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COMMITMENT, FLEXIBILITY AND MARKET OUTCOMES

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An n-firm oligopoly model, parametrized by the degree of flexibility of the technology and where firms choose the optimal scale of production (capacity) first and then a competitive stage follows, is presented. It is shown that in (Nash) equilibrium as one moves from non-flexible to flexible technologies the resulting price ranges from the Cournot price to the Bertrand price. Furthermore, if the slope of short run marginal cost is bounded, the order of magnitude of the margin of price over long run unit cost is $1/n^2$ and the speed of convergence to the efficient outcome as the number of firms grows is, in a finer sense, faster the more flexible is the technology.

1. Introduction

Consider an n-firm industry selling an homogenous product. Suppose firms have identical and constant long run unit costs, equal to c (there are no fixed costs and no capacity limits). In Cournot competition producers make quantity decisions and bring to the market what they have produced, the market price being the one that equates supply and demand. In Bertrand competition producers set prices and demand goes to the lowest priced firms which then produce to satisfy this demand. There is no residual demand left for high price producers since we are in the 'long run' and firms have no capacity limits. In the Cournot case the equilibrium price is higher than marginal cost c and in the Bertrand case all active firms charge c.

In both models the equilibrium concept is the non-cooperative equilibrium of Nash in which every firm maximizes profits given the actions of the other firms. In the Bertrand case actions are prices and in the Cournot case, quantities.[1] What seems important is to explain the margin of price over

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[1]There is an extensive literature on quantity and price Nash competition starting from the work of Cournot (1838) and Bertrand (1883). See the books by Friedman (1977) and Shubik (with Levitan) (1980).

long run unit cost, which can be taken as an indication of the degree of inefficiency of the market. Several attempts have been made to build general models which have the Bertrand and Cournot conjectures as particular cases and which yield a prediction of the market price. Bresnahan (1981) takes the conjectural variation approach, which goes back to Bowley,[2] and requires the conjectures of the firms to be consistent in the sense that conjectures and reactions should be the same. Under certain assumptions he shows that there is a unique consistent conjectures equilibrium. Grossman (1981) considers a model of free entry into an industry with large fixed costs where firms' strategies are supply functions and argues that the competitive equilibrium, when it exists, is always a Nash equilibrium in supply functions. These supply functions are, in principle, arbitrary, being rationalized as contracts with consumers. Different abilities of incumbents and potential entrants to make contracts with consumers characterize the Cournot and Bertrand worlds.[3]

We consider in this paper a two-stage process where firms first make a commitment and choose a 'capacity' of production and then a competitive, price-taking, stage follows. We take as a starting point the following interpretation of Cournot competition: firms purchase capacity at a constant marginal cost c and once capacity is set production costs are zero up to the capacity limit and infinite afterwards. Firms choose first simultaneously and independently their capacities and afterwards there is a competitive, price-taking, stage where production takes place. That is, we have a two-stage process where at the first stage firm i chooses its supply function (through its capacity choice) for the market clearing stage to follow.[4]

Think now of 'capacity' as the efficient scale of operation of the firm. That is, firm i when choosing a capacity level k is choosing a cost function with minimum average cost equal to c (the constant long run marginal cost) at an output level equal to k. Marginal production costs are zero up to k and afterwards they increase with slope proportional to λ, where λ is a positive constant. We can think that the firm before the market period buys or contracts for output k at unit cost c. If the firm wants to sell more at the second stage, it has to pay an additional cost over c. The technology is flexible, for finite λ, in the sense that once firm i is committed to a capacity

[2] See Friedman (1977) for an account of the conjectural variation approach.

[3] Along these lines, Singh and Vives (1985) consider a differentiated duopoly where firms can make only two types of binding contracts with consumers: the quantity contract and the price contract. They show that, restricting attention to subgame perfect equilibria of a two-stage game where first firms choose what type of contract to offer the consumers and afterwards they compete contingent on the chosen types of contracts, if the goods are substitutes (complements) it is a dominant strategy for firm i to choose the quantity (price) contract.

[4] Kreps and Scheinkman (1983) consider a two-stage duopoly game where first firms choose quantities (or capacities) and then compete by price à la Bertrand. Under a particular rationing rule of unsatisfied demand they show that the Cournot outcome is the only equilibrium outcome.

level k it can still produce more than k at an increasing marginal cost larger than c. When $\lambda=0$ the technology is completely flexible as there is no 'penalty' associated with producing more than k. When $\lambda=\infty$ the technology is completely inflexible and the firm cannot produce more than k. λ represents thus the degree of flexibility of the technology.

The capacity choice of firm i determines its supply in the competitive stage. The market clearing price is the one that equates total supply with demand. Therefore, firm i can evaluate the profits it will get from any capacity choice given the choice of the other firms. We look at the (Nash) equilibria of this game and, under certain assumptions, show that as λ ranges from infinity to zero, that is, as we go from a completely inflexible to a completely flexible technology, the equilibrium price ranges monotonically from the Cournot to the Bertrand price. When the technology is inflexible capacity has its full pre-commitment value and we are in the Cournot world. The more flexible the technology is the more capacity loses its pre-commitment power and the closer we are to the Bertrand world. Our theory predicts that the margin over long run marginal cost will be positively related to λ, which, in a first approximation, is the steepness of short run marginal cost after the efficient scale of operation. Larger margins will be associated to inflexible technologies, that is, with closeness to the Cournot world and smaller ones to flexible technologies and with closeness to the Bertrand world.[5]

It is well known that in our context the order of magnitude of the margin of the Cournot price over long run marginal cost c is $1/n$.[6] That is to say, the Cournot price converges to c as the number of firms grows at a rate $1/n$. We show in our model that given any positive λ the rate of convergence of the equilibrium price p_n^* to c is $1/n^2$ if the slope of marginal cost is bounded and furthermore that the convergence is faster (in a finer sense) for more flexible technologies. That is, introducing some flexibility in the technology the order of magnitude of p_n^*-c is $1/n^2$ and we can order the speed of convergence according to the flexibility of the technology in the natural way.

The plan of the paper is as follows. Section 2 establishes the benchmark equilibria: Bertrand and Cournot. Section 3 presents the model and derives the main results. The asymptotic properties of the equilibria as the number of firms grow are examined in section 4. Concluding remarks follow.

2. Long run Bertrand and Cournot equilibria

Consider an n-firm industry selling an homogenous product. Marginal costs

[5]Dixon (1985a, b) has independently developed an approach related to the one in the present paper.

[6]See, for example, Ruffin (1971) for a Cournot homogenous product setting and Vives (1985) for a differentiated demand structure.

are constant and equal to c for all firms. There are no fixed costs and no capacity limits. We are considering thus a long run situation. The constant marginal cost c may be viewed as the envelope of a family of short run U-shaped average cost functions with c as minimum average cost. Inverse demand is given by a continuous function $P(\cdot)$, which is positive, twice-continuously differentiable, strictly decreasing and concave on some bounded interval $(0,\bar{X})$. For $X \geqq \bar{X}$, $P(X)=0$. Let $D(\cdot)$ denote the demand function and $\bar{p}=P(0)$. That is, D equals P^{-1} and shares the same properties of P on the interval $(0,\bar{p})$. To avoid trivial cases suppose that $\bar{p}>c$. Under these conditions the efficient price (the one that maximizes total surplus) equals marginal cost, c, and total output is then $D(c)$. Propositions 1 and 2 characterize the Cournot and Bertrand equilibria.

In Cournot competition firms set quantities and in equilibrium each firm maximizes profit given the quantities produced by other firms.

Proposition 1 (*Cournot equilibrium*). *There is a unique and symmetric Cournot equilibrium with all the firms producing a positive quantity* $\hat{x}_n$. *It is the only root of* $P(nx)+xP'(nx)-c=0$ *in the interval* $(0,D(c)/n)$. *Furthermore the order of magnitude of the margin over marginal cost* c is $1/n$.

Proof. For the first part of the statement, see Burger (1963). For the second part notice that $\hat{x}_n$ converges to 0 as n grows since $n\hat{x}_n<D(c)$ and then from the first order condition the Cournot price, $\hat{p}_n$, converges to c since demand has bounded slope. Therefore total output, $n\hat{x}_n$, goes to $D(c)$ and the order of magnitude of $\hat{x}_n$ is $1/n$ which is the same as the order of magnitude of $\hat{p}_n-c$. That is, $n(\hat{p}_n-c)=n\hat{x}_n|P'(n\hat{x}_n)|$ converges to $D(c)|P'(D(c))|$ as n goes to infinity. □

In Bertrand competition firms set prices, and in equilibrium each firm maximizes profit given the prices set by the other firms. Let $N=\{1,2,\dots,n\}$ be the set of firms. We suppose that the firms which set the lowest price split the demand and the remaining firms do not sell anything. That is, given $(p_i)_{i\in N}$ sales of firm i are

$$x_i=\frac{D(p_i)}{l} \quad \text{if} \quad p_j \geqq p_i \quad \text{for all} \quad j\in N \quad \text{where} \quad l=\#\{j\in N: p_j=p_i\},$$

$$=0 \qquad \text{otherwise.}$$

Proposition 2 (*Bertrand equilibrium*). $(p_i)_{i\in N}$ *is a Bertrand equilibrium if and only if* $p_i \geqq c$ *for all firms and at least two firms set the price equal to marginal cost.*

Remark. When $n=2$ the unique Betrand equilibrium is $p_i=c$, $i=1,2$. In any case the unique symmetric equilibrium is $p_i=c$ for all firms. There are other equilibria when $n\geqq 3$ but if a firm sets a price larger than marginal cost in equilibrium it gets no demand and therefore it does not produce anything. Therefore the only equilibrium where all the firms are producing a positive amount is the symmetric one. We take the symmetric equilibrium as the Bertrand outcome.

3. Commitment and flexibility with competitive pricing

We suppose that firm i when choosing a capacity level k is choosing a cost function with minimum average cost equal to c (the constant long run unit cost) at an output level equal to k. That is, the capacity level is the efficient scale of operation. In particular, we assume that the total cost of firm i of producing output x when it has installed capacity k is

$$\begin{aligned} C_\lambda(x;k) &= ck && \text{if} \quad x\leqq k, \\ &= cx+\lambda V(x-k) && \text{otherwise,} \end{aligned}$$

where λ is a positive constant representing the degree of flexibility of the technology and V is a continuous non-negative valued function on $[0,\infty)$ which is strictly increasing, three times continuously differentiable and such that $V(0)=0$. Its first derivative, V', goes through the origin also and is strictly increasing and concave. For example, the function V, defined by $V(z)=(1/(\alpha+1))z^{\alpha+1}$ $(0<\alpha\leqq 1)$ for all $z\geqq 0$, satisfies our requirements.

The cost structure can be interpreted as follows. Firm i before the market period buys or contracts for output k at constant long run unit cost c. If the firm wants to sell more than k at the market stage then it has to pay an additional cost $c(x-k)+\lambda V(x-k)$, larger than long run cost if λ is positive.[7]

Marginal costs, MC, is thus zero up to k and increasing (and concave) after it,

$$\begin{aligned} MC_\lambda(x;k) &= 0 && \text{if} \quad x<k, \\ &= c+\lambda V'(x-k) && \text{if} \quad x>k. \end{aligned}$$

[7] A straightforward extension of the model is to make production costly, at a constant marginal cost w, before the efficient scale of operation is reached. Then total production cost of x units, given k, is

$$\begin{aligned} c(x;k) &= ck+wx && \text{if} \quad x\leqq k, \\ &= (c+w)x+\lambda V(x-k) && \text{otherwise.} \end{aligned}$$

Long run marginal cost equals $c+w$ and the Cournot and Bertrand outcomes are computed with $c+w$ instead of c. Propositions 3 and 4 below hold replacing c by $c+w$.

Notice that for a small positive ε, marginal cost at $k+\varepsilon$ is approximately c, since $V'(0+)=0$, and at $k-\varepsilon$ is zero. Marginal cost is discontinuous at k. Marginal cost after k is increasing with slope $\lambda V''(x-k)$, that is, proportionally to λ for any given output $x>k$. The technology is flexible, for finite λ, in the sense that once firm i is committed to a capacity level k it can still produce more than k at an increasing marginal cost larger than c. This marginal cost increases with λ for any given output $x>k$. When $\lambda=0$ the technology is completely flexible as there is no 'penalty' in producing more than k. When $\lambda=\infty$ the technology is completely inflexible and the firm cannot produce more than k.

Alternatively, we could specify the cost structure with a continuous short-run marginal cost as follows:

$$C_\lambda(x;k)=cx \qquad \text{if} \quad x \leqq k,$$
$$=cx+\lambda V(x-k) \quad \text{otherwise.}$$

Marginal cost is now constant and equal to c up to k. The interpretation being now that k is a plant design parameter chosen by the firm before the market stage which gives the maximal efficient scale of the plant. To produce more than k, the firm has to pay a penalty as before which increases with λ. With this alternative specification, all the results of the paper go through with minor modifications in the derivations. For the rest of the paper, we work with the first cost structure.

Consider a two-stage process where firms at the first stage choose independently and simultaneously their capacity levels. The second stage is competitive, firms take prices as given and the market price is the one that equates the quantity demanded with the competitive supply of the firms. Production takes place at this stage. Firm i when choosing its capacity level, k_i, is choosing its supply function, $S_i(\cdot;k_i)$, for the competitive stage to follow. Total supply is the addition of the n individual supplies and the market price is the one that clears the market. $S_i(\cdot;k_i)$ is just the inverse of marginal cost for $p>c$ and equals k_i for $p \leqq c$. Let Φ denote V'^{-1} and suppose for the sake of the argument that the range of V' is $(0,\infty)$.[8] Then,

$$S_i(p;k_i)=k_i \qquad \text{if} \quad p \leqq c,$$
$$=k_i+\Phi\left(\frac{p-c}{\lambda}\right) \quad \text{otherwise.}$$

[8]If V' is bounded above then the domain of Φ is a bounded interval and the supply of firm i is infinite for prices above a certain level. In any case the domain of Φ is an open interval: $(0,\infty)$ or of the form $(0,T)$ for some $\infty>T>0$ if V' is bounded above.

Since V' is strictly increasing, smooth and concave, Φ is going to be strictly increasing, smooth (twice-continuously differentiable) and convex.[9] Therefore, the supply of firm i will be nondecreasing and convex. Let

$$K=\sum_{i=1}^{n} k_i.$$

Total supply is then

$$\begin{aligned} S(p;K)&=K && \text{if} \quad p\leqq c,\\ &=K+n\Phi\left(\frac{p-c}{\lambda}\right) && \text{otherwise.}\end{aligned}$$

The market clearing price p is the unique price for which excess demand is not positive. Notice that $D(\cdot)$ and $S(\cdot;K)$ are continuous, $D(\cdot)$ is decreasing when positive and $S(\cdot;K)$ is non-decreasing so that they intersect once when $K\leqq\bar{X}$. Otherwise the market clearing price is zero. Let F be the function which assigns the market clearing price to every $K\in[0, D(c)]$. That is, $F(K)$ solves in p the equation $D(p)=K+n\Phi((p-c)/\lambda)$. Then,

$$\begin{aligned} P&=P(K) && \text{if} \quad K>D(c),\\ &=F(K) && \text{otherwise.}\end{aligned}$$

F is twice-continuously differentiable, strictly decreasing and concave. $F'=(D'-(n/\lambda)\phi')^{-1}$, which is negative since $D'\leqq 0$ and $\phi'\geqq 0$. $F''=((n\phi''-\lambda D'')/(\lambda D'-n\phi')^2)F'$, which is non-positive since $D''\leqq 0$ and $\phi''\geqq 0$.

Given the capacity choices of the firms, $(k_i)_{i\in N}$, if p is the market clearing price profits of firm i, π_i, equal $pS_i(p;k_i)-C_\lambda(S_i(p;k_i);k_i)$. Substituting in the value of p we get the profits of firm i in terms of the capacity choices of the firms,

$$\begin{aligned} \pi_i&=(P(K)-c)k_i && \text{if} \quad K>D(c),\\ &=(p-c)\left(k_i+\Phi\left(\frac{p-c}{\lambda}\right)\right)-\lambda V\left(\Phi\left(\frac{p-c}{\lambda}\right)\right) && \text{otherwise,}\\ & && \text{where} \quad p=F(K).\end{aligned}$$

We have thus a well defined game with firm i choosing a capacity k_i in the

[9] $\phi=V'^{-1}$. ϕ inherits smoothness from V'. $\phi'=1/V''$, which is positive since $V''>0$ and $\phi''=-V'''/(V'')^3$, which is non-negative since $V'''<0$.

interval $[0, \bar{X}]$ and getting a payoff π_i as above. We show below that with n firms in the market and given a degree λ, $\lambda \in (0, \infty)$, of flexibility of the technology, the game has a unique and symmetric Nash equilibrium (in pure strategies) where all firms choose a positive capacity k^* with an associated market price p^*. The pair (k^*, p^*) will be such that the market clears, $D(p^*) - nS_i(p^*; k^*) = 0$, and will satisfy the first order condition (FOC), which using the usual envelope result for a competitive firm is easily seen to be

$$\frac{\partial \pi_i}{\partial k_i} = S_i(p^*, k^*)F'(nk^*) + p^* - c = 0.$$

That is,

$$p^* - c = \frac{S_i(p^*, k^*)}{|F'(nk^*)|} \quad \text{where} \quad S_i(p^*, k^*) = k^* + \phi\left(\frac{p^* - c}{\lambda}\right).$$

Notice that the FOC is similar to the Cournot case, where $p - c$ equals $x_i/|D'(p)|$; x_i is the output of firm i and $D'(p)$ the slope of the residual demand that the firm faces. In our model, the margin over long run unit cost equals the supply of firm i, S_i, divided by $|F'| = (n/\lambda)\phi' - D'$ which is the absolute value of the slope of the residual demand of firm i given the capacity choices, $K_i = \sum_{j \neq i} k_j$ of the other firms. The residual demand of firm i is $D(p) - (K_{-i} + n\phi((p-c)/\lambda))$. Letting λ go to infinity, we get the Cournot outcome, $p - c = k/|D'(p)|$ and $D(p) = nk$, since $(p-c)/\lambda$ and ϕ go to zero and ϕ' is bounded above $[\phi' = 1/V''$ and $V''(0) > 0]$. Letting λ go to zero, the equilibrium price approaches the long run cost c. In fact, we can show that for any given number of firms, as λ goes from infinity to zero, the equilibrium price ranges monotonically from the Cournot price to the Bertrand price which substantiates our claim that with flexible technologies we are close to the Bertrand world and with non-flexible ones, we are close to the Cournot world.

Combining the FOC and the market clearing conditions, we get something similar to a Lerner index for the industry.

$$\frac{p^* - c}{p^*} = \frac{1}{n(\varepsilon_\lambda(p^*; k^*) - \eta(p^*))},$$

where η is the elasticity of demand and ε_λ the elasticity of supply, $\varepsilon_\lambda = (p/\lambda S_i)\phi'$.

Notice that the margin is over the long run unit cost c and not over the actual short run marginal cost. This index makes clear the welfare loss associated with the strategic behavior of firms at the first stage, since to maximize total surplus a total capacity of $D(c)$ should be chosen to obtain the efficient price c. When λ goes to infinity, the elasticity of supply goes to zero and we are in the Cournot case; when λ goes to zero the elasticity of supply goes to infinity and we are in the Bertrand case. Proposition 3 states the results.

Proposition 3. Given a positive λ and n firms in the market there is a unique and symmetric Nash equilibrium of the game where all the firms set capacity $k_n^*(\lambda)$. Let $p_n^*(\lambda)$ be the associated market clearing price, then $k_n^*(\cdot)$ and $p_n^*(\cdot)$ are smooth functions on $(0, \infty)$ and $p_n^*(\cdot)$ is strictly increasing. Furthermore as λ ranges from ∞ to 0 $p_n^*(\lambda)$ ranges from the Cournot price to the Bertrand price c.

Proof.[10] To show that there is a unique and symmetric Nash equilibrium of the game one proceeds in three steps. First, the best reply function of firm i is derived and one sees that it is strictly positive, smooth and strictly decreasing on $(0, D(c))$. Second, it is checked that an equilibrium must be symmetric and then it follows that the unique symmetric equilibrium is given by the intersection of the best reply function of firm i and the line through the origin with slope $1/(n-1)$.[11]

Step 1. The best reply of firm i is a continuous function on the non-negative reals, $g(\cdot)$, strictly positive on $[0, D(c))$ and zero otherwise. $g(\cdot)$ is continuously differentiable and strictly decreasing on $(0, D(c))$.

To prove our claim let Y be the sum of the capacities of firms other than i. If $Y \geqq D(c)$ then the market price is $P(Y)$, $P(Y) \leqq c$, so that firm i cannot make positive profits and sets $k_i = 0$ to get zero profits. If $Y < D(c)$ and firm i chooses k_i then the market price is $F(Y+k_i)$ which is larger than c for k_i small enough since $\lambda > 0$ and firm i can make positive profits by choosing $k_i < D(c) - Y$. When $0 < k_i + Y < D(c)$, $\pi_i(k_i, Y)$ is twice-continuously differentiable in both arguments by the smoothness of F. Furthermore $\pi_i(\cdot, Y)$ is strictly concave on $(0, D(c) - Y)$ since

$$\frac{\partial^2 \pi_i}{\partial k_i^2} = (k_i + \Phi)F'' + F'\left(2 + \frac{F'}{\lambda}\phi'\right)$$

is negative as $F' < 0$, $F'' \leqq 0$ and

$$2 + \frac{F'}{\lambda}\phi' > 1 + \frac{F'}{\lambda}\Phi' = \frac{\lambda D' - (n-1)\Phi'}{\lambda D' - n\Phi'} > 0.$$

[10]The assumption that V' is concave is stronger than necessary for the results to hold. All that is needed is that the elasticity of V'' be less than $(1-1/n)$ times the elasticity of V'. If V' is concave, this is always satisfied since then the elasticity of V'' is negative and $V'' > 0$ always. In terms of the ϕ function what is required is that the elasticity of ϕ' be larger than $1/n - 1$. For simplicity, we stick to our concavity assumption about V'.

[11]The proof of existence and uniqueness is standard. An alternative equivalent proof would note that the slope of the best response of firm i is negative but larger than minus one. Existence of a unique and symmetric equilibrium follows immediately.

We see thus that there is a unique best response to Y. Since the best response lies in the interval $(0, D(c)-Y)$ it is the unique solution in k of the equation $\partial\pi_i/\partial k_i|_{(k,Y)}=0$. Let $g(Y)$ be the best response to Y. According to the implicit function theorem g is continuously differentiable on $(0, D(c))$ and g' is negative since

$$\frac{\partial^2\pi_i}{\partial k_i \partial Y}=(k_i+\phi)F''+F'\left(1+\frac{F'}{\lambda}\right)\phi'<0.$$

Step 2. Let $(k_i)_{i\in N}$ be an equilibrium, then it must be the case that $K<D(c)$. Otherwise there is at least a firm which has a positive capacity and makes non-positive profits, but it can make strictly more by setting its capacity equal to zero. According to Step 1 then $k_i>0$ for all i since $k_i=g(K_{-i})$ and $K_{-i}\leqq K<D(c)$. Since $k_i>0$ and $K<D(c)$ k_i must satisfy the first order condition $(k_i+\phi)F'+p-c=0$. Subtracting the jth equation from the ith one we get $(k_i-k_j)F'=0$ which implies $k_i=k_j$ for all i and j since $F'<0$ on $(0, D(c))$.

Step 3. There is a unique symmetric equilibrium. This follows immediately: g is strictly decreasing when positive and there is a unique intersection of its graph with the line defined by the equation $Y=(n-1)k$. The unique k such that $g((n-1)k)=k$ is the symmetric equilibrium of the game.

The market clearing price is $p=F(nk)$ and (k,p) satisfy the market clearing and the first order conditions,

$$D(p)-n\left(k+\phi\left(\frac{p-c}{\lambda}\right)\right)=0$$

and

$$k+\phi\left(\frac{p-c}{\lambda}\right)+(p-c)\left(D'(p)-\frac{n}{\lambda}\phi'\left(\frac{p-c}{\lambda}\right)\right)=0.$$

The Jacobian determinant of the system is

$$\Delta=(n+1)\lambda D'-n^2\phi'+n(p-c)\left(\lambda D''-\frac{n}{\lambda}\phi''\right),$$

which is negative according to our assumptions.

Let (k_n^*, p_n^*) be the functions which assign to each $\lambda\in(0,\infty)$ the equilibrium capacity and price respectively when there are n firms in the industry. According to the implicit function theorem then $k_n^*(\cdot)$ and $p_n^*(\cdot)$ are continuously differentiable functions of λ and, after some computations,

$$\frac{\mathrm{d}p_n^*}{\mathrm{d}\lambda} = -\frac{n^2}{\Delta}\left(\phi' + \frac{p-c}{\lambda}\Phi''\right)\frac{p-c}{\lambda},$$

which is strictly positive, and

$$\frac{\mathrm{d}k_n^*}{\mathrm{d}\lambda} = -\frac{1}{\Delta}\left(\left(n\frac{p-c}{\lambda}\Phi'' - \Phi'\right)D' - n(p-c)\Phi' D''\right)\frac{p-c}{\lambda}. \quad \square$$

Remark. One may conjecture that as the technology gets more flexible firms try to keep the price up by setting smaller capacities. In that case the equilibrium capacity k^* would be increasing in λ. This turns out not to be true in general. The sign of $\mathrm{d}k^*/\mathrm{d}\lambda$ depends on the elasticities of Φ' and D'. A sufficient condition for this sign to be positive is that the elasticity of Φ' be less than $1/n$. This follows immediately from the expression of $\mathrm{d}k_n^*/\mathrm{d}\lambda$ above. Example 1 below makes clear that the sign is ambiguous.

Example 1. Let $V(z)=z^{\alpha+1}/(\alpha+1)$, $0<\alpha\leq 1$, then the elasticity of Φ' is $(1-\alpha)/\alpha$ and therefore the equilibrium capacity is increasing in λ if $(1-\alpha)/\alpha<1/n$. When $\alpha=\frac{1}{2}$ and demand is linear k_n^* is decreasing in λ.

Example 2. Let $\alpha=1$ in Example 1 then marginal cost increases linearly with slope λ after k[$V'(z)=z$ and $V''=1$]. In that case, the equilibrium capacity k^* is increasing in λ since the elasticity of ϕ' is zero. The lower bound of k^* (as λ goes to zero) is easily seen to be $((n-1)/n^2)D(c)$. If, furthermore, we assume that demand is linear, $p=a-X$, with $a>c$, we may compute explicitly the equilibrium price and capacity. These are

$$p^*=c+\frac{\lambda}{\lambda(1+n)+n^2}\bar{a} \quad \text{and} \quad k^*=\frac{\lambda+n-1}{\lambda(1+n)+n^2}\bar{a} \quad \text{where} \quad \bar{a}=a-c.$$

k^* ranges from $\bar{a}/(n+1)$ (the Cournot output) to $((n-1)/n^2)\ \bar{a}$ and p^* from $(a+nc)/(n+1)$ (the Cournot price) to c as λ ranges from ∞ to 0.

4. Asymptotic results

We know that in a Cournot market (under our assumptions) as the number of firms grows price goes to the constant unit cost c (at a rate $1/n$). This result is clear if we look at the perceived elasticity of demand for firm i. If the other firms produce Y, residual demand for firm i is $D(p)-Y$, which has an elasticity $(p/x_i)D'(p)$. As the number of firms grows, residual demand for firm i is shifting inwards and, by symmetry, the individual Cournot output is going to zero at a rate $1/n$. Price is always above marginal cost and the slope of demand is bounded away from zero, therefore the perceived elasticity of firm i goes to infinity as n goes to infinity and in the limit $p=c$.

When the technology is not completely unflexible ($\lambda<\infty$), individual production x_n goes to zero as n increases since total production is less than $D(c)$ and all firms produce the same amount. Therefore, from the FOC,

$$p_n-c=\frac{x_n}{\left|D'-\frac{n}{\lambda}\phi'\right|},$$

price goes to long run marginal cost c. We see thus that total production goes to the efficient output $D(c)$, and that the order of magnitude of individual output is $1/n$. The market price goes to the efficient price c at a rate of *at least* $1/n$. In fact, if marginal cost has bounded slope [i.e., if $V''(0)<\infty$], then the order of magnitude of $1/|D'-(n/\lambda)\phi'|$ is also $1/n$ since ϕ' is bounded away from zero ($\phi'=1/V''$). Consequently, the order of magnitude of p_n-c is $1/n^2$, and it is easily checked that $(p_n-c)n^2$ converges to $\lambda D(c)V''(0)$ as n goes to infinity. It is worth noting that the convergence is faster for more flexible technologies, that is, for smaller λ's, in the sense that the difference of the equilibrium and the efficient price times n^2 goes to a constant $[\lambda D(c)V''(0+)]$ which is monotonic in λ and ranges from ∞ to 0 as λ ranges from ∞ to 0.

The perceived elasticity of demand goes to infinity on two counts when marginal cost has bounded slope. First, as a result of individual output going to zero at a rate $1/n$, that is, as a result of the inward shifting of residual demand for firm i as the number of firms grows (like in the Cournot case). Second, as a result of the slope of residual demand going to infinity linearly with n (while in the Cournot case we did not have this effect). This is because adding some flexibility to the technology the slope of residual demand depends on the supply of the other firms, getting flatter as the number of firms increases. p_n-c equals individual output times the inverse of the slope of residual demand, each factor is of the order of magnitude of $1/n$. Therefore, p_n-c is of the order of magnitude of $1/n^2$. Proposition 4 summarizes the results.

Proposition 4. As the number of firms grows, the equilibrium price p_n^* goes to the efficient price c at a rate of at least $1/n$. If marginal costs have bounded slope the rate of convergence is $1/n^2$: $(p_n-c)n^2$ converges to $\lambda D(c)V''(0)$ as n goes to infinity.

5. Concluding remarks

We have put forth a theory of commitment and flexibility with competitive pricing in an n-firm market for an homogenous product. If the technology is inflexible (λ high) capacity is a good pre-commitment variable, we are close to the Cournot world and the margin over long run unit cost c will be high. If the technology is flexible (λ low) capacity is not such a good precommitment variable, we are close to the Bertrand world and the margin will be low. Intermediate situations will yield intermediate outcomes. Except in the case of a completely flexible technology the market price is above the long run unit cost or efficient price, and there is a welfare loss. Furthermore, the Lerner index of the industry, $L_n(\lambda)=(p_n(\lambda)-c)/p_n(\lambda)$, is positively associated with the degree of flexibility of the technology, λ, and its order of magnitude is at most $1/n$. In fact, if short run marginal costs have bounded slope the order of magnitude of $L_n(\lambda)$ is $1/n^2$ and welfare losses tend to dissipate quickly as the numb of firms in the industry grows.

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