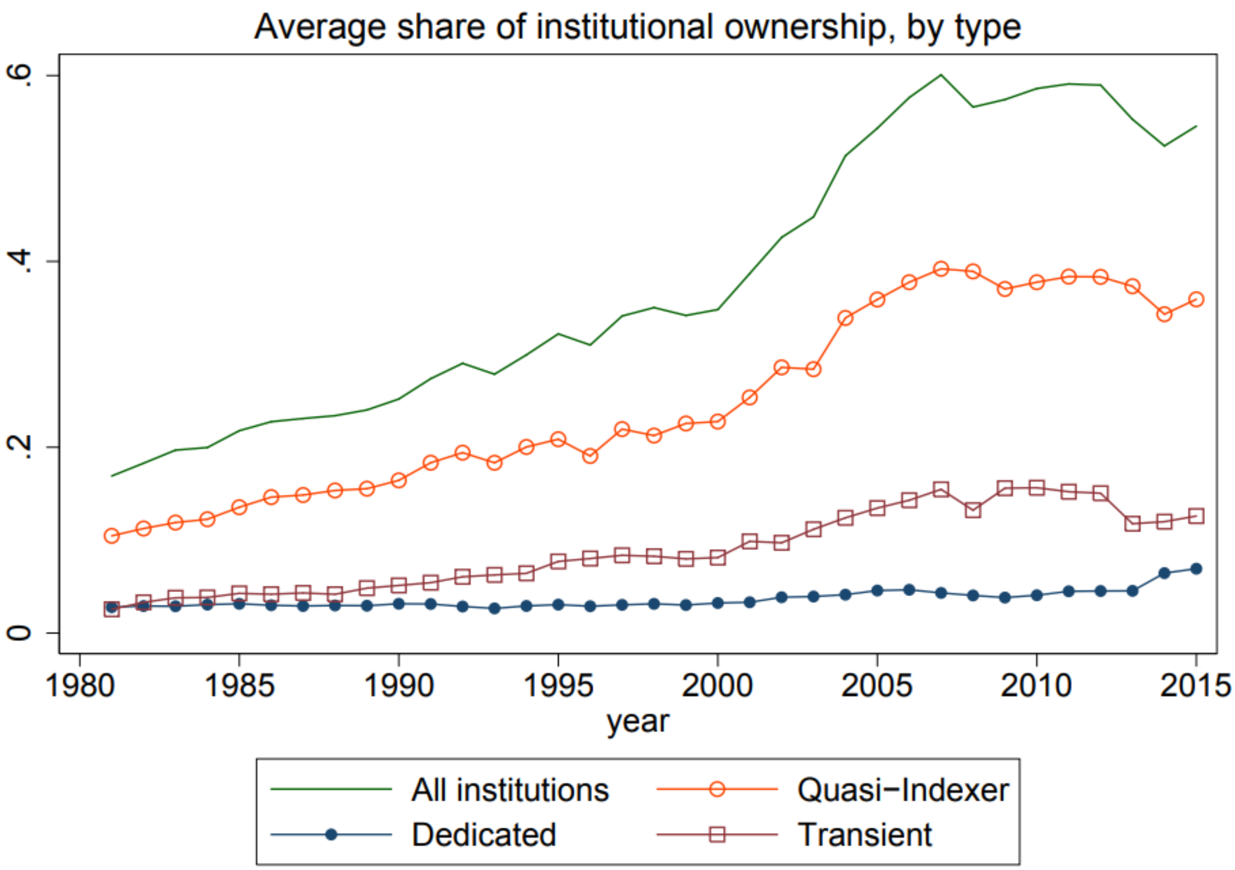


Figure A.4. Ownership share of quasi-indexers. Source: Gutiérrez and Philippon (2016).

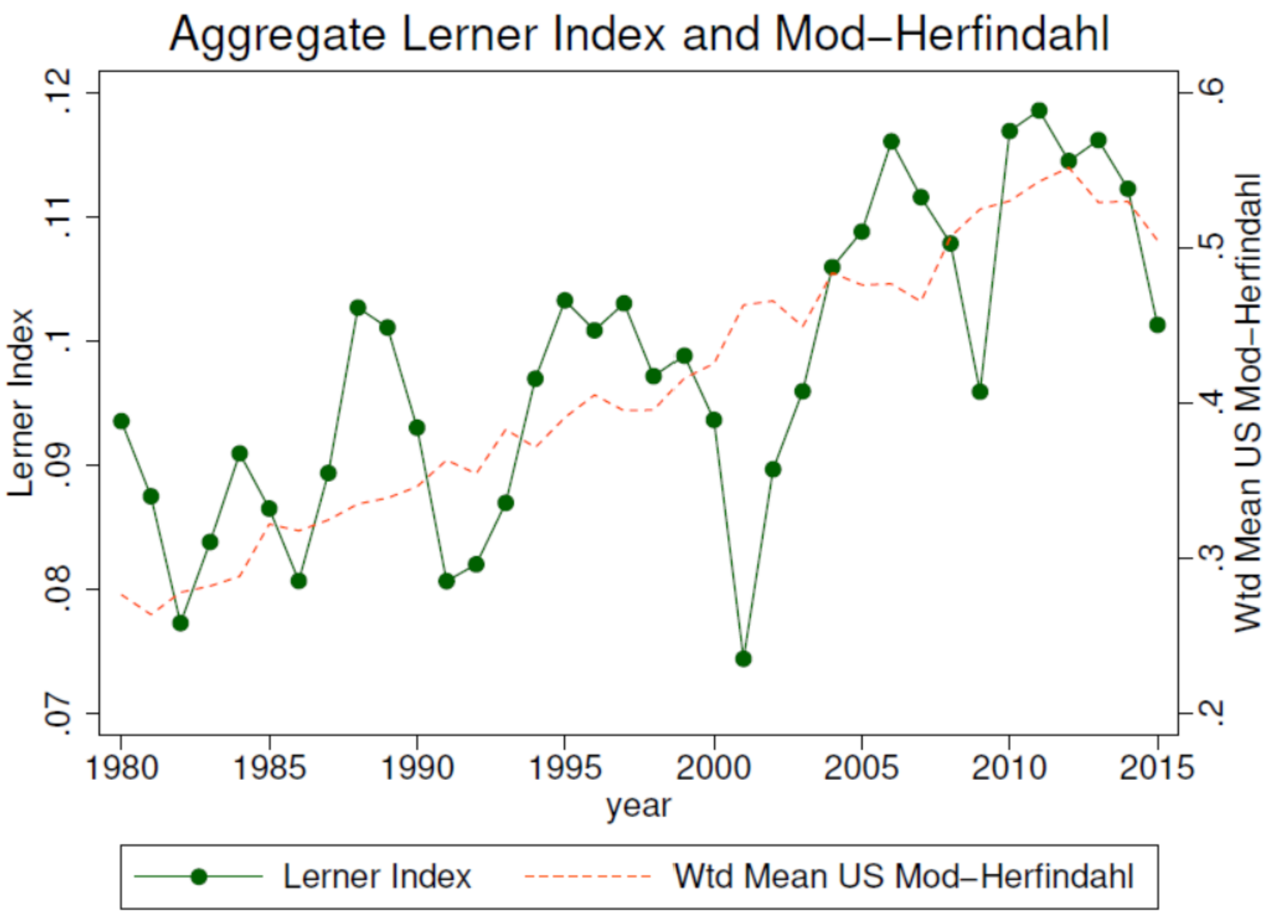


Figure A.5. Average Lerner Index and MHHI across industries (Compustat). Source: Gutiérrez and Philippon (2017).

A.2 A model with labor and capital

A.2.1 Model setup

In this section, we develop a model with two factors of production: labor and capital. There are three types of people: workers, owners, and savers. The owners and the workers are the same as in the model with only labor, and only consume at time zero. We denote the set of savers I_S, the set of workers I_W, and the set of owners I_O. Each of the sets of workers, savers, and owners has measure one. Since the workers are as in the one-sector CES model, their labor supply function is the same as in that case.

The savers do not work or own the firms. Unlike the workers and the owners, they live for two periods: the period in which the other people live, which we call period zero or "the present," and an additional period, which we call period 1 or "the future," when there is no production. The savers are born with an endowment E of the consumption good, of which they can consume a fraction in the present and lend the rest to the firms at a nominal interest rate R, so that a firm has to pay back $1 + R$ dollars in period one for every dollar borrowed in period zero. The firms can then invest those savings to purchase units of the consumption good, transforming them into productive capital at a 1:1 rate, and they can produce, sell some of their output in the goods market in period zero, and store some of their output and use it to repay the savers with interest in period one.

The utility of the savers is assumed to have constant elasticity of substitution between present and future consumption $1/\sigma$:

$$U\left(C_{0,i}, C_{1,i}\right) = \frac{C_{0,i}^{1-\sigma}}{1-\sigma} + \beta \frac{C_{1,i}^{1-\sigma}}{1-\sigma}, \tag{A.1}$$

with σ and β in $(0,1)$.

The budget constraint in period zero is $p_0 C_{0,i} + S_i = p_0 E$, and the budget constraint in period one is $p_1 C_{1,i} = (1+R) S_i$, where S_i is the level of savings of saver i. Combining the budget constraints for both periods, we get the intertemporal budget constraint of the savers:

$$C_{0,i} + \frac{C_{1,i}}{1+r} = E, \tag{A.2}$$

where r is the real interest rate, defined as $r \equiv \frac{1+R}{p_1/p_0} - 1$.

As before, there are J firms, which are large relative to the economy as a whole. The production function of firm j now depends on both labor and capital: $Y_j = F(K_j, L_j)$. Throughout this section we will assume that the production function is constant-returns to scale and Cobb-Douglas: $Y_j = A K_j^{\alpha} L_j^{1-\alpha}$, $\alpha \in (0,1)$.[34]

For simplicity, we assume that the firm owns no capital of its own at the beginning of period zero. It issues bonds to the savers for $p_0 K_j$ to purchase an amount K_j of the period zero consumption good from the savers and transform it into capital. After being used in production, the capital stock depreciates at a rate δ. The firm must also set aside and store $(r+\delta)K_j$ units of output so that it has $(1-\delta+r+\delta)K_j = (1+r)K_j$ units of output in period one. It sells these units of output in the market in period one at a price p_1 (to the savers) to pay back $\frac{p_1}{p_0}(1+r)K_j p_0 = (1+R)K_j p_0$ to the bondholders (i.e., the savers).

Thus, the profits of firm j in terms of the period zero good are:

$$\frac{\pi_j}{p_0} = F(K_j, L_j) - \frac{w}{p_0} L_j - (r+\delta) K_j. \tag{A.3}$$

The objective function of the manager of the firm is to maximize a weighted average of shareholder utilities, which in this case requires expressing profits in terms of consumption of the present good, which is the one that the owners consume:

$$\max_{L_j, K_j} \frac{\pi_j}{p_0} + \lambda \sum_{k \neq j} \frac{\pi_k}{p_0}. \tag{A.4}$$

[34] This production function is twice continuously differentiable and concave, with $F_{KK} \leq 0$, $F_{LL} \leq 0$, and $F_{KK} F_{LL} - F_{KL}^2 \geq 0$.

A.2.2 Solving the model

A.2.3 The competitive equilibrium conditional on firms' production plans

Labor supply, as before, is:

$$L = \chi^{-\frac{1}{\xi+\sigma}} \left(w/p_0\right)^{\frac{1-\sigma}{\xi+\sigma}}. \tag{A.5}$$

We can use this equation to characterize the competitive equilibrium real wage (relative to consumption in period zero) as a function of the total employment plans by the firms:

$$\omega(L) = \chi^{\frac{1}{1-\sigma}} \left(L\right)^{\frac{\xi+\sigma}{1-\sigma}}.$$

The first-order conditions for the savers yield the Euler equation:

$$C_{0,i}^{-\sigma} = \beta(1+r)C_{1,i}^{-\sigma}. \tag{A.6}$$

Solving for $C_{1,i}$ and replacing in the intertemporal budget constraint, we obtain:

$$C_{0,i}\left[1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}\right] = E. \tag{A.7}$$

Solving for $C_{0,i}$ and replacing in the budget constraint for period zero, yields an expression for the level of savings as a function of the real interest rate:

$$S_i = E - C_{0,i} = E\frac{\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}{1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}. \tag{A.8}$$

Since all savers are the same, the expression for the total supply of savings $S = \int_{i\in I_S} S_i$ is the same as that for S_i. We assume that $\sigma < 1$, so that saving is strictly increasing in r. As was the case for the real wage and labor supply, the inverse of the savings function determines the competitive equilibrium real interest as a function of the total investment plans of the firms K, which we call $\psi(K)$ and is given by:

$$\psi(K) = \left(\frac{K}{E-K}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{1}{\beta}\right)^{\frac{1}{1-\sigma}} - 1. \tag{A.9}$$

The competitive equilibrium real interest rate is strictly increasing in K, with the limit 0 as $K \to 0$, and the limit ∞ as $K \to E$.

A.2.4 Equilibrium

Let $\pi_j(K_j, L_j) = p_0 F(K_j, L_j) - p_0\left[\psi(K)+\delta\right]K_j - p_0\omega(L)L_j$. The objective function of the manager of firm j is:

$$\max_{L_j,K_j} \frac{\pi_j(K_j,L_j)}{p_0} + \lambda\sum_{k\neq j}\frac{\pi_k(K_k,L_k)}{p_0}. \tag{A.10}$$

Proposition 11. *A unique symmetric equilibrium exists and it is characterized by the solution to the system of equations:*

$$\frac{F_L\left(\frac{K}{J},\frac{L}{J}\right)-\omega(L)}{\omega(L)}=\frac{\omega'(L)L}{\omega(L)}H, \tag{A.11}$$

$$\frac{F_K\left(\frac{K}{J},\frac{L}{J}\right)-[\delta+\psi(K)]}{1+\psi(K)}=\frac{\psi'(K)K}{1+\psi(K)}H. \tag{A.12}$$

PROOF: The objective function of the firm is strictly concave. The second derivative of the objective function with respect to labor is:

$$F_{LL}-2\omega'-\omega''\left(L_j+\lambda L_{-j}\right)<0$$

since $F_{LL}<0$ and $-2\omega'-\omega''\left(L_j+\lambda L_{-j}\right)<0$ because we are assuming that labor supply is constant elasticity. The second derivative of the objective function with respect to capital is

$$F_{KK}-2\psi'-\psi''\left(K_j+\lambda K_{-j}\right)<0$$

since $F_{KK}<0$ and $-2\psi'-\psi''\left(K_j+\lambda K_{-j}\right)<0$. The latter inequality follows because $-2\psi'-\psi''\left(K_j+\lambda K_{-j}\right)=-\psi'(K)\left[2+\psi''(K)K/\psi'(K)(s_j^K+\lambda(1-s_j^K))\right]$, where s_j^K is firm j's share of capital and the expression in brackets is positive because $\psi''(K)K/\psi'(K)\geq -1$. To see this, note that $\psi'(K)=\frac{\sigma}{1-\sigma}\frac{E}{E-K}\frac{1+\psi(K)}{K}$ and $\psi''(K)=\frac{\sigma}{1-\sigma}\frac{1+\psi(K)}{K^2}\frac{E}{E-K}\left[\frac{K}{E-K}+\frac{\psi'(K)K}{1+\psi(K)}-1\right]$. Since $\frac{\psi'(K)K}{1+\psi(K)}=\frac{\sigma}{1-\sigma}\frac{E}{E-K}$, then

$$\psi''(K)K/\psi'(K)=\left(K/E+\sigma/(1-\sigma)\right)E/\left(E-K\right)-1\geq -1.$$

The cross derivative of the objective function is negative, and therefore the determinant of the matrix of second derivatives is positive, which is the last condition we needed to establish strict concavity of the objective function. From the first-order conditions, it is then clear that the reaction functions are continuous, and therefore a Nash equilibrium exists.

To prove that there is a unique symmetric equilibrium, we consider the system of FOCs when employment and capital are symmetric across firms, and show that there is a unique solution. From the FOC for labor, we can solve for labor as a function of capital, obtaining:

$$L=\left[\frac{A(1-\alpha)}{\chi^{\frac{1}{1-\sigma}}\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)}\right]^{\frac{1}{\alpha+\frac{\xi+\sigma}{1-\sigma}}}K^{\frac{\alpha}{\alpha+\frac{\xi+\sigma}{1-\sigma}}}.$$

Replacing this in the FOC for capital, we obtain an implicit equation for capital:

$$A\alpha\left[\frac{A(1-\alpha)}{\chi^{\frac{1}{1-\sigma}}\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)}\right]^{\frac{1-\alpha}{\alpha+\frac{\xi+\sigma}{1-\sigma}}}K^{-\frac{\frac{\xi+\sigma}{1-\sigma}(1-\alpha)}{\alpha+\frac{\xi+\sigma}{1-\sigma}}}-\delta-\psi(K)-\psi'(K)KH=0. \tag{A.13}$$

The limit when $K \to 0^+$ of this expression is $+\infty$, while the limit when $K \to E^-$ is $-\infty$. The derivative of this expression with respect to K is strictly negative, which implies that the there is a unique solution to the equation. The two-equation characterization of the equilibrium obtains directly from imposing symmetry in the FOCs of the firm. □

As in the one-factor case, the equilibrium is characterized by the markdown of real wages relative to the marginal product of labor being equal to the elasticity of the competitive equilibrium real wage with respect to firms' employment plans, multiplied by the MHHI. The new condition adds that the markdown of the real interest rate relative to the marginal product of capital (net of depreciation) is equal to the elasticity of the competitive equilibrium real interest rate with respect to firms' investment plans, multiplied by the MHHI.

A.2.5 Comparative statics

Proposition 12. *Suppose $\phi < 1$. Then either a decline in the number of firms J or an increase in the common ownership parameter ϕ leads to an equilibrium with lower:*

(a) capital stock K^*; (b) employment L^*; (c) real interest rate r^*; (d) real wage $(w/p)^*$; (e) output; and (f) labor share of income.

PROOF:

(a) We start by noting that the number of firms J and the common ownership parameter ϕ enter the equilibrium equation for capital only through market concentration H. We then use the equilibrium equation for capital to define capital as an implicit function of $H \in (0,1]$:

$$A\alpha \left[\frac{A(1-\alpha)}{\chi^{\frac{1}{1-\sigma}}\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)}\right]^{\frac{1-\alpha}{\alpha+\frac{\xi+\sigma}{1-\sigma}}} K(H)^{-\frac{\frac{\xi+\sigma}{1-\sigma}(1-\alpha)}{\alpha+\frac{\xi+\sigma}{1-\sigma}}} - \delta \equiv \psi(K(H)) + \psi'(K(H))K(H)H. \tag{A.14}$$

To ease the notation, we rewrite it as:

$$d_1\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)^{-d_2} K(H)^{-d_3} - \delta \equiv \psi(K(H)) + \psi'(K(H))K(H)H, \tag{A.15}$$

where $d_1 = A\alpha\left[\frac{A(1-\alpha)}{\chi^{\frac{1}{1-\sigma}}}\right]^{\frac{1-\alpha}{\alpha+\frac{\xi+\sigma}{1-\sigma}}} > 0$, $d_2 = \frac{1-\alpha}{\alpha+\frac{\xi+\sigma}{1-\sigma}} > 0$, and $d_3 = \frac{\frac{\xi+\sigma}{1-\sigma}(1-\alpha)}{\alpha+\frac{\xi+\sigma}{1-\sigma}} > 0$. Differentiating the equation and solving for $\frac{\partial K}{\partial H}$ yields the following:

$$\frac{\partial K}{\partial H} = -\frac{d_1 d_2\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)^{-d_2-1}\frac{\xi+\sigma}{1-\sigma}K(H)^{-d_3} + \psi'(K(H))K(H)}{d_1 d_3\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)^{-d_2}K(H)^{-d_3-1} + \psi''(K(H))K(H)H + \psi'(K(H))(1+H)} < 0 \tag{A.16}$$

since both the numerator and the denominator of the fraction are positive. To see why the denominator is positive, note that $\psi''KH + \psi'K(1+H) = \psi'\left[1+H+\psi''K/\psi'\right] > 0$ because $\psi''K/\psi' \geq -1$.

(b) We know that

$$L^* = \left[\frac{A(1-\alpha)}{\chi^{\frac{1}{1-\sigma}}\left(1+\frac{\xi+\sigma}{1-\sigma}H\right)}\right]^{\frac{1}{\alpha+\frac{\xi+\sigma}{1-\sigma}}} (K^*)^{\frac{\alpha}{\alpha+\frac{\xi+\sigma}{1-\sigma}}},$$

which is decreasing in H and increasing in K. Since H increases when the number of firms decreases or common ownership increases, and K decreases with them, L must decline with both lower J and higher ϕ.

(c), (d), and (e) Since the equilibrium real wage and real interest rates are increasing in L and K, they also must decline when the number of firms decreases or common ownership increases. A lower level of employment and capital also implies lower output.

(f) The labor share of income is $\frac{\omega(L)L}{F(K,L)} = \frac{1-\alpha}{1+\frac{\xi+\sigma}{1-\sigma}H}$. A decrease in the number of firms or an increase in the common ownership parameter ϕ increases H and therefore decreases the labor share. □

A.3 A general framework for oligopoly in general equilibrium, and a general proof of invariance to price normalization

In this section, we introduce a general framework for oligopoly in general equilibrium where firms are assumed to maximize a weighted average of shareholder utilities instead of profits. We show that the equilibrium does not depend on the choice of price normalization.

Consider an economy with a finite number J of firms, and a continuum I of agents (who both consume the output of the firms and provide the factors of production) of measure M. There are Q goods, including factors of production.

Consumers. Each consumer i has an endowment $\mathbf{e}_i$ of the Q goods, and ownership shares in the firms $\boldsymbol{\theta}_i$, and control shares γ_i, with ownership and control not necessarily equal. For all j, $\int_{i\in I}\theta_{ij} = 1$ and $\int_{i\in I}\gamma_{ij} = 1$. Consumer i gets utility $U_i(\mathbf{x}_i)$ from consuming a vector $\mathbf{x}_i$ of the Q goods.

Firms. Firm j has a production set $G_j \in \mathbb{R}^Q$. A production plan for firm j is a point $\mathbf{y}_j \in G_j$. Firms return profits to the owners in proportion to their ownership shares θ_{ij}.

Definition 4 (Competitive Equilibrium Relative to Firms' Production Plans). *A* price system $\mathbf{p}$ *is a non-null element of* $\mathbb{R}^l_+$. *A* competitive equilibrium relative to $(\mathbf{y}_1, \ldots, \mathbf{y}_J)$ *is a price system and an allocation pair* $(\mathbf{p}, \{\mathbf{x}_i\}_{i\in I})$ *such that:*

1. *Budget constraints are satisfied, that is, for each* $i \in I$, $\mathbf{p}\cdot\mathbf{x}_i \le \mathbf{p}\cdot\left(\mathbf{e}_i + \sum_{j=1}^J \theta_{ij}\mathbf{y}_j\right)$
2. *There is no other element in the budget set that could achieve higher utility for any consumer.*

Let the set Y be the production plan vectors $\mathbf{Y} = (\mathbf{y}_1, \ldots, \mathbf{y}_J)$ for which there is at least one equilibrium. If an equilibrium exists for a given set of production plans, then an infinite set of renormalized equilibria exist for that set of production plans, with prices multiplied by a constant.

A price function $\mathbb{P}$ assigns a price vector $\mathbb{P}(\mathbf{Y})$ to each production plan vector $\mathbf{Y} \in Y$, such that for any $\mathbf{Y}$, $(\mathbb{P}(\mathbf{Y}), \{\mathbf{x}_i\}_{i\in I})$ is a competitive equilibrium for some allocation $\{\mathbf{x}_i\}_{i\in I}$. Note that the price normalization does not necessarily imply that one of the prices is equal to one, and the normalization constant can be different for each set of production plans. A numéraire normalization is a special case of normalization in which the price of a given good is set to one.

Consumer i has indirect utility $V_i\left(\mathbf{p}, \mathbf{p}\cdot\left(\mathbf{e}_i + \sum_{j=1}^{J}\theta_{ij}\mathbf{y}_j\right)\right)$ when the price system is $\mathbf{p}$. In the following definition, we assume that a firm maximizes a control-share weighted average of consumer indirect utility functions.

Definition 5 (Cournot-Walras Equilibrium with Shareholder Representation)**.** *A Cournot-Walras equilibrium with shareholder representation is a price function $\mathbb{P}$, an allocation $\{\mathbf{x}_i^*\}_{i\in I}$, and a set of production plans $\mathbf{Y}^*$ such that:*

1. *The pair $[\mathbb{P}(\mathbf{Y}^*); \{\mathbf{x}_i^*\}_{i\in I}]$ is a competitive equilibrium relative to $(\mathbf{y}_1^*, \ldots, \mathbf{y}_J^*)$.*
2. *For each firm j, the objective function $\int_{i\in I}\gamma_{ij}V_i\left(\mathbb{P}(\mathbf{y}_j, \mathbf{y}_{-j}^*), \mathbb{P}(\mathbf{y}_j, \mathbf{y}_{-j}^*)\cdot\left(\mathbf{e}_i + \theta_{ij}\mathbf{y}_j + \sum_{k\neq j}\theta_{ik}\mathbf{y}_k^*\right)\right) di$ achieves its maximum on G_j at $\mathbf{y}_j = \mathbf{y}_j^*$ (where $\mathbf{y}_{-j}$ denotes the vector production plans of all the firms except j).*

Theorem 1 (Numeraire independence)**.** *If $\{\mathbb{P}; \{\mathbf{x}_i^*\}_{i\in I}; \mathbf{Y}^*\}$ is a Cournot-Walras equilibrium with shareholder representation, and $\tilde{\mathbb{P}}(\mathbf{Y}) = \Gamma(\mathbf{Y})\mathbb{P}(\mathbf{Y})$ for every $\mathbf{Y}\in Y$, where $\Gamma(\mathbf{Y})$ is a scalar-valued function, then $\left\{\tilde{\mathbb{P}}; \{\mathbf{x}_i^*\}_{i\in I}; (\mathbf{Y}^*)\right\}$ is also a Cournot-Walras equilibrium with shareholder representation.*

Proof. Since $\{\mathbb{P}; \{\mathbf{x}_i^*\}_{i\in I}; \mathbf{Y}^*\}$ is a Cournot-Walras equilibrium with shareholder representation, then $[\mathbb{P}(\mathbf{Y}^*); \{\mathbf{x}_i^*\}_{i\in I}]$ is a competitive equlibrium relative to $\mathbf{Y}^*$. Since $\tilde{\mathbb{P}}(\mathbf{Y}^*) = \Gamma(\mathbf{Y}^*)\mathbb{P}(\mathbf{Y}^*)$, then $\left[\tilde{\mathbb{P}}(\mathbf{Y}^*); \{\mathbf{x}_i^*\}_{i\in I}\right]$ is also a competitive equlibrium relative to $\mathbf{Y}^*$. This establishes item 1 from the definition.

To show point 2 from the definition, note that, for each firm j,

$$\int_{i\in I}\gamma_{ij}V_i\left(\mathbb{P}(\mathbf{Y}), \mathbb{P}(\mathbf{Y})\cdot\left(\mathbf{e}_i + \sum_{k=1}^{J}\theta_{ik}\mathbf{y}_k\right)\right) di$$

is equal to

$$\int_{i\in I}\gamma_{ij}V_i\left(\Gamma(\mathbf{Y})\mathbb{P}(\mathbf{Y}), \Gamma(\mathbf{Y})\mathbb{P}(\mathbf{Y})\cdot\left(\mathbf{e}_i + \sum_{k=1}^{J}\theta_{ik}\mathbf{y}_k\right)\right) di$$

for every set of production plans $\mathbf{Y}$ because $\Gamma(\mathbf{Y})$ is a scalar-valued function, and indirect utility functions are homogenous of degree zero in prices and wealth. Therefore, if

$$\int_{i\in I}\gamma_{ij}V_i\left(\mathbb{P}(\mathbf{y}_j, \mathbf{y}_{-j}^*), \mathbb{P}(\mathbf{y}_j, \mathbf{y}_{-j}^*)\cdot\left(\mathbf{e}_i + \theta_{ij}\mathbf{y}_j + \sum_{k\neq j}\theta_{ik}\mathbf{y}_k^*\right)\right) di$$

achieves its maximum on G_j at $\mathbf{y}_j^*$, then

$$\int_{i\in I}\gamma_{ij}V_i\left(\tilde{\mathbb{P}}(\mathbf{y}_j, \mathbf{y}_{-j}^*), \tilde{\mathbb{P}}(\mathbf{y}_j, \mathbf{y}_{-j}^*)\cdot\left(\mathbf{e}_i + \theta_{ij}\mathbf{y}_j + \sum_{k\neq j}\theta_{ik}\mathbf{y}_k^*\right)\right) di$$

also achieves its maximum on G_j at $\mathbf{y}_j^*$. □

The intuition of the proof is straightforward: the change of numéraire is simply a renormalization of prices, in which the constant by which prices are renormalized is different depending on the production plans of the firms. The indirect utility function does not change with renormalization, and therefore neither does the objective function of the firm, which is a weighted average of indirect utility functions.

A.4 Mapping of the general model to the particular models in the paper

A.4.1 One-sector model with only labor

- $I = I_W \cup I_O$. I_O, the group of owners, is divided into J subgroups.
- $Q = 2$ (consumption good and leisure).
- $e_{1,i} = 0$ for all i.
- $e_{2,i} = T$ if $i \in I_W$.
- $\theta_{ij} = \frac{1-\phi+\frac{\phi}{J}}{1/J}$ if $i \in I_O$ and is in group j.
- $\theta_{ij} = \frac{\frac{\phi}{J}}{1/J}$ if $i \in I_O$ and is in group k.
- $\theta_{ij} = 0$ for $i \in I_W$.
- $\gamma_{ij} = \theta_{ij}$ (proportional control assumption).
- $U_i(\mathbf{x}_i) = U_i(C_i, L_i)$ if $i \in I_W$ (where L_i is labor supply).
- $U_i(\mathbf{x}_i) = U_i(C_i) = C_i$ if $i \in I_O$.
- $G_j = \{(y_j, -L_j) | y_j \leq F(L_j)\}$, where y_j is the amount of the consumption good produced by firm j and L_j is the amount of labor used by firm j in the production process.

A.4.2 One-sector model with labor and capital

- $I = I_W \cup I_S \cup I_O$. I_O, the group of owners, is divided into J subgroups.
- $Q = 6$ (consumption good in period zero, consumption good in period 1, leisure, capital, investment storage, bonds).
- $e_{1,i} = E$ if $i \in I_S$, zero otherwise.
- $e_{3,i} = T$ if $i \in I_W$, zero otherwise.
- $e_{k,i} = 0$ for every i for $k \in \{2, 4, 5, 6\}$.
- $\theta_{ij} = \frac{1-\phi+\frac{\phi}{J}}{1/J}$ if $i \in I_O$ and is in group j.

- $\theta_{ij} = \frac{\frac{\phi}{J}}{1/J}$ if $i \in I_O$ and is in group k.

- $\theta_{ij} = 0$ for $i \in I_W$.

- $\gamma_{ij} = \theta_{ij}$ (proportional control assumption).

- $U_i(\mathbf{x}_i) = U_i(C_{0,i}, C_{1,i}, L_i) = \frac{C_{0,i}^{1-\sigma}}{1-\sigma} - \chi \frac{L_i^{1+\xi}}{1+\xi}$ if $i \in I_W$ (L_i is labor supply).

- $U_i(\mathbf{x}_i) = U_i(C_{0,i}, C_{1,i}, L_i) = \frac{C_{0,i}^{1-\sigma}}{1-\sigma} + \beta \frac{C_{1,i}^{1-\sigma}}{1-\sigma}$ if $i \in I_S$.

- $U_i(\mathbf{x}_i) = U_i(C_{0,i}, C_{1,i}, L_i) = C_{0,i}$ if $i \in I_O$.

- $G_j = \left\{(y_{0,j}, y_{1,j}, -L_j, ST_j, -K_j, B_j) | y_{0,j} \leq F(L_j, K_j) - I_j, y_{1,j} \leq (1-\delta)K_j + ST_j, K_j \leq B_j\right\}$, where $y_{0,j}$ is the amount of the period zero consumption good produced by firm j (net of storage), L_j is the amount of labor used by firm j in the production process, K_j is the amount of the zero-period consumption good (transformed into capital at a 1:1 rate) used in the production process, ST_j is the amount of output from period zero that the firm stores until period one (which is an intermediate good that is internal to the firm and therefore not traded in the market; this is a minor departure from the general framework), B_j is the amount of bonds the company sells in the bond market, and $y_{1,j}$ is the total amount of output available to the firm to sell in the goods market in period one (including the storage received from period zero).

A.4.3 Multi-sector model with only labor

- $I = I_W \cup I_O$. I_O, the group of owners, is divided into $N \times J$ subgroups.

- $Q = N + 1$ (N consumption goods and leisure).

- $e_{n,i} = 0$ for all i, $n = 1 \ldots N$.

- $e_{n+1,i} = T$ if $i \in I_W$, zero otherwise.

- $\theta_{inj} = \frac{1-\phi+\phi/NJ}{1/NJ}$ if $i \in I_O$ and is in group nj.

- $\theta_{inj} = \frac{\frac{\phi}{NJ}}{1/NJ}$ if $i \in I_O$ and is not in group nj.

- $\theta_{inj} = 0$ for $i \in I_W$.

- $\gamma_{inj} = \theta_{inj}$ (proportional control assumption).

- $U_i(\mathbf{x}_i) = U_i(\{c_{ni}\}_{n=1}^N, L_i) = \frac{\left\{\left[\sum_{n=1}^N \left(\frac{1}{N}\right)^{\frac{1}{\theta}} c_{ni}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\right\}^{1-\sigma}}{1-\sigma} - \chi \frac{L_i^{1+\xi}}{1+\xi}$ if $i \in I_W$ (L_i is labor supply).

- $U_i(\mathbf{x}_i) = U_i(\{c_{ni}\}_{n=1}^N, L_i) = \left[\sum_{n=1}^N \left(\frac{1}{N}\right)^{\frac{1}{\theta}} c_{ni}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$ if $i \in I_O$

- $G_{nj} = \{(\mathbf{0}_{n-1}, y_{nj}, \mathbf{0}_{N-n}, -L_{nj}) | y_{nj} \leq F(L_{nj})\}$, where $\mathbf{0}_k$ is a k-dimensional vector of 0's, y_{nj} is the amount of the consumption good produced by firm nj and L_{nj} is the amount of labor used by firm nj in the production process.