9. Price and information dynamics in financial markets

This chapter considers dynamic markets where informed traders use market orders and introduces strategic behavior to analyze the consequences for the informational dynamics of the market and associated trading patterns. It provides the dynamic trading counterpart of the static models presented in Section 4.3 and in Chapter 5.

Some of the questions addressed in the chapter are:

- Can herding arise in a sequential trading market?
- Is learning from past prices (technical analysis) fast or slow?
- What is the role of market makers in the price discovery process?
- Will an insider trade slowly, so as to control the potential information leakage out his trades, or will he try to make a quick "killing"?
- Under what circumstances will an insider have an incentive to dissimulate his trades? When several informed traders compete, will information revelation speed up or slow down?
- In the presence of informed traders will the adverse selection problem faced by market makers improve or worsen as trade proceeds? What are the consequences for the dynamic patterns of market depth, volatility and volume traded?
- Does it make a difference whether informed traders have long or short lived information on the fundamentals? What about if liquidity traders have discretion on when to trade?
- When will a large trader have incentives to manipulate the market or use contrarian strategies? Can a strategic larger trader slow down a price discovery process?
- Do large risk averse traders have incentives to engage in strategic hedging and if so what welfare consequences follow?

Section 9.1 addresses dynamic market order markets and the effect of market microstructure on the informational efficiency of prices. It revisits herding and the slow learning results obtained in Chapters 6 and 7 in the context of a financial market, studies the speed of information revelation in a price discovery mechanism and the role that market makers play. Section 9.2 considers strategic trading with long-lived information and reviews the seminal Kyle (1985) model and extensions. Main issues of interest are the determination of the trading strategy of large informed agents, with possible camouflage and dissimulation strategies, and their impact on market depth, volatility and volume; and the effect of competition among informed traders on the speed of information revelation. Section 9.3 examines market manipulation models and the impact of strategic behavior in the price discovery mechanism presented in Section 9.1.3. The section reviews the market manipulation literature and shows how contrarian behavior by an insider may arise to manipulate the price discovery process. Section 9.4 deals with strategic trading when information is short-lived and liquidity traders can choose when to trade. Section 9.5 studies the dynamic hedging strategies of large risk averse traders with and without private information on the fundamentals.

9.1 Sequential trading, dynamic market order markets, and the speed of learning from past prices

This section studies sequential trade markets where informed traders submit market orders and highlights the role of market makers in information revelation (the speed at which prices incorporate information). We present first the benchmark model of Glosten and Milgrom (1985) and we relate it to the results on herding of Section 6.1. We present afterwards a model of learning from past prices -a variation of the Cournot-type model considered in Section 7.4 (Vives (1993)), and finally a price discovery mechanism (Vives (1995a)).

9.1.1 Sequential trading and herding

Glosten and Milgrom (1985) present a sequential trading model in which competitive risk neutral market makers set a bid-ask spread and earn zero expected profits every period. In their model a single investor arrives each period and trades only once. He always trades one unit and therefore the competitive market maker just sets a bid and an ask price. We present a simplified version of the model with two asset values and two signals values. The investor makes an information-based trade (having received a private binary signal about the liquidation value of the stock θ, which may in turn be high or low) with probability μ and a liquidity-based trade (say, buying, selling, or abstaining with equal probability at the set prices) with the complementary probability. The history of transactions (prices and quantities) is known at any period and the type of the trader is unknown to the market makers. The ask price is the conditional expectation of θ given a buy order and past public information, and the bid price is the conditional expectation of θ given a sell order and past public information.

Adverse selection will imply a positive, and increasing in μ, bid-ask spread. This means that an increase in μ at time t increases the spread at this time. This does not mean that spreads will be period by period larger in a market with a higher μ since a larger proportion of insiders implies a larger initial spread but also faster information revelation (because of the impact of insiders on prices). That is, the comparison of spreads is ambiguous since more initial adverse selection in the market with a higher μ may be compensated by more information being revealed. Market makers, as in Section 8.1.1, on average lose money with informed traders and they balance the losses with the profits obtained from liquidity traders. The bid and ask price converge to true value as market makers accumulate information. The role of the depth parameter λ in the competitive price formation model of Section 8.1.1 is played here by the bid-ask spread. Transaction prices follow a martingale. (See Exercise 9.1.)

It is worth noting that even tough we are in an information structure similar to Bikhchandani *et al.* (1992) (see Section 6.1), in which the informed investor receives a noisy signal about the value of the stock, an informational cascade and herding will not occur with trading à la Glosten and Milgrom. The reason is that the price is a continuous public signal that keeps track of aggregate public information. Suppose that traders ignore their private signals, then the price cannot reveal any information, and both the bid and the ask prices must equal the probability that the value is high given public information (because of the zero expected profit condition). However, an informed trader has an incentive then to follow his signal and this contradicts the cascade assumption. This point has been emphasized by Avery and Zemsky (1998). With only two possible liquidation values there cannot be herding.[1] Still, Park and Sabourian (2006) show how herding can arise with three possible states when there is enough noise and traders believe that extreme outcomes are more likely that

[1] Drehmann *et al.* (2005) and Cipriani and Guarino (2005) provide experimental evidence in favor of the theoretical prediction of absence of herding by traders in the Glosten-Milgrom/Avery-Zemsky model with two states.

intermediate ones. This may happen even if signals conform to a standard monotone likelihood ratio property (see Section 1.5 in the Technical Appendix).[2] Herding and crashes arise naturally when traders are uncertain about the precision of information of other traders (like in Romer (1993), see Section 8.4.1). In this case market makers may update the price little after observing the order flow because of the uncertainty on the quality of information in the market (see Avery and Zemsky (1998) and Cipriani and Guarino (2007) –who also provide experimental evidence using financial professionals as subjects).

Another way to think about the result is to realize that competitive market making induces a payoff externality on informed traders. Indeed, investors, as well as market makers, learn from past trades. The changes in the bid-ask spread due to competitive market making (implying a payoff externality) offset the incentive to herd (because of the informational externality). As market makers learn more about the fundamental value the bid-ask spread is reduced and this entices an informed investor to use his information. We will see that a similar phenomenon happens in dynamic market order markets where the presence of a competitive market making sector speeds up learning from past prices because they make the market deeper (Vives (1995a) and Section 9.1.3).

Dow (2004) extends the Glosten-Milgrom model to incorporate expected-utility maximizing liquidity traders and shows that multiple equilibria with different endogenous levels of liquidity may arise. Equilibria have the familiar bootstrap property: if a high (low) level of liquidity is anticipated, the liquidity traders increase (decrease) their trading intensity and the spread is small (large).

9.1.2 <u>Slow learning from past prices</u>

Consider a financial market interpretation of the model in Section 7.4. Informed traders are risk neutral but face a quadratic adjustment cost in their position (which can be thought as an imperfect proxy for risk aversion). The horizon is infinite and at each period there is an independent (small) probability $1-\delta > 0$ that the ex post

[2] Recall that an informational cascade implies that agents herd (i.e. agents with the same preferences choose the same action) but the converse need not hold (see Section 6.1).

liquidation value of the risky asset θ is realized. The probability of θ not being realized at period t, δ^t, tends to zero as t tends to infinity.

Each agent of a continuum of long-lived traders receives a private noisy signal about θ at $t = 1$ and submits a market order to a centralized market clearing mechanism. At period t the information set of agent i is $\{s_i, p^{t-1}\}$. The (random) demand of noise traders (which is sensitive to price) is $u_t - p_t$, where u_t is a random intercept that follows a white noise process. Denote by Δx_t the aggregate demand of the informed traders in period t. The market clearing condition in period t is thus $u_t - p_t + \Delta x_t = 0$.

Trader i obtains profits

$$\pi_{it} = (\theta - p_t)\Delta x_{it} - \lambda(\Delta x_{it})^2 / 2$$

from the quantity Δx_{it} demanded in period t.[3] Total profits associated with the final position $\sum_{k=1}^{t} \Delta x_{ik}$ are $\sum_{k=1}^{t} \pi_{ik}$. At any period an informed trader maximizes the (expected) discounted profits with discount factor δ.

This model is formally equivalent to the one developed in Section 7.4 and therefore the slow learning (Proposition 7.4) and convergence (Proposition 7.5) results obtained apply here. Traders do learn from past prices and public information eventually reveals θ but the speed of learning is slow (at the rate $1/\sqrt{t^{1/3}}$) if there is no positive mass of perfectly informed traders. In this case the asymptotic variance of public information in relation to θ is $(3\tau_u)^{-1/3}(\lambda/\tau_\varepsilon)^{2/3}$ and it increases with the amount of noise trading, average noise in the signals, and the slope of adjustment costs. A change in the slope of adjustment costs may increase (for λ small) or decrease (for λ large) the asymptotic variance of $\Delta x_t - \Delta x_t^*$, the difference in net trading at the market and full information solutions.

[3] At any period a trader (given θ) expects a benefit per unit traded of θ. Note that the adjustment cost is exogenous while with CARA preferences, like in section 9.1.3 below, it will be endogenous and will depend, in expectation, on the degree of risk aversion times the variance of θ conditional on the information of the trader.

Technical analysis reveals information but at a slow rate. In practice this may render it ineffective. The consequence is that the market equilibrium converges to the full information equilibrium but does so slowly.

9.1.3 Price discovery, speed of learning, and market microstructure

We have seen how there may be slow learning with technical analysis. We will show now in the context of a price discovery process that the result depends on the market microstructure. We will see how market makers may accelerate the speed of learning and recover the standard $1/\sqrt{t}$ convergence rate.[4]

Consider a market with a single risky asset, with random ex post liquidation value θ, and a riskless asset, with unitary return. There are a continuum of risk-averse competitive informed agents and price sensitive noise traders. The profits of agent i with position x_i in the asset at price p are given by $\pi_i = (\theta - p)x_i$. Informed agents are risk averse, have CARA utilities, $U(\pi_i) = -\exp\{-\rho\pi_i\}$, where $\rho > 0$, and their initial wealth is normalized to zero.[5] Informed agent i submits a market order contingent on the information he has. Noise traders submit in the aggregate a price-sensitive order u – p.

Price discovery is modeled as an information tâtonnement with potentially many stages. At stage t there is a probability $\gamma_t > 0$ that the market opens, the value θ is realized and trade occurs given that there has not been trade before stage t, and with the complementary probability $1-\gamma_t$ there is no trade and the tâtonnement continues. If at the beginning of stage t the market has not opened the competitive informed agents, before knowing whether there will be trade in the period, have the opportunity to place orders and noise traders place them. These orders supersede previous orders,

4 This section follows Vives (1995b).

5 It is worth noting that for small risk, say when information about θ is very good, the decision of any risk averse von Neumann-Morgentern decision maker is well approximated by the solution with a CARA function (see, e.g. Pratt (1964). This provides robustness to the CARA model when public information ends up revealing θ.

which are understood to be cancelled if the market does not open.[6] The auctioneer or a centralized trading mechanism quote a notional price and in the next round traders can revise their orders.

Information tâtonnement processes are used in the preopening period of continuous, computerized, trading systems in several exchanges (for example, in the Paris Bourse, Toronto Stock Exchange, Bolsa de Madrid, or the Arizona Stock Exchange (AZX)). The price discovery process works as follows. Traders submit orders to the system for a certain period of time before the opening (one hour or one hour and a half) and theoretical market clearing prices are quoted periodically as orders accumulate. No trade is made until the end of the tâtonnement and at any point agents may revise their orders. This preopening auction is designed to decrease the uncertainty about prices after a period without trade. In the Deutsche Börse with the Xetra system there is an opening auction which begins with a call phase in which traders can enter and/or modify or delete existing orders before the (short) price determination phase. The indicative auction price is displayed when orders are executable.[7] The call phase has a random end after a minimum period. In the New York Stock Exchange the specialist provides some information to floor traders but there is no organized information tâtonnement to set the opening price. [8]

The information tâtonnement serves the purpose of eliciting information about the fundamental value of the asset. Notional prices convey noisy information, because of noise trading, about θ. The described process is a crude idealization of actual preopening price discovery processes and matches the situation with a random opening time (like in Xetra). In the former cases the real pattern of tâtonnement from,

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6 The model as stated may include also the case in which only the orders of noise traders are cancelled if the market does not open.

7 Otherwise, the best limit/ask limit is displayed. See Xetra Market Model Release 3 at www.exchange.de.

8 In some circumstances the revision of orders is allowed at the opening. In particular, this happens when the specialist plans to set an opening price which differs substantially from the previous close or when the order imbalance is large (see Section 1 in Stoll and Whaley (1990), Withcomb (1985) and Amihud and Mendelson (1987)). Madhavan and Panchapagesan (1998) provide evidence that the specialist facilitates price discovery. Cao, Ghysels and Hatheway (2000) provide evidence of price discovery in the preopening at Nasdaq.

say, 8.30 am to 10 am and opening at 10 am can be approximated smoothly by a sequence of probabilities $\{\gamma_t\}$ approximating a step function with no trade before 10 am and opening at 10 am. Furthermore, the possibility of a communication breakdown at some point (with increasing probability as the opening approaches), implying that the standing order "sticks" and can not be revised, is similar to the possibility of the market opening at any point in the process (with increasing probability as the end of the horizon approaches).[9]

The information tâtonnement process can be interpreted as a mechanism to elicit the aggregate information of informed agents via price quotations. It is analogous to the "dynamic information adjustment process" considered by Jordan (1982, 1985) to implement rational expectations equilibria. In both cases prices serve only as public information signals and trades are not realized until the iterative process has stopped. Similarly, Kobayashi (1977) assumes that agents trade at any period as if it were the last. [10]

9.1.3.1 Price discovery with exogenous market depth

The information set of trader i is given by $\{s_i, p^{t-1}\}$, where s_i is his private signal about θ and p^{t-1} the sequence of past price quotations. The trader places a market order $X_{it}(s_i, p^{t-1})$ and noise traders submit the aggregate price contingent order $u_t - p_t$. Taking into account that old orders are cancelled, the limit order book is thus

$$L_t(p_t) = \omega_t - p_t, \text{ with } \omega_t = x_t + u_t \text{ and } x_t = \int_0^1 X_{it}(s_i, p^{t-1})\, di$$

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9 However, in the model presented the market opens or not for everybody, meanwhile in the communication breakdown case a trader has a certain *individual* probability of being cut-off from the market.

10 Those authors consider markets with a finite number of agents with no noise added in the price system. With a finite state space convergence occurs then in a finite number of steps. In the model of this section with no noise trading and with a continuum of agents the liquidation value θ would be revealed in the first round since then p_1 would be a linear function of the average signal received by agents which, according to our convention, equals θ.

The auctioneer quotes a price to clear the market, $p_t = u_t + x_t$. As time evolves the depth of the market is fixed (at 1) and the price elastic noise traders avoid the market breaking down.

All random variables are assumed to be normally distributed. The sequence $\{u_t\}$ is independently and identically distributed with zero mean and variance σ_u^2. Private signals are given by $s_i = \theta + \varepsilon_i$, where $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$, with $\operatorname{cov}[\theta, u_t] = \operatorname{cov}[\theta, \varepsilon_i] = \operatorname{cov}[\varepsilon_i, u_t] = \operatorname{cov}[\varepsilon_i, \varepsilon_j] = 0,\ j \neq i$ for all t.

At stage t a strategy for trader i is a function that maps his private information s_i and the observed past prices p^{t-1} into desired trades. The asset is liquidated and trade realized in the period with probability γ_t and with probability $1-\gamma_t$ the trader obtains the continuation (expected) utility which, because the agent's size is negligible, is independent of his market order in period t. In consequence, the agent behaves as if the asset were to be liquidated and trade realized in the period. Myopic behavior is optimal. From the point of view of an agent the only difference between periods is in information. We restrict attention to linear equilibria. Given the preferences of traders and symmetric information structure, and similarly to Section 8.1, equilibria will be symmetric.

Traders form a price statistic to estimate θ with their past observations of price quotations and knowledge of strategies. At period t the price sequence p^t can be summarized in a public statistic $\theta_t \equiv E[\theta | p^t] = E[\theta | z^t]$, where $z_t = a_t\theta + u_t$, with a_t the response coefficient of traders to private information. In this market the price p_t is not a sufficient statistic for the information in the sequence of prices p^t because there is no competitive risk neutral market making sector. Given the CARA utility function and the optimality of myopic behavior the demand of trader i will be given by

$$X_t\left(s_i,\theta_{t-1}\right)=\frac{E\left[\theta-p_t\left|s_i,\theta_{t-1}\right.\right]}{\rho\ \text{var}\left[\theta-p_t\left|s_i,\theta_{t-1}\right.\right]}.$$

Equilibrium strategies are symmetric and linear in s_i and θ_{t-1}. In equilibrium we have that $a_t = \dfrac{\tau_\varepsilon\tau_u\left(1\text{-}a_t\right)}{\rho\left(\left(1\text{-}a_t\right)^2\tau_u+\ \tau_\varepsilon+\ \tau_{t-1}\right)}$ where $\tau_t=\tau_\theta+\tau_u\sum_{k=1}^t a_t^2$. This yields a recursive cubic equation $G_t\left(a\right)\equiv\left(1-a\right)\left(\rho a\left(\tau_\varepsilon\right)^{-1}\left(1-a\right)-1\right)\tau_u\tau_\varepsilon+\rho a\left(\tau_\varepsilon+\tau_{t-1}\right)=0$, with potentially three solutions in the interval $\left(0,1\right)$.[11] (However, for t large it can be checked that the solution is unique.) We have also that the response to public information is given by $c_t=\dfrac{\tau_{t-1}}{a_t^{-1}\tau_\varepsilon+\left(\tau_\varepsilon+\tau_{t-1}\right)\left(1-a_t\right)^{-1}}$.

It is possible to show that for any linear equilibrium sequence, as $t\to\infty$, the weight to private information $a_t\to 0$, and the response to public information $c_t\to\left(\rho\sigma_u^2+1\right)^{-1}$. The informativeness of the price statistic θ_t, $\tau_t\equiv\left(\text{var}\left[\theta\middle|\theta_t\right]\right)^{-1}$, is of the order of $t^{1/3}$ and a_t is of the order of $t^{1/3}$. The asymptotic precision $A\tau_\infty$ $(\equiv\lim_{t\to\infty}t^{-1/3}\tau_t)$ is given by $\tau_u 3^{1/3}\left(\tau_\varepsilon/\rho\right)^{2/3}$ and the stated limit results follow from the equilibrium expression for a_t and the fact that $a_t\,\tau_{t-1}\xrightarrow[t]{}\tau_u\left(\rho\sigma_\varepsilon^2\right)^{-1}$ and $t^{-1/3}\tau_t\xrightarrow[t]{}3^{1/3}\tau_u\left(\rho\sigma_\varepsilon^2\right)^{-2/3}$ (those facts follow similarly as in the proof of Claims 1 and 2 in the Appendix to Chapter 7).

As a consequence we have that θ_t converges (almost surely and in mean square) to θ as t tends to infinity and $\sqrt{t^{1/3}}\left(\theta_t-\theta\right)\xrightarrow{L}N\left(0,\left((\rho\sigma_\varepsilon^2)^2/3\right)^{1/3}\sigma_u^2\right)$. Convergence to the shared-information equilibrium, where informed agents pool their information, learn θ, and trade an amount $X\left(\theta\right)=\theta\left(1+\rho\sigma_u^2\right)^{-1}$, is at the slow rate $1/\sqrt{t^{1/3}}$.

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[11] It is worth to recall that in the Muth (1961) rational expectations model if agents are risk averse there are potentially multiple linear equilibria (see McCafferty and Driskill (1980)).

The reason for the slow learning and convergence is as before (Section 9.1.2 and Section 7.4). The responsiveness to private information a_t converges to zero as t grows since Bayesian agents decrease the weight they put on their private signals as public information becomes better and better. Finally, the asymptotic variance of $1/\sqrt{t^{1/3}}\left(\theta_t-\theta\right)$ increases with the degree of risk aversion and with the noise in the signals.

9.1.3.2 Price discovery with endogenous market depth

Let us introduce now a competitive risk neutral market making sector in the market.

Competitive market makers set p_t equal to the expectation of θ conditional on public information (including the current order flow ω_t). Current public information is given by the intercept ω_t of the order book $L_t\left(p_t\right)=\omega_t-p_t$. We have thus

$$p_t=E\left[\theta\middle|\omega^t\right],$$

where $\omega^t=\left(\omega_1,\ldots,\omega_t\right)$. The current price p_t is now a sufficient statistic of all past and current prices p^t and those are observationally equivalent to the history of noisy order flows or intercepts of the order books ω^t. We have $E\left[\theta\middle|p^t\right]=E\left[\theta\middle|p_t\right]=p_t$. Letting $\tau_t\equiv\left(\operatorname{var}\left[\theta\middle|p_t\right]\right)^{-1}$ we have that $\operatorname{var}\left[p_t\middle|p_{t-1}\right]=\left(\tau_{t-1}\right)^{-1}-\left(\tau_t\right)^{-1}$.

Restricting attention to equilibria in linear strategies it is possible to show that there is a unique linear equilibrium. The Appendix to the chapter provides a proof of the following result.

Proposition 9.1 (Vives (1995b)): With a competitive risk neutral market making sector there is a unique linear equilibrium. Traders use symmetric strategies

$$X_t\left(s_i, p^{t-1}\right) = a_t\left(s_i - p_{t-1}\right), \text{ where } a_t = \left(\rho\left(\sigma_\varepsilon^2 + \text{var}\left[p_t \middle| p_{t-1}\right]\right)\right)^{-1},$$

and prices are given by $p_t = \lambda_t \omega_t + p_{t-1}$, $\omega_t = a_t\left(\theta - p_{t-1}\right) + u_t$ with $\lambda_t = \tau_u a_t / \tau_t$, $\tau_t = \tau_\theta + \tau_u \sum_{k=1}^{t} a_k^2$, and $\text{var}\left[p_t \middle| p_{t-1}\right] = \left(\tau_{t-1}\right)^{-1} - \left(\tau_t\right)^{-1}$.

Several properties of the equilibrium are worth highlighting:

- Trader i desires to buy or sell according to whether his private estimate of θ, s_i, is larger or smaller than the market estimate, p_{t-1}. Informed traders' response to private information, a_t, depends negatively, ceteris paribus, on ρ, σ_ε^2, and $\text{var}\left[p_t \middle| p_{t-1}\right]$. The latter term affects the trading intensity because agents use market orders and face price uncertainty.
- Informed traders optimize against the linear function $p_t = \lambda_t \omega_t + p_{t-1}$, and market makers determine the price function $p_t = E\left[\theta \middle| \omega^t\right] = E\left[\theta \middle| p_t\right]$, making λ_t endogenous. It is easy to check that, ceteris paribus, an increase in the depth of the market induces risk averse informed traders to respond more to their information (since $\text{var}\left[p_t \middle| p_{t-1}\right]$ is lower). On the contrary, an increase in a_t, again ceteris paribus, induces market makers to put more weight on the order flow in setting p_t, decreasing market depth, as the order flow is more informative. An increase in the precision of prices τ_t, holding a_t constant, has the opposite effect.

The asymptotic properties of the linear equilibrium as t tends to infinity are as follows (see Exercise 9.3):

- a_t converges monotonically from below to $\left(\rho\sigma_\varepsilon^2\right)^{-1}$;
- τ_t and λ_t^{-1} tend to infinity at a rate of t;
- $\text{var}\left[p_t\right]$ converges monotonically from below to σ_θ^2;
- $\text{var}\left[p_t \middle| p_{t-1}\right]$ tends to 0;

- the expected volume traded by informed agents $E\left[\left|x_{it}\right|\right]$ converges from above to $\left(2/\pi\right)^{1/2}\left(\rho\sigma_{\varepsilon}\right)^{-1}$, and
- the expected total volume traded converges from above to $\left(2\pi\right)^{-1/2}\left(\left(\rho\sigma_{\varepsilon}\right)^{-1}+2\sigma_u\right)$.

As t grows prices become more informative about θ and τ_t increases linearly. The competitive market making sector increases the depth of the market with λ_t^{-1} growing also at the rate of t. The conditional variance of prices decreases and induces each informed trader to respond more to his information. However, the notional volume of trade of informed traders decreases since their information advantage with respect the market makers disappears as prices become more informative. In the limit informed traders lose all information advantage.

The aggregate volume of trade of informed traders against market makers is given by $E\left[\left|x_t\right|\right]=\left(2/\pi\right)^{1/2}a_t\left(1/\tau_{t-1}\right)^{1/2}$ and tends to 0 with t. From the fact that the precision of prices grows linearly with t it is immediate that

(i) p_t converges (almost surely and in mean square) to θ at a rate of $1/\sqrt{t}$, and
(ii) $\sqrt{t}\left(p_t-\theta\right)$ converges in distribution to $N\left(0,\sigma_u^2\rho^2\sigma_\varepsilon^4\right)$.

In contrast to the exogenous depth market now the responsiveness to private information a_t increases with t and converges to a positive constant. Market depth is endogenous and increasing as more tâtonnement rounds accumulate because of market makers. A risk averse trader responds more to the deviations of p_{t-1} from the private signal s_i the deeper is the market. The new information in the current price p_t, $z_t=a_t\theta+u_t$, does not vanish for t large and the order of magnitude of the precision of prices τ_t is t.

With a competitive market making sector price quotations converge to θ fast, at the rate $1/\sqrt{t}$, and the asymptotic precision of p_t is decreasing in ρ, σ_u^2 and σ_ε^2. Convergence is slower if agents are more risk averse, have less precise private information, or noise is larger. A larger ρ and σ_ε^2 makes informed agents respond less to information. This same effect follows from increased noise trading if t is large. In summary, the information tâtonnement only needs a few rounds to get close to the value of the asset since the precision of prices τ_t grows linearly with the number of rounds t. This can be taken as an indication that the public announcement of theoretical prices proves effective in resolving the uncertainty about the value of the asset (provided that market makers, or traders using limit orders, are present).[12] We will see in Section 9.3.2 how the results are affected by the presence of large strategic traders.

9.1.4 Summary

This section has demonstrated that market microstructure matters for the information revelation properties of prices. Market makers modify the depth of the market in response to the information content of the order flow and speed up information revelation by prices. In the Glosten and Milgrom (1985) model the continuous price variable avoids informational cascades and herding. In the Vives (1993 and 1995a) models market makers avoid a slow learning outcome. Indeed, technical analysis yields slow information revelation from prices. The same lesson applies to price discovery mechanisms.

9.2 Strategic trading with long-lived information

In this section we consider dynamic trading by a risk neutral large informed trader ("insider") facing noise traders and risk neutral market makers. Section 9.2.1 presents the model in Kyle (1985) and Section 9.2.2 several extensions to multiple informed traders, risk aversion, the effects of compulsory disclosure of trades by the insider as well as a connection with the Glosten-Milgrom model (Section 9.1.1).

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[12] See Exercise 9.8 where it is shown that competitive informed traders using limit orders put constant weight to their private information in the tâtonnement and therefore ensure that convergence is fast.

9.2.1 The Kyle (1985) model

Kyle (1985) considers a model where a large trader receives information and trades for T periods with a competitive risk neutral market making sector and noise traders. The model is a dynamic version of the model considered in Section 5.2 (with no competitive informed traders).

Consider a market with a single risky asset, with random (ex post) liquidation value θ, and a riskless asset, with unitary return, traded among noise traders and a large risk neutral informed trader (the "insider"), who observes θ, with the intermediation of competitive market makers. The informed trader acts strategically, that is, takes into account the effect his demand has on prices and faces a trade-off: taking positions early, and increasing profits then, leaks information to the market and diminishes profits later.

The horizon is finite (T periods). Consider period t. The insider's information is given by $\{\theta, p^{t-1}\}$, where $p^{t-1} = (p_1, ..., p_{t-1})$ is the sequence of past prices. He submits a market order contingent on the information he has: $\Delta Y_t(\theta, p^{t-1})$. Noise traders submit the aggregate order u_t. The order flow is then $\omega_t = \Delta Y_t + u_t$. Competitive risk neutral market makers set prices efficiently conditional on the observation of the order flow. That is, p_t is equal to the expectation of θ conditional on public information (including the current order flow ω_t): $p_t = E[\theta | \omega^t]$, where $\omega^t = (\omega_1, ..., \omega_t)$.

All random variables are assumed to be normally distributed with the sequence $\{u_t\}$ independently and identically distributed with zero mean and variance σ_u^2. The liquidation value $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and the sequence $\{u_t\}$ are mutually independent. Denote by π_t the profits of the insider directly attributable to his period t trade $\Delta y_t : \pi_t = (\theta - p_t)\Delta y_t$. Then the profits of the insider on trades from period t to T are $\pi_t^T = \sum_{k=t}^T \pi_k$. His initial wealth is normalized to zero.

Kyle (1985) solves the dynamic programming problem of the insider and shows that there is a unique linear recursive solution. This corresponds to a linear PBE of the dynamic game between the insider and the competitive market makers. The following proposition is a dynamic version of Proposition 5.2 (with $\mu = 1$).

Proposition 9.2 (Kyle (1985)): There is a unique linear equilibrium and, for $t = 1, 2, \ldots, T$, it is given by:

$$\Delta Y_t\left(\theta, p^{t-1}\right) = \alpha_t\left(\theta - p_{t-1}\right),$$

$$E\left[\pi_t^T \middle| \theta, p^{t-1}\right] = h_{t-1}\left(\theta - p_{t-1}\right)^2 + \delta_{t-1}, \text{ and } p_t = \lambda_t \omega_t + p_{t-1},$$

where $p_0 \equiv \bar{\theta}$, $\omega_t = \Delta y_t + u_t$, $\lambda_t = \tau_u \alpha_t / \tau_t$, and $\tau_t = \tau_\theta + \tau_u \sum_{k=1}^{t} \alpha_k^2$.

The constants α_t, h_t and δ_t are the unique solution to the difference equation system

$$h_{t-1} = 1/\left(4\lambda_t\left(1 - \lambda_t h_t\right)\right)$$

$$\alpha_t = \left(1 - 2\lambda_t h_t\right)/\left(2\lambda_t\left(1 - \lambda_t h_t\right)\right)$$

$$\delta_{t-1} = \delta_t + h_t \lambda_t^2 \tau_u^{-1}$$

subject to the boundary conditions $h_T = 0$, $\delta_T = 0$, and the second order conditions $\lambda_t\left[1 - \lambda_t h_t\right] > 0$ for $t = 1, 2, \ldots, T$.

Outline of proof: The strategy of the insider and its expected profits are obtained first as a function of the market depth parameters λ_t at a linear equilibrium. Competitive market making at a linear equilibrium yields a price process of the form $p_t = \lambda_t \omega_t + p_{t-1}$ as in Proposition 9.1. The boundary conditions $h_T = 0$, $\delta_T = 0$ just state that no profits are to be made after trade is completed. In the last period the trading intensity, as in the static model of Proposition 5.2 (with $\mu = 1$) fulfills $\alpha_T \lambda_T = 1/2$. The form of the strategy is also as in the static model with p_{T-1} taking the role of the prior mean $\bar{\theta}$: $\Delta Y_T\left(\theta, p^{T-1}\right) = \alpha_T\left(\theta - p_{T-1}\right)$. Indeed, from competitive market making we know that the price p_{T-1} is just the expected value of the fundamental given public information up to t-1. In particular, $p_{T-1} = E\left[\theta \middle| p^{T-1}\right] = E\left[\theta \middle| p_{T-1}\right]$. The properties of the price process yield immediately a quadratic value function of the form

$$E\left[\pi_T \middle| \theta, p^{T-1}\right] = h_{T-1}\left(\theta - p_{T-1}\right)^2$$

where $h_{T-1} = \alpha_T\left(1 - \alpha_T\lambda_T\right) = \frac{1}{4}\lambda_T$. Using an induction argument the recursive form of the value function follows as stated in the proposition. For any $t < T$ we will have that $\alpha_t\lambda_t < 1/2$ because the insider considers the future information leakage of the impact of his trades. The insider has an incentive to trade when the market is deep (λ_t is low) and therefore if he expects depth to be high in the future he will try to trade more intensely at later dates. The second order condition prevents the insider from destabilizing prices at auction t to make it up in excess at later auctions. When market depth is low (λ_t is high) by trading small quantities prices can be destabilized. The second order condition puts an upper bound on λ_t which is decreasing in h_t which measures the value of private information at future trading dates. The higher this value, the higher the incentives of the insider to move prices away form the fundamental value. Finally, from $\alpha_t = \left(1 - 2\lambda_t h_t\right) / \left(2\lambda_t\left(1 - \lambda_t h_t\right)\right)$ and $\lambda_t = \tau_u \alpha_t / \tau_t$ we can obtain the cubic equation in λ_t

$$\left(1 - \lambda_t^2 \tau_t / \tau_u\right)\left(1 - h_t\lambda_t\right) = 1/2\,,$$

which has three real roots, the middle one satisfying the second order condition. We can thus iterate the difference equation system backwards for a given τ_T (recall that $h_T = 0$). It is easy to see then that only one terminal value τ_T is consistent with the prior τ_θ (see Kyle (1985) for the details).

As in Sections 8.1.1 and 8.1.2 the risk neutral competitive fringe makes price p_{t-1} a sufficient statistic for public information $\omega^t = \left(\omega_1, ..., \omega_{t-1}\right)$. The insider buys or sells in period t according to whether the liquidation value θ is larger or smaller than public information p_{t-1}. Information is gradually incorporated into the price, as an outcome of the trade-off faced by the insider, as price precision τ_T increases with t but remains bounded.

The question arises about whether the insider may have incentives to introduce noise in his order. The answer is no. The reason, as in the static model (see Section 5.2.1) is that he is optimizing at any stage against a fixed conjecture on the behavior of market makers, that is, a fixed lambda. For a given market depth then it is optimal not to introduce noise in the order since the only effect of placing a noisy order is just to distort trade from its optimal level given θ. (See Exercise 9.4.) We will see below that things change if the informed trader is forced to disclose his trade at the close of the period.

The author also analyzes a continuous time version of the model letting the intervals between trades go to zero. Noise trading, as well as equilibrium prices, follows then a Brownian motion (this is due to competitive market making). A remarkable result is that market depth is constant over time, information is incorporated into prices at a constant rate with all information incorporated at the end of trading. The result is that prices converge to θ (in mean square) as the end of the horizon approaches.[13]

Back (1992) provides an extension of the model in continuous time. Back and Pedersen (1998) consider the case in which the monopolistic insider receives a flow of private information on top of an initial stock of information. The insider reveals the information slowly also. Chau and Vayanos (2007) consider a steady state infinite horizon model in which the insider receives information every period about the expected growth rate of asset dividends. As trading is more frequent, converging to continuous time, the insider chooses to reveal the information more and more quickly and the market approaches strong-form efficiency. The reason is that in the model the price impact is constant over time independently of whether the insider trades quickly or not. Impatience (in the form of discounting, information leakage or obsolescence of information) leads then to a fast trading pattern. The major difference with Kyle (1985) is that there a stock of information arrives only once.

[13] Back (1992) provides an extension of the Kyle model in continuous time.

9.2.2 Extensions

There are several other extensions available of the Kyle model, introducing competition among insiders, disclosure requirements, risk averse traders, and building bridges with the Glosten-Milgrom model (Section 9.1.1).

9.2.2.1 Competition among insiders

Holden and Subrahmanyam (1992) introduce several insiders, all observing the fundamental value, and show that the information is incorporated into prices much more quickly. In fact, all information is incorporated immediately as the interval between auctions tends to zero. This is so because equally informed agents trade more aggressively (Holden and Subrahmanyam (1994)). Moreover, if the insiders were to be risk averse they would even speed up more information revelation with a resulting increasing pattern of market depth. This may seem surprising at first glance because one could expect more cautious behavior on the part of risk averse traders. However, risk averse traders want to trade early to avoid future price uncertainty (and this makes market makers to decrease depth to protect themselves at the beginning).

Foster and Viswanathan (1996) consider several risk neutral informed agents each receiving a noisy signal of the fundamental value. The information structure is symmetric and error terms in the signals are potentially correlated. Therefore, both the cases of all insiders receiving the same signal and receiving (conditionally) independent signals are covered. The authors focus on linear recursive Markov perfect equilibria and show that the problem of forecasting the forecast of others (infinite regress) does not arise in equilibrium, neither with one player deviations in order to check for equilibrium, because a sufficient statistic for the past can be found. The latter is a consequence of the combination of the recursive structure of the model, normality and competitive market making (much as in Vives (1995a), see Section 8.2).

An interesting feature of the analysis, uncovered by simulations in examples, is that prices are less revealing the lower the correlation of private signals, and that the correlation of private signals conditional on public information decreases over time and becomes negative towards the end of the horizon with enough trading rounds. The reason is that the more similar the information traders have the more they compete and more of their information is transmitted to prices. Furthermore, the competitive market making sector is learning basically the average of the signals of the traders (all there is to learn) and this means that the covariance between individual signals conditional on public information, when close to the average signal, must be negative. Now, an informed trader will learn faster from the order flow than the market maker and this means that by trading aggressively he will reveal more to the competitors than to the market makers. Therefore, informed traders will be cautious and play a waiting game trying to induce the competitor to reveal information. Note the contrast with Holden and Subrahmanyam (1994) where informed traders cannot learn anything from each other because they receive the same signal.[14]

9.2.2.2 Disclosure of trades and dissimulation strategies

Huddart, Hughes and Levine (2001) study a version of the Kyle model where the insider has to disclose his trade before the next round of trading. In the US an insider of a firm (manager, officer, or large stockholder) has to report ex post the trades he makes on the stock of the firm in question. The report is made to the Security and Exchange Commission (SEC) and is made public immediately.[15]

Obviously, the equilibrium strategy of the insider in the Kyle (1985) model will no longer be optimal. Indeed, after the first round of trade it would be fully revealing of θ. This would induce the competitive market making sector to let depth be infinite in the second period and the insider would have the opportunity to make unbounded profits. The insider has thus to dissimulate its trade by introducing noise in his order. The authors show that there is an equilibrium in which the insider adds normally distributed noise η_t to his order at every stage except the last. This noise is uncorrelated with all other random variables in the model, with mean zero and

[14] See Back, Cao and Willard (2000) for an analysis of the model in continuous time, obtaining closed form solutions. The authors show that there is no linear equilibrium when signals are perfectly correlated, and that after some date, the market would have been more informationally efficient had there been a monopolist informed trader instead of competing informed traders.

[15] According to the Securities Exchange Act of 1934: (i) insiders have to report their trades to the SEC within ten days following the end of the month in which the trade occurs (Section 16(a)); (ii) any individual who acquires 5% or more of the stock of a firm, as well as subsequent changes to the position, must report it within ten days (Section 13(d)).

variance $\sigma^2_{\eta_t}$. The parameters of the randomization are not observable by market makers. Letting $\Delta y^t = \{\Delta y_1, ..., \Delta y_t\}$ the linear equilibrium of the T-period market is characterized, for t = 1, 2, ... , T, as follows:

$$\Delta y_t = \alpha_t\left(\theta - E\left[\theta \middle| \Delta y^{t-1}\right]\right) + \eta_t, \text{ with } E\left[\theta \middle| \Delta y^0\right] \equiv \bar{\theta}, ,$$

$$p_t = \lambda_t \omega_t + E\left[\theta \middle| \Delta y^{t-1}\right] \text{ and } \omega_t = \Delta y_t + u_t,$$

where

$$\alpha_t = \left(2\left(T - t + 1\right)\lambda_t\right)^{-1},$$

$$\lambda_t = \sqrt{\tau_u / 4T\tau_\theta} \text{ and}$$

$$\sigma^2_{\eta_t} = \sigma^2_u\left(T - t\right)/\left(T - t + 1\right).$$

Furthermore, $E\left[\theta \middle| \Delta y^t\right] = E\left[\theta \middle| \Delta y^{t-1}\right] + 2\lambda_t \Delta y_t$ and $\text{var}\left[\theta \middle| \Delta y^t\right] = \sigma^2_\theta\left(T - t\right)/T$.

The equilibrium displays several notable properties. Note first that the strategy of the insider at t is conditioned on public information $E\left[\theta \middle| \Delta y^{t-1}\right]$ since p_{t-1} is a noisy version of it. The dissimulation strategy of the insider involves setting in every period the variance of added noise in his trade equal to the variance of the information-based component, that is,

$$\sigma^2_{\eta_t} = \alpha^2_t \text{var}\left[\theta - E\left[\theta \middle| \Delta y^{t-1}\right]\right],$$

and the total variance of his trade equal to the variance of noise trading, that is, $\text{var}\left[\Delta y_t\right] = \sigma^2_u$ (all this as evaluated by the market maker). The second part of the strategy camouflages the insider behind the noise traders and the first part makes difficult to distinguish between information-based and random-based trades once they are disclosed. The conditional volatility of θ at the start of period t, $\text{var}\left[\theta \middle| \Delta y^{t-1}\right]$, is smaller than with no disclosure and its reduction overtime is constant across periods. This contrasts with the slower pace of variance reduction with no disclosure. The trading intensity of the insider is increasing overtime. The market depth parameter λ_t is constant over time. The latter is necessary to sustain the mixed strategy equilibrium (for reasons similar to the continuous time version of the Kyle (1985) model). In contrast, in the Kyle (1985) discrete time model λ_t is decreasing with t as more information is incorporated into the price. Furthermore, depth is always larger with disclosure. The reason is that with disclosure some of the trades of the insider are not information-based but randomly generated. Expected per period profits for the insider are constant over time (and equal to $\lambda_t \sigma_u^2 = 1/\sqrt{4T\tau_u\tau_\theta}$), and are lower round by round than with no disclosure (where they decline over time).[16]

9.2.2.3 Risk averse traders

Guo and Kyle (2005) consider a continuous time model over an infinite horizon with a risk averse informed trader who receives continuously new information about the dividend process, noise traders, and risk averse market makers. The informed trader and the market makers have negative exponential utility and the informed faces quadratic trading costs. They characterize linear Bayesian equilibria and use the results to explain financial anomalies. The key to explain them, as in Section 8.2, is the presence of risk averse market makers who ask compensation for bearing risk. This explains "excess volatility". The model can explain also the momentum and reversal puzzles. Stock returns tend to show positive short-term autocorrelation (momentum) but negative long-term autocorrelation (reversal).[17] The explanation is as follows. The orders from the informed trader are positively autocorrelated in the short run because he wants to smooth his trade over time in order to minimize trading and market impact costs. This may dominate the negative autocorrelation of orders from liquidity traders. However, the informed trader is risk averse, his order is negatively related to his inventory of the stock and private information is mean-reverting. His position will be mean-reverting in the long run therefore. Together with the mean

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16 All results are obtained with simulations and can be shown to hold analytically when T = 2. When T = 2 it is possible to show also that the trading intensity of the insider is larger period by period with disclosure.

17 See, for example, Jegadeesh (1990), Jegadeesh and Titman (1993), Fama and French (1988) and Poterba and Summers (1986, 1988). See Daniel *et al.* (1998) for a literature review.

reverting position of liquidity traders this explains the long run negative autocorrelation in stock returns.

The model provides an explanation of the anomalies that does not rely on the irrationality of traders. An alternative behavioral explanation is provided, for example, by Daniel, Hirshleifer and Subrahmanyam (1998) based on the overconfidence of investors about the precision of their information and biased self-attribution (according to which when an investor receives public information that confirms his beliefs, the confidence level increases more than it decreases when he receives disconfirming information). Other behavioral explanations of the anomalies have been provided by Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999).

9.2.2.4 Kyle meets Glosten and Milgrom

Back and Baruch (2004) extend the Glosten and Milgrom (1985) model (Section 9.1.1) to consider a single informed trader who uses market orders and decides, in continuous time, the optimal trading times. Uninformed buy and sell orders arrive as a Poisson process with constant and exogenous arrival intensities. Market makers are competitive and risk neutral, post bid and ask prices and see individual trades. The authors show that if the liquidation value of the risky asset follows a Bernoulli distribution and the informed trader knows the liquidation value, there is an equilibrium in which he follows a mixed strategy between trading and waiting. This means that both informed and uninformed traders arrive stochastically from the perspective of market makers, as assumed in the Glosten and Milgrom model. Interestingly, the equilibrium in this version of the Glosten and Milgrom model is close to the equilibrium in the continuous-time version of the Kyle (1985) model when uninformed traders arrive frequently and the trade size is small. The parallel here to the gradual trade of the insider in Kyle (1985) is the probabilistic waiting to trade of the informed trader. It is shown also that the bid-ask spread is approximately twice "lambda" times the order size in the Kyle-type model. It is worth noting also that the informed trader, in some circumstances, may randomize over trades that go against his information. This occurs even though, in contrast to Huddart et al. (2001), trades are not disclosed ex post.

The results by Back and Baruch (2004) are important because they show that the more tractable Kyle (1985) model (with discrete batch auctions) is consistent also with the more common case where market makers set bid-ask prices and see individual trades. The model is generalized by Back and Baruch (2007) to encompass both multiple order sizes and limit order markets. Both the informed trader and discretionary liquidity traders submit market orders and choose between block orders or a series of small orders (i.e. use "work orders").[18] Liquidity providers are assumed to be competitive and risk neutral. The aim of the analysis is to compare floor exchanges, where a uniform price is established and an open limit order book, where there is discriminatory pricing since each limit order executes at its limit price. With risk neutral competitive liquidity providers, in the first case prices are the expectation of the fundamental value conditional on public information and order size; in the second case, ask (bid) prices are "upper (lower) tail" expectations of the fundamental value; i.e. expectations conditional on the size of the demand (supply) being at least (at most) the size of the order (see Section 5.3.2).

The model allows for larger traders to work their orders and pool with small traders. It is shown that in a floor exchange it is never an equilibrium for all traders to use block orders. That is, any equilibrium must involve at least partial pooling. Furthermore, if traders can submit orders an instant apart –effectively with no execution difference from a block trade- then the block-order equilibrium in the limit-order market is equivalent to a fully pooling worked-order equilibrium on the floor exchange. The incentive for large traders to pool with small ones is that if they do not (i.e. in a separating equilibrium), and since small orders supposedly have a lessened adverse selection problem, prices for small orders would be more favorable. However, in a pooling equilibrium in the floor exchange, market makers can not know whether after an order there will be more from the same trader in the same direction. In consequence, ask prices will also be upper-tail expectations as in the limit order market. The authors claim that their floor exchange model is a good representation of trade in the CBOE and that the hybrid design in the NYSE (with the proposed use of

[18] Chordia and Subrahmanyam (2004) study order imbalances and stock returns in a competitive model where liquidity traders have an incentive to split their orders across periods.

uniform pricing when a market order walks up the book) shares the features of a uniform-price market.

9.2.3 Summary

The main learning points of the section are the following:

- A large informed trader (an "insider") has incentives to trade slowly so as not to reveal too much information and keep an informational advantage over uninformed traders and market makers. As the number of trading rounds increases information is incorporated in the price at a constant rate and risk neutral market makers keep a constant market depth.
- The insider will try to camouflage behind liquidity traders but has no incentive to introduce noise in his order to confuse market makers.
- Competition among strategic informed traders speeds up information revelation when they have symmetric information; otherwise informed traders may play a waiting game trying to induce the competitor to reveal information.
- If an insider has to disclose his trades then he does have an incentive to dissimulate his trades by randomizing optimizing his camouflage behind liquidity traders and obscuring the separation of information-based from liquidity-based trades. Disclosure increases market depth and information revelation, and decreases the expected profits of the insider.
- Financial market anomalies, such as the momentum and reversal in stock returns, can be explained with rational risk averse traders.
- Quote-driven markets (like in Glosten and Milgrom (1985)) and order-driven markets (like in Kyle (1985)) have a close connection when in the former uninformed traders arrive frequently and the trade size is small. This has important implications for the equivalence of trading in a floor exchange as compared to trading in a limit-order market, and for hybrid markets as well.

9.3 Market manipulation and price discovery

We have seen how an insider may have incentives to dissimulate trades if he has to disclose them before trading again. A distinct possibility is market manipulation according to which an agent takes covert actions attempting to change the terms of trade in his favor. I provide first a quick survey of the literature on the topic, and I analyze the possibilities of manipulation in the price discovery process studied in Section 9.1.3.

9.3.1 Market manipulation in the literature

The stock-price manipulation literature can be classified according to whether manipulation is based on actions that change the value (or the perceived value) of the asset, or based on releasing misleading information, or purely based on trade. Examples, of the first type are given in Vila (1989), of the second in Vila (1989) and Benabou and Laroque (1992), and of the third type in Hart (1977), Jarrow (1992), Allen and Gale (1992), Allen and Gorton (1992), Kumar and Seppi (1992), Fishman and Hagerty (1995), and Chakraborty and Yilmaz (2004). Still, we can add another dimension in the classification according to whether the trader that manipulates the market is informed or uninformed. For example, Allen and Gorton (1992) explain price manipulation by an uninformed agent in the presence of asymmetries in noise trading (noise selling is more likely than noise buying) or asymmetries in whether buyers or sellers are informed (with short-sale constraints to exploit good news is easier than to exploit bad news). In Allen and Gale (1992) or Fishman and Hagerty (1995), an uninformed trader can pretend to be informed to manipulate the price and make money. In Fishman and Hagerty (1995) uninformed insiders may exploit the inability of market makers to distinguish trades of uninformed agents from those of insiders with private information. For example, the uninformed insider may buy shares, imitating an insider who has received good news, move prices up, and sell the shares after disclosure of the trade. Goldstein and Guembel (2007) show how the allocation role of prices opens the possibility of market manipulation. The authors explain how an uninformed trader may want sell a stock when the price guides the investment decisions of the firm. The informativeness of the stock price diminishes and the trader profits from the investment distorsion.

Contrarian behavior, or trade against one's information, is obtained in some instances in the literature. John and Narayanan (1997) develop a variation of the model of Fishman and Hagerty (1995) in which mandatory disclosure of trades of corporate insiders gives them under certain assumptions incentives to manipulate the market using a contrarian strategy. According to this strategy the insider trades in a first period against his information to unwind his position in a second period. In their model agents are restricted to trade only one unit, the fundamental value follows a two-state distribution, and market makers fix prices before seeing the order flow. The insider either manipulates when he receives good news or when he receives bad news depending on whether the probability of receiving good news is lower or higher. Foster and Viswanantan (1994) provide an example of a duopoly where information has a common and a private component and where the better informed agent tries to minimize the learning of the lesser informed one. This market manipulation may lead to contrarian behavior by the better informed trader if the private and common signals have very disparate realizations (something that happens with low probability). Chakraborty and Yilmaz (2004) find that insiders may trade in the wrong direction when there is uncertainty about their presence in the market and there is a large number of periods before information is revealed.

9.3.2 Strategic behavior and price discovery

We will study here an instance of price manipulation by a strategic trader in a price discovery mechanism used in opening auctions.[19] More specifically, we consider the price discovery process with a random opening time for the market studied in Section 9.1.3.2 but now on top of the competitive informed traders, with mass $1-\mu$ each with constant degree of risk aversion ρ, competitive risk neutral market makers, and noise traders, there is a large risk neutral informed trader, with mass μ who knows the liquidation value θ. This model belongs to the third class of manipulation models with trade-based manipulation. In our case the objective of the strategic informed trader (the "insider"), who has accurate information on the liquidation value of the asset and this is known to other agents, is to neutralize the informative trades that competitive informed agents make. In order to this the insider will use a contrarian strategy. The model will make clear that introducing a random opening time, like in Xetra, limits but does not eliminate the incentives to manipulate the market.

The insider's information set at round t is given by $\{\theta, p^{t-1}\}$, where $p^{t-1} = (p_1, ..., p_{t-1})$, is the sequence of past prices. Denote his desired position at t by y_t. The information

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[19] This section is based on Medrano and Vives (2001). Hillion and Suominen (2004) explain the incentives of brokers to manipulate the prices at the close to "look good" in front of customers and provide evidence of strategic behavior at the close of the Paris Bourse.

set of competitive informed trader i is given by $\{s_i, p^{t-1}\}$, where s_i is his private signal about θ, and his (symmetric) market order of the type $X_t(s_i, p^{t-1})$. Noise traders submit the aggregate order u_t.

All random variables are assumed to be normally distributed with the same properties as in Section 9.1.3.2. In particular we use the usual convention that given θ, the average signal of the competitive informed agents $\tilde{s} = (1-\mu)^{-1} \int_\mu^1 s_i di$ equals (a.s.) θ (i.e. errors cancel out in the aggregate, $\int_\mu^1 \varepsilon_i \, di = 0$).[20] The pooled information of informed agents reveals θ. We can interpret the insider of size μ as emerging from a coalition of small informed traders (of measure μ) who decide to form a cartel of investors and pool their information.

The order flow is then $\omega_t = \mu y_t + \int_\mu^1 X_t(s_i, p^{t-1}) di + u_t$. Competitive market makers set $p_t = E[\theta | \omega^t]$, where $\omega^t = (\omega_1, ..., \omega_t)$. If the market opens at stage t, θ is realized, trade occurs and this is the end of the story. Otherwise the tâtonnement continues. All trades are notional until the market opens. Informed traders can revise their orders before the market opens. In general they will have incentives to do so once they receive more public information since this helps them to predict better the net value $\theta - p$. Furthermore, the insider may be able to manipulate the information contained in prices and may have incentives to do so.

At stage t a strategy for the insider is a function that maps his private information θ and the observed past prices p^{t-1} into a market order $Y_t(\theta, p_{t-1})$ given that p_{t-1} is a sufficient statistic for past public information because of the presence of the risk neutral competitive market making sector. He knows that the asset will be liquidated and trade realized in the period with probability γ_t, obtaining in expectation $E[(\theta - p_t)\mu y_t | \theta, p_{t-1}]$, and with the complementary probability $1-\gamma_t$ he will obtain the continuation expected profit $E[\pi_{t+1} | \theta, p_{t-1}]$ which depends on his market order in

[20] See Section 3.1 in the Technical Appendix for a justification of the convention.

period t. Therefore, at stage t the insider will face the following expected profit conditional on his information:

$$E\left[\pi_t \middle| \theta, p_{t-1}\right] = \gamma_t E\left[(\theta - p_t)\mu y_t \middle| \theta, p_{t-1}\right] + (1-\gamma_t) E\left[\pi_{t+1} \middle| \theta, p_{t-1}\right].$$

Restricting attention to equilibria in linear strategies it is possible to obtain a full characterization of equilibrium behavior. In equilibrium competitive traders, given their preferences and symmetric information structure, will use a symmetric strategy.

<u>Proposition 9.3</u> (Medrano and Vives (2001)): Linear equilibria are characterized as follows for $t = 1, 2, \ldots, T$:

$$Y_t(\theta, p_{t-1}) = \alpha_t(\theta - p_{t-1}),\ Y_t$$

$$X_t(s_i, p_{t-1}) = a_t(s_i - p_{t-1}),$$

$$p_t = \lambda_t \omega_t + p_{t-1}, \text{ and } \omega_t = A_t(\theta - p_{t-1}) + u_t,$$

where $p_0 = \overline{\theta}$, $\lambda_t = \tau_u A_t / \tau_t$, $A_t = \mu\alpha_t + (1-\mu)a_t$ and $\tau_t = \tau_\theta + \tau_u \sum_{k=1}^{t} A_k^2$. At stage t, the insider's expected continuation profit is given by

$$E\left[\pi_{t+1} \middle| \theta, p_{t-1}\right] = \mu\left(h_t(\theta - p_t)^2 + \delta_t\right).$$

The constants a_t, α_t, h_t and δ_t are the solution to the difference equation system

$$a_t = \left(\rho\left(\tau_\varepsilon^{-1} + \tau_{t-1}^{-1} - \tau_t^{-1}\right)\right)^{-1}$$

$$\alpha_t = \left(\left[1-(1-\mu)\lambda_t a_t\right]/\left[2\mu\lambda_t\right]\right)\left(\left[\gamma_t - 2(1-\gamma_t)\mu\lambda_t h_t\right]/\left[\gamma_t - (1-\gamma_t)\mu\lambda_t h_t\right]\right)$$

$$h_t = (1-\lambda_{t+1}A_{t+1})\left(\gamma_{t+1}\alpha_{t+1} + (1-\gamma_{t+1})h_{t+1}(1-\lambda_{t+1}A_{t+1})\right)$$

$$\delta_t = (1-\gamma_{t+1})\left(\delta_{t+1} + \lambda_t^2 h_{t+1}/\tau_u\right)$$

subject to the boundary conditions $h_T = 0$, $\delta_T = 0$, $2\mu\alpha_T\lambda_T = 1-(1-\mu)\lambda_T a_T$ and the second order conditions $\lambda_t\mu\left(\gamma_t - (1-\gamma_t)\mu\lambda_t h_t\right) > 0$ for t = 1, 2, …, T.

Corollary: At a linear equilibrium the following inequalities hold for any t: $0 < a_t < \tau_\varepsilon / \rho$, $A_t > 0$, $\lambda_t > 0$, $0 < \mu\lambda_t h_t < \gamma_t / (1-\gamma_t)$, $0 < \left(1-(1-\mu)\lambda_t a_t\right) < 1$ and $0 < 1 - \lambda_t A_t < 1$.

The proof of the proposition is a variation of the arguments in Kyle (1985) and Vives (1995b). Note, however, that the proposition does not assert the existence of a linear equilibrium. In fact, to show existence and uniqueness of the linear equilibrium in the model is not a trivial task. This is a difference equation system with T periods and two unknowns in each period ($\{a_t, \alpha_t\}$ or $\{a_t, \lambda_t\}$). Solving the difference equation system is complicated since we cannot find a way to iterate the dynamic equation system backwards like in Kyle (1985) or Holden and Subrahmanyam (1992), nor forward as in Vives (1995b). The reason is that the insider's responsiveness to private information at stage t (α_t) depends on all his futures trading intensities $(\alpha_{t+1}, \ldots, \alpha_T)$ via h_t, while the responsiveness to private information of the competitive informed agents (a_t) depends on all their past trading intensities $(a_1, \ldots, a_{t-1})$ via τ_{t-1}. However, we may expect that the linear equilibrium exists and is unique for all parameter configurations because this is the case for the extreme cases $\mu = 0$ (Vives (1995b)) and $\mu = 1$ (with a monopolistic informed trader), and also when $T = 2$ and γ_1 is close to zero. Furthermore, systematic simulations performed in a wide range of parameter values have always produced a unique (linear) equilibrium. When informed traders submit demand schedules, instead of market orders, the dynamics are simplified and it is possible to show that there is a unique linear equilibrium (see Exercise 9.8).

We characterize now trading volume for further reference. As in Section 5.2.1 we define the total volume traded at stage t, denoted by TV_t, as the sum of the absolute values of the demands coming from the different agents in the model divided by *2*.[21] Its expectation is given by

$$E[TV_t] = \left((1-\mu)E\left[\left|X_t(s_i, p_{t-1})\right|\right] + \mu E\left[\left|Y_t(\theta, p_{t-1})\right|\right] + E\left[|\omega_t|\right] + E\left[|u_t|\right]\right)/2 .$$

[21] See the explanations for trading volume in Chapter 4 and in Section 5.2.1.

The behavior of the total trading volume is driven by the behavior of the volume traded by informed (competitive and strategic) agents. As in Exercise 5.7 in a linear equilibrium it is easy to check that

$$E\left[\left|X_t\left(s_i, p_{t-1}\right)\right|\right] = \left(2/\pi\right)^{1/2} a_t \left(\tau_\varepsilon^{-1} + \tau_{t-1}^{-1}\right)^{1/2},$$

$$E\left[\left|Y_t\left(\theta, p_{t-1}\right)\right|\right] = \left(2/\pi\right)^{1/2} \left(\alpha_t^2 / \tau_{t-1}\right)^{1/2},$$

$$E\left[\left|\omega_t\right|\right] = \left(2/\pi\right)^{1/2} \left(\sigma_u^2 + A_t^2 / \tau_{t-1}\right)^{1/2}, \text{ and}$$

$$E\left[TV_t\right] = \left(1/2\pi\right)^{1/2} \left(\left(1-\mu\right) a_t \left(\tau_\varepsilon^{-1} + \tau_{t-1}^{-1}\right)^{1/2} + \mu\left(\alpha_t^2 / \tau_{t-1}\right)^{1/2} + \left(\sigma_u^2 + A_t^2 / \tau_{t-1}\right)^{1/2} + \sigma_u\right).$$

When $\mu = 1$ and there is a monopolistic insider it is possible to show the existence of a unique linear equilibrium. Then insider faces a more stark version of the insider's trade-off in the Kyle model (Section 9.2). At stage t his future profit will decrease by placing a market order if there is no trade, because of the information leaked to the market makers. By not submitting an order if trade occurs, his future profit will be zero because θ will have been revealed.[22] The optimal market order, which balances the two effects, implies a trading intensity that is lower than in the one-shot model where there is trading with probability one. In our monopolistic market $\lambda_t \alpha_t < 1/2$ for all t < T and $\lambda_T \alpha_T = 1/2$ (since for t = T the model becomes like the static Kyle model).

The large informed agent refrains from trading too aggressively because there is a positive probability that there is no trade. This suggests that his trading intensity α_t should be increasing in the probability γ_t (and this is confirmed by the simulations). An important result is that for the central case where the probability is of the type $\gamma_t = \gamma^{T-t}$, and in contrast to the competitive economy (where μ = 0 as in Section 9.1.3.2), no matter how long the horizon is the price precision is bounded above (and

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[22] It never pays to set $\alpha_t < 0$ because it is dominated by $\alpha_t = 0$. If there is trade, with $\alpha_t < 0$ the insider makes negative profits while it makes zero with $\alpha_t = 0$; if there is no trade, with $\alpha_t = 0$ no information is revealed to the market makers while with $\alpha_t < 0$ some information is revealed. When $\gamma_t = 0$ it is optimal not to trade (set $\alpha_t = 0$).

the bound depends only on the parameter γ). The monopolistic insider prevents the full revelation of θ no matter how many rounds the tâtonnement has.

A simulation analysis (assuming that $\gamma_t = \gamma^{T-t}$) uncovers the following properties comparing the monopolistic version of the model with the competitive version.

- <u>The responsiveness to private information</u> increases monotonically with t; in the monopolistic case at an accelerating rate and in the competitive case at a decelerating rate.
- <u>The informativeness of prices</u> τ_t increases monotonically with t; in the monopolistic case at an accelerating rate close to the opening and in the competitive case at the rate of t.
- <u>Market depth</u> $(\lambda_t)^{-1}$ tends to infinity at a rate of t in the competitive equilibrium; in the monopolistic equilibrium, in general, it decreases during the first rounds of the tâtonnement and then increases as the probability that there will be trade tends to one.
- <u>The unconditional volatility of prices</u> $\mathrm{var}[p_t]$ increases monotonically towards σ_θ^2 in both cases. However, in the competitive economy $\mathrm{var}[p_t]$ gets close to σ_θ^2 in the first few rounds of tâtonnement while it is close to zero in the monopolistic economy (because market depth is extremely high).
- <u>Expected trading volume:</u> In the competitive economy, the expected volume traded by informed agents is decreasing for t large, while in the monopolistic economy it increases monotonically.[23]

When the insider and the competitive informed sector coexist the insider's responsiveness to private information α_t may be negative for t not too close to the end of the horizon. This may be interpreted as an attempt to manipulate the market because the insider goes against what his private information suggests, buying when

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[23] It should be clear that $E\left[\left|Y_t(\theta, p_{t-1})\right|\right] = (2/\pi)^{1/2} (\alpha_t^2 / \tau_{t-1})^{1/2}$ will be increasing if the rate of increase of α_t is sufficiently high in relation to the increase in prices τ_{t-1}.

$\theta < p_{t-1}$ and selling when $\theta > p_{t-1}$. The insider attempts to neutralize the information incorporated in prices from the demands of the competitive informed traders. The more information market makers have, the lower the speculative profits of the insider. This is how the insider can manipulate the informativeness of prices τ_t and the depth of the market $(\lambda_t)^{-1}$. Both $\tau_t = \tau_{t-1} + \tau_u A_t^2$ and $\lambda_t^{-1} = \tau_t / (\tau_u A_t)$ depend on the average of the trading intensities of the strategic and the competitive informed agents, $A_t = \mu\alpha_t + (1-\mu) a_t$. We have that $a_t > 0$ as long as $\tau_\varepsilon > 0$. By setting $\alpha_t < 0$ the insider can decrease A_t and he will do so if γ_t is sufficiently low. In particular, when there is no danger of the market opening ($\gamma_t = 0$) the insider trades in a way that no information is revealed by neutralizing the response of competitive informed agents ($\alpha_t = -(1-\mu) a_t / \mu$ and $A_t = 0$).[24] At stage $t < T$, if $0 < \gamma_t < 1$, the insider must balance reducing the informativeness of prices by choosing a low (and possibly negative) trading intensity α_t, and trading intensely (choosing α_t close to the static equilibrium value) to obtain a high profit if trades are executed. If $\gamma_t = 1$, the insider behaves as in the static version of the model. At stage t we should expect that the insider's incentives to manipulate the market decrease in the probability of trading γ_t, or equivalently, α_t increasing in γ_t (and since γ_t is increasing in t, α_t should be increasing in t).

The simulations performed for the case $\gamma_t = \gamma^{T-t}$ corroborate the analysis and conjectures above:[25] α_t is increasing in t and in γ, τ_t is strictly convex in t; and, provided T is large enough: (i) $\alpha_t < 0$ for t low; (ii) $\mathrm{var}[p_t | p_{t-1}]$ may be hump-

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[24] However, it never pays the strategic informed trader to let $A_t < 0$ choosing $\alpha_t < -(1-\mu) a_t / \mu$. This is worse than choosing $\alpha_t = -(1-\mu) a_t / \mu$, since in the first case the expected loss in case trade is realized would be higher and the future expected profit in case there is no trading would be lower (because then the price does reveal some information while if $\alpha_t = -(1-\mu) a_t / \mu$ it does not).

[25] We have explored the behavior of the model with $\gamma_t = \gamma^{T-t}$ in the following parameter grid: ρ in $\{1, 2, 4\}$, τ_u, τ_θ and τ_ε in $\{.5, 1, 2\}$, γ in $\{.2, .3, .5, .6, .7, .8\}$, μ in $\{.2, .5, .8\}$ with T up to 30 rounds.

shaped or increasing and a_t U-shaped or decreasing in t; and (iii) the total expected trading volume is U-shaped in t. Further simulations support the conjecture that for $\mu > 0$, as in the case of a monopolistic insider ($\mu = 1$), for any given γ there is an upper bound for the price precision $\overline{\tau}$, no matter the length T of the horizon.[26] A larger size of the insider μ implies a lower limit value for the price precision and this limit is attained in fewer rounds of trade. Indeed, when μ increases the average responsiveness to information A_t tends to decrease and this impacts negatively on the informativeness of prices. A larger insider tends decrease the price precision and the expected volume traded.

The insider manipulates the market at the beginning of the price adjustment process (result (i)). As a consequence, the informativeness of prices is very low during the first stages and increases quite fast as t gets close to T. Result (iii) is driven by the fact that the insider's expected trading volume is U-shaped. The expected volume traded by informed traders (ignoring the volume traded among competitive informed agents) equals $\left(\mu|\alpha_t| + (1-\mu)a_t\right)\left(\mathrm{var}\left[\theta|p_t\right]\right)^{1/2}$. For γ not too high this volume will have a U-shaped temporal pattern because $|\alpha_t|$ does and dominates. This in turn dominates the decreasing tendency of $\mathrm{var}\left[\theta|p_t\right]$. (See Exercise 9.7 for an explanation for result (ii).)

Finally, it is worth to remark that the general pattern of results obtained hold also in the case that the strategic and the competitive informed agents use demand schedules instead of market orders. In the presence of the insider there is market manipulation, price precision is bounded above and volume is U-shaped. (See Exercise 9.8)

The theoretical results obtained in this section are in line with the empirical analysis of the preopening period in the Paris Bourse by Biais, Hillion and Spatt (1999). Those authors find that the preopening period is active, in particular close to the opening.

26 Simulations have been performed in the range of parameters: ρ in $\{1, 2, 4\}$, τ_u, τ_θ and τ_ε in $\{.5, 1, 5\}$, μ between .01 and 1 with a step of .05 and γ with the same step from .01 until .5. In this range the upper bound for τ is attained in 30 rounds or less. For γ's up to .7 and μ's no smaller than .2 the upper bound for τ is attained in 40 rounds or less.

The last 15 minutes before the opening (say 9.45 to 10 am in the Paris Bourse) are the most active order placement period in the day (including therefore the period with real trade). Trading at the opening amounts to about 10% of the total trading of the day. About half of the preopening orders are serious orders for sure since they are actually executed and about 60% of those are executed at the opening and not later. Therefore, the preopening order flow is directly linked to the opening price. The average size of orders placed in the preopening period increases as we get closer to the opening and large traders place sometimes not aggressive orders and tend to modify their orders. The volume of trade has typically a U-shaped form dropping after the first round to increase sharply later when approaching the opening. Biais, Hillion and Spatt do not reject the hypothesis of semi-strong efficiency for prices close to the opening. Before that the hypothesis that prices do not reflect any information cannot be discarded.[27] The speed of learning from prices is of the order of $t^{3/2}$, in the second part of the preopening, where t is the number of rounds in the tâtonnement. This means that the precision of prices grows more than linearly towards the end of the process (recall that a price precision of the order of t^k is associated to a speed of learning of $t^{k/2}$). This speed of learning is easy to generate in the theoretical model.[28]

The interaction between a strategic informed trader and a sector of competitive informed agents in the model presented in this section yields outcomes consistent with the empirical evidence available from the Paris Bourse. Indeed, we have seen how the presence of the insider slows down at first and later accelerates the transmission of information by prices. The price precision tends to increase sharply towards the end of the tâtonnement. However, the price does not fully reveal the fundamental value of the asset no matter how many rounds the tâtonnement has. Furthermore, trading volume displays a U-shaped pattern driven by the insider's activity.

9.3.3 Summary

Market manipulation is a distinct possibility when there are large traders in the market. In the preopening period of a price discovery process we have seen how a

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[27] A similar result is obtained by Sola (1999) with data from the Bolsa de Madrid.

[28] If we fit a curve of the type Kt^k to $\tau_t - \tau_\theta$ we find easily values for k close to 3 for a range of periods in which τ_t is significantly different from τ_θ.

strategic informed trader has incentives to use a contrarian strategy to suppress the information leakage from the price deriving from the competitive behavior of other informed traders. This manipulation is understood by everyone in the market to happen in equilibrium.

9.4 Strategic trading with short-lived information

Up to now we have studied the impact of long-lived information on price informativeness, volatility and volume in the presence of strategic traders and noise traders which were given no choice of when to trade. However, a strategic trader may possess also short-lived information and "noise" traders may have at least some discretion about when to trade.

It has been observed that the average intraday volume and variance of price changes in the NYSE is U-shaped. Admati and Pfleiderer (1988) try to explain this pattern considering a T-period dynamic trading model where the information of n insiders is short-lived and where there are some liquidity traders that can choose when to trade. The basic idea is that the intraday trading patterns for volume and price volatility may be explained by the incentives of liquidity and informed traders to cluster their trades.

The liquidation value of the single risky asset is given by $\theta = \bar{\theta} + \sum_{t=1}^{T} \theta_t$ where $\bar{\theta}$ is a known parameter, $\{\theta_t\}$ are independently normally distributed random variables with mean zero and variance σ_t^2. There are n_t informed traders in period t and they all see the same signal about the innovation next period, $s_t = \theta_{t+1} + \varepsilon_t$ where ε_t is normally distributed with mean 0 and variance $\sigma_{\varepsilon_t}^2$. Error terms in signals and innovations in the fundamental value are mutually independent. The value of θ_t becomes known at the beginning of period t and therefore the information of the informed traders is short lived. Denote by $Y_t(s_t)$ the market order of an informed trader in period t.

There are two types of liquidity traders in period t. The usual noise traders, trading according to a normal random variable u_t, and m discretionary liquidity traders who can choose when to trade within a time interval (say the trading day). When discretionary liquidity trader j trades he has a demand of z_j shares that cannot be split during the trading period. His demand in period t z_{jt} is z_j if he trades and 0 otherwise. All traders submit market orders and a competitive risk neutral market making sector sets prices upon observing the order flow and public information. Denote by y_{it} the order of informed trader i in period t. The order flow in period t is given by

$$\omega_t = \sum_{i=1}^{n_t} y_{it} + \sum_{j=1}^{m} z_{jt} + u_t .$$

It is assumed that

$$\{z_1,...,z_m, u_1,...,u_{T-1}, \theta_1,...,\theta_T, \varepsilon_1,...,\varepsilon_{T-1}\}$$

are mutually independent normally distributed random variables.

The competitive market making sector in period t sets prices conditional on the order flow ω_t and on public information $\theta^t \equiv \{\theta_1,...,\theta_t\}$, $p_t = E\left[\theta \middle| \omega_t, \theta^t\right]$.
Given normality this yields immediately:

$$p_t = E\left[\theta \middle| \theta^t\right] + \lambda_t \omega_t = \bar{\theta} + \sum_{k=1}^{t} \theta_k + \lambda_t \omega_t$$

where $\lambda_t = \mathrm{cov}\left[\theta_{t+1}, \omega_t\right] / \mathrm{var}\left[\omega_t\right]$. The inverse of the parameter λ_t is as usual a measure of the depth of the market.

In period t the n_t informed traders compete taking into account the price rule of the market makers and trader i demands $y_{it} = \alpha_t s_t$. In equilibrium[29]

$$\alpha_t = \sqrt{\frac{\Psi_t}{n_t \mathrm{var}\left[s_t\right]}} ,$$

where $\Psi_t = \mathrm{var}\left[\sum_{j=1}^{m} z_{jt} + u_t\right]$ is the variance of total liquidity trading in the period, and

$$\lambda_t = \frac{\sigma_{t+1}^2}{n_t + 1} \sqrt{\frac{n_t}{\Psi_t \mathrm{var}\left[s_t\right]}} .$$

[29] The derivation is similar to the derivation of equilibrium in Exercise 5.3 (but note that there the informed traders use demand schedules instead of market orders).

Not surprisingly, an increase in total liquidity demand increases the sensitivity of informed traders to their information, as they can camouflage better behind noise traders, and market depth, as the order flow becomes less informative. As the number of informed traders n_t increases, each one of them responds less to his information, since they all receive the same signal, and market depth increases, as they are not able to restrict their trade enough to control the information leakage in the order flow. With more informed traders the adverse selection problem of market makers is less severe because of competition among the informed.

A discretionary liquidity trader will choose to trade when the cost of trading is lowest, that is, when his losses are minimal. The expected losses for a liquidity trader trading z_j in period t are

$$E\left[(p_t - \theta) z_j \middle| \omega^{t-1}, \theta^t, z_j\right] = \lambda_t z_j^2$$

where $\omega^{t-1} \equiv \{\omega_1, ..., \omega_{t-1}\}$, after substituting for the expressions of p_t and θ and using the independence assumptions made. Therefore, liquidity traders would like to trade when λ_t is lowest. This means that discretionary liquidity trades like to trade when the market is deep and this happens when there are a lot of other liquidity traders (λ_t is decreasing in Ψ_t). It is not difficult to see that liquidity traders face a coordination problem and that there will exist multiple equilibria. If in period t there is a lot of discretionary liquidity trading then λ_t will be low and this will attract more liquidity traders. There is always an equilibrium where all discretionary liquidity trading happens in the same period and generically only this type of equilibrium is possible. Indeed, if for some parameters of the model there is trade in two periods, implying that the two periods must have the same market depth, a small perturbation of $\text{var}\left[z_j\right]$, for example, would tip the balance towards the period with a strictly deeper market. Furthermore, insiders like to trade also in a deep market to disguise better their trades (indeed, α_t is increasing in ψ_t).[30]

[30] For other models with a coordination problem of investors concentrating trade in a single market or at certain times see, respectively, Pagano (1989) with a model with no asymmetric information, and Foster and Viswanathan (1990).

This means that the concentration of discretionary liquidity trading in one period will induce more trading by informed traders and explains a peak in volume. In fact, this peak in volume will occur even if the rate at which information becomes public is constant (say $\sigma_t^2 = 1$ without loss of generality), signals have the same precision, $\tau_{\varepsilon_t} = \tau_\varepsilon$, and the amount of nondiscretionary noise trading is constant, $\text{var}\left[u_t\right] = \sigma_u^2$, in any period. (See Exercise 9.9.) However, the model does not explain volatility changes. Indeed, if there are the same number of informed traders in any period, $n_t = n$ (and $\tau_{\varepsilon_t} = \tau_\varepsilon$, $\sigma_t^2 = 1$) for all t, then the volatility of price changes, is given by

$$\text{var}\left[p_t - p_{t-1}\right] = \frac{\tau_\varepsilon}{1+\tau_\varepsilon}\left(\tau_\varepsilon^{-1} + \frac{n}{1+n} + \frac{1}{1+n}\right) = 1.$$

And, indeed, the informativeness of prices about the dividend innovation

$$\left(\text{var}\left[\theta_{t+1} \middle| p_t\right]\right)^{-1} = \left(1 + \frac{n\tau_\varepsilon}{1+\tau_\varepsilon + n}\right)^{-1}$$

is also constant across time periods. All this holds irrespective of ψ_t and therefore is true for the period in which liquidity trading is concentrated. Indeed, as in the Kyle (1985) static model (see Section 5.2.1) the informativeness of prices, and therefore conditional volatility in a semi-strong efficient market, is independent of the amount of noise trading. More liquidity trading entices risk neutral informed traders to trade more intensely and this is just sufficient to keep the order flow with the same information content. Note that with no informed trading $p_t - p_{t-1} = \theta_t$ and therefore $\text{var}\left[p_t - p_{t-1}\right] = \text{var}\left[\theta_t\right] = 1$. When there is informed trading the result is the same. To generate more interesting volatility patterns we need to have different numbers of informed traders in different periods.

The authors go on to study the incentives to acquire information and endogenize the number of insiders. Whenever the number of insiders is known in equilibrium concentrated trading patterns are reinforced. This is so because more liquidity trading incentivates entry of informed traders and, because of enhanced competition among them, lowers trading costs for liquidity traders. This reinforces the incentive of discretionary liquidity traders to trade in the period to start with. Indeed, the expected profits of an informed trader in period t just match the (share of) expected losses of (discretionary and nondiscretionary) liquidity traders:

$$\pi\left(n_t,\psi_t\right)=\frac{\lambda_t}{n_t}\psi_t=\frac{1}{1+n_t}\left(\frac{\tau_\varepsilon\ \psi_t}{\left(1+\tau_\varepsilon\right)n_t}\right)^{1/2},$$

which are increasing in the total amount of liquidity trading in the period $\psi_t \equiv \operatorname{var}\left[\sum_{j=1}^{m} z_{jt}+\ u_t\right]$. At a free entry equilibrium where traders have to pay F to become informed $\pi\left(n_t,\psi_t\right)\geq F$ and entry by one more informed trader would induce an equilibrium with negative net expected profits. Now with more information about the dividend process concentrated when liquidity trading is high we have that volatility and volume are positively correlated.

In summary, when liquidity traders have discretion about when to trade they will tend to concentrate their trading in periods where market depth is high and this will become a self-reinforcing process where more liquidity traders and informed traders, with short-lived information, will do so also, generating volume and volatility peaks.

9.5 Strategic hedging

The models we have considered so far display the strategic behavior of traders privately informed about the fundamental value of the risky asset. The motivation for trade for those large traders is to exploit their information advantage. Risk averse large traders may trade also for insurance motives when receiving a shock to their endowments or to hedge an investment. What are the consequences of strategic behavior when large traders trade for an insurance motive?

We study the case of the endowment shock of each strategic trader being the only private information and examine the consequences for the speed of trading and welfare losses, as well as possibilities to manipulate the market. The case when the strategic trader has information about the fundamental value and wants to obtain insurance to hedge his investment has been examined in Section 5.4. Chau (2002) examines the dynamic incentives of a strategic trader to exploit his private information about the fundamental value as well as controlling his inventory after an endowment shock.

We will display two patterns of trading by a large risk averse trader that suffers an endowment shock. In the first we will see how private information about the endowment shock lead to slow trading and potentially large welfare losses. In the second we will uncover an instance of market manipulation in the presence of noise traders.

Vayanos (1999) considers a dynamic model with n CARA risk averse infinitely lived strategic agents who receive an endowment shock every period and want to insure against dividend risk. Dividends follow a random walk, dividend information is public, and the endowment shock to a trader is the only private information. Traders submit (continuous) demand schedules every period to a centralized market clearing mechanism and all random variables are normally distributed. There is no noise trading. The model may fit inter-dealer markets, where dealers want to share their inventory risk and participants are large. In the linear Nash equilibrium in demand functions studied prices are fully revealing of the endowment shocks because there is no noise trading.

A first result is that agents trade slowly even when the time between trades tends to zero. This is so because making use of more trading opportunities the price impact would be very important. Indeed, a trade in one direction would indicate many more trades in the same direction. To avoid a strong price impact large agents trade slowly. The result would go away, and trade would accelerate as the interval between trades decreases, as in the Coase conjecture, if endowment shocks were to be public information. According to the Coase conjecture a durable goods monopolist sells fast and prices converge to marginal cost fast as the interval between trades diminishes. Private information on endowments drives the slow trading result by increasing the price impact of a trade. Indeed, in equilibrium if a trader were to sell more shares then the other traders would incorrectly infer that he has received a larger endowment shock and would expect more sales in the future. This does not happen when endowments are public information as when in an inter-dealer market dealers are required to disclose the trades received from their customers. With public information on endowments there is in fact a continuum of equilibria because then traders know the market clearing price and are indifferent about which demand function to submit for out of equilibrium prices (this is as in any demand function model with no uncertainty, see footnote 1 in Section 5.1.1). The author in this case selects an equilibrium using a perturbation technique.

A second result is that the welfare loss due to strategic behavior increases, in contrast to the public information case, as the time between trades shrinks. A third result is that the welfare loss is of the order of $1/n^2$ for a fixed length of the interval between trades (this is the same that in a k-double auction, see Section 2.5.1) but that as this time interval shrinks to zero the welfare loss is of the order of $1/n$. Dynamic trading with strategic behavior may imply a slower convergence to efficiency than static competition as the number of traders increases. The results may shed some light on the debate about the welfare properties of continuous versus discrete auctions in the organization of stock markets.

Vayanos (2001) studies a stationary model where one risk averse large trader receives a privately observed endowment shock every period and submits a market order to a competitive risk averse market making sector in the presence of noise traders. As before, information about asset payoffs is public and the large trader trades for an insurance motive. It is shown that after receiving an endowment shock the large trader reduces his risk exposure (and shares risk with the market makers) either by selling at a decreasing rate over time or, more surprisingly, by selling first to achieve optimal risk sharing and then engaging in a round trip transaction selling some more shares to buy them back later. The second pattern, which happens when there is enough noise trading and the large trader is not very risk averse compared to market makers, has a manipulation flavor. Indeed, in the second pattern of trade market makers are misled by the first sale, thinking that has originated with the noise traders. However, the large trader knows that this is not the case and therefore that the price will fall. He exploits the situation then by selling and buying back when the price has fallen. It is found also that when the time between trades tends to zero the information about the endowment of the large trader is reflected in the price very quickly.

In short, large risk averse traders when hedging endowment shocks which are private information will trade slowly, even when trade is very frequent. A consequence is that the welfare loss due to strategic behavior may increase as trade becomes more frequent and that as the number of traders increases an efficient outcome is approached more slowly than with one shot trading. Furthermore, large risk averse traders, when hedging endowment shocks which are private information, may have incentives to manipulate the market trying to mislead market makers, much as large traders with privileged information about the fundamental value of the asset.

Summary

The chapter has examined the dynamics of competitive market order markets and has allowed for strategic behavior. The main insights are:

- Learning from past prices (technical analysis) may be very slow, as in the canonical model of learning from others, if market depth is fixed exogenously.
- If competitive market makers set the depth of the market then informational cascades are not possible and learning from prices in price discovery processes is faster.
- A large informed trader has incentives to trade slowly so as not to reveal too much information and keep an informational advantage over uninformed traders and market makers. The insider will try to camouflage behind liquidity traders but has no incentive to introduce noise in his order to confuse market makers.
- Competition among strategic informed traders speeds up information revelation when they have symmetric information; otherwise informed traders may play a waiting game trying to induce the competitor to reveal information.
- If an insider has to disclose his trades ex post then he will dissimulate by randomizing to optimize his camouflage behind liquidity traders and obscure the separation of information-based from liquidity trades. Disclosure increases market depth and information revelation, and decreases the expected profits of the insider.
- Market manipulation is a distinct possibility when there are large traders in the market. A strategic informed trader may have incentives to use a contrarian strategy to suppress the information leakage from the price deriving from the competitive behavior of other informed traders.
- When liquidity traders have discretion about when to trade they will tend to concentrate their trading in periods where market depth is high and this will

become a self-reinforcing process where informed traders, with short-lived information, as well as more liquidity traders will do so also, generating volume and volatility peaks.

- Large risk averse traders hedging endowment shocks which are private information will trade slowly, even when trade is very frequent, and may have incentives to manipulate the market trying to mislead market makers.

Appendix

Proposition 9.1: With a competitive risk neutral market making sector there is a unique linear equilibrium. Traders use symmetric strategies $X_t\left(s_i, p^{t-1}\right) = a_t\left(s_i - p_{t-1}\right)$, where $a_t = \left(\rho\left(\sigma_\varepsilon^2 + \operatorname{var}\left[p_t \middle| p_{t-1}\right]\right)^{-1}\right)$, and prices are given by $p_t = \lambda_t \omega_t + p_{t-1}$, $\omega_t = a_t\left(\theta - p_{t-1}\right) + u_t$ with $\lambda_t = \tau_u a_t / \tau_t, \tau_t = \tau_\theta + \tau_u \sum_{k=1}^{t} a_k^2$, and $\operatorname{var}\left[p_t \middle| p_{t-1}\right] = \left(\tau_{t-1}\right)^{-1} - \left(\tau_t\right)^{-1}$.

Proof: At a linear equilibrium and given our assumptions all random variables are normally distributed. Maximization of a CARA utility function by trader i yields then at stage t

$$X_t\left(s_i, p^{t-1}\right) = \frac{E\left[\theta - p_t \middle| s_i, p^{t-1}\right]}{\rho \operatorname{var}\left[\theta - p_t \middle| s_i, p^{t-1}\right]},$$

where $p_t = E\left[\theta \middle| \omega^t\right]$ from the competition among market makers, and where ω_t is the t period order flow. The expression is independent of i and therefore equilibria will be symmetric.[31] Similarly than in Section 8.1.1 we obtain that $p_t - p_{t-1} = \lambda_t \omega_t$, where $\lambda_t = \tau_u a_t / \tau_t$, $\omega_t = a_t\left(\theta - p_{t-1}\right) + u_t$ and a_t is the weight to private information in the t period strategy. (In fact, just take the expression for $p_t - p_{t-1}$ in Section 8.1.1 and replace Δa_t by a_t since now trade is notional and at period t the orders from period t-1 are cancelled). It is immediate that

$$E\left[\theta \middle| s_i, p_t\right] = \left(\tau_\varepsilon s_i + \tau_t p_t\right) / \left(\tau_\varepsilon + \tau_t\right), \text{ and } \left(\operatorname{var}\left[\theta \middle| s_i, p_t\right]\right)^{-1} = \tau_\varepsilon + \tau_t$$

Furthermore, since $p_t = \lambda_t\left(a_t\theta + u_t\right) + \left(1 - \lambda_t a_t\right) p_{t-1}$, we have that $\theta - p_t = \left(1 - \lambda_t a_t\right)\left(\theta - p_{t-1}\right) - \lambda_t u_t$. It follows that in the estimation of $\theta - p_t$; $s_i - p_{t-1}$ is sufficient with respect to the information $\left\{s_i, p^{t-1}\right\}$. Furthermore,

$$E\left[\theta - p_t \middle| s_i, p^{t-1}\right] = \left(1 - \lambda_t a_t\right) \tau_\varepsilon \left(\tau_\varepsilon + \tau_{t-1}\right)^{-1} \left(s_i - p_{t-1}\right).$$

Now, we have that

31 See the related discussion on the symmetry of linear equilibria in Section 8.1.

$$X_t\left(s_i, p_{t-1}\right) = \frac{E\left[\theta - p_t \middle| s_i, p_{t-1}\right]}{\rho \operatorname{var}\left[\theta - p_t \middle| s_i, p_{t-1}\right]} = \frac{(1-\lambda_t a_t)E\left[\theta - p_{t-1} \middle| s_i, p_{t-1}\right]}{\rho\left[(1-\lambda_t a_t)^2 \operatorname{var}\left[\theta \middle| s_i, p_{t-1}\right] + \lambda_t^2 \sigma_u^2\right]}.$$

Using the expressions for $E\left[\theta - p_t \middle| s_i, p_{t-1}\right]$ and $\operatorname{var}\left[\theta \middle| s_i, p_{t-1}\right]$, we obtain:

$$a_t = \rho\left(\sigma_\varepsilon^2 + a_t \lambda_t \operatorname{var}\left[\theta \middle| p_{t-1}\right]\right)^{-1} = \rho\left(\sigma_\varepsilon^2 + \operatorname{var}\left[p_t \middle| p_{t-1}\right]\right)^{-1}.$$

This yields the recursive cubic equation $F_t\left(a_t\right) \equiv \left(\rho\left(\tau_\varepsilon\right)^{-1} a_t - 1\right)\tau_{t-1} + \rho\lambda_t a_t^2 = 0$. We show that the equation $F_t\left(a\right) = 0$ has a unique positive root, which lies in the interval $\left(0, \left(\rho\sigma_\varepsilon^2\right)^{-1}\right)$. It is clear that positive roots must lie in $\left(0, \left(\rho\sigma_\varepsilon^2\right)^{-1}\right)$. It can be easily checked that $F_t\left(0\right) < 0$, $F_t\left(\left(\rho\sigma_\varepsilon^2\right)^{-1}\right) > 0$, and that $F_t\left(a\right) = 0$ implies $F_t'\left(a\right) > 0$. It follows then that there is a unique positive root.♦

Exercises

*9.1. Sequential trading à la Glosten and Milgrom. Consider the model in Section 9.1.1 with $\theta \in \{0,1\}$ and private binary, conditionally independent, signals $s_i \in \{s_L, s_H\}$ with $\Pr\left(s_H \middle| \theta = 1\right) = \Pr\left(s_L \middle| \theta = 0\right) = \ell > 1/2$. Given the public belief about the value (probability that the value is high given public information) write the zero expected profit condition for a buy and for a sale for a market maker, and find the bid and ask equilibrium prices. Check that transaction prices, but not the quoted bid and ask prices, follow a martingale. Show that in equilibrium an informed trader buys if and only if he receives a high signal; and that as trading periods accumulate, the public belief converges to the true value and the bid-ask spread converges to zero.
Solution: See Glosten and Milgrom (1985).

9.2. The Glosten and Milgrom model with transaction costs. Consider the model of Section 9.1.1. Suppose that traders have to pay a transaction cost if they want to buy or sell the asset. Show that in this case an informational cascade in which all informed traders abstain from trading occurs almost surely. Furthermore, show that in this case the price does not converge to the fundamental value of the asset. (*Hint*: Recall what happens in Exercise 9.1.)

Solution: See Cipriani and Guarino (2006).

9.3 Asymptotic properties of the price discovery process (Section 9.1.3.2). Check the following asymptotic properties as t tends to infinity of the linear equilibrium in Proposition 9.1: a_t converges monotonically from below to $\left(\rho\sigma_\varepsilon^2\right)^{-1}$; τ_t and λ_t^{-1} tend to infinity at a rate of t; $\mathrm{var}\left[p_t\right]$ converges monotonically from below to σ_θ^2; $\mathrm{var}\left[p_t \middle| p_{t-1}\right]$ tends to 0; the expected volume traded by informed agents $E\left[\left|x_{it}\right|\right]$ converges from above to $(2/\pi)^{1/2}\left(\rho\sigma_\varepsilon\right)^{-1}$, and the expected total volume traded $E\left[TV_t\right]$converges from above to $(2\pi)^{-1/2}\left(\left(\rho\sigma_\varepsilon\right)^{-1}+2\sigma_u\right)$.

Solution: From the analysis of the recursive cubic equation $F_t(a)=\left(\rho\tau_\varepsilon^{-1}a-1\right)\tau_{t-1}+\rho\lambda_t a^2$, which yields the equilibrium a_t (see the proof of Proposition 9.1 in the Appendix), it follows that a_t is increasing and therefore τ_t (and $\left(\lambda_t\right)^{-1}=\tau_t/\tau_u a_t$) tend to infinity with t. In consequence, $\mathrm{var}\left[p_t \middle| p_{t-1}\right]$ tends to 0 and a_t tends to $\left(\rho\sigma_\varepsilon^2\right)^{-1}$. This implies that τ_t and $\left(\lambda_t\right)^{-1}$ are of the order of t. Furthermore, $\mathrm{var}\left[p_t\right]=\sigma_\theta^2-\mathrm{var}\left[\theta \middle| p_t\right]$ tends monotonically upwards to σ_θ^2 since $\mathrm{var}\left[\theta \middle| p_t\right]=\left(\tau_t\right)^{-1}$ tends monotonically to zero. With respect to trading volume, both $E\left[\left|x_{it}\right|\right]=(2/\pi)^{1/2}a_t\left(1/\tau_\varepsilon+1/\tau_{t-1}\right)^{1/2}$ and $\left(\rho\tau_\varepsilon^{-1}a-1\right)$

$E\left[TV_t\right]=(1/2\pi)^{1/2}\left(a_t\left(1/\tau_\varepsilon+1/\tau_{t-1}\right)^{1/2}+\left(\sigma_u^2+\left(a_t\right)^2/\tau_{t-1}\right)^{1/2}+\sigma_u\right)$ will be decreasing for t large (since then the term $1/\tau_{t-1}$ dominates) and the results follow.

9.4 A noisy strategy for the insider in the Kyle (1985) model? Consider the model of Section 9.2 and suppose that the insider in period t can add to his order normally distributed noise η_t uncorrelated with all other random variables in the model, with mean zero and variance $\sigma^2_{\eta_t}$. Market makers observe the aggregate order flow and do not observe $\sigma^2_{\eta_t}$. Show that it does not pay to set $\sigma^2_{\eta_t} > 0$ at any period.

Answer: The insider draws a realization of η_t and places the order $\Delta Y_t\left(\theta, p^{t-1}\right) = \alpha_t\left(\theta - p_{t-1}\right) + \eta_t$. The market makers do not observe $\sigma^2_{\eta_t}$ but have a conjecture (in equilibrium correct) about the insider's choice of $\sigma^2_{\eta_t}$. If the last period T the optimal level of added noise is zero because it can not affect market depth (derived from the fixed conjecture of market makers) and it distorts trading. The reasoning applies then to stage T-1 and so on.

**9.5 <u>The effect of trade disclosure in the Kyle (1985) model</u>. Consider a two period version of Kyle (1985) model presented in Section 9.2. Derive a closed-form solution for the equilibrium parameters. Consider now the case where at t = 1 the trade of the insider is disclosed. Derive the equilibrium in this case and compare it to the equilibrium in the Kyle model.

Answer: See Huddart et al. (2001).

*9.6 <u>Long lived information as a durable good</u>. Consider a two period trading model where competitive informed traders coexist with competitive risk neutral makers and noise traders as in Section 8.1.2 with T = 2. There is an information monopolist who has perfect knowledge of the fundamental value θ and who decides every period how much noise to add to the private signals sold to the traders in order to maximize his revenue. The analyst reveals information truthfully. (A static version of this model is given in Exercise 5.10.)

(1) Find out how much would a trader be willing to pay for a private signal of precision τ_{ε_t}, t = 1, 2.

(2) Find the sequence of optimal precisions of the signals sold $\left\{\tau^*_{\varepsilon_1}, \tau^*_{\varepsilon_2}\right\}$ from the point of view of the analyst and show that $\tau^*_{\varepsilon_1} < \hat{\tau}_\varepsilon < \tau^*_{\varepsilon_2}$ where $\hat{\tau}_\varepsilon = \rho^{-1}\sqrt{\tau_\theta / \tau_u}$ is the optimal precision in the static problem (Exercise 5.10). Draw an analogy of the long lived information seller with a durable goods monopolist seller that reduces durability

in the first period (Bulow (1986)) and markets a new product in the second period that makes the first period product obsolete (Waldman (1993)).

(3) Compare the solution in (2) with a market with a single risk neutral insider submitting a market order in each period (as in Proposition 9.2 with T = 2). Show that at t = 2 the insider trades less aggressively than a competitive informed trader when sold a signal of equilibrium precision and this makes the market in this period thinner and the price less informative than when there is an information monopolist. Draw an analogy with the durable goods monopolist producer that can rent the good instead of selling it in order to diminish intertemporal self-competition.

Solution: (1) Using the properties of CARA utilities (see Section 2.4 in the Technical Appendix) the maximum prices that a trader is willing to pay for observing signals with precisions $\{\tau_{\varepsilon_1}, \tau_{\varepsilon_2}\}$ (when the other traders are observing signals of precision $\{\tau_{\varepsilon_1}, \tau_{\varepsilon_2}\}$) are given by $\phi_1 = (2\rho)^{-1}\left(\ln\left(1+\tau_{\varepsilon_1}\tau_1^{-1}\right)+\ln\left(1+\tau_{\varepsilon_1}\tau_2^{-1}\right)\right)$ for τ_{ε_1} and $\phi_2 = (2\rho)^{-1}\ln\left(1+\tau_{\varepsilon_2}\left(\tau_2+\tau_{\varepsilon_1}\right)^{-1}\right)$, with τ_t the precision of prices at the competitive equilibrium: $\tau_t = \tau_\theta + \tau_u\rho^{-2}\sum_{k=1}^{t}\tau_{\varepsilon_k}^2, t = 1,2$ (see Section 8.1.2). (2) Optimize ϕ_1 and ϕ_2. (3) Use the equilibrium parameters for the T = 2 Kyle (1985) model derived in Huddart *et al.* (2001) as in Exercise 9.5. (See Cespa (2007).)

9.7 Conditional volatility in the preopening market. Consider the tatônnement model of Section 9.1.3 and explain why the conditional volatility of prices $\text{var}\left[p_t | p_{t-1}\right]$ may be hump-shaped or increasing in t, implying that the responsiveness to information of the competitive agents a_t is U-shaped or decreasing, respectively.

Answer: Use the fact that the conditional volatility of prices is increasing (decreasing) if and only if information revelation accelerates (decelerates) as t increases. More precisely, $\text{var}\left[p_t | p_{t-1}\right]$ is increasing (decreasing) in n if and only if $\text{var}\left[\theta | p_t\right]$ is concave (convex) in t.[32] When γ is low information revelation accelerates as the

[32] Note also that if $\text{var}\left[\theta | p_t\right]$ is concave in t then $\tau_t = \left(\text{var}\left[\theta | p_t\right]\right)^{-1}$ is convex in t.

tâtonnement progresses ($var\left[\theta|p_t\right]$ is concave in t). Otherwise, for larger γ's, $var\left[\theta|p_t\right]$ is first concave and then convex in t, implying that $var\left[p_t|p_{t-1}\right]$ is first increasing and then decreasing in t.

**9.8 <u>Equilibrium with demand schedules in the preopening market</u>. Consider a version of the preopening model of Section 9.3 in which both the strategic and the competitive informed traders submit demand schedules instead of market orders. At stage t, informed agent i submits a demand schedule $X_t\left(p_t;s_i,p^{t-1}\right)$, contingent on the private signal s_i he has and the past history of prices. Similarly, the insider's strategy at t is a demand function contingent in his private information and past prices, $Y_t\left(p_t;\theta,p^{t-1}\right)$. As before, noise traders are assumed to submit at round t the order u_t and market makers quote prices efficiently on the basis of public information and the aggregate limit order book, which is just a noisy version of the aggregate orders of informed agents. Competitive market making implies that p_t is a sufficient statistic for public information and therefore the strategies can be written as $X_t\left(s_i,p_t\right)$ and $Y_t\left(\theta,p_t\right)$. Show that there exists a unique linear equilibrium characterized by for t = 1, …, T:

$$\begin{aligned}
&X_t\left(s_i,p_t\right)=a\left(s_i-p_t\right)\\
&Y_t\left(\theta,p_t\right)=\alpha_t\left(\theta-p_t\right)\\
&p_t=\lambda_t\omega_t+p_{t-1}
\end{aligned}$$

where $a=\tau_\varepsilon/\rho$, $\omega_n=A_t\left(\theta-p_{t-1}\right)+u_t$, $\lambda_t=\tau_u A_t/\tau_t$, $A_t=\mu\alpha_t+\left(1-\mu\right)a$, and $\tau_t=\tau_\theta+\tau_u\sum_{k=1}^{t}A_k^2$

At stage t, the strategic informed trader's expected continuation profit is given by $E\left[\pi_{t+1}|\theta,p_t\right]=\mu h_t\left(\theta-p_t\right)^2+\mu\delta_t$.

The constants α_t, h_t and δ_t are given by the solutions to the difference equation system

$$\alpha_t = \left(\gamma_t - 2\left(1-\gamma_t\right)\mu\lambda_t h_t\right) / \left(2\gamma_t\mu\lambda_t\right)$$
$$h_t = \left(1-\lambda_{t+1}A_{t+1}\right)^2 \gamma_{t+1}\alpha_{t+1} + \left(1-\gamma_{t+1}\right)h_{t+1}$$
$$\delta_t = \gamma_{t+1}\alpha_{t+1} + \left(1-\gamma_{t+1}\right)h_{t+1}\left(\lambda_{t+1}\sigma_u\right)^2 + \left(1-\gamma_{t+1}\right)\delta_{t+1}$$

subject to the boundary conditions $h_T = 0$, $\delta_T = 0$, $2\mu\alpha_T\lambda_T = 1$ and the second order conditions $\lambda_t > 0$ for all t = 1, 2,..., T.

Hint: Get inspiration from the derivation of the equilibrium in the static model with demand schedules in Section 5.2.3. Follow the steps of the proof of Proposition 9.3 and note that the strategy of competitive informed traders is stationary, and therefore the difference equation system that characterizes the equilibrium parameters can be iterated backwards as in the proof of Proposition 9.2 (Kyle (1985)).

9.9 Volume peaks with discretionary liquidity traders. Show in the model of Section 9.4 that expected trading volume peaks in the period where discretionary liquidity trading is concentrated when $\sigma_t^2 = 1$, $\tau_{\varepsilon_t} = \tau_\varepsilon$ and $\mathrm{var}\left[u_t\right] = \sigma_u^2$ for any t.

Solution: Immediate once the expression for expected trading volume is written (get inspiration from the expressions for volume in Exercise 5.7).

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