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On the Efficiency of Bertrand and Cournot Equilibria with Product Differentation

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In a differentiated products setting with n varieties it is shown, under certain regularity conditions, that if the demand structure is symmetric and Bertrand and Cournot equilibria are unique then prices and profits are larger and quantities smaller in Cournot than in Bertrand competition and, as n grows, both equilibria converge to the efficient outcome at a rate of at least $1/n$. If Bertrand reaction functions slope upwards and are continuous then, even with an asymmetric demand structure, given any Cournot equilibrium price vector one can find a Bertrand equilibrium with lower prices. In particular, if the Bertrand equilibrium is unique then it has lower prices than any Cournot equilibrium. *Journal of Economic Literature* Classification Numbers: 022, 611. © 1985 Academic Press, Inc.

1. INTRODUCTION

It is a well-established idea that Bertrand (price) competition is more efficient than Cournot (quantity) competition. In fact with an homogenous product and constant marginal costs the Bertrand outcome involves pricing at marginal cost. This is not the case with differentiated products where margins over marginal cost are positive even in Bertrand competition. Shubik showed in a model with a linear and symmetric demand structure that the margin over marginal cost is larger in Cournot competition, and that, under certain conditions, as the number of varieties grows equilibrium prices go to marginal cost in either Bertrand or Cournot competition (see Shubik [16, Chaps. 7 and 9]). This note generalizes the first result to a general demand structure (not necessarily linear and/or symmetric) and the second to a general symmetric demand structure. We give sufficient conditions to guarantee the existence and uniqueness of both types of equilibria. These conditions are strong but otherwise usual in the oligopoly

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literature. Roberts and Sonnenschein [13] have shown that non-existence problems may arise with well-behaved preferences. Marginal costs are assumed constant to insure the existence of pure strategy Bertrand equilibria.[1] We follow the Chamberlinian tradition (Chamberlin [2]) and consider an industry selling differentiated substitute products in which each good is in competition with every other one.

In Section 3 the utility foundations of the demand structure are provided. It is assumed that utility is separable and linear in the numéraire (which is a standard assumption in partial equilibrium welfare analysis)[2] and some analogies with production theory are exploited. It is shown in Section 4 that if the demand structure is symmetric (and Bertrand and Cournot equilibria are unique) then prices and profits are larger and quantities smaller in Cournot than in Bertrand competition (Proposition 1). If Bertrand reaction functions are upward sloping (and continuous) then (even with an asymmetric demand structure) given any Cournot equilibrium price vector one can find a Bertrand equilibrium with lower prices (Proposition 2).[3] In particular, if the Bertrand equilibrium is unique then it has lower prices than any Cournot equilibrium. Section 5 deals with the asymptotic properties of the equilibria. It is shown that with a symmetric demand structure Cournot and Bertrand prices go to marginal cost at least at the rate $1/n$, where n is the number of goods, provided that there is a bounded demand for the industry as a whole and that inverse demands have bounded slopes. Some notation is introduced in Section 2.

2. Notation

Given a set $A \subset R^n$, $\operatorname{int} A$ denotes its interior and $\operatorname{bd}(A)$ denotes its boundary. Set theoretic union is denoted by $\bigcup$. For a function $U: R^n_+ \to R$, $DU(x)$ will denote the vector of first derivatives, $(\partial_i U(x))_{i=1}^n$ and $D^2U(x)$ the Hessian matrix of U, with entries $\partial_{ij} U(x)$; all evaluated at the point x. The vector inequality $\gg$ means strict inequality for every component. If z is a vector in R^n, z_{-i} stands for the vector derived from z by deleting the ith component.

[1] Mixed strategies are needed to insure existence of equilibria with price competition when marginal costs are increasing. Since a firm will not produce more than its competitive supply payoff relevant demands are contingent demands. (See Shubik [16] and Shapley [15].)

[2] See Spence [18], for example.

[3] Related results are obtained by Cheng [3], Hathaway and Rickard [9], Okuguchi [12] and Singh and Vives [17].

3. The Demand Structure

There are n differentiated goods in our monopolistic sector. We have a representative consumer which maximizes $\{U(x)-\sum_{i=1}^{n} p_i x_i\colon\ x\in R_+^n\}$, where p_i is the price of good i and $U(\cdot)$ is a $\mathbf{C}^3$ (differentially) strictly concave utility function on R_+^n. That is, $D^2U(x)$ is negative definite for all $x\in R_+^n$. Note that we assume $U(\cdot)$ to be differentiable at the boundary of R_+^n. Furthermore $\partial_i U(x)$ is positive in a non-empty, bounded region of R_+^n, X_i and, letting $X=\bigcap_{i=1}^{n} X_i$, $\partial_{ij}U(x)<0$ for $x\in \operatorname{int} X$ for any i and j (which is a reasonable assumption if the goods are substitutes). Given positive prices the solution to the maximization problem of our consumer will lie in X. The first order conditions (FOC) of the consumer problem are $\partial_i U(x)\leqslant p_i$ $(i=1,..., n)$, with equality if $x_i>0$. The inverse demand system f will be a continuous function on R_+^n. For $x\in X$ prices will be strictly positive and out of X one or more prices will be zero. Consider good i. f_i restricted to int X will be of class $\mathbf{C}^2$, decreasing in all its arguments, $\partial_j f_i<0$ for all j, and cross effects are symmetric, $\partial_j f_i=\partial_i f_j$, $j\neq i$. (All these properties follow from noting that the Jacobian of f restricted to int X is just $D^2U(\cdot)$ restricted to int X and that $\partial_{ij}U<0$ for $x\in\operatorname{int} X$.)

The demand system, h, is defined on R_+^n and satisfies:

(1) h is a continuous function on R_+^n.

(2) Let $P_i=\{p\in R_+^n\colon\ h_i(p)>0\}$; then h is of class $\mathbf{C}^2$ on $R_{++}^n\setminus\bigcup_{i=1}^{n}\operatorname{bd}(P_i)$. h_i is decreasing in its own price whenever $h_i(p)>0$. If $h_i(p)$ and $h_j(p)$ are positive, cross effects are symmetric, $\partial_j h_i=\partial_i h_j$.

(3) $p_i\leqslant\partial_i U(0,..., 0)$ for $p_i\in P_i$. That is, P_i is bounded along the ith axis.

Condition (1) follows by continuity and strict concavity of $U(\cdot)$. Condition (2) follows from the smoothness of $U(\cdot)$ and the FOC using the Inverse Function Theorem (extend U to a $\mathbf{C}^3$ function defined on an open set containing R_+^n). Downward sloping demand and symmetry of cross effects follow from the negative definiteness and symmetry of $D^2U(\cdot)$. Condition (3) follows from the FOC noting that $\partial_i U(h(p))\leqslant\partial_i U(0,..., 0)<\infty$. The first inequality is true since $\partial_{ii}U$ and $\partial_{ij}U$ are negative and the second since $U(\cdot)$ is differentiable at $(0,..., 0)$.

We assume, furthermore, that the goods are gross substitutes. That is, h_i is increasing in the price of firm j, $\partial_j h_i>0$, $j\neq i$, whenever $h_i(p)$ and $h_j(p)$ are positive.[4] Note then that for $p\in\operatorname{int} P$ the Jacobian matrix of h, J_h is negative definite, since it is the inverse of $D^2U(\cdot)$, and has positive off-

[4] For $n=2$ concavity of $U(\cdot)$ and $\partial_{12}U<0$ imply that the goods are gross substitutes since for $p\in P$, $\partial_2 h_1=-\partial_{12}U/\Delta$, where $\Delta=\det D^2U$, which is positive.

diagonal entries and therefore J_h has a dominant negative diagonal in the McKenzie sense (see McKenzie [10, Theorem 2′]).

The properties of our demand system, except for the symmetry of cross effects, are those assumed by J. Friedman in his treatment of demand with product differentation (see Friedman [7]).

EXAMPLE. Let $U: R^2_+ \to R$ be defined by

$$U(x) = \alpha_1 x_1 + \alpha_2 x_2 - \tfrac{1}{2}(\beta_1 x_1^2 + 2\gamma x_1 x_2 + \beta_2 x_2^2)$$

with all the parameters positive, $\beta_1\beta_2 - \gamma^2 > 0$ and $\alpha_i \beta_j - \alpha_j \gamma > 0$, $i \neq j$, $i = 1, 2$. Then

$$X = \{x \in R^2_+ : \alpha_1 - \beta_1 x_1 - \gamma x_2 > 0, \alpha_2 - \beta_2 x_2 - \gamma x_1 > 0\},$$

the inverse demand system being

$$p_1 = \alpha_1 - \beta_1 x_1 - \gamma x_2$$
$$p_2 = \alpha_2 - \beta_2 x_2 - \gamma x_1$$

on X.

$$P = \{p \in R^2_+ : a_1 - b_1 p_1 + c p_2 > 0, a_2 - b_2 p_2 + c p_1 > 0\}$$

(where $a_1 = (\alpha_1 \beta_2 - \alpha_2 \gamma)/\Delta$, $b_1 = \beta_2/\Delta$, $c = \gamma/\Delta$, $\Delta = \beta_1\beta_2 - \gamma^2$, and similarly for a_2 and b_2). The direct demands on P are

$$x_1 = a_1 - b_1 p_1 + c p_2$$
$$x_2 = a_2 - b_2 p_2 + c p_1.$$

(α_1, α_2) is the maximal element of the closure of P.

Our representative consumer maximizes surplus, $CS = U(x) - \sum_{i=1}^n p_i x_i$. We can make an analogy with production theory and think that our consumer produces utils out of the consumption inputs. In that case the consumer is maximazing profits with a technology represented by $U(\cdot)$ and with the prices of utils normalized to be one. Therefore the demand system arising from this maximization will have the properties of an input demand system and consumer surplus, CS, as a function of prices will be the analog of the profit function. We know from production theory that CS will be a convex function of prices and that $\partial CS(p)/\partial p_i = -h_i(p)$ for all i.

4. Bertrand Competition Is More Efficient than Cournot Competition

Suppose that we have n firms each producing a different variety at constant marginal costs. Since $X_i(P_i)$ is bounded along the ith axis we can take the strategy space of a quantity (price) setting firm to be a compact interval. Suppose that any firm can make positive profits even when the competitors' prices equal marginal costs. This insures that in equilibrium each firm will produce a positive amount. Consider prices net of marginal cost. Profits of firm i in terms of prices are $\pi_i(p) = p_i h_i(p)$ and in terms of quantities, $\hat{\pi}_i(x) = f_i(x)\, x_i$. Quasiconcavity of $\pi_i(\cdot)$ with respect to p_i and of $\hat{\pi}_i(\cdot)$ with respect to x_i insures the existence of Bertrand and Cournot equilibria. Uniqueness requires stronger assumptions.

The following assumptions insure that Bertrand and Cournot best reply mappings are contractions and therefore Bertrand and Cournot equilibria exist and are unique and stable (see Friedman [7]). Let $P = \bigcap_{i=1}^{n} P_i$.

(A.1) $(1-\varepsilon)\, \partial_{ii}\pi_i(p) + \sum_{j \neq i} |\partial_{ij}\pi_i(p)| < 0$ for all $p \in \operatorname{int} P$, for some $\varepsilon > 0$, and

(A.2) $(1-\delta)\, \partial_{ii}\hat{\pi}_i(x) + \sum_{j \neq i} |\partial_{ij}\hat{\pi}_i(x)| < 0$ for all $x \in \operatorname{int} X$, for some $\delta > 0$.

Assumptions A.1 and A.2 are very strong, particularly the one of the quantity setting model (A.2) when the products are close to perfect substitutes. Assumption A.2 is not satisfied for an homogenous product market with downward sloping concave demand if there are more than two firms (the Cournot equilibrium is still unique though). In what follows we use A.2 only to insure the uniqueness of the Cournot equilibrium. Alternatively one could assume that inverse demand for good i depends only on x_i and on the sum of the quantities of the other firms, $\sum_{j \neq i} x_j$, and that the Cournot reaction functions have negative slope larger than -1. The uniqueness of the Cournot equilibrium is easily established then. Let p^B and p^C denote respectively Bertrand and Cournot equilibrium price vectors. Both are going to be strictly positive and will satisfy the corresponding FOC. In the Bertrand case this is $\partial_i \pi_i(p) = 0$. This equation gives implicitly the Bertrand reaction function of firm i, $p_i = R_i^B(p_{-i})$, provided $p \in P$. $R_i^B(\cdot)$ will be increasing in all its arguments (upward sloping) if $\partial_{ij}\pi_i = p_i \partial_{ij} h_i + \partial_j h_i$ is positive for all $j \neq i$. The Cournot problem for firm i in price space is to choose a price p_i to maximize

$$\pi_i(p) \text{ subject to } h_j(p) = x_j, \qquad j \neq i.$$

The FOC of this problem is

$$\partial_i \pi_i - p_i [\partial_j h_i]_{j \neq i}\, J_{h_{-i}}^{-1}\, [\partial_i h_j]_{j \neq i} = 0$$

where $J_{h_{-i}}$ is the Jacobian matrix of h_{-i}. Noting than $\partial_j h_i = \partial_i h_j$ we have that at the Cournot prices p^C, $\partial_i \pi_i(p^C) < 0$ since p_i^C is positive and $J_{h_{-i}}$ is negative definite.

Bertrand competition is viewed as more "competitive" than Cournot competition. An intuitive reason behind this view is that (since the goods are substitutes) in Cournot competition each firm expects the others to cut prices in response to price cuts, while in Bertrand competition the firm expects the others to maintain their prices; therefore, Cournot penalizes price cutting more. One should expect Cournot prices to be higher than Bertrand prices. This is indeed the case either if the utility function is symmetric and Bertrand and Cournot equilibria are unique or if the Bertrand reaction functions slope upwards (which is reasonable if the goods are substitutes) and the Bertrand equilibrium is unique. An immediate consequence is that consumer surplus, *CS*, is higher under Bertrand competition. Total surplus derived from an output vector x is just $U(x)$. In the symmetric case Bertrand quantities are higher than Cournot quantities and therefore total surplus is higher and firms' profits are larger under Cournot competition. If there are multiple Bertrand equilibria given any Cournot equilibrium price vector we can find a Bertrand equilibrium with lower prices. Propositions 1 and 2 state these results.

In the symmetric case the demand system will be symmetric too. Let all prices equal p and let $g(p)$ be the demand for any product. That is $g(p) = h_i(p,..., p)$ for any i. Then since X and P are bounded there exists $\bar{p}$ and $\bar{x}$ such that $g(\bar{p}) = 0$ and $g(0) = \bar{x}$. Furthermore g is $\mathbf{C}^2$ on $(0, \bar{p})$ and $g' < 0$ since $U(\cdot)$ is a symmetric differentially strictly concave function. g would correspond to the Chamberlinian *DD* curve for "simultaneous movements in the prices of all goods." The following assumption insures that g is concave:

(A.3) $\partial_{ii} h_i(p) + \sum_{j \neq i} |\partial_{ij} h_i(p)| \leqslant 0$ for $p_i = q$, $q \in (0, \bar{p})$, for any i.

PROPOSITION 1. *If* $U(\cdot)$ *is symmetric and* A.1, A.2 *and* A.3 *hold then prices and profits are larger and quantities smaller in Cournot than in Bertrand competition.*

Proof. The unique equilibrium (Cournot and Bertrand) will be symmetric. It is enough to consider firm 1. Let $\phi(p) = \partial_1 \pi_1(p,..., p)$. The Bertrand price, p^B, solves $\phi(p) = 0$ (note that $\phi' < 0$ according to A.1). On the other hand we know that at the Cournot price, p^C, $\phi(p^C) < 0$. Therefore $p^C > p^B$. Note that $x^B > x^C$ since both are on g and $g' < 0$. Let $\tilde{\pi}(p) = pg(p)$ and note that $\tilde{\pi}$ is strictly concave in p since $g'' \leqslant 0$ and $g' < 0$. Consider the price a monopolist would charge, p^M. p^M solves $\tilde{\pi}'(p) = 0$. At the Cournot solution $\tilde{\pi}'(p^C) = p^C g'(p^C) + g(p^C)$, which is positive. To see this note that since $U(\cdot)$ is symmetric and at the Cournot solution all prices equal p^C,

$\partial_i h_i = \partial_1 h_1$ and $\partial_j h_i = \partial_2 h_1$ for all $i, j \neq i$. Then $\tilde{\pi}'(p^C) = p^C \partial_1 h_1 + h_1 + p^C(n-1)\,\partial_2 h_1$ which, using the Cournot FOC, equals

$$p^C \partial_2 h_1 (\partial_2 h_1 \mathbf{1}_{n-1} J_{h_{-1}}^{-1} \mathbf{1}_{n-1} + n - 1)$$

where $\mathbf{1}_{n-1}$ is an $(n-1) \times 1$ vector of ones. Inverting $J_{h_{-1}}$ one gets

$$\mathbf{1}_{n-1} J_{h_{-1}}^{-1} \mathbf{1}_{n-1} = \frac{n-1}{\partial_1 h_1 + (n-2)\,\partial_2 h_1}$$

and therefore

$$\partial_2 h_1 \mathbf{1}_{n-1} J_{h_{-1}}^{-1} \mathbf{1}_{n-1} + n - 1 = (n-1) \frac{\partial_1 h_1 + (n-1)\,\partial_2 h_1}{\partial_1 h_1 + (n-2)\,\partial_2 h_1},$$

which is positive since $\partial_1 h_1 + (n-1)\,\partial_2 h_1 = g'$ and $g' < 0$. We conclude that $\tilde{\pi}'(p^C) > 0$ since $\partial_2 h_1 > 0$. Summing up, we have $\tilde{\pi}'(p^M) = 0$, $\tilde{\pi}'(p^C) > 0$ and $\tilde{\pi}'$ strictly decreasing since $\tilde{\pi}$ is strictly concave. We conclude $p^M > p^C$. Since $p^C > p^B$, this implies $\pi^C > \pi^B$. Q.E.D.

PROPOSITION 2. *Assume, for all i, that $\pi_i(p)$ is strictly quasiconcave in p_i whenever the demand for the ith good is positive and that $R_i^B(\cdot)$ is nondecreasing in all its arguments, then given any Cournot equilibrium price vector one can find a Bertrand equilibrium with lower prices.*

Proof. Let p^C be a Cournot equilibrium price vector. We know that $\partial_i \pi_i(p^C) < 0$ for all i and that $p^C \in P$. The Bertrand reaction function of firm i, $p_i = R_i^B(p_{-i})$, is defined implicitly by $\partial_i \pi_i(p) = 0$ provided $p \in P$. It will be continuous since $\pi_i(p)$ is strictly quasiconcave in p_i. Again because of π_i's quasiconcavity, $\partial_i \pi_i(p) < 0$ means that $p_i > R_i^B(p_{-i})$. Therefore $p_i^C > R_i^B(p_{-i}^C)$ for all i. Let $R^B = (R_1^B, ..., R_n^B)$ and $p^1 = R^B(p^C)$, then $p^C \gg p^1$. Note that $p^1 \in P$ since any firm can make positive profits even when the competitors charge prices equal to marginal cost. Since R_i^B is nondecreasing for all i, $R^B(p^C) \geqslant R^B(p^1)$. Let $p^2 = R^B(p^1)$ and keep applying R^B to obtain a decreasing sequence p^t in P which converges since prices must be nonnegative.[5] Say p^t converges to p^*. p^* must satisfy $p^* = R^B(p^*)$, and therefore it is a Bertrand equilibrium, since $R^B(\cdot)$ is continuous. We conclude that $p^C \gg p^*$. Q.E.D.

Under the assumptions of Proposition 2 multiple Bertrand and Cournot equilibria may exist. Existence of the Bertrand equilibrium is guaranteed and there will be at least one Cournot equilibrium as long as $\hat{\pi}_i(x)$ is

[5] Similar arguments can be found in Deneckere and Davidson [5, pp. 13–14] and Spence [18, p. 221].

quasiconcave in x_i. Note that if there is a unique Bertrand equilibrium then it has lower prices than any Cournot equilibrium. Assumption A.1 insures the uniqueness of the Bertrand equilibrium.

5. Asymptotic Results[6]

Suppose that there is a countable infinity of potential commodities. Our representative consumer has preferences over them defined by a sequence of utility functions $\{U^n\}$, $U^n: R^n_+ \to R$, where each $U^n(\cdot)$ is symmetric and satisfies the assumptions of Section 2. For any given n, consider the program $\text{Max}\{U^n(x) - p\sum_{i=1}^n x_i,\ x \in R^n_+\}$ and let $x_i = g^n(p)$ $(i = 1,..., n)$ be its solution. We know that there exist $\bar{x}_n > 0$ and $\bar{p}_n > 0$ such that $\bar{x}_n = g^n(0)$ and $g^n(\bar{p}_n) = 0$, and that g^n is downward sloping. We assume

(A.4) there exists $\bar{p} > 0$ and $k > 0$ such that $\bar{p}_n \leqslant \bar{p}$ and $n\bar{x}_n \leqslant k$ for all n;

(A.5) there exists $c > 0$ such that $|\partial_{ii} U^n(x,..., x)| \leqslant c$ for all i $(i = 1,..., n)$, for all n and for all positive x.

Assumption A.4 means that there is a bounded demand for the varieties produced by the industry. The Chamberlinian *DD* curve for "simultaneous movements in the prices of all goods," $g^n(\cdot)$, shifts inwards as the number of varieties increase. Assumption A.5 implies that inverse demands have bounded slopes along the 45° line. This is immediate since $\partial_i f_i(x) = \partial_{ii} U(x)$. Suppose now that for any n there are unique Bertrand and Cournot equilibria (A.1 and A.2 are sufficient for this to hold). Since $U^n(\cdot)$ is symmetric the unique equilibria will be symmetric too. Denote them (p_n^B, x_n^B) and (p_n^C, x_n^C), respectively.

Proposition 3. *As n goes infinity p_n^B and p_n^C go to marginal cost at a rate of at least $1/n$.*

Proof. (p_n^C, x_n^C) satisfy the Cournot FOC $p_n^C = x_n^C |\partial_i f_i^n(x_n^C)|$ for any i. Therefore $np_n^C = nx_n^C |\partial_i f_i^n(x_n^C)| \leqslant kc$ from A.4 and A.5. Furthermore from Proposition 1 we know that $p_n^C > p_n^B$ and therefore $np_n^B \leqslant kc$ also. Q.E.D.

The intuition of the result should be clear. As we put more commodities in a limited market where the absolute values of the slopes of the inverse demand functions are bounded above, the substitute goods come closer together and demand elasticities go to infinity. This holds for Bertrand or

[6] For asymptotic results in a Cournot homogenous product setting see Ruffin [14] for the case of exogenous n and Novshek [11] for the free entry case.

Cournot competition. Although Bertrand is always more efficient than Cournot, the order of magnitude of their departure from efficiency is the same for both and so the rate of convergence to the efficient outcome is equal in both cases.

EXAMPLES. Let

$$U^n(x)=\alpha\sum_{i=1}^{n}x_i-\frac{1}{2}\left(\beta\sum_{i=1}^{n}x_i+2\gamma\sum_{j\neq i}x_ix_j\right),$$

where $\beta>\gamma>0$ and $\alpha>0$; and let marginal costs be constant and equal to m for all firms. Then for positive demands,

$$f_i^n(x)=\alpha-\beta x_i-\gamma\sum_{j\neq i}x_j \qquad (i=1,\dots,n)$$

$$h_i^n(p)=a_n-b_np_i+c_n\sum_{j\neq i}p_j \qquad (i=1,\dots,n),$$

where $a_n=\alpha/(\beta+(n-1)\gamma)$, $b_n=(\beta+(n-2)\gamma)/(\beta+(n-1)\gamma)(\beta-\gamma)$, and $c_n=\gamma/(\beta+(n-1)\gamma)(\beta-\gamma)$. $g^n(p)=(\alpha-p)/(\beta+(n-1)\gamma$, $\bar{p}=\alpha$, and $\bar{x}_n=\alpha/(\beta+(n-1)\gamma)$.

It is easily checked that

$$n(p_n^C-m)\underset{n}{\longrightarrow}\beta\frac{\alpha-m}{\gamma},$$

$$n(p_n^B-m)\underset{n}{\longrightarrow}(\beta-\gamma)\frac{\alpha-m}{\gamma}.$$

(Note that $\partial_i f_i^n=\beta$ for all n and $\partial_i h_i^n=b_n$, which tends to $1/(\beta-\gamma)$.) Having a limited market for the monopolistic industry is not enough for our result if the slopes of the inverse demand functions are unbounded. An example by Shubik (1980) illustrates this point.

Let

$$U^n(x)=\frac{\alpha}{\beta}\sum_i x_i-\frac{1}{2\beta}\left(\sum x_i\right)^2-\frac{n}{2\beta(1+\gamma)}\left[\sum x_i^2-\frac{(\sum x_i)^2}{n}\right],$$

where α, β, and γ are positive constants. Then for positive demands,

$$f_i^n(x)=\frac{\alpha}{\beta}-\frac{n+\gamma}{\beta(1+\gamma)}x_i-\frac{\gamma}{\beta(1+\gamma)}\sum_{i\neq j}x_j \qquad (i=1,\dots,n).$$

$$h_i^n(p)=\frac{1}{n}\left(\alpha-\beta\left[p_i+\gamma\left(p_i-\frac{1}{n}\sum_i p_i\right)\right]\right) \qquad (i=1,\dots,n).$$

Note that $|\partial_i h_i^n| = -\beta(1+\gamma(1-1/n))/n$, which goes to zero as $n \to \infty$, and that $ng^n(p) = \alpha - \beta p$, which means that we have a limited market. Certainly x_n^C and x_n^B go to zero as n goes to infinity but the corresponding prices do not go to the constant marginal cost. It is easily seen that the demand elasticity for any good does not go to infinity as n increases. We are thus in the Chamberlinian situation, where although there are many "small" firms each one of them has some market power and prices are above marginal cost.[7]

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[7] See Dixit and Stiglitz [6] and Hart [8] for a formalization of the Chamberlinean "large group" monopolistic competition.