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Excess entry, vertical integration, and welfare

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This article provides a systematic analysis of the welfare effects of vertical integration by a monopolist input supplier into a monopolistically competitive downstream industry. We give sufficient conditions on consumer preferences that lead to Pareto-improving vertical integration and demonstrate a close relationship between assumptions on preference for variety, excess entry in monopolistically competitive markets, and the welfare effects of vertical integration: Excess entry in downstream markets tends to give rise to Pareto-improving vertical integration. We extend the analysis to vertical oligopoly and access price regulation.

1. Introduction

■ The treatment of vertical mergers and vertical restraints has been an important issue for competition policy. However, there has been much controversy about which types of anticompetitive effects may arise from vertical integration. Traditionally, it has been suspected that vertical integration of an input supplier into downstream production reduces competition either directly through consolidation in the downstream market or indirectly by foreclosing upstream competitors' access to downstream firms. Such concerns are reflected in the Celler-Kefauver Act, which brought vertical (and conglomerate) mergers in the United States under the control of the Clayton Act. German competition law reflects similar concerns, prohibiting vertical mergers that significantly reduce competition between firms at one level in the supply chain.[1]

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[1] See Kühn (1997) and references therein on the restrictions on vertical agreements that have been implemented by the Federal Cartel Office in Germany.

In contrast to this legal tradition, it has been widely argued that vertical integration should be ignored altogether by competition policy.[2] This view is based on the observation that the initial problem is caused by a lack of competition at one stage of the supply chain and should therefore best be corrected by inducing more competition at this stage. In practice such a solution is often not feasible.[3] For policy purposes it is therefore important to consider whether the prevention of vertical integration can be a second-best measure for a competition authority. Is it worth preventing integration when a competition policy authority cannot induce more competition in upstream or downstream markets?

In this article we specifically analyze the question whether competition policy should prevent vertical integration by an upstream monopolist to safeguard downstream competition. Vertical integration is interpreted as any complete vertical merger or any set of vertical restraints that eliminates the externalities between the upstream and the downstream firms as, for example, double marginalization.[4] The welfare effects of such vertical integration are difficult to study analytically because two completely distinct allocations have to be compared. As a result, the literature has concentrated on solving for equilibrium allocations in specific examples, either analytically (Dixit, 1983; Perry and Groff, 1985) or numerically (Mathewson and Winter, 1983). While this work has shown, for the case of upstream monopoly and varying forms of downstream monopolistic competition, that both welfare-improving vertical integration and welfare-reducing vertical integration are possible, it has failed to identify the fundamental assumptions on preferences driving the results.[5,6] When theoretical models yield such conflicting results, we need to gain a deeper understanding of the social tradeoffs that are involved before we can formulate sensible competition policy recommendations. In particular, we would like to obtain interpretable conditions on preferences that would suggest whether welfare-increasing or welfare-reducing outcomes are more likely.

In this article we are able to derive such conditions adopting a change in perspective from the earlier literature. Past work started from the observation that the allocation achieved in a disintegrated market generally leads to reductions in output *and* variety relative to the *first-best* allocation. Since vertical integration tends to increase output because double marginalization is eliminated but also tends to restrict variety because the upstream monopolist can eliminate entry for rent-seeking purposes, the comparison with the first-best solution suggested a general tradeoff between the output and variety effects of vertical integration. We show that this conjecture is incorrect. Under a large

[2] This policy debate is unresolved. The 1984 Merger Guidelines and 1985 Vertical Restraints Guidelines, which considered vertical integration as competitively neutral or procompetitive, were rescinded by the Clinton administration in 1993. See Riordan and Salop (1995) for a suggestion to balance efficiency and anticompetitive effects of vertical mergers and a fairly complete list of the relevant literature.

[3] For example, if there are increasing returns to scale in production the industry may be a natural monopoly. Regulatory intervention either through entry subsidies or direct control would in most cases be inefficient because of informational problems associated with such intervention.

[4] This interpretation follows Mathewson and Winter (1984). Competition policy has sometimes restricted vertical restraints that could act as substitutes for vertical merger like nonlinear pricing. Examples include regulatory limitations on nonlinear pricing in contracts between U.S. car makers and their retailers (Smith, 1982; Bresnahan and Reiss, 1985) and restrictions on franchise fees in Germany (see Kühn, 1997).

[5] Perry and Groff (1985) show for a representative consumer model with CES preferences that vertical integration by a monopolist into a monopolistically competitive industry decreases welfare. The opposite has been shown for examples of spatial oligopoly downstream by Dixit (1983) (unit demands) and Bru (1991) (constant expenditure demand), and numerically by Mathewson and Winter (1983).

[6] Gallini and Winter (1983) is closest to our article in the sense that they relate the differences between private and social incentives for vertical integration to the elasticity of the demand function. However, they have to impose that demand is separable in price and the number of firms.

class of preferences there is no welfare tradeoff between the increase in output and the reduction in variety. The reason is that, from the appropriate *second-best* perspective, variety is often excessive so that the variety reduction induced by vertical integration is welfare improving.

Since excess-entry results are closely related to assumptions on preference for variety, one would hope to derive general sufficient conditions for welfare-improving or welfare-decreasing vertical integration from properties of preference for variety alone. However, the intuition that relates the welfare effects of vertical integration to the excess provision of variety relies on the assumption that vertical integration leads to an increase in total output. Despite the fact that all previously worked out examples conform to this pattern of comparative statics, in the literature there is no general result on the comparative statics of vertical integration into an imperfectly competitive downstream industry.[7] In this article we derive general comparative statics results for the limiting case of small costs of entry in the downstream market. In this case the equilibrium allocations under vertical integration and vertical disintegration are close so that we can use differential calculus to derive a full set of comparative statics results. Vertical integration increases output per firm and total output, and it decreases variety as well as price. These results do not require any assumptions about preference for variety. We can thus meaningfully ask what the relationship is between the welfare effects of vertical merger and conditions on preference for variety.

Conditional on the standard comparative statics results, we derive a precise and interpretable preference characterization that allows a sharp distinction between environments in which vertical integration is welfare improving and welfare decreasing. It relies on one property of preference for variety alone: the degree of "increasing preference for variety." The latter measures the extent to which consumers become relatively more concerned about variety at higher consumption levels. If there is a sufficiently strong degree of increasing preference for variety, complete vertical integration leads to a Pareto improvement on the fully disintegrated solution. Conversely, vertical integration is welfare decreasing when there is decreasing preference for variety.

This condition on preferences translates directly into an intuition about the role of excess variety in determining the welfare effects of vertical integration. To make this intuition rigorous, we decompose small moves in the direction of vertical integration into a small change in variety given total output and a small change in total output. Our analysis concentrates on the welfare effect of reduced variety moving along the constant output locus. We show that it is positive, whenever a vertically integrated monopolist would generate excess variety.

An integrated monopolist always generates less excess variety for any given total output than free entry downstream under vertical disintegration. A move toward vertical integration may therefore eliminate excess variety and move production closer to the second best (for a given level of total output). If, in addition, output rises vertical integration will be Pareto improving.[8] Since excess variety is provided by a monopolist

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[7] There is a substantial literature on the comparative statics of vertical integration by a monopolist input producer into a competitive industry with variable input proportions (see Schmalensee, 1973; Warren-Boulton, 1974; Westfield, 1981; and Quirmbach, 1986). It shows that vertical integration leads to increases in the final goods price if the elasticity of substitution in inputs is sufficiently high.

[8] Excess-entry results in the models of Spence (1977) and Dixit and Stiglitz (1977) depend on the increasing preference for variety condition (see Kühn and Vives, 1996). Anderson, de Palma, and Nesterov (1995) show that excess entry is the normal case in discrete-choice models.

only if there is a sufficient degree of increasing preference for variety, we have demonstrated a tight relationship between (second-best) excess-entry results and welfare-increasing vertical integration. The analysis thus provides a formalization of Perry's (1989) conjecture that in models that generate "more entry" at the retailing stage, vertical integration is more likely to lead to welfare improvements.

In our opinion, increasing preference for variety has to be considered the normal case (see Kühn and Vives, 1996). Therefore, these results give stronger theoretical support for competition policies that do not intervene in vertical agreements. To study the robustness of such conclusions, we extend our analysis to vertical oligopoly and show that a similar comparison holds between completely integrated and disintegrated markets when there is Cournot competition upstream. In addition, the comparative statics and welfare results derived for the model with upstream monopoly and almost frictionless downstream markets carry over in our setting to an almost competitive upstream market and any size of fixed costs downstream.

Another extension with more immediate policy relevance studies the regulation of two-part pricing. In industries like telecommunications in which a naturally monopolistic network serves as an input for value-added (monopolistically competitive) services on the network, the regulation of access prices seems a formidable task for a regulatory authority. Regulation of access prices may in practice be considered undesirable. In such a case our analysis provides guidance for *price structure* regulation, suggesting that the regulated firm running the network should be allowed to use access prices that include fixed access charges in addition to traffic-dependent fees if preference for variety is increasing.

In Section 2 we discuss our measure of preference for variety and define the notions of increasing and decreasing preference for variety in the context of the Spence-Dixit-Stiglitz model of product differentiation. Section 3 develops a model of vertically related markets with downstream monopolistic competition. Section 4 discusses the comparative statics effects of vertical integration. Sections 5 contains our main results. We provide sufficient conditions for Pareto-improving complete vertical integration, characterize the preference conditions for welfare-improving and welfare-reducing vertical integration, and interpret these conditions in terms of excess-entry results. Section 6 contains extensions to upstream oligopoly and nonlinear pricing. Section 7 concludes.

2. Preference for variety and consumer demand

■ We consider the class of preferences, first analyzed by Spence (1976a, 1976b, 1977) and Dixit and Stiglitz (1977), described by the utility function $u(z, y) = G(z) + y$, where z is a composite commodity and y is the amount of a numeraire good. We assume $G(z)$ to be increasing with constant elasticity $(1 - \gamma) \in (0, 1)$. The amount of the composite commodity z is determined by the subutility function $z = \int_{i=0}^{\infty} x_i V(x_i)\, di$, where x_i is the amount consumed of variety i. We can interpret utility as being given by a weighted average of all individual outputs. We assume that there is preference for variety, so that the weight on a variety i, $V(x_i)$, is decreasing in the output x_i. The function $f(x) = xV(x)$ is smooth, increasing, and strictly concave for all $x > 0$, continuous at $x = 0$ and $f(0) = 0$. Let n be the mass of varieties consumed in strictly positive quantities. We will be interested in allocations in which all such varieties are consumed in equal amounts, i.e., $z(n, x) = nxV(x) \equiv QV(x)$, where Q is the total output consumed.

To derive our results we need a characterization of the intensity of preferences for variety represented by a given utility function. For this purpose we define the *degree* of preference for variety, $\nu(x)$, as the absolute value of the elasticity of $V(x)$: $\nu(x) = |[V'(x)x]/[V(x)]|$. It measures the relative "contribution of variety" to the total utility change induced by adding another variety dn, holding the level of production per firm, x, fixed. The total utility change from adding a variety in this way is given by $G'(z)V(x)xdn$. This gain can be decomposed into two parts. First there is a *variety effect*. It is measured as the utility gain from increasing the number of varieties, holding total output Q constant: $G'(z)V(x)xv(x)dn$. Secondly there is an *output effect*. It is measured as the utility gain due to an increase in total output, holding variety constant. Since the total output increase is $dQ = xdn$, the output effect is given by

$$G'(z)[V(x) + (Q/n)V'(Q/n)]dQ = G'(z)V(x)x[1 - v(x)]dn.$$

The two effects add up to give the total effect of adding an additional variety at constant output per firm. The ratio of the variety effect to the total utility gain from adding a variety equals $v(x)$.

The degree of preference for variety, $v(x)$, thus measures the proportion of the utility gain from adding a variety that can be attributed to spreading output across more firms. It lies between zero and one. If $v(x) = 0$, there is no preference for variety, and the composition of Q is irrelevant for consumer utility. If $v(x) = 1$, the function $f(x) = xV(x)$ is constant, so utility is affected only by variety n, not by changes in the output per firm. Crucial for the analysis of the welfare effects of entry and of vertical integration is how the degree of preference for variety changes with the amount of variety provided by the market. This is measured by the elasticity of $v(x)$,

$$\epsilon^v(x) = [v'(x)x]/[v(x)].$$ [9]

For further reference we define some important properties of preference for variety:

Definition 1. There is "increasing (decreasing) preference for variety" at x if $v(x)$ is increasing (decreasing) at x, i.e., if $\epsilon^v(x) > 0$, ($\epsilon^v(x) < 0$). We say that preference for variety is "strongly increasing" at x if $k(x) \equiv \epsilon^v(x) - v(x) > 0$.

Increasing preference for variety means that at low levels of total consumption a consumer cares less about variety increases (relative to total output increases) than at high consumption levels. For the analysis of the welfare effects of vertical integration, a slightly stronger property of preferences, "strongly increasing preference for variety," plays an important role. Both properties are implied for the relevant domain by ρ-concavity of the function $V(x)$ with $\rho \geq -1$.[10]

The inverse demand for any variety i in terms of the numeraire (the Chamberlinian dd-curve) is $p(z, x_i) = G'(z)V(x_i)(1 - v(x_i))$. The absolute value of its elasticity with respect to x_i is denoted $\sigma(x_i)$. Inverse industry demand as a function of output per (produced) variety (the Chamberlinian DD-curve) is given by $P(n, x) \equiv p(z(n, x), x)$. The absolute value of its elasticity with respect to x is given by $\sigma(x) + \gamma(1 - v(x))$.[11] This is the elasticity of demand that a vertically integrated firm producing n varieties faces when deciding on output per variety. Since a vertically integrated firm will decide on the output of all varieties at once, it has an effect on $z(n, x)$, which is represented by the term $\gamma(1 - v(x))$. The elasticity $\sigma(x)$ has to be smaller than one at any

[9] We will generally denote the elasticity of a function ξ with respect to x by $\epsilon^{\xi}(x) = [\xi'(x)x]/[\xi(x)]$.

[10] A function $f(x)$ is said to be ρ-concave if $(f(x)^{\rho})[-(f(x)^{\rho})]$ is concave for $\rho > 0$ $[\rho < 0]$. If a function is $\hat{\rho}$-concave it is also ρ-concave, for $\rho > \hat{\rho}$ (see Caplin and Nalebuff, 1991). In our context, ρ-concavity of $V(x)$ is an appealing global measure for the degree of increasing preference for variety.

[11] The derivation of these elasticities is given in the Appendix. The elasticities of demand for single varieties and industry demand are completely determined by the local measures of preference for variety, $v(x)$ and $k(x)$.

optimal allocation for downstream firms. Hence, we restrict ourselves to preferences with $\sigma(x) < 1$ for all $x \geq 0$. We also assume that firm and industry demand functions are not too convex:

Assumption 1. $-(P_{xx}x/P_x) \leq 2$ and $-(p_{xx}x/p_x) \leq 1 + \nu(x)$.

Assumption 1 guarantees that single-firm and industry profits are concave in x.[12]

In some parts of the article we analyze the behavior for limiting cases at which equilibrium output per firm x is close to zero (Proposition 2 and Corollary 1). For those parts of the analysis we impose a weak regularity condition on preferences at the origin: we assume that the functions describing preference for variety look similar to constant elasticity functions at the origin. Let $\lim_{x\to 0} \nu(x) \equiv \nu(0)$; then we assume[13]

Assumption 2. There exists $\infty > \rho_1 > 0$ and $\infty > \kappa_1 > 0$ such that $\lim_{x\to 0} f(x)/x^{\rho_1} = \kappa_1$ and there exists $\infty > \rho_2 > 0$ and $\infty > \kappa_2 > 0$ such that $\lim_{x\to 0} [\nu(x) - \nu(0)]/x^{\rho_2} = \kappa_2$.

Assumption 2 is a purely technical requirement that allows us to compare all the admitted preferences according to order relations at the origin.[14] With these technical assumptions in place, we now introduce our model of a vertically related market.

3. A model of vertically related markets

■ We consider an industry with a homogeneous upstream input Q, which can be produced at constant marginal cost c. Downstream production transforms inputs one to one into units of output at zero marginal costs and fixed costs F. Each downstream firm i can produce exactly one variety facing demand $p(z, x_i)$. There is free entry. In the input market, downstream firms face a per-unit wholesale price w at which they can buy arbitrary amounts of inputs, so that their profits are given by

$$\pi_i = [p(z, x_i) - w]x_i - F. \tag{1}$$

Since each downstream firm can sell only one variety, the downstream market is monopolistically competitive. Each firm i that has decided to enter the market maximizes (1) with respect to x_i, so that the solution must be symmetric among active firms. Furthermore, firms make zero profits due to free entry. A monopolistically competitive equilibrium in the downstream market is therefore characterized by

$$\begin{aligned} [p(z, x) - w] - p(z, x)\sigma(x) &= 0 \\ [p(z, x) - w]x &= F. \end{aligned} \tag{2}$$

We assume that there is a single monopolist producing the upstream good.[15] Then

[12] For very convex individual demand functions an increase in marginal costs for all firms in an imperfectly competitive industry decreases the marginal incentives for production sufficiently to reduce competition and increase industry profits (see Scade, 1987; Kimmel, 1992; and Vives, 1999). The slightly stronger assumption on $p(z, x)$ excludes this counterintuitive possibility.

[13] To save on notation we will define for any function $\xi(x)$ the shorthand notation $\xi(0) \equiv \lim_{x\to 0} \xi(x)$.

[14] The assumption means that $f(x) = V(x)x$ and $\nu(x) - \nu(0)$ have asymptotic expansions at $x = 0$ with leading terms that are constant elasticity functions (see Hubbard and West, 1991). It is not very restrictive for our purposes. Some examples for preferences that satisfy these assumptions are $f(x) = x^\rho e^{\kappa x}$ with $\rho \in (0, 1)$ and $\kappa < (1 - \rho)/(1 + \rho)\rho$, $f(x) = xe^{-x^\kappa}$ with $\kappa \in (0, 1)$, $f(x) = x^\kappa(1 - x)$ with $\kappa \in (0, 1)$, and $f(x) = x(1 - x^\kappa)$ with $\kappa > 0$.

[15] We relax this assumption in Section 6.

the outcome does not depend on whether the upstream firm sets the wholesale price w or the quantity of the input Q. However, it will be helpful for the exposition to think of the upstream firm as a quantity setter. The wholesale price w is then determined by the condition that in equilibrium, input supply has to equal input demand, i.e., $Q = \int_0^\infty x_i\, di$. Since downstream firms make zero profits by the free-entry condition, the upstream monopolist's problem is to maximize industry profits with respect to Q.

Any input quantity Q supplied induces a unique monopolistically competitive equilibrium in the downstream market. To see this, note that eliminating w from (2) yields

$$P(n, x)x\sigma(x) = F. \tag{3}$$

This determines all the combinations of n and x that can be induced as a downstream monopolistically competitive equilibrium outcome. Equation (3) has a unique solution $\hat{n}(x)$ for any $x > 0$, since $P(n, x)$ is strictly decreasing in n, becomes arbitrarily large for $n \to 0$, and tends to zero for $n \to \infty$. The schedule $\hat{n}(x)$ implicitly defined by (3) is strictly increasing, with elasticity $(\partial\hat{n}/\partial x)(x/n) = [1 - \phi(x)]/\phi(x)$, where

$$\phi(x) \equiv \gamma[1 - \sigma(x) + \gamma v(x) + \epsilon^{\sigma}(x)]^{-1}$$

lies between zero and one.[16]

Figure 1 shows the schedule $\hat{n}(x)$ and a constant total output locus $nx = Q$. Since $\hat{n}(x)$ is continuous, strictly increasing, and $\hat{n}(x) \to 0$ if $x \to 0$, there exists a unique intersection of $\hat{n}(x)$ for any constant output locus with $Q \geq 0$. Since any choice of Q determines a unique point on the increasing schedule $\hat{n}(x)$, maximizing industry profits with respect to Q is equivalent to setting x taking $\hat{n}(x)$ as a constraint. Therefore, the upstream monopolist solves

$$\max_x \Pi(\hat{n}(x), x) = [p(\hat{n}(x)xV(x), x) - c]\hat{n}(x)x - \hat{n}(x)F, \tag{4}$$

yielding the first-order condition

$$\Pi_x(\hat{n}(x), x) + \Pi_n(\hat{n}(x), x)\frac{n}{x}\frac{1 - \phi(x)}{\phi(x)} = 0, \tag{5}$$

where

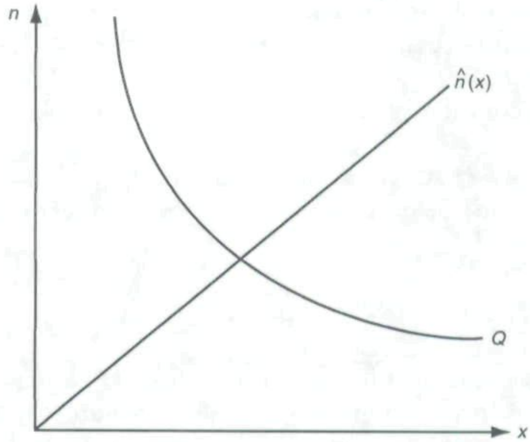
$$\Pi_x(x, n) = p(z, x)[1 - \sigma(x) - [1 - v(x)]\gamma]n - cn \tag{6}$$

and

$$\Pi_n(x, n) = p(z, x)[1 - \gamma]x - cx - F. \tag{7}$$

[16] An increase in x along $\hat{n}(x)$ must be induced by lowering w. A lower w should be expected to lead to higher profitability for a given number of firms n, increasing entry incentives. This intuition fails only for very convex demand functions, which are excluded by Assumption 1 (see footnote 12). See the Appendix (Lemma A1) for a formal proof.

FIGURE 1



Substituting for $p(z, x)$ from (3) into the first-order condition (5) and rearranging terms yields

$$F = \frac{\sigma(x)}{1 - \gamma - \sigma(x) + \gamma v(x)\phi(x)} cx, \tag{8}$$

which determines the equilibrium level of output per firm in the disintegrated market, $\hat{x}$. Given the output per firm, (3) determines equilibrium variety $\hat{n}(\hat{x})$ and total output $\hat{Q} = \hat{n}(\hat{x})\hat{x}$.

In a *vertically integrated* market, the upstream monopolist directly controls output per firm and variety so that he will set (6) and (7) equal to zero. We can now construct equilibrium conditions analogous to those of the vertically disintegrated case. Eliminating c from the system of first-order conditions, we obtain an equation analogous to (3) in the vertically disintegrated case:

$$P(n, x)x[\sigma(x) - \gamma v(x)] = F, \tag{9}$$

determining a schedule $n^*(x)$. Solving (7) for $p(z, x)$ and substituting in (6) gives an analogous equation to (8), which determines output per firm in an integrated market, x^*:

$$F = \frac{\sigma(x) - \gamma v(x)}{1 - \gamma - \sigma(x) + \gamma v(x)} cx. \tag{10}$$

Substituting x^* in (9) determines equilibrium variety, $n^*(x^*)$, and total output, $Q^* = n^*(x^*)x^*$, in the vertically integrated market. To determine the comparative statics effects of vertical integration, we compare first the equations determining output per firm under the two regimes and subsequently the equations for the optimal determination of variety given equilibrium outputs per firm.

4. The comparative statics of vertical integration

■ In this section we analyze the comparative statics of vertical integration in terms of output and variety choices. In all of the earlier literature, output per variety increases and variety decreases as a result of vertical integration. The intuition for this is that every downstream firm faces a double marginalization problem when deciding on x_i, since $w > c$ under vertical disintegration. On the other hand, free entry means that variety is provided competitively, so that a monopolist would be expected to restrict variety when vertically integrating. Unfortunately, it proves remarkably difficult to obtain these results for a general model. In this section we show that the basic intuition for the results is correct, when considering the incentives of the upstream monopolist to induce small changes in variety and output around the vertically disintegrated solution. However, this does not suffice to determine the global comparative statics of vertical integration. To obtain such results we analyze the case in which vertical integration and vertical disintegration allocations are "close." For this case we obtain the intuitive results for the whole class of preferences considered.

We first confirm the basic intuition that at the vertically disintegrated solution, industry profits could be increased by increasing output per firm (because of double marginalization) and decreasing variety (because of the competitive externality of free entry). To do so it is useful to identify the horizontal and vertical externalities generated under vertical disintegration from the point of view of the upstream firm (see Mathewson and Winter, 1984). For that purpose, write total profits in the industry as

$$\Pi(n, x) = n[(w - c)x] + n[(P(n, x) - w)x - F], \tag{11}$$

where the first term gives upstream profits and the second term downstream profits. We now show that in equilibrium $\Pi_x > 0 > \Pi_n$, i.e., industry profits could locally be increased by an increase in the output per firm and a reduction in variety.

Given the wholesale price w, the decisions of downstream firms with respect to x are subject to two externalities. There is a vertical externality due to double marginalization, since the downstream firms do not take into account the margin $(w - c)$ that the upstream firm earns on every unit sold. There is a horizontal externality (due to downstream competition) because downstream firms ignore part of their effect on prices, namely on $z(n, x)$, so that $P_x(n, x) < p_x(z, x)$. Hence, the marginal change in industry profit from an increase in output per firm at the downstream monopolistic equilibrium is given by

$$\Pi_x(n, x) = n(w - c) - nP(n, x)\gamma(1 - \nu(x)). \tag{12}$$

The first term in (12) is the vertical externality, and the second term is the horizontal externality induced by competition among retailers. Similarly, we can decompose the marginal effect of increasing variety at a downstream equilibrium into a vertical and horizontal externality:

$$\Pi_n(n, x) = (w - c)x - P(n, x)x\gamma. \tag{13}$$

Downstream firms ignore upstream profits due to marginal entry (the first term) and do not internalize the effect of price reductions on other firms in the market induced by entry (the second term). From (12) and (13) it follows directly that the product $\Pi_n \cdot (n/x) < \Pi_x$, because $nP(n, x)\gamma\nu(x) > 0$. Now note from (5) that Π_x and the product $\Pi_n \cdot (n/x)$ must have opposite signs because $[1 - \phi(x)]/\phi(x) > 0$, so that $\Pi_x > 0$ and $\Pi_n < 0$ as claimed.

We can, therefore, usefully think about vertical disintegration as leading to a net horizontal (competitive) externality generated by free entry in the downstream market and a net-vertical (double marginalization) externality generated in downstream output decisions. Intuitively, the double-marginalization problem arises on the dimension along which downstream firms have market power, that is, the decision about x, while along the dimension where there is perfect competition (i.e., free entry), the monopolist would like to limit activity. While this intuition, based on the local incentives, is suggestive, it does not imply the global comparative statics results for vertical integration we are interested in. We will now show to what extent this intuition can be extended to global comparative statics results.

The only general global comparative statics result that is available is on output per firm. It can be obtained from direct comparison of equations (8) and (10):

Proposition 1. Output per variety is strictly increased through vertical integration, i.e., $x^* > \hat{x}$.

Proof. Since $\sigma(x) > \sigma(x) - \gamma v(x)$ and $\phi(x) < 1$, the right-hand side of (8) strictly exceeds the right-hand side of (10) for every x. Furthermore, the right-hand side of (10) is strictly increasing in x by the first part of Assumption 1. It follows that $x^* > \hat{x}$. *Q.E.D.*

Proposition 1 implies that vertical integration always leads to an efficiency gain in the sense that economies of scale are better exploited. Unfortunately it is more difficult to compare the amount of variety. Vertical integration does lead to lower incentives to provide variety in the following sense: For any given output per firm, the vertically integrated monopolist would provide less variety than would be generated from free entry in a vertically disintegrated market. To see this, fix x at the same level in equations (3) and (9). For any given (n, x) the left-hand side of equation (9) is greater than the left-hand side of (3). Furthermore, this expression is monotonically decreasing in n in both cases. Hence, $n^*(x) < \hat{n}(x)$. The problem for the analysis arises because the integrated monopolist may have higher incentives to provide variety at higher output levels per firm, and output levels rise as a result of vertical integration $(x^* > \hat{x})$.

To obtain more general comparative statics results for vertical integration, we look at cases in which allocations generated by vertical integration and disintegration are close. This is true for small fixed costs F downstream. As can be seen quite easily from (8) and (10), output per firm coincides for the two market structures as $F \to 0$ since $\lim_{F\to 0} x^* = \lim_{F\to 0} \hat{x} = 0$, because $\sigma(x) - \gamma v(x) > 0$ for all $x > 0$ by Assumption 1.[17] For this case we can use differential methods and extend the comparative statics results of Perry and Groff (1985) for CES preferences to general preference structures.

Proposition 2. Suppose that F is small. Then vertical integration reduces variety, increases total output, and decreases the final goods price. More formally, there exists $\overline{F} > 0$, such that for all $F \in (0, \overline{F})$: $n^* < \hat{n}$, $Q^* > \hat{Q}$, and $P(z^*, x^*) < P(\hat{z}, \hat{x})$.

Proof. See the Appendix.

The comparative statics results confirm our general intuition about the effects of vertical integration. Integration leads to an increase in output through the elimination of double marginalization and a decrease in variety because the monopolist obtains market power over the addition of new varieties to the market. From the perspective

[17] The limiting case we consider has a natural interpretation. For small F there are arbitrarily small barriers to entry. With increasing preference for variety, that means downstream firms lose all market power and price converges to marginal cost. (This is not the case for decreasing preference for variety.)

of the consumer, the loss in variety leads to a utility loss. But this is at least partially compensated through the fall in prices because of the elimination of double marginalization. Because of these features, earlier authors have conjectured a general tradeoff between production and variety effects of vertical integration.

5. The welfare effects of vertical integration

■ **Is there a welfare tradeoff?** The previous literature on vertical integration of an upstream monopolist into a downstream monopolistically competitive market has suggested a general tradeoff between increasing the output per firm and a potential loss in variety from allowing such integration. We will show in this section that there is generally no such tradeoff: Vertical integration may eliminate both double marginalization and an excessive number of downstream firms. To understand better why this is the case, it is useful to change the perspective of the analysis. Up to now we have thought about the downstream market as distorting the decision about output per firm and variety relative to the integrated market. However, to derive results on the welfare effects of vertical integration, it turns out to be more useful to think about how downstream monopolistic competition determines the composition of Q in terms of variety and output per firm relative to any given level of Q. Looking at the problem from this angle allows one to see that vertical integration primarily eliminates the business-stealing effect of entry, yielding in many cases welfare improvements. We then give sufficient conditions on preference for variety to yield Pareto-improving and welfare-reducing vertical mergers respectively and interpret these findings in the light of excess-entry results in monopolistic competition.

Defining welfare in terms of total output Q and variety n, we can write social welfare at symmetric allocations as

$$S(Q, n) = G\left[QV\left(\frac{Q}{n}\right)\right] + y - [cQ + nF]. \tag{14}$$

$S(Q, n)$ is strictly concave in (Q, n). To evaluate the global changes in welfare induced by vertical integration, consider some path $\{Q(t), n(t)\}$, $t \in [0, 1]$, between the disintegration allocation $(\hat{Q}, \hat{n})$ and the full integration allocation (Q^*, n^*) with the property that $(Q(0), n(0)) = (\hat{Q}, \hat{n})$ and $(Q(1), n(1)) = (Q^*, n^*)$. The total change in welfare is given by

$$\Delta S = S(Q^*, n^*) - S(\hat{Q}, \hat{n}) = \int_0^1 \left\{S_Q(Q, n)\frac{\partial Q}{\partial t} + S_n(Q, n)\frac{\partial n}{\partial t}\right\} dt. \tag{15}$$

Equation (15) yields a decomposition of the welfare change due to vertical integration into a "total output effect" and a "variety effect," which we can write more intuitively as

$$\Delta S = \int_0^1 \{P(Q, x) - c\}\frac{dQ}{dt}\, dt + \int_0^1 \{G'(z)V(x)xv(x) - F\}\frac{dn}{dt}\, dt, \tag{16}$$

where $x = Q/n$ and $z = QV(x)$. The first integral captures the "total output effect." It measures the change in total output weighted by the marginal contribution of total output to welfare, $\{P(Q, x) - c\}$. The second integral describes the "variety effect."

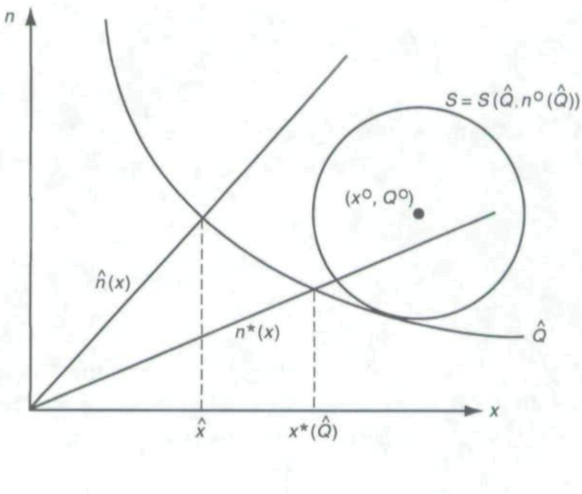
The expression $G'(z)V(x)xv(x)$ gives the marginal social benefit of increasing variety holding total output Q constant, while F is the marginal cost of variety.

Expression (16) gives some intuition for why there is significant scope for welfare-improving vertical mergers. Suppose that vertical integration increases total output and decreases variety, as it does in all the cases for which we have analytical results. We can then choose some path $\{Q(t), n(t)\}$ that is monotonic in t. It is easy to see that the total output effect is positive. Since $P(Q^*, x^*) > c$ and both Q and x are increasing along the path, the price cost margin is positive and the increase in total output will be unambiguously contributing to a welfare increase. Our analysis below therefore concentrates on the variety effect.

The variety effect will be positive, if in the range considered the marginal variety in the market has lower social value than F. This can easily happen in markets with market power. For example, for given Q a monopolist may choose to over- or under-supply variety. There is, however, a systematic reason why excessive production of variety is more likely in a monopolistically competitive market: Part of the benefit from downstream entry derives from a redistribution of profits from existing firms to the entrant. This is often called the "business-stealing effect." From the point of view of industry profits this effect always leads to excess entry given the amount produced upstream Q. To see this in our model, consider Figure 2, where we have plotted both the schedules $n^*(x)$ and $\hat{n}(x)$ against a constant output locus. Since $P(n, x)$ is strictly decreasing in n, it can be directly seen from (9) and (3) that the schedule $n^*(x)$ always lies to the right and below the schedule $\hat{n}(x)$. Hence, for given Q the upstream monopolist systematically would want to produce at higher scale and lower variety than the monopolistically competitive downstream market. The business-stealing effect of entry leads to excessive entry from the point of view of the upstream firm. Maintaining higher scale downstream would require downstream firms to make profits, which attracts entry into the market.

Since business stealing involves a pure transfer between downstream firms, this effect also tends to give excessive incentives for entry from a social point of view. Vertical integration eliminates the business-stealing effect and in this sense better aligns social and private incentives. This is the reason why vertical integration may not involve a tradeoff from a social point of view. However, business stealing is not the only distortion to the choice of variety. The exercise of market power by a monopolist can

FIGURE 2



lead to excessive or insufficient variety for given Q. If the latter is the case, business stealing by entering downstream firms may be desirable from a social point of view in order to compensate the incentives of the upstream monopolist to limit variety. In contrast, welfare can only be improved from eliminating business stealing if the monopolist produces excess variety. Which distortion an integrated monopolist induces will depend on the properties of preference for variety. Below we show that the welfare effects of vertical integration are essentially determined by the degree of increasing preference for variety. We relate these findings to results on excess entry, formalizing the intuition on the variety effect developed above.

□ **The main welfare results.** We now show that the welfare results of vertical integration essentially depend on the degree of increasing preference for variety. If the degree of increasing preference for variety is sufficiently large, vertical integration is Pareto improving whenever the output effect is positive:

Proposition 3. Suppose total output increases through vertical integration, i.e., $Q^* > \hat{Q}$. Then complete vertical integration is Pareto improving if $k(x) \geq \gamma(1 - \nu(x))$ for all x. This condition is satisfied if $V(x)$ is $-(1 - \gamma)$-concave.

Proof. See the Appendix.

Proposition 3 shows that whenever vertical integration leads to an increase in total production, it only depends on the degree of increasing preference for variety whether vertical integration is Pareto improving. For all the cases for which we can determine the comparative statics of output, the condition that output rises is satisfied so that we obtain a tight relationship between the degree of increasing preference for variety and the welfare effects of vertical integration:

Corollary 1. There exists $\overline{F} > 0$ such that for all $F \in (0, \overline{F})$, vertical integration is Pareto improving if $\epsilon^v(0) > \gamma$, i.e., if there is a sufficient degree of increasing preference for variety at the origin.

So far we have shown that for a sufficient degree of increasing preference for variety, we generally obtain Pareto improvements from vertical integration. To complete our argument that the main determinant for the welfare effects of vertical integration is the degree of increasing preference for variety, we will now show the converse result: Imposing a mild condition on the third derivative of downstream firm demand, we can show that vertical integration leads to welfare reductions whenever there is decreasing preference for variety.

Proposition 4. Suppose $[\partial\epsilon^\sigma(x)]/\partial x > -\sigma'(x)$. Then vertical integration is strictly welfare decreasing if preferences exhibit decreasing preference for variety, i.e., $\epsilon^v(x) \leq 0$.

Proof. See the Appendix.

As Perry and Groff (1985) have shown in the constant-elasticity case, cost savings from vertical integration and falling prices are not large enough to outweigh the utility loss to consumers from reduced variety. This result extends generally to decreasing preference for variety for which the utility loss due to reduced variety becomes even more pronounced. Increasing preference for variety is therefore necessary to generate welfare-increasing vertical integration. Together these results establish a close relationship between the notion of increasing preference for variety and welfare-improving vertical integration. We will now show how this relationship can be interpreted in terms of excess entry, providing a more formal analysis of the intuition based on the business-stealing effect developed earlier.

□ **An interpretation of the results in terms of excess entry.** Above we have split the welfare analysis of vertical integration into a total-output effect and a variety effect. We have shown that in all analytically tractable cases total output increases, leading to a positive total-output effect. Our results on welfare-improving vertical integration are therefore primarily about the sign of the variety effect. We can interpret the variety effect as arising from a second-best problem of maximizing welfare for a given amount of input production Q. If, for given total output Q, the marginal benefit of increasing variety is smaller at an allocation than the cost of providing it, F, then the downstream market generates "excess variety." This is analogous to the usual analysis of excess entry in models of monopolistic competition.[18] In those models excess entry is defined relative to a second-best social optimum that takes either price setting by firms or free entry as a constraint. For our analysis a fixed Q is taken as the constraint. This can be interpreted as fixing the choice of production in the input market and looking for the optimal allocation downstream conditional on this choice. Hence, the distortion in the composition of total output Q can be analyzed in an analogous way to the standard analysis of excess entry, allowing us to relate excess-entry results in monopolistic competition to the welfare effects of vertical integration.

More formally, we want to evaluate $S_n(Q, n)$, for any given Q, at the allocations generated by downstream monopolistic competition or by industry profit maximization. Consider first the case of downstream monopolistic competition. Then, evaluating the marginal value of variety at the monopolistically competitive outcome given Q yields

$$S_n(Q, \hat{n}) = G'(QV(\hat{x}))V(\hat{x})\hat{x}v(\hat{x}) - F = -G'(QV(\hat{x}))V(\hat{x})v(\hat{x})\hat{x}k(\hat{x}), \qquad (17)$$

where the second equality arises from substituting for F from equation (3).[19] It turns out that the combination of output per firm and variety that maximizes industry profits for fixed Q is given by equation (9). Substituting for F from this equation yields the corresponding marginal value of variety at the monopoly solution for given Q:

$$S_n(Q, n^*) = -G'(QV(x^*))V(x^*)v(x^*)x^*[k(x^*) - \gamma(1 - v(x^*))]. \qquad (18)$$

Equations (17) and (18) reflect the fact that for a given Q a monopolist has more incentives to reduce variety than downstream firms do. The monopolist takes into account the horizontal externalities of entry and output setting in the downstream market, which is reflected by the term $\gamma(1 - v(x))$ in (18). Notice that this term is a measure of the business-stealing effect that drives a tendency toward excess entry in the downstream market. If γ is small, there is little business stealing from entry and industry, and firm incentives almost coincide. Similarly, if preference for variety is very large, i.e., $v(x)$ close to one, each firm becomes a local monopolist, and the business-stealing effect is arbitrarily small. Expressions (17) and (18) immediately yield the following:

Lemma 1. In a vertically disintegrated market there is excess variety if and only if preferences exhibit strongly increasing preference for variety at $\hat{x}$, i.e., $k(\hat{x}) > 0$. There exists excess variety at the completely integrated outcome if and only if $k(x^*) > [1 - v(x^*)]\gamma$.

[18] See, for example, the contributions by Dixit and Stiglitz (1977), Koenker and Perry (1981), Mankiw and Whinston (1986), and Corchón (1991).

[19] Since this analysis is for any given output level Q, the expressions $\hat{n}$, $\hat{x}$, n^*, and x^* should be understood as functions of the total output level, not as their values at $\hat{Q}$ and Q^* respectively. We suppress this distinction at this point to save on notation.

This lemma implies that whenever $k(x) > \gamma(1 - \nu(x))$ for all x, both a vertically disintegrated market and a vertically integrated market would exhibit excess variety in equilibrium in the sense that, given the respective levels of total output, welfare could be enhanced by decreasing variety.

We can now use this insight about the relative incentive for generating excess variety of a monopolist and a monopolistically competitive market to interpret Proposition 3. Consider Figure 2. Suppose we hold total output fixed at the level of vertical disintegration $\hat{Q}$. If $k(\hat{x}) > 0$, we know that there is excess variety at the vertically disintegrated solution. Since the monopolist has a greater incentive to reduce variety for fixed total output than do the downstream firms, he will push the allocation in the direction of the second-best welfare optimum, i.e., $n^*(\hat{Q}) < \hat{n}(\hat{Q})$. The condition $k(x) - \gamma(1 - \nu(x)) > 0$ for all x guarantees that the monopolist generates excess variety at $n^*(\hat{Q})$, and by concavity of $S(Q, n)$ in n, welfare must increase from this move. The move from the vertically disintegrated situation at $(\hat{n}, \hat{Q})$ to the optimal allocation of the monopolist given $\hat{Q}$, $(n^*(\hat{Q}), \hat{Q})$, will therefore eliminate some excess entry and move the allocation closer to the social second best relative to the constraint that $\hat{Q}$ has to be produced. Furthermore, if the integrated monopolist produces more total output than is produced under vertical disintegration, there will also be an upward move in Q along the schedule $n^*(x)$. Since output per firm and total output both increase along $n^*(x)$, this move will again be welfare enhancing. Effectively the move along $\hat{Q}$ eliminates excess variety, while the move along $n^*(x)$ eliminates double marginalization.

Interestingly enough, consumers also benefit from this move because the savings in production costs are passed on to them through increased production and consequently lower prices. To see the price reduction, write price along the constraint as $p(\hat{Q}V(x), x)$ and differentiate with respect to n, yielding $-[p(n, x)]/n[\gamma\nu(x) - \sigma(x)] > 0$, where $x = \hat{Q}/n$. The total effect on consumer surplus, which we denote by $CS(Q, n)$, arises from a cost of reduced variety and a benefit from lower prices:

$$\begin{aligned} \frac{\partial CS(\hat{Q}, n)}{\partial n} &= -\frac{Q}{n^2}\left[\frac{\partial G(\hat{Q}V(x))}{\partial x} - \frac{\partial p(\hat{Q}V(x), x)}{\partial x}\hat{Q}\right] \\ &= -\frac{p(\hat{Q}V(x), x)\hat{Q}}{n}\frac{\nu(x)}{1 - \nu(x)}\{k(x) - \gamma(1 - \nu(x))\}. \end{aligned} \tag{19}$$

This shows that the condition that generally guarantees that an integrated monopolist would generate excess variety independently of the fixed costs of downstream production is the same as needed for a consumer to benefit from the elimination of excess variety. Therefore, vertical integration that eliminates excess variety will be Pareto improving.

The case for welfare-improving vertical integration gets weaker if a monopolist does not find it optimal to extract rents through an excessive number of varieties sold at high prices. If $k(x) < \gamma(1 - \nu(x))$, the monopolist will produce less variety than is socially optimal given $\hat{Q}$. This may still induce a welfare improvement when the monopolist would generate only small reductions in variety relative to the second-best welfare optimum. The condition in Proposition 3 is therefore significantly stronger than what is needed to generate welfare-improving and Pareto-improving vertical integration. It should be clear, however, that vertical integration always produces welfare losses due to variety reductions whenever $k(x) < 0$. Then there is insufficient variety in the first place, and vertical integration aggravates the problem. When preferences have constant elasticity of substitution, then $k(x) = -\nu(x)$ and the variety reduction is large enough to overcome the gains from the elimination of double marginalization.

We can therefore tightly relate excess-entry results in the literature on monopolistic competition to the welfare effects of vertical integration. As Perry (1989) conjectured, preferences that lead to a stronger tendency for excess entry will lead to more favorable results about the welfare effects of vertical integration. This can be nicely illustrated for the case of small F. Since $\lim_{x\to 0} k(x) = \epsilon^{\nu}(0)$, the condition comes down to determining the sign of $\epsilon^{\nu}(0) - \gamma$. When the degree of increasing preference for variety is large, i.e., $\epsilon^{\nu}(0) > \gamma$, vertical integration is Pareto improving. For small degrees of increasing preference for variety, market power over entry starts to become a potential problem. For decreasing preference for variety, $\epsilon^{\nu}(0) = 0$ and welfare losses result.[20] The condition for welfare-improving vertical integration is slightly tighter than that for excess entry under downstream monopolistic competition because of the horizontal market power established by the integrating firm over the provision of variety.

□ **Interpreting "strongly increasing preference for variety."** We have seen that the property of strongly increasing preference for variety is crucial in generating Pareto-improving vertical integration. What exactly does this condition mean? In contrast to the property of increasing preference for variety, we lack a good intuition for the stronger property. For this reason it is helpful to look for an indirect interpretation. Suppose a benevolent social planner would be selling the upstream input, trying to induce the welfare optimum. It is easy to show that he can implement the first-best allocation through a two-part tariff. He would clearly want to subsidize production by setting a wholesale price, w, below marginal cost in order to counteract market power in the downstream market. But whether he subsidizes or taxes entry through the fixed component of the tariff, A, depends on whether there is strongly increasing preference for variety or not:

Proposition 5. Let (w^0, A^0) be the two-part tariff that implements the social optimum. Then $w^0 < c$. The socially optimal allocation is implemented with a strictly positive franchise fee if and only if there is strongly increasing preference for variety at x^0, i.e., if $k(x^0) > 0$, then $A^0 > 0$, and if $k(x^0) < 0$, then $A^0 < 0$.

Our notion of excess variety is therefore equivalent to a statement that entry into the downstream market should be taxed. Since the upstream monopolist can implement the vertically integrated solution through a two-part tariff with a positive franchise fee, we can think of vertical integration in the case of strongly increasing preference for variety as eliminating double marginalization through a lower w and inducing a welfare-improving tax on entry.

6. Extending the analysis

■ **Vertical oligopoly.** One of the limitations of the analysis above is that it has been derived for a situation of monopoly upstream. However, we can extend the analysis to allow for Cournot oligopoly in the upstream market and compare symmetric vertically integrated markets with symmetric vertically disintegrated markets. Suppose there is an integer number of m firms in the upstream market. In an integrated market, each upstream firm decides simultaneously about its total production Q_i, $\sum_i Q_i = Q$, and the

[20] This corresponds nicely to conventional analyses of excess entry in the Spence-Dixit-Stiglitz model. Taking price setting by downstream firms as the second-best constraint, there is excess entry if and only if $\epsilon^{\nu}(x) > 0$. For all standard second-best criteria the condition $\epsilon^{\nu}(0) > 0$ is sufficient for excess entry whenever F is sufficiently small (see Kühn and Vives, 1996).

degree of variety n_i.[21] With all firms vertically integrated, an upstream firm then maximizes

$$\left[G'\left(Q_iV(x_i) + \sum_{j\neq i} Q_jV(x_j)\right)f'(x_i) - c - \frac{F}{x_i}\right]Q_i \tag{20}$$

with respect to Q_i and n_i, where $x_i = Q_i/n_i$. Solving the first-order conditions, we obtain an analogous condition to (10) for the determination of output per firm at a symmetric equilibrium:

$$F = \frac{\sigma(x) - \dfrac{\gamma}{m}v(x)}{1 - \dfrac{\gamma}{m} - \left[\sigma(x) - \dfrac{\gamma}{m}v(x)\right]}cx. \tag{21}$$

Variety is then determined by the equation $p(\sigma(x) - (\gamma/m)v(x))x = F$. The decision of the integrated firm is therefore taken as if the firm were a monopolist facing a market-demand function with a lower elasticity γ/m of the function $G'(z)$. This is because the firm does not take into account the effects of its output and variety decisions on other manufacturers.

We model the vertically disintegrated market in a way that concentrates on the tradeoff between double marginalization and excess variety. We assume that each manufacturer i can contract with retailers from a distinct pool i of retailers. This excludes the possibility of competition between manufacturers for retailers, which would give an additional incentive for production of output in the first stage of the disintegrated market, an effect that is distinct from the effects discussed so far.[22] Then the game proceeds as follows. Each upstream firm first sets its output Q_i. Each downstream firm in pool i then receives a price quote w_i, $i = 1, \ldots, m$, for the input from exactly one firm and has to decide whether to enter the market selling product i or not and how much of the product to sell. In making its decision, a retailer takes the wholesale prices w_j charged to other retailer pools as given. This means that if upstream firm i increases Q_i, it believes that n_j and x_j will remain unchanged for all $j \neq i$. This means that rival potential downstream firms do not observe this change by i. In equilibrium all beliefs are confirmed. By symmetry of the downstream equilibrium, each downstream firm contracting with the same manufacturer in equilibrium sells the same output x_i. Since downstream firms earn zero profits in equilibrium, total upstream profits are given by

$$(w_i(Q_i) - c)Q_i = \left(P_i(Q_i, x_i) - c - \frac{F}{x_i}\right)Q_i, \tag{22}$$

where $P_i(Q_i, x_i) = p(\Sigma_j\ Q_jV(x_j), x_i)$. Upstream firms are facing the inverse demand function $w(Q_i) = P(Q_i, x(Q_i)) - F/[x(Q_i)]$, where $x(Q_i)$ is determined by the downstream equilibrium locus $P_i(Q_i, x_i)\sigma(x_i)x_i = F$. Each upstream firm therefore maximizes

[21] By preference for variety it will be optimal for upstream firm i to have equal production across its n_i brands, so that all of them have output x_i. Given Q_i, variety of firm i is therefore fully determined by x_i.

[22] For very competitive upstream markets this effect dominates the double marginalization effect, generating a negative output effect from vertical integration. However, there is still a positive variety effect if and only if $k(x) > 0$. The formulation in the article excludes this effect because a full discussion would be beyond the scope of this article.

$(P(Q_i, x(Q_i)) - c - F/[x(Q_i)])Q_i$ with respect to Q_i. Using the first-order condition and substituting $P(Q_i, x_i)$ from the equilibrium locus, we can determine output per firm x in a symmetric equilibrium between all manufacturers by the equation

$$F = \frac{\sigma(x)}{1 - \frac{\gamma}{m} - \sigma(x) + \frac{\gamma}{m}\nu(x)\phi_m(x)} cx, \tag{23}$$

where $\phi_m(x) = (\gamma/m)[1 - \sigma(x) + (\gamma/m)\nu(x) + \epsilon^{\sigma}(x)]^{-1}$. This is the same equation as (8), substituting γ by γ/m. All the comparative statics and welfare results for small F that we derived earlier will therefore go through substituting γ by γ/m. In addition, this model allows us to derive results for the limiting case in which the upstream market is almost competitive while the downstream market has substantial barriers to entry in the form of a high F.

Proposition 6. Suppose that there is sufficient competition in the upstream market, i.e., m is large. Then a fully vertically integrated market produces higher total output, higher output per downstream firm, lower variety, and lower price than a vertically separated market does. Furthermore, the vertically integrated market is Pareto better than the vertically separated market if there is increasing preference for variety, i.e., $\epsilon^{\nu}(x) > 0$.

Proof. See the Appendix.

The intuition for this proposition is fairly simple. For slight deviations from perfect competition upstream, double marginalization leads to a positive total output effect that is proportional to $\nu(x)$. Furthermore, at the limit as $m \to \infty$, the disintegrated market and the integrated market produce excess variety if and only if $k(x) > 0$. Any increase in market power upstream will lead vertically integrated firms to limit entry, which has a positive effect if and only if there is excess variety in the limit as $m \to \infty$. The welfare benefit from this effect is the same multiple of $k(x)$ as the output effect is of $\nu(x)$. Hence, the total effect is proportionate to $k(x) + \nu(x) = \epsilon^{\nu}(x)$.

Note that the above is clearly not a full analysis of vertical integration with upstream oligopoly. However, for a more complete analysis we would have to study asymmetric market structures, which is beyond the scope of this article. It should, however, be clear that the effects should be similar in such an analysis. With an upstream Cournot market, integration should, at least for small F, lead to an increase in total output of that firm and of overall output. But there will also be a decrease in variety of that firm, which would probably not be compensated by more variety in other firms. We therefore conjecture that the welfare effects of vertical integration would still essentially be driven by excess variety at the downstream level.

□ **Regulating the structure of access prices.** Another limitation of our approach is that we have only compared completely disintegrated structures with fully vertically integrated markets. The problem with analyzing partial vertical integration is that there are many ways of slightly moving in the direction of vertical integration, each of them capturing a different aspect of policy relevance. One way to think about partial vertical integration is to look at markets in which an upstream monopolist owns a given number of downstream varieties. One could then ask the question whether a limit on ownership of downstream assets can be welfare improving. Since such an analysis again requires the study of asymmetric market configurations, we have not undertaken it here.

However, we can also interpret the difference between vertical integration and disintegration in our model as arising from different regulatory restrictions on permitted two-part tariffs. With complete disintegration the monopolist is restricted to charging a franchise fee of $A = 0$; with complete integration he is allowed to charge a franchise fee A^* that allows the implementation of the full-integration solution. We can then interpret small moves in the highest permitted franchise fee A as "small moves" in the direction of vertical integration (or disintegration). Such an analysis has direct policy relevance for the issue of access pricing. Direct regulation of access prices in network industries like telecommunications is often a formidable task. However, even if the regulator refrains from interfering with access conditions directly, he may want to regulate the structure of access prices. The question is then whether a regulatory body should restrict the monopolistic owner of a network to linear access pricing or allow at least some use of a franchise fee.

We will now show that with increasing preference for variety, there is always scope for Pareto improvements by allowing some fixed element in the access tariff. In particular, a small increase in the franchise fee from an initial two-part tariff $(w(0), 0)$ to $(w(A), A)$, with $A > 0$, will be Pareto improving:

Proposition 7. Suppose there is increasing preference for variety and $[\partial\epsilon^{\sigma}(x)]/\partial x + \sigma'(x) < 0$. Then there exists $A^+ > 0$ such that $S(A^+) > S(0)$ and $CS(A^+) > CS(0)$ for all $A \in [0, A^+)$.

Proof. Consumer surplus is given by $CS(z, x) = G(z) - G'(z)z[1 - \nu]$, which is increasing in z and x. From the proof of Proposition 4 in the Appendix, $z(A)$ and $x(A)$ are strictly increasing at $A = 0$ if $[\partial\epsilon^{\sigma}(x)]/\partial x + \sigma'(x) < 0$. Thus, $CS(A)$ is strictly increasing in A at $A = 0$. Since $\pi(A)$ is strictly increasing in A, $S(A)$ is also increasing at $A = 0$. By continuity of all of these functions in A, the proposition follows. *Q.E.D.*

This result implies for a market like telecommunications that, in the absence of regulation of marginal access prices, the regulatory body should permit a monopolistic network operator to set some positive fixed access charge.

7. Conclusion

■ In this article we have analyzed the link between assumptions on preference for variety, excess entry, and excess variety results in monopolistic competition, and the welfare effects of vertical integration. We have formalized the close relationship between excess-entry results and Pareto-improving vertical integration. The presence of excess entry and, in particular, excess variety in monopolistic competition generates a situation in which there is no tradeoff between eliminating double marginalization and limiting variety from a second-best perspective. In this case, variety reduction through vertical integration simply eliminates excess variety. These gains are partly passed on to consumers in the form of lower prices. Hence there is a large class of preferences for which we obtain Pareto improvements. Our analysis allowed us to reinterpret the negative welfare results of Perry and Groff (1985) as resulting from the fact that CES preferences represent a boundary case between increasing and decreasing preference for variety, for which the downstream market always produces insufficient variety. We have argued that the plausible class of preferences for welfare analysis is one with increasing preference for variety. This leads us to an evaluation of competition policy toward vertical restraints that is much more favorable than that of Perry and Groff (1985). Finally, we observe that Pareto-improving vertical integration is associated with preferences that lead to niche markets. This corresponds well to the positive welfare results of vertical integration obtained for the models of spatial downstream markets analyzed by Mathewson and Winter (1983) and Dixit (1983). Our results can therefore be seen as a formalization of the relationship between excess entry in monopolistic competition and welfare-improving vertical integration conjectured by Perry (1989).

Appendix

■ Proofs of Propositions 2, 3, 4, and 6 follow.

We first derive the elasticities of individual firm demand and industry demand. Since

$$p(z, x) = G'(z)V(x)(1 - v(x)),$$

the elasticity of individual firm demand is given by

$$\sigma(x_i) = -\frac{\partial p(z, x_i)}{\partial x_i}\frac{x_i}{p} = \frac{v(x_i)}{1 - v(x_i)}[1 + k(x_i)]. \tag{A1}$$

Industry demand is given by $P(n, x) = G'(nxV(x))V(x)(1 - v(x))$. Hence, the elasticity is given by

$$-\frac{\partial P(n, x)}{\partial x}\frac{x}{p} = \sigma(x) + (1 - v(x))\gamma. \tag{A2}$$

Next, we prove the properties of the schedule $\hat{n}(x)$, which is the constraint for the upstream firm's maximization problem in the disintegrated market.

Lemma A1. The schedule $\hat{n}(x)$ is increasing and has elasticity $(\partial\hat{n}/\partial x)(x/n) = [1 - \phi(x)]/\phi(x)$, with $0 < \phi(x) < 1$.

Proof. To see that the schedule schedule $\hat{n}(x)$ is strictly increasing, totally differentiate (3) to obtain

$$P(n, x)\sigma(x)\gamma\frac{x}{n}dn = P(n, x)\sigma(x)[1 - \sigma(x) - (1 - v(x))\gamma + \epsilon^{\sigma}(x)]dx. \tag{A3}$$

This can be reduced to:

$$\frac{d\hat{n}}{dx}\frac{x}{n} = \frac{[1 - \sigma(x) + v(x)\gamma + \epsilon^{\sigma}(x)]}{\gamma} - 1 = \frac{1}{\phi(x)} - 1 = \frac{1 - \phi(x)}{\phi(x)}. \tag{A4}$$

The second part of Assumption 1 implies $\epsilon^{\sigma}(x) > \sigma(x) - v(x)$. Therefore, the expression in square brackets exceeds $\gamma + (1 - \gamma)(1 - v(x))$. This implies $1 > \phi(x) > 0$ and therefore that the schedule is strictly increasing. *Q.E.D.*

To proceed to the proof of Proposition 2 it is necessary to first prove some intermediate results about the behavior of the relevant functions at zero. These are summarized in Lemmas A2–A9. We will use the notation $\xi(0) \equiv \lim_{x\to 0} \xi(x)$ to indicate the limit of a function of x at zero.

Lemma A2. $v(0) = \sigma(0)$.

Proof.

$$v(0) = \lim_{x\to 0}\left(1 - \frac{f'(x)x}{f(x)}\right) = \lim_{x\to 0}\left(1 - \frac{f'(x) + f''(x)x}{f'(x)}\right) = \lim_{x\to 0}\left(-\frac{f''(x)x}{f'(x)}\right) = \sigma(0). \tag{A5}$$

Q.E.D.

Lemma A3. $\lim_{x\to 0} v(x)\epsilon^{v}(x) = 0$.

Proof. From the definition of $\sigma(x)$ we have

$$\epsilon^{v}(x)v(x) = (\sigma(x) - v(x))(1 - v(x)). \tag{A6}$$

Since by Lemma A2 $v(0) = \sigma(0)$, the claim follows. *Q.E.D.*

Note that Lemma A3 states in particular that $v(0) > 0 \Rightarrow \epsilon^{v}(0) = 0$ and, conversely,

$$\epsilon^{\nu}(0) = \lim_{x\to 0} \sigma(x)/\nu(x) - 1 > 0 \Rightarrow \nu(0) = 0.$$

Furthermore, it follows that $\epsilon^{\nu}(0) \geq 0$, since for $\nu(0) = 0$, we must have $\nu'(0) \geq 0$, given that $\nu(x) \geq 0$.

Lemma A4. Suppose that $\xi(x)$ is a smooth function on $(0, \infty)$ continuous at $x = 0$ such that there exists $\infty > \rho > 0$ and $\infty > \kappa > 0$ such that $\lim_{x\to 0} \xi(x)/x^{\rho} = \kappa$. Then, $\rho = \epsilon^{\xi}(0)$.

Proof. By L'Hôpital's rule we have

$$\kappa = \lim_{x\to 0} \frac{\xi(x)}{x^{\rho}} = \lim_{x\to 0} \frac{\dfrac{\xi'(x)x}{\xi(x)}}{\rho} \frac{\xi(x)}{x^{\rho}} = \lim_{x\to 0} \frac{\epsilon^{\xi}(x)}{\rho}\cdot\kappa. \tag{A7}$$

Hence $\lim_{x\to 0} [\epsilon^{\xi}(x)]/\rho = 1$ or $\rho = \epsilon^{\xi}(0)$. *Q.E.D.*

The next lemma is a subcase of Lemma A4, but for the following arguments it is useful to state it explicitly. Let $\rho \in (0, \infty)$ be such that $\lim_{x\to 0} [\nu(x) - \nu(0)]/x^{\rho} = \kappa \in (0, \infty)$. By Assumption 2, such a ρ exists. We then have the following property:

Lemma A5. $\lim_{x\to 0} [\epsilon^{\nu}(x) + \{[\partial\epsilon^{\nu}(x)]/\partial x\}\{x/[\epsilon^{\nu}(x)]\}] = \rho$. If $\epsilon^{\nu}(0) > 0$, then $\epsilon^{\nu}(0) = \rho$.

Proof. By L'Hôpital's rule,

$$\kappa = \lim_{x\to 0} \frac{\nu(x) - \nu(0)}{x^{\rho}} = \lim_{x\to 0} \frac{\nu(x)\epsilon^{\nu}(x)}{\rho x^{\rho}}. \tag{A8}$$

Since by Lemma A3 $\lim_{x\to 0} \nu(x)\epsilon^{\nu}(x) = 0$, we can apply L'Hôpital's rule again to obtain

$$\kappa = \lim_{x\to 0} \frac{\nu(x)\epsilon^{\nu}(x)}{\rho x^{\rho}}\cdot\frac{1}{\rho}\left[\epsilon^{\nu}(x) + \frac{\partial\epsilon^{\nu}(x)}{\partial x}\frac{x}{\epsilon^{\nu}(x)}\right]. \tag{A9}$$

Since, by definition, $\kappa = \lim_{x\to 0} [\nu(x) - \nu(0)]/x^{\rho}$, the first part of the lemma follows directly from (A8). To show the second part, note that for $\epsilon^{\nu}(0) > 0$ we have $\nu(0) = 0$ and hence

$$\kappa = \lim_{x\to 0} \frac{\nu(x)\epsilon^{\nu}(x)}{\rho x^{\rho}} = \epsilon^{\nu}(0)\cdot\lim_{x\to 0} \frac{\nu(x)}{\rho x^{\rho}} = \epsilon^{\nu}(0) \lim_{x\to 0} \frac{1}{\rho}\frac{\nu(x)\epsilon^{\nu}(x)}{\rho x^{\rho}} = \epsilon^{\nu}(0)\frac{\kappa}{\rho}, \tag{A10}$$

which proves the second part of the lemma. *Q.E.D.*

Lemma A6. $\epsilon^{\sigma}(0) = \epsilon^{\nu}(0)$.

Proof. By differentiating

$$\sigma(x) = \nu(x)\left(1 + \frac{\epsilon^{\nu}(x)}{1 - \nu(x)}\right) \tag{A11}$$

and multiplying both sides by $x/[\sigma(x)]$ we obtain

$$\begin{aligned} \epsilon^{\sigma}(x) &= \epsilon^{\nu}(x) + \frac{\epsilon^{\nu}(x)}{1 - \nu(x) + \epsilon^{\nu}(x)}\left(\frac{\partial\epsilon^{\nu}(x)}{\partial x}\cdot\frac{x}{\epsilon^{\nu}(x)} + \frac{\epsilon^{\nu}(x)\nu(x)}{1 - \nu(x)}\right) \\ &= \epsilon^{\nu}(x) + \frac{1}{\dfrac{1 - \nu(x)}{\epsilon^{\nu}(x)} + 1}\left(\frac{\partial\epsilon^{\nu}(x)}{\partial x}\cdot\frac{x}{\epsilon^{\nu}(x)} + \epsilon^{\nu}(x) - \frac{\epsilon^{\nu}(x)}{1 - \nu(x)}\right). \end{aligned} \tag{A12}$$

Suppose $\nu(0) < 1$. If $\epsilon^{\nu}(0) = 0$, then taking limits on both sides we note that the term in brackets goes to ρ by Lemma A5 and the term premultiplying the brackets goes to zero. If $\epsilon^{\nu}(0) > 0$, then the term premultiplying the brackets is positive and finite. Since in this case $\nu(0) = 0$ by Lemma A3, the term in brackets converges to $\rho - \rho = 0$ by Lemma A5. It remains to be shown that the proposition holds for $\nu(0) = 1$. Since $\epsilon^{\nu}(0) = 0$ by Lemma A3, we can use L'Hôpital's rule to obtain

$$\lim_{x\to 0} \frac{\epsilon^{v}(x)}{1 - v(x)} = \lim_{x\to 0} -\frac{\dfrac{\partial \epsilon^{v}(x)}{\partial x}}{v'(x)} = \lim_{x\to 0} \frac{\dfrac{\partial \epsilon^{v}(x)}{\partial x} \cdot \dfrac{x}{\epsilon^{v}(x)}}{v(x)} = \rho, \tag{A13}$$

where the last equality follows from Lemma A5. Hence, the term in brackets in (A12) converges again to $\rho - \rho = 0$, and the term premultiplying it is positive and finite. Hence, we have proved the lemma. *Q.E.D.*

Lemma A7. $\lim_{x\to 0} [(\sigma'(x) - \gamma v'(x))x]/[\sigma(x) - \gamma v(x)] = \epsilon^{v}(0)$.

Proof.

$$\lim_{x\to 0} \frac{(\sigma'(x) - \gamma v'(x))x}{\sigma(x) - \gamma v(x)} = \lim_{x\to 0} \left\{ \epsilon^{\sigma}(x) \frac{\sigma(x)}{\sigma(x) - \gamma v(x)} - \epsilon^{v}(x) \frac{\gamma v(x)}{\sigma(x) - \gamma v(x)} \right\} = \lim_{x\to 0} \epsilon^{v}(x), \tag{A14}$$

where the last equality follows from Lemma A6 and the fact that $\epsilon^{v}(0) \geq 0$ implies $0 \leq \lim_{x\to 0} [v(x)]/[\sigma(x)] \leq 1$. *Q.E.D.*

Lemma A8. $\lim_{F\to 0} [x^*(F)]/[\hat{x}(F)] > 1$.

Proof. First note that $\lim_{F\to 0} x^*(F) = \lim_{F\to 0} \hat{x}(F) = 0$. Define $\psi^*(x^*) \equiv 1 - \gamma - \sigma(x^*) + \gamma v(x^*)$ and $\hat{\psi}(\hat{x}) \equiv 1 - \gamma - \sigma(\hat{x}) + \gamma v(\hat{x})\phi(\hat{x})$. Taking the ratio of (8) and (10) we obtain

$$\frac{\hat{x}}{x^*} = \frac{\sigma(x^*) - \gamma v(x^*)}{\sigma(\hat{x})} \frac{\hat{\psi}(\hat{x})}{\psi^*(x^*)}. \tag{A15}$$

Taking $[\sigma(x^*)]/[\sigma(\hat{x})]$ out of the first expression on the right-hand side, premultiplying both sides in (A15) by $([\hat{x}(F)]/[x^*(F)])^{\epsilon^{v}(0)}$, and taking limits, we obtain

$$\begin{aligned} \lim_{F\to 0} \left(\frac{\hat{x}}{x^*}\right)^{1+\epsilon^{v}(0)} &= \lim_{F\to 0} \left[\left(\frac{\hat{x}(F)}{x^*(F)}\right)^{\epsilon^{v}(0)} \cdot \frac{\sigma(x^*(F))}{\sigma(\hat{x}(F))} \right] \lim_{F\to 0} \left(1 - \gamma \frac{v(x^*(F))}{\sigma(x^*(F))}\right) \lim_{F\to 0} \left(\frac{\hat{\psi}(\hat{x})}{\psi^*(x^*)}\right) \\ &\leq 1 - \gamma \lim_{F\to 0} \left(\frac{v(x^*(F))}{\sigma(x^*(F))}\right) < 1. \end{aligned} \tag{A16}$$

To see that the first inequality holds, consider the first and the last term in the second line of (A16). By Assumption 2 and Lemma A4, there exists κ such that $\lim_{x\to 0} [\sigma(x)]/[x^{\epsilon^{v}(0)}] = \kappa$, where $\kappa = v(0)$ if $v(0) > 0$ (which implies $\epsilon^{v}(0) = 0$, by Lemma A3). Hence, the first term converges to one. Considering the last term in the second line, we observe $\lim_{F\to 0} ([\hat{\psi}(\hat{x})]/[\psi^*(x^*)]) \leq 1$, and the first inequality follows. The last inequality holds because $\lim_{x\to 0} [v(x)]/[\sigma(x)] = 1/[1 + \epsilon^{v}(0)] \leq 1$ by Lemma A3. Hence, $\lim_{F\to 0} (\hat{x}/x^*) < 1$. *Q.E.D.*

Lemma A9. $\lim_{F\to 0} (\partial x^*/\partial F)(F/x^*) = \lim_{F\to 0} (\partial \hat{x}/\partial F)(F/\hat{x}) = 1/[1 + \epsilon^{v}(0)]$.

Proof. Totally differentiating (10) in x^* and F, we obtain

$$\lim_{F\to 0} \frac{\partial x^*}{\partial F} \frac{F}{x^*} = \lim_{F\to 0} \left\{ 1 + \epsilon^{\sigma-\gamma v}(x^*) - \frac{\psi^{*\prime}(x^*)x^*}{\psi^*(x)} \right\}^{-1} = \{1 + \epsilon^{v}(0)\}^{-1}, \tag{A17}$$

where the last equality holds because $\lim_{F\to 0} \epsilon^{\sigma-\gamma v}(x^*) = \epsilon^{v}(0)$ by Lemma A7 and because

$$\psi^{*\prime}(x^*)x^* = -\epsilon^{\sigma-\gamma v}(x^*)[\sigma(x^*) - \gamma v(x^*)],$$

which by Lemmas A2, A3, and A7 goes to zero as $F \to 0$, since $x^*(F)$ tends to zero. Totally differentiating (8) yields

$$\lim_{F\to 0} \frac{\partial \hat{x}}{\partial F} \frac{F}{\hat{x}} = \lim_{F\to 0} \left\{ 1 + \epsilon^{\sigma}(\hat{x}) - \frac{\hat{\psi}'(\hat{x})\hat{x}}{\hat{\psi}(\hat{x})} \right\}^{-1} = \{1 + \epsilon^{v}(0)\}^{-1}, \tag{A18}$$

where the result again follows from Lemmas A2, A3, and A7 *Q.E.D.*

Proof of Proposition 2. Taking the ratio of the equilibrium conditions for $\hat{n}$ and n^*, i.e., the ratio of (3) and (9), and using the fact that $p(z, x) = z^{-\gamma}V(x)[1 - v(x)]$, $z = QV(x)$, we can write the ratios of levels of the endogenous variables as

$$\left(\frac{\hat{Q}}{Q^*}\right)^{\gamma} = \left(\frac{x^*}{\hat{x}}\right)^{\nu(0)(1-\gamma)}\left[\frac{V(\hat{x})\hat{x}^{\nu(0)}}{V(x^*)(x^*)^{\nu(0)}}\right]^{(1-\gamma)}\frac{1-\nu(\hat{x})}{1-\nu(x^*)}\cdot\frac{\hat{\psi}(\hat{x})}{\psi^*(x^*)} \tag{A19}$$

$$\left(\frac{\hat{n}}{n^*}\right)^{\gamma} = \left(\frac{x^*}{\hat{x}}\right)^{\gamma+\nu(0)(1-\gamma)}\left[\frac{V(\hat{x})\hat{x}^{\nu(0)}}{V(x^*)(x^*)^{\nu(0)}}\right]^{(1-\gamma)}\frac{1-\nu(\hat{x})}{1-\nu(x^*)}\cdot\frac{\hat{\psi}(\hat{x})}{\psi^*(x^*)} \tag{A20}$$

$$\frac{\hat{P}}{P^*} = \frac{\psi^*(x^*)}{\hat{\psi}(\hat{x})}, \tag{A21}$$

where (A19), (A20), and (A21) represent the same equation.

Consider first the case $\nu(0) > 0$. Then by the first part of Assumption 2 and by Lemma A4,

$$\lim_{x\to 0} V(x)x^{\nu(0)} = \lim_{x\to 0}\,[f(x)]/[x^{1-\nu(0)}] = \lim_{x\to 0}\,[f(x)]/[x^{\epsilon^f}(0)] = \kappa > 0.$$

Since $\lim_{F\to 0} x^*(F) = \lim_{F\to 0} \hat{x}(F) = 0$, it follows that the term in square brackets in (A19) and (A20) converges to one and hence, taking logs and then limits on both sides of (A19), we obtain

$$\begin{aligned}\lim_{F\to 0}\left\{\ln\left(\frac{\hat{Q}}{Q^*}\right)^{\gamma}\right\} &= \nu(0)(1-\gamma)\ln\left(\frac{1}{1-\gamma}\lim_{F\to 0}\frac{\psi^*(x^*)}{\hat{\psi}(\hat{x})}\right) + \ln\left(\lim_{F\to 0}\frac{\hat{\psi}(\hat{x})}{\psi^*(x^*)}\right)\\ &= \nu(0)(1-\gamma)\ln\left(\frac{1}{1-\gamma}\right) + (1-\nu(0)(1-\gamma))\ln\left(\frac{1-\nu(0)}{1-(1-\gamma)\nu(0)}\right),\end{aligned} \tag{A22}$$

where the second equality follows from the fact that $\lim_{F\to 0}\,[\hat{\psi}(\hat{x})]/[\psi^*(x^*)] = [1-\nu(0)]/[1-(1-\gamma)\nu(0)]$. We will now show that the right-hand side of (A22) is strictly increasing in $(1-\gamma)$ and zero at $\gamma = 0$, proving that the ratio $\hat{Q}/Q^*$ is smaller than one in the limit as $F \to 0$. Differentiating with respect to $(1-\gamma)$ yields

$$\frac{\partial}{\partial(1-\gamma)}\lim_{F\to 0}\left\{\ln\left(\frac{\hat{Q}}{Q^*}\right)^{\gamma}\right\} = \nu(0)\ln\left(\frac{1}{1-\gamma}\right) - \nu(0)\ln\left(\frac{1-\nu(0)}{1-(1-\gamma)\nu(0)}\right) + \nu(0)(1-\nu(0)) > 0 \tag{A23}$$

for all $\gamma > 0$. Furthermore, $\lim_{\gamma\to 0}\lim_{F\to 0}\ln(\hat{Q}/Q^*)^{\gamma} = 0$ from (A22). Hence, $\lim_{F\to 0}(\hat{Q}/Q^*)^{\gamma} < 1$ for all $\gamma > 0$. The result for n is proved similarly by noting that

$$\lim_{F\to 0}\left(\frac{\hat{n}}{n^*}\right)^{\gamma} = \left(\frac{1}{1-\gamma}\right)^{\gamma+\nu(0)(1-\gamma)}\left(\frac{1-\nu(0)}{1-(1-\gamma)\nu(0)}\right)^{1-\gamma-\nu(0)(1-\gamma)}, \tag{A24}$$

$\lim_{\gamma\to 0}\{\lim_{F\to 0}(\hat{n}/n^*)\} = 1$, and $\partial/\partial\gamma\{\lim_{F\to 0}(\hat{n}/n^*)^{\gamma}\} > 0$. Since P falls in x and in Q, it follows directly that $P^* < \hat{P}$.

Now consider the case $\nu(0) = 0$. Since $\lim_{F\to 0}\,[\hat{\psi}(\hat{x})]/[\psi^*(x^*)] = 1$ in this case, we have

$$\lim_{F\to 0}\hat{Q}/Q^* = \lim_{F\to 0}\hat{P}/P^* = 1 < \lim_{F\to 0}\hat{n}/n^*.$$

Since P falls in x and in Q, we only have to prove that $\hat{Q}/Q^* < 1$ for $F > 0$ but small.

$$\ln\frac{Q^*}{\hat{Q}} = \frac{\ln\dfrac{[\psi^*(x^*)V(x^*)^{1-\gamma}(1-\nu(x^*))]}{[\hat{\psi}(\hat{x})V(\hat{x})^{1-\gamma}(1-\nu(\hat{x}))]}}{\gamma}$$

$$\begin{aligned}\frac{\partial}{\partial F}\left\{\ln\frac{Q^*}{\hat{Q}}\right\} &= \frac{1}{\gamma}\left\{[\epsilon^{\psi^*}(x^*) - \sigma(x^*) + \gamma\nu(x^*)]\frac{dx^*}{dF}\frac{1}{x^*} - [\epsilon^{\hat{\psi}}(\hat{x}) - \sigma(\hat{x}) + \gamma\nu(\hat{x})]\frac{d\hat{x}}{dF}\frac{1}{\hat{x}}\right\}\\ &= \frac{1}{\gamma}\left\{-\left[\frac{\epsilon^{\sigma-\gamma\nu}(x^*) + \psi^*(x^*)}{1+\epsilon^{\sigma-\gamma\nu}(x^*) - \epsilon^{\psi^*}(x^*)}\right]\frac{1}{x^*}\right.\\ &\left.\quad + \left[\frac{\epsilon^{\sigma-\gamma\nu}(\hat{x}) + \hat{\psi}(\hat{x})}{1+\epsilon^{\sigma-\gamma\nu}(\hat{x}) - \epsilon^{\hat{\psi}}(\hat{x})} + \gamma\frac{[1-\phi(x)]\dfrac{\nu(\hat{x})}{\sigma(\hat{x})-\gamma\nu(\hat{x})}\epsilon^{\nu}(\hat{x}) - \phi'(\hat{x})\hat{x}\dfrac{\nu(\hat{x})}{\sigma(\hat{x})-\gamma\nu(\hat{x})}\epsilon^{\nu}(\hat{x})}{1+\epsilon^{\sigma-\gamma\nu}(\hat{x}) - \epsilon^{\hat{\psi}}(\hat{x})}\right]\frac{1}{\hat{x}}\right\}.\end{aligned} \tag{A25}$$

Premultiplying by $F^{1/1+\epsilon^v(0)}$ on both sides and taking limits on both sides yields

$$\lim_{F\to 0} F^{1/[1+\epsilon^v(0)]}\frac{\partial}{\partial F}\left\{\ln\frac{Q^*}{\hat{Q}}\right\} = \frac{1}{\gamma[1+\epsilon^v(0)]}\left\{[\epsilon^v(0)+1-\gamma]\left[\frac{1}{\hat{a}}-\frac{1}{a^*}\right]+\frac{\gamma}{1+\epsilon^v(0)}\epsilon^v(0)\frac{1}{\hat{a}}\right\} > 0, \quad \text{(A26)}$$

where $\hat{a} \equiv \lim_{F\to 0}\hat{x}(F)F^{-1/[1+\epsilon^v(0)]}$ and $a^* \equiv \lim_{F\to 0} x^*(F)F^{-1/[1+\epsilon^v(0)]}$ and the inequality follows because $\hat{a} < a^*$ by Lemma A8. *Q.E.D.*

Proof of Proposition 3. Let $C(x) = c + F/x$ and $P(Q, x) = p(QV(x), x)$. We can therefore write the maximization problem of the vertically integrated monopolist as

$$\max_{Q,x} \Pi(Q, x) = [P(Q, x) - C(x)]Q \quad \text{(A27)}$$

with first-order conditions

$$P(Q^*, x^*)[1-\gamma] - C(x^*) = 0 \qquad P_x(Q^*, x^*) - C'(x^*) = 0. \quad \text{(A28)}$$

On the other hand, it is straightforward to show that at the vertically disintegrated solution with upstream monopoly, we have

$$P(\hat{Q}, \hat{x}) - C(\hat{x}) = w - c \qquad P_x(\hat{Q}, \hat{x}) - C'(\hat{x}) = \frac{1}{\hat{x}}P(\hat{Q}, \hat{x})v(\hat{x})\gamma. \quad \text{(A29)}$$

Hence, at the vertically disintegrated solution, $P_x(\hat{Q}, \hat{x}) - C'(\hat{x}) > 0$. Furthermore, it is straightforward to show that $P_{xx}(\hat{Q}, \hat{x}) - C''(\hat{x}) < 0$ whenever $P_x(\hat{Q}, \hat{x}) - C'(\hat{x}) > 0$. We can now separate the move from vertical disintegration to vertical integration into two steps. First, we fix $\hat{Q}$ and increase x until we reach the point $P_x(\hat{Q}, x^*(\hat{Q}) - C'(x^*(\hat{Q}) = 0$, which is the optimal choice of the upstream monopolist given production $\hat{Q}$. By assumption we know that $Q^* \geq \hat{Q}$. Therefore, in a second step we increase Q along the schedule implicitly defined by $P_x(Q, x) - C'(x) = 0$ until Q^* is reached. Since $P_{xQ}(Q, x) > 0$, output per firm, x, is increasing along this schedule as well. Define consumer surplus as $\widehat{CS}(Q, x) = CS(Q/x, x)$. The total change in consumer surplus is therefore given by

$$\Delta CS = \int_{\hat{x}}^{x^*(\hat{Q})} \widehat{CS}_x(\hat{Q}, x)\, dx + \int_{\hat{Q}}^{Q^*}\left[\widehat{CS}_Q(Q, x) + \widehat{CS}_x(Q, x)\frac{dx^*(Q)}{dQ}\right] dQ, \quad \text{(A30)}$$

where $[dx^*(Q)]/[dQ] > 0$ denotes the change of x along the schedule defined by $P_x(Q, x) - C'(x) = 0$. Since $CS = S - \Pi = G(QV(x)) - P(Q, x)Q$, equation (A30) can be written as

$$\begin{aligned}\Delta CS = &\int_{\hat{x}}^{x^*(\hat{Q})} \frac{P(\hat{Q}, x)}{x}\hat{Q}\frac{v(x)}{1-v(x)}[k(x) - \gamma(1-v(x))]\, dx \\ &+ \int_{\hat{Q}}^{Q^*} \frac{P(Q, x)}{x}Q\frac{v(x)}{1-v(x)}[k(x) - \gamma(1-v(x))]\frac{dx^*(Q)}{dQ}\, dQ \\ &+ \int_{\hat{Q}}^{Q^*} \{P(Q, x)[(1-v(x))^{-1} - (1-\gamma)\}\, dQ.\end{aligned} \quad \text{(A31)}$$

If $\kappa(x) > \gamma(1 - v(x))$, the first and second term in (A31) is strictly positive. Furthermore, note that $(1 - v(x))^{-1} - (1 - \gamma) > \gamma$ and $P > 0$, so that the last term in (A31) is strictly positive as well.

Let us finally show the last part of the proposition. It is straightforward to show that ρ-concavity of $V(x)$ implies that $\kappa(x) > [1 - (1 - \rho)v(x)]$. Suppose that $V(x)$ is $-(1 - \gamma)$-concave. Then, $\kappa(x) > 1 - (2 - \gamma)v(x)$ and on the relevant range,

$$1 > \sigma(x) = \frac{v(x)}{1-v(x)}[1 + \kappa(x)] > \frac{v(x)}{1-v(x)}[2 - (2-\gamma)v(x)].$$

By this fact,

$$(1 - \nu(x)) > 2\nu(x) - (2 - \gamma)\nu(x)^2$$

or, equivalently,

$$\frac{1}{\nu(x)} > 2 + \gamma\frac{\nu(x)}{1 - \nu(x)}.$$

Hence, $\nu(x) < \frac{1}{2}$. But then

$$k(x) > \gamma(1 - \nu(x)) + (1 - \gamma) - 2(1 - \gamma)\nu(x) = \gamma(1 - \nu(x)) + (1 - \gamma)(1 - 2\nu(x)) > \gamma(1 - \nu(x)),$$

which proves the proposition. *Q.E.D.*

To prove Propositions 4 and 7 we slightly generalize the problem in the vertically separated market so that the monopolist can charge a two-part tariff with a maximal franchise fee of A in addition to the wholesale price w. Hence, downstream firms face a two-part tariff (w, A). Then for any A the downstream equilibrium must lie on a locus $p(z, x)\sigma(x)x = F + A$, which determines a modified increasing schedule $\hat{n}(x; A)$, which becomes the constraint for the upstream monopolist's maximization problem. The schedule $\hat{n}(x; A)$ is shifted downward through increases in A, and there exists $A^* > 0$ such that the monopolist can implement the fully integrated solution. The maximal franchise fee A thus defines a family of maximization problems for the upstream monopolist and the downstream retailers between the fully disintegrated outcome and the fully integrated outcome.

Let $\tilde{z}(x, A) \equiv \hat{n}(x; A)V(x)x$ and define the first-order condition in (5) as $\alpha(\tilde{z}(x, A), x) = 0$ for all A. We will first determine the changes in x and z as A is increased.

First we have

$$\frac{dx}{dA} = -\frac{\alpha_z}{\alpha_x + \alpha_z\dfrac{\partial\tilde{z}}{\partial x}} \cdot \frac{\partial\tilde{z}}{\partial A}, \tag{A34}$$

where the denominator is negative since we are at an optimal choice of x. Since $\hat{n}(x; A)$ is increasing in A, we have $\partial\tilde{z}/\partial A < 0$. Therefore, sgn $dx/dA = -$ sgn α_z. Straightforward differentiation yields

$$\alpha_z z = -\gamma p(z, x)\mu(x, z) < 0, \tag{A35}$$

where $\mu(x, z) = [(1 - \gamma)(1 + \epsilon^\sigma - \sigma) - \gamma(\sigma - \nu)]$. To see that $\mu(x, z) > 0$, consider two cases. If there is decreasing preference for variety, $\nu > \sigma$ and the claim follows directly from $\epsilon^\sigma > \sigma - \nu$. If there is increasing preference for variety, rewrite $\mu(x, z)$ as $(1 - \gamma)\epsilon^\sigma + (1 - \gamma)(1 - \sigma) - \gamma(\sigma - \nu)$. This consolidates to $(1 - \gamma)\epsilon^\sigma + [1 - \sigma - \gamma(1 - \nu)]$. In equilibrium the second term in brackets must be positive, while the first term in brackets exceeds $\sigma - \nu$, which exceeds zero with increasing preference for variety.

Secondly, the slope of $z(A)$ has the same sign as α_x, since

$$\frac{dz}{dA} = -\frac{\alpha_x}{\alpha_z}\frac{dx}{dA}. \tag{A36}$$

Differentiating α and substituting in the downstream market constraint, we have

$$\alpha_x x^2 = -[1 - \gamma - \sigma(x) + \nu(x)\gamma]A + \gamma\nu(x)\epsilon^\nu(x)\frac{\Pi(n, x)}{n} + \delta(n, x), \tag{A37}$$

where $\delta(n, x) = x\Pi_n(n, x)[[\partial\epsilon^\sigma(x)]/\partial x + \sigma'(x)]$. With this in place we can now prove Propositions 4 and 7.

Proof of Proposition 4. To establish the proof of Proposition 4, note that the assumptions in the proposition imply $\delta \geq 0$ and $\epsilon^\nu(x) \leq 0$. Hence, $\alpha_x < 0$ for all $A \geq 0$ and therefore $z(A)$ is monotonically decreasing on $[0, A^*]$. To establish the welfare result, first note that arg $\max_z \hat{S}(z, x(0)) > z(0) > z(A^*)$ and $x^0 < x(0) < x(A^*)$. Divide the move from vertical disintegration to vertical integration into two steps. First keep z constant at $z(0)$. Then, increasing x from $x(0)$ to $x(A^*)$ reduces welfare by concavity of $\hat{S}(z, x)$ in x. Now fix x at $x(A^*)$. Then, decreasing z from $z(0)$ to $z(A^*)$ reduces welfare by concavity of $\hat{S}(z, x)$ in z. Hence, welfare is lower under vertical integration than under vertical disintegration. *Q.E.D.*

Proof of Proposition 6. To prove Proposition 6 we first fully specify the model of vertical disintegration. First, each upstream firm sets its output Q_i. We assume that each downstream firm can only buy the input from a single upstream manufacturer. Then downstream firms, taking the upstream prices w_i, $i = 1, \ldots, m$, as given, play a monopolistically competitive equilibrium, choosing which manufacturer's retailer to enter as and choosing the optimal output given w_i. The input price w_i is set to clear the ith input market. Since a retailer decides to enter as the retailer for a specific upstream seller, there is a monopolistically competitive market for each upstream seller in which the equilibrium is symmetric and satisfies

$$\left[p_i\left(Q_iV(x_i) + \sum_{j \neq i} Q_jV(x_j), x_i\right) - w_i\right]x_i = F \quad \text{and} \tag{A38}$$

$$p_i - w_i = \sigma(x_i)p_i. \tag{A39}$$

The downstream constraint that the disintegrated manufacturer faces is given by

$$p_i\sigma(x_i)x_i = F. \tag{A40}$$

By symmetry of the downstream equilibrium, each downstream firm contracting with manufacturer i in equilibrium sells the same output x_i. Since downstream firms earn zero profits in equilibrium, total upstream profits are given by

$$(w_i(Q_i) - c)Q_i = \left(P_i(Q_i, x_i) - c - \frac{F}{x_i}\right)Q_i, \tag{A41}$$

where $P_i(Q_i, x_i) = p(\sum_j Q_jV(x_j), x_i)$. Upstream firms are facing the inverse demand function

$$w(Q_i) = P(Q_i, x(Q_i)) - F/[x(Q_i)],$$

where $x(Q_i)$ is determined by the downstream equilibrium locus $P_i(Q_i, x_i)\sigma(x_i)x_i = F$. Each upstream firm therefore maximizes $(P(Q_i, x(Q_i)) - c - F/[x(Q_i)])Q_i$ with respect to Q_i. Maximizing this with respect to Q_i under constraint (A40) generates the condition on $\hat{x}_i$ in the text. The equations for the integrated manufacturers are generated from symmetric solutions of maximizing profits with respect to Q_i and n_i and rearranging in the same way as in the monopoly case.

To simplify notation we use the change of variable $\mu = 1/m$, so that $\mu \to 0$ is equivalent to $m \to \infty$. Let $\hat{\psi}(\hat{x}) = 1 - \mu\gamma - \sigma(\hat{x}) + \mu\gamma v(\hat{x})\phi_m(\hat{x})$ and $\psi^*(x^*) = 1 - \mu\gamma - \sigma(x^*) + \mu\gamma v(x^*)$. As in the case of small F, it is immediate that $\lim_{\mu\to 0} x^* = \lim_{\mu \to 0} \hat{x} = \bar{x}$. Since $\psi^*(x^*)$ and $\hat{\psi}(\hat{x})$ converge to the same number $(1 - \sigma(\bar{x}))$ for $\mu \to 0$, it is immediate that $\lim_{\mu\to 0} \ln(Q^*/\hat{Q}) = 0$ and allocations exactly coincide in the competitive limit. To analyze what happens off the limit totally differentiate (23) and (21) with respect to x and μ to obtain

$$\frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}} = \frac{\dfrac{\partial\hat{\psi}(\hat{x})}{\partial\mu}}{\hat{\psi}(\hat{x})}\frac{1}{1 + \epsilon^{\sigma} - \epsilon^{\hat{\psi}}} \quad \text{and} \tag{A42}$$

$$\frac{dx^*}{d\mu}\frac{1}{x^*} = \left[\frac{\gamma v(x^*)}{\sigma(x^*) - \mu\gamma v(x^*)} + \frac{\dfrac{\partial\psi^*}{\partial\mu}}{\psi^*(x^*)}\right]\frac{1}{1 + \epsilon^{\sigma} - \epsilon^{\psi^*}}, \tag{A43}$$

where $\lim_{\mu\to 0} \epsilon^{\hat{\psi}} = \lim_{\mu\to 0} \epsilon^{\psi^*} = -[\sigma(\bar{x})]/[1 - \sigma(\bar{x})]\epsilon^{\sigma}(\bar{x})$. Hence,

$$\begin{aligned}\lim_{\mu\to 0}\left[\frac{dx^*}{d\mu}\frac{1}{x^*} - \frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}}\right] &= \frac{1 - \sigma(\bar{x})}{1 - \sigma(\bar{x}) + \epsilon^{\sigma}(\bar{x})}\lim_{\mu\to 0}\left[\frac{\gamma v(x^*)}{\sigma(x^*) - \mu\gamma v(x^*)} + \frac{\dfrac{\partial\psi^*}{\partial\mu}}{\psi^*(x^*)} - \frac{\dfrac{\partial\hat{\psi}(\hat{x})}{\partial\mu}}{\hat{\psi}(\hat{x})}\right] \\ &= \gamma\frac{v(\bar{x})}{1 - \sigma(\bar{x}) + \epsilon^{\sigma}(\bar{x})}\frac{1}{\sigma(\bar{x})} > 0.\end{aligned} \tag{A44}$$

The log ratio of total output changes in μ as

$$\frac{\partial \ln\left(\dfrac{Q^*}{\hat{Q}}\right)}{\partial \mu} = \frac{1}{\gamma}\left\{[\epsilon^{\psi^*} - \sigma + \gamma\nu]\frac{dx^*}{d\mu}\frac{1}{\mu} - [\epsilon^{\hat{\psi}} - \sigma + \gamma\nu]\frac{d\hat{x}}{d\mu}\frac{1}{\mu}\right\} + \frac{1}{\gamma}\left\{\frac{\dfrac{\partial\psi^*(x^*)}{\partial\mu}}{\psi^*(x^*)} - \frac{\dfrac{\partial\hat{\psi}(\hat{x})}{\partial\mu}}{\hat{\psi}(\hat{x})}\right\}, \tag{A45}$$

and hence

$$\begin{aligned}\lim_{\mu\to 0}\frac{\partial \ln\left(\dfrac{Q^*}{\hat{Q}}\right)}{\partial \mu} &= -\frac{1}{\gamma}\left[\frac{\sigma[1-\sigma+\epsilon^\sigma] - (1-\sigma)\gamma\nu}{1-\sigma(\overline{x})}\right]\lim_{\mu\to 0}\left[\frac{dx^*}{d\mu}\frac{1}{x^*} - \frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}}\right] - \frac{1}{\gamma}\frac{\gamma\nu(\overline{x})}{1-\sigma(\overline{x})}\\ &= -\left[\frac{\sigma[1-\sigma+\epsilon^\sigma] - (1-\sigma)\gamma\nu}{1-\sigma(\overline{x})}\right]\frac{\nu(\overline{x})}{1-\sigma(\overline{x})+\epsilon^\sigma(\overline{x})}\frac{1}{\sigma(\overline{x})} + \frac{\nu(\overline{x})}{1-\sigma(\overline{x})}\\ &= \gamma\nu(\overline{x})\frac{\nu(\overline{x})}{1-\sigma(\overline{x})+\epsilon^\sigma(\overline{x})}\frac{1}{\sigma(\overline{x})} > 0.\end{aligned} \tag{A46}$$

Furthermore, since

$$\frac{\partial \ln\left(\dfrac{n^*}{\hat{n}}\right)}{\partial\mu} = \frac{\partial \ln\left(\dfrac{Q^*}{\hat{Q}}\right)}{\partial\mu} - \frac{\partial \ln\left(\dfrac{x^*}{\hat{x}}\right)}{\partial\mu}, \tag{A47}$$

we have

$$\lim_{\mu\to 0}\frac{\partial \ln\left(\dfrac{n^*}{\hat{n}}\right)}{\partial\mu} = -(1-\gamma)\nu(\overline{x})\frac{\nu(\overline{x})}{1-\sigma(\overline{x})+\epsilon^\sigma(\overline{x})}\frac{1}{\sigma(\overline{x})} < 0. \tag{A48}$$

To complete the comparative statics of vertical integration, we obtain

$$\begin{aligned}\lim_{\mu\to 0}\frac{\partial \ln\dfrac{P^*}{\hat{P}}}{\partial\mu} &= \lim_{\mu\to 0}\left\{\epsilon^{\hat{\psi}}\frac{d\hat{x}}{d\mu}\frac{1}{\mu} - \epsilon^{\psi^*}\frac{dx^*}{d\mu}\frac{1}{\mu} + \frac{\dfrac{\partial\hat{\psi}(\hat{x})}{\partial\mu}}{\hat{\psi}(\hat{x})} - \frac{\dfrac{\partial\psi^*(x^*)}{\partial\mu}}{\psi^*(x^*)}\right\}\\ &= \frac{\sigma(\overline{x})}{1-\sigma(\overline{x})}\epsilon^\sigma(\overline{x})\lim_{\mu\to 0}\left[\frac{dx^*}{d\mu}\frac{1}{x^*} - \frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}}\right] - \gamma\frac{\nu(\overline{x})}{1-\sigma(\overline{x})}\\ &= \frac{\sigma(\overline{x})}{1-\sigma(\overline{x})}\epsilon^\sigma(\overline{x})\gamma\frac{\nu(\overline{x})}{1-\sigma(\overline{x})+\epsilon^\sigma(\overline{x})}\frac{1}{\sigma(\overline{x})} - \gamma\frac{\nu(\overline{x})}{1-\sigma(\overline{x})} = -\frac{\gamma\nu(\overline{x})}{1-\sigma(\overline{x})+\epsilon^\sigma(\overline{x})} < 0,\end{aligned} \tag{A49}$$

which is strictly smaller than zero.

The welfare result then follows by a simple calculation:

$$\begin{aligned}&\lim_{\mu\to 0}\frac{\partial}{\partial\mu}[S(Q^*, x^*) - S(\hat{Q}, \hat{x})]\\ &= \lim_{\mu\to 0}\left[S_Q(Q^*, x^*)Q^*\frac{\partial Q^*}{\partial\mu}\frac{1}{\mu} - S_Q(\hat{Q}, \hat{x})\hat{Q}\frac{\partial\hat{Q}}{\partial\mu}\frac{1}{\mu} + S_x(Q^*, x^*)x^*\frac{\partial x^*}{\partial\mu}\frac{1}{\mu} - S_x(\hat{Q}, \hat{x})\hat{x}\frac{\partial\hat{x}}{\partial\mu}\frac{1}{\mu}\right]\\ &= S_Q(\overline{Q}, \overline{x})\overline{Q}\lim_{\mu\to 0}\frac{\partial\left(\dfrac{Q^*}{\hat{Q}}\right)}{\partial\mu} + S_x(\overline{Q}, \overline{x})\overline{x}\lim_{\mu\to 0}\left[\frac{dx^*}{d\mu}\frac{1}{x^*} - \frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}}\right]\\ &= \left[G'(\overline{Q}V(\overline{x}))V(\overline{x})\overline{Q} - c\overline{Q} - \frac{F}{x}\overline{Q}\right]\lim_{\mu\to 0}\frac{\partial\left(\dfrac{Q^*}{\hat{Q}}\right)}{\partial\mu} + \left[G'V'\overline{Q}\,\overline{x} + \overline{Q}\frac{F}{\overline{x}}\right]\lim_{\mu\to 0}\left[\frac{dx^*}{d\mu}\frac{1}{x^*} - \frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}}\right]\end{aligned} \tag{A50}$$

$$= G'V(\overline{x})\nu(\overline{x})\overline{Q}\left\{\lim_{\mu\to 0}\frac{\partial\left(\dfrac{Q^*}{\hat{Q}}\right)}{\partial\mu} + k(\overline{x})\lim_{\mu\to 0}\left[\frac{dx^*}{d\mu}\frac{1}{x^*} - \frac{d\hat{x}}{d\mu}\frac{1}{\hat{x}}\right]\right\}$$

$$= G'V(\overline{x})\nu(\overline{x})\overline{Q}\,\gamma\frac{\nu(\overline{x})}{1 - \sigma(\overline{x}) + \epsilon^{\sigma}(\overline{x})}\frac{1}{\sigma(\overline{x})}\{\nu(\overline{x}) + k(\overline{x})\},$$

which will always exceed zero if $k(x) + \nu(\overline{x}) = \epsilon^{\nu}(\overline{x}) > 0$. *Q.E.D.*

References

ANDERSON, S.P., DE PALMA, A., AND NESTEROV, Y. "Oligopolistic Competition and the Optimal Provision of Products." *Econometrica,* Vol. 63 (1995), pp. 1281–1301.

BRESNAHAN, T.F. AND REISS, P.C. "Dealer and Manufacturer Margins." *RAND Journal of Economics,* Vol. 16 (1985), pp. 253–268.

BRU, L. "Vertical Integration and Welfare in Spatial Markets." Mimeo, IAE, Barcelona, 1991.

CAPLIN, A. AND NALEBUFF, B. "Aggregation and Social Choice: A Mean Voter Theorem." *Econometrica,* Vol. 59 (1991), pp. 1–24.

CORCHÓN, L.C. "Monopolistic Competition: Equilibrium and Optimality." *International Journal of Industrial Organization,* Vol. 9 (1991), pp. 441–453.

DIXIT, A., "Vertical Integration in a Monopolistically Competitive Industry." *International Journal of Industrial Organization,* Vol. 1 (1983), pp. 63–78.

——— AND STIGLITZ, J. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review,* Vol. 67 (1977), pp. 297–308.

GALLINI, N.T. AND WINTER, R.A. "On Vertical Control in Monopolistic Competition." *International Journal of Industrial Organization,* Vol. 1 (1983), pp. 275–286.

HUBBARD, J.H. AND WEST, B.H. *Differential Equations: A Dynamical Systems Approach. Part I.* New York: Springer Verlag, 1991.

KIMMEL, S. "Effects of Cost Changes in Oligopolists' Profits." *Journal of Industrial Economics,* Vol. 40 (1992), pp. 441–449.

KOENKER, R.W. AND PERRY, M.K. "Product Differentiation, Monopolistic Competition, and Public Policy." *Bell Journal of Economics,* Vol. 12 (1981), pp. 217–231.

KÜHN, K.-U. "Germany." In E.M. Graham and D. Richardson, eds., *Global Competition Policy.* Washington, D.C.: Institute for International Economics, 1997.

——— AND VIVES, X. "Excess Entry Revisited." Mimeo, IAE, Barcelona, 1996.

MANKIW, N.G. AND WHINSTON, M.D. "Free Entry and Social Inefficiency." *RAND Journal of Economics,* Vol. 17 (1986), pp. 48–58.

MATHEWSON, G.F. AND WINTER, R.A. "Vertical Integration by Contractual Restraints in Spatial Markets." *Journal of Business,* Vol. 56 (1983), pp. 497–517.

——— AND ———. "An Economic Theory of Vertical Restraints." *RAND Journal of Economics,* Vol. 15 (1984), pp. 27–38.

PERRY, M.K. "Vertical Integration: Determinants and Effects." In R. Schmalensee and R. Willing, eds., *Handbook of Industrial Organization, Vol. 1.* Amsterdam: North-Holland, 1989.

——— AND GROFF, R.H. "Resale Price Maintenance and Forward Integration into a Monopolistically Competitive Industry." *Quarterly Journal of Economics,* Vol. 100 (1985), pp. 1294–1311.

QUIRMBACH, H.C. "The Path of Price Changes in Vertical Integration." *Journal of Political Economy,* Vol. 94 (1986), pp. 1110–1119.

RIORDAN, M.H. AND SALOP, S.C. "Evaluating Vertical Mergers: A Post-Chicago Approach." *Antitrust Law Journal,* Vol. 63 (1995), pp. 513–568.

SCHMALENSEE, R. "A Note on the Theory of Vertical Integration." *Journal of Political Economy,* Vol. 81 (1973), pp. 442–449.

SEADE, J. "Profitable Cost Increases and the Shifting of Taxation: Equilibrium Responses of Markets in Oligopoly." University of Warwick Working Paper no. 260, 1987.

SMITH, R.L. "Franchise Regulation: An Economic Analysis of State Restrictions on Automobile Distribution." *Journal of Law and Economics,* Vol. 25 (1982), pp. 125–157.

SPENCE M. "Product Selection, Fixed Costs and Monopolistic Competition." *Review of Economic Studies,* Vol. 43 (1976a), pp. 217–235.

———. "Product Differentiation and Welfare." *American Economic Review,* Vol. 66 (1976b), pp. 407–414.

———. "Nonprice Competition." *American Economic Review,* Vol. 67 (1977), pp. 255–259.

VIVES, X. *Oligopoly Pricing: Old Ideas and New Tools,* Cambridge, Mass.: MIT Press, 1999.

WARREN-BOULTON, F.R. "Vertical Control with Variable Proportions." *Journal of Political Economy,* Vol. 82 (1974), pp. 783–802.

WESTFIELD, F.M. "Vertical Integration: Does Product Price Rise or Fall?" *American Economic Review,* Vol. 71 (1981), pp. 334–346.