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Implementation in Economies with a Continuum of Agents

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We study a general implementation problem for exchange economies with a continuum of players and private information, and test the robustness of the results for sequences of approximating finite economies. Assuming that the designer knows the distribution of the characteristics in the economy we consider continuous and unique implementation in both its equilibrium and dominant strategies versions and obtain results for general self-selective (first- and second-best) allocations. An upper hemicontinuity property of Bayesian equilibria of approximating economies for continuous mechanisms is demonstrated. Using this, we can, for example, conclude that if a given *continuous* mechanism implements *uniquely* a Walrasian allocation in the continuum economy then all Bayesian equilibria of large approximation economies will (with probability close to one) yield an allocation which is almost ex-post Pareto optimal.

1. INTRODUCTION

In this paper we study a general implementation problem for (exchange) economies with a continuum of players. A first objective is to construct mechanisms which implement first- or second-best allocations in large economies. A second objective is to relate incentive compatibility results in continuum economies with results pertaining to approximating sequences of large economies.

We have argued elsewhere (see Mas-Colell (1984a, b), Vives (1988)) that strategic games with a continuum of players constitute a useful technique in economics. The analysis tends to become relatively straightforward and the conclusions are not too misleading when applied to realistic situations (or so one hopes). The present paper reinforces this methodological point.

The approximation results have a parallel in similar research programmes which have been successfully developed in relation to the core (Aumann (1964), Debreu and Scarf (1963), Hildenbrand (1974), Anderson (1978)), and to Cournot-Nash equilibria (Gabszewicz and Vial (1972), Roberts (1980), Novshek and Sonnenschein (1978), Mas-Colell (1982)).

The importance of finding mechanisms to implement first- and second-best allocations for public policy problems cannot be overstressed. Particularly relevant in public finance is the design of tax and subsidy schemes which improve upon market allocations and attain constrained optima. We obtain positive implementation results in large, continuum economies with private information but no macroscopic uncertainty. Nevertheless, given that the continuum assumption is an idealization of large but finite economies, it is imperative to test the robustness of our results for the upper tail of sequences of approximating economies. This we do for finite economies formed by successive independent random draws from the continuum population. An implication of the approximation exercise is that this type of asymmetric information need not preclude the (approximate) attainment of first-best allocations in large economies.

In our study of the implementation problem we differ from previous work (e.g. Hammond (1979), Champsaur and Laroque (1982), Groves and Ledyard (1987), Makowski and Ostroy (1988)) in two respects. First, we make the problem more difficult by insisting on *unique implementation by continuous mechanisms.* This is because we want our results to be relevant, i.e. sufficiently robust, to apply to large, finite economies. Second, we make it easier by assuming that *the (probability) distribution of characteristics is known to the designer.* This is in keeping with the Bayesian tradition: in our set-up there is no aggregate uncertainty and thus, at the continuum limit, knowing the prior distribution implies the knowledge of the realized distribution of characteristics. It has the added virtue of steering us away from impossibility results.

Under the basic assumptions of our model (a continuum of agents and a known distribution of characteristics) there is a close conceptual relationship between implementation by anonymous mechanisms and tax systems as in Mirrlees (1971). This has been emphasized by Hammond (1979), Guesnerie (1981), and Dierker and Haller (1990) (who extend the analogy to large but finite economies). (See also Blackorby and Donaldson (1988), for an application to the optimal provision of medical care.) In this interpretation our mechanisms are decentralized in the sense that the desired outcomes are attained by confronting consumers with the same budget constraint. It is this property that gives considerable intuitive appeal to implementation results in terms of optimal tax schemes.

We pose the implementation problem as one of finding a deterministic continuous mechanism such that *truth-telling* is an equilibrium and such that any other possible equilibrium is equivalent in utility terms (unique implementation). We also study a stronger requirement for implementation, namely, that truth-telling be a *dominant* strategy. Our results are sufficient for many applications but not complete, especially when it comes to dominant implementation. Many problems are left open. We defer specific comments to the main text. Our proofs proceed by the concrete construction of mechanisms and some of these may be of interest in their own right (some indeed are, and have been, well known for many years).

Before stating the main results obtained we remark that there is a necessary condition for an allocation to be implementable in the continuum limit, namely, it has to be *self-selective* (no agent envies the net trade of another agent whose characteristics can be claimed by him).

We find that in a continuum economy self-selective and pure (non-random) allocations can always be implemented via a subscription-type mechanism when the possible types of agents are finite (Proposition 3). Self-selective, pure allocations can also be implemented in dominant strategies either if preferences are homothetic and endowments publicly known, or if they are quasi-linear and vectors of non-numeraire commodities can be produced at some cost (Section 11). For two-good economies, or if all goods are normal, Walrasian allocations can be implemented in dominant strategies (Proposition 5 and 6 respectively). This is also true if the Walrasian equilibrium is unique for any possible economy (Proposition 4). With respect to finite approximating economies we show that, for continuous mechanisms, any accumulation point of a sequence of Bayesian equilibria of finite economies (obtained by independent sampling from the limit economy) is supported in the equilibrium set of the continuum economy (that is, the Bayesian equilibrium correspondence is upper hemicontinuous at the continuum limit, Proposition 2). We emphasize (and we provide an example) that the continuity of the mechanism is essential for this result.

The upper hemicontinuity property is useful for proving results of the following kind. Given the unique implementation results at the continuum limit and the fact that Walrasian allocations are pure and self-selective, it follows that there exist allocation mechanisms with the property that the Bayesian equilibria of large economies (with traders' characteristics independently drawn) are such that nearly every realization is almost Pareto optimal and individually rational (for nearly every consumer).

As an illustration of the potential application of our approach consider a cartel in an industry with a continuum of firms with unknown, strictly convex costs but known distribution, trying to implement the monopoly allocation (variants of this problem are studied in detail in Kihlstrom and Vives (1992), who also give examples of real-life large cartels). The problem, obviously, is to devise a system which induces firms to reveal truthfully their types (costs) to the cartel manager. The solution is to create a market for production duties giving each firm as endowment the per capita monopoly revenue and output, interpreted as a production duty. It is easy to see that this market will have a unique Walrasian equilibrium for any distribution of declared costs and therefore (according to Proposition 4) it can be implemented in dominant strategies. The cartel manager just has to ask the firms their types and allocate production duties and transfers according to the equilibrium of the duties market for the declared types. More efficient firms will buy, at a *negative* price, production duties from less efficient ones and produce more than their given duty endowment. In essence, to guarantee truthful revelation of types, (relatively) inefficient firms must pay to keep their production low, and (relatively) efficient firms must be compensated to induce them to produce high levels of output. The right balance in order to avoid misrepresentations is struck at the (unique) competitive price of the production duties market. The proposed mechanism realizes trivially the monopoly outcome since aggregate production must equal the given aggregate duty endowment. Further, for a large finite number of firms, *any* Bayesian equilibrium of the mechanism will yield, with probability not far from one, allocations which are nearly collusive.

It is worth recalling that from d'Aspremont, Cremer, and Gerard-Varet (1979), we know that, for independently drawn economies with quasi-linear utilities, full ex-post optimality (but not individual rationality) can always be guaranteed (see Maskin (1986) and d'Aspremont and Gerard-Varet (1987) for relaxation of the hypotheses and illustrative examples). Hence we do not assume quasi-linearity and independence can be weakened. What is essential for our approach is that uncertainty becomes small in the aggregate, so developing an implementation theory for economies with a continuum of traders and macroscopic uncertainty remains to be done.

The plan of the paper is as follows. Sections 2 to 5 set up the basic definitions and concepts about mechanisms in continuum economies. Sections 6 and 7 deal with Bayesian equilibria of finite economies and the upper hemicontinuity proposition. Section 8 defines the implementation problem. Section 9 deals with incentive compatible mechanisms (equilibrium implementation) and Sections 10 and 11 with strong incentive compatibility (dominant implementation). Concluding remarks follow.

2. INDIVIDUAL CHARACTERISTICS

We deal with exchange economies whose agents have characteristics (i.e. preference-endowments pairs) continuously indexed by a *compact, metric set of parameters* A. We denote by u_a, ω_a the characteristics of $a \in A$. The commodity space is R^l. Every consumption set is R^l_+ and every u_a is continuous, monotone and strictly concave. For example, let $A = [0, 1] \times \Theta$, where $[0, 1]$ stands for agents' labels, and Θ is a space of fundamental agents' characteristics (a set of parameters which characterize utility functions and endowments such that there is a continuous mapping from Θ to pairs (u, ω)).

We denote by $J \subset A \times A$ the *set of feasible misrepresentations*, i.e. an agent of characteristics a can claim to be of characteristics a' if and only if $(a, a') \in J$. We assume J is a closed subset of $A \times A$ containing the diagonal $\{(a, a)\colon a \in A\}$. The set J embodies part of the information available to the designer. As an example, if agents fall into m classes of publicly known characteristics A_j, $1 \leqq j \leqq m$, we can take $A = A_1 \cup \cdots \cup A_m$ and $J = (A_1 \times A_1) \cup \cdots \cup (A_m \times A_m)$. Notice that in the general case it is possible that type a can pretend to be a', a' can pretend to be a'' but that for type a it is not a feasible misrepresentation to pretend to be of type a''. If it helps, the reader should keep in mind the simple case $J = A \times A$.

3. MECHANISMS

For the concept of an allocation mechanism see, for example, Hurwicz (1960), Hammond (1979), or Dubey, Mas-Colell and Shubik (1980).

We only consider (deterministic) *direct* mechanisms, that is, mechanisms for which agents' messages are the characteristics A themselves.

Let $P(A)$ be the space of (Borel) probability measures on A, then an (anonymous) mechanism is a function $G'\colon A \times P(A) \to R^l$ such that $\int G(a, \upsilon) d\upsilon(a) \leqq 0$ for all $\upsilon \in P(A)$. This is a condition of aggregate, or public, feasibility.

The mechanism so defined is anonymous because the allocation assigned to an agent depends only on its declaration and the distribution of declarations υ. Nevertheless, anonymity can be easily relaxed using the set J introduced earlier. For example, take $A = [0, 1] \times \Theta$, and let J, the subset of $A \times A$ of feasible misrepresentations, be $J = \{(s, \theta), (s', \theta')\colon s' = s\}$.

The space $P(A)$ is endowed with the weak convergence topology and we assume that our mechanisms are continuous. This requirement guarantees that the continuum economies perform the role of proper idealizations of large economies. We will comment more on the continuity requirement later.

Remark 3.1. Our consideration of A as the message space is not without loss of generality. Although our set-up is Bayesian, our insistence on continuous mechanisms means that, strictly speaking, we cannot appeal to the revelation principle. This is so because in going from an indirect, perhaps continuous, mechanism to the corresponding direct mechanism, continuity is only guaranteed if equilibrium strategies can be chosen continuously on individual characteristics and distributions of characteristics. This is not always so. For example, the well-known trading mechanisms of Shapley and Shubik (see, for example, Shapley (1976) and Shapley and Shubik (1977)) are continuous but equilibrium is not necessarily unique and the equilibrium selection required for the direct mechanisms can not therefore be made continuously.

Remark 3.2. The analysis that follows could be extended to a setting with a general (metric) message space B, a set of feasible misrepresentations of the type $J' \subset A \times B$, and a mechanism $G\colon B \times P(B) \to R^l$.

Remark 3.3. Because we permit a feasibility inequality our definition permits mechanisms which allow credible threats to destroy resources.

Remark 3.4. There are other conceivable formulations of the feasibility properties to be imposed on G. A weaker formalization would require aggregate feasibility (i.e. $\int G(a, v)dv \leqq 0$) only for $v = \bar{v}$ (or, in other words, only at equilibrium). This is done in, for example, Mirrlees (1971) and Guesnerie (1981) and it has a commitment interpretation: the agents of the economy believe that the designer will honour his obligations even if this means bringing resources from outside the system. We do not want to go so far and thus, in our implementation problem there is always aggregate feasibility. We do not insist, however, on private feasibility. Outside of equilibrium, mechanisms can yield final consumptions with negative components. We are aware that the usual convention in the literature of letting utility be minus infinity when the net trade is individually infeasible, to which we have adhered, presents problems of interpretation. We believe that if private feasibility is insisted upon then the problem should be posed in the context of publicly known endowments. Otherwise it may be too strong to require that a mechanism should generate privately feasible outcomes for all possible economies (that is, for all possible v). As we go along we will comment from this perspective on the private feasibility properties of the mechanisms we study.

4. CONTINUUM ECONOMIES

A *continuum economy* is specified simply by $\bar{v} \in P(A)$, interpreted as the distribution of characteristics of a non-atomic continuum of agents. Let a continuum economy $\bar{v}$ be given.

An *allocation* is a joint distribution μ on $A \times R^l_+$ having the property that $\mu_1 = \bar{v}$ and $\int i d\mu_2 \leqq \int \omega_a d\bar{v}(a)$, where μ_1 (resp. μ_2) is the marginal of μ on A (resp. R^l) and i denotes the identity map. An equivalent way to express the resource balance constraint is:

$$\int_{A \times R^l_+} (x - \omega_a)\mu(da \times dx) \leqq 0.$$

The following definition corresponds to the usual definition of Pareto optimality in terms of distributions on the space of agents' characteristics. An allocation μ is *Pareto optimal* if there is no $\tau \in P(A \times R^l \times R^l)$ such that $\tau_{12} = \mu$, τ_{13} is an allocation, $\tau(\{a, y, y'): u_a(y') \geqq u_a(y)\}) = 1$ and $\tau(\{a, y, y'): u_a(y') > u_a(y)\}) > 0$. Of course, τ_{12} (resp. τ_{13}) stands for the projection of τ on the first and second (resp. first and third) co-ordinates of $A \times R^l \times R^l$.

An allocation μ is Walrasian if there is a $p \neq 0$, $p \in R^l_+$ such that $p \cdot y = p \cdot \omega_a$ whenever $(a, y) \in \text{supp}\ \mu$ and $p \cdot v > p \cdot \omega_a$ whenever $(a, y) \in \text{supp}\ \mu$ and $u_a(v) > u_a(y)$.

An allocation μ is *self-selective*, or "anonymous", if $u_a(y) \geqq u_a(x - \omega'_a + \omega_a)$ whenever (a, y), $(a', x) \in \text{supp}\ \mu$ and $(a, a') \in J$. It is *strictly self-selective* if, in addition, we have equality only if $a' = a$. By convention here and later we put $u_a(z) = -\infty$ whenever some component of z is negative. Note that a Walrasian allocation is always self-selective (in fact, because of strict concavity, it is strictly self-selective).

An allocation μ is *pure* if supp μ is contained in the graph of a function $h: A \to R^l$. Because supp μ is closed we can, without loss of generality, assume that if μ is pure then it is supported in the graph of a continuous function. Note that every Pareto optimal allocation must be pure (because of the strict concavity of the utility functions).

The concepts of this section are all familiar. For self-selective allocations see, among others, Champsaur and Laroque (1981, 1982), Hammond (1979), Mas-Colell (1985 Chapter 7, 1987). For the measure-theoretic approach see Hildenbrand (1974).

5. EQUILIBRIUM CONCEPTS FOR MECHANISMS IN CONTINUUM ECONOMIES

In this section we consider as given a continuous mechanism $G: A \times P(A) \to R^l$ and a continuum economy $\bar{\upsilon}$.

Denote by $\eta \in P(A \times A)$ the distributions on $A \times A$. The first (resp. second) factor of $A \times A$ is interpreted as the true (resp. declared) characteristics. Thus a $\eta \in P(A \times A)$ is in the nature of a strategy play by all agents. Denote by η_1, η_2 the marginal of η on the first and second factor of $A \times A$, respectively.

An $\eta \in P(A \times A)$ is an *equilibrium* if $\eta_1 = \bar{\upsilon}$ supp $\eta \subset J$, and for any $(a, a') \in \text{supp } \eta$ we have $u_a(\omega_a + G(a', \eta_2)) \geqq u_a(\omega_a + G(a'', \eta_2))$ for every $a'' \in A$ such that $(a, a'') \in J$.

An $\eta \in P(A \times A)$ is a *truth-telling* (*or truthful*) *equilibrium* if it is an equilibrium supported on $\Delta = \{(a, a): a \in A\}$. If G has a truth-telling equilibrium we will say G is *incentive compatible* (IC). Denote by $\bar{\eta}$ the (uniquely determined) measure which is supported in Δ and has $\bar{\eta}_1 = \bar{\upsilon}$.

We will also be interested in a stronger solution concept. We say that G is *strongly incentive compatible* (SIC) if for any υ and $a \in \text{supp } \bar{\upsilon}$ we have that $u_a(\omega_a + G(a, \upsilon)) \geqq u_a(\omega_a + G(a', \upsilon))$ for every $a' \in A$ such that $(a, a') \in J$. That is, truth-telling is a dominant strategy. Of course, if G is SIC then $\bar{\eta}$ is an equilibrium.

Given an equilibrium η denote by $\mu[\eta] \in P(A \times R^l)$ the allocation induced by η. If the mechanism is to be emphasized we put $\mu[\eta, G]$. Note that if η is a truthful equilibrium, i.e. $\eta = \bar{\eta}$, then $\mu[\eta]$ is pure.

6. FINITE ECONOMIES: BAYESIAN EQUILIBRIUM

The concept of Bayesian equilibrium for a finite game is familiar (see, for example, Myerson (1985)). In this paper we shall concentrate on games induced by mechanisms acting on economies in which agents are privately informed and have characteristics independently and identically distributed. We will comment later (at the end of the next section) on the possibility of relaxing this.

Given a distribution $\bar{\upsilon}$ we let a *finite economy* be a random sample from A according to $\bar{\upsilon}$. More precisely, a finite economy of size N is specified by a collection of $\bar{\upsilon}$-i.i.d. random variables $(a_1, \ldots, a_N)$.

Suppose now that a mechanism $G: A \times P(A) \to R^l$ is given. We can let G act on a finite economy. As in the continuum economy, the second argument of G is the distribution of declared types. A *Bayesian equilibrium* is then an N-tuple $(\eta_1, \ldots, \eta_N)$ such that for every i:

(a) $\eta_i \in P(A \times A)$ and supp $\eta_i \subset J$, that is, each η_i is a joint probability distribution over pairs (a_i, a'_i) which consist of the true characteristic a_i and the declaration a'_i,

(b) $\eta_{i,1} = \bar{\upsilon}$, and

(c) for every $(a_i, a'_i) \in \text{supp } \eta_i$, a'_i solves:

$$\text{Max} \int u_{a_i}\left(\omega_{a_i} + G\left(a''_i, \frac{1}{N}(\delta_{a'_1} + \cdots + \delta_{a'_{i-1}} + \delta_{a''_i} + \delta_{a''_{i+1}} + \cdots + \delta_{a'_N})\right)\right)$$
$$\times d(\eta_{1,2} \times \cdots \times \eta_{i-1,2} \times \delta_{a''_i} \times \eta_{i+1,2} \times \cdots \times \eta_{N,2})$$

over all a''_i such that $(a_i, a''_i) \in J$. $\quad (*)$

Here $\eta_{j,1}$, $\eta_{j,2}$ are, respectively, the marginals of η_j on its first and second co-ordinate.

If at the equilibrium $(\eta_1, \ldots, \eta_N)$ we have $\eta_i(\{a, a'): a = a')\} = 1$ for every i then we say that equilibrium is truthful. It is well known (revelation principle) that any equilibrium for a mechanism G can be made (in the sense of preserving the same payoffs) into a truthful equilibrium of a modified mechanism G'. It is not generally true however that if G is continuous then G' will also be. Moreover, G' may have equilibria that are very different from the equilibria of G. In the approach to implementation in finite economies taken in this paper it is essential that the mechanism G be defined with independence of the particular limit economy $\bar{v} \in P(A)$. Because of this when we study finite economies we do not insist on the equilibrium being truthful.

If η_i is supported on the graph of a function $h: A \to A$ then it has the interpretation of a pure strategy, otherwise it should be thought of as mixed.

If at an equilibrium $(\eta_1, \ldots, \eta_N)$ we have $\eta_i = \eta_j$, all i, j, we say that the equilibrium is *symmetric.*

To establish the existence of an equilibrium is simple enough:

Proposition 1. *If G is continuous then every finite economy has an equilibrium.*

Proof. We prove the existence of a symmetric equilibrium. Recall that A is compact by assumption. For any $\eta \in P(A \times A)$ let $\Phi(\eta) \subset P(A \times A)$ be the set of measures η' such that, for every $(a, a') \in \operatorname{supp} \eta'$, a' solves the problem $(*)$ for $\eta_i = \eta$, all i. It is clear that this set is non-empty and convex and that the correspondence $\eta \to \Phi(\eta)$ is u.h.c. Therefore, a fixed point exists by the Ky Fan-Glicksberg Fixed Point Theorem (Ky Fan (1952), Glicksberg (1952)). ||

7. UPPER HEMICONTINUITY AT THE CONTINUUM LIMIT

The probability measure $\bar{v} \in P(A)$ is as in the previous section.

The aim of this section is to establish that if for any N we have an equilibrium $(\eta_1^N, \ldots, \eta_N^N)$ then in some appropriate sense the sequence converges to an equilibrium of the limit economy. Because of the continuity of G this is not difficult. An appeal to the law of large numbers will do the trick.

We begin by defining a metric ρ on $P(A \times A)$ as follows. Let $\{f_m\}$ be a countably dense family of continuous functions $f_m: A \times A \to [0, 1]$, then let $\rho(\eta, \eta') = \sum_m (1/2^m)|\int f_m d\eta - \int f_m d\eta'|$. This metric is different from the better known Prohorov metric but induces exactly the same notion of convergence.

For every N let $\tau^N \in P(P(A \times A))$ be the distribution of sample realizations (with every agent weighted equally) of $(\eta_1^N, \ldots, \eta_N^N)$, that is τ^N is the distribution induced by $(\eta_1^N, \ldots, \eta_N^N)$ on the variable

$$\frac{1}{N} \sum_{i=1}^{N} \delta_{(a_i, a_i')} \in P(A \times A).$$

Proposition 2. *For every* $\varepsilon > 0$ *there is some* $\bar{N}$ *such that if* $N > \bar{N}$ *then* $\tau^N(\{\eta: \rho(\eta, \eta') \leqq \varepsilon$ *for some equilibrium* η' *of the continuum economy* $\bar{v}\}) \geqq 1 - \varepsilon$. *In other words, any accumulation point of* $\{\tau^N\}$ *is supported in the equilibrium set of the continuum economy.*

Proof. It is enough to show that if $N^{-1} \sum_{i=1}^{N} \eta_i^N$ converges to some η' then the conclusion of the Proposition holds for this η' (remember that $P(A \times A)$ is compact and that therefore by extracting a subsequence if necessary we can always guarantee that the previous sum converges). In particular, we are asserting that η' must be an equilibrium of the continuum economy.

Let $f_m : A \times A \to [0, 1]$ be an arbitrary continuous function. For every N consider the random variable

$$\frac{1}{N}\sum_{i=1}^{N} f_m(a_i, a_i')\left[= \int f_m d\left(\frac{1}{N}\sum_{i=1}^{N} \delta(a_i, a_i')\right)\right].$$

Its expectation is

$$\frac{1}{N}\sum_{i=1}^{N} \int f_m d\eta_i^N.$$

Note that its variance cannot exceed $1/N$.

By Chebyshev's inequality (see Breiman (1968 p. 6)) we have

$$\tau^N\left[\left|\frac{1}{N}\sum_{i=1}^{N} f_m(a_i, a_i') - \frac{1}{N}\sum_{i=1}^{N}\int f_m d\eta_i^N\right| > \delta\right] \leqq \frac{1}{\delta^2 N} \quad \text{for any } \delta > 0.$$

Fix an ε and choose an M sufficiently large for $\sum_{m=M}^{\infty} 1/2^m < \varepsilon/4$. Let now $\delta = \varepsilon/4M$ and, finally, choose $\bar{N}$ such that $M/\delta^2\bar{N} < \varepsilon$ and $\rho(N^{-1}\sum_{i=1}^{N} \eta_N^i, \eta') < \varepsilon/4$ for all $N > \bar{N}$.

Combining all this we get, for $N > \bar{N}$,

$$\tau^N\left[\left|\frac{1}{N}\sum_{i=1}^{N} f_m(a_i, a_i') - \frac{1}{N}\sum_{i=1}^{N}\int f_m d\eta_i^N\right| \leqq \frac{\varepsilon}{4M} \text{ for every } m \leqq M\right] \geqq 1 - \varepsilon,$$

or

$$\tau^N\left[\left\{\eta : \rho\left(\eta, \frac{1}{N}\sum_{i=1}^{N} \eta_i^N\right) \leqq \frac{\varepsilon}{2}\right\}\right] \geqq 1 - \varepsilon.$$

Hence, we conclude that for any $\varepsilon > 0$ there is $\bar{N}$ such that if $N > \bar{N}$ then $\tau^N(\{\eta : \rho(\eta, \eta') \leqq \varepsilon\}) \geqq 1 - \varepsilon$.

It remains to show that η' is an equilibrium of the continuum game. To this effect let $(\bar{a}, \bar{a}') \in \operatorname{supp} \eta'$. Then $(\bar{a}(N), \bar{a}'(N)) \to (\bar{a}, \bar{a}')$ for some sequence satisfying $(\bar{a}(N), \bar{a}(N)) \in \operatorname{supp}[N^{-1}\sum_{i=1}^{N} \eta_i^N]$ (the supp is lower hemicontinuous on the measures).

Without loss of generality we can take $(\bar{a}(N), \bar{a}'(N)) \in \operatorname{supp} \eta_i^N$. This implies that $\bar{a}'(N)$ solves problem (*) of Section 6. Denote $u_N = u_{\bar{a}(N)}$, $\omega_N = \omega_{\bar{a}(N)}$. With a slight abuse of notation we can write this conclusion as:

$$\int u_N\left[\omega_N + G(\bar{a}(N), \eta_2 - \frac{1}{N}(\delta_{a_1'} - \delta_{\bar{a}'(N)}))\right] d\tau^N(\eta)$$
$$\geqq \int u_N\left[\omega_N + G(a'', \eta_2 - \frac{1}{N}(\delta_{a_1'} - \delta_{a''}))\right] d\tau^N(\eta) \quad \text{for every } a'' \in A.$$

The result of the last paragraph says that for large N the probability of τ^N is almost entirely concentrated very near a single outcome, namely η'. More precisely, $\tau^N \to \delta_{\eta'}$ weakly. Appealing to continuity we can take limits and conclude

$$\begin{aligned} u_{\bar{a}}(\omega_{\bar{a}} + G(\bar{a}', \eta')) &= \int u_{\bar{a}}(\omega_{\bar{a}} + G(\bar{a}', \eta_2)) d\delta_{\eta} \\ &\geqq u_{\bar{a}}(\omega_{\bar{a}} + G(a'', \eta_2)) d\delta_{\eta'} \\ &= u_{\bar{a}}(\omega_{\bar{a}} + G(a'', \eta')) \quad \text{for all } a'' \in A. \end{aligned}$$

Therefore, η' is an equilibrium for the continuum game. ||

Remark 7.1. The previous proposition generalizes to the situation where a finite economy is formed by N independently and identically distributed groups of m agents. Suppose indeed that $\zeta \in P(A^m)$ is the probability distribution of a single group. Then the statement of Proposition 2 remains valid when we take the continuum economy $\bar{\upsilon}$ to be $\bar{\nu} = m^{-1} \sum_{i=j}^{m} \zeta_j$, where ζ_j is the marginal of ζ on the j-th factor. Of course, finite economies have size mN and equilibria are of the form $(\beta_1^N, \ldots, \beta_N^N)$ where $\beta_i^N \in P(A^{2m})$.

Remark 7.2. We should stress that the straightforward generalization suggested in the previous remark would *not* yet allow for common values. That is, for the possibility that the utility of an agent depends on the type of another. In a recent paper, Gul and Postlewaite (1992) have investigated several implementation issues and the limiting behavior of a model where there is a basic group of agents with interrelated utilities which is then replicated in an i.i.d. manner.

Remark 7.3. We have taken the information available to every agent to be minimal (i.e. they only know their type). In a sense this is the worst possible case. Allowing for more information (e.g. sampling information) will not change the conclusions. In fact they become easier to obtain because we need to rely less on the law of large numbers in order to eliminate uncertainty from individual decision problems. When there is perfect information the upper hemicontinuity result follows from Green (1984).

Remark 7.4. The fact that enough independent shocks remove aggregate uncertainty is of course well known and has been used before in equilibrium analysis (e.g. Hildenbrand (1971), Malinvaud (1972)).

Remark 7.5. There are mechanisms (penalty-type, for example) which can be sensibly defined in the continuum economy but do not satisfy the continuity assumption. To them we could not directly apply the reasoning or the results of this section. Since those are the ultimate justification of our approach we will insist on continuous implementation.

8. THE IMPLEMENTATION PROBLEM IN CONTINUUM ECONOMIES

For this and the next two sections we consider a fixed continuum economy $\bar{\upsilon}$.

It is an obvious consequence of the definition of equilibrium that if η is an equilibrium for G then $\mu[\eta, G]$ is a self-selective allocation. Moreover, if η is truthful, i.e. $\eta = \bar{\eta}$, then $\mu[\bar{\eta}, G]$ is also pure. Conversely, if μ is a self-selective, pure allocation is there a (continuous) mechanism G for which $\bar{\eta}$ is an equilibrium and $\mu = \mu[\bar{\eta}, G]$? It is not difficult to give a positive answer to this question but this is not, from our point of view, the most interesting question. If the equilibrium (or, more precisely, the utility allocation induced by the equilibrium) is not unique then the upper hemicontinuity result of the last section can not be used to conclude that the equilibria of the finite approximation are close to the limit equilibria we are interested in. Hence, we are led to formulate the general implementation problem as:

Implementation problem. *Given a self-selective, pure allocation μ, is there a continuous mechanism G such that*: (1) *truth-telling is an equilibrium and* (2) *any other equilibrium of G is utility-equivalent to μ?*

We emphasize that we impose the continuity requirement on G because we are interested in the relationship between large finite and continuum economies. Indeed, it should be clear how the previous results (Sections 6 and 7) on finite economies combine with implementation results for the continuum economy. Suppose, for example, that we have a mechanism G which implements at the limit a Walrasian allocation. Then *any* Bayesian equilibrium of a large economy will with probability close to one yield an allocation which is (ex-post) nearly Pareto optimal and individually rational. (To minimize technicalities we skip a precise definition of nearly optimal; but see Proposition 2 to deduce one.)

We should stress that the above type of result constitutes the payoff of our insistence on unique and continuous implementation in the limit. Continuity of the mechanism yields upper hemicontinuity of the equilibrium correspondence. With uniqueness, upper hemicontinuity yields automatically continuity and therefore we get strong results for the finite but large situation. Without uniqueness we would need to prove lower hemicontinuity which is more difficult and, anyhow, would yield a result only for some equilibria of the approximating economies. There may be cases nevertheless where unique implementation is not needed. Suppose, for example, that we are only interested in guaranteeing (near) Pareto optimality. Then it is sufficient that given G all the equilibria of the continuum economy be Pareto optimal. Indeed, in this case Proposition 2 implies that Bayesin equilibria of large economies which approximate the limit economy will be close to one of the equilibria of the limit economy, hence nearly Pareto optimal.

Note that even if for the continuum the implementation is truthful the approximating equilibria need not be, although there will be a (convergent) sequence of equivalent (direct) mechanisms in which to tell the truth is a (Bayesian) equilibrium strategy.

To generalize the theory to allow for production does not seem difficult (provided that the technology is public information). The same remark applies to public goods. This should not come as a surprise. It is only when the mechanism is required to respect individual rationality relative to the given declarations, and for any possible distribution of declarations, that the private and the public good theory on the continuum differ drastically (see, e.g. Groves and Ledyard (1987)).

There is a stronger version of the implementation problem where we add the condition that G be strongly incentive compatible. Telling the truth must be then a dominant strategy. We call this the *dominant implementation problem.*

The two versions are discussed in turn in Sections 9 and 10. For the stronger dominant implementation problem we only have general results for the case where μ is Walrasian. For general self-selective μ, the central interest of second best theory, the dominant implementation problem remains open, although we report some partial results in Section 11 below.

Remark 8.1. The study of the (continuous) implementation problem for non-pure μ would require the consideration of stochastic mechanisms and/or mechanisms with more general strategy spaces. Suppose, for example, that A is connected. Then an allocation which does not have a connected support (and which, therefore, can not be pure) can not be implemented via our continuous non-stochastic mechanisms with strategy spaces being the space A of agents' types. It is possible however that it can be implemented using another (non-connected) strategy space.

9. INCENTIVE COMPATIBLE MECHANISMS: EQUILIBRIUM IMPLEMENTATION

We offer a general solution to the implementation problem for the case where A is finite. Without any loss of conceptual generality we assume that supp $\bar{\upsilon} = A$.

Proposition 3. *Suppose that A is finite. For every self-selective pure allocation $\bar{\mu}$ there is a continuous $G(\cdot)$ which implements $\bar{\mu}$. That is, telling the truth is an equilibrium at $\bar{\upsilon}$ and, moreover, this equilibrium yields the allocation $\bar{\mu}$. In addition, all equilibria are payoff equivalent and if $\bar{\mu}$ is strictly self-selective then the equilibrium is unique.*

Proof. Denote $A = \{a_1, \ldots, a_m\}$ and $\bar{\upsilon} = \bar{\upsilon}(\{a_j\})$. Recall that $\bar{\upsilon}_j > 0$ for all j. Let y_j be the consumption corresponding to a_j in supp $\bar{\mu}$. Put $\hat{y}_j = y_j - \omega_j$. For any $\upsilon \in P(A)$, denote $\upsilon_j = \upsilon(\{a_j\})$, and let $G(a_j, \upsilon) = \hat{y}_j$ if $\upsilon_j = \bar{\upsilon}_j$ for all j; otherwise let

$$G(a_j, \upsilon) = \hat{y}_j - \left[\frac{\text{mas}\,\{0, \upsilon_j - \bar{\upsilon}_j\}}{\sum_1 \max\{0, \upsilon_i - \bar{\upsilon}_i\}}\right] \frac{1}{\upsilon_j} [\textstyle\sum_{i=1}^m \upsilon_i \hat{y}_i]^+$$

Note: for any vector z, z^+ (resp. z^-) denotes its positive (resp. negative) part.

The function G satisfies feasibility since $\sum_{i=1}^m \upsilon_i G(a_i, \upsilon) \leqq 0$.

It is easy to see that G is continuous (with respect to υ; there is nothing to prove for the argument a). Indeed, the only possible question arises with continuity at $\bar{\upsilon}$. Consider a sequence υ^n converging to $\bar{\upsilon}$. Then $\hat{y}_j - (\sum_{i=1}^m \upsilon_i^n \hat{y}_i)^+ / \upsilon_j^n \leqq G(a_j, \upsilon^n) \leqq \hat{y}_j$. As n goes to infinity we have the desired result since $(\sum_{i=1}^m \bar{\upsilon}_i \hat{y}_i)^+ = 0$.

Telling the truth is an equilibrium since we are implementing a self-selective allocation and a single agent is negligible. To see that it is (up to utility equivalence) unique observe that if $\upsilon \neq \bar{\upsilon}$ then there must be j and j' such that υ'_j, and some agents (precisely, a positive measure set) of type j declare as of type j'. If $\sum_{i=1}^m \upsilon_i \hat{y}_i \neq 0$ the agents in this group would not be maximizing utility. Therefore $\sum_{i=1}^m \upsilon_i \hat{y}_i = 0$ and so everybody must be getting the same level of utility at υ as at $\bar{\upsilon}$.

The argument for the strictly self-selective case should be clear. Notice that the only possibility for non-uniqueness in the argument above comes from indeterminacy in the self-selective assignment of individuals to indifferent net trades. ||

For the general case of a compact, metric A there is no difficulty in extending the above mechanism as follows. We can represent the pure allocation to be implemented as a continuous function $y(a)$ assigning net trades to agents' characteristics. For any $\upsilon \in P(A)$ let then $(\upsilon - \bar{\upsilon})^+$ be the positive part of the signed measure $(\upsilon - \bar{\upsilon})$. Denote by $g_\upsilon(a)$ the Radon-Nikodym derivative of $(\upsilon - \bar{\upsilon})^+$ relative to υ: $g_\upsilon(a)$ is defined so that $[\upsilon - \bar{\upsilon}]^+(B) = \int_B g_\upsilon(a)\, d\upsilon$ for all measurable B. Finally, define a mechanism $G(a, \upsilon)$ by

$$G(a, \upsilon) = y(a) - g_\upsilon(a)\left[\frac{1}{[\upsilon - \bar{\upsilon}]^+(A)}\left(\int y d\upsilon\right)^+\right].$$

It can be verified, without much difficulty, that if A is finite then this is precisely the mechanism proposed in the proof of Proposition 3. Moreover, as in that proof, one can verify that G implements the allocation $y(a)$ uniquely.

The difficulty with the above mechanism, however, is that G is not continuous. The mechanism depends on the ability to decompose $\upsilon - \bar{\upsilon} = (\upsilon - \bar{\upsilon})^+ - (\upsilon - \bar{\upsilon})^-$. This lattice decomposition is not continuous (except for A finite) for the weak convergence topology on $\upsilon - \bar{\upsilon}$. For example: suppose υ_n is purely atomic, $\bar{\upsilon}$ the Lebesgue measure on $[0, 1]$, and $v_n \to \bar{v}$. Then $[v_n - v]^+ = v_n \to \bar{v}$. But $v_n \to \bar{v}$ implies $(v_n - \bar{v}) \to 0$, contradicting continuity since $0^+ = 0$. The next example shows that for the subscription mechanism considered this failure of continuity may imply the breakdown of the approximation conclusion of Proposition 2.

Example. The context is a standard two-good exchange economy. We have a continuum of consumers indexed in the unit interval by $a \in [0, 1]$ (with Lebesgue measure $\bar{v}$). All consumers have the same endowments, which we normalize to the origin.

Consider now the allocation $y(a) = (1-2a, 2a-1)$, represented in Figure 1. Let $Y = \{y(a): a \in [0, 1]\}$. We proceed to specify the preferences of the consumers. The zero utility indifference curve is the same for all of them. It has a sharp kink at the point $r = (-1, 0)$, represented in the figure. The unit utility indifference levels are different across consumers and are such that the choice of consumer a on the set Y is precisely $y(a)$. For this economy the allocation $y(a)$ is self-selective, pure and Pareto optimal.

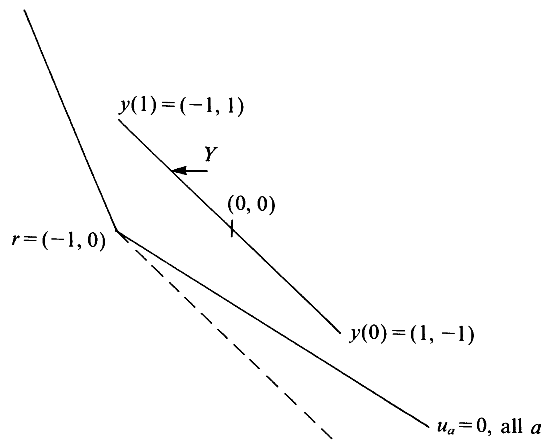

FIGURE 1

The subscription mechanism G described above implements $y(\cdot)$ uniquely. However, consider the finite approximations. Because in the finite case the message distribution will always be purely atomic it is immediate that the mechanism for N people will give to player i (let a_i be the declaration of player i in the finite approximation) the net trade:

$$y(a_i) - \frac{1}{N}(\textstyle\sum_j y(a_j))^+,$$

that is, the deficit is split equally. We now show that there may be sequences of Bayesian equilibria for this mechanism with a limit not equal to $y(\cdot)$.

Suppose that irrespective of true characteristics everybody declares $a = 1$. Then, if N is large the effective budget set faced by any consumer is the broken segment indicated in the figure (the effective budget set and the segment Y supporting the allocation $y(a)$ are almost parallel for N large). Then we can see in the figure how it is optimal for every agent to choose the point r, that is, to declare $a = 1$. This constitutes an equilibrium in which, for every N, every player pays a tax of one unit of the first good. The result is a limit very far from $y(a)$ or, for that matter, from Pareto optimality.

Remark 9.1. We owe to John Geanakoplos the observation that if with a finite number of agents the actual distribution of types is, for some reason, known then a version of the mechanism proposed in Proposition 3 will still work. An equivalent situation is when information on characteristics is *de facto* publicly known but *de jure* unusable (except statistically).

Remark 9.2. The mechanism proposed in Proposition 3 does not satisfy private feasibility. It would be interesting to know how the mechanism can be modified to ensure private feasibility when endowments are publicly known.

10. STRONGLY INCENTIVE COMPATIBLE MECHANISMS: DOMINANT IMPLEMENTATION OF FIRST-BEST ALLOCATIONS

In this section we assume that $\bar{\mu}$ is *Walrasian.* For $J = A \times A$ it is well known that under natural conditions every self-selective, Pareto optimal allocation is Walrasian (see the references in Section 4). We are considering thus the general first-best implementation problem.

We begin by clearing up a possible source of misunderstanding. We only require that our SIC mechanism yield an optimal allocation ($\bar{\mu}$ to be more specific) for the economy $\bar{\upsilon}$ (in the terminology of Hurwicz (1972), and Groves and Ledyard, (1987), our mechanism is parametric, i.e. it is designed with knowledge of $\bar{\upsilon}$). We do *not* demand that for any υ the allocation $\mu[\upsilon]$, induced by $a \to G(a, \upsilon) + \omega_a$ and υ, be optimal for υ. This is a stronger implementation problem which, at least for the case $J = A \times A$, has been well studied (see references of Section 4 and, also, Makowski and Ostroy (1988)). For $J = A \times A$ and under some additional natural conditions, the solution mechanism would have the property that $\mu[\upsilon]$ is Walrasian for υ. Since it is well known that the Walrasian equilibrium correspondence does not have in general a continuous selection this provides an impossibility result for the non-parametric problem (see Champsaur and Laroque (1982)). For some restricted domains, however, this approach is successful and consequently solves also our implementation problem. The following proposition is well known (the basic idea goes back to Roberts and Postlewaite (1976)):

Proposition 4. *Suppose that A is such that any economy $\upsilon \in P(A)$ has a unique Walrasian equilibrium. Then the mechanism $G(a, \upsilon) = \varphi_a(p(\upsilon), p(\upsilon) \cdot \omega_a) - \omega_a$, where φ_a is the demand function of a and $p(\upsilon)$ a Walrasian price vector for υ, is strongly incentive compatible, continuous, and implements $\bar{\mu}$ uniquely at $\bar{\upsilon}$.*

Proof. Obvious. ||

What about general domains A? For $l = 2$, i.e. two commodities, we have a complete answer.

Proposition 5: *Let $l = 2$ and suppose that $\bar{\mu}$ is a Walrasian allocation. Then there is a continuous strongly incentive compatible mechanism $G(a, \upsilon)$ that implements $\bar{\mu}$ uniquely at $\bar{\upsilon}$.*

Proof. An old and familiar face will turn out to do the job, namely rationing the long side of the market (see Benassy (1982, Chapter 2)). Let p be the Walrasian price vector corresponding to $\bar{\mu}$ and denote by $y(a)$ the excess demand corresponding to p and a. Given any υ in $P(A)$ choose the good j so that $\int y^j(a) d\upsilon \geqq 0$. The equation

$\int \min\{y^j(a), s\}dv = 0$ has a unique solution $s(v)$ (if $\int y^j(a)dv = 0$ put $s(v) = \max_{a\in A} y^i(a)$). Moreover, this $s(v)$ is continuous on v. Let then $G^j(a, v) = \min\{y^j(a), s(\nu)\}$ and $p\cdot G(a, \nu) = 0$. This defines G. It is not difficult to see it has the desired properties. ||

Remark 10.1. We have not been able to find a generalization of the mechanism in the previous proof for $l > 2$. The reasons are the well known (see Benassy (1982)): spillover effects—constraints in some market may lead to misrepresentation in other markets.

For $l > 2$ we can, however, offer a different solution for the case where every demand function φ_a is strictly normal (i.e. if $\varphi^j_a(p, w) > 0$ then $\varphi^j_a(p, w') < \varphi^j_a(p, w)$ for $w' < w$).

Proposition 6. *Suppose that, for every $a \in A$, φ_a is strictly normal. Then for any Walrasian allocation $\bar{\mu}$ there is a continuous strongly incentive compatible mechanism $G(a, v)$ that implements $\bar{\mu}$ uniquely at $\bar{v}$.*

Proof. Let p be a price vector corresponding to $\bar{\mu}$. Extend $\varphi_a(p, w)$ to be defined for negative w by putting $\varphi_a(p, w) = -\lambda e$ and $p\cdot\varphi_a(p, w) = -\lambda p\cdot e$ where $e = (1, \dots, 1)$. Denote by $t(v)$ the smallest t such that $\int(\varphi_a(p, p\cdot\omega_a - t) - \omega_a)dv \leqq 0$. This $t(v)$ always exists and because of strict normality it is continuous on v. The mechanism is then $G(a, v) = \varphi_a(p, p\cdot\omega_a - t(v)) - \omega_a$. It is easy to verify that it has all the desired properties. ||

Remark 10.2. Simple as Proposition 4 is it is nonetheless quite useful. As an example, we consider in more detail the problem discussed in the introduction (variants of which are studied in Kihlstrom and Vives (1992)). There is a cartel formed by a continuum of firms $a \in [0, 1]$. Every firm has a non-observable strictly convex cost function and cost functions are distributed according to $\bar{v}$. The monopoly production for the cartel is $\bar{Q}$ and the monopoly revenue is $\bar{R}$. Can a system be derived which produces the quantity $\bar{Q}$ efficiently (hence realizes the monopoly profit) and is incentive compatible? This is how it can be done. Give $(\bar{Q}, \bar{R})$ to every firm as its initial endowment. Here $\bar{Q}$ is understood as a production duty. Except for monotonicity with respect to $\bar{Q}$ (and this is inessential) we are then exactly in the framework of Proposition 4 (firms' utilities, i.e. profits, are, of course, quasi-linear in money and therefore equilibrium is unique). In consequence, a strongly incentive compatible and efficient mechanism will consist in asking firms for their type and exchanging the production duties so as to clear the production quotas market for the distribution of declared types. Firms are given $\bar{R}$ as revenue and are required to produce $\bar{Q}$ but can sell units of their production duty at a negative price to reduce the production obligation. In the equilibrium of the production duties market (relatively) inefficient firms pay to reduce their production while (relatively) efficient firms are paid to increase their production. This contrasts with the mechanisms considered in Kihlstrom and Vives (1992), where firms obtain revenue from two sources: selling output in the market or receiving a transfer payment from the cartel manager. In this case, in order to guarantee truthful revelation of types, (relatively) inefficient firms must be bribed to keep their production low.

Remark 10.3 For the mechanisms in this section there is no essential difficulty with private feasibility (i.e. non-negative constraints in final consumption) if endowments are publicly known. Indeed, for Proposition 4 and 5 we only have to take $J = \{(a', a): \omega_{a'} = \omega_a\}$

and verify that the mechanism proposed will then be privately feasible. For Proposition 6 the mechanism suggested will guarantee private feasibility if all agents have the same endowments. If endowments are different but publicly known, the mechanism can be modified easily so as to ensure private feasibility.

11. STRONGLY INCENTIVE COMPATIBLE MECHANISMS: PARTIAL RESULTS ON DOMINANT IMPLEMENTATION OF SELF-SELECTIVE ALLOCATIONS

As we have already mentioned we have no general results on the implementability of an arbitrary self-selective, pure μ. Some very partial results are possible. We can give a positive answer if endowments are publicly known, i.e. $\omega_a = \omega_{a'}$ for all $(a, a') \in J$, and every $u_a, a \in A$, is homothetic (thanks to E. Maskin for this observation); or if preferences are quasi-linear and vectors of non-numeraire commodities are producible at some cost. We consider the two cases in turn. (As remarked in Section 4 if an allocation μ is pure we may as well assume it is supported on the graph of a continuous function h: $A \to R^l$.)

Homothetic case: Let $G(a, \upsilon) = \alpha(\upsilon)\ \ h(a) - \omega_a$, where $\alpha(\upsilon) = \text{Max}\,\{\alpha: \int (\alpha h(a) - \omega_a)d\upsilon \leqq 0\}$. This clearly defines a continuous mechanism. Further, to tell the truth is dominant since a type a agent by pretending to be a' obtains $\alpha(\upsilon)h(a')$ (since $\omega_a = \omega_{a'}$ for all $(a, a') \in J$) and by telling the truth obtains $\alpha(\upsilon)h(a)$. Given homotheticity and self-selection, lying can not report a higher utility for any reported distribution υ.

Quasi-linear case: Let $G(a, \upsilon) = h(a) - \alpha(\upsilon)e^l - \omega_a$ where $e^l = (0, \ldots, 0, 1)$ and $\alpha(\upsilon) = C([\int (h(a) - \omega_a)d\upsilon]^+)$, with $C(\cdot)$ being the (publicly owned) cost function, in terms of the numeraire commodity l. Again, by construction, this defines a continuous mechanism. Linearity of the preferences in terms of the numeraire implies then that telling the truth is a dominant strategy. This is so since the self-selection condition implies that (for any declared distribution υ) to lie is not worthwhile because in any case the same amount is subtracted in utility terms $(\alpha(\upsilon)e^l)$.

12. CONCLUSION

It may be useful as a conclusion to recapitulate the problems that have been left pending or unsolved. We do so from the more specific to the more general.

(1) Proposition 3 on equilibrium implementation needs generalization to the case where A is not finite. This will require the use of a mechanism different to the subscription device used in its proof.

(2) The treatment of the dominant implementation problem has been left in a very unsatisfactory state. For first best allocations the results have some generality, but for second best we have not achieved much.

(3) We have only tackled the implementation of pure self-selective allocations. As indicated, handling general self-selective allocations will require the consideration of stochastic mechanisms and quite possibly of message spaces different from own characteristics.

(4) We have not concerned ourselves with desirability properties of mechanisms other than their ability to implement. Typically, however, there may be many mechanisms accomplishing this task (some, for example, in the nature of revelation mechanisms while others not). It may be useful to consider more refined criteria of selection. One such

could be the rate of convergence of the finite approximation to the limit, or, in the case of the implementation of first-best allocations, the rate of convergence of the welfare loss to zero.

(5) Finally, we have not studied the implementation problem when individual shocks can be significantly correlated, that is, when there are macroeconomic shocks and the allocation of the limit continuum economy can be stochastic.

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