Econometrica Supplementary Material

SUPPLEMENT TO "STRATEGIC SUPPLY FUNCTION COMPETITION WITH PRIVATE INFORMATION": PROOFS, SIMULATIONS, AND EXTENSIONS

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CONTENTS

THIS APPENDIX provides proofs, extensions, complementary simulations, and connections to the literature of the analysis and results presented in the main text. Section S.1 provides the convergence concepts, proofs, and further connections with the literature of replica markets (Section 4). Section S.2 develops the analysis of Bayesian Cournot equilibrium as well as its asymptotic properties as the market grows large. Section S.3 contains a development and details of the simulations of the model in the paper (Section S.3.1 corresponding to Section 3.2 in the paper (the basic model), Section S.3.2 corresponding to Section 4 (replica markets), and Section S.3.3 corresponding to the comparison with the Cournot model). Section S.4 analyzes information acquisition, Section S.5 deals with demand uncertainty (Section 5.4) and Section S.6 provides the proof of Proposition 8 of the model with a public signal (Section 5.5).

S.1. REPLICA MARKETS

This section provides the convergence concepts for replica markets (Section 4), proofs, and further connections with the literature.

S.1.1. *Measures of Speed of Convergence and Notation for Large Markets*

We say that the sequence (of real numbers) b_n is of the *order* n^v, with v a real number, whenever $n^{-v}b_n \longrightarrow_n k$ for some nonzero constant k. We say that the sequence of random variables $\{y_n\}$ converges in *mean square* to

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zero at rate $1/\sqrt{n^r}$ (or that y_n is on the order of $1/\sqrt{n^r}$) if $E[(y_n)^2]$ converges to zero at rate $1/n^r$ (i.e., $E[(y_n)^2]$ is of the order of $1/n^r$). Given that $E[(y_n)^2] = (E[y_n])^2 + \mathrm{var}[y_n]$, a sequence $\{y_n\}$ such that $E[y_n] = 0$ and $\mathrm{var}[y_n]$ is of the order of $1/n$ and converges to zero at rate $1/\sqrt{n}$. We use the subscript n to emphasize the dependence on n of average random variables $\tilde{\theta}_n = (\sum_{i=1}^n \theta_i)/n$, $\tilde{s}_n = (\sum_i s_i)/n = \tilde{\theta}_n + (\sum_i \varepsilon_i)/n$, and $\tilde{t}_n = E[\tilde{\theta}_n|\tilde{s}_n]$.

PROOF OF PROPOSITION 6: Recall that the n subscript denotes the n-replica market.

(i) I show that $E[(p_n - p_n^{PT})^2]$ tends to 0 at the rate of $1/n^2$. Note first that from the equation $g_n(c; M) = 0$ in the n-replica market defining c_n (just replace β with β/n in $g(c; M) = 0$), we have that $c_n \longrightarrow_n (\lambda^{-1} - \beta^{-1}M_\infty)(M_\infty + 1)^{-1}$, $M \longrightarrow_n M_\infty \equiv \sigma_\varepsilon^2((1-\rho)\sigma_\theta^2)^{-1}$ if $\rho > 0$, and $c_n \longrightarrow_n \lambda^{-1}$ if $\rho = 0$. It is immediate then that the order of the distortion $d_n = (\beta^{-1}n + (n-1)c_n)^{-1}$ is $1/n$. Let us show first the order for the price difference $E[(p_n - p_n^{\mathrm{PT}})^2]$. Recall that the price-taking allocation coincides with the full information efficient allocation. We know that $p_n - p_n^{\mathrm{PT}} = \beta(\tilde{x}_n^o - \tilde{x}_n)$ and from the proof of Proposition 4 that $E[(\tilde{x}_n - \tilde{x}_n^o)^2] = ((\beta+\lambda)^{-1} - (\beta+\lambda+d_n)^{-1})^2 E[(\alpha - \tilde{t}_n)^2]$, where $E[(\alpha - \tilde{t}_n)^2] = (\alpha - \bar{\theta})^2 + \frac{((1+(n-1)\rho)\sigma_\theta^2)^2}{((1+(n-1)\rho)\sigma_\theta^2+\sigma_\varepsilon^2)n}$ is of the order of a constant. The order of $E[(p_n - p_n^{\mathrm{PT}})^2]$ and of $E[(\tilde{x}_n - \tilde{x}_n^o)^2]$ is the order of $((\beta+\lambda)^{-1} - (\beta+\lambda+d_n)^{-1})^2$, which is $1/n^2$ since d_n is on the order of $1/n$.

(ii) With regard to efficiency, from the proof of Proposition 4 (and with analogous notation), we know that $E[(u_{in} - u_{in}^o)^2] = (\lambda^{-1} - (\lambda+d_n)^{-1})^2 E[(t_{in} - \tilde{t}_n)^2]$, where $E[(t_i - \tilde{t})^2] = \frac{(1-\rho)^2(n-1)\sigma_\theta^4}{n(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2)}$ is of the order of a constant. It follows that $E[(u_{in} - u_{in}^o)^2]$ is of order $1/n^2$ since $(\lambda^{-1} - (\lambda+d_n)^{-1})^2$ is. We conclude that $(\mathrm{ETS}_n^o - \mathrm{ETS}_n)/n = ((\beta+\lambda)E[(\tilde{x}_n - \tilde{x}_n^o)^2] + \lambda E[(u_{in} - u_{in}^o)^2])/2$ is of order $1/n^2$. *Q.E.D.*

REMARK S.1: If we want to keep aggregate uncertainty ($\mathrm{var}[\tilde{\theta}_n]$) constant when lowering market concentration (increasing n, which means decreasing σ_θ^2 appropriately since $\mathrm{var}[\tilde{\theta}_n]$ is decreasing in n), then c_n will be smaller than when we allow $\mathrm{var}[\tilde{\theta}_n]$ to vary (this is so since M increases by more when we keep $\mathrm{var}[\tilde{\theta}_n]$ constant). This means that the distortion d_n will be larger.

REMARK S.2: In Biais, Martimort, and Rochet (2000), increasing the number of market makers reduces market power, but the limit market has features of a monopolistically competitive equilibrium in which market makers charge a positive markup but make zero profits because they trade an infinitesimal amount. There is a spread even for small trades, the reason being that, under discriminatory pricing, market makers do not know whether the informed trader will want to buy more than the marginal unit. An infinitesimal order has a discrete impact on the price because it conveys a noninfinitesimal amount of information. This is why the bid–ask spread subsists even for very small orders (see Section 5.3.2 in Vives (2008)). This is not the case under uniform pricing as in our equilibrium.

S.2. COURNOT COMPETITION

In this section we study Bayesian Cournot competition. We first characterize equilibrium (in its strategic and price-taking versions), and then consider replica markets and asymptotic results.

S.2.1. *Equilibrium*

Consider the market as in Section 2, with $\rho \in [0, 1]$, but now seller i sets a quantity contingent on his information $\{s_i\}$.[1] The seller has no other source of information and, in particular, does not condition on the price. The expected profits of seller i conditional on receiving signal s_i and assuming seller j, $j \neq i$, uses strategy $X_j(s_j)$, are

$$E[\pi_i|s_i] = x_i\left(P\left(\sum_{j\neq i} X_j(s_j) + x_i\right) - E[\theta_i|s_i]\right) - \frac{\lambda}{2}x_i^2.$$

From the FOC of the optimization of seller i, we obtain

$$p - (E[\theta_i|s_i] + \lambda x_i) = \beta x_i.$$

Given that the profit function is strictly concave and the information structure is symmetric, equilibria will be symmetric.

We can also define a price-taking Bayesian Cournot equilibrium in which each seller sets a quantity, but does not realize his influence on the price. In this case, seller i chooses x_i to maximize

$$E[\pi_i|s_i] = x_i(p - E[\theta_i|s_i]) - \frac{\lambda}{2}x_i^2,$$

yielding FOC

$$p - (E[\theta_i|s_i] + \lambda x_i) = 0.$$

The following proposition characterizes the Bayesian Cournot equilibrium (denoted by a superscript C) and the price-taking Bayesian Cournot equilibrium (denoted by a superscript CPT). Both equilibria are different from their supply function counterparts (except in the knife-edge case for which $c = 0$

[1]See Vives (2002) for related results when cost parameters are independent and identically distributed (i.i.d.) and Vives (1988) for the common value case.

at the SFE) since there is no conditioning in the market price. Note that the Bayesian Cournot equilibrium exists even if $\rho = 1$ since there is no learning from prices.

PROPOSITION S.1: *Let $\rho \in [0, 1]$. There is a unique Bayesian Cournot equilibrium and a unique price-taking Bayesian Cournot equilibrium. They are symmetric and affine in the signals. Letting $\xi \equiv \sigma_\theta^2/(\sigma_\theta^2 + \sigma_\varepsilon^2)$, the strategies of the sellers are given (respectively) by*:

$$X^{\mathrm{C}}(s_i) = b^{\mathrm{C}}(\alpha - \bar{\theta}) - a^{\mathrm{C}}(s_i - \bar{\theta}),$$

where $a^{\mathrm{C}} = \frac{\xi}{2\beta+\lambda+\beta(n-1)\rho\xi}$ and $b^{\mathrm{C}} = \frac{1}{\lambda+\beta(1+n)}$, and by

$$X^{\mathrm{CPT}}(s_i) = b^{\mathrm{CPT}}(\alpha - \bar{\theta}) - a^{\mathrm{CPT}}(s_i - \bar{\theta}),$$

where $a^{\mathrm{CPT}} = \frac{\xi}{\beta+\lambda+\beta(n-1)\rho\xi}$ and $b^{\mathrm{CPT}} = \frac{1}{\lambda+\beta}$.

For a proof, see the proof of Proposition 2.1 in Vives (2008).

S.2.2. *Replica Markets and Convergence*

We study convergence to price-taking and its speed as the economy is replicated. Consider thus the replica market as in Section 4 with inverse demand $P_n(y) = \alpha - \beta y/n$. The equilibria are then given as in Proposition S.1, replacing β with β/n. The following proposition characterizes the convergence of the Bayesian Cournot equilibrium to a price-taking equilibrium. Here $\mathrm{ETS}_n^{\mathrm{C}}$ denotes the expected total surplus at the (price-taking) Bayesian Cournot equilibrium in the n-replica market and $\mathrm{ETS}_n^{\mathrm{CPT}}$ denotes that in the price-taking Bayesian Cournot equilibrium. It is worth remarking that $\mathrm{ETS}_n^{\mathrm{CPT}}$ does not, in general, attain the expected total surplus at the efficient (full information) allocation ETS_n^o since the price-taking Bayesian Cournot equilibrium with $\lambda > 0$ is not full information efficient except if $\rho = 0$ (Vives (2002)).[2]

In general we have that $\mathrm{ETS}_n^o > \mathrm{ETS}_n^{\mathrm{CPT}}$, since the price-taking Bayesian Cournot equilibrium does not aggregate information, and as the market grows large, there is no convergence to a full information equilibrium for $\rho > 0$. A consequence of the result is that for a given $\rho > 0$ and for large enough n, we always have that $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n > 0$. This is so since as n grows, the SFE, but not the Cournot equilibrium, converges to the (full information)

[2]With constant marginal costs, the Bayesian Cournot equilibrium does replicate the full information outcome under some regularity conditions (see Palfrey (1985) and Vives (1988)). A price-taking Bayesian Cournot equilibrium is team optimal (i.e., maximizes total expected surplus subject to the constraint that sellers use decentralized (quantity) strategies in information; see Vives (1988)).

first best. However, Proposition 7 holds for the (Bayesian) Cournot equilibrium (with price p_n^{C} and expected total surplus $\mathrm{ETS}_n^{\mathrm{C}}$), replacing ETS_n^o with $\mathrm{ETS}_n^{\mathrm{CPT}}$, where $\mathrm{ETS}_n^{\mathrm{CPT}}$ stands for the expected total surplus at the price-taking (Bayesian) Cournot equilibrium.

PROPOSITION S.2: *Let* $\rho \in [0, 1]$. *As the market grows large, the market price* p_n^{C} *at the Bayesian Cournot equilibrium converges in mean square to the price-taking Bayesian Cournot price* p_n^{CPT} *at the rate of* $1/n$. (*That is,* $E[(p_n^{\mathrm{C}} - p_n^{\mathrm{CPT}})^2] \to 0$ *at the rate of* $1/n^2$.) *The difference* $(\mathrm{ETS}_n^{\mathrm{CPT}} - \mathrm{ETS}_n^{\mathrm{C}})/n$ *is on the order of* $1/n^2$.

PROOF: Let us show first the order for the price difference $E[(p_n^{\mathrm{C}} - p_n^{\mathrm{PT}})^2]$. We know that $p_n^{\mathrm{C}} - p_n^{\mathrm{PT}} = \beta(\tilde{x}_n^{\mathrm{CPT}} - \tilde{x}_n^{\mathrm{C}})$ and letting $k_i \equiv E[\theta_i | s_i]$ and $\tilde{k}_n = (\sum_{i=1}^n k_i)/n$, it is easily checked that

$$E[(\tilde{x}_n^{\mathrm{CPT}} - \tilde{x}_n^{\mathrm{C}})^2] = ((\beta + \lambda)^{-1} - (\beta + \lambda + \beta n^{-1})^{-1})^2 E[(\alpha - \tilde{k}_n)^2],$$

where $\tilde{k}_n = \xi \tilde{s}_n + (1 - \xi)\bar{\theta}$. It follows that $E[(\alpha - \tilde{k}_n)^2]$ is of the order of a constant since $\mathrm{var}[\tilde{k}_n^2] = \xi^2 \mathrm{var}[\tilde{s}_n]$ is. The order of $E[(p_n^{\mathrm{C}} - p_n^{\mathrm{CPT}})^2]$ and of $E[(\tilde{x}_n^{\mathrm{CPT}} - \tilde{x}_n^{\mathrm{C}})^2]$ is the order of $((\beta + \lambda)^{-1} - (\beta + \lambda + \beta n^{-1})^{-1})^2$, which is $1/n^2$.

Similarly as in Section 3 (see Lemma 1 in Vives (2002)), we can decompose the deadweight loss at the Bayesian Cournot equilibrium in relation to the price-taking allocation, letting $u_{in}^{\mathrm{C}} \equiv x_{in}^{\mathrm{C}} - \tilde{x}_n^{\mathrm{C}}$ and $u_{in}^{\mathrm{CPT}} \equiv x_{in}^{\mathrm{CPT}} - \tilde{x}_n^{\mathrm{CPT}}$, as

$$\begin{aligned}&(\mathrm{ETS}_n^{\mathrm{CPT}} - \mathrm{ETS}_n^{\mathrm{C}})/n \\ &\quad = \big((\beta + \lambda)E[(\tilde{x}_n^{\mathrm{CPT}} - \tilde{x}_n^{\mathrm{C}})^2] + \lambda E[(u_{in}^{\mathrm{CPT}} - u_{in}^{\mathrm{C}})^2]\big)/2.\end{aligned}$$

It is easily checked also that $E[(u_{in}^{\mathrm{CPT}} - u_{in}^{\mathrm{C}})^2] = (\lambda^{-1} - (\lambda + \beta n^{-1})^{-1})^2 E[(k_i - \tilde{k}_n)^2]$. The order of $E[(k_i - \tilde{k}_n)^2]$ will be the same as $\xi^2 E[(s_i - \tilde{s}_n)^2]$, which is the order of a constant. Since the order of $(\lambda^{-1} - (\lambda + \beta n^{-1})^{-1})^2$ is $1/n^2$, it follows that the order of $E[(u_{in}^{\mathrm{CPT}} - u_{in}^{\mathrm{C}})^2]$ is also $1/n^2$. We conclude that $(\mathrm{ETS}_n^{\mathrm{CPT}} - \mathrm{ETS}_n^{\mathrm{C}})/n$ is of order $1/n^2$. *Q.E.D.*

S.3. SIMULATIONS

This section provides details and further results on the simulations performed with the basic and with the replica models, as well as providing a comparison with the Cournot model.

S.3.1. *The Basic Model*

This subsection presents details and a complementary illustration of the welfare simulations in Section 3.2 of the basic model (Section 2). Simulations

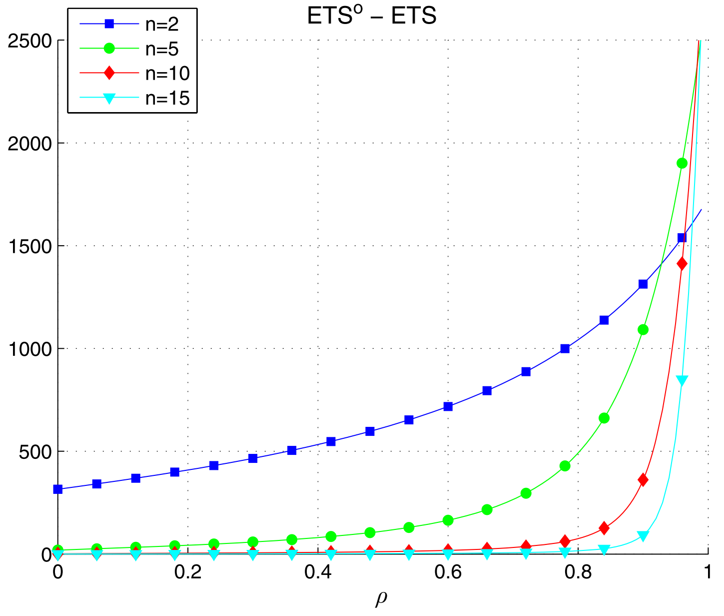

FIGURE S.1A.—$E[\text{DWL}] \equiv \text{ETS}^{o} - \text{ETS}$ as a function of ρ for different values of n (with parameters $\beta = \lambda = 1$ and $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$).

were performed in the following base case parameter grid: β in $\{0.5, 1\}$, λ in $\{0.5, 1, 5\}$, $\rho \in [0, 0.99]$ with step size 0.01, σ_ε^2 in $[0, 10]$, and σ_θ^2 in $[0.01, 10]$ with step size 1, and $n \in [2, 5, 10, 15, 20, 25]$. In this range of simulations and with $\bar{\theta} = 20$ and $\alpha = 200$, the probability of a negative output is at most 13% in either the strategic or price-taking equilibrium, as well as in the Cournot equilibrium. We have that 97% of points in the grid have a maximal probability of negative output of less than 1%.

Figure S.1a depicts the evolution of $E[\text{DWL}]$ as ρ increases for different numbers of sellers and complements in Figure 2. Note that $E[\text{DWL}]$ is higher for $n = 3$ than for $n = 2$ when ρ is close to 1. In this case, we have, in fact, that $d(n = 3) > d(n = 2)$. Indeed, when ρ is close to 1, $c < 0$ and d need not be decreasing in n. When $c < 0$, the simulations show that a possible pattern is for $(n - 1)c$ to have a U-shaped form with n (and, therefore, $d = (\beta^{-1} + (n - 1)c)^{-1}$—a hump-shaped form). Simulations have been extended to the range of parameters β and λ in $[1, 10]$, with step size 1 and n up to 30.

Increasing ρ may decrease $E[\text{DWL}]$ when σ_ε^2 is small for a range of ρ bounded away from 1 (see Figure S.1b) and increasing σ_ε^2 may decrease $E[\text{DWL}]$ when ρ is small (see Figure S.1c).

The welfare loss due to private information $\text{ETS}^{f} - \text{ETS}$ is typically increasing in ρ or σ_ε^2 except possibly for small values of those parameters (see Figure S.1d). An example where $\text{ETS}^{f} - \text{ETS}$ decreases in σ_ε^2 when ρ is small is

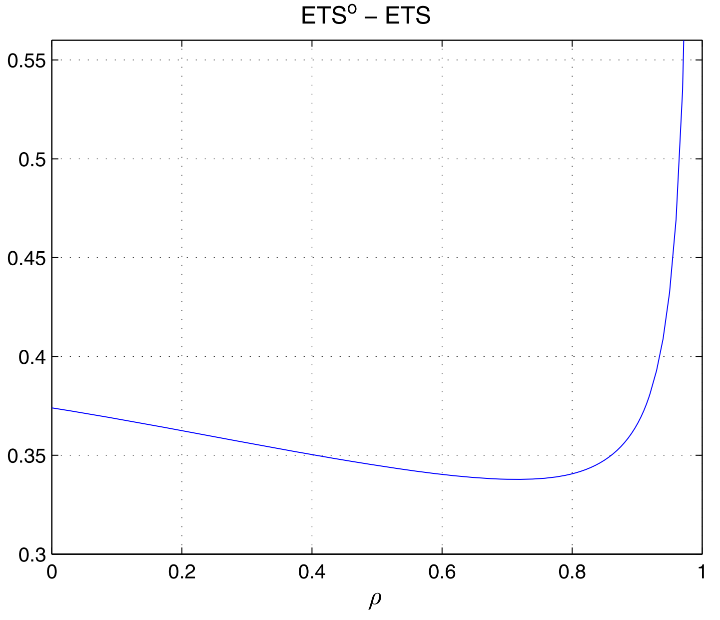

FIGURE S.1B.—E[DWL] as a function of ρ (with parameters $\sigma_\varepsilon^2 = 0.01$, $\beta = \lambda = 1$, $\sigma_\theta^2 = 1$, $n = 4$, $\alpha = 20$, and $\bar{\theta} = 5$).

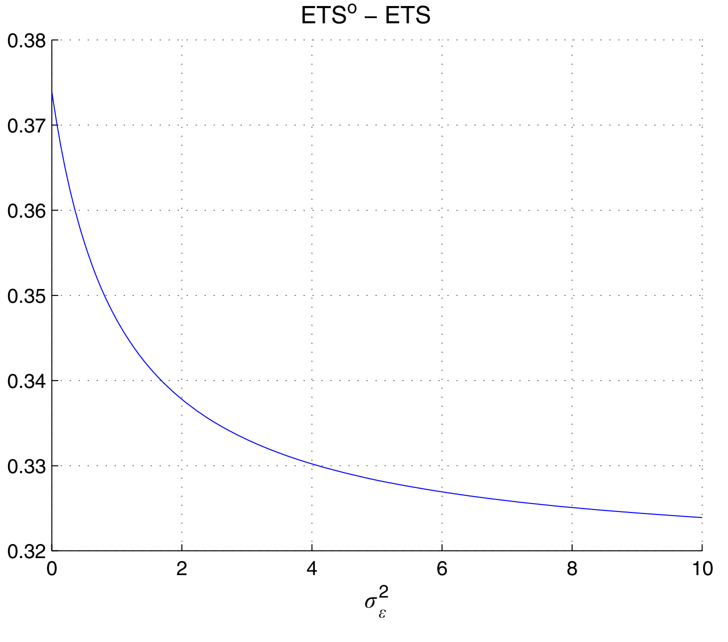

FIGURE S.1C.—E[DWL] as a function of σ_ε^2 (with parameters $\rho = 0.01$, $\beta = \lambda = 1$, $\sigma_\theta^2 = 1$, $n = 4$, $\alpha = 20$ and $\bar{\theta} = 5$).

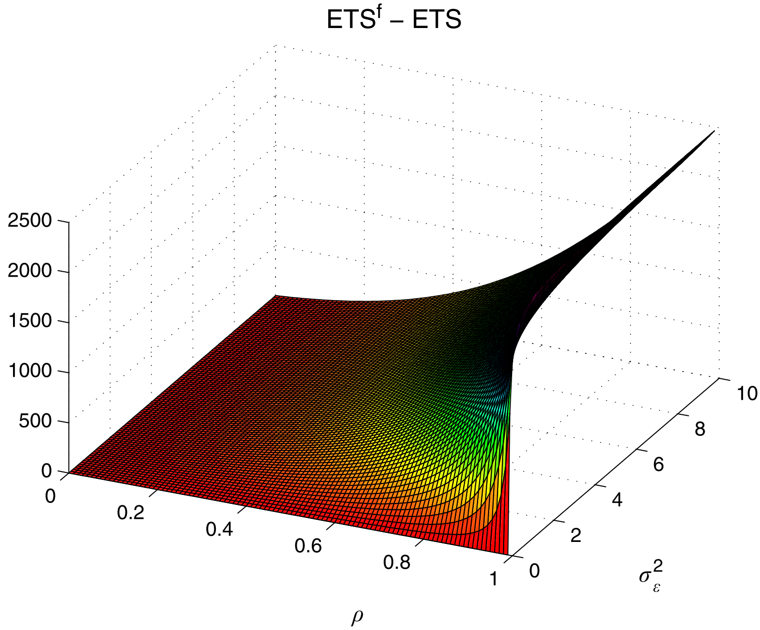

FIGURE S.1D.—$\text{ETS}^f - \text{ETS}$ as a function of ρ and σ_ε^2 (with parameters $\beta = \lambda = 1$, $\sigma_\theta^2 = 1$, $n = 4$).

the parameter constellation $\rho = 0.01$, $\lambda = 1$, $\beta = 5$, $\sigma_\theta^2 = 8$, and $n = 10$ (and $\bar{\theta} = 20$, $\alpha = 200$) for σ_ε^2 large enough.

Expected profits $E[\pi_i]$ increase in ρ (when $\sigma_\varepsilon^2 > 0$) or in σ_ε^2 (when $\rho > 0$) provided ρ or σ_ε^2 are not too small (see Figure S.2). Otherwise $E[\pi_i]$ may decrease in ρ or σ_ε^2, and this will tend to be so for σ_θ^2 large (see Figure S.3). Recall that $E[\pi_i]$ decreases in ρ when $\sigma_\varepsilon^2 = 0$ and in σ_ε^2 when $\rho = 0$ (Proposition 4(iii)).

Figure S.2 depicts the outcome of a typical simulation of $E[\pi_i]$. We can also check that $E[\pi_i]$ decreases in ρ and σ_ε^2 when close to $(0, 0)$.

Furthermore, increasing σ_ε^2 may decrease $E[\pi_i]$ when ρ is small (always when $\rho = 0$; see Proposition 4(iii)); for example, this happens when $\beta = 1$, $\lambda = 5$, $\rho = 0.01$, $\sigma_\theta^2 = 5$, $n = 2$, $\alpha = 200$, and $\bar{\theta} = 20$.

S.3.2. *Replica Markets*

This subsection presents details and complementary results on the welfare simulations of the replica market model (Section 4).

Simulations were performed for the base case β and λ in $\{1, 5\}$, $\rho \in [0.01, 0.99]$ with step size 0.01, σ_ε^2 and σ_θ^2 in $[0.01, 10.01]$ with step size 2, and $n \in [2, 82]$ with step size 20. (When necessary we extended the simulations to ρ in $[0.001, 0.999]$ with step size 0.001, to σ_ε^2 in $[0, 10]$ with step size 0.01,

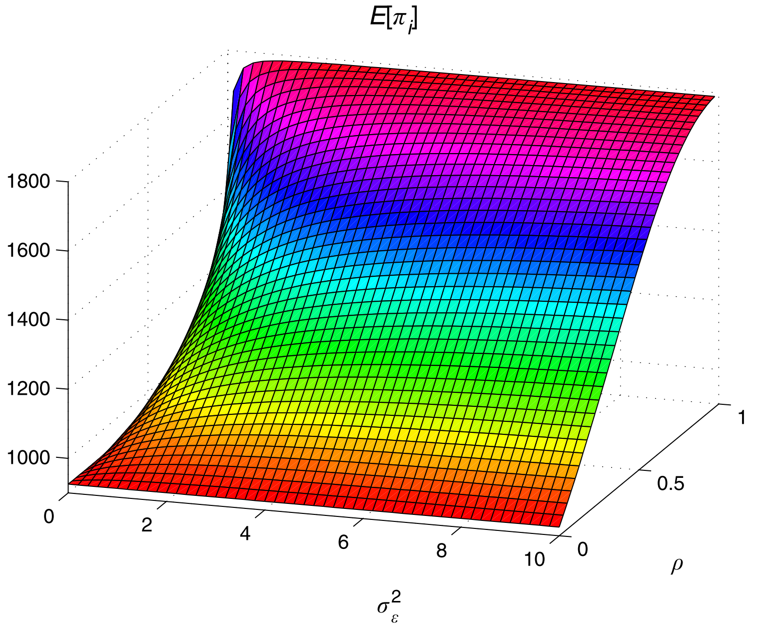

FIGURE S.2.—$E[\pi_i]$ as a function of ρ and σ_ε^2 (with parameters $\beta = \lambda = 1$, $\sigma_\theta^2 = 1$ and $n = 4$).

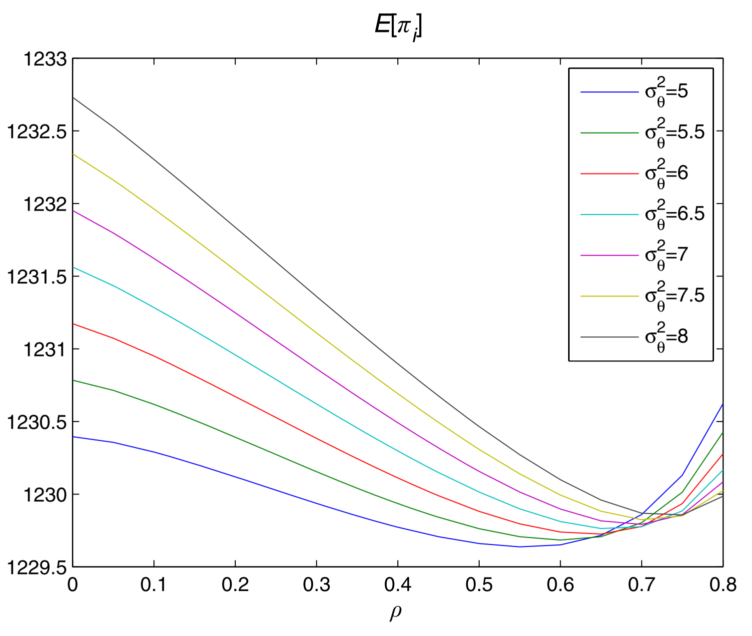

FIGURE S.3.—$E[\pi_i]$ as a function of ρ for different values of σ_θ^2 (with parameters $\beta = \lambda = 0.5$, $\sigma_\varepsilon^2 = 0.01$, and $n = 5$).

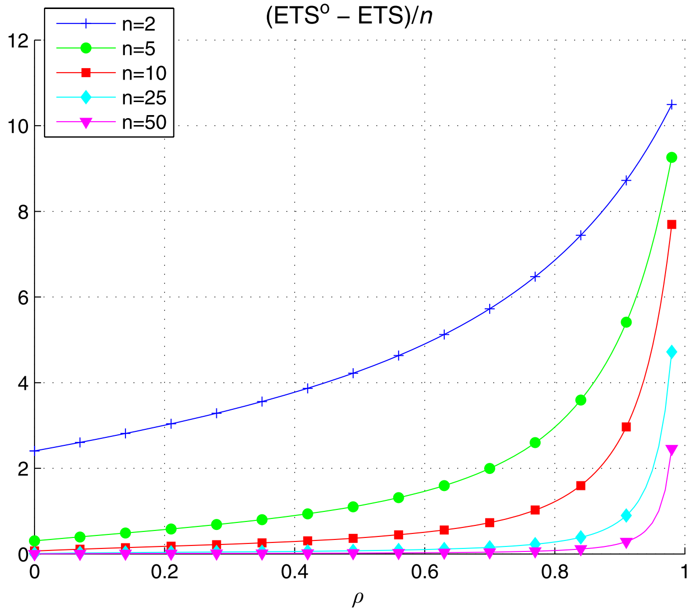

FIGURE S.4A.—$E[\mathrm{DWL}_n]/n \equiv (\mathrm{ETS}_n^o - \mathrm{ETS}_n)/n$ as a function of ρ for different values of n (with parameters $\beta = \lambda = 1$, $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, $\bar{\theta} = 30$, and $\alpha = 50$).

and to large n.) In this range of simulations and with $\bar{\theta} = 30$ and $\alpha = 50$, the probability of a negative output is at most 15% in either the SFE (strategic or competitive versions) or the Cournot equilibrium, and the upper bounds are attained only when $\beta = 5$ and $\lambda = 1$. Otherwise the probabilities of negative output tend to be very low.

Figures S.4a and S.4b provide the counterparts to Figure S.1a and Figure 2 for the replica market. Figure S.4a displays $E[\mathrm{DWL}_n]/n \equiv (\mathrm{ETS}_n^o - \mathrm{ETS}_n)/n$ as a function of ρ for different values of n. It is worth noting that $E[\mathrm{DWL}_n]/n$ is monotone in n and this is a general feature of the simulations.

Figure S.4b displays $E[\mathrm{DWL}_n]/n$ as a function of σ_ε^2 for different values of ρ. Note that the effect of increases in σ_ε^2 are small when ρ is small.

The result of Figure S.4a generalizes and $E[\mathrm{DWL}_n]/n$ decreases as the market gets large, and the rate of decrease is found to be slow for low σ_θ^2. The result is driven by the decrease in d_n with n (which overwhelms the effects that when averaging over predictions, $E[(\alpha - \tilde{t}_n)^2]$ may increase with n and that $E[(t_i - \tilde{t}_n)^2]$ does increase with n). Typically, the speed of convergence of the deadweight loss to zero (in terms of the constant of convergence) is slower when ρ is larger; that is, the limit as n tends to infinity of $n(\mathrm{ETS}_n^o - \mathrm{ETS}_n)$ is increasing with ρ. (This is so since $(\mathrm{ETS}_n^o - \mathrm{ETS}_n)$ is typically increasing in ρ for any n and the limit of $n(\mathrm{ETS}_n^o - \mathrm{ETS}_n)$ as n tends to infinity is well defined.)

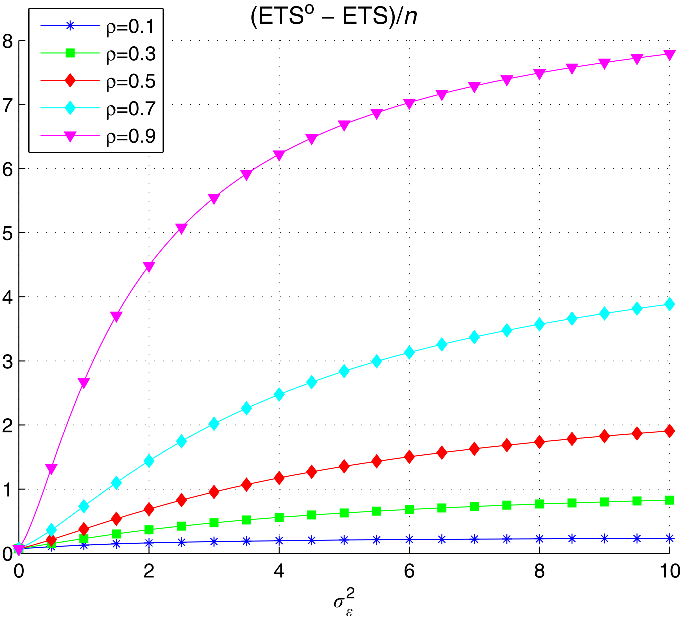

FIGURE S.4B.—$E[\mathrm{DWL}_n]/n$ as a function of σ_ε^2 for different values of ρ (with parameters $\beta=\lambda=1$, $\bar{\theta}=30$, $\alpha=50$, $\sigma_\theta^2=1$, and $n=10$).

The simulations also suggest that the (per capita) deadweight loss at the full information equilibrium $(\mathrm{ETS}_n^o - \mathrm{ETS}_n^f)/n$ (or deadweight loss due to standard market power) also decreases with n. This conclusion is driven by the fact that it can be checked analytically that c_n^f increases in n and, therefore, d_n^f decreases in n. However, the (per capita) deadweight loss due to the private-information-induced market power (that is, $(\mathrm{ETS}_n^f - \mathrm{ETS}_n)/n$) may increase with n for n low. This is because the deadweight loss due to standard market power falls more sharply with n at the beginning than the deadweight loss due to the private-information-induced market power.[3]

S.3.3. *Comparison With Cournot*

It is worth comparing the efficiency of the Cournot market ($\mathrm{ETS}^{\mathrm{C}}$) relative to the supply function market (denoted now by $\mathrm{ETS}^{\mathrm{SF}}$). A typical pattern for n not too large is for $\mathrm{ETS}^{\mathrm{SF}} - \mathrm{ETS}^{\mathrm{C}}$ to be positive for ρ close to zero and to be negative for ρ close to 1, being zero at the point for which the supply function equilibrium calls for a vertical supply and both equilibria coincide.

[3]For example, this happens from $n=2$ to $n=4$ when $\beta=\lambda=1$, $\bar{\theta}=30$, $\alpha=50$, $\sigma_\theta^2=1$, $\sigma_\varepsilon^2=10$, and $\rho=0.9$.

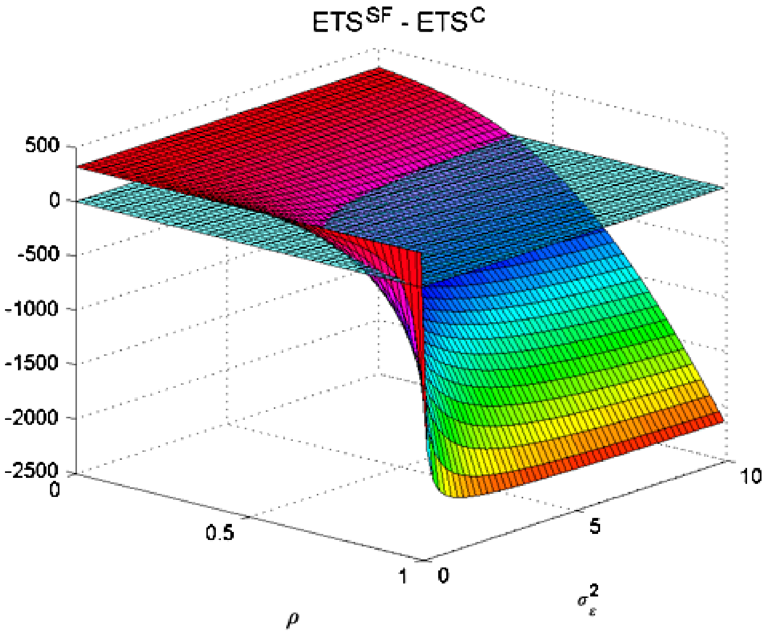

FIGURE S.5.—Efficiency differential between supply function and Cournot equilibria $\mathrm{ETS}^{\mathrm{SF}} - \mathrm{ETS}^{\mathrm{C}}$ as a function of ρ and σ_ε^2 (with parameters $\beta = \lambda = 1$, $\sigma_\theta^2 = 1$, $n = 4$, $\alpha = 200$, and $\bar{\theta} = 20$).

Furthermore, when signals are perfect ($\sigma_\varepsilon^2 = 0$) or close to perfect, we have that $\mathrm{ETS}^{\mathrm{SF}} - \mathrm{ETS}^{\mathrm{C}} > 0$. (See Figure S.5.) For σ_ε^2 or ρ small, sellers at the supply function market act with full information and have less market power. In the (Bayesian) Cournot equilibrium, sellers do not act with full information (see Section S.2).[4] For larger ρ and $\sigma_\varepsilon^2 > 0$, supply functions slope downward and sellers in the supply function market have more market power, and this may dominate the information effect.

It is worth comparing the efficiency of the Cournot market ($\mathrm{ETS}_n^{\mathrm{C}}$) relative to the supply function market ($\mathrm{ETS}_n^{\mathrm{SF}}$) in per capita terms. For large n, we have $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n > 0$ (except for ρ close to 1). (See Figure S.6a.)

The typical pattern of $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n$ as a function of σ_ε^2 is similar to that for ρ whenever $c < 0$ obtains for the parameters under consideration, being positive for σ_ε^2 small, negative for σ_ε^2 large, and zero at the point for which the supply function equilibrium calls for a vertical supply. See Figure S.6b.

Finally, and as an illustration of the theoretical result, for a given ρ we check that $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n > 0$ for large n. See Figure S.7.

[4]However, with constant marginal costs, the Bayesian Cournot equilibrium does replicate the full information outcome under some regularity conditions (see Palfrey (1985) and Vives (1988)).

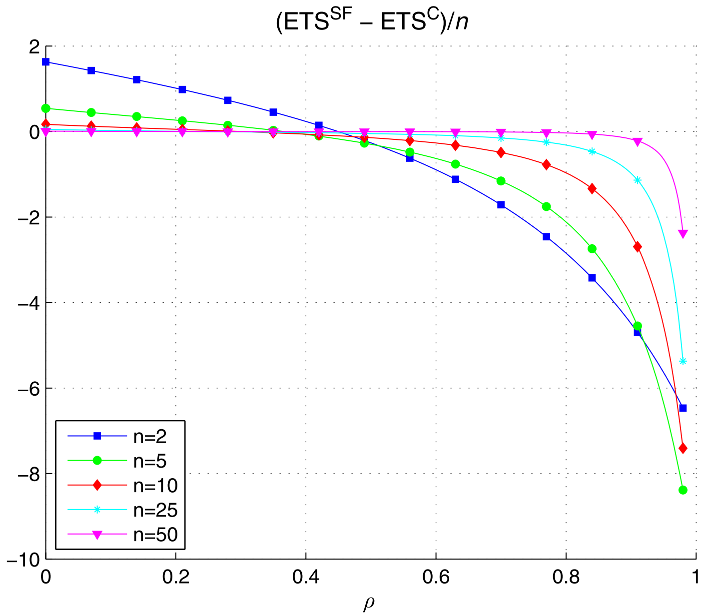

FIGURE S.6A.—Efficiency differential $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n$ between the supply function and the Cournot equilibria as a function of ρ for different values of n (with parameters $\beta = \lambda = 1$, $\bar{\theta} = 30$, $\alpha = 50$, and $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$).

S.4. INFORMATION ACQUISITION

This section studies the incentives to acquire information in the supply function market.

Do sellers have incentives to gather information? If the sellers receive the private signals for free in the course of their activity, for example, then in a privately revealing equilibrium, each seller has an incentive to rely on his private signal even though the price also provides information. Indeed, for seller i, the signal s_i still helps to estimate θ_i even though p reveals $\tilde{s}$.

Consider the model of Section 2 and suppose now that private signals have to be purchased at a cost, which is increasing and convex in the precision $\tau_\varepsilon \equiv 1/\sigma_\varepsilon^2$ of the signal, according to a smooth function $H(\cdot)$ that satisfies $H(0) = 0$, $H' > 0$ for $\tau_\varepsilon > 0$, and $H'' \geq 0$. There are, thus, nonincreasing returns to information acquisition. A strategy for seller i is a pair $(\tau_{\varepsilon_i}, X_i(\cdot, \cdot))$ that determines the precision purchased and the supply function strategy. Note that we consider the case where no seller observes the precision purchased by other sellers and we look, therefore, at a simultaneous move game where each seller chooses his precision and the supply function.

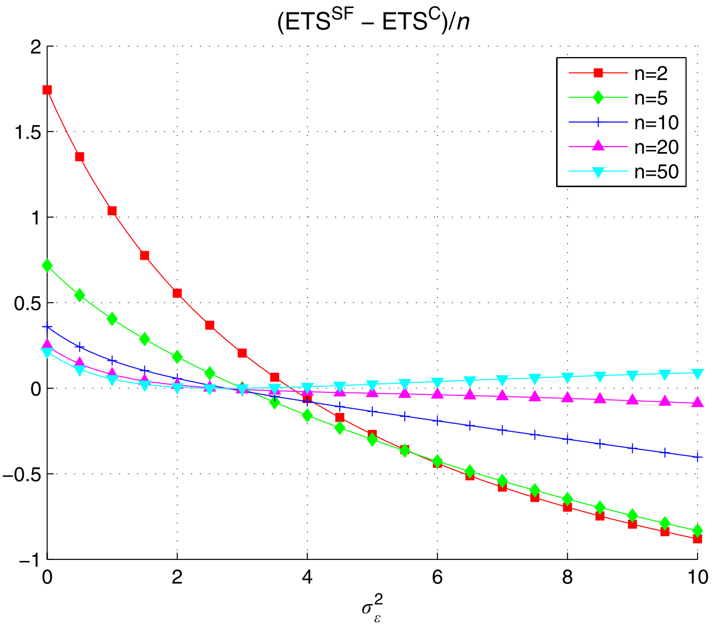

FIGURE S.6B.—Efficiency differential $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n$ between the supply function and the Cournot equilibria as a function of σ_ε^2 for different values of n (with parameters $\beta = \lambda = 1$, $\bar{\theta} = 30$, $\alpha = 50$, $\sigma_\theta^2 = 5$, and $\rho = 0.5$).

We analyze the symmetric Nash equilibria of the game. Note that for $(\tau_{\varepsilon_i}, X_i(\cdot,\cdot))_{i=1,\dots,n}$ to be a pure equilibrium of the game, $(X_i(\cdot,\cdot))_{i=1,\dots,n}$ needs to be the equilibrium of a game for a given precision tuple $(\tau_{\varepsilon_i})_{i=1,\dots,n}$.

Since we are interested in studying a symmetric equilibrium, we assume that any seller other than i, $j \neq i$, has the same precision, $1/\sigma_\varepsilon^2$, and the same coefficients, denoted by (b, a, c), for the candidate equilibrium supply function $X(s_j, p) = b - as_j + cp$, $j \neq i$. Seller i has precision $1/\sigma_{\varepsilon_i}^2$. Provided that $1 + \beta(n-1)c > 0$ and exactly as in Section 2, we obtain an optimal supply function for seller i for given supply functions of the rivals,

$$X_i(s_i, p) = (p - E[\theta_i | s_i, p])/(d + \lambda)$$

with $d = (\beta^{-1} + (n-1)c)^{-1}$. Now, as in the proof of Proposition 1, from the point of view of seller i, the price is informationally equivalent to $h_i \equiv \beta b(n-1) - \alpha + (1 + \beta(n-1)c)p + \beta x_i = \beta a \sum_{j \neq i} s_j$ and, therefore,

$$\begin{pmatrix} \theta_i \\ s_i \\ h_i \end{pmatrix} \sim N\left(\left(\begin{matrix} \bar{\theta} \\ \bar{\theta} \\ \beta a(n-1)\bar{\theta} \end{matrix}\right),\right.$$

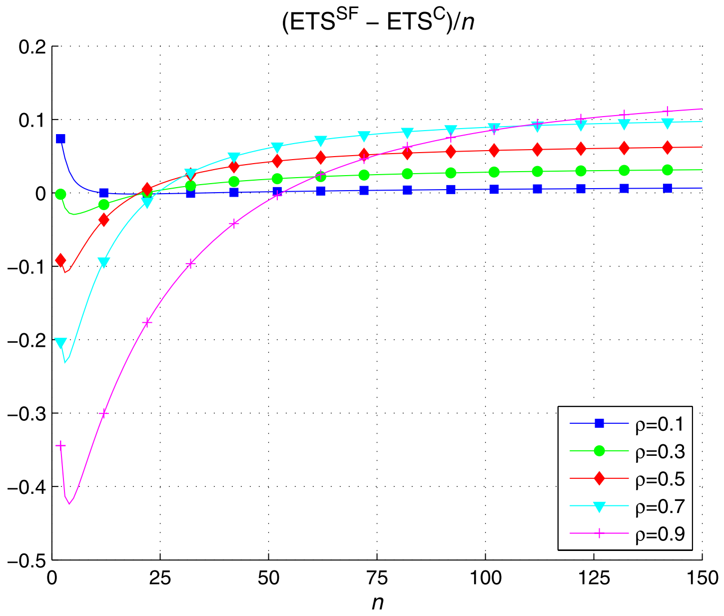

FIGURE S.7.—Efficiency differential $(\mathrm{ETS}_n^{\mathrm{SF}} - \mathrm{ETS}_n^{\mathrm{C}})/n$ between the supply function and the Cournot equilibria as a function of n (with parameters $\beta = \lambda = 1$, $\bar{\theta} = 15$, $\alpha = 20$, $\sigma_\theta^2 = 1$, and $\sigma_\varepsilon^2 = 5$).

$$\left(\begin{matrix} \sigma_\theta^2 & \sigma_\theta^2 & (n-1)\rho\sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_{\varepsilon_i}^2 & (n-1)\rho\sigma_\theta^2 \\ (n-1)\rho\sigma_\theta^2 & (n-1)\rho\sigma_\theta^2 & (n-1)((\sigma_\theta^2 + \sigma_\varepsilon^2) + (n-2)\rho\sigma_\theta^2) \end{matrix}\right)\Bigg),$$

and using the projection theorem for normal random variables, we obtain

$$E(\theta_i|s_i, h_i) = \bar{\theta} + \frac{\sigma_\theta^2(\sigma_\theta^2 + (n-2)\rho\sigma_\theta^2 + \sigma_\varepsilon^2) - (n-1)(\rho\sigma_\theta^2)^2}{\Delta_i}(s_i - \bar{\theta}) + \frac{\rho\sigma_\theta^2\sigma_{\varepsilon_i}^2}{\Delta_i}(h_i - \beta a(n-1)\bar{\theta}),$$

where $\Delta_i \equiv (\sigma_\theta^2 + \sigma_{\varepsilon_i}^2)((1 + (n-2)\rho)\sigma_\theta^2 + \sigma_\varepsilon^2) - (n-1)\sigma_\theta^4\rho^2$.

Using the expression for the supply function of i and for h_i, and identifying coefficients with the candidate strategy $X_i(s_i, p) = b_i - a_i s_i + c_i p$, we obtain

$$a_i = \frac{(\sigma_\theta^2(\sigma_\theta^2 + (n-2)\rho\sigma_\theta^2 + \sigma_\varepsilon^2) - (n-1)(\rho\sigma_\theta^2)^2)a}{(d + \lambda)a\Delta_i + \rho\sigma_\theta^2\sigma_{\varepsilon_i}^2},$$

$$b_i = -\frac{a((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2)\sigma_{\varepsilon_i}^2\bar{\theta} + \rho\sigma_\theta^2\sigma_{\varepsilon_i}^2((n-1)b - \alpha\beta^{-1})}{(d+\lambda)a\Delta_i + \rho\sigma_\theta^2\sigma_{\varepsilon_i}^2},$$

and

$$c_i = \frac{a\Delta_i - \rho\sigma_\theta^2\sigma_{\varepsilon_i}^2 d}{(d+\lambda)a\Delta_i + \rho\sigma_\theta^2\sigma_{\varepsilon_i}^2}.$$

Given $(\tau_\varepsilon, X(\cdot,\cdot))_{j\neq i}$ with $X(s_j, p) = b - as_j + cp$, we obtain the optimal supply strategy for seller i when he has precision $\tau_{\varepsilon_i} \equiv 1/\sigma_{\varepsilon_i}^2$. From the form of the optimal supply, it follows that the seller's expected profits from trading are given by

$$E[\pi_i] = \left(d + \frac{\lambda}{2}\right)E\big[(X_i(s_i, p))^2\big].$$

Note that $E[\pi_i]$ is a function of $(b, a, c, \sigma_\varepsilon^2, \sigma_{\varepsilon_i}^2)$ since $d = (\beta^{-1} + (n-1)c)^{-1}$. After some lengthy manipulations, it can be checked that

$$\begin{aligned}
E[\pi_i] = {} & \big(2(1 + c\beta(n-1))\big(2\beta + \lambda(1 + c\beta(n-1))\big)\big)^{-1} \\
& \times \Big(\big(\sigma_\theta^4(1-\rho)(\rho(n-1)+1) + \sigma_\theta^2\sigma_\varepsilon^2 \\
& + \sigma_{\varepsilon_i}^2\big(\sigma_\theta^2(\rho(n-2)+1) + \sigma_\varepsilon^2\big)\big)^{-1} \\
& \times \big(\sigma_\theta^2(\sigma_\theta^2(1-\rho)(\rho(n-1)+1) + \sigma_\varepsilon^2\big) \\
& \times \big(\sigma_\theta^2\big(a\beta\rho(n-1) - (1 + c\beta(n-1))\big)^2 \\
& + a^2\beta^2(n-1)\big(\sigma_\theta^2(1-\rho)(\rho(n-1)+1) + \sigma_\varepsilon^2\big)\big) \\
& + \sigma_{\varepsilon_i}^2(n-1)\big(\sigma_\theta^2\rho(1 + c\beta(n-1)) \\
& - a\beta(\sigma_\theta^2(\rho(n-2)+1) + \sigma_\varepsilon^2)\big)^2\big) \\
& + \big((\alpha - \overline{\theta}) - (b + \overline{\theta}(c-a))\beta(n-1)\big)^2\Big).
\end{aligned}$$

When optimizing, seller i chooses $\sigma_{\varepsilon_i}^2$ and takes as given $(b, a, c, \sigma_\varepsilon^2)$. The marginal benefit of acquiring precision $\tau_{\varepsilon_i} \equiv 1/\sigma_{\varepsilon_i}^2$, $\partial E[\pi_i]/\partial\tau_{\varepsilon_i}$, evaluated at a symmetric solution $\tau_{\varepsilon_i} = \tau_\varepsilon$, is equal to

$$\psi(\tau_\varepsilon) \equiv \frac{1}{2(2d+\lambda)}\frac{(\tau_\varepsilon(1-\rho)(1+\rho(n-1)) + \tau_\theta)^2}{(\tau_\varepsilon(1-\rho) + \tau_\theta)^2(\tau_\varepsilon(1+\rho(n-1)) + \tau_\theta)^2},$$

which is decreasing in τ_ε for a given c or d.[5] Interior symmetric equilibria are characterized by the solution of $\psi(\tau_\varepsilon) - H'(\tau_\varepsilon) = 0$ with c given by the largest solution to the quadratic equation $g(c; M) = 0$ for a given τ_ε. We know from Proposition 1 that there is a solution that fulfills $1 + \beta nc > 0$ provided that $-(n-1)^{-1} < \rho < 1$. From Claim A.2, we know that $c \to \widehat{c}$ as $\sigma_\varepsilon^2 \to \infty$ or $\tau_\varepsilon \to 0$ (and $\widehat{c}$ is decreasing in ρ and independent of τ_θ). Let $d(0+) \equiv \lim_{\tau_\varepsilon \to 0} d(\tau_\varepsilon)$. Then $\psi(0) \equiv \lim_{\tau_\varepsilon \to 0} \psi(\tau_\varepsilon) = (2(2d(0+) + \lambda)\tau_\theta^2)^{-1} > 0$ and $\psi(\tau_\varepsilon) \to 0$ as $\tau_\varepsilon \to \infty$. If $H'(0) < \psi(0)$ for $\rho < 1$, there is an interior solution $\tau_\varepsilon^* > 0$ to the equation

$$\phi(\tau_\varepsilon) \equiv \psi(\tau_\varepsilon) - H'(\tau_\varepsilon) = 0$$

since $\phi(0) > 0$, $\phi(\infty) < 0$, and $\phi(\cdot)$ is continuous.

If $H'(0) \geq \psi(0)$, then there can be no information acquisition in a symmetric equilibrium and, in fact, there is no equilibrium (with $H'(0)$ not too high). We have that $\tau_\varepsilon^* = 0$ at a candidate equilibrium but this cannot be an overall equilibrium since if other sellers do not purchase information, then the price contains no additional information for a seller and it will pay a single seller to get information (with $H'(0)$ not too high). This is akin to the Grossman–Stiglitz (1980) paradox on the impossibility of an informationally efficient market. As parameters β, λ, ρ, and n move in such a way that $\psi(0) \downarrow H'(0)$, then $\tau_\varepsilon^* \to 0$ and the linear supply function equilibrium collapses.

The following proposition characterizes the equilibrium in the information acquisition game.[6]

PROPOSITION S.3: *Let* $-(n-1)^{-1} < \rho < 1$, $\psi(0) \equiv (2(2d(0+) + \lambda)\tau_\theta^2)^{-1}$, *and* $d(0+) = (\beta^{-1} + (n-1)\,\widehat{c})^{-1}$. *There is a symmetric equilibrium in the game with costly information acquisition provided that* $H'(0) < \psi(0)$. *At equilibrium, sellers buy a positive precision of information* $\tau_\varepsilon^* > 0$.

REMARK S.3: Existence obtains in particular if $H'(0) = 0$ or the prior is diffuse enough (τ_θ small) even if the number of sellers is large and/or ρ is close to 1. If $H'(0) > 0$, for any $-(n-1)^{-1} < \rho < 1$ and n, we have $\tau_\varepsilon^* > 0$ if $H'(0)$ or τ_θ is small enough. As $\rho \to 1$, we have that $\psi(0) \to (2(2\beta n + \lambda)\tau_\theta^2)^{-1}$ since $\widehat{c} \to -1/\beta n$ (see Claim A.2) and $d(0+) \to \beta n$. Therefore, for ρ close to 1 and a large number of sellers, we need a very diffuse prior so as to have positive precision acquisition (at the unique equilibrium). If $H'(0) > 0$, then for ρ close to 1 and n large enough, there is no purchase of information. However, the

[5] It can be checked that $E[\pi_i]$ is strictly concave in τ_{ε_i} if $d > 0$ (which is always the case in equilibrium).

[6] Jackson (1991) showed the possibility of fully revealing prices in a common value environment with costly information acquisition and a finite number of agents, and under some specific parametric assumptions.

situation changes, as we will see, in the natural case of a large market where the number of buyers and sellers grows together (in the replica economy).

The intuition for the result is as follows. The marginal benefit of acquiring precision τ_ε declines with the level of precision acquired and is positive for $\tau_\varepsilon = 0$ with a finite number of sellers even if parameters are very correlated ($\psi(0) > 0$). This is so since, even with high correlation, a seller who purchases a signal improves the information on his random cost parameter despite learning the signals of the other sellers through the price. When the number of sellers is large, the improvement will be small, but if the seller can purchase a little bit of precision at a small cost, he will do so. Furthermore, the more diffuse is the prior, the higher is the marginal value of information.

Let us consider the replica economy of Section 4 (with $\rho \in [0, 1)$). Replacing β with β/n, we obtain that $\psi_n(0) \equiv (2(2d_n(0+) + \lambda)\tau_\theta^2)^{-1}$, where $d_n(0+) = (n\beta^{-1} + (n-1)\widehat{c}_n)^{-1}$. Proposition S.3 applies with $\psi_n(0)$ instead of $\psi(0)$. Now, for any n, as $\rho \to 1$, we have that $\widehat{c} \to -1/\beta$, $d_n(0+) \to \beta$, and $\psi_n(0) \to (2(2\beta + \lambda)\tau_\theta^2)^{-1}$. For ρ close to 1 for any number of sellers (size of replica), we need the same degree of diffusion of the prior to have positive precision acquisition.

What happens in a large market (that is, a market where the limit $n \to \infty$ is taken first)?

When the number of buyers and sellers grows together, we can consider the large market limit case as $n \to \infty$. It is possible to show then that whenever the marginal cost of acquiring precision at zero is positive, there is an upper bound on the degree of correlation of costs (strictly less than 1 and increasing in the diffuseness of the prior) below which there is an information acquisition equilibrium. (See Vives (2011).)

REMARK S.4: We could also consider the case where information acquisition is observable in a first stage of the game and then, at a second stage, sellers compete in supply functions for given precisions. In this case, we add a strategic effect of information acquisition and the characterization is much more involved. However, it is possible at least to check that the results are similar for ρ close to 1. (For example, we also have that $\psi_n(0) \to (2(2\beta + \lambda)\tau_\theta^2)^{-1}$ as $\rho \to 1$.)

SUMMARY: If the signals are costly to acquire and agents face a convex cost of acquiring precision, then it is possible to show that each seller will have an incentive to purchase some precision for any correlation of the cost parameters that is not perfect, provided that at symmetric solutions the marginal cost of acquiring precision for zero precision is less than the marginal benefit. This will happen, for example, whenever the marginal cost of acquiring precision is zero for zero precision or when the prior is diffuse enough (since then the marginal benefit of acquiring precision at zero precision is large) even if the number of sellers is large and/or ρ is close to 1. The result is obtained in the case where no seller observes the precision purchased by other sellers and considers, therefore, a simultaneous move game where each seller chooses his precision and the supply function. In the case of a large market where the number of buyers and sellers grows together whenever the marginal cost of acquiring precision at zero is positive, there is an upper bound on the degree of correlation of costs (increasing in the diffuseness of the prior) below which there is an information acquisition equilibrium.

S.5. DEMAND UNCERTAINTY

This section deals with the case where demand is uncertain and characterizes the SFE, providing the analysis and full results for Section 5.2.

Let $P(y) = \alpha + u - \beta y$ with $u \sim N(0, \sigma_u^2)$ independent of the other random variables in the model. The analysis of the equilibrium proceeds as in Section 2.1, positing candidate linear strategies $X(s_j, p) = b - as_j + cp$, $j \neq i$. Now the intercept of residual demand $p = I_i - dx_i$ for seller i is given by

$$I_i \equiv d\left((\alpha + u)\beta^{-1} - (n-1)b + a\sum_{j\neq i} s_j\right)$$

$$\text{with} \quad d \equiv (\beta^{-1} + (n-1)c)^{-1},$$

where I_i is informationally equivalent to $h_i \equiv u + \beta a \sum_{j\neq i} s_j$.

S.5.1. *Characterization of the Equilibrium*

The following proposition provides a full statement of the results presented in Section 5.2 (including Proposition 7).

PROPOSITION S.4: *Let $\lambda > 0$. For any $\rho \in [-(n-1)^{-1}, 1]$, $\sigma_u^2 > 0$, and $\sigma_\varepsilon^2 \geq 0$, there exists a SFE. It is given by $X(s_i, p) = (p - E[\theta_i | s_i, p])/(d + \lambda)$ with $0 < d < \beta n$.*

(i) *Let $\sigma_\varepsilon^2 > 0$. Then when $\rho \in (0, 1]$ or $\rho \in (-(n-1)^{-1}, 0)$, d is a root of the sixth degree polynomial*

$$\begin{aligned}\Gamma(d) = {} & \rho Y(d)(\Lambda(d)(1-\rho) + dn(d+\lambda+n\beta))^2\sigma_u^2 \\ & + \beta^2\sigma_\theta^2(1-\rho)(\rho(n-1)+1) \\ & \times (n\rho - M(1-\rho))(n-1)^{-1}(d-n\beta)^2\Lambda(d),\end{aligned}$$

where

$$\begin{aligned}\Lambda(d) &= -(M+n)d^2 - \big((\lambda - n\beta)M + n(\beta(n-2)+\lambda)\big)d \\ &\quad + n\beta\lambda(M+1), \\ Y(d) &= -d^2 - (\beta(n-2)+\lambda)d + \beta\lambda,\end{aligned}$$

and

$$M \equiv \frac{\rho\sigma_\varepsilon^2 n}{(1-\rho)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)},$$

that belongs to the interval (Y_2, Λ_2) *when* $\rho \in (0,1]$ *or belongs to* (Λ_2, Y_2) *when* $\rho \in (-(n-1)^{-1}, 0)$, *with* $Y_2 \equiv$ *largest root of* $Y(d)$ *and* $\Lambda_2 \equiv$ *largest root of* $\Lambda(d)$. *The root is unique if* $\rho \in (0,1]$ *or* $\rho \in (-(n-1)^{-1}, 0)$ *and* $n > 3$. *When* $\rho = -(n-1)^{-1}$ *and* $\sigma_\varepsilon^2 > 0$, d *is a root* (*unique if* $n > 3$) *in* $(0, Y_2)$ *of the sixth degree polynomial* $\frac{\Gamma(d)}{((n-1)\rho+1)(n\rho - M(1-\rho))}$. *When* $\rho\sigma_\varepsilon^2 = 0$, *then* $d = \Lambda_2 = Y_2 = d^f$.

(ii) *As* $\sigma_u^2 \to 0$, $d \to \Lambda_2$, *and as* $\sigma_u^2 \to \infty$, $d \to Y_2 = d^f$. *In the cases with a unique equilibrium*, *when* $\sigma_\varepsilon^2 > 0$, *then d increases in* ρ *for* $\rho \in (-(n-1)^{-1}, 1)$, *and when* $\rho\sigma_\varepsilon^2 > 0$ (*resp.* $\rho\sigma_\varepsilon^2 < 0$), *then d is decreasing* (*resp. increasing*) *with* σ_u^2.

(iii) *If* $\rho\sigma_\varepsilon^2 > 0$, *then* $\partial d/\partial\sigma_\varepsilon^2 > 0$ *if* $\sigma_\varepsilon^2 \le \sigma_\theta^2$.

COROLLARY: *We have that* $c = (d^{-1} - \beta^{-1})/(n-1)$ *with* $c > -M((1+M)\beta n)^{-1}$ (*and* $\lambda^{-1} > c$ *when* $\rho \ge 0$),

$$a = \frac{(n\beta - d)(n\rho - M(1-\rho))}{(n-1)\rho((1-\rho)\Lambda(d) + dn(d+\lambda+n\beta))} > 0,$$

$$b = \left(-\frac{(n\beta - d)}{d(n-1)(d+\lambda+n\beta)} + a\right)\bar{\theta} - \frac{\alpha}{d\beta(n-1)(d+\lambda+n\beta)}Y(d)$$

and when $\rho\sigma_\varepsilon^2 > 0$, *both a and c increase in* σ_u^2, *and c is nonmonotone in* σ_ε^2.

REMARK S.5: Simulations suggest that the equilibrium is also unique when $\rho < 0$ and $n = 2, 3$.

REMARK S.6: The largest root of $Y(d)$ is $Y_2 = d^f$, the full information solution when $\sigma_u^2 = 0$ (as in Proposition 1 when $\rho\sigma_\varepsilon^2 = 0$). In equilibrium, therefore, $d > d^f$ when $\rho\sigma_\varepsilon^2 > 0$ and $d < d^f$ when $\rho\sigma_\varepsilon^2 < 0$.

REMARK S.7: The largest root Λ_2 of $\Lambda(d)$ is the solution for d in Proposition 1 (where $\sigma_u^2 = 0$). Therefore, as $\sigma_u^2 \to 0$, the equilibrium d tends to the value of when $\sigma_u^2 = 0$.

REMARK S.8—Conditions for the Noise Independence Property (Equilibrium Independent of σ_u^2): (i) When $\rho\sigma_\varepsilon^2 = 0$ ($M = 0$), then $\Lambda(d) = nY(d)$ and $d = Y_2 = \Lambda_2 = d^f$. (ii) When $\sigma_\varepsilon^2 \to \infty$ and $\sigma_u^2 > 0$, then again $d \to d^f$ (yielding $X(p) = c^f(p - \bar{\theta})$). When $\rho = 0$, then $d = d^f$. If $\rho \in (0,1]$ or $-(n-1)^{-1} < \rho < 0$, then $(n\rho - M(1-\rho)) \to 0$ as $\sigma_\varepsilon^2 \to \infty$ and $\Gamma(d) = 0$ if and only if $Y(d) = 0$. If $\rho = -(n-1)^{-1}$, then

$$\frac{\Gamma(d)}{((n-1)\rho+1)(n\rho - M(1-\rho))\sigma_\varepsilon^2} \to \frac{\beta^2(n-1)(2d+\lambda)^2}{\sigma_\theta^2}Y(d)n\sigma_u^2$$

as $\sigma_\varepsilon^2 \to \infty$ and $d \to Y_2 = d^f$ (recall that the SOC is $2d + \lambda > 0$). It is easily checked that this limit is also the equilibrium when $\sigma_\varepsilon^2 = \infty$ ($\tau_\varepsilon = 0$).

CLAIM S.1: *The coefficient of h_i in $E(\theta_i|s_i, h_i)$ increases in ρ if $\sigma_\varepsilon^2 > 0$.*

REMARK S.9: When $\lambda = 0$, we need $d > 0$ to fulfill the SOC $2d > 0$. We have that $Y_2 = 0$ and $\Lambda_2 = \frac{-n\beta(n-2-M)}{M+n}$, and since $M + n > 0$, for a SFE to exist, we need $n - 2 - M < 0$, proving Remark 7 in Section 5.2.

CLAIM S.2: *With inelastic noisy demand u, $\rho = 1$, $n > 2$, and $\lambda = 0$, there is a unique SFE*:

$$d = \frac{n\sigma_\theta^2}{(2\sigma_\varepsilon^2 + n\sigma_\theta^2)}\left(\frac{n\sigma_\varepsilon^2}{(n-1)(n-2)\sigma_u^2}\right)^{1/2}$$

and

$$(nc)^{-1} = \frac{\sigma_\theta^2}{2\sigma_\varepsilon^2 + n\sigma_\theta^2}\left(\frac{(n-1)n\sigma_\varepsilon^2}{(n-2)\sigma_u^2}\right)^{1/2}.$$

S.5.2. *Proof of Results*

PROOF OF PROPOSITION S.4(i)—Existence and Uniqueness of a SFE: Given linear strategies of rivals $X(s_j, p) = b - as_j + cp$, $j \neq 1$, seller i faces a residual inverse demand

$$p = \alpha + u - \beta(n-1)(b + cp) + \beta a \sum_{j\neq i} s_j - \beta x_i.$$

Provided $1 + \beta(n-1)c > 0$, it follows that $p = I_i - dx_i$ where

$$I_i \equiv d\left((\alpha + u)\beta^{-1} - (n-1)b + a\sum_{j\neq i} s_j\right) \quad \text{and}$$

$$d \equiv (\beta^{-1} + (n-1)c)^{-1}.$$

Note that (s_i, I_i) is informationally equivalent to (s_i, h_i), where $h_i = u + \beta a \sum_{j\neq i} s_j$ and, therefore, $E(\theta_i|s_i, I_i) = E(\theta_i|s_i, h_i)$ and

$$\operatorname{var}[s_i, h_i] = \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & \beta a(n-1)\rho\sigma_\theta^2 \\ \beta a(n-1)\rho\sigma_\theta^2 & \sigma_u^2 + (\beta a)^2((n-1)(\sigma_\theta^2 + \sigma_\varepsilon^2) + (n-1)(n-2)\rho\sigma_\theta^2) \end{pmatrix}$$

From the Gaussian updating formulæ, it follows that

$$
\begin{aligned}
&E(\theta_i|s_i, h_i)\\
&\quad = \bar{\theta}\\
&\qquad + \frac{(\sigma_u^2 + a^2\beta^2(n-1)(\sigma_\theta^2(1-\rho)((n-1)\rho+1)+\sigma_\varepsilon^2))\sigma_\theta^2}{\Delta}(s_i - \bar{\theta})\\
&\qquad + \frac{(n-1)\beta\rho a\sigma_\theta^2\sigma_\varepsilon^2}{\Delta}(h_i - \beta a(n-1)\bar{\theta}),
\end{aligned}
$$

where $\Delta \equiv \sigma_u^2(\sigma_\theta^2 + \sigma_\varepsilon^2) + a^2\beta^2(n-1)(\sigma_\varepsilon^2 + \sigma_\theta^2(1-\rho))(\sigma_\theta^2((n-1)\rho+1) + \sigma_\varepsilon^2)$.

From the first order condition of the optimization problem for seller i, we obtain the optimal supply $X(s_i, p) = (p - E[\theta_i|s_i, p])/(d+\lambda)$ with second order condition $2d + \lambda > 0$. Taking into account the expression of the price and the symmetry of the equilibrium, we obtain $h_i = p(1+\beta nc) - \alpha + \beta nb - \beta as_i$. Substituting in the expression for $E(\theta_i|s_i, h_i)$ and in the optimal supply for seller i, noting that $X(s_i, p) = b - as_i + cp$ and identifying coefficients, we have the following system of equations for a, b, and c where $d = (\beta^{-1} + (n-1)c)^{-1}$:

$$
\begin{aligned}
a &= \frac{\sigma_\theta^2(a^2\beta^2(n-1)(1-\rho)(\sigma_\theta^2(\rho(n-1)+1)+\sigma_\varepsilon^2)+\sigma_u^2)}{\Delta(d+\lambda)},\\
b &= (d+\lambda)^{-1}\\
&\quad \times \Big(\Big(\sigma_\theta^2\big(a^2\beta^2(n-1)(1-\rho)\big(\sigma_\theta^2(\rho(n-1)+1)+\sigma_\varepsilon^2\big)+\sigma_u^2\big)\bar{\theta}\\
&\quad + (n-1)\rho\beta a\sigma_\theta^2\sigma_\varepsilon^2(\alpha - n\beta b + n\beta a\bar{\theta})\Big)\\
&\quad /\Delta - \bar{\theta}\Big),\\
c &= \left(1 - \frac{(n-1)\rho\beta a\sigma_\theta^2\sigma_\varepsilon^2(n\beta c+1)}{\Delta}\right)(d+\lambda)^{-1}.
\end{aligned}
$$

From the expression for c, it follows that

$$
\begin{aligned}
a^2 &= \sigma_u^2((d+\lambda)^{-1} - c)\\
&\quad /\big((n-1)\rho\beta\sigma_\varepsilon^2(n\beta c+1) - \beta^2(n-1)(1-\rho)\\
&\quad \times \big(\sigma_\theta^2(\rho(n-1)+1)+\sigma_\varepsilon^2\big)((d+\lambda)^{-1} - c)\big),
\end{aligned}
$$

and using this expression in the first equation for a, we obtain

$$a = (1 + n\beta c)(1 + (n-1)\beta c)\sigma_\theta^2 (f(c))^{-1},$$

where

$$\begin{aligned} f(c) \equiv\ & c^2 \lambda \beta^2 (n-1)^2 (\sigma_\theta^2 (1-\rho) + \sigma_\varepsilon^2) \\ & + c\beta(n-1)\big(2(\sigma_\theta^2 + \sigma_\varepsilon^2)(\beta + \lambda) + \sigma_\theta^2 \rho(\beta(n-2) - \lambda)\big) \\ & + (\sigma_\theta^2 + \sigma_\varepsilon^2)(2\beta + \lambda) + \sigma_\theta^2 \beta\rho(n-1). \end{aligned}$$

Using the previous two expressions for a^2 and a in the first equation for c and after tedious computations, using $d = (\beta^{-1} + (n-1)c)^{-1}$, when $\rho \in (0, 1]$, or $\rho \in (-(n-1)^{-1}, 0)$, we obtain the equilibrium d as the solution to $\Gamma(d) = 0$ as defined in the proposition.

Let Λ_1 and Λ_2 (Υ_1 and Υ_2, respectively) denote the smallest and largest roots of the quadratic equations $\Lambda(d)$ ($\Upsilon(d)$, respectively). The discriminant of the quadratic equations is always positive for $\lambda > 0$. We have that

$$\Upsilon_2 = \beta - \frac{1}{2}\lambda - \frac{1}{2}n\beta + \frac{1}{2}\sqrt{\beta^2(n-2)^2 + \lambda^2 + 2n\beta\lambda}$$

and

$$\begin{aligned} \Lambda_2 = \ & \big(-\lambda(M+n) + n\beta(M-n+2) \\ & + \sqrt{n^2(M-n+2)^2\beta^2 + 2n\lambda(M+n)^2\beta + \lambda^2(M+n)^2}\big) \\ & \big/(2(M+n)). \end{aligned}$$

It follows that $\Upsilon_2 = \Lambda_2$ for $\rho\sigma_\varepsilon^2 = 0$ since then $M = 0$. Let $\sigma_\varepsilon^2 > 0$. Then $\Upsilon_1 < \Lambda_1 < -(\lambda/2) < 0 < \Upsilon_2 < \Lambda_2$ for $\rho \in (0, 1]$ and $\Lambda_1 < \Upsilon_1 < -(\lambda/2) < 0 < \Lambda_2 < \Upsilon_2$ for $\rho \in (-(n-1)^{-1}, 0)$. In addition, $\Gamma(d) > 0$ ($\Gamma(d) < 0$) for all $d \in (\frac{-\lambda}{2}, \Upsilon_2]$ and $\Gamma(d) < 0$ ($\Gamma(d) > 0$) for all $d \geq \Lambda_2$ for $\rho \in (0, 1]$ ($\rho \in (-(n-1)^{-1}, 0)$). Therefore, we conclude that a root of $\Gamma(d)$ that satisfies $d > -\lambda/2$ (i.e., the SOC $2d + \lambda > 0$) exists and belongs to the interval (Υ_2, Λ_2) if $\rho \in (0, 1]$ or to the interval (Λ_2, Υ_2) if $\rho \in (-(n-1)^{-1}, 0)$.

I show now that for $\rho \in (0, 1]$ or $\rho \in (-(n-1)^{-1}, 0)$ and $n > 3$, d is the unique root of the sixth degree polynomial $\Gamma(d) = 0$. To obtain uniqueness of the equilibrium, we show that $\Gamma'(d) < 0$ (resp. $\Gamma'(d) > 0$) for any root of $\Gamma(d)$ belonging to the interval (Υ_2, Λ_2) for $\rho \in (0, 1]$ (resp. to the interval (Λ_2, Υ_2) for $\rho \in (-(n-1)^{-1}, 0)$ and $n > 3$). This fact guarantees the uniqueness of the equilibrium. Note that $\Upsilon(d) < 0$ and $\Lambda(d) > 0$ for $d \in (\Upsilon_2, \Lambda_2)$ for $\rho > 0$, and $\Upsilon(d) > 0$ and $\Lambda(d) < 0$ for $d \in (\Lambda_2, \Upsilon_2)$ for $\rho < 0$.

We have that

$$\begin{aligned}\Gamma'(d) = \rho\sigma_u^2\bigl(&Y'(d)\bigl(\Lambda(d)(1-\rho) + dn(d+\lambda+n\beta)\bigr)^2 \\ &+ 2Y(d)(\Lambda(d)(1-\rho) + dn(d+\lambda+n\beta)) \\ &\times (\Lambda'(d)(1-\rho) + n(2d+\lambda+n\beta))\bigr) \\ &+ \frac{\beta^2\sigma_\theta^2(1-\rho)(\rho(n-1)+1)(\rho(M+n)-M)}{(n-1)} \\ &\times (2(d-n\beta)\Lambda(d) + (d-n\beta)^2\Lambda'(d)).\end{aligned}$$

Using the fact that in a zero of $\Gamma(d)$,

$$\begin{aligned}\rho\sigma_u^2 = &-\frac{\beta^2\sigma_\theta^2(1-\rho)(\rho(n-1)+1)(\rho(M+n)-M)}{(n-1)} \\ &\times \frac{(d-n\beta)^2\Lambda(d)}{Y(d)(\Lambda(d)(1-\rho) + dn(d+\lambda+n\beta))^2},\end{aligned}$$

we obtain

$$\begin{aligned}\Gamma'(d) = &\frac{\beta^2\sigma_\theta^2(1-\rho)(\rho(n-1)+1)(\rho(M+n)-M)}{(n-1)} \\ &\times \Bigl(-\frac{(d-n\beta)^2\Lambda(d)}{Y(d)}Y'(d) \\ &- 2(\Lambda'(d)(1-\rho) + n(2d+\lambda+n\beta)) \\ &\times \frac{(d-n\beta)^2\Lambda(d)}{\Lambda(d)(1-\rho) + dn(d+\lambda+n\beta)} \\ &+ 2(d-n\beta)\Lambda(d) + (d-n\beta)^2\Lambda'(d)\Bigr).\end{aligned}$$

If $\rho \in (0,1]$, then $\rho(M+n) - M > 0$. Hence, the first factor of the previous product is positive. Concerning the second factor, note that by adding the first and the fourth terms, we obtain

$$\begin{aligned}&-\frac{(d-n\beta)^2\Lambda(d)}{Y(d)}Y'(d) + (d-n\beta)^2\Lambda'(d) \\ &\quad = \frac{(d-n\beta)^2}{Y(d)}(M\beta(n-1)(2d\lambda + \lambda^2 + 2d^2 + n\beta\lambda)).\end{aligned}$$

Since $d > Y_2$, we know that $Y(d) < 0$, which implies that the previous expression is negative. On the other hand, adding the second and the third terms yields

$$
\begin{aligned}
&-2(\Lambda'(d)(1-\rho)+n(2d+\lambda+n\beta))\frac{(d-n\beta)^2\Lambda(d)}{\Lambda(d)(1-\rho)+dn(d+\lambda+n\beta)} \\
&\quad +2(d-n\beta)\Lambda(d) \\
&\quad =\frac{2(d-n\beta)\Lambda(d)}{\Lambda(d)(1-\rho)+dn(d+\lambda+n\beta)} \\
&\quad \times \big((d(-d+2n\beta)(-M+M\rho+n\rho)+n\beta(-\rho+n\rho+1)\lambda \\
&\quad +n^2\beta^2((1-\rho)(M+2)+n\rho))\big).
\end{aligned}
$$

Since $d<\Lambda_2$, we know that $\Lambda(d)>0$ and $d<n\beta$, which implies that the previous expression is negative. Combining all these results, it follows that $\Gamma'(d)<0$ at any zero of $\Gamma(d)$ belonging to the interval (Y_2,Λ_2) if $\rho\in(0,1]$.

If $\rho\in(-\frac{1}{n-1},0)$, then $\rho(M+n)-M<0$. Hence, in the equilibrium expression of $\Gamma'(d)$, the first factor is negative. Concerning the second factor, adding the first and the third terms, we obtain

$$
\begin{aligned}
&-\frac{(d-n\beta)^2\Lambda(d)}{Y(d)}Y'(d)+2(d-n\beta)\Lambda(d) \\
&\quad =\frac{(d-n\beta)\Lambda(d)}{Y(d)}\big(-\beta(n-2)(\lambda+n\beta)-d(\lambda+\beta(3n-2))\big),
\end{aligned}
$$

which is negative since $\Lambda(d)<0$, $Y(d)>0$, and $d<n\beta$ whenever $d\in(\Lambda_2,Y_2)$. Adding the second and the fourth terms, we obtain

$$
\begin{aligned}
&-\frac{(d-n\beta)^2\Lambda(d)}{\Lambda(d)(1-\rho)+dn(d+\lambda+n\beta)}2(\Lambda'(d)(1-\rho)+n(2d+\lambda+n\beta)) \\
&\quad +(d-n\beta)^2\Lambda'(d) \\
&\quad =\frac{(d-n\beta)^2}{\Lambda(d)(1-\rho)+dn(d+\lambda+n\beta)}\times g(d),
\end{aligned}
$$

where

$$
\begin{aligned}
g(d)\equiv{}&\big(2(M+n)d+(M\lambda+n\lambda+n\beta(-M+n-2))\big) \\
&\times\frac{M-\rho(M+n)}{M+n}\Lambda(d) \\
&-\frac{1}{M+n}n^2\beta(M+1) \\
&\times\big(2n\beta(M-n+2)d+\lambda(\lambda(M+n)+n\beta(3M+n+2))\big).
\end{aligned}
$$

Straightforward computations yield $\Lambda(d)(1-\rho)+dn(d+\lambda+n\beta)>0$ whenever $d\in(\Lambda_2, \Upsilon_2)$. Hence, if we show that $g(d)<0$ whenever $d\in(\Lambda_2, \Upsilon_2)$, then we can conclude that the second factor of $\Gamma'(d)$ is negative, and hence, $\Gamma'(d)>0$.

Note that the first term of $g(d)$ is negative since $M-\rho(M+n)>0$ whenever $\rho\in(-\frac{1}{n-1}, 0)$ and $\Lambda(d)<0$ whenever $d\in(\Lambda_2, \Upsilon_2)$. Now, if $n>3$, then $d<\Upsilon_2\leq\frac{\lambda(\lambda(M+n)+n\beta(3M+n+2))}{2n\beta(-M+n-2)}$. In this case, the second term of $g(d)$ is negative and hence $g(d)<0$ whenever $d\in(\Lambda_2, \Upsilon_2)$.

If $\rho=0$ or $\rho=-(n-1)^{-1}$, then $\Gamma(d)\equiv 0$. If $\rho=-(n-1)^{-1}$ (and $\sigma_\varepsilon^2>0$), it can be checked directly that d is a root in $(0, \Upsilon_2)$ of the polynomial:

$$
\begin{aligned}
&\Omega(d)\\
&\quad\equiv\frac{\Gamma(d)}{((n-1)\rho+1)(n\rho-M(1-\rho))}\\
&\quad=\frac{(\sigma_\theta^2d^2+(\sigma_\theta^2(\lambda-n\beta)-2\beta\sigma_\varepsilon^2(n-1))d-\beta\lambda(\sigma_\varepsilon^2(n-1)+n\sigma_\theta^2))^2}{\sigma_\theta^2\sigma_\varepsilon^2(n-1)^2}\\
&\qquad\times\Upsilon(d)n\sigma_u^2\\
&\qquad+\beta^2\sigma_\theta^2(d-n\beta)^2\frac{n}{(n-1)^2}\Lambda(d).
\end{aligned}
$$

We have that $\Lambda_2=0$ and $\Lambda_1<\Upsilon_1<\frac{-\lambda}{2}<0=\Lambda_2<\Upsilon_2$. In addition, $\Omega(d)>0$ for all $d\in(\frac{-\lambda}{2}, 0]$ and $\Omega(d)<0$ for all $d\geq\Upsilon_2$. Therefore, a root of $\Omega(d)$ that satisfies $d>\frac{-\lambda}{2}$ exists and belongs to the interval $(0, \Upsilon_2)$. The root can be shown to be unique if $n>3$ with a development parallel to the case $\rho\in(-\frac{1}{n-1}, 0)$ (in this case $\Omega(d)$ is decreasing around the equilibrium value).

When $\rho\sigma_\varepsilon^2=0$, we have that $M=0$, and from the equations for the coefficients a, b, and c, it follows that $c=(d+\lambda)^{-1}$ and $d=\Upsilon_2$ (note also that when $M=0$, $\Lambda(d)=n\Upsilon(d)$).

Note that we know from Propositions 1 and 2 that as ρ ranges from $-(n-1)^{-1}$ to 1, M ranges from -1 to ∞ and Λ_2 ranges from 0 to βn. Therefore, when $\sigma_u^2>0$, then $0<d<\beta n$ for $\rho\in[-(n-1)^{-1}, 1]$. *Q.E.D.*

PROOF OF PROPOSITION S.4(ii): I show that (a) in the cases where the equilibrium is unique and $\rho\sigma_\varepsilon^2>0$ (resp. $\rho\sigma_\varepsilon^2<0$), then d is decreasing (resp. increasing) with σ_u^2, (b) as $\sigma_u^2\to 0$, then $d\to\Lambda_2$, (c) as $\sigma_u^2\to\infty$, then $d\to\Upsilon_2$, and (d) when $\sigma_\varepsilon^2>0$, then $\rho\in(-(n-1)^{-1}, 1)$, $n>3$ if $\rho<0$, then d is increasing with ρ.

(a) Let $\sigma_\varepsilon^2>0$. The result follows since in the cases where the equilibrium is unique and $\rho\in(0, 1]$ (resp. $\rho\in(-\frac{1}{n-1}, 0)$ and $n>3$), $\Gamma(d)$ is decreasing (resp. increasing) around the equilibrium value, and in the relevant range for d, then $\operatorname{sgn}\{\partial\Gamma(d)/\partial\sigma_u^2\}=\operatorname{sgn}\{\rho\Upsilon(d)\}<0$ (since it can be checked that if $\rho>-(n-1)^{-1}$, then $\Lambda(d)(1-\rho)+dn(d+\lambda+n\beta)>0$ for $d\in[\Lambda_2, \Upsilon_2]$). In the case

$\rho = -(n-1)^{-1}$ and $n > 3$, $\Omega(d)$ is decreasing around the equilibrium value and $\text{sgn}\{\partial\Omega(d)/\partial\sigma_u^2\} = \text{sgn}\{Y(d)\} > 0$ (since it can be checked that $\sigma_\theta^2 d^2 + (\sigma_\theta^2(\lambda - n\beta) - 2\beta\sigma_\varepsilon^2(n-1))d - \beta\lambda(\sigma_\varepsilon^2(n-1) + n\sigma_\theta^2) < 0$ for $d \in [\Lambda_2, Y_2]$). It follows that the same result as in the case $\rho \in (-\frac{1}{n-1}, 0)$ holds.

(b) We have that when $\rho \in (-(n-1)^{-1}, 1)$ as $\sigma_u^2 \to 0$, then $\Gamma(d) \to$ nonzero constant $\times (d - n\beta)^2 \Lambda(d)$ and we know that $d < \Lambda_2 < n\beta$ for $\rho > 0$ and $d < Y_2 < n\beta$ for $\rho < 0$. Therefore, in the limit, $d = \Lambda_2 > 0$. When $\rho = 1$, as $\sigma_u^2 \to 0$, then $\Gamma(d) \to$ nonzero constant $\times (d - n\beta)^2(-d^2 - (\lambda - n\beta)d + n\beta\lambda)$ with unique root fulfilling the SOC $d = n\beta$ and, therefore, $d \to n\beta$ (recall that $\Lambda_2 \to n\beta$ as $\rho \to 1$ from Proposition 1). When $\rho = -(n-1)^{-1}$, then $\Omega(d) \to \beta^2\sigma_\theta^2 n(n-1)^{-2}(d - n\beta)^2\Lambda(d)$ as $\sigma_u^2 \to 0$. Since $d < \Lambda_2 < n\beta$ for $\sigma_u^2 > 0$, it follows that $d \to \Lambda_2 = 0$.

(c) As $\sigma_u^2 \to \infty$, then $\sigma_u^{-2}\Gamma(d) \to \rho Y(d)(\Lambda(d)(1-\rho) + dn(d + \lambda + n\beta))^2$ if $\rho > -(n-1)^{-1}$, and

$$\begin{aligned}\frac{\Omega(d)}{\sigma_u^2} \to \Big(&\sigma_\theta^2 d^2 + \big(\sigma_\theta^2(\lambda - n\beta) - 2\beta\sigma_\varepsilon^2(n-1)\big)d \\ &- \beta\lambda\big(\sigma_\varepsilon^2(n-1) + n\sigma_\theta^2\big)\Big)^2 \\ &/(\sigma_\theta^2\sigma_\varepsilon^2(n-1)^2)Y(d)n\end{aligned}$$

if $\rho = -(n-1)^{-1}$. In both cases it follows that $d \to Y_2$ as $\sigma_u^2 \to \infty$ (since for $d \in [\Lambda_2, Y_2]$, $(\Lambda(d)(1-\rho) + dn(d + \lambda + n\beta))^2 > 0$ and $(\sigma_\theta^2 d^2 + (\sigma_\theta^2(\lambda - n\beta) - 2\beta\sigma_\varepsilon^2(n-1))d - \beta\lambda(\sigma_\varepsilon^2(n-1) + n\sigma_\theta^2))^2 > 0$).

(d) I show that whenever $\sigma_\varepsilon^2 > 0$, $\rho \in (-(n-1)^{-1}, 1)$, $n > 3$ if $\rho < 0$, then d is increasing with ρ. Let $\rho > 0$. Since in equilibrium $\Gamma'(d) < 0$ we know that $\text{sgn}\{\partial d/\partial\rho\} = \text{sgn}\{\partial\Gamma/\partial\rho\}$. It is possible to show that when $\Gamma(d) = 0$, we have that

$$\frac{\partial\Gamma}{\partial\rho} = \frac{\sigma_u^2 Y(d)(\Lambda(d)(1-\rho) + dn(d + \lambda + n\beta))}{\Lambda(d)(\rho(n-1)+1)(1-\rho)} W(d),$$

where

$$\begin{aligned}W(d) = &-(\rho(M+n) - M)n((1-\rho)(M+2) + n\rho)(Y(d))^2 \\ &- (\rho(M+n) - M) \\ &\times \beta\big((\rho-1)^2(n-1)^2 M + n(n-2-2\rho(n-1))\big) \\ &\times (2d+\lambda)Y(d) \\ &- M\beta^2(1-\rho)(n-1)(\rho(n-1)+1) \\ &\times ((1-\rho)(n-1)M + n)(2d+\lambda)^2.\end{aligned}$$

Since $Y(d) < 0$, $\Lambda(d) > 0$, and $\Lambda(d)(1-\rho) + dn(d + \lambda + n\beta) > 0$ for $d \in (Y_2, \Lambda_2)$, and $\rho(n-1) + 1 > 0$, then $\text{sgn}\{\partial\Gamma/\partial\rho\} = -\text{sgn}\{W\}$. An elaborate analysis shows that $W(d) < 0$. Note that $(\rho(M+n)-M) > 0$ since $M \in [0, \frac{\rho n}{1-\rho})$ for $\rho \geq 0$, $((1-\rho)(M+2)+n\rho) > 0$, and $((1-\rho)(n-1)M+n) > 0$. We distinguish two cases: If $(\rho-1)^2(n-1)^2M+n(n-2-2\rho(n-1)) \leq 0$, then the three terms of $W(d)$ are negative; if $(\rho-1)^2(n-1)^2M+n(n-2-2\rho(n-1)) > 0$, note that

$$\begin{aligned}&-(\rho(M+n)-M)\beta\big((\rho-1)^2(n-1)^2M+n(n-2-2\rho(n-1))\big)\\&\quad\times(2d+\lambda)Y(d)\\&\quad-M\beta^2(1-\rho)(n-1)(\rho(n-1)+1)((1-\rho)(n-1)M+n)\\&\quad\times(2d+\lambda)^2 > W(d),\end{aligned}$$

and taking into account that $Y(d) = (\Lambda(d)+M(d+\lambda)(d-n\beta))n^{-1}$, gives

$$\begin{aligned}&-(\rho(M+n)-M)\beta\big((\rho-1)^2(n-1)^2M+n(n-2-2\rho(n-1))\big)\\&\quad\times(2d+\lambda)n^{-1}\Lambda(d)+M\beta(2d+\lambda)K(d) > W(d),\end{aligned}$$

where

$$\begin{aligned}K(d) = {}&-(\rho(M+n)-M)\big((\rho-1)^2(n-1)^2M\\&+n(n-2-2\rho(n-1))\big)\frac{(d+\lambda)(d-n\beta)}{n}\\&-\beta(1-\rho)(n-1)(\rho(n-1)+1)\\&\times((1-\rho)(n-1)M+n)(2d+\lambda).\end{aligned}$$

Since $\Lambda(d) > 0$ and $(\rho-1)^2(n-1)^2M+n(n-2-2\rho(n-1)) > 0$, we obtain $W(d) < M\beta(2d+\lambda)K(d)$. Since $Y(d) < 0$, we have that $-(\beta(n-2)+\lambda)d+\beta\lambda < d^2$. Using this inequality in the expression of $K(d)$, it follows that

$$\begin{aligned}K(d) < {}&\frac{\beta(n-1)(2d+\lambda)}{n}\big((\rho-1)^3(n-1)^2M^2\\&+n(\rho-1)(2n-3\rho(n-1)-3)M-n^2(\rho^2(n-1)+1)\big).\end{aligned}$$

It is easy to see that the last factor is negative whenever $(\rho-1)^2(n-1)^2M+n(n-2-2\rho(n-1)) > 0$. This implies that in this case we also obtain $W(d) < 0$.

Let $\rho < 0$. Then in equilibrium $\Gamma'(d) > 0$ and, therefore, $\operatorname{sgn}\{\partial d/\partial\rho\} = \operatorname{sgn}\{-\partial\Gamma/\partial\rho\}$. Since $Y(d) > 0$ and $\Lambda(d) < 0$ for $d \in (\Lambda_2, Y_2)$, then $\operatorname{sgn}\{\partial\Gamma/\partial\rho\} = -\operatorname{sgn}\{W\}$. We have that $W(d) < 0$ for $d \in (\Lambda_2, Y_2)$ since the three terms of $W(d)$ are negative. Note that since $M \in (\frac{\rho n}{1-\rho}, 0]$ for $\rho \leq 0$ and $(\rho(n-1)+1) > 0$, we have that $((1-\rho)(M+2)+n\rho) > 0$ and $(\rho(M+n)-M) > 0$, $((1-\rho)(n-1)M+n) > 0$. Furthermore, $Y(d) > 0$ and $((\rho-1)^2(n-1)^2M+$

$n(n-2-2\rho(n-1))) > 0$ (since it is increasing in M and it is nonnegative when $M = \frac{\rho n}{1-\rho}$ and $\rho \in (-(n-1)^{-1}, 0)$). It follows that $\partial \Gamma/\partial \rho < 0$ and $\partial d/\partial \rho > 0$). *Q.E.D.*

PROOF OF CLAIM S.1: The coefficient of h_i in $E(\theta_i|s_i, h_i)$ is $\nu \equiv \frac{(n-1)\beta\rho a \sigma_\theta^2 \sigma_\varepsilon^2}{\Delta}$ (where $\Delta > 0$). We show it increases in ρ if $\sigma_\varepsilon^2 > 0$. Tedious algebra using the expressions for a and c in the proof of Proposition S.4(i) and the definitions of d and Δ shows that $\nu = -\frac{Y(d)}{\beta(n\beta - d)}$. It follows that $\frac{\partial \nu}{\partial \rho} = \frac{\partial \nu}{\partial d}\frac{\partial d}{\partial \rho}$ and $\frac{\partial \nu}{\partial d} = -\frac{\varphi(d)}{\beta(d-n\beta)^2}$, where $\varphi(d) \equiv d^2 - 2n\beta d - n\beta^2(n-2) - \lambda\beta(n-1) < 0$ in the relevant range (since $\varphi(d)$ is decreasing in d, as $d - \beta n < 0$, and $\varphi(0) < 0$) and, therefore, $\frac{\partial \nu}{\partial d} > 0$ and $\frac{\partial \nu}{\partial \rho} > 0$ since we know from (d) in the Proof of Proposition S.4(ii) that $\partial d/\partial \rho > 0$. *Q.E.D.*

PROOF OF PROPOSITION S.4(iii): Let $\rho\sigma_\varepsilon^2 > 0$. Then I show that $\partial d/\partial \sigma_\varepsilon^2 > 0$ if $\sigma_\varepsilon^2 \le \sigma_\theta^2$ (and note from Claim S.1 that $\operatorname{sgn}\{\partial \nu/\partial \sigma_\varepsilon^2\} = \operatorname{sgn}\{\partial d/\partial \sigma_\varepsilon^2\}$). From the expression for M and the fact that in equilibrium $\Gamma'(d) < 0$, we have that

$$\operatorname{sgn}\{\partial d/\partial \sigma_\varepsilon^2\} = \operatorname{sgn}\{\partial d/\partial M\} = \operatorname{sgn}\{\partial \Gamma/\partial M\}.$$

Furthermore,

$$\frac{\partial \Gamma}{\partial M} = \rho\sigma_u^2 Y(d)(\Lambda(d)(1-\rho) + dn(d+\lambda+n\beta)) \times \frac{l(d)}{(\rho(M+n)-M)\Lambda(d)},$$

where

$$l(d) = n\beta(1-\rho)(2d+\lambda)(\rho(n-1)+1)\Lambda(d) - dn(\rho(M+n)-M)(d+\lambda+n\beta)(d+\lambda)(n\beta-d)$$

and $\operatorname{sgn}\{\partial \Gamma/\partial M\} = -\operatorname{sgn}\{l(d)\}$ since $Y(d) < 0$ and $\Lambda(d) > 0$ whenever $d \in (Y_2, \Lambda_2)$. We have that $l(d)$ is a polynomial of degree 4 with coefficient $n(\rho(M+n)-M) > 0$, and, therefore, tends to ∞ when $d \to \pm\infty$. Furthermore,

$$l(-\lambda) = -n^2\beta^2\lambda^2(1-\rho)(n-1)(\rho(n-1)+1) < 0,$$
$$l(0) = n^2\beta^2\lambda^2(1-\rho)(\rho(n-1)+1)(M+1) > 0,$$

and

$$l(\Lambda_2) = -(\rho(M+n)-M)dn(d+\lambda+n\beta)(d+\lambda)(n\beta-d) < 0.$$

There is a root in $(-\infty, -\lambda)$, another in $(-\lambda, 0)$, and another in (Λ_2, ∞).

In addition, $\text{sgn}\{l(Y_2)\} = \text{sgn}\{(1-\rho)(n\rho(n-1)+2)M - n\rho\}$. Therefore, if $M \leq n\rho(1-\rho)^{-1}(\rho(n-1)+2)^{-1}$ (or equivalently $\sigma_\varepsilon^2 \leq \sigma_\theta^2$), we have $l(Y_2) \leq 0$ and $l(d) < 0$ for $d \in (Y_2, \Lambda_2)$. It follows that $\text{sgn}\{\partial\Gamma/\partial M\} = -\text{sgn}\{l(d)\} > 0$ when $\sigma_\varepsilon^2 \leq \sigma_\theta^2$. *Q.E.D.*

PROOF OF CLAIM S.2: Consider noise of the form βu in the model with inverse demand $P(y) = \alpha + \beta u - \beta y$, $\rho = 1$ and $\lambda = 0$. Note that $y = \beta^{-1}(\alpha + \beta u - p) \to u$ as $\beta \to \infty$. Consider the expression for $\Gamma(d)$ in the proof of Proposition S.4 and let $\rho = 1$ and $\lambda = 0$. Note that $(1-\rho)M = \frac{n\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + n\sigma_\theta^2}$ and that now $\text{var}[\beta u] = \beta^2\sigma_u^2$. Then it can be checked that $\Gamma(d)/\beta^5 \to \Xi(d)$ as $\beta \to \infty$, where $\Xi(d) = d(-d^2(n-2)(2\sigma_\varepsilon^2 + n\sigma_\theta^2)\sigma_u^2 + n^3\sigma_\theta^4\sigma_\varepsilon^2(n-1)^{-1})$. The positive solution to $\Xi(d) = 0$ fulfilling the SOC is the desired d and since $d = ((n-1)c)^{-1}$ with inelastic demand, we obtain the result for $(nc)^{-1}$. *Q.E.D.*

S.6. PUBLIC SIGNAL

This section provides the proof of Proposition 8, and states and proves Claim S.3.

PROOF OF PROPOSITION 8: If sellers other than i use the strategy $X(s_j, p) = b - as_j - er + cp$, for seller i, from the market clearing condition, the price is informationally equivalent to $h_i \equiv \beta(b - er)(n-1) - \alpha + (1 + \beta(n-1)c)p + \beta x_i = \beta a \sum_{j \neq i} s_j$. We have that $E[\theta_i | s_i, r, p] = E[\theta_i | s_i, r, h_i]$ and

$$\begin{pmatrix} \theta_i \\ s_i \\ r \\ h_i \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{\theta} \\ \bar{\theta} \\ \bar{\theta} \\ (n-1)\bar{\theta} \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \text{var}[\tilde{\theta}] & (n-1)\rho\sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & \text{var}[\tilde{\theta}] & (n-1)\rho\sigma_\theta^2 \\ \text{var}[\tilde{\theta}] & \text{var}[\tilde{\theta}] & \text{var}[\tilde{\theta}] + \sigma_\delta^2 & (n-1)\,\text{var}[\tilde{\theta}] \\ (n-1)\rho\sigma_\theta^2 & (n-1)\rho\sigma_\theta^2 & (n-1)\,\text{var}[\tilde{\theta}] & (n-1)(\sigma_\theta^2 + \sigma_\varepsilon^2 + (n-2)\rho\sigma_\theta^2) \end{pmatrix}\right],$$

and we obtain

$$E[\theta_i | s_i, r, h_i] = \frac{n\sigma_\delta^2\sigma_\varepsilon^2}{n\sigma_\delta^2(\sigma_\theta^2(1 + (n-1)\rho) + \sigma_\varepsilon^2) + \sigma_\varepsilon^2\sigma_\theta^2(1 + (n-1)\rho)}\bar{\theta}$$

$$
\begin{aligned}
&+\big(n^2\sigma_\delta^2\sigma_\theta^2((1-\rho)(1+(n-1)\rho)\sigma_\theta^2+\sigma_\varepsilon^2)\\
&+\sigma_\varepsilon^2\sigma_\theta^4(1-\rho)(n-1)(1+(n-1)\rho)\big)\\
&\big/\big((n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)\\
&+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))n(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2)\big)s_i\\
&+\frac{\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho)}{n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho)}r\\
&+\sigma_\varepsilon^2\sigma_\theta^2(n^2\sigma_\delta^2\rho-\sigma_\theta^2(1-\rho)(1+(n-1)\rho))\\
&\big/\big((n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)\\
&+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))n(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2)\beta a\big)h_i.
\end{aligned}
$$

From the FOC, $p-E[\theta_i|s_i,r,h_i]=(d+\lambda)x_i$, $d=(\beta^{-1}+(n-1)c)^{-1}$, $x_i=b-as_i-er+cp$, and the expression for h_i, we can obtain a system of equations to identify the coefficients of the linear strategy:

$$
\begin{aligned}
a&=\frac{(1-\rho)\sigma_\theta^2}{(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2)}(d+\lambda)^{-1},\\
b&=\frac{1}{1+Q}\bigg(\frac{\alpha}{\beta n}Q\\
&\quad-\frac{n\sigma_\delta^2\sigma_\varepsilon^2}{n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho)}\\
&\quad\times(d+\lambda)^{-1}\bar{\theta}\bigg),\\
e&=\frac{\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho)}{n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho)}\\
&\quad\times\frac{1}{(d+\lambda)(1+Q)},
\end{aligned}
$$

where

$$
Q=\frac{\sigma_\varepsilon^2(n^2\sigma_\delta^2\rho-\sigma_\theta^2(1-\rho)(1+(n-1)\rho))}{(1-\rho)(n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))},
$$

and

$$
c=\left((d+\lambda)^{-1}-\frac{Q(1+\beta nc)}{\beta n}\right)
$$

yields a quadratic equation in c exactly as in Proposition 1 replacing M with Q: $g(c; Q) = 0$. Its largest solution fulfills the SOC. Comparative static properties follow immediately from the expression of Q. Recall that $-(n-1)^{-1} < \rho < 1$ and let $\sigma_\varepsilon^2 > 0$. We obtain,

$$\frac{\partial Q}{\partial \sigma_\delta^2} = \frac{n\sigma_\varepsilon^2\sigma_\theta^2(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)(1+(n-1)\rho)^2}{(1-\rho)(n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))^2} > 0,$$

$$\frac{\partial Q}{\partial \rho} = \frac{n\sigma_\delta^2\sigma_\varepsilon^2((n^2\sigma_\delta^2 + \sigma_\theta^2(1+(n-1)\rho)^2)\sigma_\varepsilon^2 + n^2\sigma_\delta^2\sigma_\theta^2(1+(n-1)\rho^2))}{(1-\rho)^2(n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))^2} > 0,$$

and

$$\frac{\partial Q}{\partial \sigma_\theta^2} = -\frac{n\sigma_\delta^2\sigma_\varepsilon^2(1+(n-1)\rho)(n^2\sigma_\delta^2\rho + \sigma_\varepsilon^2(1+(n-1)\rho))}{(1-\rho)(n\sigma_\delta^2(\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2)+\sigma_\varepsilon^2\sigma_\theta^2(1+(n-1)\rho))^2}.$$

Therefore,

$$\operatorname{sgn}\left\{\frac{\partial Q}{\partial \sigma_\theta^2}\right\} = -\operatorname{sgn}\{n^2\sigma_\delta^2\rho + \sigma_\varepsilon^2(1+(n-1)\rho)\}$$

and $n^2\sigma_\delta^2\rho + \sigma_\varepsilon^2(1+(n-1)\rho) \geq 0$ if and only if $\rho \geq -((n-1)+n^2(\sigma_\delta^2/\sigma_\varepsilon^2))^{-1}$.

Furthermore, from the expression for Q, it follows that

$$\begin{aligned}\operatorname{sgn}\{\partial Q/\partial \sigma_\varepsilon^2\} &= \operatorname{sgn}\{Q\} \\ &= \operatorname{sgn}\{(n^2\rho\sigma_\delta^2 - \sigma_\theta^2(1-\rho)(1+(n-1)\rho))\}.\end{aligned}$$ *Q.E.D.*

CLAIM S.3: *When $\sigma_\varepsilon^2 > 0$, the absolute value of the weight on h_i in $E[\theta_i|s_i, r, h_i]$ increases in $|\rho|$ and σ_ε^2.*

PROOF: The coefficient of h_i in $E[\theta_i|s_i, r, h_i]$ equals $(d+\lambda)Q/\beta n$ and is, therefore, increasing in Q since d is increasing in Q. It follows that when $\sigma_\varepsilon^2 > 0$, the coefficient is increasing in ρ since then Q is increasing in ρ, and when $Q > 0$ ($Q < 0$), it is increasing (decreasing) in σ_ε^2 since then Q is increasing (decreasing) in σ_ε^2. We conclude that the absolute value of the weight on h_i in $E[\theta_i|s_i, h_i]$ increases in $|\rho|$ and σ_ε^2. *Q.E.D.*

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