Information and learning in markets
The impact of market microstructure
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Introduction and lecture guide

Information is at the heart of the economy today. Despite this there is still a lively debate over whether markets aggregate the dispersed information of agents in the economy or whether they are at the mercy of herd behavior and fads and in the hands of short-term speculators and insiders. The debate has old roots and goes back at least to the exchange between Hayek and Lange in the 1930s about the economic viability of socialism. While Lange (1936, 1937) argued that socialism was viable because a competitive allocation can be replicated by a central planner, Hayek contended that the superiority of markets was to be found in their ability to aggregate the dispersed information of agents in the economy (Hayek (1945, p. 526)):

“The mere fact that there is one price for any commodity – or rather that local prices are connected in a manner determined by the cost of transport, etc. – brings about the solution which (it is just conceptually possible) might have been arrived at by one single mind possessing all the information which is in fact dispersed among all the people involved in the process.”

In contrast to Hayek, Keynes in his General Theory pioneered the view of the stock market as a beauty contest where investors try to guess average opinion, instead of fundamentals, and end up chasing the crowd (Keynes, 1936, p. 136):

“…professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitor as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view.”
This view is consistent with people behaving like a herd of penguins that follow a first individual that jumps off a cliff. Indeed, it has been claimed that markets are in the hands of short-term speculators, insiders, and manipulators who induce bubbles and crashes. To this it may be added that traders are not rational to start with and that market outcomes are heavily influenced by psychological biases, behavioral rules and persistent mistakes made by decision makers.

The central issue is whether we can trust markets aggregate information in a world where information is dispersed. Does the stock market reflect the underlying fundamentals of the traded firms? Or, to put a concrete example, does the price in the futures market for wheat aggregate the relevant information of producers and speculators?

The view of Hayek is optimistic in the sense that we can trust markets to provide the right signals for decision making. Indeed, Hayek thought that the price system is a marvel because of its power in aggregating information.

In this book we will analyze in depth the information aggregation properties of markets and the impact of specific trading arrangements (the market microstructure) on the information aggregation process and the quality or performance of the market.

Hayek’s ideas are the basis for the rational expectations models that explain how rational agents make optimal inferences from prices, and other public statistics, about the relevant parameters they are uncertain.1 The statistics that aggregate the dispersed information in the economy are indicators of aggregate activity such as output, employment, investment or trade volume. The result is that learning from market statistics makes up for the lack of knowledge of traders and producers.

The book will try to disentangle whether the two views of markets, informational efficiency or herding, are compatible. Do prices in the stock market reflect all available information in the hands of traders and fundamental values … or prices are in the hands of short-term speculators, insiders, manipulators, gurus and subject to

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1 See Radner (1979) and Grossman (1989).
fads, herding behavior, and bubbles? We will explain how rational well-informed agents may disregard their private information and make persistent errors when acting. We will see indeed that we do not need irrational traders with behavioral biases and making persistent mistakes to explain, for instance, crises and bubbles, or financial market anomalies like excess volatility. The book therefore does not cover the, otherwise very interesting, contributions of the behavioral finance literature. 2

Thinking a little bit more generally we are asking in what circumstances social learning, that is, learning from others, from the actions of other people, leads to information revelation and efficient outcomes. This cuts across more disciplines than markets and economics and extends to sociology and political science. Indeed, markets are not the only mechanism to aggregate information. Deliberation is an old method. Information technology and Internet have provided with the world wide web a range of instruments to aggregate information, from blogs to wikis to the open source movement, that hint at an information aggregation revolution.3 Those instruments are based on the desire of people to build a reputation, attain status, or simply driven by altruism and conformation to a social norm. They have proved effective although they are vulnerable to manipulation and attack. Markets work based on another mechanism: people put their money were their mouth is and, on average, have proved tremendously effective in aggregating information. Prediction markets, betting on the outcome of an election for example, have emerged recently combining the features of markets and information technology aggregating procedures.4 In markets agents learn from other agents with the intermediation of the

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3 "Wikis" are web pages that can be freely edited by any user that has access to them. A leading example is Wikipedia, a free online encyclopedia that can be edited by anyone. In open source projects the source material (e.g. computer code) is available freely for other people to use and improve (typically under the condition that the outcome will be made available also freely). See Sunstein (2006).

price system or market aggregates like market shares. For example, a high market share of a brand may provide a signal of the quality of the product similarly to when we buy a best seller because we think that the book must be good. Sometimes people learn directly from the actions of other people. Depositors may run on a bank when they see others doing so, or tourists may decide not to patronize an empty restaurant when visiting a city.

All these information aggregation mechanisms share the Hayekian idea of the power of many minds in collecting information. For example, the “blogosphere” is seen by some analysts as the newest mechanism for society to pool knowledge.\(^5\) It is tempting to think that Hayek’s “one single mind possessing all the information which is in fact dispersed among all the people” is represented today by the world wide web. The main point, however, is that, much according to Hayek’s vision, the world wide web is not akin to a central planner but to a much decentralized institution with many minds at work. Furthermore, it is yet to be seen whether some of these new information aggregation methods can match the accuracy and economy of knowledge of the price system.

**Approach of the book**

This book takes the perspective of explaining aggregation of information and learning in markets with rational agents who understand market conditions and make the most of it. In particular, our agents will be Bayesian and will learn accordingly. We will leave aside therefore bounded rationality and non-Bayesian methods of learning. We concentrate on Bayesian rational models because of the discipline they introduce in the analysis, because they deliver results, including the explanation of market

even the price forecasts in the (relatively small) internal market set up by Hewlett-Packard were closer to the actual sales of the company than the official forecast.

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\(^5\) From the introduction to the Becker-Posner blog at [www.becker-posner-blog.com/archives/2004/12/](http://www.becker-posner-blog.com/archives/2004/12/): “Blogging is a major new social, political, and economic phenomenon. It is a fresh and striking exemplification of Friedrich Hayek’s thesis that knowledge is widely distributed among people and that the challenge to society is to create mechanisms for pooling that knowledge. The powerful mechanism that was the focus of Hayek’s work, as of economists generally, is the price system (the market). The newest mechanism is the “blogosphere”.”
anomalies, and because there is evidence that those models are useful in explaining the behavior of agents. Indeed, a Bayesian model requires making explicit the assumptions of the learning agent and the context of learning. It makes life harder for the researcher when trying to explain a phenomenon. Furthermore, despite the challenges to the Bayesian model coming from the experimental literature emphasizing boundedly rational and error-prone heuristics to explain cognitive judgments under uncertainty, there is recent evidence of the Bayesian optimality of human cognition in realistic scenarios as well as in experimental settings. Market pressure may imply also aggregate behavior consistent with Bayesian prediction even allowing for substantial departures for individuals agents.

Our approach will bring together results from the rational expectations literature and recent analysis of herding phenomena in a coherent game-theoretic framework. This framework will be used both when considering competitive or strategic agents. The advantage of such an approach is that the details of the environment are modeled precisely and outcomes are the results of the actions of the agents and the information they have. The approach in the book is game-theoretic but this is not a book of “learning in games”, it emphasizes the consequences of market interaction and social learning for informational and economic efficiency. The book puts together therefore dynamic models of rational expectations with Bayesian learning models, including the herding variety.

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6 See Tversky and Kahneman (1971, 1974), Kahneman, Slovic, and Tversky (1982), and Camerer (1995) and Rabin (1998) for surveys on error prone heuristics (such as making strong inferences from small samples -“law of small numbers”; confirmatory bias –where agents ignore information that contradicts beliefs held; and “representativeness” where agents tend to neglect prior probabilities). Griffiths and Tenenbaum (2006) present recent evidence of the Bayesian optimality of human cognition in realistic scenarios, and Anderson and Holt (1997), Hung and Plott (2001), Cipriani and Guarino (2005), and Drehmann et al. (2005) evidence in experimental settings.

7 Sandroni (2000, 2005) shows that Bayesian agents drive behavioral non-Bayesian agents out of the market (see the discussion in Section 7.1.1). See Jamal and Sunder (1996) and Ackert, Church and Shehata (1997) for experimental evidence.

8 See Fudenberg and Levine (1998) for a treatment of this topic.

9 In the macroeconomics field, see Evans and Honkapohja (2001) for a treatment of adaptive learning models and Ljungqvist and Sargent (2000) for an exhaustive study or recursive methods. See Guesnerie (2001 and 2006) for an assessment of rational expectations models.
The strategy of the book is to analyze the different topics considering a basic workhorse asymmetric information model in a linear-quadratic mean-variance environment. Most results will be obtained in the context of a model where optimal actions end up being a linear function of the information agents have. The model is developed in two basic versions: a partial equilibrium production market and a financial market. The book examines information aggregation mechanisms, progressing from simple to complex environments: from static to dynamic models, from competitive to strategic agents, from simple market strategies (such as non-contingent orders or quantities) to complex ones (such as price contingent orders or demand schedules).

This is a book that focuses on theoretical models.\textsuperscript{10} A non-exhaustive outline of topics with some of the basic ideas covered in the book follows.

Outline of selected themes and issues

*Herding, rational expectations and the underlying information externality*

If actions of others suggest strongly a certain action then a rational individual may ignore his own information and follow the crowd or herd. When this happens information is lost and it is not revealed to others. There is no further accumulation of information and an “informational cascade” obtains. The result is that the outcome may be very inefficient. This is an insight derived from the sequential prediction model considered in the herding literature\textsuperscript{11} which has been suggested to apply to investors and decision makers in markets. In principle, the results would give support to the pessimistic view of markets and would seem to contradict directly the rational

\textsuperscript{10} The reader is refereed to Hasbrouck (2007) for a recent survey of empirical market microstructure models.

\textsuperscript{11} See Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992), and Chamley (2004) for an assessment.
expectations literature with its emphasis on the informational properties of markets following Hayek’s ideas.

We provide an analytical framework that makes apparent that both rational expectations and herding models have in common a basic information externality. Indeed, a trader or agent when acting does not take into account the informational benefits of his action on the other traders/agents. When the trader/agent cannot fine tune his action to his information (e.g. with discrete actions) and receives signals of bounded strength then herding on a wrong action occurs with positive probability. This cannot happen with a rich choice set, in which case information will end up being revealed. However, even then with noisy observations revelation will be slow.

*Slow learning from others and welfare*

Traders learning about the fundamentals of the market will typically learn slowly over time from a noisy public statistic (e.g. the price). A rational (Bayesian) trader will respond to increased price informativeness with decreased weight to his private information when formulating his trade. This will mean that less of his private information will be incorporated into the public statistic. The result will be that the increase in informativeness in the public statistic will be smaller the larger is his absolute level, slowing down information revelation. Learning from others has a self-correcting property.

What are the welfare consequences? Welfare analysis has been typically neglected in herding models. We introduce an appropriate welfare benchmark with private information. This is team-efficiency where agents use decentralized decision rules but have as common objective the welfare of a representative agent. The information externality implies that the market outcome is inefficient with respect to this team benchmark which internalizes the externality. Herd behavior or informational cascades are extreme manifestations of the self-correcting aspect of learning from others. With discrete action spaces and signals of bounded strength public information may end up overwhelming the private signals of the agents, who may (optimally) choose not to act on their information. More in general, “herding” means that agents put too little weight on their private information with respect to the team benchmark and results in under-accumulation of public information. An interesting possibility is
that more public information may hurt. The reason is that a higher amount of public information may reduce the effort to collect private information. A similar analysis can be performed in rational expectations models. The bottom line is that the welfare analysis of “rational expectations” and “herding” models is not qualitatively different.

Information externalities and payoff externalities
With social interaction there is typically both an informational externality and a payoff externality. In a pure prediction model there is only an information externality. The question is how do the informational and the payoff externalities interact? In the first place we show in a static model that informational efficiency is not the same as economic efficiency. We will see that prices in a rational expectations equilibrium of a market with substitute products will tend to convey “too little” information when the informational role of prices prevails over its “index-of-scarcity” role and “too much” in the opposite situation. The market will be team-efficient only in exceptional circumstances (i.e. when the information externality vanishes).

Explaining market dynamics
Do prices converge to full-information values as trading periods accumulate? Is the market an effective price discovery mechanism in the presence of asymmetric information? If so, how fast? How long does the adjustment process takes? How does the payoff externality modify the result of slow learning from others in prediction models?

The results obtained in prediction models generalize to market settings. Traders learn the unknown parameters and prices converge to full information values with repeated interaction in a stationary market environment. However, learning an unknown parameter and converging to full information equilibrium is not equivalent. For example, with serial correlation in period shocks it is possible to learn an unknown parameter at a lower speed than converge to a full information equilibrium. In non-stationary environments learning and convergence are not equivalent.

We find that in market models where traders learn from prices about the information of other traders about a valuation parameter, like in a financial market, learning may be fast because of the payoff externality induced by market makers. Market makers
induce a deeper market as more information about the fundamental value is revealed by the trading process. With discrete actions, as in the simple prediction models, there is no herding; with a rich choice set and noise in public information convergence to a full information equilibrium is fast because risk averse informed traders respond more intensely to their private signals as market makers induce a deeper market. The role of a competitive market making sector proves crucial for price discovery. Its presence implies that current prices reflect all public information.

The impact of market microstructure
The information aggregation properties of the market depend on the details of market organization or market microstructure. First of all, information aggregation depends on whether the market mechanism is smooth, like a Cournot market, or has the winner-take-all feature, like auction or voting mechanisms. The latter aggregate information better than the former both in the sense that they aggregate information in more circumstances and that they do so more economically, that is, without the necessary concourse of a very large number of agents. Second, information aggregation depends on whether agents use non-contingent strategies, like market orders, or price-contingent ones, like limit orders or demand schedules in financial markets. The more complex contingent strategies do better at aggregating information. This also affects the incentives to acquire information. Third, it also matters whether the market is order- or quote-driven. In order-driven markets informed traders move first while in quote-driven markets uninformed market makers move first.12

Can “market anomalies” be explained?
Market anomalies, seemingly inconsistent with rational expectations models, can in fact be explained without recourse to the irrationality of traders. Excess volatility, where asset prices are more volatile than fundamental values, can be consistent with dynamic trading models where market makers are risk averse and do not accommodate fully shocks. The analysis of patterns of stock prices, or “technical

12 The term market microstructure in finance was coined by Garman (1976) in a piece of work about market making. Brunnermeier (2001) provides a nice introduction to market microstructure models in finance.
analysis”, is valuable if the current price is not a sufficient statistic for public information about the fundamental value. This is typically so except if market makers are risk neutral. However, learning from past prices may be slow. The presence of traders with short horizons may reduce the informational efficiency of the market and explain incentives for investors to herd in information acquisition. For example, short-term traders may care more about the information that other short-term traders have than about the fundamentals, like in Keynes’ beauty contest, because of strategic complementarities in information acquisition. Similarly, discrepancies between the average expectations of investors and stock prices, which sometimes may look like a bubble, can be rationalized in the presence of short-term traders who induce multiple market equilibria. More in general, price dynamics in financial markets may have a “Keynesian” or a “Hayekian” flavor depending on parameters such as the degree of persistence of shocks or the amount of residual uncertainty on the liquidation value of the asset.

**Crises and crashes**
Sudden and significant drop in asset prices, even in the absence of major news, may be explained with our models. For example, by an abrupt information revelation about the quality of information of investors brought by a small price movement; by a misinterpretation of a price drop due to underestimation of the extent of portfolio trading generating multiple equilibria; or by a liquidity shortage as a result of an underestimation by market timers of the degree of dynamic hedging activity. Crises that arise from a coordination problem of investors, such as exchange rate crises or bank runs, leading to multiple equilibria can be explained with the interaction of private and endogenously generated public information. Policy implications may be derived from the analysis. For example, it need not be true that more transparency is always good since it may coordinate the expectations of investors in a bad equilibrium.

**The impact of large traders**
Large informed traders are more cautious when responding to their private signals because they are aware of the price impact of their trades. The result is that prices are less informative in their presence. Insiders, when forced to reveal their trades, will engage in dissimulation strategies. Otherwise, they may have incentives to manipulate
the market and slow down information revelation. For example, they may do so by using a contrarian strategy to neutralize the potential revelation of information by the trades of competitive informed agents. The effects of insider trading are multiple. Insider trading generates adverse selection, inducing market makers to protect themselves by making the market thinner, and advances the resolution of uncertainty, increasing price informativeness. The first effect tends to be bad for investment and welfare. The impact of the second effect, under risk aversion, depends on what is the alternative to insider trading. If the alternative is that information is disclosed then insider trading may be good because too much information disclosure destroys insurance opportunities. If the alternative is that no information is collected then insider trading will typically be bad for welfare.

A relevant issue from the point of view of policy is whether market power or asymmetric information loom larger in accounting for welfare losses in relation to full information competitive equilibria. The general result is that market power dissipates faster then asymmetric information as a market grows large and supports more traders, and therefore in moderately sized markets asymmetric information is likely to be a larger source of the welfare loss. This is the case in Cournot markets for example.

Some conclusions
Apparently contending theories, such as market informational efficiency and herding, build in fact on the same principles of Bayesian decision making. The upshot is that we do not need “irrational” agents to explain herding behavior, crises, and crashes. Traders may be rational but learn slowly or even in some occasions disregard private information and end up making persistent errors when acting; or be trapped in a coordination failure. However, informational and economic efficiency need not coincide and in general will not. In any case the impact of market microstructure on the informational efficiency of prices is crucial. The stock market conveys information about the fundamentals of firms although the misalignment of stock prices and fundamentals is possible but typically not extremely long lived.

Outline of chapters
In Chapters 1, 2, 3 and 7 we consider mostly “real” partial equilibrium markets, with Cournot competition as the leading example, although we study also auctions, supply function competition and its relation to rational expectations equilibria (in Chapter 3). In Chapters 4, 5, 8 and 9 we consider financial markets and spend time explaining how the market microstructure affects trading and the informational efficiency of prices. The equivalent of Cournot competition in a financial setting is when traders submit market orders, while the equivalent of supply function competition is when traders submit demand schedules to a centralized trading mechanism. Chapters 1, 3, 4 and 6 deal with competitive models and chapters 2, 5, and 9 (mostly) with models of strategic traders. Chapters 1 to 5 deal with static models and chapters 6 to 9 with dynamic models. Chapter 6 builds a bridge between stylized social learning models and dynamic rational expectations models.

Let us review in some more detail the content of the chapters and the organization of the book.

Chapters 1 and 2 study market mechanisms, such as auctions or Cournot markets, in which traders do not have the opportunity to condition their actions on prices or other market statistics. This provides a benchmark in which traders can use only their private information to decide how much to trade. In this framework we study under what circumstances the market replicates the outcome of shared information equilibrium. If it does we say that the market aggregates information. Chapter 1 studies first in detail large Cournot markets with demand uncertainty and asymmetric information and uncovers when information aggregation obtains, providing a taxonomy and a welfare analysis of the value of information in a general setting. When information aggregation obtains in a “large” market a second issue of interest is how large a market is needed for the result. In any case, as a market grows large strategic behavior vanishes and price-taking obtains.

Chapter 2 analyzes the convergence properties of finite markets to price-taking equilibria as the market gets larger and the number of agents grows. When the limit is first best efficient, with full information aggregation, we compare the rates of information aggregation in different market mechanisms and disentangle the sources of the welfare loss at the market solution in terms of market power and private
information. Both Chapter 1 and Chapter 2 compare the information aggregation properties of smooth market mechanisms, like Cournot markets, with auctions.

Chapter 3 introduces the concept of rational expectations equilibrium (REE), with its different variants, fully revealing REE, partially revealing REE, and noisy REE, in the context of a static partial equilibrium homogenous product market with demand and cost uncertainty and a continuum of firms. It is shown that not all REE are implementable. That is, not all REE can be seen to be the outcome of a well-specified game among market participants. A game form is considered then in which firms compete in supply functions and Bayesian equilibrium in supply functions is studied. The chapter goes on to perform a welfare analysis of REE introducing the appropriate team-efficiency benchmark that internalizes information externalities. That is, the chapter checks the alignment of informational efficiency and allocative and productive efficiency. The information externality at the market equilibrium is characterized and related to economic efficiency. The chapter also considers the information aggregation properties of the double auction mechanism.

Chapters 4 and 5 look at the basic static financial market models with asymmetric information and review the impact of different market microstructures with competitive (in Chapter 4) and strategic (in Chapter 5) players. The standard competitive noisy rational expectations financial market is presented along with variations in which informed traders submit market orders to market makers. A model is presented that has as special cases virtually all the competitive models in the literature. The potential paradox of informationally efficient markets when information is costly to acquire is addressed. Chapter 5 goes on to study strategic behavior in different market structures with simultaneous and sequential order placement, uniform and discriminatory pricing, and distinguishing the cases in which informed traders move first and those in which uninformed traders move first.

Chapter 6 considers social learning models. It presents first the basic sequential decision herding model and variations. It highlights the assumptions that drive the results and develops a basic smooth model of learning from others that serves as a benchmark for the analysis and that is close to rational expectations models. It is found that the basic driving forces in herding and rational expectations models are the
same. A self-correcting property of learning from others and its implications in terms of information revelation and its speed are at the center of the chapter. Furthermore, the information externality of social learning is characterized and optimal learning studied. Results are extended to the case where information acquisition is costly and the value of public information studied. Finally, a static version of the model of learning from others with a rational expectations flavor is presented and a welfare analysis is performed.

Chapter 7 deals with dynamic information aggregation models and studies their convergence properties as the number of trading periods grows without bound. The basic model considered is a repeated Cournot model with an uncertain demand parameter and period specific shocks with firms receiving private signals about the unknown demand. The rate of convergence to full-information equilibrium is characterized in the cases of independent and correlated shocks to demand. Results for the latter case make clear that learning an unknown parameter is not equivalent to converging to full-information equilibrium. A variation of the model with an unknown cost parameter is considered and it shown that learning and convergence to full-information equilibrium may be slow. The latter model is a model of learning from others and extends the results of Chapter 6 to an environment with payoff externalities.

Chapters 8 and 9 study dynamic competitive and strategic models of financial markets. They address the difficult issue of solving for and characterizing an equilibrium when risk averse traders have private information and a long horizon. The resolution of the problem illuminates trading and price temporal patterns and serves as a benchmark to compare with the case in which informed speculators have short horizons. A detailed analysis of the impact of market microstructure on the dynamic properties of market quality parameters, such as price volatility and informativeness, market depth, and volume traded, is performed. These chapters provide explanations for technical analysis, trading with no news, excess volatility, and market crashes, as well as the impact of short-term traders. We address also the interaction between the potential coordination failure of investors, e.g. in an exchange rate crisis, and endogenously determined market signals to generate multiple equilibria. All this is done without introducing any irrationality on the part of traders. The properties of a
price discovery mechanism are studied in both competitive and strategic versions showing the importance of the market microstructure and the presence of strategic traders in the speed of price discovery. The trading strategies of large informed traders are analyzed, including the possibility of dissimulation of trades and market manipulation.

The Technical Appendix presents in an accessible way the basic tools needed to follow comfortably the material in the book. These include information structures with particular attention to the Gaussian and related models, convergence properties of random processes, and game-theoretic concepts (such as Bayesian equilibrium). It includes in particular a concise self-contained development of the linear-Gaussian model.

Road map and lecture guide
The book may be read according to different field itineraries:

- Real markets: Chapters 1, 2, 3, 6, 7.
- Financial markets (Market Microstructure in the strict sense): Sections 1.1, 1.2, 3.1, chapters 4, 5, 6, 8, 9.

Alternatively, it may be read according to the following segments:
1. Static models: Chapters 1, 2, 3, 4, 5.
2. Dynamic models: Chapters 6, 7, 8, 9.

Or
i) Competitive models: Chapters 1, 3, 4, 6, 7, 8, Section 9.1.
ii) Strategic models: Chapter 2, Section 3.4, chapters 5, 9.

Readers familiar with Bayesian updating techniques and the basics of rational expectations interested only in financial markets may jump directly to Chapter 4 after reading Sections 1.1, 1.2, and 3.1. Readers who are on top familiar with static models may jump directly to Chapter 6. (Readers who are on top familiar with dynamic information models may consider stopping reading here.)
The level of difficulty of chapters is pretty homogenous with dynamic models being in general more demanding. Chapters 2 and 7 are more demanding and may be skipped in a first reading without major loss of continuity. The same applies to sections which are more advanced (marked with a *). Instructors who use the text for advanced undergraduates or master students with little formal background in economics may want to introduce the major results with the examples which follow usually the major propositions, and present the general results later on. Each chapter is closed by a summary and an appendix with proof of results in the text, and worked out exercises. The level of difficulty of problems varies from very challenging (marked **) because of inherent difficulty or the amount of work they require; challenging (marked *); and easy (with no mark).
References


Some of the material presented in the book has been adapted from the following publications:


