Information and Learning in Markets

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Chapter 4

Rational Expectations and Market Microstructure in Financial Markets

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Plan of the Chapter

In this chapter we look at:

1. Some definitions related to the microstructure of stock markets.
2. Formal analysis of how information is *(i)* impounded into and *(ii)* reflected by prices in static, competitive markets
   1. Does it make a difference if informed traders move first?
   2. Do prices reflect information or noise?
   3. What determines the liquidity , the volume and the volatility of a market?
   4. What drives the incentives to acquire information?
3. Formal analysis of how the welfare of different market participants depends on the informational properties of the market.

4.1 Market Microstructure
4.1.1 Types of Orders

Main types of orders:♣

- **Market Orders**
  - Specifies a quantity to be bought or sold at whatever price the market determines.
  - It incorporates price execution risk.
  - Akin to a quantity strategy in a Cournot Market.
- **Limit Orders**
  - Specifies a quantity to be bought (sold) and a *limit* price below (above) which to carry the transaction.
  - Limits price execution risk, but the transaction could be delayed or not executed at all if the conditioning price cannot be matched.
- **Stop Orders**
  - Like a limit order but with "inverted" limits, specifying a quantity to be sold (bought) and a limit price below (above) which to carry the transaction. If the price goes below (above) a certain limit, the asset is sold (bought) to "stop" losses (to profit from raising prices).

4.1 Market Microstructure

4.1.2 Trading Systems (I)

Main trading systems:

- **Order-driven**
  - Traders place orders before prices are set either by market makers or by a centralized mechanism or auction.
  - Trading can be continuous or in batches at discrete intervals.
  - In many continuous systems the order submission is against a limit order book where orders have accumulated.
  - Batch auction to open continuous trading (e.g. Paris Bourse, Deutsche Börse, Tokyo Stock Exchange).
- **Quote-driven**
  - Market makers set bid and ask prices (i.e. the price at which they are willing to buy and sell the asset) and traders submit orders.
  - Continuous dealer market: a trader can get immediate execution from the market maker.
- Many trading mechanisms feature both systems $\Rightarrow$ trade at NYSE starts with a batch auction and then continues as a dealer market.

4.1 Market Microstructure
4.1.2 Trading Systems (II)

Adverse selection problem:

- Market makers face an adverse selection problem as traders may possess private information on the asset return.
- Order-driven system has a *signalling* flavour since the (potentially) informed party moves first.
- Quote-driven system has a *screening* flavour since the (potentially) uninformed party moves first proposing a schedule to informed traders.

4.1 Market Microstructure

4.1.2 Trading Systems (III)

- **Pricing rule**
  - Uniform pricing: all units are transacted at the same price $\Rightarrow$ Batch auctions.
  - Discriminatory pricing: different units can be sold at different prices $\Rightarrow$ Limit order book.
- **Transparency**
  - Information on current quotes.
  - Information on past quotes and transaction sizes (“ticker tape”).
- **Fragmentation**
  - Fragmented: different transactions are cleared by different dealers at (potentially) different prices.
  - Centralized: all transactions are cleared at the same quote.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

Competitive rational expectations equilibrium model with differential information (Hellwig (1980), Grossman and Stiglitz (1980), Admati (1985), and Vives (1995)).♣

Model

- Single, risky asset with random liquidation value θ and riskless asset (with unitary return) are traded by
- Risk averse agents in the interval $[0, 1]$ endowed with the Lebesgue measure and "noise traders."
- The utility derived by a trader i for the profit $\pi_i = (\theta - p)x_i$ of buying x_i units of the asset at price p is of the CARA type: $U(\pi_i) = -\exp\{-\rho_i \pi_i\}$, where $\rho_i > 0$ is the CARA coefficient.
- Initial wealth of each trader i is normalized to 0 (wlog).
- Trader i is endowed with a piece of private information about θ. Noise traders are assumed to trade for liquidity reasons submitting a random trade u.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- Suppose that a fraction of traders $\mu \in [0,1]$ receives a private signal s_i about θ while the complementary fraction does not.
- Both classes of traders condition their orders on the price p.
- The information set of an informed trader is thus $\{s_i, p\}$, while that of an uninformed trader is $\{p\}$. Let $\rho_i = \rho_I > 0, \forall i \in [0,\mu]$ and $\rho_i = \rho_U \geq 0, \forall i \in (\mu, 1]$.
- All random variables are normally distributed: $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, $s_i = \theta + \epsilon_i$, $\epsilon_i \sim N(0, \sigma_\epsilon^2)$, and $u \sim N(0, \sigma_u^2)$ (where θ and ϵ_i, and u are pairwise independent).
- Convention: given θ the average signal of a positive mass μ of agents $(1/\mu)\int_0^\mu s_i di = \theta$ a.s.
- The distributional assumptions are common knowledge.
- Notation: we denote the precision of x by $\tau_x = (1/\sigma_x^2)$.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

We look for symmetric equilibria in linear strategies.

Definition

A symmetric rational expectations equilibrium (REE) is a set of trades, contingent on the information traders have $\{X_I(s_i, p) \text{ for } i \in [0, \mu]; X_U(p) \text{ for } j \in (\mu, 1]\}$, and a price functional $P(\theta, u)$ such that:

1. Markets clear:

$$\int_0^{\mu} X_I(s_i, p) di + \int_{\mu}^{1} X_U(p) dj + u = 0 \, \text{(a.s.)}.$$

2. Traders in $[0, 1]$ optimize:

$$X_I(s_i, p) \in \arg\max_z E\left[U_i((\theta - p)z) | s_i, p\right]$$
$$X_U(p) \in \arg\max_z E\left[U_j((\theta - p)z) | p\right],$$

for $i \in [0, \mu]$, $j \in (\mu, 1]$.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- Traders correctly conjecture the relationship between the price $P(\cdot,\cdot)$ and the couple (θ, u), and on the basis of it, they update their beliefs. As the price is not invertible in the signal, the equilibrium is *noisy*.
- Grossman (1976). Case of a market with a finite number of informed traders, no uninformed traders, and no noise: the price is strong-form efficient.
- The equilibrium has paradoxical features: demands are independent of private signals and prices!
  1. Demands are independent of private signals because the price is fully revealing, that is, the price is a sufficient statistic for θ.
  2. Demands are also independent of prices because a higher price apart from changing the terms of trade (classical substitution effect) also raises the perceived value of the risky asset (information effect).
- In the model the two effects exactly offset each other (see Admati (1989)).

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- However, this equilibrium is not implementable: the equilibrium cannot be derived from the equilibrium of a well-defined trading game.
- For example, how is it that prices are sufficient statistics for the private information in the economy?
- In the Grossman economy each trader is not *informationally small*: his signal is not irrelevant when compared with the pooled information of other traders.
- There is a natural game in demand schedules which implements partially revealing REE in the presence of noise as a Bayesian equilibrium in the continuum economy.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- Note that with a continuum of traders each agent is informationally "small."
- In the continuum economy there is always a trivial FRREE where traders are indifferent about the amounts traded and end up taking the counterpart in the aggregate of noise traders. This FRREE is not implementable and would not be an equilibrium if we were to insist that prices be measurable in excess demand functions as in Anderson and Sonnenschein (1982).

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

Suppose traders use demand schedules as strategies (parallel to firms using supply functions as strategies).

- At the interim stage, once each trader has received his private signal, traders submit demand schedules contingent on their private information (if any), noise traders place their orders, and then an auctioneer finds a market clearing price (as in (i) of the above definition of a REE).
- We will study the linear Bayesian equilibria of the demand schedule game.
- Traders optimize taking into account the (equilibrium) relationship of prices with the random variables in the environment (θ and u).
- Trader i's strategy is a mapping from his private information to the space of demand functions (correspondences more generally). Let $X_I(s_i, \cdot)$ be the demand schedule chosen by an informed trader when he has received signal s_i.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- When the signal of the trader is s_i and the price realization is p the desired position of the agent in the risky asset is then $X_I(s_i, p)$.
- Similarly, for an uninformed trader the chosen demand schedule is represented by $X_U(p)$.
- Noise traders' demands aggregate to the random variable u.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- We restrict attention to linear equilibria with price functional of the form $P(\theta, u)$.
- Linear Bayesian equilibria in demand functions will be necessarily noisy (i.e. $\partial P/\partial u \neq 0$) since, as we have argued, a fully revealing equilibrium is not implementable.
- If traders receive no private signals then the price will not depend on the fundamental value.

4.2 Competitive Rational Expectations Equilibria
4.2.1 The CARA-Gaussian Model

Let us determine a trader's optimal strategy (Reminder):

- Every trader i's profit $\pi_i = (\theta - p)x_i$, is conditionally normally distributed given the assumption of price linearity in θ and u.
- As a consequence, a trader i chooses his trade in order to maximize:

$$\begin{aligned} E\left[U(\pi_i)|G\right] &= E\left[-\exp\{-\rho_i \pi_i\}|G\right] \\ &= -\exp\left\{-\rho_i \left(E[\pi_i|G] - \frac{\rho_i}{2}\mathrm{Var}[\pi_i|G]\right)\right\}. \end{aligned}$$

- Which is equivalent to the maximization of

$$E[\pi_i|G] - \frac{\rho_i}{2}\mathrm{Var}[\pi_i|G] = E[(\theta - p)|G]x_i - \frac{\rho_i}{2}\mathrm{Var}[(\theta - p)|G]x_i^2.$$

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

- This is a concave problem whose first order condition yields:

$$x_i = \frac{E[\theta|G] - p}{\rho_i \mathrm{Var}[\theta|G]}, \quad G = \{s_i, p\} \ (G = \{p\}) \text{ for the informed (uninformed).}$$

- Owing to the assumed symmetric signal structure for informed traders, demand functions will be symmetric in equilibrium.
- Substituting the optimal demand function into the trader i's objective function yields:

$$-\exp\left\{-\rho_i\left(E[\pi_i|G] - \frac{\rho_i}{2}\mathrm{Var}[\pi_i|G]\right)\right\} = -\exp\left\{-\frac{(E[\theta - p|G])^2}{2\mathrm{Var}[\theta - p|G]}\right\}.$$

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

To solve for the equilibrium one can either:

1. Conjecture a linear equilibrium price functional $p = P(\theta, u)$.
2. Using this conjecture, compute traders' updated beliefs about θ.
3. Using these, determine demand functions and, imposing market clearing, find the actual relationship between p and (θ, u).
4. Finally, impose that the price conjecture must be self-fulfilling to pin down the coefficients of the price functional.

Or

1. Conjecture equilibrium linear strategies for traders.
2. Using this conjecture, and imposing market clearing, find the (linear) relationship between p and (θ, u).
3. Use it to update beliefs about θ and determine demand functions.
4. Finally, identify the coefficients of the demand functions imposing consistency between conjectured and actual strategies.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

The following proposition characterizes the linear REE:

Proposition

Let $\rho_I > 0$ and $\rho_U > 0$. There is a unique Bayesian linear equilibrium in demand functions. It is given by:

$$X_I(s_i, p) = a(s_i - p) - b_I(p - \bar{\theta}),$$
$$X_U(p) = -b_U(p - \bar{\theta}),$$

where $a = (\rho_I)^{-1}\tau_\epsilon$, and

$$b_I = \frac{\tau_\theta}{\rho_I + \mu\tau_\epsilon\tau_u(\mu\rho_I^{-1} + (1-\mu)\rho_U^{-1})}, \qquad b_U = \frac{\rho_I}{\rho_U} b_I.$$

In addition, $p = \bar{\theta} + \lambda z$, where $z = \mu a(\theta - \bar{\theta}) + u$, and

$$\lambda = \frac{1}{\mu(a + b_I) + (1-\mu)b_U}.$$

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

Using the second approach we start by

1. Conjecture equilibrium strategies of the form $X_I(s_i, p) = as_i - c_I p + \hat{b}_I$, $X_U(p) = -c_U p + \hat{b}_U$.
2. Impose market clearing:

$$\int_0^\mu X_I(s_i, p)\,di + \int_\mu^1 X_U(p)\,dj + u = 0,$$

and obtain $p = \lambda(\mu a\theta + u + \tilde{b})$, where $\tilde{b} = \mu\hat{b}_I + (1-\mu)\hat{b}_U$, and $\lambda = (\mu c_I + (1-\mu)c_U)^{-1}$. Let $\mu a > 0$, then the equilibrium price is a linear transformation of the random variable $\hat{z}$:

$$\hat{z} \equiv \theta + \frac{1}{\mu a}u = \frac{p - \lambda\tilde{b}}{\lambda\mu a}.$$

Hence, $\mathrm{Var}[\theta|p] = \mathrm{Var}[\theta|\hat{z}]$ and using standard normal results we have

$$\mathrm{Var}[\theta|\hat{z}] \equiv \tau^{-1} = (\tau_\theta + \tau_u(\mu a)^2)^{-1}$$

$$E[\theta|p] \equiv E[\theta|\hat{z}] = \frac{\tau_\theta\bar{\theta} + (\mu a)^2\tau_u\hat{z}}{\tau} = \frac{\tau_\theta\bar{\theta} + \mu a\tau_u\lambda^{-1}(p - \lambda\tilde{b})}{\tau}.$$

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

3. From optimization of the CARA utility function for the uninformed we have
$$X_U(p) = \frac{E[\theta|p] - p}{\rho_U \mathrm{Var}[\theta|p]} = -c_U p + \hat{b}_U,$$
and identifying coefficients yields
$$c_U = \frac{1}{\rho_U}\left(\tau - \frac{\mu a \tau_u}{\lambda}\right) \text{ and } \hat{b}_U = \frac{\tau_\theta \bar{\theta} - \mu a \tau_u \tilde{b}}{\rho_U}.$$

4. From optimization of the CARA utility function for the informed we have
$$X_I(s_i, p) = \frac{E[\theta|s_i, p] - p}{\rho_U \mathrm{Var}[\theta|s_i, p]} = a s_i - c_I p + \hat{b}_I,$$
where $(\mathrm{Var}[\theta|s_i, p])^{-1} = \tau + \tau_\epsilon$. Furthermore
$$E[\theta|s_i, p] = E[\theta|s_i, \hat{z}] = \frac{\tau_\epsilon s_i + \tau_\theta \bar{\theta} + (\mu a)^2 \tau_u \hat{z}}{\tau + \tau_\epsilon}$$
$$= \frac{\tau_\epsilon s_i + \tau_\theta \bar{\theta} + \mu a \tau_u \lambda^{-1}(p - \lambda \tilde{b})}{\tau + \tau_\epsilon}$$

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model

4. Identifying coefficients: $a = \rho_I^{-1}\tau_\epsilon$, $c_I = \rho_I^{-1}(\tau_\epsilon + \tau - \mu a \tau_u \lambda)$ and $\hat{b} = \rho_I^{-1}(\tau_\theta \bar{\theta} - \mu a \tau_u \tilde{b})$.
5. It follows that

$$\lambda = \frac{1 + \mu a(\mu\rho_I^{-1} + (1-\mu)\rho_U^{-1})\tau_u}{\mu a + (\mu\rho_I^{-1} + (1-\mu)\rho_U^{-1})\tau} \text{ and } \tilde{b} = (\lambda^{-1} - \mu a)\bar{\theta}.$$

6. From these expressions we obtain $\hat{b}_I = b_I\bar{\theta}$ where

$$b_I = \frac{\tau_\theta}{\rho_I + \mu\tau_\epsilon\tau_u(\mu\rho_I^{-1} + (1-\mu)\rho_U^{-1})}, \text{ and } c_I = a + b_I,$$

   and $\hat{b}_U = b_U\bar{\theta}$, where $b_U = c_U = \rho_I(\rho_U)^{-1} b_I$ and the expressions for the strategies follow.
7. The expression for the price $p = \lambda z + \bar{\theta}$, with $z = \mu a(\theta - \bar{\theta}) + u$ follows from $p = \lambda(\mu a \theta + u + \tilde{b})$ and the expressions for λ and $\tilde{b}$.

QED

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model-Remarks

- Uninformed traders

$$X_U(p) = -b_U(p - \bar{\theta}),$$

  - Since $b_U > 0$, uninformed traders buy (sell) when the price is above (below) the prior expectation: they "lean against the wind" as market makers do.
  - Uninformed traders face an adverse selection problem: they do not know whether they are trading with a more informed trader or with a noise trader. If $\rho_U = \rho_I$, b_U decreases in μ.

- Informed traders

$$X_I(s_i, p) = a(s_i - p) - b_I(p - \bar{\theta})$$

  - Trade for two reasons: they speculate on private information and also
  - Act as market makers

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model-Remarks

- Price
  - Expressing traders demands as follows:

$$X_I(s_i, p) = \rho_I^{-1}(\tau + \tau_\epsilon)(E[\theta|s_i, p] - p)$$
$$X_U(p) = \rho_U^{-1}\tau(E[\theta|p] - p)$$

    and imposing market clearing yields:

$$p = \frac{\rho_I^{-1}(\tau + \tau_\epsilon)\int_0^\mu E[\theta|s_i, p]\,di + (1-\mu)\rho_U^{-1}\tau E[\theta|p] + u}{\mu\rho_I^{-1}(\tau + \tau_\epsilon) + (1-\mu)\rho_U^{-1}\tau},$$

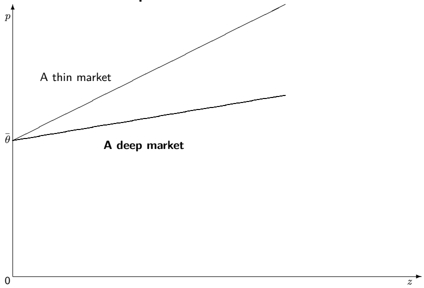
  - The price is a weighted average of investors' expectations about the liquidation value (plus a noise component reflecting the risk premium required to absorb noise traders' demand).
  - If $\rho_U \to 0$, $p \to E[\theta|p]$ and the price is semi-strong efficient.
  - If $\mu = 0$, $p = \bar{\theta} + \rho_U \tau_\theta^{-1} u$.
  - If $\mu = 1$, $p = \int_0^1 E[\theta|s_i, p]\,di + \rho_I(\tau + \tau_\epsilon)^{-1} u$.

4.2 Competitive Rational Expectations Equilibria
4.2.1 The CARA-Gaussian Model-Remarks

- Market depth ♣
  - It is captured by λ^{-1}, i.e. the change in price due to a 1-unit change in noise traders' demand:

$$\lambda = \frac{1}{\mu(a + b_I) + (1 - \mu)b_U}.$$

  - The market is *deep* (*thin*) when a unit change in u has little (large) effect on the price.



Depth increases with τ_θ and decreases with τ_u and ρ_U.

4.2 Competitive Rational Expectations Equilibria
4.2.1 The CARA-Gaussian Model-Remarks

- Price informativeness ♣
  - The random variable $z = \mu a(\theta - \bar{\theta}) + u$ captures the informational content of the price.
  - Note that $E[p] = \bar{\theta}$. However, the price is biased in the sense that $E[\theta|p] \neq p$.
  - Using the uninformed strategy $X_U(p) = -\rho_I(\rho_U)^{-1}(E[\theta|p] - p)$, yields

    $$E[\theta|p] - p = \frac{\rho_I b_I}{\tau}(\bar{\theta} - p).$$

    The price is below (above) its "public expectation" whenever uninformed traders buy (sell).
  - Price precision is captured by

    $$(\mathrm{Var}[\theta|p])^{-1} \equiv \tau = \tau_\theta + (\mu a)^2 \tau_u.$$

  - It reflects the "amount" of information contained in the price. If the price is fully revealing, $p = \theta$ and τ is infinite. If p is pure noise, $\tau = \tau_\theta$.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model-Remarks

- Volatility ♣
  - It is captured by
$$\mathrm{Var}[p] = \lambda^2 \frac{\tau}{\tau_u \tau_\theta}.$$
  - It depends negatively on market depth λ^{-1} and positively on price precision τ, prior volatility τ_θ^{-1} and noise trading τ_u^{-1}.
- Expected traded volume of informed ♣
  - It is given by
$$E\left[\left|\int_0^\mu X_I(s_i, p)\, di\right|\right] =$$
$$\mu\left(\sigma_\theta^2 a^2 \left(1-(a+b_I)\lambda\mu\right)^2 + \sigma_u^2 (a+b_I)^2 \lambda^2\right)^{1/2} \sqrt{\frac{2}{\pi}},$$
since if $x \sim N(0, \sigma_x^2)$, then $E[|x|] = \sigma_x \sqrt{2/\pi}$.
  - If $\sigma_u \to 0$, $b_I \to 0$, and $\lambda \to 1/(\mu a)$ and the expected volume of informed traders vanishes.
  - In this case price precision goes to infinity and informed traders completely lose their informational advantage.
  - No trade theorem.

4.2 Competitive Rational Expectations Equilibria

4.2.1 The CARA-Gaussian Model-Examples

- No informed traders: $\mu = 0$.
  - This is a REE without asymmetric information.
  - In this case, $b_U = \tau_\theta / \rho_U$ and $\lambda = 1/b_U$.
  - If $\mu > 0$, then $b_U < \tau_\theta / \rho_U$, owing to the adverse selection problem.
- No uninformed traders: $\mu = 1$.
  - This case corresponds to the limit equilibrium of Hellwig (1980).
  - Informed traders "make" the market, $a = \rho_I^{-1} \tau_\epsilon$, $b_I = (\rho_I + a\tau_u)^{-1} \tau_\theta$, $\tau = \tau_\theta + a^2 \tau_u$, and

$$\lambda = \frac{\rho_I + a\tau_u}{\tau + \tau_\epsilon}.$$

- Competitive risk-neutral market makers: $\rho_U \to 0$.
  - This corresponds to the static model in Vives (1995).
  - In this case, informed withhold from market making, the price is semi-strong efficient $E[\theta|p] = p$, $\lambda = \mu a \tau_u / \tau$, and $\tau = \tau_\theta + (\mu a)^2 \tau_u$.

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

Grossman and Stiglitz (1980) ♣

- Suppose all informed traders observe the same signal s and that $\theta = s + \epsilon$, with $s \sim N(\bar{s}, \sigma_s^2)$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$ independent.
- The liquidation value if the sum of two components, one of which (s) is observable at a cost k.
- Suppose (s, u, ϵ) are jointly normally distributed and that $\rho_I = \rho_U = \rho$.
- Noise trading has mean = -1.

With the above assumptions s is sufficient for $\{s, p\}$ to estimate θ. Hence, an informed trader only uses the private signal when estimating θ:

$$E[\theta|s, p] = E[\theta|s] = s$$
$$\mathrm{Var}[\theta|s, p] = \mathrm{Var}[\theta|s] = \sigma_\epsilon^2,$$

and the informed strategy is given by

$$X_I(s) = a(s - p), \;\; a = (\sigma_\epsilon^2 \rho)^{-1}.$$

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

- For a given fraction of uninformed traders $1-\mu$, market clearing requires that $\mu X_I(s,p) + (1-\mu)X_U(p) + u = 0$.
- The unique equilibrium price that arises is given by

$$P(s,u) = \alpha_1 + \alpha_2 w,$$

  for some α_1, $\alpha_2 > 0$, where $w = s + (\mu a)^{-1}u$ is the informational content of the price.
- Observing the equilibrium price uninformed traders infer w and can forecast s with a precision $(\mathrm{Var}[s|w])^{-1} = \tau_s + (\mu a)^2 \tau_u$.

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

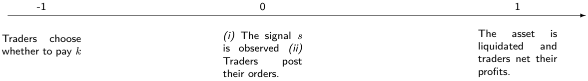
Special cases

- Suppose $\mu = 0$. Then, there is no information available on θ in the market and $p = \bar{\theta} - \rho\sigma_\theta^2$.
- Suppose $\sigma_u = 0$. Then, there is a fully revealing equilibrium in which $p = s - a^{-1}$, each trader demands one unit of the asset and absorbs the deterministic supply $\bar{u} = -1$. The equilibrium is implementable.

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

Suppose that prior to trading agents decide whether to acquire or not the signal s, paying a cost k:



Which fraction μ of agents will acquire the signal?

- An informed trader has an expected utility

$$E[U(\pi_I)|s,p] = -\exp\left\{-\frac{(s-p)^2}{2\sigma_\epsilon^2}\right\}. \quad (1)$$

- An uninformed trader, on the other hand

$$E[U(\pi_U)|p] = -\exp\left\{-\frac{(E[\theta|p]-p)^2}{2\mathrm{Var}[\theta|p]}\right\}. \quad (2)$$

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

- Ex-ante the two will be indifferent when the expected value of (1) (taking into account the cost k) will equal the expected value of (2): (Proof)

$$\frac{E[U(\pi_I)]}{E[U(\pi_U)]} = \exp\{\rho k\}\sqrt{\frac{\mathrm{Var}[\theta|s]}{\mathrm{Var}[\theta|p]}} \equiv \exp\{\rho k\}\sqrt{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \mathrm{Var}[s|p]}}.$$

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

Let

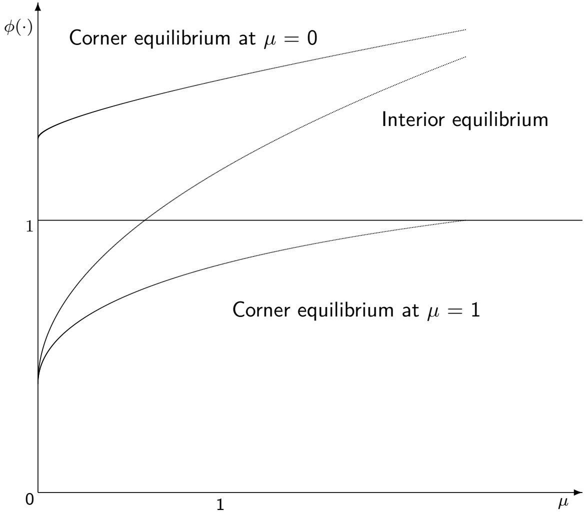
$$\phi(\mu) = \exp\{\rho k\}\sqrt{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \mathrm{Var}[s|p]}},$$

An equilibrium in the overall game is given by

1. If $\phi(\mu^*) = 1$, then $\mu^* \in [0,1]$ is an equilibrium.
2. If $\phi(1) < 1$, $\mu^* = 1$ is an equilibrium.
3. If $\phi(0) > 1$, $\mu^* = 0$ is an equilibrium.

4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox



4.2 Competitive Rational Expectations Equilibria

4.2.2 Information acquisition and the Grossman-Stiglitz paradox

The Paradox

- What happens when $\sigma_u \to 0$?
  - As the market becomes less noisy, the price becomes *more* informative.
  - As the price becomes more informative, *less* traders acquire the signal.

  Thus, price informativeness does not change!
- What is the equilibrium value for μ (for k small) in this case?
  - Suppose $\mu^* = 0$. Then, $\phi(\mu) = \exp\{\rho k\}\sqrt{\sigma_\epsilon^2/(\sigma_\epsilon^2 + \sigma_s^2)} < 1$ (for k small). Hence, $\mu^* > 0$.
  - Suppose $\mu^* > 0$. Then, as $\mathrm{Var}[\theta|w] = \mathrm{Var}[\theta|s]$, $\phi(\mu) = \exp\{\rho k\} > 1$. Hence, $\mu^* = 0$!

  In other words, in this case there is no equilibrium: *In the absence of noise, no one has an incentive to acquire private information. However, if nobody observes the private signal, there are incentives for a single trader to purchase information.*

4.2 Competitive Rational Expectations Equilibria

Strategic Complementarity and Multiplicity of Equilibria

There are several attempts in the literature to introduce strategic complementarity in information acquisition in variants of the Grossman and Stiglitz model.

1. Barlevy and Veronesi (2000, 2007) and Chamley (2007).
2. Ganguli and Yang (2006)
3. Lundholm (1988)
4. Veldkamp (2006)

4.2 Competitive Rational Expectations Equilibria

Summary

1. Informed agents trade both to *profit* from private information and to *exploit* price deviations from fundamentals.
2. Uninformed agents act as market makers and trade less aggressively because of adverse selection.
3. Prices equal a weighted average of investors' expectations about the fundamental value plus noise.
4. Market makers protect themselves from adverse selection by reducing market liquidity when they are more risk averse and/or there is less precise public information. The opposite happens when there is more noise trading.
5. The informativeness of prices increases with the risk tolerance-adjusted informational advantage of informed traders, with the proportion of informed traders, and decreases with the volatility of fundamentals and the amount of noise trading. There is strategic substitutability in information acquisition.

4.2 Competitive Rational Expectations Equilibria

Summary

6. The volatility of prices depends, ceteris paribus, negatively on market depth, and positively on price precision, prior volatility, and noise trading. In any case volatility increases with the degree of risk aversion of uninformed traders and with prior volatility.
7. Departures from the standard model introducing private signals on noise trading, or correlation on fundamentals and noise trading, or correlation in the error terms of private and public signals, introduce multiple (linear) equilibria in the financial market and, potentially, strategic complementarity in information acquisition. Another way to obtain strategic complementarity in information acquisition is with economies of scale in information production.

4.3 Informed Traders Move First
The Model

Suppose (Vives, 1995 and Medrano, 1996) that in the competitive model with differential information ♣

- A proportion $\nu \in [0,1]$ of traders submits limit orders: $X(s_i, p)$ for $i \in [0, \nu]$.
- A proportion $1-\nu \in [0,1]$ of traders submits market orders: $Y(s_i)$, for $i \in (\nu, 1]$.
- The price is set by a sector of competitive and risk-neutral market makers that observes the aggregate book $L(\cdot)$ and (Bertrand competition) sets
$$p = E[\theta | L(\cdot)].$$
- All traders have the same degree of risk aversion: ρ.

4.3 Informed Traders Move First
The Model

- Assume that any $i \in [0, \nu]$ submits an order $X(s_i, p) = a(s_i - \bar{\theta}) + \zeta(p)$, while any $j \in (\nu, 1]$ trades according to $Y(s_j) = c(s_j - \bar{\theta})$, where $\zeta(\cdot)$ is a linear function of the price.
- The noisy limit order book schedule is given by

$$L(p) = \int_0^\nu X(s_i, p)di + \int_\nu^1 Y(s_j)dj + u = z + \nu\zeta(p),$$

  where $z = A(\theta - \bar{\theta}) + u$, and $A = \nu a + (1 - \nu)c$
- Market makers observe $L(\cdot)$, infer z and set $p = E[\theta|z]$:

$$p = \bar{\theta} + \lambda z,$$

  with $\lambda = A\tau_u/\tau$, $\tau = (\mathrm{Var}[\theta|z])^{-1} = \tau_\theta + A^2\tau_u$.
- Linearity of the price implies that $p = E[\theta|z] = E[\theta|p]$.
- Owing to the presence of risk-neutral market makers, total volatility is constant:

$$\begin{aligned}\mathrm{Var}[\theta] &= E\left[\mathrm{Var}[\theta|p]\right] + \mathrm{Var}\left[E[\theta|p]\right] \\ &= \mathrm{Var}[\theta|p] + \mathrm{Var}[p]\end{aligned}$$

4.3 Informed Traders Move First

The Model

- As a consequence ex-ante price volatility is given by

$$\begin{aligned}\mathrm{Var}[p] &= \mathrm{Var}[\theta] - \mathrm{Var}[\theta|p] \\ &= \tau_\theta^{-1} - \tau^{-1},\end{aligned}$$

  increasing in price precision.
- An increase in price precision advances the resolution of uncertainty increasing price variance $\Rightarrow$ price volatility proxies for uncertainty resolution (not the same without risk neutral dealers).

4.3 Informed Traders Move First
The Equilibrium

Proposition

There is a unique linear Bayesian equilibrium. It is given by

$$X(s_i, p) = a(s_i - p)$$
$$Y(s_j) = c(s_j - \bar{\theta}),$$

where $a = \rho^{-1}\tau_\epsilon$, $c = (\rho(\sigma_\epsilon^2 + \mathrm{Var}[p]))^{-1}$, *and*

$$p = E[\theta|z]$$
$$= \bar{\theta} + \lambda z,$$

$z = A(\theta - \bar{\theta}) + u$, $\lambda = A\tau_u/\tau$, $A = \nu a + (1-\nu)c$.

4.3 Informed Traders Move First

Market Orders vs. Limit Orders

- Due to the presence of risk-neutral dealers, limit order traders withdraw from market making and concentrate on speculating on the difference between the signal and the price weighted by $a = \rho^{-1}\tau_\epsilon$.
- Market order traders speculate on the difference between the private signal and the ex-ante mean taking into account the joint effect of σ_ϵ^2, ρ, and $\mathrm{Var}[p]$:

$$c = \left(\rho\left(\sigma_\epsilon^2 + \mathrm{Var}[p]\right)\right)^{-1}.$$

  As $\mathrm{Var}[p] > 0$ price execution risk dampens market order traders' reaction to information and $c < a$.
- If $\nu \uparrow$, $A \uparrow$ and
  1. $\mathrm{Var}[p] \uparrow$
  2. $\tau \uparrow$

  the effect on λ is ambiguous.

4.3 Informed Traders Move First

Market Orders vs. Limit Orders

Suppose traders differ in terms of risk aversion and private signal precision. Then if placing a limit order entails a small positive cost:

- Traders with "high" risk-tolerance adjusted informational advantage (high $\rho_i^{-1}\tau_{\epsilon_i}$) place demand schedules
- Traders with "low" risk-tolerance adjusted informational advantage (low $\rho_i^{-1}\tau_{\epsilon_i}$) place market orders

As in Verrecchia (1982) the former trade more aggressively, thus benefit more from observing the information contained in the price and are willing to pay to get it.

4.3 Informed Traders Move First

Market Orders vs. Limit Orders

Suppose $\nu = 1$ then

- If $\tau_u^{-1} \uparrow$ then *(i)* $\tau \downarrow$ and *(ii)* $\lambda^{-1} \uparrow$: as the market becomes noisier prices are less informative but liquidity improves as adverse selection is less severe.
- If $\tau_\epsilon^{-1} \uparrow$ or $\rho \uparrow$ *(i)* $\tau \downarrow$ and *(ii)* $\lambda^{-1} \downarrow\uparrow$: as traders risk-tolerance adjusted informational advantage decreases, less information is impounded in the price. This at first worsens depth (as price precision decreases) but then it improves it, as adverse selection becomes less severe.
- Expected aggregate volume of informed traders is $E[|\int_0^1 X(s_i, p)di|] = (2\pi)^{1/2} a\sqrt{\tau^{-1}}$. It increases with noise trading (τ_u^{-1}) and decreases in the noisiness of private information (τ_ϵ^{-1}). When $\tau_\epsilon \to 0$, $(2\pi)^{1/2} a\sqrt{\tau^{-1}} \to 0$, market makers absorb the order imbalance without updating the price: $\lambda = 0$ and $p = \bar{\theta}$.

If $\nu = 0$ the model becomes the financial counterpart of a static Cournot model.

4.3 Informed Traders Move First

Market Orders vs. Limit Orders

Related literature:

- Brown and Zhang (1997) a market with traders using limit orders induces more informational price efficiency than one with traders using market orders since in the former execution price risk is moderated.
- Chakravaty and Holden (1995) analyze this choice by an informed trader in a quote-driven system. In this case the informed trader may exploit limit orders by submitting a market order.
- Foucault (1999) analyzes the choice in a dynamic model and concludes that it is better to place a limit (market) order when the spread is large (tight). This analysis is extended by Goettler, Parlour and Rajan (2005).
- Harris and Hasbrouck (1996) and Biais, Hillion and Spatt (1995) provide evidence consistent with the last two theoretical pieces.
- Wald and Horrigan (2005) analyze the choice of a risk averse investor between a limit and a market order and estimate the parameters of the model with NYSE data.

Summary

1. The presence of a competitive risk neutral market making sector induces prices to reflect all publicly available information. Prices are volatile because they are informative.
2. As a consequence, total volatility is constant and equal to the volatility of fundamentals. An increase in informativeness of prices only brings forward the resolution of uncertainty.
3. Sequential and simultaneous order placement need not yield different outcomes. This is so in the presence of competitive risk neutral market makers.
4. Risk averse traders using market orders are more cautious when responding to their information than limit order (demand schedule) traders because they are subject to price volatility.
5. As a result, when the proportion of traders using demand schedules increases, so does price precision and volatility (and the impact on market depth is ambiguous).
6. Whenever there is a differential fixed cost to submit a demand schedule instead of a market order, traders with a large risk tolerance-adjusted informational advantage place demand schedules while the others place market orders.

4.4 Hedgers and Producers in a Futures' Market

Up to now we have considered markets where some exogenous noise traders are present and drive the trade. ♣

- Their presence is motivated by unspecified liquidity reasons and allows for REE not to be fully revealing as well as trade in the presence of asymmetric information.
- This is unsatisfactory because the decisions of noise traders are not modeled, it is not explained why these traders are willing to lose money in the market, and consequently a proper welfare analysis cannot be performed.
- In this section we endogenize the presence of noise traders with risk averse hedgers.
- We present a variation of the model of Section 4.2 replacing noise traders by risk-averse competitive hedgers and assuming that all informed traders receive the same signal (we follow Medrano and Vives (2007)).

4.4 Hedgers and Producers in a Futures' Market

- The risky asset is a futures contract for a good (say agricultural product or raw material) with future random spot price θ.
- The futures contract trades at price p.
- Producers want to hedge their production in the futures market at $t = 2$ and obtain private information at $t = 1$ about the future value of the product once the production process has been set (say, the seeds have been planted) at $t = 0$ (see the timeline).

4.4 Hedgers and Producers in a Futures' Market

Preview of Results

- The private information of producers can not help production decisions, because it comes too late, but allows them to speculate in the futures market.
- This creates adverse selection in the future's market where uninformed speculators (market makers) and other hedgers operate.
- This will tend to diminish the hedging effectiveness of the futures market and diminish consequently the output of risk averse producers (since they will be able to hedge less of their production).
- The adverse selection is aggravated with more precise private information. Adverse selection is eliminated if the signal received by producers is made public.
- However, more public information may decrease production (and the expected utility of all traders) because it destroys insurance opportunities (this is the "Hirshleifer effect").
- The model also shows under what circumstances hedgers have demands of the "noise trader" form.

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

Model

- A single risky asset (the futures contract), with random liquidation value θ (the future spot price), and a riskless asset, with unitary return, are traded among a continuum of risk-averse competitive uninformed speculators (market makers), a continuum of risk-averse competitive hedgers, and a continuum of risk-averse competitive informed speculators.
- The risky asset is traded at a price p and thus generates a return $\theta - p$.

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

Informed Traders. Continuum of informed traders with mass one who produce a good with random future spot price θ. Each informed trader:

- Receives a private signal $s = \theta + \epsilon$, where θ and ϵ are independent, and $E[\epsilon] = 0$.
- Produces q bearing a cost $C(q) = c_1 q + c_2 q^2/2$ where $c_1, c_2 \geq 0$.
- Is risk averse with CARA utility $U_I(W_I) = -\exp\{-\rho_I W_I\}$, $\rho_I > 0$, where $W_I = \theta q - C(q) + (\theta - p)x_I$ is his final wealth when buying x_I futures contracts. His position in the futures market is then $q + x_I$.
- Submits a demand schedule contingent on the private information s he observes. If $x_I > 0$ ($x_I < 0$) he is a net buyer (net supplier) of futures. In equilibrium we will see that $E[x_I] < 0$: informed sell on average.

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

An informed trader has three motives to trade in the futures market:

1. to hedge part of the risk coming from his production q ($\theta q - C(q)$ is the random value of the producer's endowment before trading in the securities' market that needs to be hedged)
2. speculative reasons (exploit private information about θ)
3. speculates on differences between prices and the expected value of θ (i.e. for market making purposes).

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

Market Makers. There is a continuum of competitive uninformed speculators (or market makers) with unitary mass.

- The final wealth of a representative market maker buying shares at price p is given by $W_U = (\theta - p)x_U$, where his initial non-random wealth is normalized to zero.
- Market makers trade in order to obtain profits by absorbing some of the risks that the informed traders and hedgers try to hedge (their trades are not motivated by any informational advantage or any need of hedging).
- A representative market maker is risk averse with CARA utility $U_U(W_U) = -\exp\{-\rho_U W_U\}$, $\rho_U > 0$ and submits a demand schedule.
- Since they have rational expectations, they use their observation of the price to update their beliefs about θ.

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

Hedgers. There is a continuum of competitive hedgers with unitary mass, indexed in the interval $[0, 1]$. Hedger j:

- Has an initial endowment u_j of an asset with future (random) value z correlated with θ. This could be the random production of a related good which is not traded in a futures market.
- Has final wealth $W_j = u_j z + (\theta - p)x_j$ when buying x_j shares at price p.
- Is risk-averse with CARA utility $U_H(W_j) = -\exp\{-\rho_H W_j\}$, $\rho_H > 0$.
- Privately observes u_j and places a demand schedule contingent on his private information u_j: $X_H(u_j, \cdot)$.
- We assume that $u_j = u + \eta_j$, where u and η_j are independent (and η_i is independent of η_j for all $i \neq j$).

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

- Assume that $\int_0^1 \eta_j dj = 0$ a.s.. As a result, $\int_0^1 u_j dj = u$ a.s., so that u is the aggregate risky endowment of the hedgers.
- A hedger uses the observation of the price to update his beliefs about θ.

Hedgers' main motive to trade is to reduce risks. However, the endowment shock to hedger j is his private information and therefore their demand has also a speculative component.

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

Timing♣ At $t = 0$, producers choose the level of production q. The level of production q is public information. At $t = 1$, each producer receives a private signal s about θ and hedger j an endowment shock u_j, and the demand schedules of all traders placed. At $t = 2$, the market clearing price is set and trade occurs. Finally, at $t = 3$, the terminal values z and θ are realized and agents consume.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
Producers set output q.	Signal s and endowment shocks u_j are received and demand schedules are placed.	The market clears and trade takes place.	z and θ and payoffs are realized.

4.4 Hedgers and Producers in a Futures' Market

4.4.1 A Futures Market with Hedgers

Distributional Assumptions.

- All random variables are assumed to be normally distributed: $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, $z \sim N(\bar{z}, \sigma_z^2)$, $u \sim N(u, \sigma_u^2)$, $\epsilon \sim N(0, \sigma_\epsilon^2)$, and $\eta_j \sim N(0, \sigma_\eta^2)$ for all j.
- Without loss of generality, we assume that z may be written as $z = \sigma_z((r_{\theta,z}/\sigma_\theta)\theta + \sqrt{1 - r_{\theta,z}^2 y})$, where $r_{\theta,z}$ is the correlation coefficient between z and θ, and $y \sim N(0,1)$ is independent of any other variable in the model.
- Assume $\mathrm{Cov}[\theta, u] = \mathrm{Cov}[s, u] = \mathrm{Cov}[\theta, u_j] = \mathrm{Cov}[s, u_j] = \mathrm{Cov}[\theta, \epsilon] = \mathrm{Cov}[\theta, \eta_j] = \mathrm{Cov}[u, \eta_j] = \mathrm{Cov}[s, \eta_j] = \mathrm{Cov}[\epsilon, u] = \mathrm{Cov}[\epsilon, \eta_j] = 0$ for all j and $\mathrm{Cov}[\eta_i, \eta_j] = 0$ for all $j \neq i$.
- Let ξ denote the square of the correlation coefficient between s and θ, $\xi \equiv \sigma_\theta^2/(\sigma_\theta^2 + \sigma_\epsilon^2)$, and let ξ_u denote the square of the correlation coefficient between u and u_j, $\xi_u \equiv \sigma_u^2/(\sigma_u^2 + \sigma_\eta^2)$.

The subscript I will refer to the informed traders; the subscript U will refer to the uninformed speculators, and the subscript H will refer to the hedgers.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

- Restrict attention to Bayesian linear equilibria with price functional of the form $P(s, u)$.
- In order to find the linear equilibrium: *(i)* we posit candidate linear strategies *(ii)* derive the linear relationship between prices and the underlying random variables *(iii)* work through the optimization problems of traders to derive their demands and *(iv)* finally determine the coefficients of the linear strategies.
- Given the information structure and traders' preferences, equilibria will be symmetric.
- With hindsight strategies can be written as follows:

$$X_I(s, p) = a(s - \bar{\theta}) + b_I(\bar{\theta} - p) - \gamma_I q$$
$$X_U(p) = b_U(\bar{\theta} - p) - \gamma_U q$$
$$X_H(p, u_j) = b_H(\bar{\theta} - p) - \delta u_j - \gamma_H q,$$

where $a, b_I, b_U, b_H, \gamma_I, \gamma_U, \gamma_H$, and δ are endogenous, non-random parameters.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

- The market-clearing condition is

$$X_U(p) + X_H(p,u) + X_I(p,s) = 0,$$

where $X_H(p,u) = \int_0^1 X_H(p,u_j)dj = b_H(\bar{\theta} - p) - \delta u - \gamma_H q$, is the hedgers' aggregate demand.

- Given the posited linear strategies, the equilibrium price is a linear function of the private information s, the hedgers' random aggregate endowment u, and production q:

$$p = \bar{\theta} - \Gamma q + \frac{a(s-\theta) - \delta u}{\Lambda},$$

$\Gamma = (\gamma_I + \gamma_u + \gamma_H)/\Lambda$, and $\Lambda = b_I + b_U + b_H$.

- Hedger j chooses x_j to maximize $E[U_H(W_j)|p,u_j]$, where $W_j = u_j z + (\theta - p)x_j$ or

$$-\exp - \left\{ E[W_j|u_j,p] - \frac{\rho_H}{2}\mathrm{Var}[W_j|u_j,p] \right\},$$

$E[W_j|u_j,p] = u_j E[z|u_j,p] + (E[\theta|u_j,p] - p)x_j$ and

$\mathrm{Var}[W_j|u_j,p] = u_j^2 \mathrm{Var}[z|u_j,p] + x_j^2 \mathrm{Var}[\theta|u_j,p] - 2u_j x_j \mathrm{Cov}[z,\theta - p|u_j,p]$.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

From FOC:

$$X_H(u_j, p) = \frac{E[\theta - p|p, u_j] - \rho_H u_j \mathrm{Cov}[z, \theta - p|p, u_j]}{\rho_H \mathrm{Var}[\theta - p|p, u_j]}.$$

Hedger j's demand can be decomposed in two:

1. *Speculative demand*: $(\rho_H \mathrm{Var}[\theta - p|p, u_j])^{-1} E[\theta - p|p, u_j]$, which depends on q (because this helps reading the information about s in the price) and on u_j provided that $\xi_u > 0$ (because then u_j contains information on u which in turn helps to recover information about s in the price) and
2. *Hedge supply*: $-(\mathrm{Var}[\theta - p|p, u_j])^{-1}\mathrm{Cov}[z, \theta - p|p, u_j]u_j = -(\sigma_{\theta,z}/\sigma_\theta^2)u_j$. The amount of the hedger's initial endowment (u_j) that is hedged in the market is proportional to the correlation between the value of the hedger's asset z and the return of the risky security $\theta - p$ conditional on the hedger's information $\{p, u_j\}$.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

For an uninformed, optimization yields:

$$X_U(p) = \frac{E[\theta - p|p]}{\rho_U \mathrm{Var}[\theta - p|p]},$$

Note that

- All the speculators will place the same demand schedule (since all of them have the same information), so that the speculators' aggregate demand $X_U(p)$ will be given by the same expression.
- The demand will depend on q because the knowledge of q is needed to infer information about s from the price.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

For an informed the optimization problem is:

$$\max_{x_I} E[-\exp\{-\rho_I W_I\}|s, p],$$

where $W_I = \theta q - C(q) + (\theta - p)x_I$. Given normality this is equivalent to maximizing:

$$E[W_I|s,p] - \frac{\rho_I}{2}\mathrm{Var}[W_I|s,p] = qE[\theta|s] + x_I(E[\theta|s] - p) - \frac{\rho_I}{2}(x_I + q)^2\mathrm{Var}[\theta|s].$$

Note that

- The price does not provide an informed trader with any further information about θ over and above the signal s and therefore $E[\theta|s,p] = E[\theta|s]$ and $\mathrm{Var}[\theta|s,p] = \mathrm{Var}[\theta|s]$.
- However, although the price has no information to aggregate, it is still useful from the informed trader's point of view since it allows him to infer the exact amount of noise trading.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

If $\rho_I \mathrm{Var}[\theta|s] > 0$, then

$$X_I(s,p) = \frac{E[\theta|s] - p}{\rho_I \mathrm{Var}[\theta|s]} - q,$$

where $E[\theta|s] = \bar{\theta} + \xi(s - \bar{\theta})$ and $\mathrm{Var}[\theta|s] = (1-\xi)\sigma_\theta^2$. Then

$$\begin{aligned} X_I(s,p) &= \frac{1}{\rho_I \sigma_\epsilon^2}(s-p) + \frac{1}{\rho_I (1-\xi)\sigma_\theta^2}(\bar{\theta} - p) - q \\ &= a(s - \bar{\theta}) + b_I(\bar{\theta} - p) - q, \end{aligned}$$

where $a = 1/\rho_I \sigma_\epsilon^2$, and $b_I = 1/(\rho_I(1-\xi)\sigma_\theta^2)$.
An informed trader's position can be decomposed in two terms:

1. *Speculative Demand:* $(\rho_I \mathrm{Var}[\theta|s])^{-1}(E[\theta|s] - p)$, according to which the informed trader buys (sells) if his estimate of the asset liquidation value is greater (lower) than the price.
2. *Hedge Supply:* q. Since the representative informed agent is strictly risk averse and price-taker he hedges all the endowment risk, $\gamma_I = 1$ (provided that he is imperfectly informed, i.e. $\sigma_\epsilon^2 > 0$ or $\xi < 1$).

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

Proposition

If $\xi < 1$ there is a unique linear Bayesian equilibrium. It is characterized by

$$X_I(s,p) = a(s-\bar{\theta}) + b_I(\bar{\theta} - p) - q$$
$$X_U(p) = b_U(\bar{\theta} - p) - \gamma_U q$$
$$X_H(p,u_j) = b_H(\bar{\theta} - p) - \delta u_j - \gamma_H q$$
$$p = \bar{\theta} - \Gamma q + \frac{a(s-\theta) - \delta u}{\Lambda},$$

$\Gamma = (1+\gamma_U+\gamma_H)/\Lambda$, *and* $\Lambda = b_I + b_U + b_H$ *and* $a, b_I, b_U, b_H, \gamma_U, \gamma_H$, *and* δ *are endogenous, non-random parameters.*

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

- The expected price is equal to the prior expected liquidation value minus a risk premium: $\bar{p} = \bar{\theta} - \Gamma q$.
- The risk premium is positive and is directly proportional to the level of the endowment of informed traders (production), where $\Gamma = (\gamma_U + \gamma_H + \gamma_I)/\Lambda$.
- The equilibrium parameter $\Lambda = b_I + b_U + b_H$ is related to market depth. In terms of our previous lambda we have that $\lambda \equiv \partial p / \partial u = -\delta / \Lambda$. The market is deeper the more traders respond to price movements and the less hedgers react to their endowment shock.
- The price is informationally equivalent to $a(s - \bar{\theta}) - \delta u$ and therefore information (s) and the aggregate endowment shock (u) are the sources of price volatility.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

- As before the price precision is $\tau \equiv (\mathrm{Var}[\theta|p])^{-1}$ where, since the price is informationally equivalent also to $\theta + \epsilon - (\delta/a)u$

$$\tau = \tau_\theta + \frac{1}{\tau_\epsilon^{-1} + \delta^2(a^2\tau_u)^{-1}}.$$

  The price contains information about θ if and only if traders with information on fundamentals trade on the basis of that information (i.e. $a > 0$).
- Thus, it is natural to expect that the higher the traders' sensitivity to information on fundamentals, the more informative the price. This is true in equilibrium.
- Producers are, on average, net suppliers of the risky asset: $E[x_I] = q((a + b_I)\Gamma - 1) < 0$. Since the risk premium is positive, the ex ante expected value of the speculative demand is positive but the hedge supply $-q$ is larger in equilibrium.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

It is possible to show the following patterns:

- increasing ξ increases the trading signal sensitivity of informed producers (a) and decreases the price responsiveness of market makers (b_U) and hedgers (b_H).
- Uninformed traders protect themselves by reducing market depth when the informed have a signal of better quality. The first effect dominates and drives price precision $\tau \equiv (\mathrm{Var}[\theta|p])^{-1}$ and price volatility $\mathrm{Var}[p]$ upwards.

4.4 Hedgers and Producers in a Futures' Market

4.4.2 Equilibrium in the Futures Market

- The equilibrium parameter δ/Λ (the inverse of market depth) is hump-shaped as a function of ξ: $\Lambda = b_I + b_U + b_H$, where b_U and b_H are strictly decreasing in ξ and b_I is strictly increasing in ξ. For ξ low (high) the former (latter) effect dominates.
- If $\xi = 1$ (perfect information) or $\rho_I = 0$ (risk-neutrality of informed), the only possible equilibrium would be characterized by $p = E[\theta|s]$. The informed are indifferent about what to trade since $p = E[\theta|s]$. The market makers are also indifferent if $\xi = 1$ (since then $p = \theta$ and they face no risk), and they do not trade if $\rho_I = 0$ (since then $p = E[\theta|p]$ but they face risk). This would constitute a fully revealing REE but it is not implementable in demand functions.

4.4 Hedgers and Producers in a Futures' Market

4.4.3 Hedgers and Noise Traders

The market microstructure models that we have studied assume the existence of noise traders, agents that trade randomly for unspecified liquidity reasons.

- Are there circumstances in which rational expected utility maximizing agents give rise to demands for assets of the "noise trader" form?
- Are expected losses an appropriate measure of their welfare?
- The answer is that the order flow will contain an exogenous supply u (independent of any deep parameter of the model) whenever z is perfectly correlated with θ and the risk tolerance-adjusted informational advantage of a hedger is vanishingly small (ξ_u/ρ_H tending to 0).

4.4 Hedgers and Producers in a Futures' Market

4.4.3 Hedgers and Noise Traders

- This happens if hedgers are infinitely risk averse ($\rho_H \to \infty$) or if there is no correlation between each individual endowment shock u_j and the average u ($\xi_u \to 0$).
- In the first case hedgers just get rid of all the risk associated to their endowment and supply u in the aggregate.
- In the second, hedgers are exactly like market makers because they have no informational advantage. In the aggregate they supply again u but now they take a speculative position also.
- In both cases we can evaluate their expected utility.

4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- For a given q, a producer's ex ante expected utility can be seen to be given by the product of three terms:
  1. The utility derived from the speculative demand $|SG_I|$,
  2. the utility derived from the insurance achieved via the hedge supply $|IG_I|$, and
  3. the utility coming from production $\exp\{-\rho_I(q\bar{\theta} - C(q) - (\rho_I/2)q^2\sigma_\theta^2)\}$.

  or:

$$J_I(q) = -|SG_I||IG_I|\exp\{-\rho_I(q\bar{\theta} - C(q) - (\rho_I/2)q^2\sigma_\theta^2)\},$$

  where

$$|SG_I| = \left\{1 + \frac{\rho_I^2(1-\xi)\sigma_\theta^2(\xi\sigma_\theta^2 + \delta^2(b_U + b_H)^{-2}\sigma_u^2)}{(\rho_I(1-\xi)\sigma_\theta^2 + (b_U + b_H)^{-1})^2}\right\}$$

$$|IG_I| = \exp\{-(\rho_I^2/2)\sigma_\theta^2 dq^2\}.$$

- The key endogenous parameter d represents the hedging effectiveness of the market. It is a complicated expression of the deep parameters of the model.

4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- The speculative term has two components:
- The term $\xi\sigma_\theta^2$ associated to gains from private information and the term $\delta^2(b_U + b_H)^{-2}\sigma_u^2$ to gains from market making.
- The private information gains disappear, obviously, when there is no private information ($\xi = 0$).
- The optimal production level maximizes

$$q\bar{\theta} - C(q) - \frac{1}{2}\rho_I\sigma_\theta^2 q^2(1-d),$$

and is given by

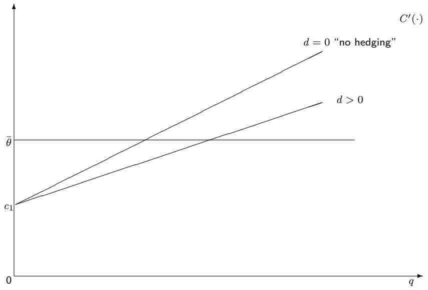
$$q^* = \frac{\bar{\theta} - c_1}{c_2 + \rho_I\sigma_\theta^2(1-d)}.$$

- Comparative statics can be conducted via numerical simulations.

4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- The direct impact of an increase in risk aversion ρ_I or underlying risk σ_θ^2 is to decrease q^*.
- An increase in the cost parameters c_1, c_2, unambiguously decreases production.
- When the market is totally ineffective in hedging, or there is no future's market, $d = 0$ and $q^* = q_0 \equiv (c_2 + \rho_I \sigma_\theta^2)^{-1}(\bar{\theta} - c_1)$ (see Figure). This happens as $\xi \to 1$.
- The parameter d is decreasing in ξ. Better private information implies worse hedging.



4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- A producer's ex ante expected utility may be written as the product of the speculative component with production and insurance gains

$$J_I(q^*) = -|SG_I| \exp\{-\rho_I(\bar{\theta} - c_1)q^*/2\}.$$

- The speculative component of utility is hump-shaped in ξ.
- For low ξ an increase in signal precision improves speculative benefits but for high ξ the opposite happens because information revelation is "too strong."
- Production and insurance gains are decreasing in ξ because q^* is decreasing in ξ.
- The result is that $J_I(q^*)$ is decreasing with ξ for "normal" values of parameters or hump shaped with ξ for more extreme parameter configurations (high noise scenarios).

4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- The uninformed speculators' ex ante expected utility EU_U can be seen to increase, for given $\mathrm{Var}[E[\theta|p] - p]$ and $\mathrm{Var}[\theta|p]$, with the risk premium Γqw hich is nothing else but the expected margin $E[\theta - p] = \Gamma q$.
- The risk premium decreases as ξ increases and this leads to a decrease in EU_U. In all cases considered in the simulations we find that EU_U is decreasing in ξ.
- The expressions for the expected utility of a hedger EU_H are complicated but an increase in ξ typically decreases EU_H because q is decreasing in ξ.
- Note that EU_H tends to increase also with the risk premium or, equivalently, decrease with $\bar{p} = \bar{\theta} - \Gamma q$.
- Indeed, when a hedger hedges his endowment the return is precisely p and a higher expected level of p increases the risk borne by the agent.

4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- Interestingly, when the precision of information is high market depth increases with ξ but EU_H decreases.
- This means that looking at the usual cost of trading in noise trading models is misleading and this happens precisely when the demands of hedgers are close to the noise trader form, that is, when ξ_u/ρ_H is small.
- In short, for a very wide range of parameter values we have that more private information is Pareto inferior because it aggravates the adverse selection problem and reduces the hedging effectiveness of the futures market and production.

4.4 Hedgers and Producers in a Futures' Market

4.4.4 Production, Insurance, and Private Information

- This means that all market participants would prefer that there is no private information in the market.
- The question arises whether this is true also of public information. With a public signal adverse selection is eliminated and this should increase market depth but the impact on production is ambiguous ex ante.
- The reason is that a public signal also reduces insurance opportunities (the "Hirshleifer effect").
- Again all market participants may end up losing with more information.

4.4 Hedgers and Producers in a Futures' Market

Summary

1. The typical (and unmodeled) noise trader behavior corresponds to a very risk averse rational hedger.
2. This implies that the usual welfare analysis of noise trader models based on the losses that those traders make, and which depends on market depth, may be misleading.
3. Indeed, market depth may increase but still the expected utility of hedgers may decrease.
4. Private information creates adverse selection and may decrease the welfare of all market participants because it reduces the hedging effectiveness of the market.
5. The same may happen with public information because of the destruction of insurance opportunities. The consequence is that more information may be Pareto inferior.

Summary

- In this chapter we have examined static financial market models in the frame of rational expectations with asymmetric information.
- A general theme of the chapter is that market microstructure matters when it comes to the informational properties of prices in financial markets and how uninformed traders protect themselves from informed trading by making the market less liquid.
- A recurrent result is how risk aversion for competitive traders makes agents cautious when trading and responding to their private signals.
- In Chapter 5 we will see how market power for strategic traders plays a similar role to risk aversion for competitive traders.

Summary

The main insights from the standard model with a unique linear REE are as follows.

1. Prices reflect private information about the returns of the asset through the trades of investors but typically not perfectly. Indeed, prices reflect the fundamentals and noise or shocks to preferences of investors.
2. A perfect informationally efficient market is impossible whenever information is costly to acquire.
3. Market makers and other uninformed agents face an adverse selection problem if some traders possess private information and they protect themselves by increasing the bid-ask spread and reducing market depth. If market makers are risk averse then price volatility increases with their degree of risk aversion.
4. The informativeness of prices increases with the risk tolerance-adjusted informational advantage of informed traders, with the proportion of informed, and decreases with the volatility of fundamentals and the amount of noise trading.
5. Information acquisition displays strategic substitutability.

Summary

6. The presence of a risk neutral competitive fringe of market makers with no privileged information makes prices reflect all public available information. That is, it makes the market semi-strong informationally efficient market and the price of the risky asset equals the expected fundamental value given publicly available information.
7. In a semi-strong efficient market:
    1. Prices are volatile because they are informative about fundamentals.
    2. Total volatility is constant and a more informative price just advances the resolution of uncertainty.
8. Risk averse traders using market orders are more cautious than limit order traders because the former bear price risk. As a consequence, if the proportion of traders using limit orders or demand schedules (instead of market orders) increases prices are more informative and more volatile (and the impact on market depth is ambiguous).
9. Whenever there is a differential fixed cost to submit a demand schedule instead of a market order, traders with a large risk tolerance-adjusted informational advantage place demand schedules while the others place market orders.

Summary

10. Noise trader demands are close to demands by rational utility maximizing hedgers with a large degree of risk aversion. It is possible then that market depth increases coexist with decreases in the expected utility of hedgers.
11. An increase in either private or public information may be Pareto inferior because the hedging effectiveness of the market is impaired. With private information this happens because of adverse selection and with public information because of the Hirshleifer effect.

- Departures from the standard model (e.g. private signals on noise trading, or correlation on fundamentals and noise trading, or correlation in the error terms of private and public signals) introduce multiple (linear) equilibria in the financial market and, potentially, strategic complementarity in information acquisition.
- Another way to obtain strategic complementarity in information acquisition is with economies of scale in information production. The end result may be multiple equilibria in both the financial market and the market for information acquisition. This allows for a rich pattern of explanations of different phenomena in financial markets.

Appendix

If $z \sim N(\mu, \sigma^2)$, then

$$E[\exp\{rz\}] = \exp\left\{r\mu + r^2\frac{\sigma^2}{2}\right\},$$

for r constant. ♣

Appendix

Proposition

The expected utility of an informed trader conditional on public information (the price) and taking into account the cost k of getting the signal is given by:

$$E[U(\pi_I)] = \exp\{\rho k\}\sqrt{\frac{\text{Var}[\theta|s]}{\text{Var}[\theta|p]}}E[U(\pi_U)].$$

Proof

We know that

$$E[U(\pi_I)|p] = \exp\{\rho k\}E\left[-\exp\left\{-\frac{(s-p)^2}{2\sigma_\epsilon^2}\right\}|p\right].$$

Let $y = (2\sigma_\epsilon^2)^{-1}(s-p)$. Then

$$E[y|p] = \frac{1}{\sqrt{2\sigma_\epsilon^2}}(E[s|p]-p) = \frac{1}{\sqrt{2\sigma_\epsilon^2}}(E[\theta|p]-p),$$

Appendix

and

$$\mathrm{Var}[y|p] = \frac{1}{2\sigma_\epsilon^2}\mathrm{Var}[s|p] = \frac{1}{2\sigma_\epsilon^2}(\mathrm{Var}[\theta|p] - \sigma_\epsilon^2).$$

We know that for a normal random variable $y|p \sim N(E[y|p], \mathrm{Var}[y|p])$ the following holds:

$$E[\exp\{-y^2\}|p] = \frac{1}{\sqrt{1 + 2\mathrm{Var}[y|p]}} \exp\left\{-\frac{E[y|p]^2}{1 + 2\mathrm{Var}[y|p]}\right\},$$

and therefore

$$E\left[-\exp\left\{-\frac{(s-p)^2}{2\sigma_\epsilon^2}\right\}|p\right] = \sqrt{\frac{\mathrm{Var}[\theta|s]}{\mathrm{Var}[\theta|p]}} \exp\left\{-\frac{(E[\theta|p]-p)^2}{2\mathrm{Var}[\theta|p]}\right\}.$$

The result follows since

$$E[U(\pi_U)|p] = -\exp\left\{-\frac{(E[\theta|p]-p)^2}{2\mathrm{Var}[\theta|p]}\right\}.$$

♣